ワイルフェルミオンの非断熱なポンプとニールセン・二宮の定理の破れ

University of Tokyo

Sho HIGASHIKAWA

Collaborator

RIKEN CEMS

Masaya NAKAGAWA

University of Tokyo, RIKEN CEMS

Masahito UEDA

熱場の量子論とその応用 30/08/2017

Message

Exotic topological band structure can be realized in periodically driven (Floquet) system.

e.g. **Weyl fermion**

・Topological semimetal

・**Single Weyl fermion**: **Impossible** to realize in equilibrium lattice system (Nielsen-Ninomiya theorem)

T. Kitagawa, et.al., Phys. Rev. B 82, 235114 (2010).
M. S. Rudner, et.al., Phys. Rev. X 3, 031005 (2013).
N. Nielsen and M. Ninomiya, Nucl. Phys. B 185, 20 (1981), 193, 173 (1981).

Floquet theory

- **Periodically driven (Floquet)** system

$i\,\partial_t \psi_F(t) = H(t)\psi_F(t)$

$H(t)$: driving Hamiltonian ($H(t+T) = H(t), T$: period)

→ Difficult to analyze ☹

→ Map to **static** system.

1. Floquet operator $\boldsymbol{U_F}$, 2. Effective Hamiltonian $\boldsymbol{H_{\mathrm{eff}}}$

Floquet theory

・Periodically driven (Floquet) system: $i\partial_t \psi_F(t) = H(t)\psi_F(t)$

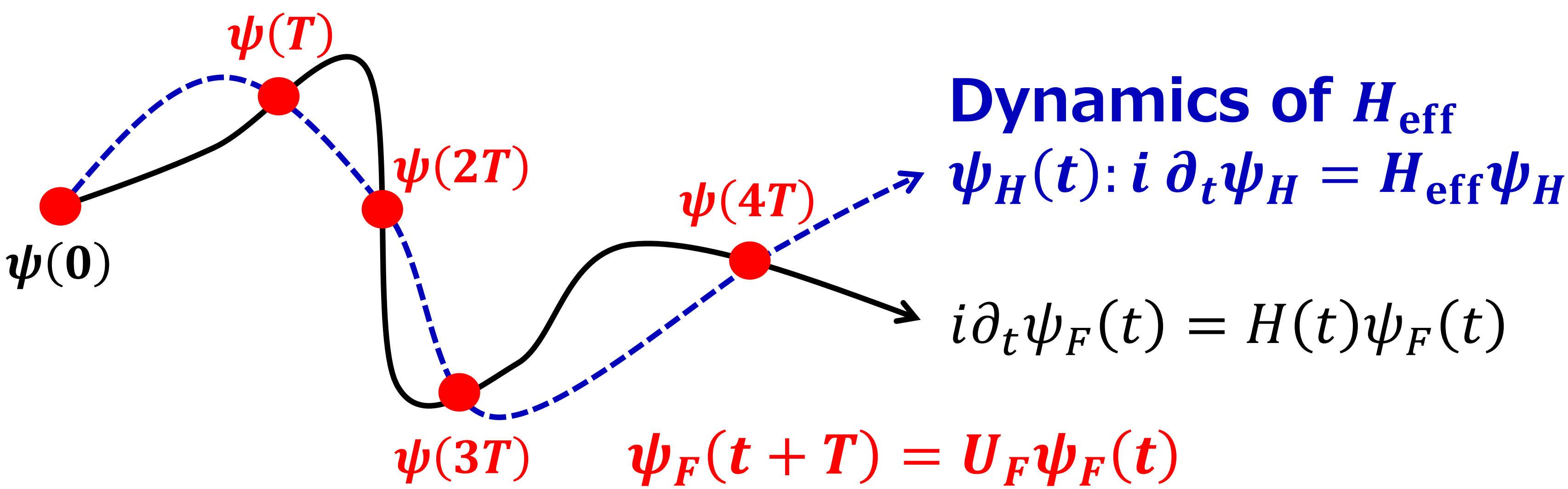
1. Floquet operator $\boldsymbol{U_F} \coloneqq \boldsymbol{\mathcal{T}\exp\left(-i\int_0^T H(t)dt\right)}$

   Time evolution in one period: $\psi_F(t+T) = U_F\psi_F(t)$

   → **stroboscopic description** $\boldsymbol{\psi_F(0), \psi_F(T), \psi_F(2T), \psi_F(3T)\cdots}$

2. Effective Hamiltonian $\boldsymbol{H_{\mathrm{eff}}}$ $(U_F \eqqcolon \exp(-iH_{\mathrm{eff}}T))$

   Floquet dynamics mimic dynamics of $\boldsymbol{H_{\mathrm{eff}}}$: $\boldsymbol{\psi_F(nT) = \psi_H(nT)}$.



