Diffusion of non-Gaussian fluctuations of conserved charges

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Y. Ohnishi et al., in prep.

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Beam-Energy Scan

- Hadrons
- Color SC
Beam-Energy Scan

LHC → RHIC BES

J-PARC

Hadrons

Color SC

Grand Canonical Ensemble

Au+Au

00-05%
05-10%
10-20%
20-30%
30-40%
40-60%
50-80%

Cleymans
Andronic

STAR Preliminary

T_{ch} (GeV)

µ_{B} (GeV)

STAR 2012
Non-Gaussian Cumulants @ BES

STAR, PRL (early 2014)

STAR, CPOD (Nov., 2014)
**$\Delta\eta$ Dependence @ ALICE**

**D-measure**

$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- $D \sim 3-4$ Hadronic
- $D \sim 1-1.5$ Quark

**ALICE**

**PRL 2013**

hadronic

suppression

rapidity window
Time Evolution of Fluctuations

① continues to change until kinetic freezeout due to diffusion.

② changes due to a conversion $y \rightarrow \eta$ at kinetic freezeout

“Thermal Blurring”
Time Evolution of Fluctuations

1. continues to change until kinetic freezeout due to diffusion.
2. changes due to a conversion \( y \rightarrow \eta \) at kinetic freezeout
   "Thermal Blurring"
Thermal Blurring
Under Bjorken picture,

\[ \frac{p_z}{E} = v_z = \frac{z}{t} \]

coordinate-space rapidity of medium

momentum-space rapidity

coordinate-space rapidity of individual particles
Thermal distribution in $\eta$ space

$w = \frac{m}{T}$

- pions $w \approx 1.5$
- nucleons $w \approx 9$

- blast wave
- flat freezeout surface
Centrality Dependence

Is the centrality dependence understood solely by the thermal blurring at kinetic f.o.?

More central \rightarrow \begin{cases} \text{lower } T \\ \text{larger } \beta \end{cases} \rightarrow \text{Weaker blurring}
Centrality Dependence

Assumptions:
- Centrality independent cumulant at kinetic f.o.
- Thermal blurring at kinetic f.o.

Centrality dep. of blast wave parameters can qualitatively describe the one of $\langle \delta N_Q^2 \rangle$
Diffusion + Thermal Blurring, Non-Gaussian Cumulants
How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta \eta$?

- suppression
- or
- enhancement
How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta \eta$?

- suppression
- enhancement
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[ \partial_\tau n = D \partial^2_\eta n + \partial_\eta \xi(\eta, \tau) \]

Fluctuation of \( n \) is Gaussian in equilibrium
How to Introduce Non-Gaussianity?

**Stochastic diffusion equation**

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

Fluctuation of \( n \) is Gaussian in equilibrium.

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - discretization of \( n \) **our choice**

**REMARK:** Fluctuations measured in HIC are almost Poissonian.
A Brownian Particle’s Model

Hadronization (specific initial condition)

Freezeout

Time evolution

Initial distribution + motion of each particle
→ cumulants of particle # in \( \Delta \eta \)
A Brownian Particle’s Model

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Initial distribution + motion of each particle → cumulants of particle # in $\Delta \eta$
Baryons in Hadronic Phase

hadronize
chem. f.o.

10~20 fm

kinetic f.o.

time

Baryons behave like Brownian pollens in water
Diffusion + Thermal Blurring

Hadronization

Kinetic f.o. (coordinate space)

Kinetic f.o. (momentum space)

Total diffusion: \( P(x - x'') = \int dx' P_1(x - x')P_2(x' - x'') \)

- Diffusion + thermal blurring = described by a single \( P(x) \)
- Both are consistent with Gaussian \( \rightarrow \) Single Gaussian
\( \Delta \eta \) Dependence: 4\(^{th} \) order

\[
\frac{\langle \delta N^4 \rangle(\eta)}{\langle \delta N^4 \rangle(0)}
\]

**MK, NPA (2015)**

**Initial Condition**

\[
D_4 = \frac{\langle Q^4_{(net)} \rangle_c}{\langle Q_{(tot)} \rangle}
\]

\[
b = \frac{\langle Q^2_{(net)} Q_{(tot)} \rangle_c}{\langle Q_{(net)} \rangle}
\]

\[
c = \frac{\langle Q^2_{(tot)} \rangle_c}{\langle Q_{(tot)} \rangle}
\]

\[
D_2 = \frac{\langle Q^2_{(net)} \rangle_c}{\langle Q_{(tot)} \rangle} = 0.5
\]

Characteristic \( \Delta \eta \) dependences!
\[ \Delta \eta \text{ Dependence: } 4^{\text{th}} \text{ order} \]

