格子QCDによる
空間相関から迫る
中間子熱変化と壊れた対称性の回復

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in collaboration with

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熱場の量子論とその応用、2015年8月31日--9月2日
Introduction

Thermal fluctuation in QCD

Modifications of hadrons
- sequential melting pattern of quarkonium and open-flavor mesons
  - e.g. $J/\psi$ suppression
  - Matsui and Satz (1986)

Restorations of broken symmetries
- restored pattern of chiral and $U_A(1)$ symmetries
  - the nature of phase transition
  - Pisarski and Wilczek (1984)

Theoretical understanding in lattice QCD simulations from spatial correlation functions

Previous: strange-charm

This work: including up/down at widely $T$ range

PRD91 (2015) 5, 054503
Hadronic excitation on Lattice

Temporal correlation function:
\[ G^T(\tau, T) = \int d^3x \langle J^\dagger_H(0, 0) J_H(\tau, x) \rangle \xrightarrow{\tau \to \infty} A e^{-m_0 \tau} \]
...difficult due to the limitation \( \tau < 1/T \)

Spatial correlation function:
\[ G^S(z, T) = \int_0^{1/T} d\tau \int dx dy \langle J^\dagger_H(0, 0) J_H(\tau, x) \rangle \xrightarrow{z \to \infty} A e^{-M(T)z} \]
\( M(T) \): screening mass

No limitation to spatial direction: more sensitive to in-medium modification
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Spectral function

\[ G^T(\tau, T) = \int_0^\infty d\omega \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \sigma(\omega, T) \]

\( \sigma(\omega, T) \) e.g.) reconstruction of \( \sigma \): MEM

\[ G^S(z, T) = \int_0^\infty \frac{2d\omega}{\omega} \int_{-\infty}^{\infty} dp_z e^{ip_z z} \sigma(\omega, p_z, T) \]

No \( T \) dependence in Kernel: direct probe of thermal modification of \( \sigma \)

\[ G^S(z, T)/G^S(z, T = 0) \]
Hadronic excitation on Lattice

Parity partner of meson states

**Vector (vector and axial-vector)**
\[
\begin{align*}
\bar{\psi} \gamma_i \psi & \quad 1^+ \\
\bar{\psi} \gamma_i \gamma_5 \psi & \quad 1^- \\
& \quad M_V(T)
\end{align*}
\]

**Scalar (pseudo-scalar and scalar)**
\[
\begin{align*}
\bar{\psi} \gamma_5 \psi & \quad 0^+ \\
\bar{\psi} \psi & \quad 0^- \\
& \quad M_S(T)
\end{align*}
\]

Degeneracy of parity partners: indicator of symmetry restorations
**Hadronic excitation on Lattice**

**Parity partner of meson states**

**Vector** (vector and axial-vector)

\[
\psi \gamma_i \psi \\
\psi \gamma_5 \psi \\
M_V(T)
\]

**Scalar** (pseudo-scalar and scalar)

\[
\psi \gamma_5 \psi \\
\psi \psi \\
M_S(T)
\]

Degeneracy of parity partners: **indicator of symmetry restorations**

**Behavior in limiting cases:**

At low \( T \), bound state: \( M(T) \sim m_0 \) pole mass at \( T=0 \)

\[
\sigma(\omega, 0, 0, p_z, T) \sim \delta(\omega^2 - p_z^2 - m_0^2)
\]

At \( T \sim T_C \), in-medium modification and/or dissolution
degeneracy of parity partner states

At \( T \to \infty \), free quark-antiquark pair:

\[
M \to 2\sqrt{m_q^2 + (\pi T)^2}
\]

with the lowest Matsubara frequency
Lattice simulations

- Setup in HISQ
- Modifications of Mesons
- Restorations of broken symmetries
Highly Improved Staggered Quark

Reduction of taste violation
Control of cutoff effects
Bazavov et al. `11, Hot-QCD `11, `14

Lattice parameters
- 2+1 flavor QCD
  (charm quenched)
- $m_s$: physical, $m_l/m_s = 1/20$
  ($m_\pi \sim 160$ MeV, $m_K \sim 504$ MeV)
- $N_\tau = 8$ ($T = 110—207$ MeV)
  10 ($T = 139—166$ MeV)
  12 ($T = 149—400$ MeV)
  keeping $N_s/N_\tau = 4$
- $32^4–48^3 \times 64$ at $T = 0$
- scale: $f_k$ input
- calculating quark-line connected part of meson correlators