Band topology in Floquet system

- Periodically driven system + **lattice** → momentum $\boldsymbol{k}$:

→ $U_F = \sum_{\boldsymbol{k}} U(\boldsymbol{k}),\ H_{\text{eff}}(\boldsymbol{k}) = \sum_{\boldsymbol{k}} h(\boldsymbol{k})$

Time evolution: $U(\boldsymbol{k}) = \sum_{\alpha} e^{-i\tilde{\epsilon}_{\alpha}(\boldsymbol{k})T} |\boldsymbol{k},\alpha\rangle\langle\boldsymbol{k},\alpha|$

Effective Bloch Hamiltonian: $h(\boldsymbol{k}) = \sum_{\alpha} \tilde{\epsilon}_{\alpha}(\boldsymbol{k}) |\boldsymbol{k},\alpha\rangle\langle\boldsymbol{k},\alpha|$

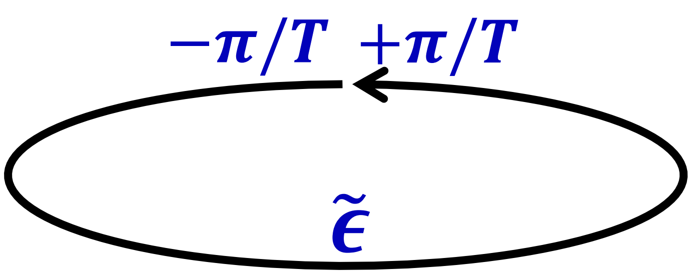
→ **Band topology**

$\tilde{\epsilon}_{\alpha}(\boldsymbol{k})$:Quasi energy

→ Ambiguity of $2\pi/T$: $\mathrm{e}^{-i\tilde{\epsilon}_{\alpha}(\boldsymbol{k})T} = \mathrm{e}^{-i(\tilde{\epsilon}_{\alpha}(\boldsymbol{k})+\frac{2\pi}{T})T}$

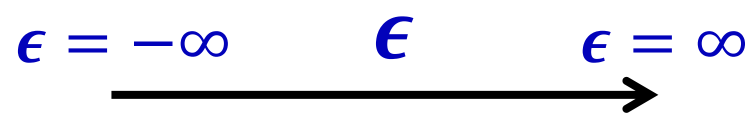
<u>Periodicity in quasi energy</u>

→ **Unique topology**



c.f. Equilibrium system

No periodicity



1 dimensional analogue of the story

	Equilibrium (lattice) system	**Periodically driven system**
1 dim. spinless fermion	Single **chiral** fermion → **Impossible** **1 dim Nielsen-Ninomiya**	Single **chiral** fermion → **Possible** **Thouless pump**

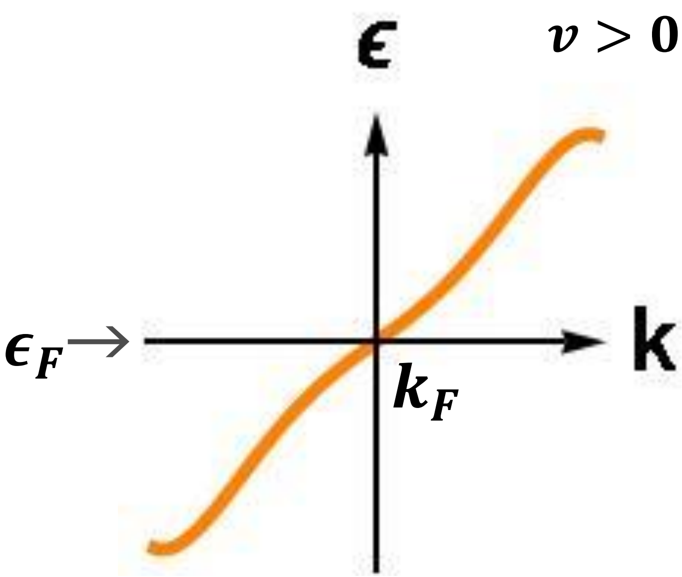
T. Kitagawa, et.al., Phys. Rev. B 82, 235114 (2010).

