Fano resonance through Higgs bound states in tunneling of Nambu-Goldstone modes

Shunji Tsuchiya (Tohoku Institute of Technology)

Collaborators: Takeru Nakayama (ISSP, Tokyo Univ.)
Tetsuro Nikuni (Tokyo Univ. of Science)
Ippei Danshita (Yukawa Institute, Kyoto Univ.)

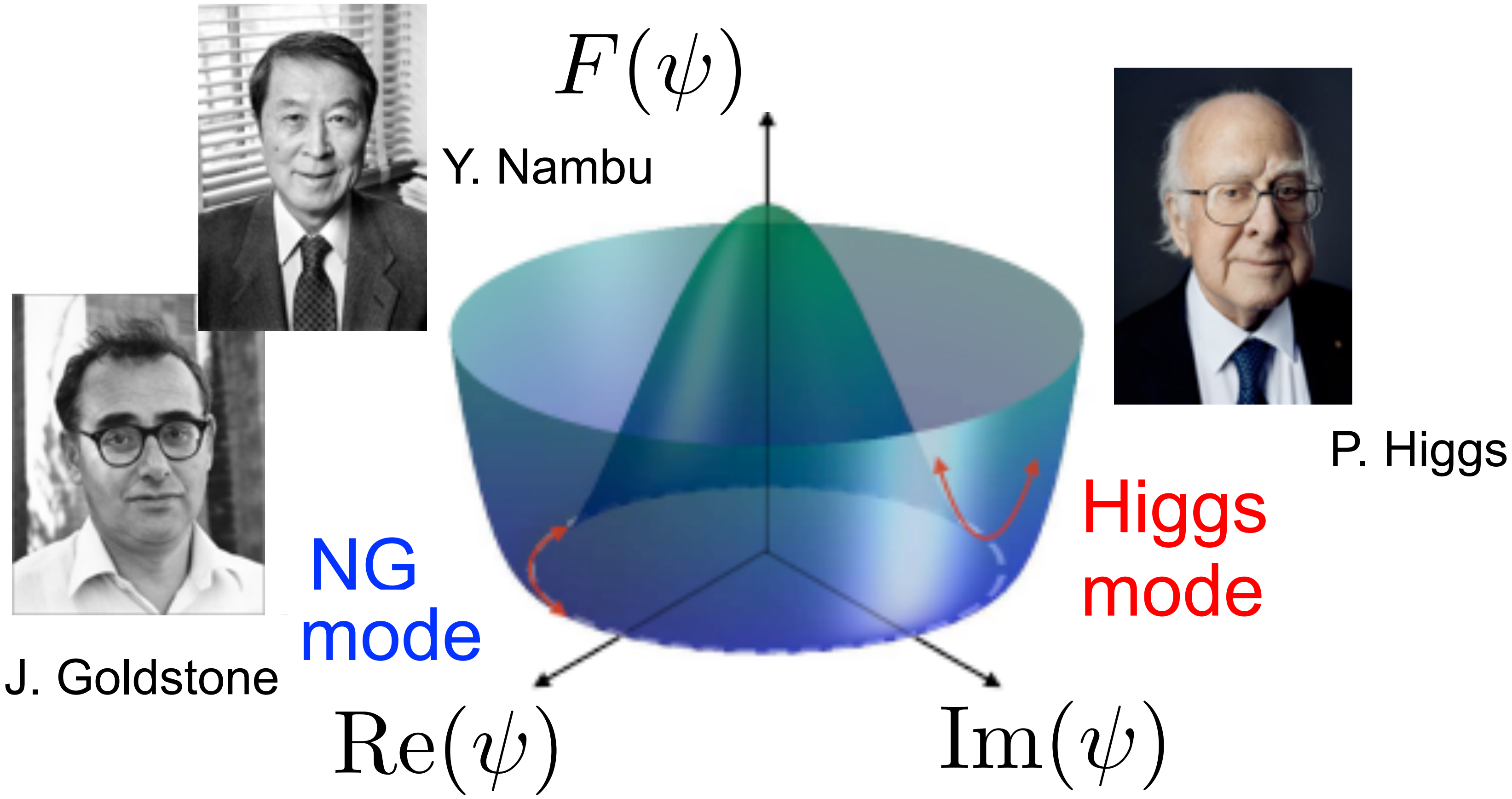
arXiv: 1503.01516

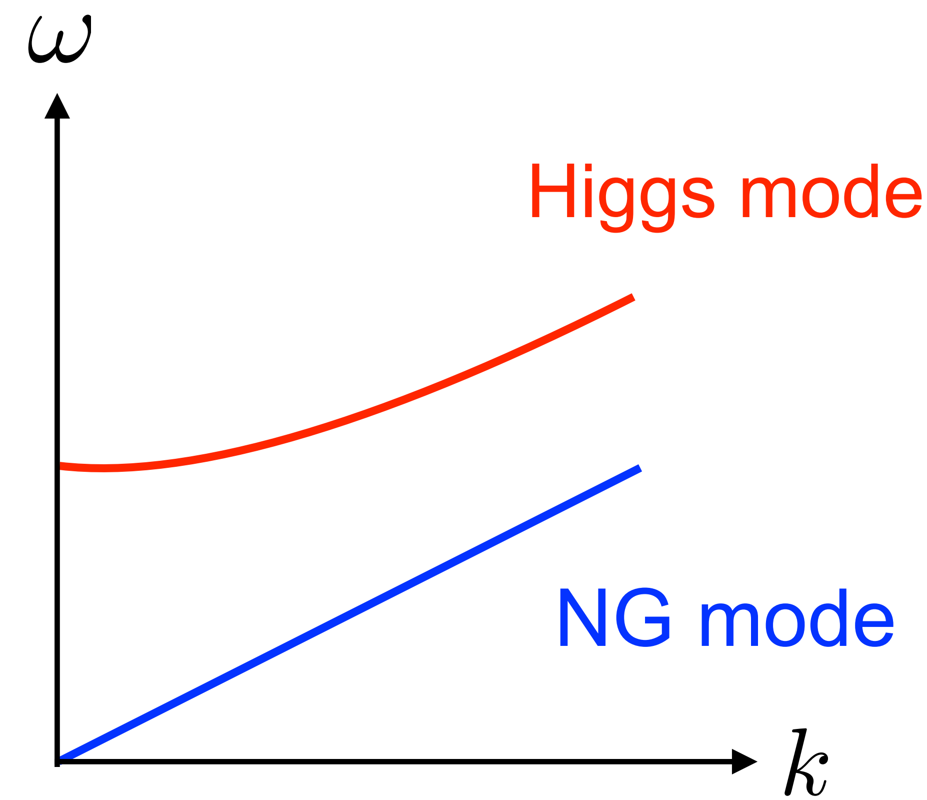
基研研究会「熱場の量子論とその応用」

@YITP, 京都

Sep. 1, 2015

Spontaneous symmetry breaking and collective modes





