Massive renormalization scheme and perturbation theory at finite temperature

QFT Workshop
YITP, Kyoto
September 1st, 2015

Based on work done with N. Wschebor, arXiv 1409.4795
(Phys.Lett. B741 (2015) 310-315)

European Research Council

Jean-Paul Blaizot, IPhT- Saclay

IPhT SACLAY

What is the origin of the strongly coupled character of the quark-gluon plasma?

A puzzling situation

- The coupling constant is not small, but not huge $\alpha_s \sim 0.3 \div 0.4$
- Strict perturbation does not work, but successful resummations exist
- Our present understanding of early stages of HI collisions relies on weak coupling concepts

Clue?

- «Strong coupling» behavior may appear at weak coupling, when many degrees of freedom contribute coherently (e.g. collective phenomena, BCS, CGC, etc)
- The quark-gluon plasma is a multiscale system

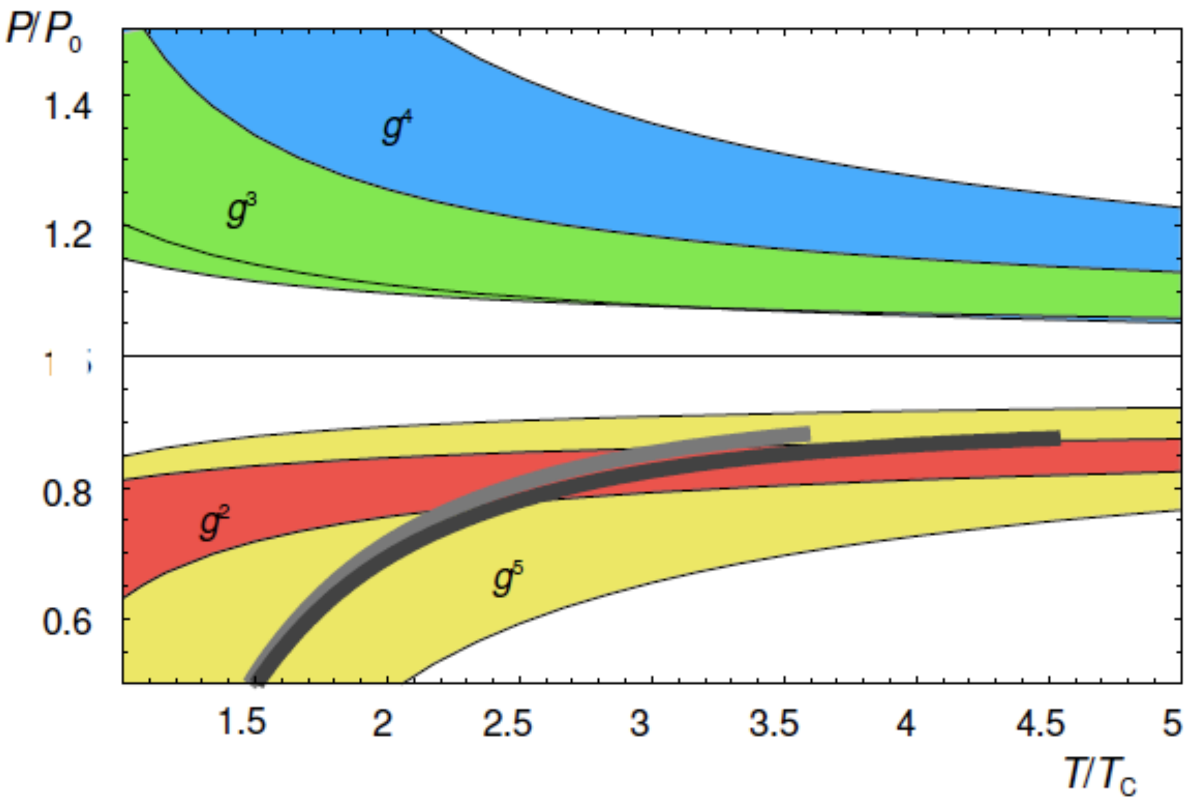
Strict perturbation theory breaks down

But

- this has (almost) nothing to do with QCD

QCD at finite temperature

Perturbation theory is ill behaved



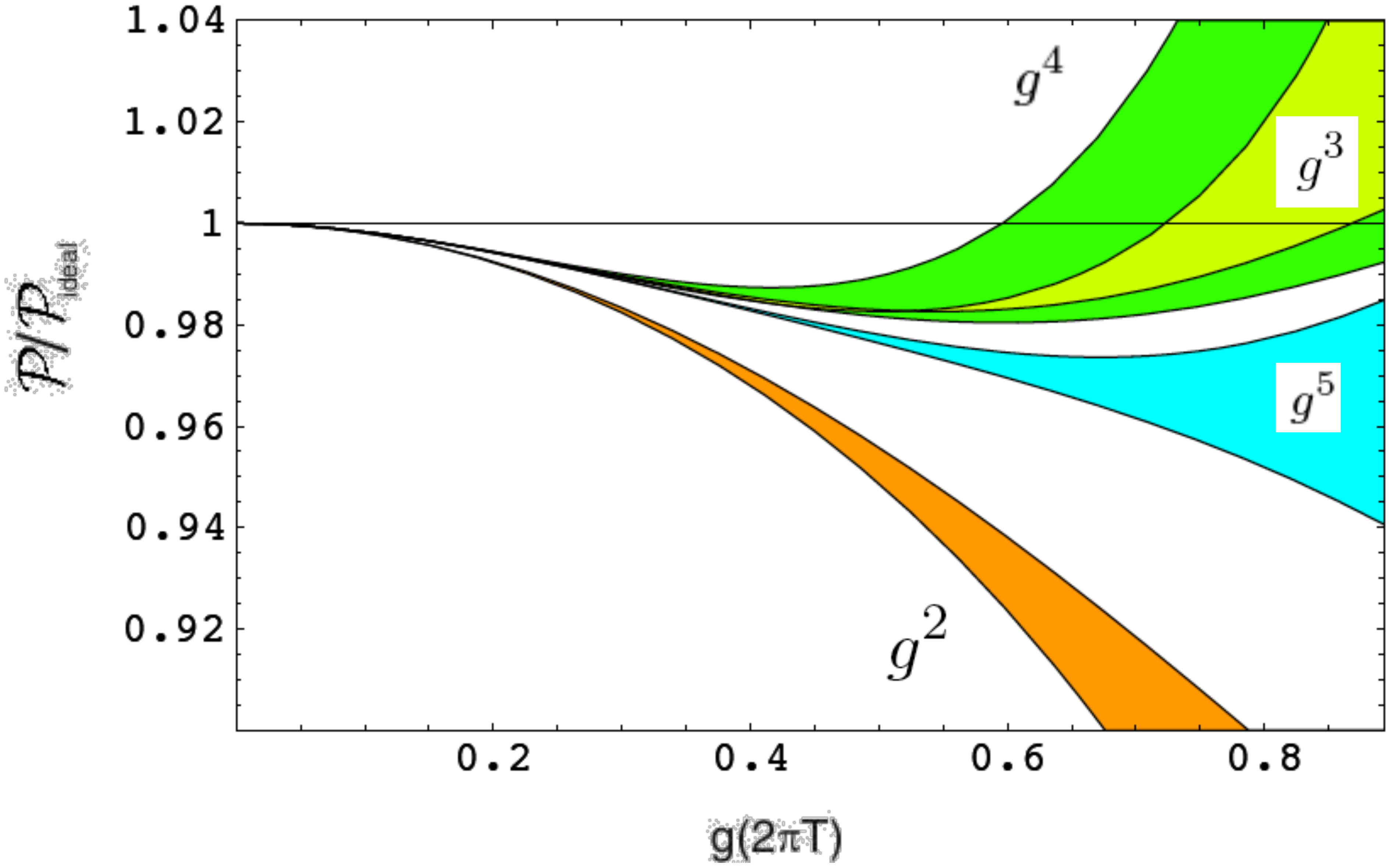
Perturbation theory:

g^2: Shuryak; Chin (1978)
g^3: Kapusta (1979)
$g^4 \ln g$: Toimela (1983)
g^4: Arnold, Zhai (1994)
g^5: Zhai, Kastening (1995), Braaten, Nieto (1996)
$g^6 \ln g$: Kajantie, Laine, Rummukainen, Schröder (2002)
g^6 (partly): Di Renzo, Laine, Miccio, Schröder, Torrero (2006)