1 dim chiral fermion & Nielsen-Ninomiya theorem

- Spinless fermion in 1 dimension
  → 2 types of (chiral) fermions near fermi surface

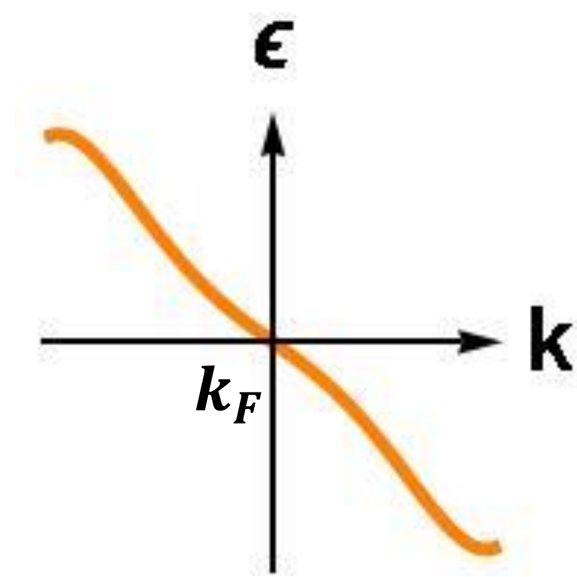
Chirality+1 fermion

$$\epsilon(k) = +v(k - k_F)$$



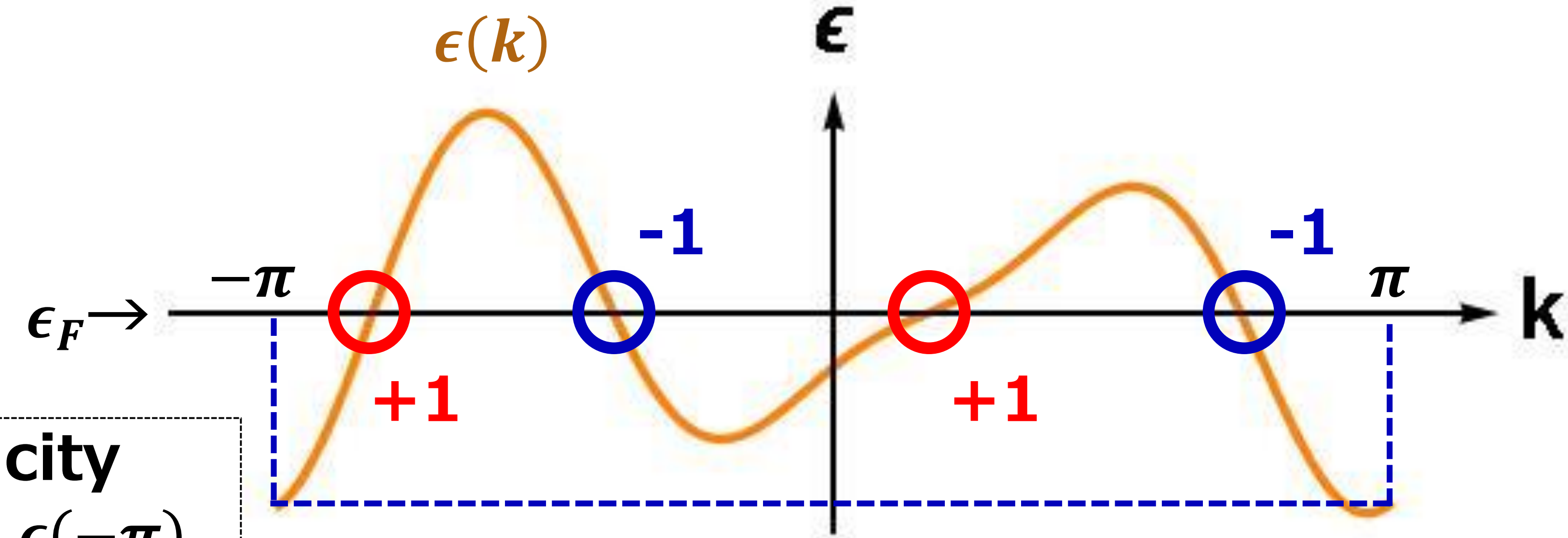
Chirality-1 fermion

$$\epsilon(k) = -v(k - k_F)$$



1 dim chiral fermion & Nielsen-Ninomiya theorem

- 1 dim lattice system
  → Single chiral fermion: Impossible !
  (1 dim. Nielsen-Ninomiya theorem)