Higgs and NG modes are ubiquitous associated with spontaneous symmetry breaking.

- Nambu-Goldstone mode - massless phase mode

  pions, magnons, phonons, Bogoliubov mode in BECs …

- Higgs mode - massive amplitude mode

  in Standard Model, SCs, SFs, magnets, CDW materials …

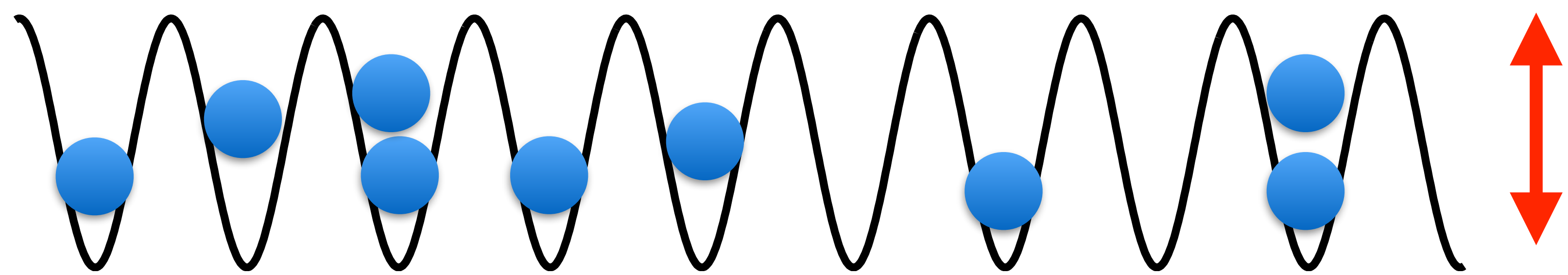
\- growing interest in Higgs modes in condensed matter physics

The 'Higgs' amplitude mode at the two-dimensional superfluid / Mott insulator transition