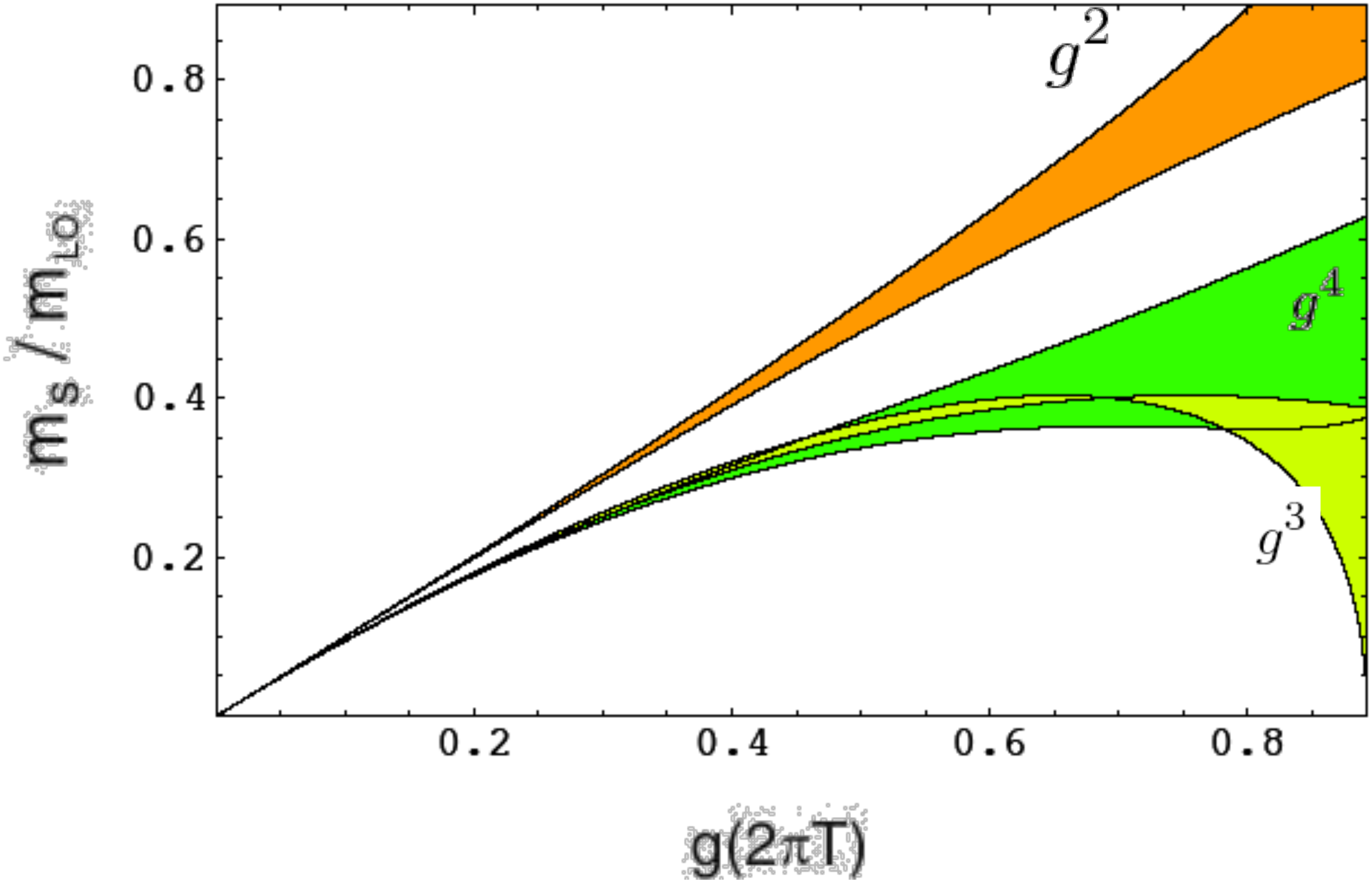
Lattice data: G. Boyd et al. (1996); M. Okamoto et al. (1999).

Similar difficulty in the case of scalar field theory at finite temperature

Scalar field theory with quartic coupling

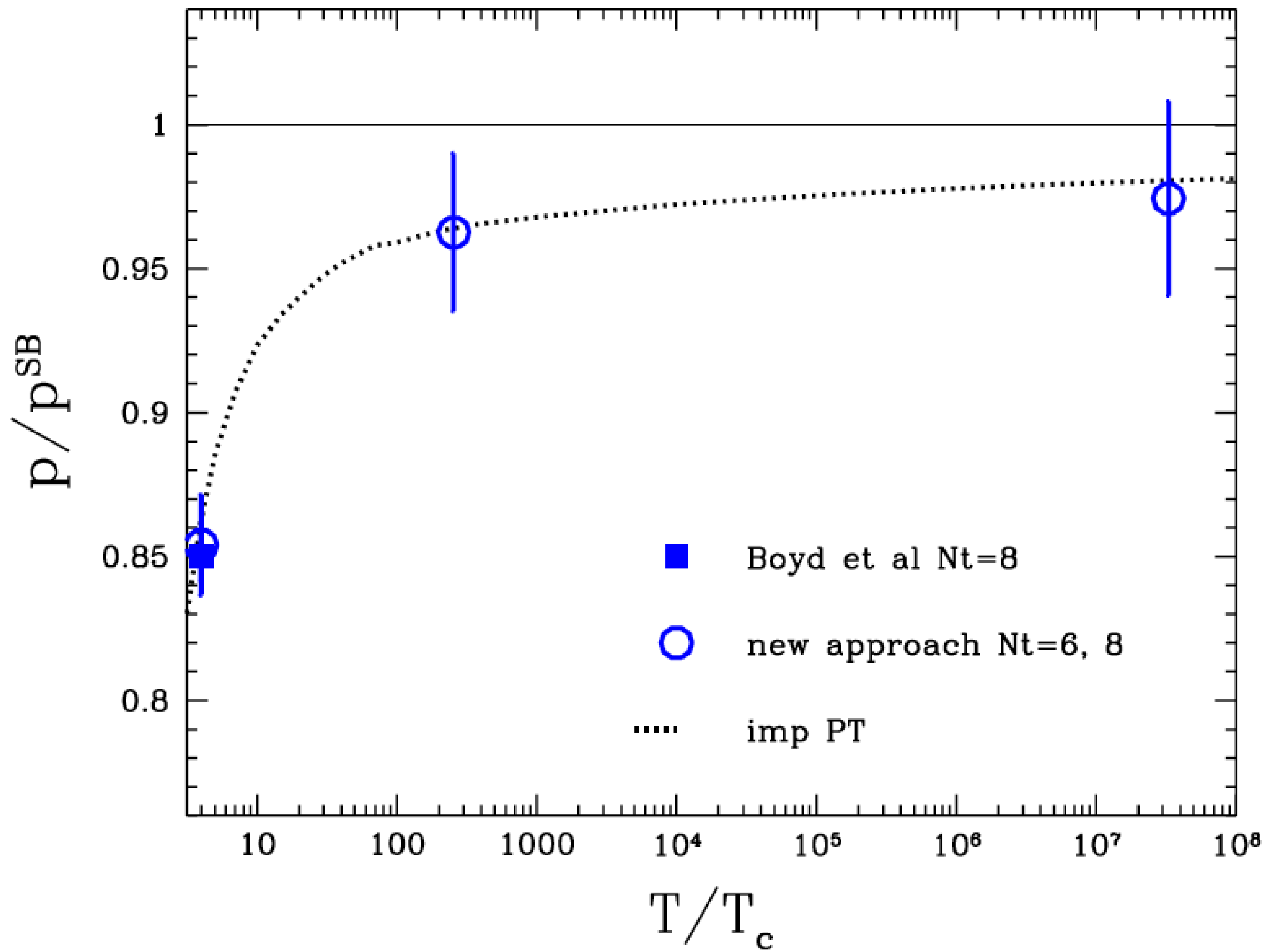


Scalar field theory with quartic coupling



Still, asymptotic freedom works !

