Fano resonance through Higgs bound states in tunneling of Nambu-Goldstone modes

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Spontaneous symmetry breaking and collective modes

Higgs and NG modes are ubiquitous associated with spontaneous symmetry breaking.

- **Nambu-Goldstone mode** - massless phase mode
  - pions, magnons, phonons, Bogoliubov mode in BECs …

- **Higgs mode** - massive amplitude mode
  - in Standard Model, SCs, SFs, magnets, CDW materials …

- growing interest in Higgs modes in condensed matter physics
The ‘Higgs’ amplitude mode at the two-dimensional superfluid/Mott insulator transition

Cold bosons in an optical lattice - Higgs mode excited by modulating lattice depth in time

Observation of the Higgs mode in the vicinity of the SF-MI phase transition point in 2d
Superfluid-Mott insulator transition

Bose-Hubbard model

\[ \mathcal{H} = -J \sum_{i,j} b_i^\dagger b_j + \frac{U}{2} \sum_i b_i^\dagger b_i^\dagger b_i b_i - \mu \sum_i b_i^\dagger b_i \]

- \( J \): hopping
- \( U > 0 \): on-site interaction

\( J \gg U \)

\( J \ll U \)

\( J/U \) can be tuned by light intensity

Effective low-energy theory

- Deep in the SF regime $J \gg U$

\[ i \frac{\partial \phi}{\partial t} = -\frac{\nabla^2}{2m} \phi + g|\phi|^2 \phi \]

amplitude and phase modes have the same gapless dispersion

= NG (Bogoliubov) mode

No Higgs mode

C. Varma, JLTP (2002)

- In the vicinity of the 2nd order SF-MI transition

Time-dependent Ginzburg-Landau eq.

\[ iK_0 \frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2}{2m^*} + r_0 + u_0|\psi|^2 \right) \psi. \]

$\psi$: SF order parameter

Sachdev, “Quantum Phase Transitions”
Higgs and NG modes at the SF-MI transitions

TDGL eq. \( iK_0 \frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2}{2m_*} + r_0 + u_0|\psi|^2 \right) \psi. \)

- \( K_0 = 0 \) particle-hole symmetric
- TDGL eq. is invariant under \( \psi \leftrightarrow \psi^* \)

- Emergent Lorentz invariance

\[ \psi(\mathbf{x}, t) = \sqrt{-r_0/u_0 + \delta n(\mathbf{x}, t)} e^{i\delta \theta(\mathbf{x}, t)} \quad \psi_0 = \sqrt{-r_0/u_0} \]

linearize w.r.t \( \delta n(\mathbf{x}, t) \) : amplitude fluctuations \( \delta \theta(\mathbf{x}, t) \) : phase fluctuations

amplitude (Higgs) : \( \omega^2 = c^2 k^2 + \Delta^2 \)

phase (NG) : \( \omega^2 = c^2 k^2 \)

\[ \Delta \equiv \sqrt{-2r_0/W_0} \quad c \equiv (2m_* W_0)^{-1/2} \]

we consider 3D wherein long-lived Higgs exits
Effects of disorder

- Disorder plays a crucial role in various condensed matter systems - Anderson localization, Bose glass, ...
- Elementary excitations localized around disorder potentials - Andreev bound states in SCs, edge modes and Majorana bound states in topologically non-trivial systems (TIs, TSCs), ...
- We study transport properties of collective modes through a potential barrier - the simplest disorder.

Disorder in the BH model

- Bose-Hubbard model

\[ \mathcal{H} = - \sum_{i,j} J_{i,j} b_i^\dagger b_j + \frac{U}{2} \sum_i b_i^\dagger b_i^\dagger b_i b_i - \sum_i \mu_i b_i^\dagger b_i \]

- Two kinds of disorder in the BH model
  1) diagonal disorder: inhomogeneous on-site potential \( V_i \)

\[ \mu_i = \mu_0 - V_i \]


2) off-diagonal disorder: inhomogeneous hopping amplitude \( J_{ij} \)

\[ J_{ij} = J + J'_{ij} \]


- We propose to introduce two kinds of external potential that tune the two kinds of disorder independently in cold-atom experiments.
2) inhomogeneous hopping amplitude

Homogeneous lattice potential

Potential barrier

Combined potential

\[ V_{\text{ext}}(x) \]

modulation in lattice height

\[ J_{ij} = J + J'_{ij} \]

additional lattice potential with Gaussian profile

no change of on-site potential -> fixed \( \mu \)
2) inhomogeneous hopping amplitude

- Homogeneous lattice potential
- Potential barrier
- Combined potential

modulation in lattice height

\[ J_{ij} = J + J_{ij}' \]

- TDGL equation including effects of the potential
  - We assume absence of the 1st order time-derivative term far from the potential barrier: particle-hole symmetry

leading contribution to the linear term \( v_r(x) = -2J'(x) \)

\[ -W_0 \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2}{2m_*} + r_0 + v_r + u_0 |\psi|^2 \right) \psi \]