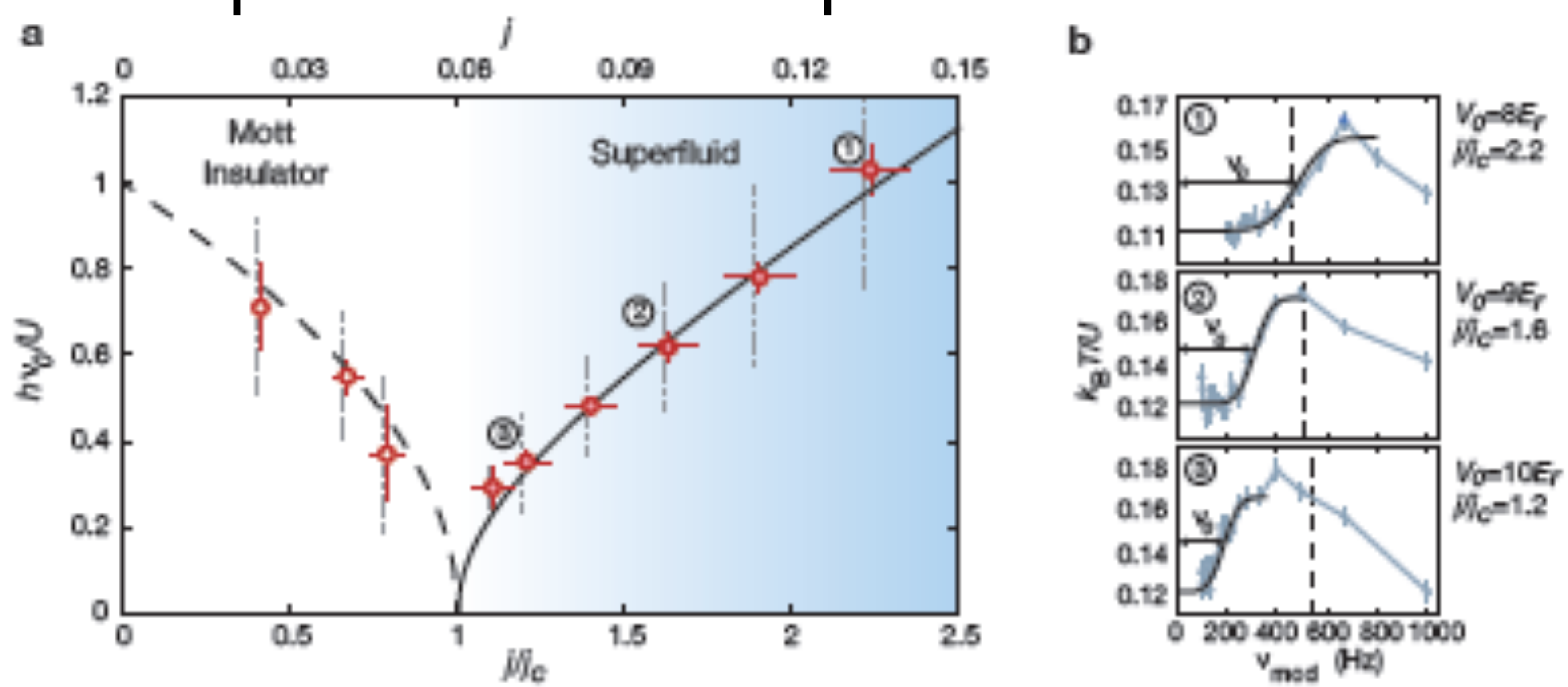
Manuel Endres[1], Takeshi Fukuhara[1], David Pekker[2], Marc Cheneau[1], Peter Schauβ[1], Christian Gross[1], Eugene Demler[3], Stefan Kuhr[1,4] & Immanuel Bloch[1,5]

Nature 487, 454 (2012)

- Cold bosons in an optical lattice - Higgs mode excited by modulating lattice depth in time



- Observation of the Higgs mode in the vicinity of the SF-MI phase transition point in 2d