Pressure for SU(3) YM theory at (very) high temperature



(from G. Endrodi et al, arXiv: 0710.4197)

A continuing effort

- Calculate higher orders....

Pressure in scalar theory is known up to order $O\left(g^8 \ln g\right)$

J.O. Andersen, L. Kyllingstad and L.E. Leganger, arXiv:0903.4596

- reorganize perturbation theory, resum, 2PI, NPRG, HTL pert. th. etc)

DIMENSIONAL REDUCTION

Integration over the hard modes $(gT \leq \Lambda_E \leq T)$

$$\mathcal{L}_E = \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \mathrm{Tr}\left[D_i, A_0\right]^2 + m_E^2\mathrm{Tr}A_0^2 + \lambda_E\left(\mathrm{Tr}A_0^2\right)^2 + \cdots$$

$$D_i = \partial_i - ig_E A_i$$

In leading order $\quad g_E \approx g\sqrt{T} \qquad m_E \approx gT \qquad \lambda_E \approx g^4 T$

Integration over the soft modes $(g^2T \leq \Lambda_M \leq gT)$

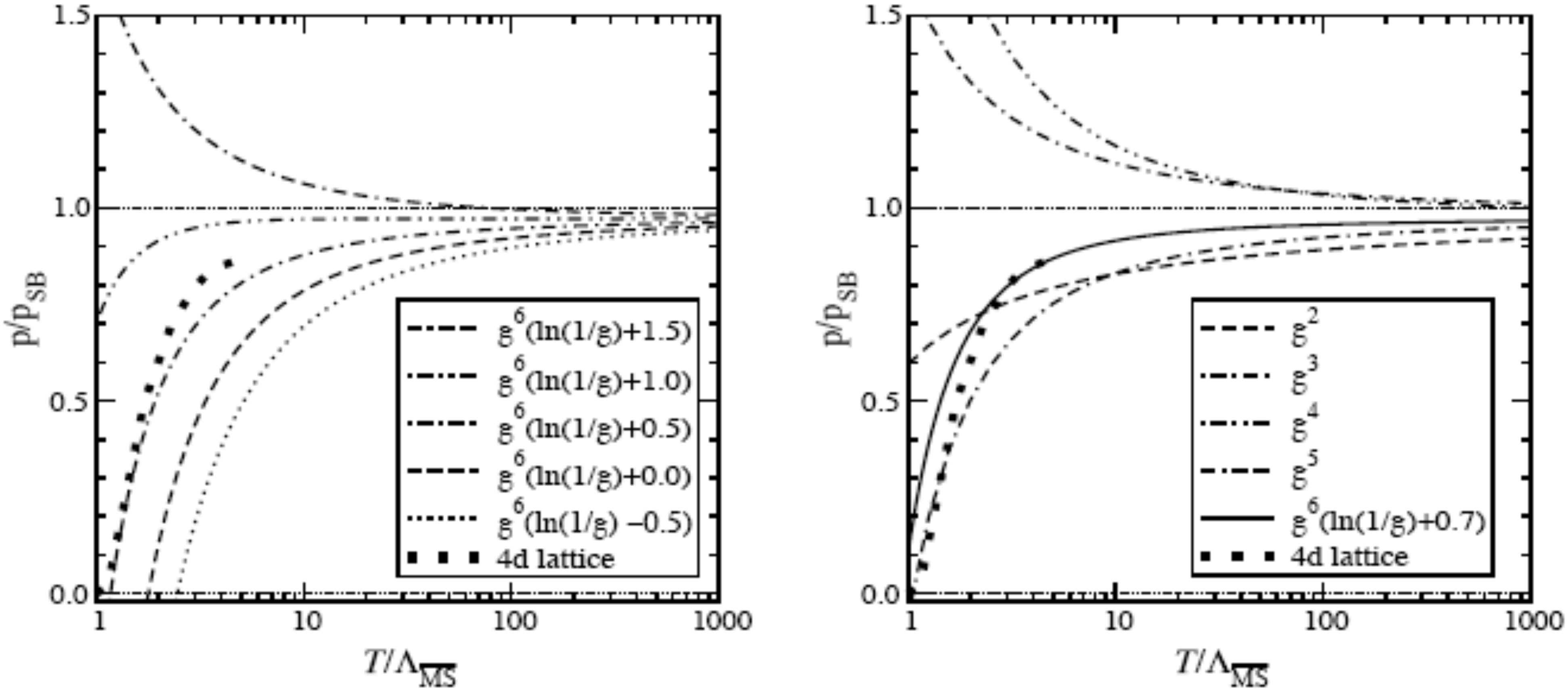
$$\mathcal{L}_M = \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \cdots \qquad D_i = \partial_i - ig_M A_i \qquad g_M^2 \sim g_E^2$$

Non perturbative contribution

E. Braaten and A. Nieto, *Phys. Rev.* D 51 (1995) 6990 , *Phys. Rev.* D 53 (1996) 3421

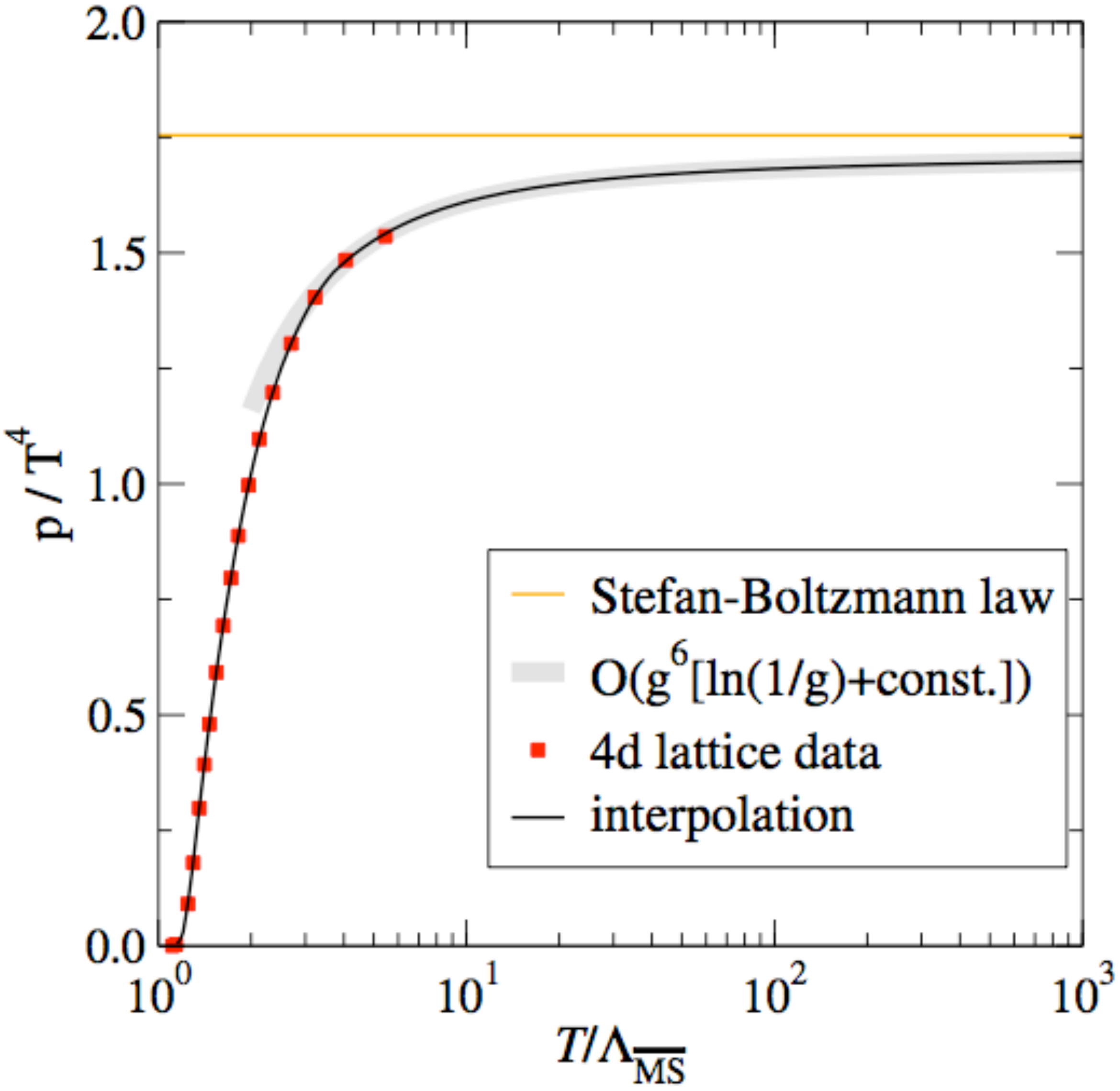
$$p = T^4 \left[c_0 + c_2\, g^2 + c_3\, g^3 + \left(c_4' \ln g + c_4 \right) g^4 + c_5\, g^5 + c_6\, g^6 \right]$$