Periodicity
$\epsilon(\pi) = \epsilon(-\pi)$

Chirality+1 fermion   Chirality-1 fermion

1 dimensional analogue of the story

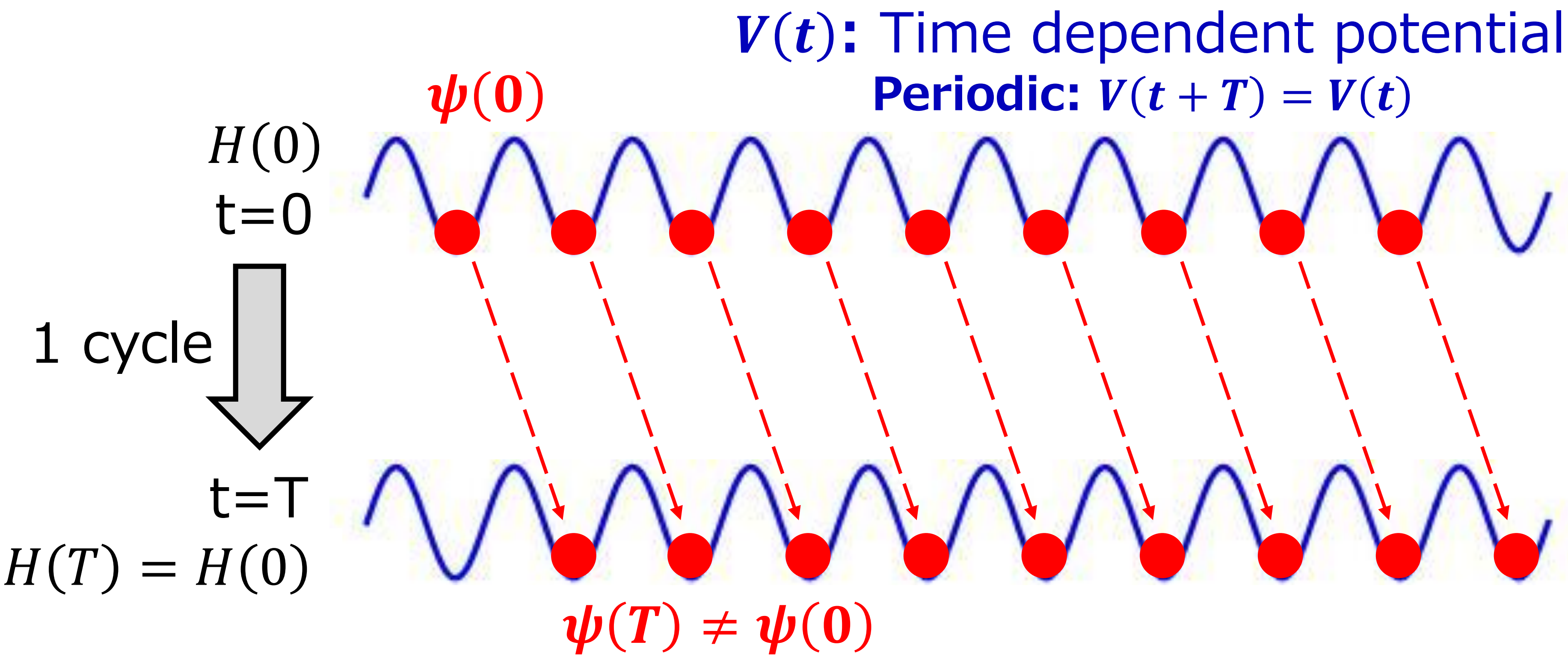
	Equilibrium (lattice) system	**Periodically driven system**
1 dim. spinless fermion	Single **chiral** fermion → **Impossible** **1 dim Nielsen-Ninomiya**	Single **chiral** fermion → **Possible** **Thouless pump**

T. Kitagawa, et.al., Phys. Rev. B 82, 235114 (2010).
D. J. Thouless, Phys. Rev. B 27 6083 (1983).

Thouless pump as periodically driven system

- Thouless pump
  - 1 dim lattice + adiabatic parameter change $H(t)$

    Periodic: $\boldsymbol{H(t+T)=H(t)}$
  - 1 cycle → **1 site translation:** $\boldsymbol{\psi(T)=\exp(-ika)\psi(0)}$