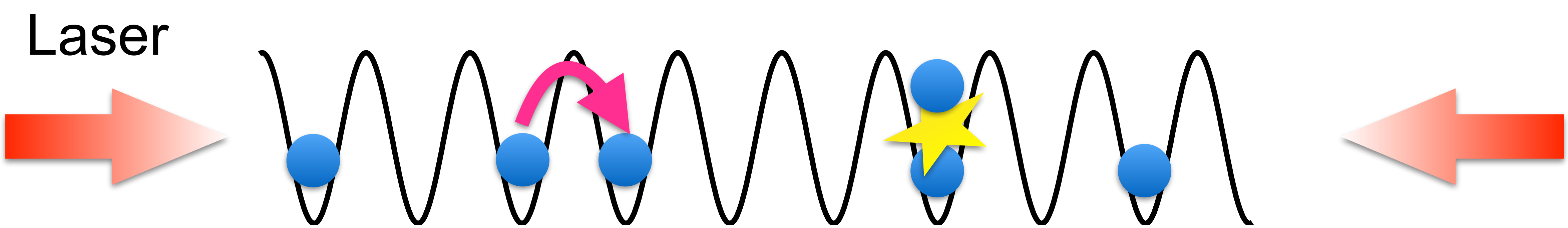


Superfluid-Mott insulator transition

- Bose-Hubbard model

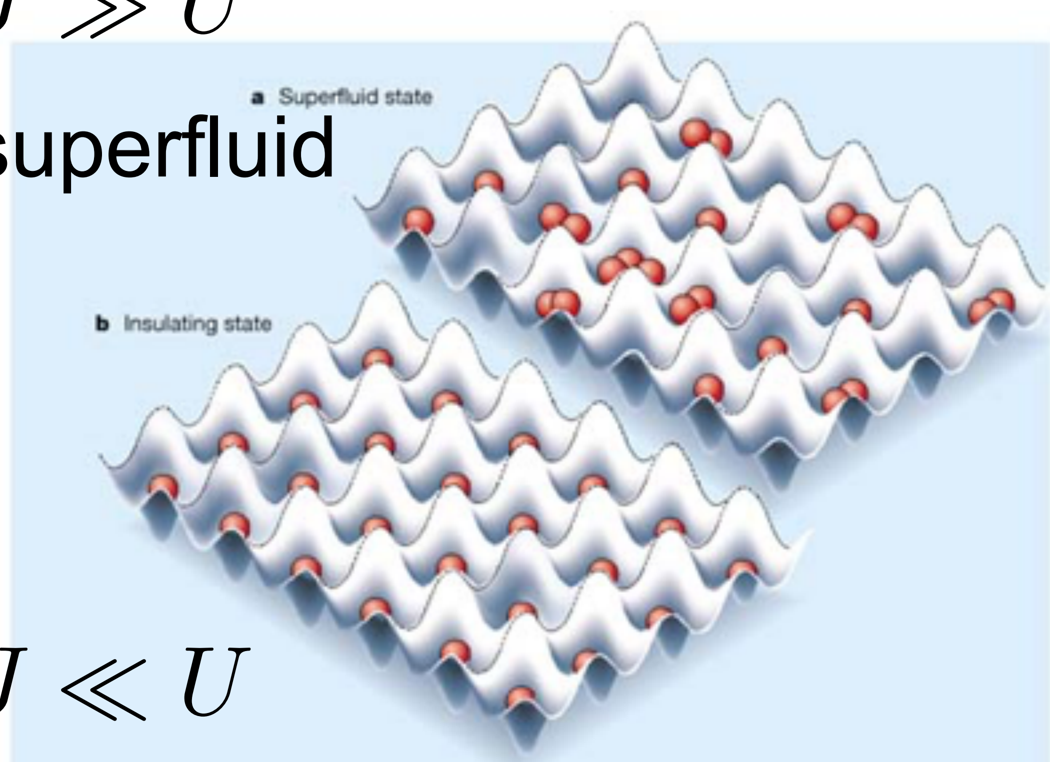
$$\mathcal{H} = -J \sum_{i,j} b_i^\dagger b_j + \frac{U}{2} \sum_i b_i^\dagger b_i^\dagger b_i b_i - \mu \sum_i b_i^\dagger b_i$$