$$c_6 = N_c^3\, \frac{N_c^2 - 1}{(4\pi^4)} \left[\left(\frac{215}{12} - \frac{805}{768}\, \pi^2 \right) \ln \frac{1}{g} + 8\, \delta \right]$$



K. Kajantie, M. Laine, K. Rummukainen, and Y. Schröder, *Phys. Rev. Lett.* 86 (2001) 10 , *Phys. Rev.* D 65 (2002) 045008 , *Phys. Rev.* D 67 (2003) 105008 , *JHEP* 0304 (2003) 036

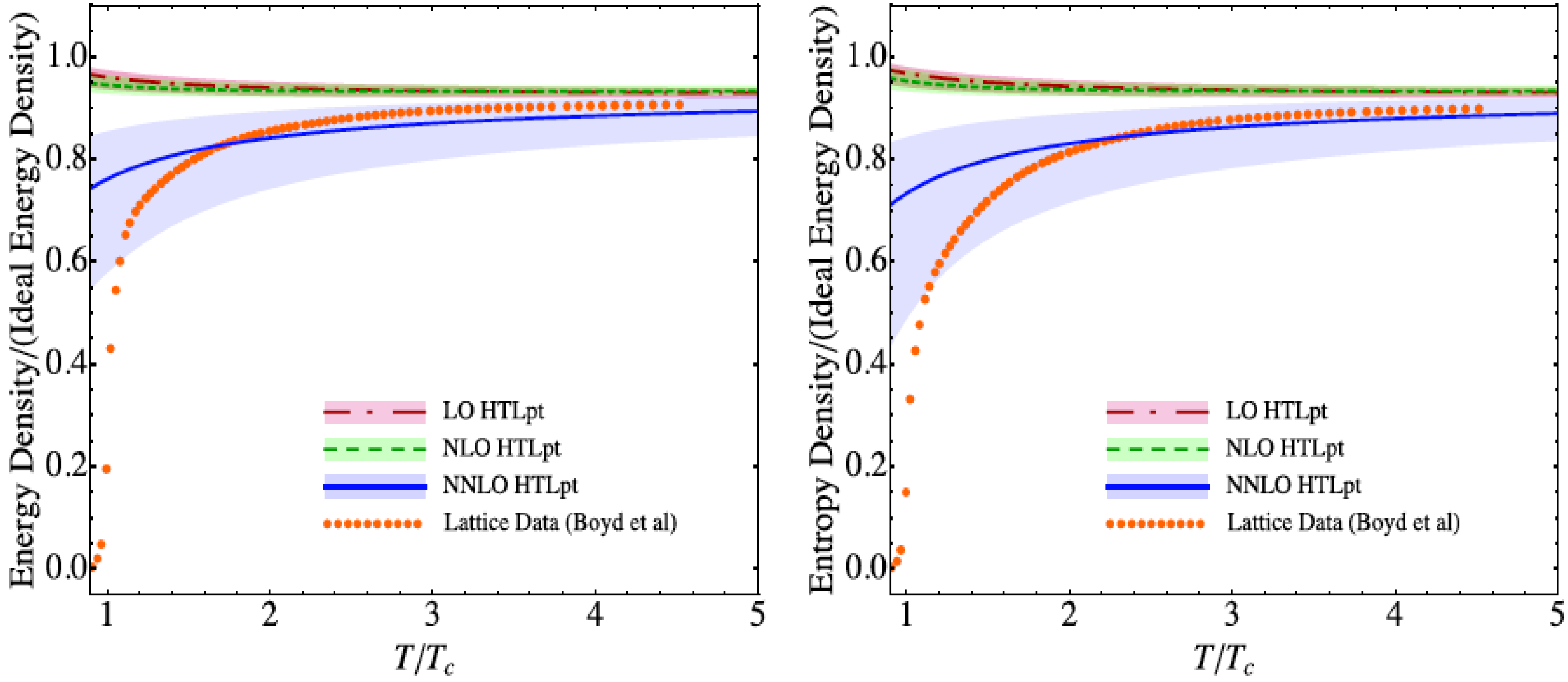
State of the art in high order perturbative calculations



(from M. Laine, Y Schroeder, hep-ph/0603048)

$N_c = 3$ Pure-Glue NNLO Energy and Entropy – BN Mass

From the free energy we can evaluate other thermodynamic variables using standard relations: $\mathcal{P} = -\mathcal{F}, \mathcal{E} = \mathcal{F} - T\frac{d\mathcal{F}}{dT}, \mathcal{S} = -\frac{d\mathcal{F}}{dT}$.



(from J. O. Andersen, Nan Su, M. Strickland, arXiv: 1005.1603

Why is pertubation theory so bad at finite temperature while it is fine at zero temperature (for comparable values of the coupling constant) ?

Expansion parameter and thermal fluctutations

$$S[\varphi] = \int_0^\beta d\tau \int d^dx \Big\{ \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi + \frac{{m_{\mathrm{B}}}^2}{2}\varphi^2 + \frac{g_{\mathrm{B}}^2}{4!}\varphi^4 \Big\}$$

$$\langle\varphi^2\rangle_\kappa \approx \int^\kappa \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{n_{\boldsymbol{p}}}{E_{\boldsymbol{p}}} \approx T\kappa$$

$$n_{\boldsymbol{p}} = \frac{1}{\mathrm{e}^{E_{\boldsymbol{p}}/T} - 1}$$

$$\gamma_\kappa \sim \frac{g^2\langle\varphi^2\rangle_\kappa}{\kappa^2} \sim \frac{g^2 T}{\kappa}$$

Suggests a breakdown of perturbation theory when $\kappa \lesssim g^2 T$

Weakly AND strongly coupled ...

Degrees of freedom with different wavelengths are differently coupled.