Thouless pump as periodically driven system

- 1 cycle → 1 site translation: $\boldsymbol{\psi(T) = \exp(-ik)\psi(0)}$

  Floquet operator U_F [$\psi(T) = U_F\psi(0)$]

  → Floquet operator $U_F = \exp(-ik)$

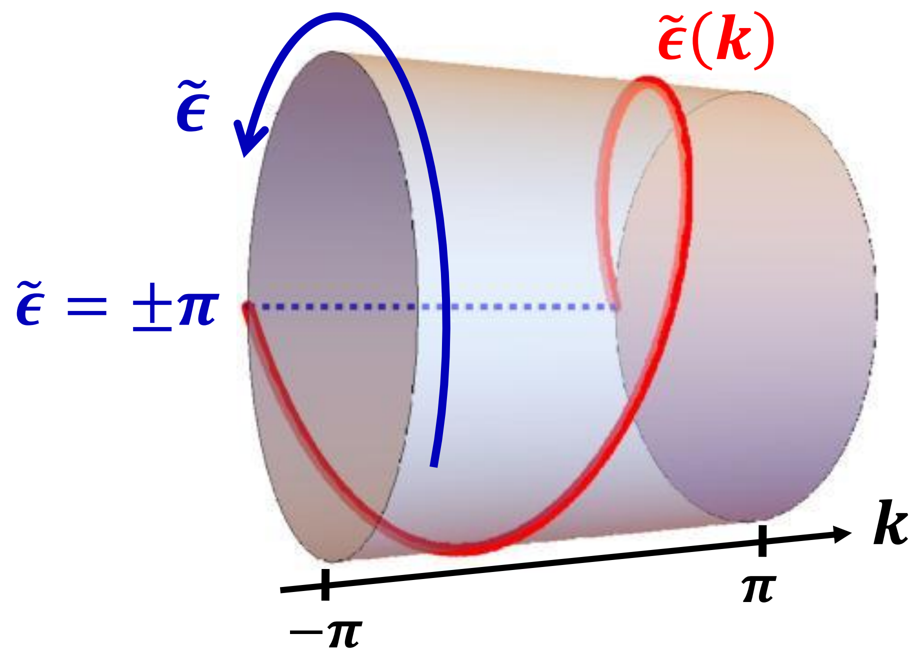
  → Effective Hamiltonian $H_{\text{eff}} = i \log U_F = k$

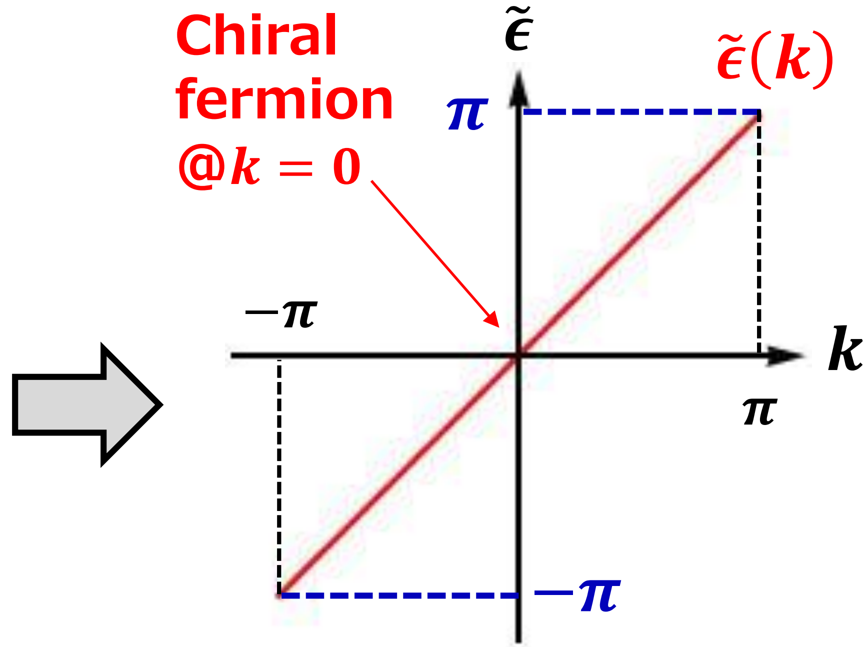
※ $T = 1, a = 1$

Single chiral fermion at $k = 0$
→ Chiral transport in Thouless pump

Change in Topology

$$U_F = \exp(-i\tilde{\epsilon}(k)) = \exp[-i(\tilde{\epsilon}(k) + 2\pi)]$$





1 dimensional analogue of the story

	Equilibrium (lattice) system	**Periodically driven system**
1 dim. spinless fermion	Single chiral fermion → Impossible 1 dim Nielsen-Ninomiya	Single chiral fermion → Possible Thouless pump
3 dim. spinful fermion	**Single** Weyl fermion → Impossible **3-dim Nielsen-Ninomiya**	Single Weyl fermion → Possible