Initial Condition

\[
D_4 = \frac{\langle Q^4_{\text{net}} \rangle_c}{\langle Q_{\text{tot}} \rangle_c} \\
b = \frac{\langle Q^2_{\text{net}} Q_{\text{tot}} \rangle_c}{\langle Q_{\text{net}} \rangle_c} \\
c = \frac{\langle Q^2_{\text{tot}} \rangle_c}{\langle Q_{\text{tot}} \rangle_c} \\
D_2 = \frac{\langle Q^2_{\text{net}} \rangle_c}{\langle Q_{\text{tot}} \rangle_c} = 0.5
\]

\[ \Delta \eta = 1.0 \text{ at ALICE} \]

\[ \Delta \eta = 1.6 \text{ at ALICE} \]

\[ \Delta \eta = 1.0 \text{ baryon #} \]

\[ D \sim M^{-1} \]
$\Delta \eta$ Dependence: 3rd order

Initial Condition

\[ D_3 = \frac{\langle Q^3_{\text{net}} \rangle_c}{\langle Q_{\text{net}} \rangle} \]

\[ a = \frac{\langle Q_{\text{net}} \rangle \langle Q_{\text{tot}} \rangle}{\langle Q_{\text{net}} \rangle} \]

\[ D_2 = \frac{\langle Q^2_{\text{net}} \rangle_c}{\langle Q_{\text{tot}} \rangle} = 0.5 \]

$D \sim M^{-1}$

$\Delta \eta = 1.0$ at ALICE

$\Delta \eta = 1.6$ baryon #
4th order : Large Initial Fluc.

\[ \langle \delta N^4(\eta) \rangle / \langle \delta N^4(0) \rangle \]

\[ D_4 = 4, \ D_2 = 1 \]

\[ b=2 \quad b=1 \quad b=0 \]

\[ \Delta \eta / \sqrt{2D\tau} \]

\[ \Delta \eta = 1.0 \quad \text{at ALICE} \]
\[ \Delta \eta = 1.6 \quad \text{at ALICE} \]

\[ \Delta \eta = 1.0 \quad \text{baryon #} \]

MK, NPA (2015)

Initial Condition

\[ D_4 = \frac{\langle Q^4_{(net)} \rangle_c}{\langle Q_{(tot)} \rangle} = 4 \]

\[ b = \frac{\langle Q^2_{(net)} Q_{(tot)} \rangle_c}{\langle Q_{(net)} \rangle} \]

\[ c = \frac{\langle Q^2_{(tot)} \rangle_c}{\langle Q_{(tot)} \rangle} \]

\[ D_2 = \frac{\langle Q^2_{(net)} \rangle_c}{\langle Q_{(tot)} \rangle} = 1 \]

\[ D \sim M^{-1} \]
Non-monotonic dependence on $\Delta \eta$?
Summary

Plenty of information in $\Delta \eta$ and centrality dependences of various cumulants

$$\langle N^2_Q \rangle_c, \langle N^3_Q \rangle_c, \langle N^4_Q \rangle_c, \langle N^2_B \rangle_c, \langle N^3_B \rangle_c, \langle N^4_B \rangle_c, \langle N^2_S \rangle_c, \ldots$$

and those of non-conserved charges, mixed cumulants…

With $\Delta \eta$ dep. we can explore
- primordial thermodynamics
- non-thermal and transport property
- effect of thermal blurring
Future Studies

- **Experimental side:**
  - rapidity window dependences
  - baryon number cumulants
  - consistency between RHIC and LHC

- **Theoretical side:**
  - rapidity window dependences in dynamical models
  - description of non-equilibrium non-Gaussianity
  - accurate measurements on the lattice

- **Both sides:**
  - Compare theory and experiment carefully
  - Do not use a fixed $\Delta \eta$ cumulant for comparison!!!
Very Low Energy Collisions

- Large contribution of global charge conservation
- Violation of Bjorken scaling

Fluctuations at low $\sqrt{s}$ should be interpreted carefully! Comparison with statistical mechanics would not make sense…
Initial Condition @ Hadronization

Hadronization (initial condition)

Boost invariance / infinitely long system
Local equilibration / local correlation

\[ \langle \bar{Q}^2 \rangle_c, \langle \bar{Q}^3 \rangle_c, \langle \bar{Q}^4 \rangle_c, \langle \bar{Q}^2 Q_{(tot)} \rangle_c, \langle Q_{(tot)}^2 \rangle_c, \langle Q_{(net)} Q_{(tot)} \rangle_c \]

suppression owing to local charge conservation
strongly dependent on hadronization mechanism
Thermal distribution in $\eta$ space

Blast wave squeezes the distribution in rapidity space

$w = \frac{m}{T}$

- pions $w \approx 1.5$
- nucleons $w \approx 9$

• blast wave
• flat freezeout surface
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