\( v_r(x) \) : a standard potential term that does not break p-h symmetry
1) inhomogeneous on-site potential

- Homogeneous lattice potential
  \[ V_{\text{opt}}(x) \]
- Potential barrier
  \[ V_{\text{bar}}(x) \]
- Combined potential
  \[ V_{\text{opt}}(x) + V_{\text{bar}}(x) \]

\[ \mu_i = \mu_0 \]

- Optical dipole potential
- Modulation of on-site potential

\[ \mu_i = \mu_0 - V_i \]

**TDGL equation including effects of the potential**

- Leading contribution to the 1st order time-derivative term
  \[ v_K(x) = -2W_0V(x) \]

\[
iv_K \frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2}{2m_*} + r_0 + u_0|\psi|^2 \right) \psi
\]

**\[ v_K(x) \]** breaks p-h symmetry!
Effective 1D setting

\[ iv_K(x) \frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2 \psi}{2m_*} + r_0 + v_r(x) + u_0 |\psi|^2 \right) \psi \]

\[ \tilde{\psi} = \psi / (-r_0/u_0)^{1/2}, \quad \tilde{t} = t (-r_0/W_0)^{1/2}, \]
\[ \tilde{v}_r = -v_r/r_0, \quad \tilde{v}_K = v_K / (-r_0 W_0)^{1/2} \]
\[ \tilde{x} = x/\xi. \]

\[ \xi \equiv (-m_* r_0)^{-1/2} : \text{coherence length} \]

we assume delta-function potentials:

\[ v_K = V_K \delta(x) \quad v_r = V_r \delta(x) \]

Potentials varying in the order of lattice spacing \( d \) can be well approximated by the delta-function potentials because of \( \xi \gg d \) near the phase boundary.
Linearized TDGL equation

\[ i v_K \frac{\partial \psi}{\partial t} - \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2}{2} - 1 + v_r + |\psi|^2 \right) \psi \]

we assume fluctuations of the order parameter only in the x direction

\[ \psi(x, t) = \psi_0(x) + U(x)e^{-i\omega t} + V^*(x)e^{i\omega^* t} \]

Static eq. (same as the static GP eq.)

\[ \left( -\frac{1}{2} \frac{d^2}{dx^2} - 1 + |\psi_0|^2 + v_r(x) \right) \psi_0(x) = 0 \]

\[ S(x) = U(x) - V(x) \propto \delta \theta(x), \quad T(x) = U(x) + V(x) \propto \delta n(x) \]

**NG:**

\[ \left( -\frac{1}{2} \frac{d^2}{dx^2} - 1 + |\psi_0|^2 + v_r(x) \right) S(x) = \omega^2 S(x) - \omega v_K(x) T(x) \]

**Higgs:**

\[ \left( -\frac{1}{2} \frac{d^2}{dx^2} - 1 + 3|\psi_0|^2 + v_r(x) \right) T(x) = \omega^2 T(x) - \omega v_K(x) S(x) \]

Higgs and NG modes are locally coupled via \( v_K(x) \)
Higgs bound states

Static solution for \( v_r(x) = V_r \delta(x) \)

\[ \psi_0(x) = \tanh(|x| + x_0) \]

we set \( v_K(x) = 0 \)

\[
\begin{align*}
\left(-\frac{1}{2} \frac{d^2}{dx^2} - 1 + |\psi_0|^2 + v_r(x)\right) S(x) &= \omega^2 S(x) - \omega v_K(x) T(x) \\
\left(-\frac{1}{2} \frac{d^2}{dx^2} - 1 + 3|\psi_0|^2 + v_r(x)\right) T(x) &= \omega^2 T(x) - \omega v_K(x) S(x)
\end{align*}
\]

= Schrödinger eq.

decoupled Higgs and NG mode due to p-h symmetry

Bound-state solutions of amplitude fluctuation \( T(x) \) **below**
the bulk Higgs gap due to the deep condensate potential
Higgs bound states

\[
\left( -\frac{1}{2} \frac{d^2}{dx^2} - 1 + 3|\psi_0|^2 + V_r \delta(x) \right) T(x) = \omega^2 T(x)
\]

\[\psi_0(x) = \tanh(|x| + x_0)\]

\[V_r = 1.5\]

\[\Delta : \text{bulk Higgs gap}\]
Scattering of collective modes

Scattering of NG modes with energy below the bulk Higgs gap $E < \Delta = \sqrt{2}$ incident to the potential barriers

$$v_K = V_K \delta(x) \quad v_r = V_r \delta(x)$$

NG: $S(x) = \begin{cases} 
(\gamma(x) + i k_s) e^{i k_s x} + r_{ng} (\gamma(x) - i k_s) e^{-i k_s x} & (x < 0) \\
\text{Incident} & \\
t_{ng} (\gamma(x) - i k_s) e^{i k_s x} & (x > 0) \\
\text{Reflected} & \\
\gamma(x) = \tanh(|x| + x_0) & k_s = \sqrt{2E} \\
\text{Transmitted} & \\
\end{cases}$