Mesons contents

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$J^P$</th>
<th>$u\bar{d}$</th>
<th>$u\bar{s}$</th>
<th>$u\bar{c}$</th>
<th>$s\bar{s}$</th>
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<th>$c\bar{c}$</th>
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<tbody>
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<td>$\gamma_5$</td>
<td>0$^-$</td>
<td>$\pi$</td>
<td>$K$</td>
<td>$D$</td>
<td>$(\eta_{s\bar{s}})$</td>
<td>$D_s$</td>
<td>$\eta_c$</td>
</tr>
<tr>
<td>1</td>
<td>0$^+$</td>
<td>$K_0^*$</td>
<td>$D_0^*$</td>
<td>$-$</td>
<td>$D_{s0}^*$</td>
<td>$\chi_{c0}$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>1$^-$</td>
<td>$\rho$</td>
<td>$K^*$</td>
<td>$D^*$</td>
<td>$\phi$</td>
<td>$D_s^*$</td>
<td>$J/\psi$</td>
</tr>
<tr>
<td>$\gamma_i\gamma_5$</td>
<td>1$^+$</td>
<td>$K_1$</td>
<td>$D_1$</td>
<td>$f_1(1420)$</td>
<td>$D_{s1}$</td>
<td>$\chi_{c1}$</td>
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Meson spectra at $T = 0$ (input: ★)

Highly Improved Staggered Quarks
Probe of thermal modifications of spectral function

\[ \frac{G^S(z, T)}{G^S(z, T = 0)} \simeq 1 \] the same \( \sigma \) at \( T = 0 \), or \( \neq 1 \) modified

\[ T_c = (154 \pm 9) \text{ MeV} \]

Pseudo-scalar

\[ J^P = 0^- \]

\[ \bullet \frac{G^S(z, T)}{G^S(z, 0)} \simeq 1 \text{ at short distance} \quad \text{physics: not sensitive to} \ T \]

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**Ratio of spatial correlation functions**

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- modification at \( T < T_c \)
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- modification at \( T < T_c \), explicit flavor dependence at \( T > T_c \)
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\[ T_c = (154 \pm 9) \text{ MeV} \]
\[ \Delta M(T) = M(T) - m_0 \sim \text{change of “binding energy”} \]

Pseudo-scalar \( J^P = 0^- \)

- \( u\bar{d}, u\bar{s}, u\bar{c} \): explicit thermal modification below \( T_c \),
- similar modification pattern at \( T < T_c \),
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$\Delta M(T) = M(T) - m_0 \sim \text{change of “binding energy”}$

- $u\bar{d}, u\bar{s}, u\bar{c}$: explicit thermal modification below $T_c$,
- similar modification pattern at $T < T_c$,
- explicit flavor dependence at $T > T_c$
- $s\bar{s}, s\bar{c}$: slight modification below $T_c$
- $c\bar{c}$: stable beyond $T_c$

PRD91 (2015) 5, 054503
Restoration of broken symmetries

Degeneracy of vector partners $\rightarrow$ restoration of chiral symmetry
Degeneracy of scalar partners $\rightarrow$ (effective) restoration of $U_A(1)$ symmetry

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Restoration of broken symmetries

Large distance behavior of spatial correlator

\[ G^S(z, T) \xrightarrow{z \to \infty} A e^{-M(T)z} \]

Light-unflavored \( u\bar{d} \)

- Vector partner degenerates at \( T \sim 1.0T_c -- 1.1T_c \)
- Scalar partner degenerates at \( T \sim 1.4T_c -- 1.6T_c \)

chiral: restored, \( U_A(1) \): broken at \( T_c \), no dependence on lattice spacing

Open: \( N_\tau = 8 \)
Black: \( N_\tau = 10 \)
Filled: \( N_\tau = 12 \)

\[ m_l/m_s = 1/20 \]
Restoration of broken symmetries

Large distance behavior of spatial correlator:

\[ \langle S(0, T) \rangle \to z \to \infty, \quad A e^{-M(T)z} \]

Light-unflavored:

- Vector partner degenerates at \( T \approx 1.0 \) \( T_c \)
- Scalar partner degenerates at \( T \approx 1.4 \) \( T_c \)

\[ \text{chiral: restored, } U_A(1): \text{broken at } T_c, \text{ no dependence on lattice spacing} \]

\[ N_\tau = 12, \quad m_l/m_s = 1/27 \]
Summary

In-medium mesons from spatial correlation function

- Sensitive to thermal effect at finite \( T \) on lattice
  - Direct probe of modification of meson spectral function
  - Indicator of restorations of broken symmetries

(2+1)-flavor QCD lattice simulations with HISQ of ratio: \( G^S(z, T)/G^S(z, T = 0) \), screening mass: \( G^S(z, T) \xrightarrow{z \to \infty} Ae^{-M(T)z} \)

- \( u\bar{d}, u\bar{s}, u\bar{c} \): explicit thermal modification below \( T_c \),
  - similar modification pattern below \( T_c \),
  - explicit flavor dependence above \( T_c \)
- \( s\bar{s}, s\bar{c} \): slight modification below \( T_c \)
- \( c\bar{c} \): stable beyond \( T_c \)

- Degeneracies of parity partners
  - chiral: restored, \( U\_A(1) \): broken at \( T_c \)
  - in continuum and physical quark mass (preliminary)

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