J : hopping $\quad U > 0$: on-site interaction



$J \gg U$

superfluid



$J \ll U$

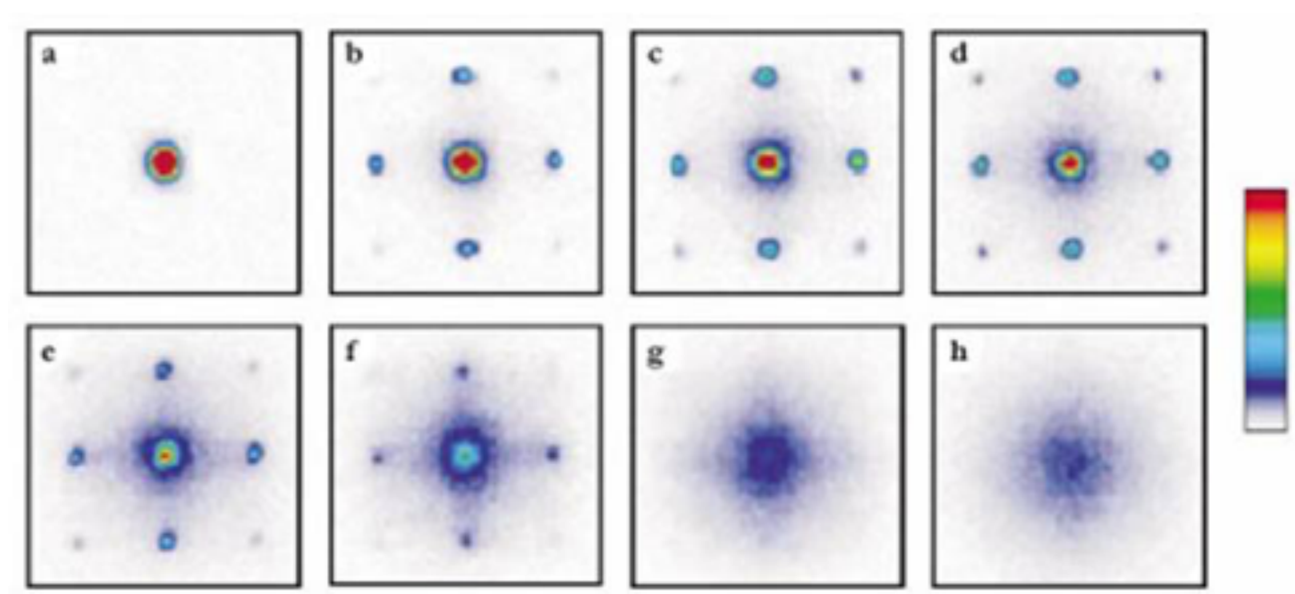
Mott insulator

J/U can be tuned by light intensity

SF

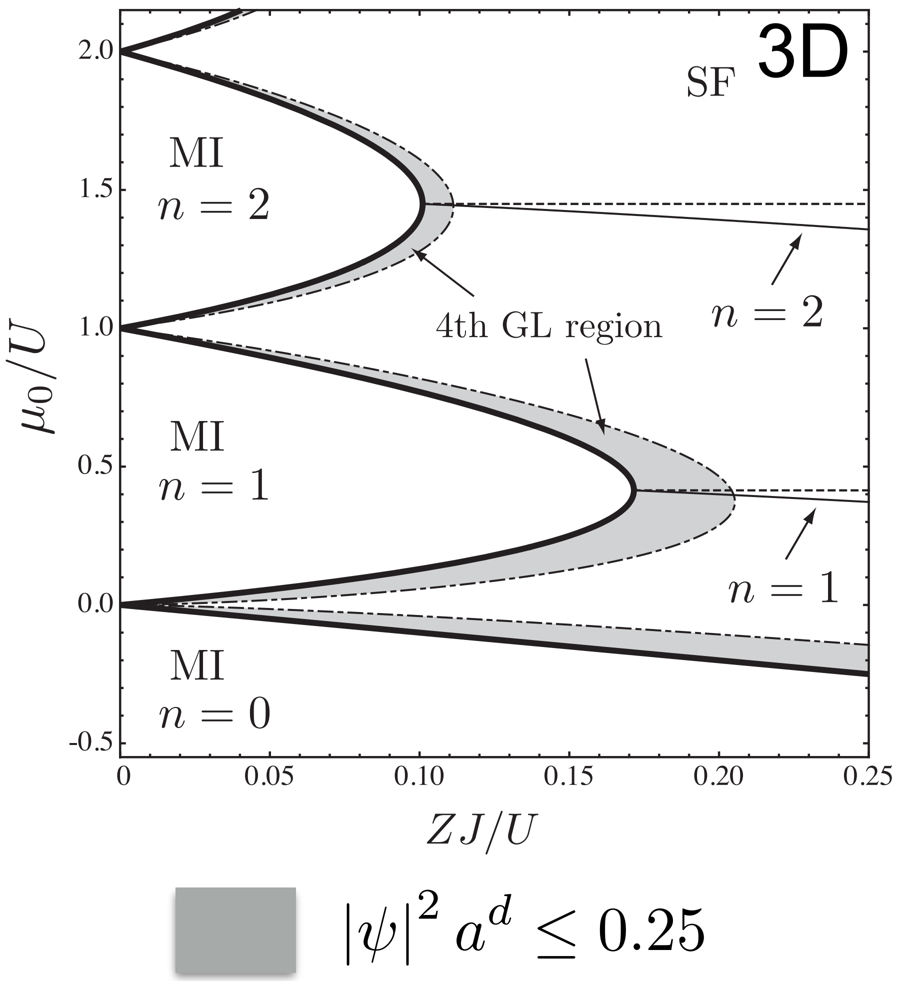
MI

^{87}Rb



M.Greiner et al., Nature 415, 39 (2002)

Effective low-energy theory



- Deep in the SF regime $J \gg U$

Gross-Pitaevskii eq.

$$\boxed{i\frac{\partial \phi}{\partial t} = -\frac{\nabla^2}{2m}\phi + g|\phi|^2\phi} \qquad \langle b_i \rangle = \phi$$

amplitude and phase modes have the same gapless dispersion

= NG (Bogoliubov) mode

No Higgs mode C. Varma, JLTP (2002)

