摩擦モデルの非平衡統計力学アプローチ

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Sec 1. Introduction: a. Dissipative Model.

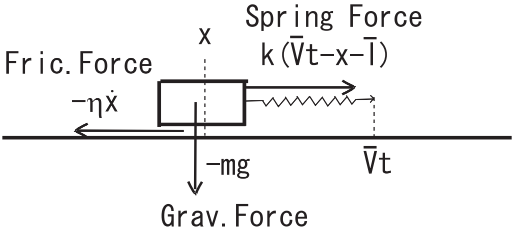

Figure: *1 The spring-block model, (2).*

Sec 1. Introduction: b. Dissipative Model.

Frictional Forces

$$Fri = -\eta \dot{x} \text{ (rain drop, Fig.1)}, \ -m\frac{\kappa \ \mathrm{sgn}(\dot{x})}{1+2\alpha|\dot{x}|} \text{ (stick slip, present work)},$$

m : block mass M, k : spring const. MT^{-2},

frictional parameters $\alpha = 2.5 \ \ TL^{-1}$, $\kappa = 1.0 \ \ LT^{-2}$,

$\bar{\ell}$: block length L, $\bar{V}$: Velocity of spring top LT^{-1} .

1. Burridge and Knopoff, Bull. Seismol. Soc.Am.1967
2. Carlson and Langer PRL, PRA 1989
   'Mechanical model of an earthquake fault'
3. Mori and Kawamura, J. Geoph. Res. 2006
   'Simulation study of the one-dimensional Burridge-Knopoff model of earthquakes'

Sec 1. Introduction: d. Energy with Dissipation

The classical equation of the dissipative block (Stick-Slip).

$$\ddot{x} + \frac{\kappa \ \mathrm{sgn}(\dot{x})}{1 + 2\alpha|\dot{x}|} + \omega^2 x = \omega^2(\bar{V}t - \bar{\ell}). \qquad (2)$$

This has been solved numerically by Runge-Kutta method (Continuous Time Method).

Energy conservation equation :

$$H[\dot{x}, x] \equiv \frac{1}{2}\dot{x}^2 + \frac{\omega^2}{2}x^2 + \omega^2\bar{\ell}x + \int_0^t \frac{\kappa \ |\dot{x}|}{1 + 2\alpha|\dot{x}|} d\tilde{t}\Big|_{4th}$$

$$- \omega^2 \bar{V} \int_0^t \tilde{t}\dot{x} d\tilde{t}\Big|_{5th} = \left(\frac{1}{2}\dot{x}^2 + \frac{\omega^2}{2}x^2 + \omega^2\bar{\ell}x\right)\Big|_{t=0} = E_0 \ . \qquad (3)$$

Three types of energy: 1 [4th] Dissipative energy (hysteresis); 2 [5th] External work (hysteresis); 3 Others. $0 \leq \tilde{t} \leq t$.

Sec 2. Spring-Block Model a.Discrete Morse Flow Theory

n-th Energy Function

$$
\begin{aligned}
K_n(x) = V(x) - hnk\bar{V}x + m\frac{\kappa\ \mathrm{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha|x_{n-1} - x_{n-2}|/h}\ x \\
+\frac{m}{2h^2}(x - 2x_{n-1} + x_{n-2})^2\ ,\ V(x) = \frac{kx^2}{2} + k\bar{\ell}x,
\end{aligned} \tag{4}
$$

x: general position L, x_{n-1}: $(n-1)$th , x_{n-2}: $(n-2)$th ,
h: 1 step interval T

Sec 2. Spring-Block Model b. Variat. Principle

Minimal Energy Priciple $\delta K_n(x)/\delta x|_{x=x_n} = 0.$

$$\frac{k}{m}(x_n + \bar{\ell} - nh\bar{V}) + \frac{1}{h^2}(x_n - 2x_{n-1} + x_{n-2}) + \frac{\kappa\ \mathrm{sgn}(x_{n-1} - x_{n-2})}{1 + 2\alpha|x_{n-1} - x_{n-2}|/h} = 0\ ,\ \omega \equiv \sqrt{\frac{k}{m}}\ , \quad (5)$$

where $n = 2, 3, 4, \cdots, N-1, N.$ Discrete Morse Flow Theory
Recursion relation among n-th, (n-1)-th and (n-2)-th
$K_n(x_n)/m \equiv \mathcal{E}_n$: *DMF energy*.
Parameters: $\bar{V} = 0.1, \bar{\ell} = 1, \omega = 1.0, \kappa = 1.0, \alpha = 2.5$
1 Step Interval: $h = 2.5 \times 10^{-3}$, Total Step Number: $N = 2 \times 10^4$
($h \cdot N = 50$ Total Step Length('Time'))
Initial condition: $x_0 = -\bar{\ell}$, $(x_1 - x_0)/h = 0$.