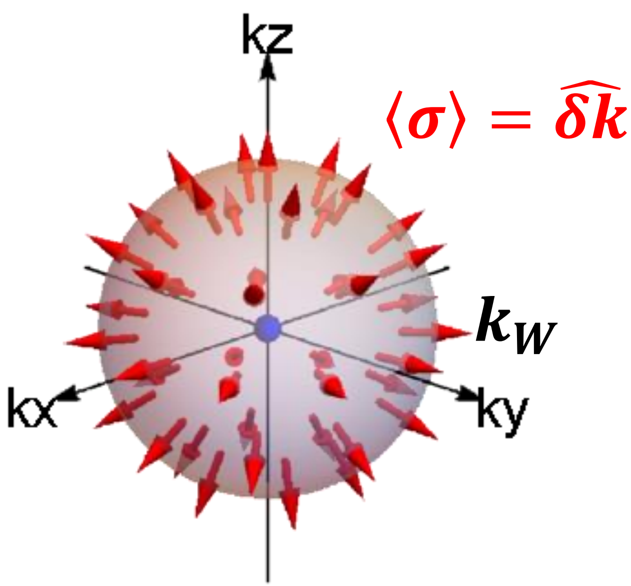
Weyl fermion(WF) & chiral magnetic effect

- Spinful $(\sigma_x, \sigma_y, \sigma_z)$ fermion, 3D (k_x, k_y, k_z)
  Semimetal: $\boldsymbol{h}(\boldsymbol{k}_W) = \boldsymbol{0}$ → **Dispersion near** $\boldsymbol{k}_W$**:** $\boldsymbol{\delta k} = \boldsymbol{k} - \boldsymbol{k}_W \simeq \boldsymbol{0}$ **?**

Left handed WF

- $\boldsymbol{h}(\boldsymbol{k}) \simeq +\boldsymbol{\sigma} \cdot \boldsymbol{\delta k}$

1. Spin momentum locking
   $$\langle \boldsymbol{k} | \boldsymbol{\sigma} | \boldsymbol{k} \rangle = +\widehat{\boldsymbol{\delta k}}$$
2. Hedgehog

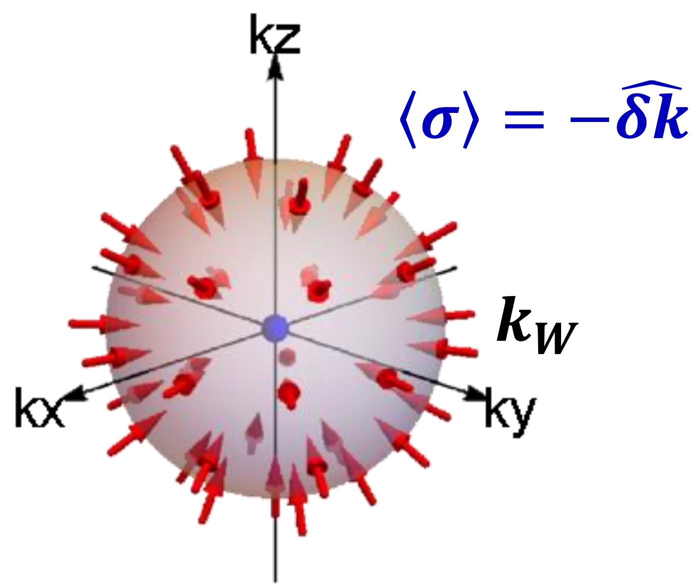


Right handed WF

- $\boldsymbol{h}(\boldsymbol{k}) \simeq -\boldsymbol{\sigma} \cdot \boldsymbol{\delta k}$

1. $\langle \boldsymbol{k} | \boldsymbol{\sigma} | \boldsymbol{k} \rangle = -\widehat{\boldsymbol{\delta k}}$
2. Antihedgehog

$$\boldsymbol{\sigma} \cdot \boldsymbol{\delta k} = \boldsymbol{\sigma}_x \boldsymbol{\delta k}_x + \boldsymbol{\sigma}_y \boldsymbol{\delta k}_y + \boldsymbol{\sigma}_z \boldsymbol{\delta k}_z$$



