

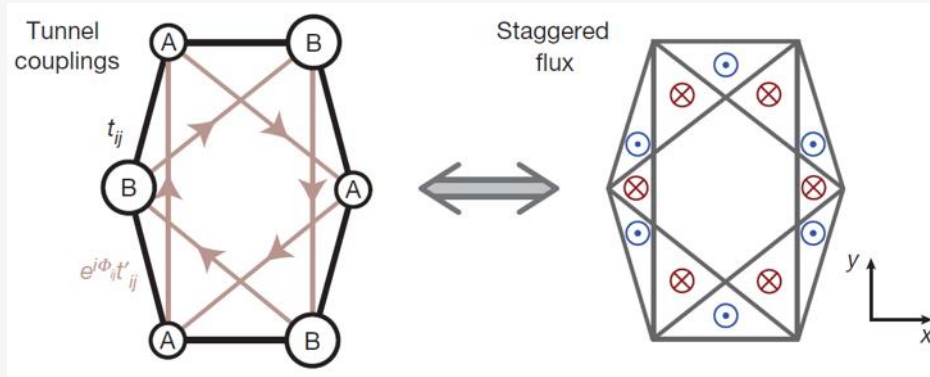
# Ground state phase diagram of a hard-core boson model in a honeycomb optical lattice



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# Experimental realization of the Haldane model



G. Jotzu et al., Nature **515**, (2014).



Inversion symmetry

Time-reversal symmetry

Quantum Hall effect

We study a system of

**dipolar hard-core bosons in a honeycomb optical lattice**

$$H_{\text{HBHM}} = J_1 \sum_{\langle i,j \rangle} (a_i^\dagger a_j + h.c.) + J_2 \sum_{\langle\langle i,j \rangle\rangle} (e^{i\phi} a_i^\dagger a_j + h.c.)$$

$$+ V \sum_{\langle i,j \rangle} n_i n_j \left( -\mu \sum_i n_i \right)$$

NN repulsive interaction

Complex NNN hopping

Geometrical frustration

$$H_{\text{HBHM}} \longrightarrow Z_{\text{HBHM}} = \int [d\delta\rho_{bi}][d\delta\rho_{hi}][d\theta_{bi}][d\theta_{hi}][d\lambda_i] e^{-S_{\text{HBHM}}}$$

$$a_i = h_i^\dagger b_i$$

$$(b_i^\dagger b_i + h_i^\dagger h_i) |\text{Phys}\rangle = |\text{Phys}\rangle$$

$$b_i = \sqrt{\rho_{bi} + \delta\rho_{bi}} e^{i\theta_{bi}}$$

$$h_i = \sqrt{\rho_{hi} + \delta\rho_{hi}} e^{i\theta_{hi}}$$

$$S_{\text{HBHM}} = \int d\tau [S_\tau + S_H + S_I + \sum_i U_\tau (\delta\rho_{bi}^2 + \delta\rho_{hi}^2)]$$

$$S_\tau = \sum_i [i(\partial_\tau \theta_{bi} + \lambda_i) \delta\rho_{bi} + i(\partial_\tau \theta_{hi} + \lambda_i) \delta\rho_{hi}] \quad S_I = V \sum_{\langle i,j \rangle} \rho_{bi} \rho_{bj}$$

$$S_H = 2 J_1 \sum_{\langle i,j \rangle} \sqrt{\rho_{bi} \rho_{hi} \rho_{bj} \rho_{hj}} \cos(\theta_i - \theta_j)$$

$$+ 2J_2 \sum_{\langle\langle i,j \rangle\rangle} \sqrt{\rho_{bi} \rho_{hi} \rho_{bj} \rho_{hj}} \cos(\theta_i - \theta_j - \phi)$$



$$S_{\text{HBHM}} = \sum_{\ell=0}^{N_\tau-1} \left[ \sum_i -\frac{1}{2U_\tau \Delta\tau} [\cos(\theta_{bi,\ell+1} - \theta_{bi,\ell} + \lambda_{i,\ell}) + \cos(\theta_{hi,\ell+1} - \theta_{hi,\ell} + \lambda_{i,\ell})] \right. \\ \left. + \frac{1}{2} J_1 \Delta\tau \sum_{\langle i,j \rangle} \sin(2\chi_{i,\ell}) \sin(2\chi_{j,\ell}) \cos(\theta_{i,\ell} - \theta_{j,\ell}) \quad \text{\textbf{(2 + 1)D XXZ model}} \right. \\ \left. + \frac{1}{2} J_2 \Delta\tau \sum_{\langle\langle i,j \rangle\rangle} \sin(2\chi_{i,\ell}) \sin(2\chi_{j,\ell}) \cos(\theta_{i,\ell} - \theta_{j,\ell} - \phi) \right. \\ \left. + V \Delta\tau \sum_{\langle i,j \rangle} \sin^2(\chi_{i,\ell}) \sin^2(\chi_{j,\ell}) - \mu \Delta\tau \sum_i \sin^2(\chi_{i,\ell}) - \Delta\tau \sum_i \ln(\sin(2\chi_{i,\ell})) \right]$$