- In the vicinity of the 2nd order SF-MI transition

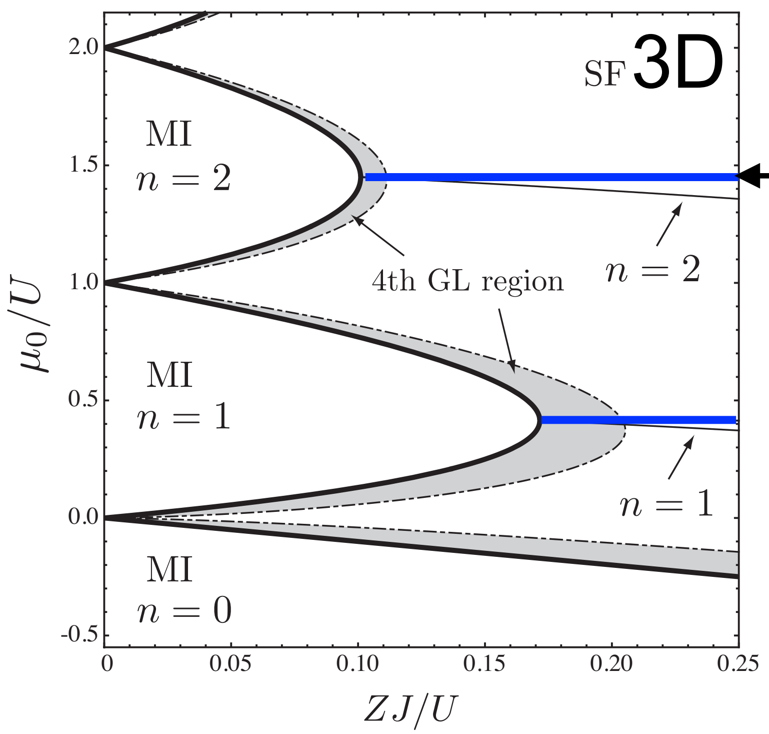
Time-dependent Ginzburg-Landau eq.

Sachdev, “Quantum Phase Transitions”

$$\boxed{iK_0\frac{\partial \psi}{\partial t} - W_0\frac{\partial^2 \psi}{\partial t^2} = \left(-\frac{\nabla^2}{2m_*} + r_0 + u_0|\psi|^2\right)\psi.}$$

ψ : SF order parameter

Higgs and NG modes at the SF-MI transitions



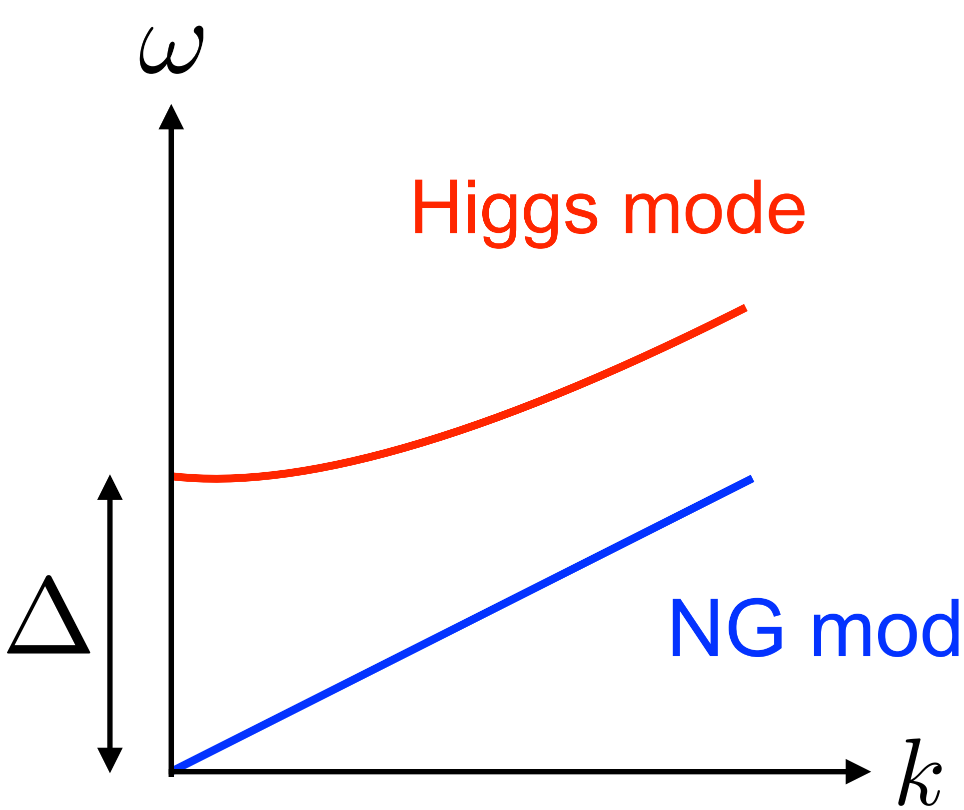
TDGL eq. $$\cancel{iK_0\frac{\partial\psi}{\partial t}} - W_0\frac{\partial^2\psi}{\partial t^2} = \left(-\frac{\nabla^2}{2m_*} + r_0 + u_0|\psi|^2\right)\psi.$$