Expansion parameter

$$\gamma_\kappa = \frac{g^2\langle\phi^2\rangle}{\kappa^2} \qquad \langle\phi^2\rangle_\kappa \sim \kappa T \quad (\kappa \lesssim T)$$

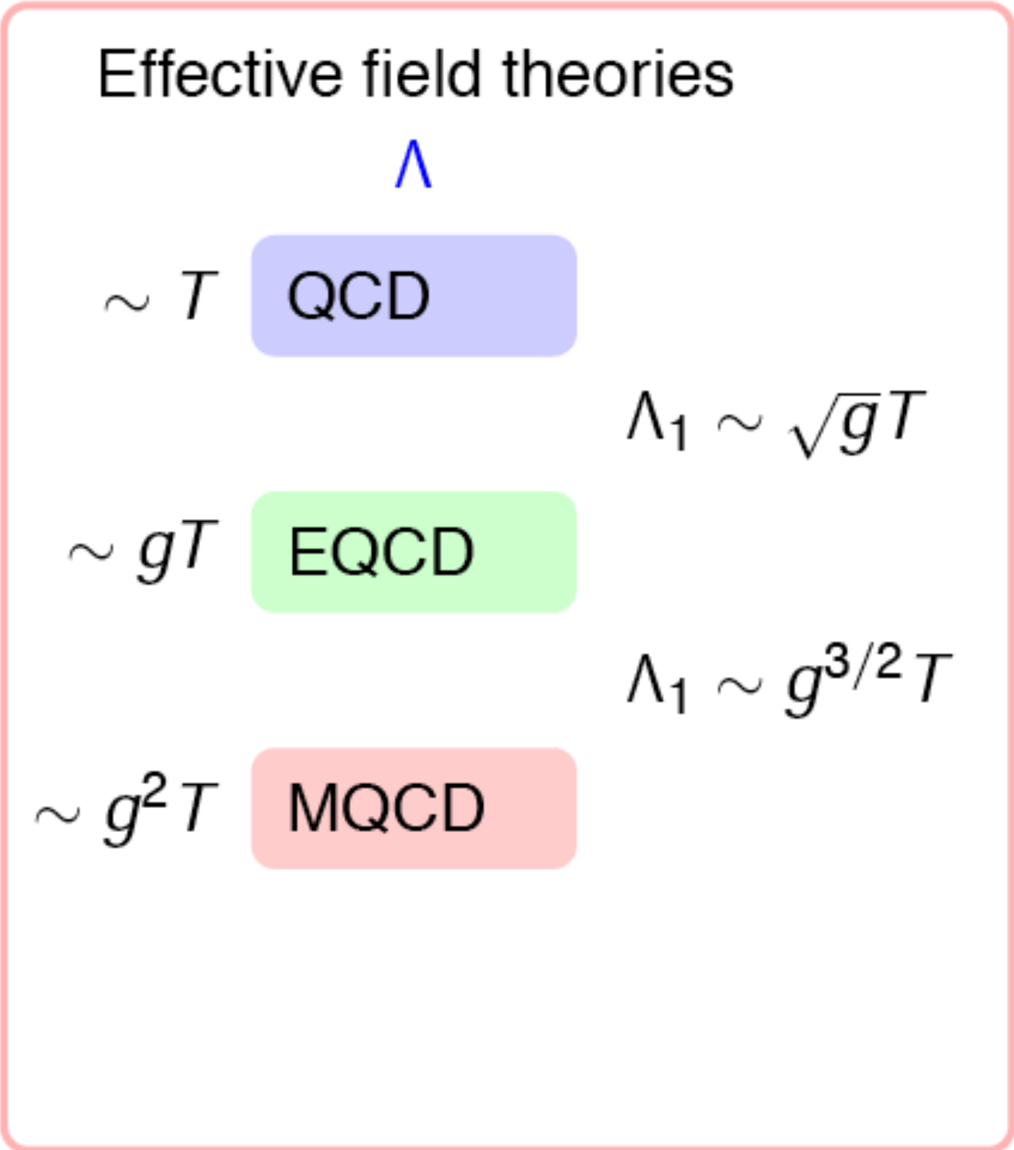
Dynamical scales

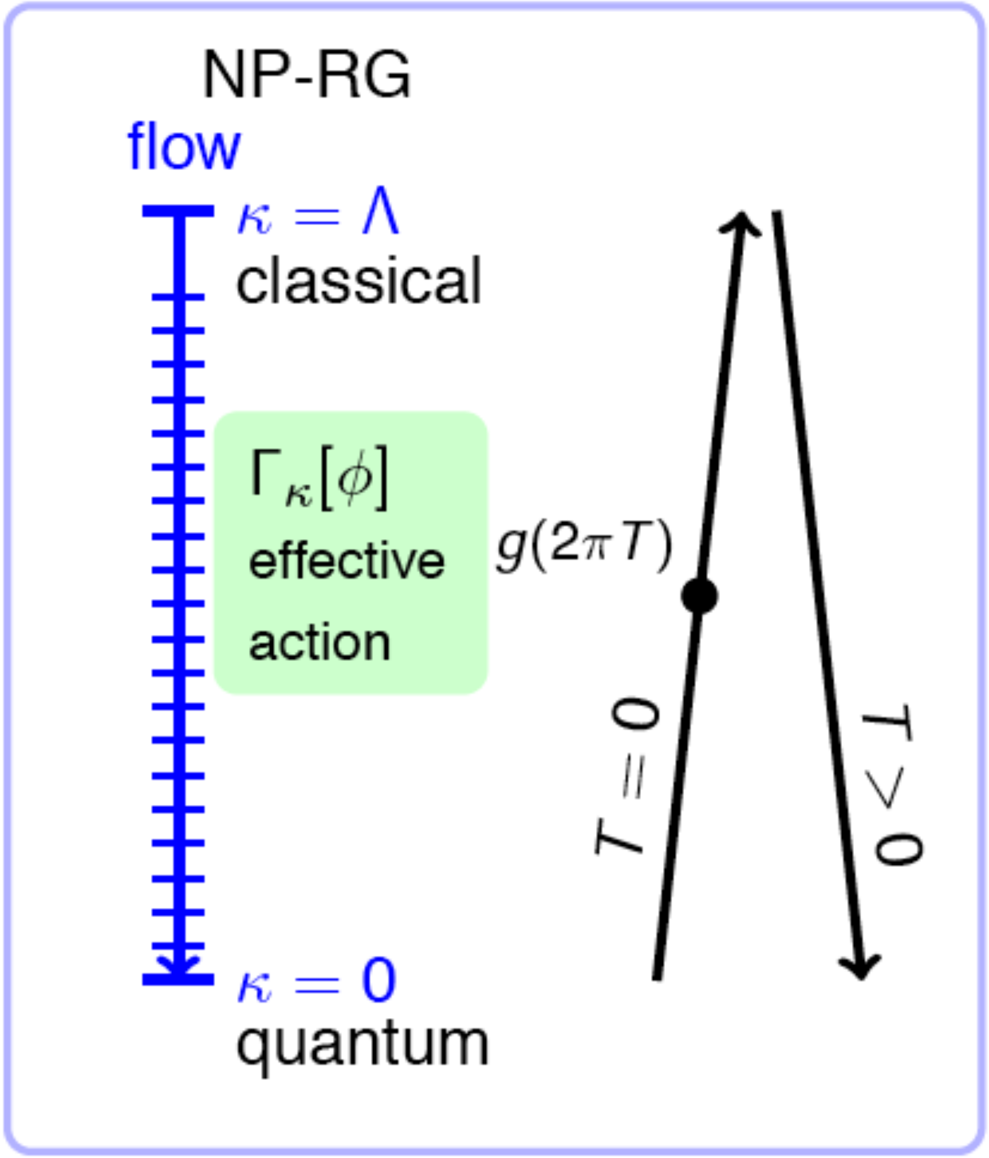
$$\kappa \sim T \qquad \gamma_\kappa \sim g^2$$

$$\kappa \sim gT \qquad \gamma_\kappa \sim g$$

$$\kappa \sim g^2 T \qquad \gamma_\kappa \sim 1$$

Non perturbative renormalization group





Courtesy, A. Ipp

Non perturbative renormalization group at finite temperature

(J.-P B, A. Ipp, N. Wschebor, 2010)

effective action

$$\partial_t \Gamma_\kappa[\phi] = \frac{1}{2} T \sum_n \int \frac{d^3\mathbf{q}}{(2\pi)^3} \partial_t R_\kappa(Q) G_\kappa(Q;\rho)$$ =

propagator $G_\kappa^{-1}(Q;\rho) \equiv \Gamma_\kappa^{(2)}(\mathbf{Q}, -\mathbf{Q};\rho) + R_\kappa(Q)$

regulator

four-momentum vectors:

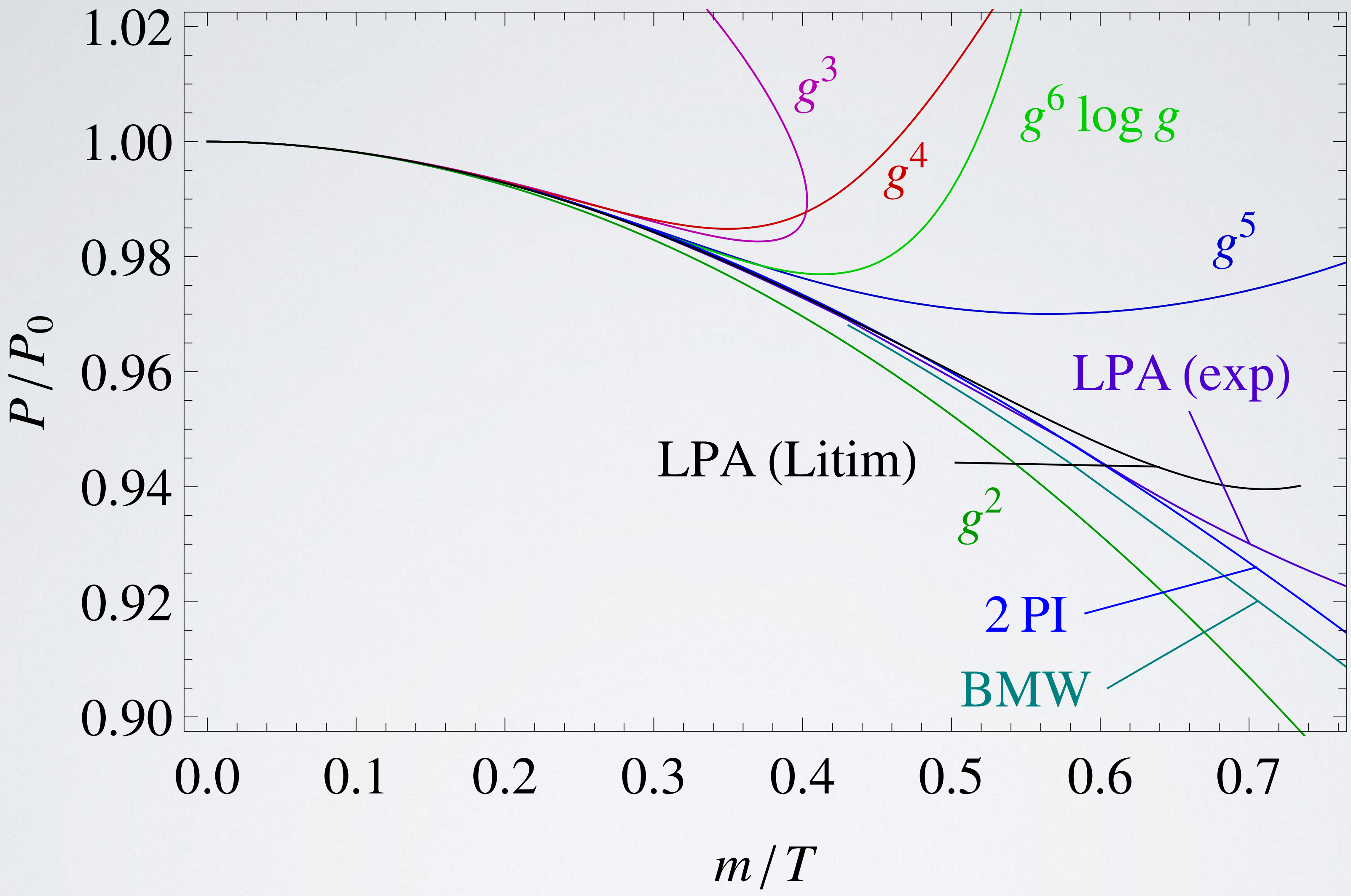
$$\mathbf{Q} = (\omega_n, \mathbf{q})$$

$$Q = |\mathbf{Q}|$$

Matsubara frequency $\omega_n = 2\pi n T$

momentum derivative: $\partial_t \equiv \kappa\, \partial_\kappa$

scalar field: $\rho \equiv \frac{1}{2}\phi^2$



JPB, A. Ipp, N. Wschebor, arXiv:1007.0991

JPB, A. Ipp, R. Mendez Galain, N. Wschebor, arXiv: hep-ph/0610004

$$\gamma_\kappa \sim \frac{g^2 \langle \varphi^2 \rangle_\kappa}{\kappa^2} \sim \frac{g^2 T}{\kappa}$$

The coupling grows with decreasing k

But !

- Dimensional reduction at high temperature

$$\kappa \frac{d\gamma_\kappa}{d\kappa} = -\gamma_\kappa + \frac{3}{16}\gamma_\kappa^2$$

- Dynamical generation of a thermal mass

$$m \sim gT$$

Massive, decoupling, scheme

$$S[\varphi] = \int_0^\beta d\tau \int d^d x \left\{ \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi + \frac{m_{\mathrm{B}}{}^2}{2}\varphi^2 + \frac{g_{\mathrm{B}}^2}{4!}\varphi^4 \right\}$$

Renormalization conditions (at finite temperature)

$$m^2 \qquad = \Gamma^{(2)}(\boldsymbol{p} = \boldsymbol{0}, \omega = 0, T)$$

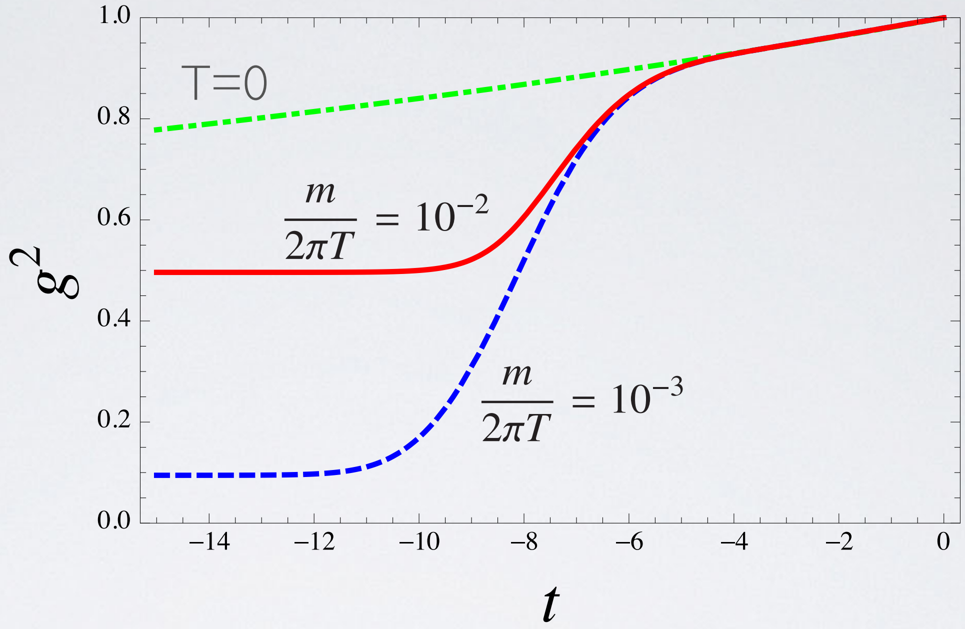
$$1 \qquad = \frac{d\Gamma^{(2)}}{d\boldsymbol{p}^2}(\boldsymbol{p}^2 = \mu^2, \omega = 0, T)$$