Weyl fermion & chiral magnetic effect

・Single Weyl fermion + magnetic field $\boldsymbol{B}$

→ Current $\boldsymbol{j}$ flows parallel to $\boldsymbol{B}$

→ Left handed $\boldsymbol{j}_L = +C\boldsymbol{B}$, Right handed $\boldsymbol{j}_R = -C\boldsymbol{B}$

・3D lattice system

→ Equal number of left & right handed fermions (Nielsen-Ninomiya theorem)

→ $\boldsymbol{j}_{\mathrm{tot}} = \boldsymbol{j}_L + \boldsymbol{j}_R = \boldsymbol{j}_L + (-\boldsymbol{j}_L) = \boldsymbol{0}$: No chiral current $\boldsymbol{j}_{\mathrm{tot}} \parallel \boldsymbol{B}$ ☹ ☹

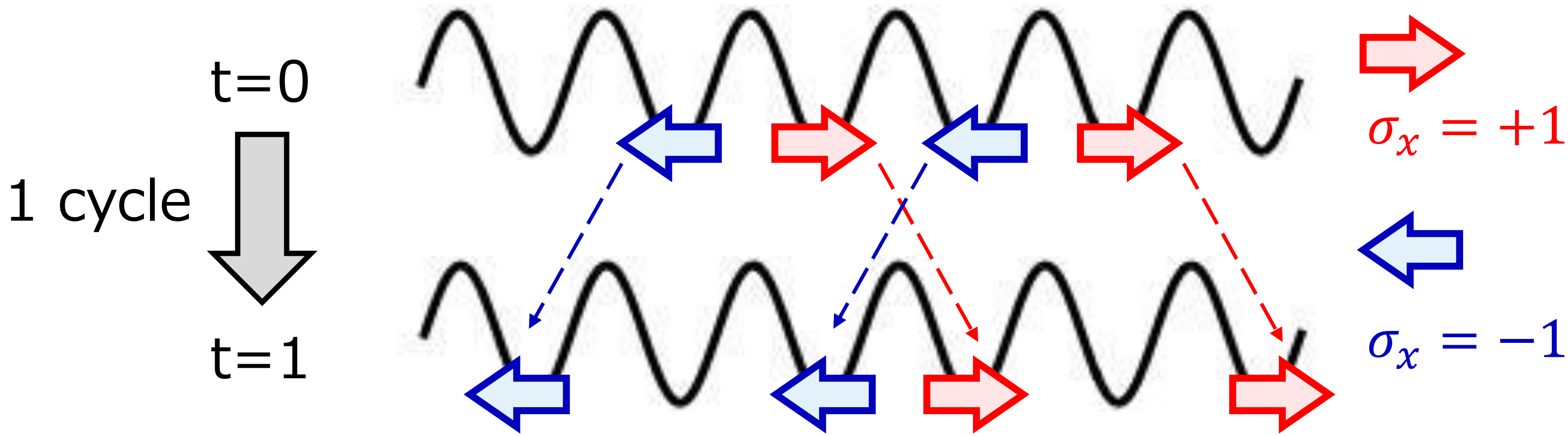
K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008).
A. A. Zyuzin and A. A. Burkov, Phys. Rev. B 85, 165110 (2012).
A. A. Zyuzin, S. Wu, and A. A. Burkov, Phys. Rev. B 86, 115113 (2012).