$K_0 = 0$ particle-hole symmetric

TDGL eq. is invariant under $\psi \leftrightarrow \psi^*$

- Emergent Lorentz invariance

$$\psi(\boldsymbol{x},t) = \sqrt{-r_0/u_0 + \delta n(\boldsymbol{x},t)}\, e^{i\delta\theta(\boldsymbol{x},t)} \qquad \psi_0 = \sqrt{-r_0/u_0}$$



linearize w.r.t $\delta n(\boldsymbol{x},t)$: amplitude fluctuations
$\delta\theta(\boldsymbol{x},t)$: phase fluctuations

amplitude (Higgs) : $\omega^2 = c^2k^2 + \Delta^2$

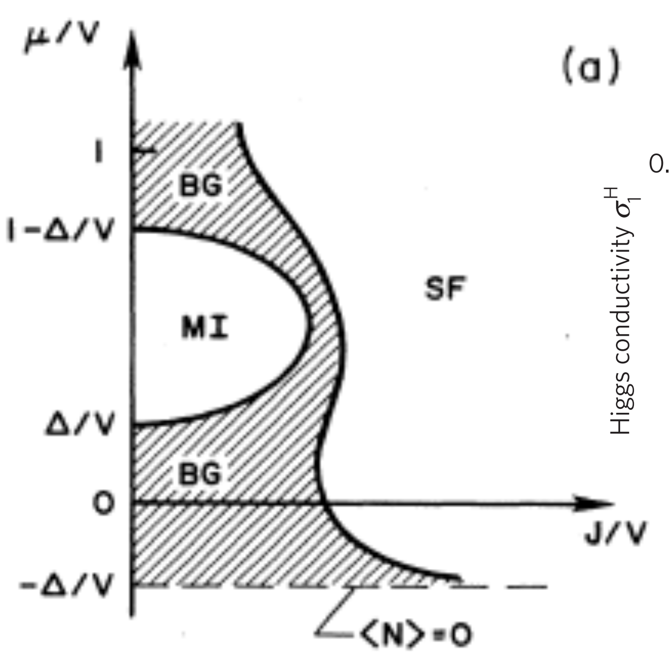
phase (NG) : $\omega^2 = c^2k^2$

$\Delta \equiv \sqrt{-2r_0/W_0} \qquad c \equiv (2m_* W_0)^{-1/2}$

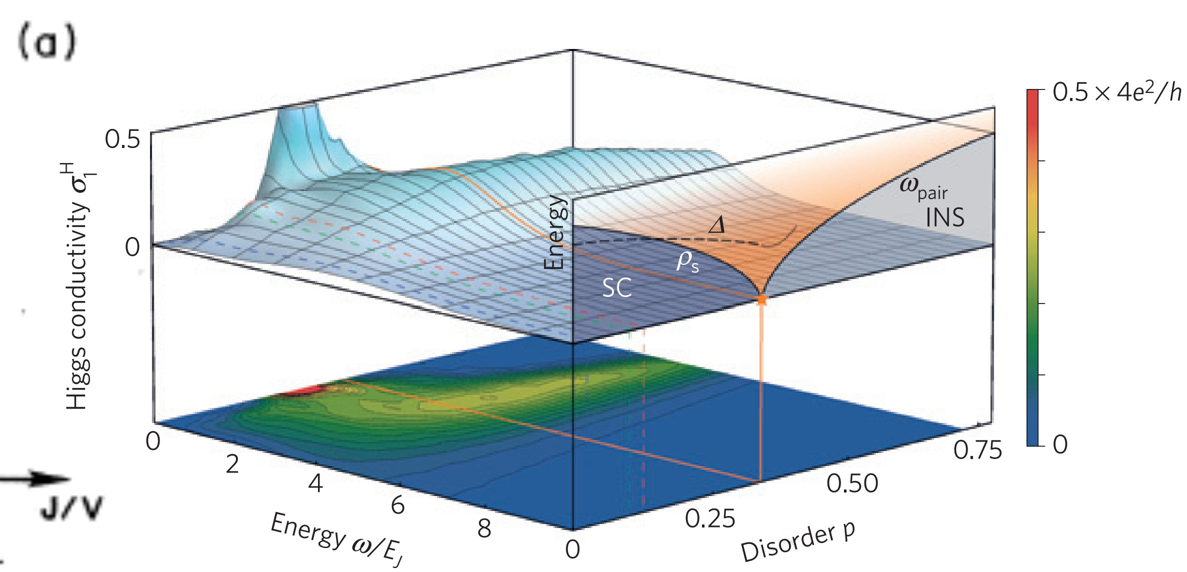
we cosider 3D wherein long-lived Higgs exits

