Physical Modeling of Growing Cellular Mosaic Patterns in Fish Retina

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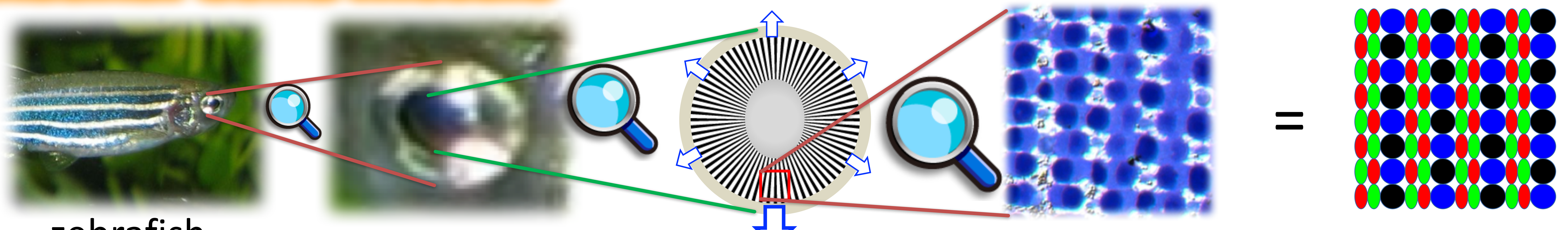
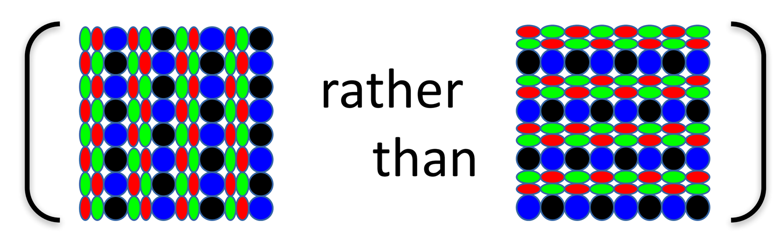
Retinal Cone Mosaic



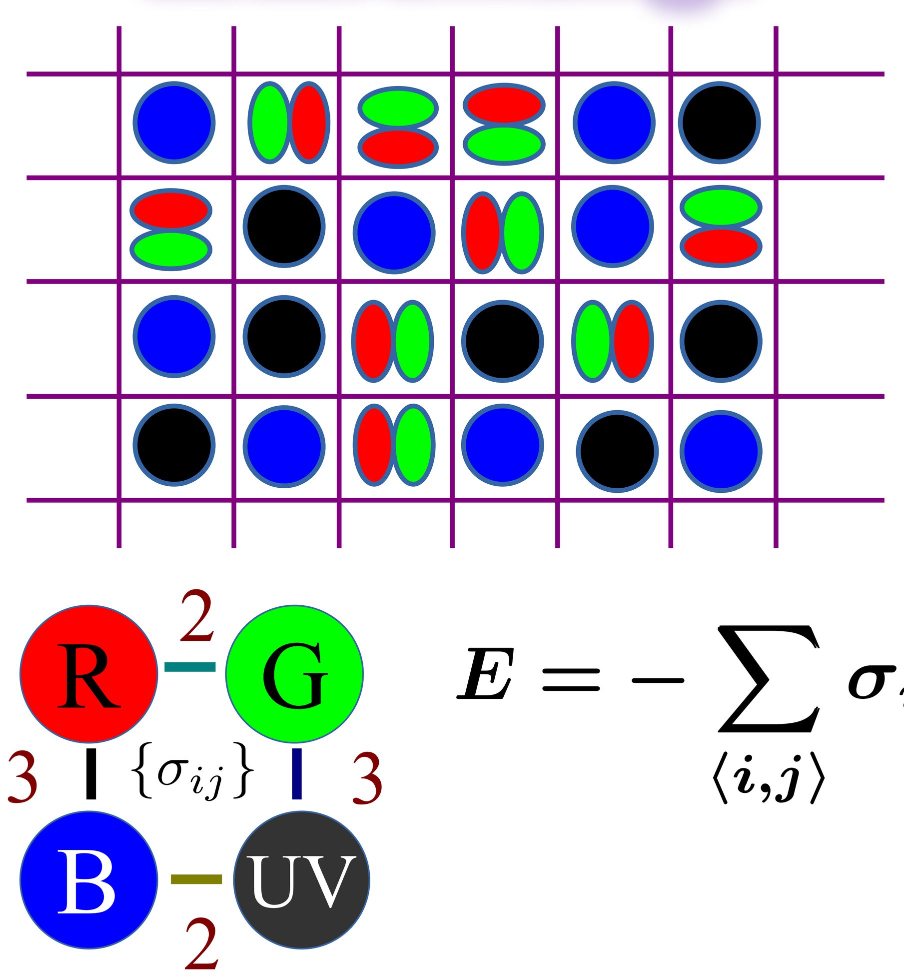

(actual size)

- How is the patterns generated ?
- How is the fixed directionality realized ?