$$g^2 \qquad = \Gamma^{(4)}(\boldsymbol{p}_{sym}^2 = \mu^2, \omega_i = 0, T)$$

Issues

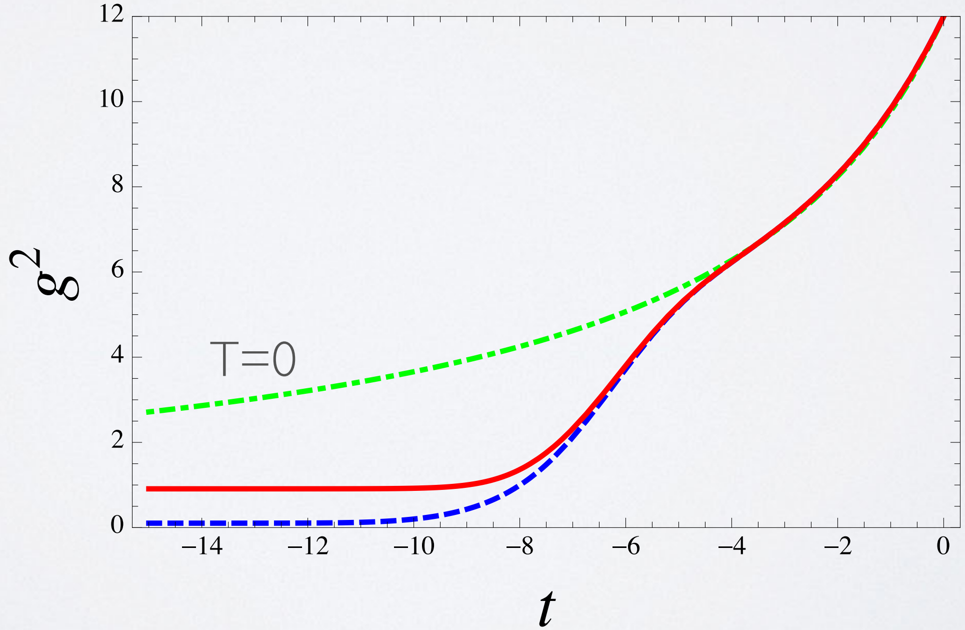
- Temperature dependent counterterms
- Fixing the parameters (m, g)

One-loop running in massive scheme



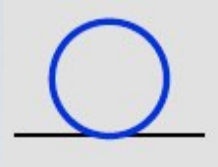
$$g^2 = 1$$

$$t = \ln(\mu/\Lambda)$$



$$g^2 = 12$$

Leading order calculation



$$\Gamma^{(2)}(\boldsymbol{p},\omega,T) = m^2 + \delta m^2 + \boldsymbol{p}^2 + \frac{g^2 T}{2}\sum_n \int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{1}{\omega_n^2 + \boldsymbol{q}^2 + m^2}$$

$$I(m) \equiv T\sum_n \int_{\boldsymbol{q}} \frac{1}{\omega_n^2 + \boldsymbol{q}^2 + m^2} = \int_{\boldsymbol{q}} \frac{1 + 2n_{\boldsymbol{q}}}{2E_{\boldsymbol{q}}} \equiv I_0(m) + I_T(m)$$

$$\Gamma^{(2)}(\boldsymbol{p}=0,\omega=0,T) = m^2 + \delta m^2 + \frac{g^2}{2} I(m)$$

The renormalization condition implies $\delta m^2 = -\frac{g^2}{2} I(m)$

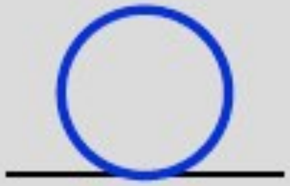
Relate thermal mass to zero temperature mass

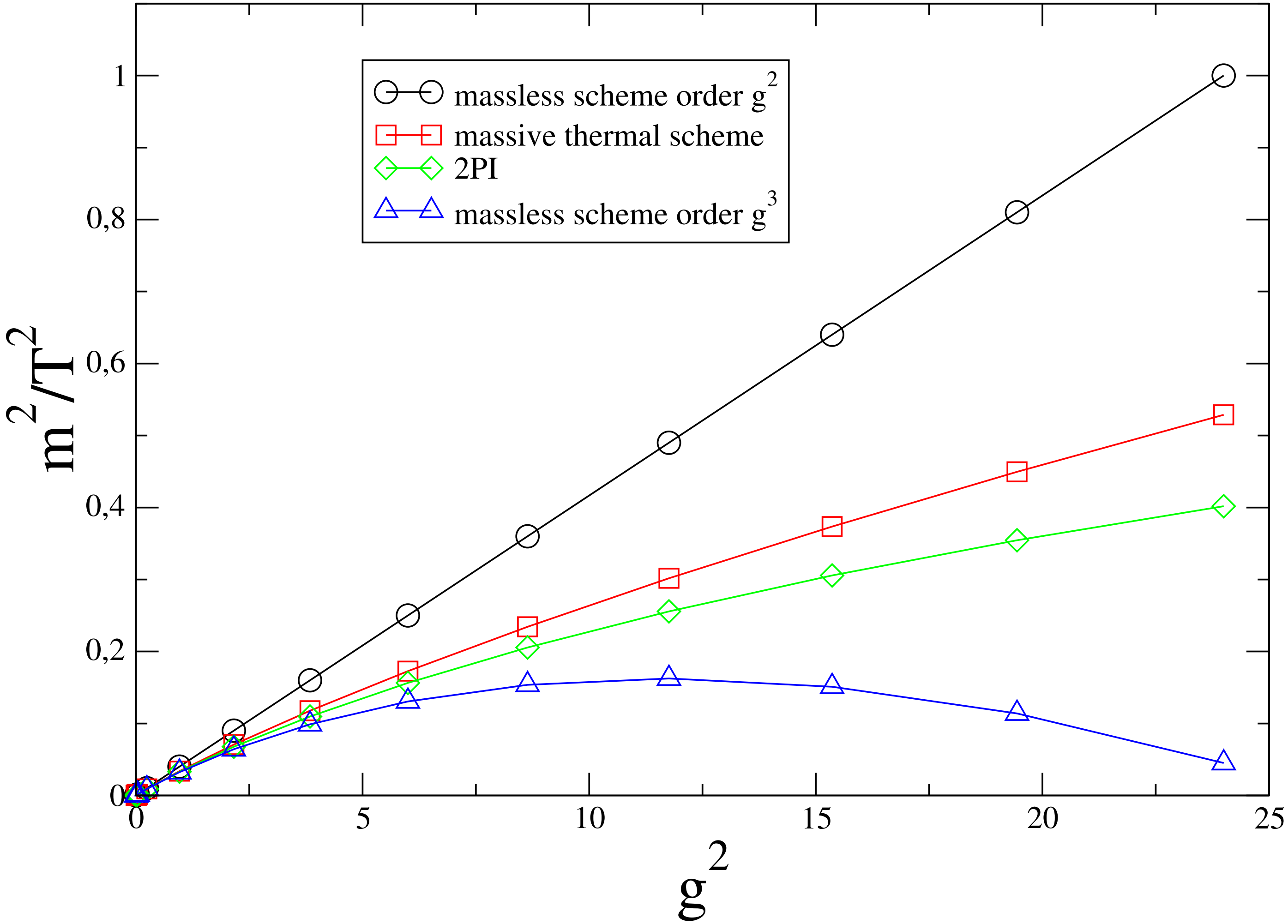
$$\Gamma^{(2)}(\boldsymbol{p}=0,\omega=0,T=0)=m^2+\delta m^2+\frac{g^2}{2}I_0(m)=m^2-\frac{g^2}{2}I_T(m)$$

Note: unusual calculation !

