Fixed point structure of the Abelian Higgs model

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Fixed point structure of the Abelian Higgs model

- Microscopic theory of superconductivity (BCS):
  $\longrightarrow$ attractive interaction between electrons via phonon exchange (4-point interaction)
- Ginzburg-Landau theory (effective model of Cooper pairs):

$$\mathcal{F} = \mathcal{F}_0 + \frac{1}{2}|(\vec{\nabla} - ie\vec{A})\phi|^2 + r|\phi|^2 + \lambda|\phi|^4 + \frac{1}{2}|\vec{\nabla} \times \vec{A}|^2$$

- Condensation at T_c $(r = 0)$ $\Rightarrow$ superconductivity

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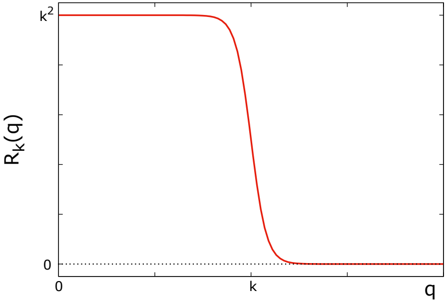
- Condensation at T_c $(r = 0) \Rightarrow$ superconductivity
- Order of the transition?
  $\longrightarrow$ mean field theory: 2nd order
  $\longrightarrow$ 1-loop calculation of the effective action: 1st order
  $\longrightarrow$ RG analysis in the ϵ-expansion: 1st order
  $\longrightarrow$ Monte-Carlo simulations: 1st or 2nd order
  [ratio between penetration depth and correlation length]

- Functional RG: follows the idea of Wilsonian renorm. group

$$Z_k[J] = \int \mathcal{D}\phi e^{-\left(\mathcal{S}[\phi]+\int J\phi+\int \frac{1}{2}\phi R_k \phi\right)}$$

- R_k: IR regulator function
- Requirements:
  1., scale separation
  (suppress modes $q \lesssim k$)
  2., $R_k \longrightarrow \infty$ if $k \longrightarrow \infty$
  3., $R_k \longrightarrow 0$ if $k \longrightarrow 0$



- Gradually moving k from Λ to 0, fluctuations are getting integrated out

- β-functions can be obtained in arbitrary dimensions $\longrightarrow$ no need of the ϵ-expansion

$$\begin{aligned}
\beta_\lambda|_{d=4-\epsilon} &= -\epsilon\bar{\lambda}_k + \frac{54\bar{e}_k^4 - 18\bar{e}_k^2\bar{\lambda}_k + 5\bar{\lambda}_k^2}{24\pi^2} \\
\beta_\lambda|_{d=3} &= -\bar{\lambda}_k + \frac{72\bar{e}_k^4 - 72\bar{e}_k^2\bar{\lambda}_k + 10\bar{\lambda}_k^2}{9\pi^2} \\
\beta_{e^2}|_{d=4-\epsilon} &= -\epsilon\bar{e}_k^2 + \frac{1}{24\pi^2}\bar{e}_k^4 \\
\beta_{e^2}|_{d=3} &= -\bar{e}_k^2 + \frac{4}{15\pi^2}\bar{e}_k^4
\end{aligned}$$

- New fixed points appear in $d=3$! They are capable of describing the 2nd order nature of the phase transition.
- Results are in decent agreement with MC simulations.

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