Physical Modeling

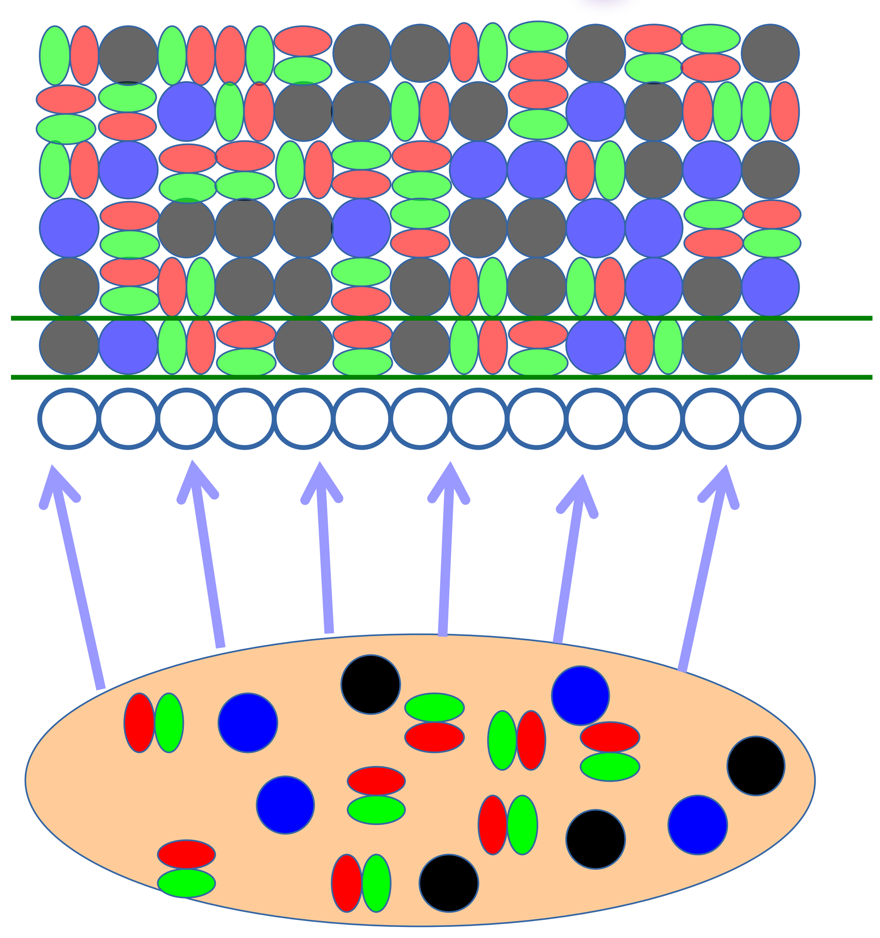
Local bindings



$$E = -\sum_{\langle i,j \rangle} \sigma_{ij}$$

+

Growth row-by-row

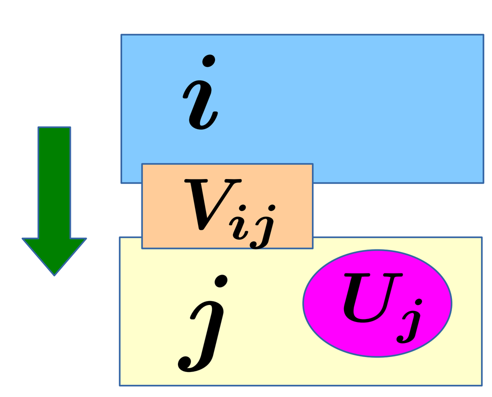


Markovian Growth Model

Transition matrix

$$\mathcal{T}_j{}^i = \frac{\exp(-E_{ij}/T)}{\sum_k \exp(-E_{ik}/T)}$$

fluctuation (T)