Higgs: $T(x) = \begin{cases} 
\begin{align*}
 r_h (3 \gamma(x)^2 + 3 \kappa_t \gamma(x) + \kappa_t^2 - 1) e^{\kappa_t x} & (x < 0) \\
t_h (3 \gamma(x)^2 + 3 \kappa_t \gamma(x) + \kappa_t^2 - 1) e^{-\kappa_t x} & (x > 0)
\end{align*} \\
\text{decay at infinity} & \\
\kappa_t = \sqrt{4 - 2E^2} & \\
\text{Boundary condition at x=0} &
\end{cases}$
Tunneling property of NG modes

Transmission probability:

\[ T(E) = \frac{1}{1 + \frac{2E^2}{(2E^2+1)^2} V_{\text{eff}}(E)^2} \quad E < \Delta \]

Effective potential:

\[ V_{\text{eff}}(E') = (1 - V_K^2 f(E)) V_r \]

- Perfect transmission of NG mode in the low energy limit anomalous tunneling


- Characteristic asymmetric peak near the energy of the Higgs bound state \( E_+ \)


\[ (V_r, V_K) = (4, 4) \]

\[ (V_r, V_K) = (4, 2) \]

Fano resonance
Tunneling property of NG modes

Transmission probability:
\[ T(E) = \frac{1}{1 + \frac{2E^2}{(2E^2 + 1)^2} V_{\text{eff}}(E)^2} \]

Effective potential:
\[ V_{\text{eff}}(E) = (1 - \frac{V_K^2}{E} f(E)) V_r \approx V_r - \frac{\alpha V_K^2}{E - E_+} V_r \quad E \approx E_+ \]

- Direct scattering
- Resonant scattering involving excitation of the even Higgs bound state

\[ f(E \rightarrow E_+) \rightarrow \infty \quad \Rightarrow \quad V_{\text{eff}} \rightarrow \infty \quad \Rightarrow \quad T \rightarrow 0 \]

\[ 1 - V_K^2 f(E) = 0 \quad \Rightarrow \quad V_{\text{eff}} = 0 \quad \Rightarrow \quad T = 1 \]

Destructive interference

- Interference between directly scattered waves within continuum and resonantly scattered waves mediated by discrete states
- Fano resonance
The asymmetric peak is manifestation of the Fano resonance of the NG mode mediated by the even Higgs bound state.

\[
V_{\text{eff}}(E) = (1 - V_K^2 f(E)) V_r
\]

\[
T(E) = \frac{1}{1 + \frac{2E^2}{(2E^2 + 1)^2} V_{\text{eff}}(E)^2}
\]

\[
f(E \to E_+) \to \infty \quad \Rightarrow \quad V_{\text{eff}} \to \infty \quad \Rightarrow \quad T \to 0
\]

\[
1 - V_K^2 f(E) = 0 \quad \Rightarrow \quad V_{\text{eff}} = 0 \quad \Rightarrow \quad T = 1
\]
Summary

• TDGL eq. including effects of two kinds of potential barriers - inhomogeneous on-site potential and hopping amplitude.

\[ i\nu K(x) \frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left( -\frac{\nabla^2 \psi}{2m_*} + r_0 + v_r(x) + u_0|\psi|^2 \right) \psi \]

• Localized Higgs bound states below the bulk Higgs gap

• Fano resonance of NG mode mediated by the Higgs bound states

Outlook  Higgs bound states in other condensed matter systems, e.g. disordered SCs

Higgs modes in condensed matter physics

- Raman spectroscopy in NbSe$_2$
  - First observation of Higgs mode!

NbSe$_2$: CDW transition at 40K and SC transition at 7.2 K

- A new peak at the frequency twice of the SC gap arises below SC $T_c$. 
- Littlewood and Varma developed a microscopic theory by extending the BCS-Nambu theory. They found the peak due to amplitude oscillations of superconducting gap - Higgs mode.


Higgs gap (mass) = $2\Delta$ in BCS (s-wave)

Sooryakumar and Klein, PRL 45, 660 (1980)
Quantum Magnets under Pressure: Controlling Elementary Excitations in TiCuCl$_3$

\[ \mathcal{H} = \sum_i J(p)S_{i,l}S_{i,r} + \sum_{ij,m,m'} J_{ij}(p)S_{i,m}S_{j,m'} \]

- **Triplon** (broken valence bond excitations)
- **Longitudinal mode** (Higgs)
- **Spin wave** (NG)
- **Neel paramagnet**
- **Triplon**
- **Spin wave condensation**
- **Triplon** (broken valence bond excitations)
- **Higgs + NG mode**