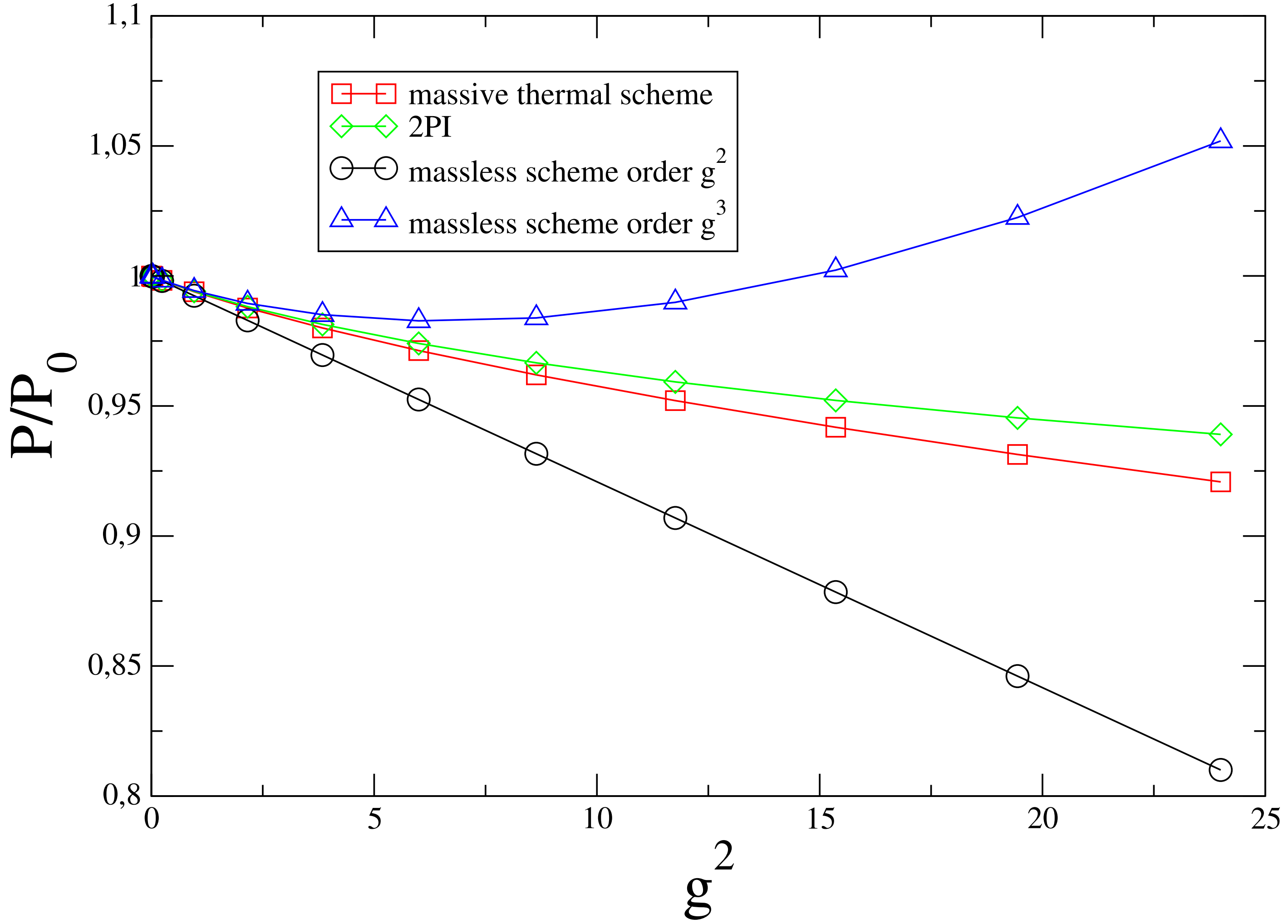
Self-consistent equation for the thermal mass

$$m_0^2=m^2-\frac{g^2}{2}I_T(m)$$

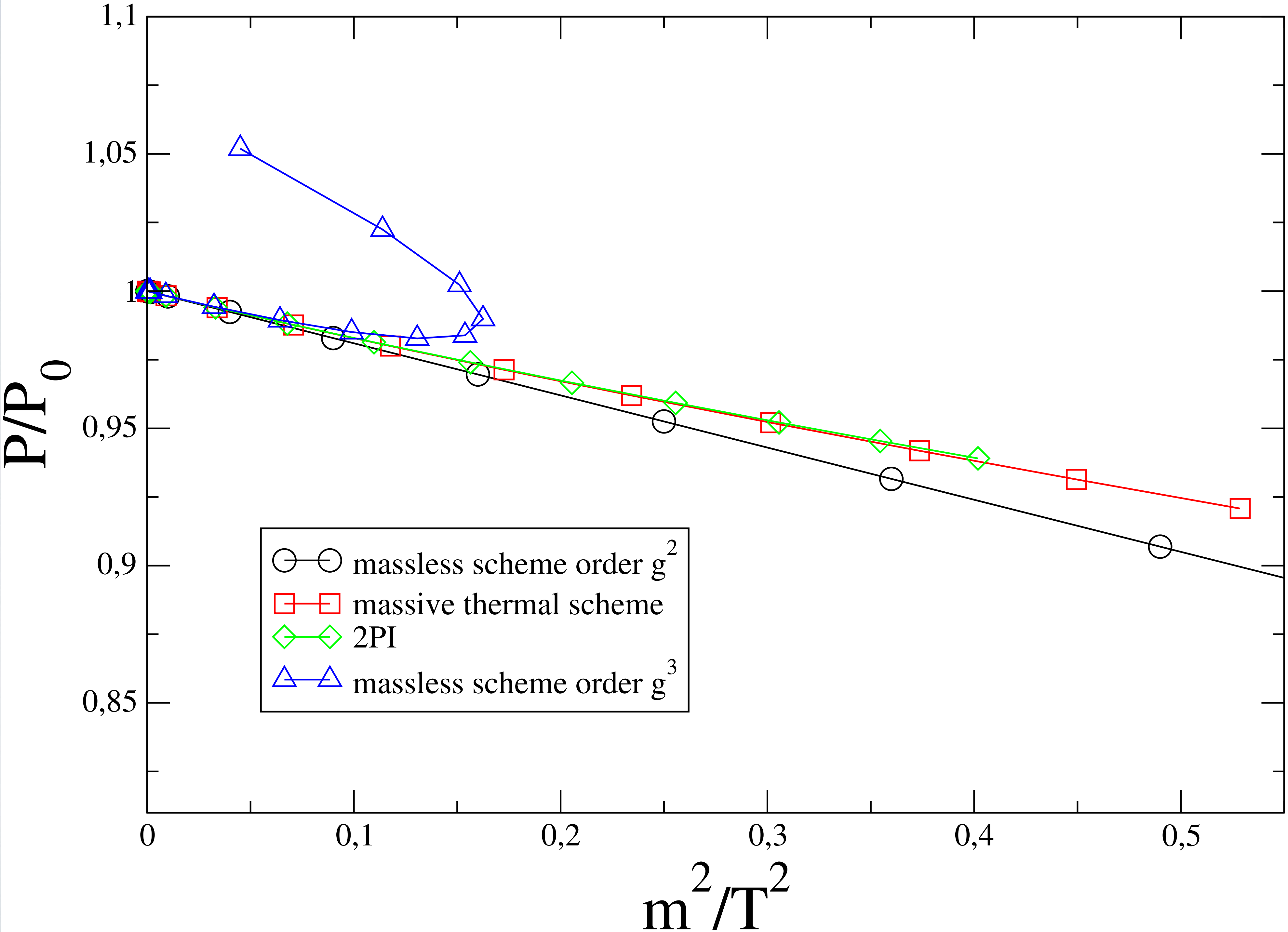




massless scheme order g^2
massive thermal scheme
2PI
massless scheme order g^3
m^2/T^2
g^2
0
0,2
0,4
0,6
0,8
1
0
5
10
15
20
25



massive thermal scheme
2PI
massless scheme order g^2
massless scheme order g^3
P/P_0
g^2
1,1
1,05
1
0,95
0,9
0,85
0,8
0
5
10
15
20
25


1,1
1,05
1
0,95
0,9
0,85
P/P0
0
0,1
0,2
0,3
0,4
0,5
m2/T2
massless scheme order g2
massive thermal scheme
2PI
massless scheme order g3

Summary

- An appropriate choice of renormalization scheme can greatly improve perturbation theory at finite temperature
- The proposed massive scheme leads to a well behaved perturbative expansion
- The idea of expanding around a massive theory is not new (screened perturbation theory, optimized perturbation theory, etc), but the present implementation is conceptually and technically simpler.
- Higher order corrections are being calculated.