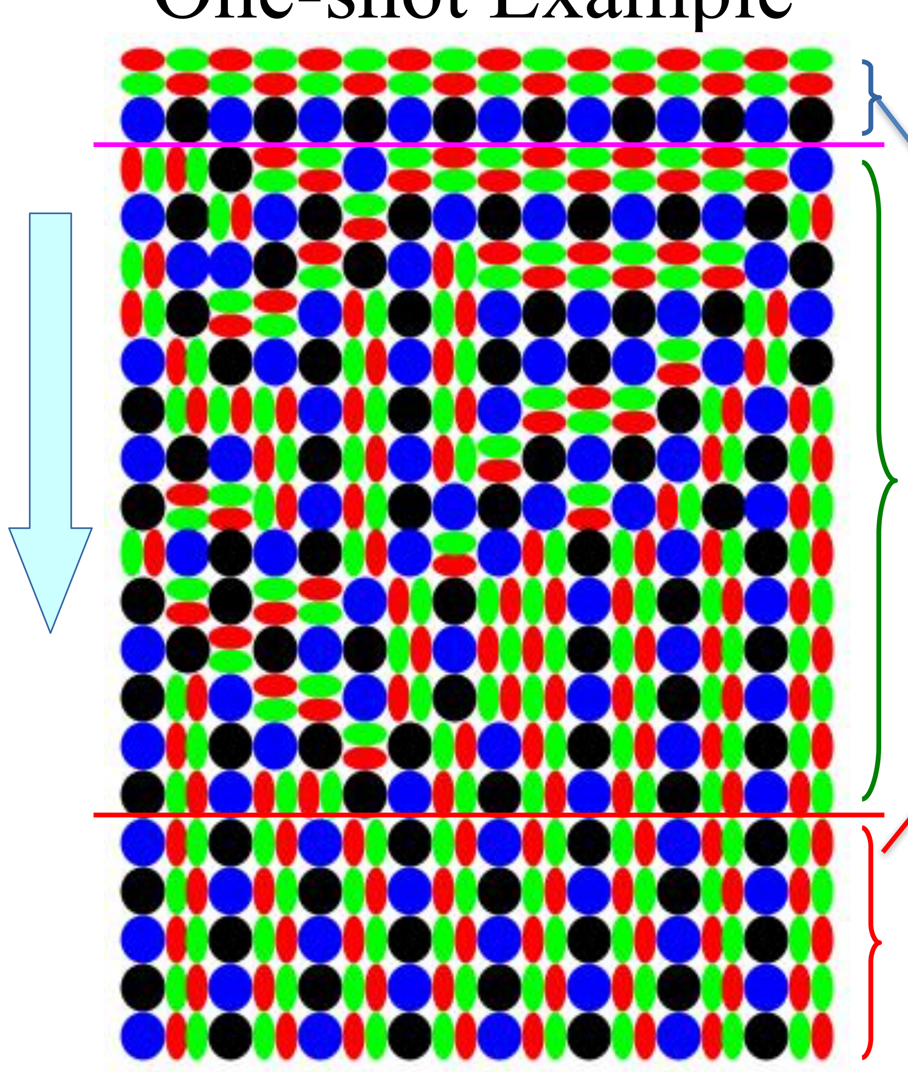
$$(E_{ij} = -U_j - V_{ij})$$

Intra (U_j), Inter (V_{ij})