Helical pump

- "Spinful" Thouless pump: 1dim lattice + spin d.o.f

  +1 site translation for $\boldsymbol{\sigma_x = +1}$

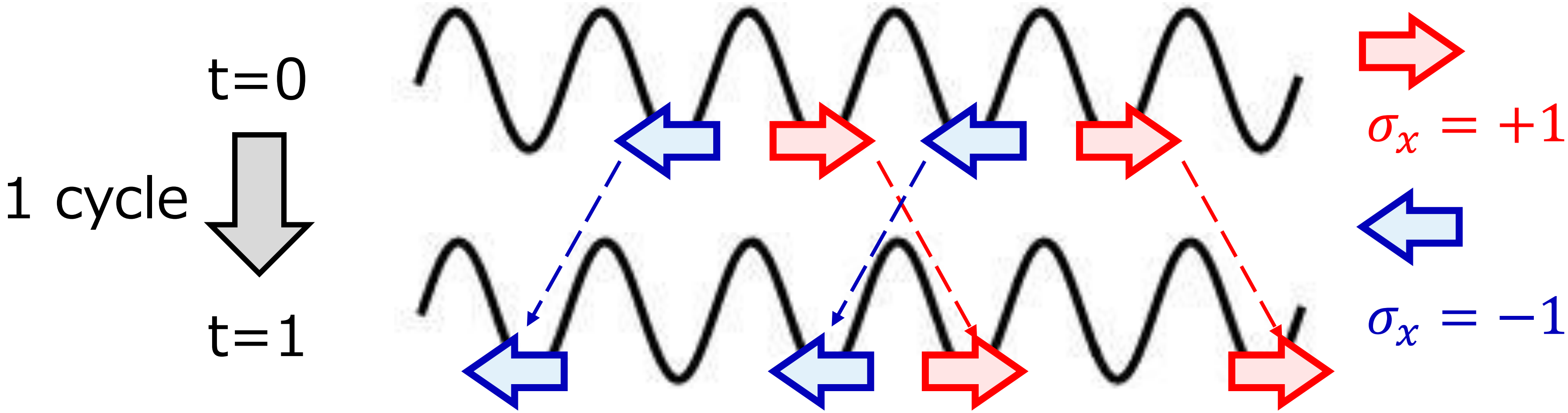
  & **-1** site translation for $\boldsymbol{\sigma_x = -1}$
- Possible to realize in ultracold atomic gas



※ $\boldsymbol{T = 1, a = 1}$

J. Budich, Y. Hu, and P. Zoller, Phys. Rev. Lett 118, 105302 (2017).

Helical pump



- Time evolution: $U(k_x) = e^{-ik_x}|k_x,+\rangle\langle k_x,+| + e^{ik_x}|k_x,-\rangle\langle k_x,-|$

$U(k_x) = \exp(-i\sigma_x k_x) \to H_{\mathrm{eff}}(k_x) = i\log U(k_x) = \sigma_x k_x$

Helical fermion

$H = \sigma_x k_x$

↕

Helical pump

$U_1(k) = \exp(-i\sigma_x k_x)$

Supplemental Materials

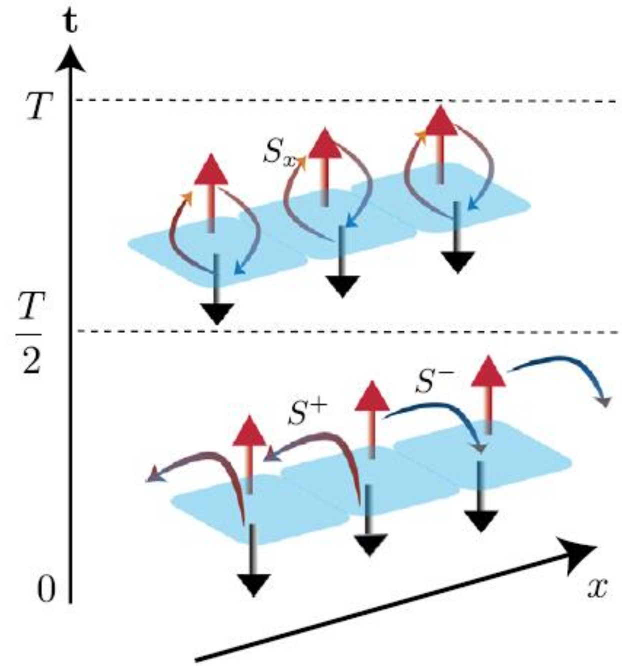
Helical pump in ultracold atomic gas

- $U(k_x) = \exp(-ik_x\sigma_x) = \exp(-iH_2T/2)\exp(-iH_1T/2)$

= successive application of

$H_1 = -(\pi/T)\left(\sigma_x \cos k_x - \sigma_y \sin k_x\right)$: spin-flip hopping

$H_2 = (\pi/T)\sigma_x$: on-site spin flip



J. Budich, Y. Hu, and P. Zoller, Phys. Rev. Lett 118, 105302 (2017).

Relation to high-frequency expansion

$H(t) = \overline{H}(\omega t)$ (ω: frequency, $t_0 = \omega^{-1}$)
High-frequency limit ($t_0 \to 0$): $H(t) \to H_0$ (static Hamiltonian)