Sec 2. SB Model e. Movement x_n, DMF result

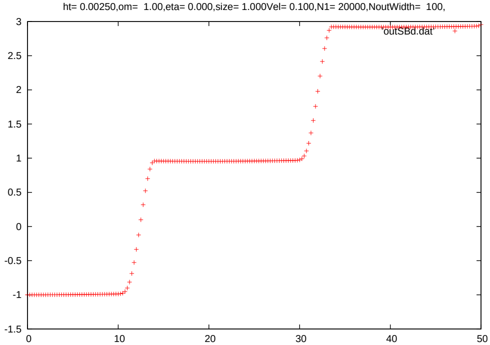

Figure: 4 Movement, x_n. The DMF solution (5) correctly reproduces the continuous-time solution: Stick region and slip region appear.

Sec 2. SB Model g. Dissipative Energy, DMF result

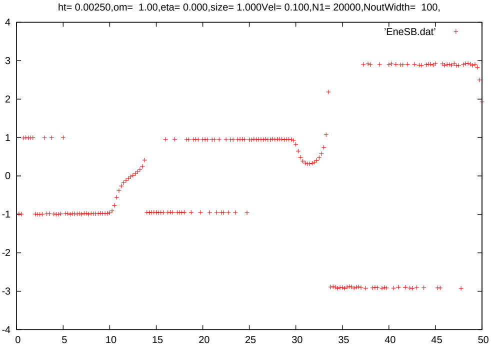

Figure: 6 Dissipative Energy. Stick intervals: 2 energy states $\pm\epsilon$ for each stick region. ϵ is 'quantized'. Slip intervals: connect $-\epsilon$ of a stick region to $+\epsilon'$ of the next stick one.

Sec 2. SB Model o. Frictional Force, DMF result

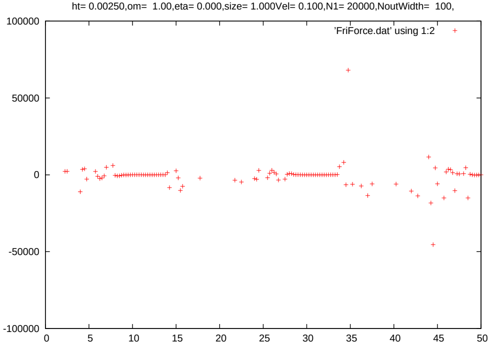

Figure: 14 *Frictional Force* Total force $F_n \equiv (\mathcal{E}_n - \mathcal{E}_{n-1})/(x_n - x_{n-1})$; Spring force $F_n^{sp} = \omega^2 * (Vnh - x_n - \bar{\ell})$; Friction force $Fri_n \equiv F_n - F_n^{sp}$. Fluctuating step-interval and steady one are repeatedly occurring. The interval distribution is similar to velocity-ratio (p.) and frictional energy (q.).

Sec 2. SB Model q. Frictional Energy, DMF result

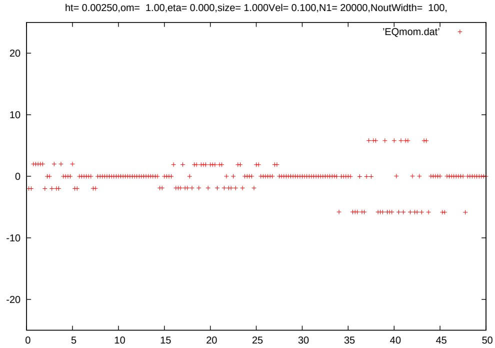

Figure: 17 *Frictional Energy* $FriE(n) \equiv Fri_n * (x_n - x_{n-1})$. Energy is 'quantized' in the fluctuating regions. The interval distribution is not the stick-slip one.