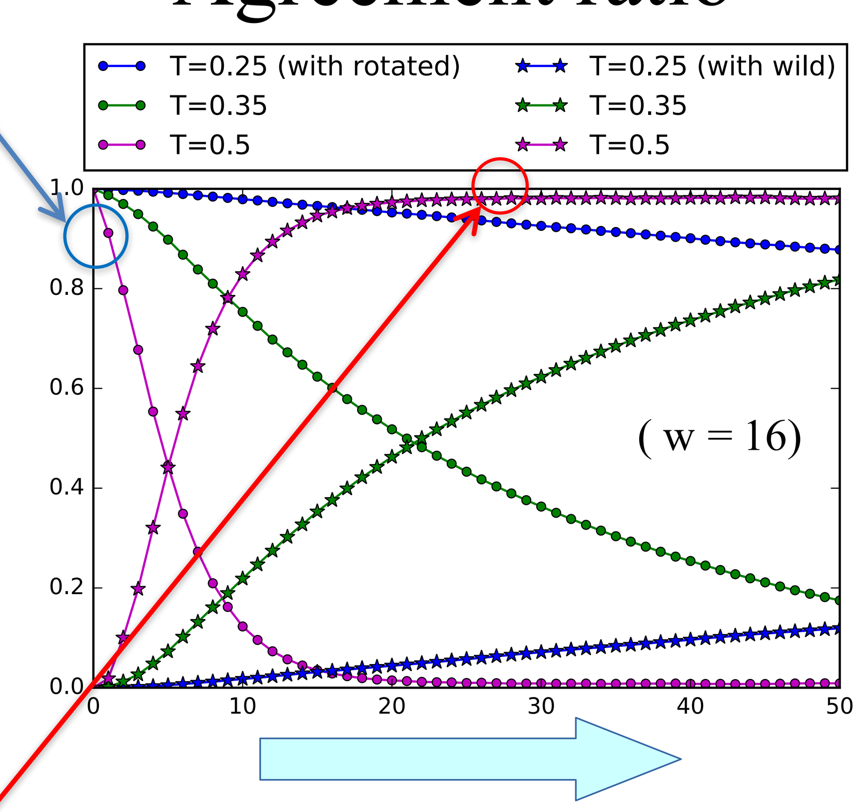
Results

Simulation from rotated initial state

One-shot Example



Agreement ratio



Rotated stripes automatically break down & arrive at stable wild-type pattern !

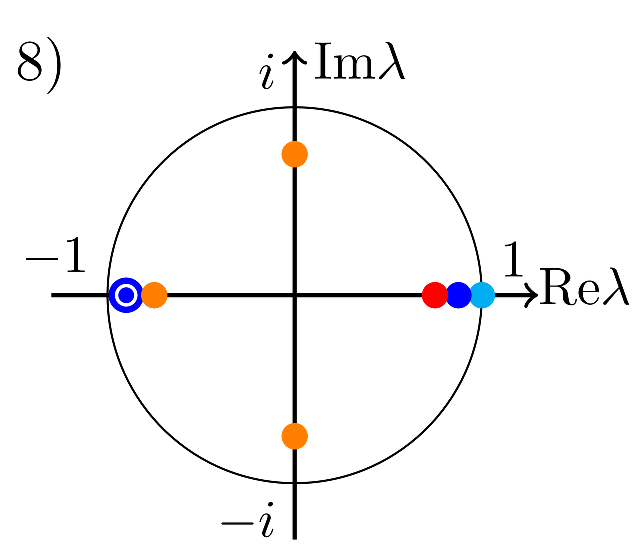
Eigen spectrum of transition matrix

Wild-type Stripe | Rotated Stripe $\quad (g \equiv e^{-1/T})$

$$
\begin{aligned}
\vec{v}_1 &= (\ 1,\ 1,\ 1,\ 1,\ 0,\ 0,\ 0,\ 0,\ \dots) + \mathcal{O}(g)\\
\vec{v}_2 &= (\ 1,\ -1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ \dots) + \mathcal{O}(g)\\
\vec{v}_3 &= (\ 0,\ 0,\ 1,\ -1,\ 0,\ 0,\ 0,\ 0,\ \dots) + \mathcal{O}(g)\\
\vec{v}_4 &= (\ 1,\ 1,\ -1,\ -1,\ 0,\ 0,\ 0,\ 0,\ \dots) + \mathcal{O}(g)\\
\vec{v}_5 &= (\ -1,\ -1,\ -1,\ -1,\ 1,\ 1,\ 1,\ 1,\ \dots) + \mathcal{O}(g)\\
\vec{v}_{6,7} &= (\ 0,\ 0,\ 0,\ 0,\ 1,\ \mp i,\ -1,\ \pm i,\ \dots) + \mathcal{O}(g)\\
\vec{v}_8 &= (\ 0,\ 0,\ 0,\ 0,\ 1,\ -1\ ,1,\ -1,\ \dots) + \mathcal{O}(g)
\end{aligned}
$$

$(w \geq 8)$

$$
\begin{aligned}
\lambda_1 &= 1,\\
\lambda_{2,3} &= -1 + \mathcal{O}(g^3),\\
\lambda_4 &= 1 - \mathcal{O}(g^3),\\
\lambda_5 &= 1 - \mathcal{O}(g^2)\\
\lambda_{6,7} &= \pm\left(1 - \mathcal{O}(g^2)\right)i + \dots\\
\lambda_8 &= -1 + \mathcal{O}(g^2) + \dots
\end{aligned}
$$



Rotated stripes always correspond to smaller eigenvalues.