Effects of disorder

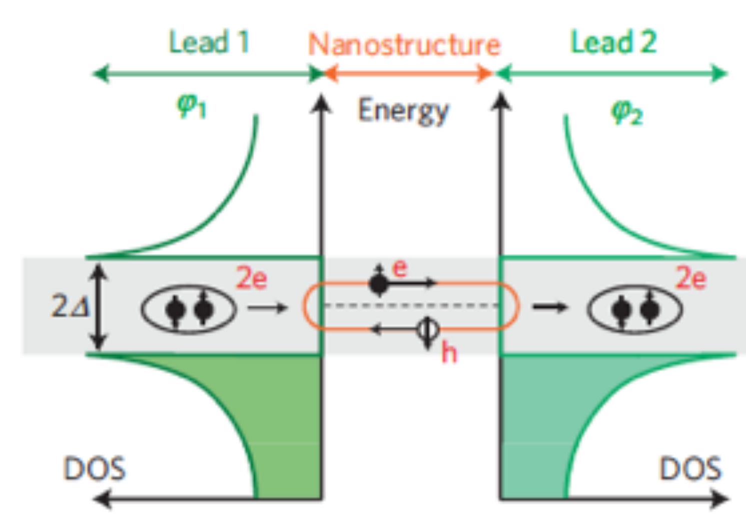
- Disorder plays a crucial role in various condensed matter systems - Anderson localization, Bose glass, ..
- Elementary excitations localized around disorder potentials - Andreev bound states in SCs, edge modes and Majorana bound states in topologically non-trivial systems (TIs, TSCs), …
- We study transport properties of collective modes through a potential barrier - the simplest disorder.



M. P. A. Fisher, et al., PRB (1989)



D. Sherman, *et al.*, Nat. Phys. (2015)



J-D. Pillet, *et al.*, Nat. Phys. (2010)

Disorder in the BH model

- Bose-Hubbard model

$$\mathcal{H} = -\sum_{\boldsymbol{i},\boldsymbol{j}} J_{\boldsymbol{i},\boldsymbol{j}} b_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{j}} + \frac{U}{2} \sum_{\boldsymbol{i}} b_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{i}} b_{\boldsymbol{i}} - \sum_{\boldsymbol{i}} \mu_{\boldsymbol{i}} b_{\boldsymbol{i}}^{\dagger} b_{\boldsymbol{i}}$$

- Two kinds of disorder in the BH model

1) diagonal disorder: inhomogeneous on-site potential V_i

$$\mu_i = \mu_0 - V_i$$

M. P. A. Fisher, PRB 40, 546 (1989)

2) off-diagonal disorder: inhomogeneous hopping amplitude J_{ij}

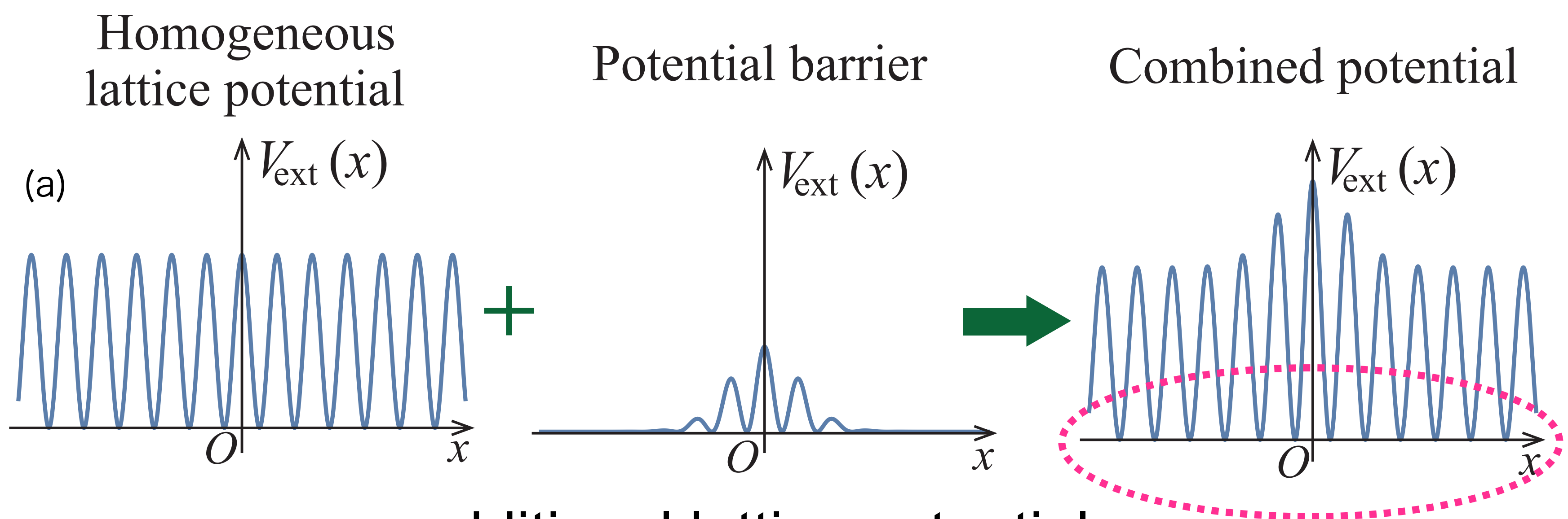
$$J_{ij} = J + J'_{ij}$$

N. Prokof'ev and B. Svistunov, PRL 92, 15703 (2004)

P. Sengupta and S. Haas, PRL 99, 050403 (2007)

- We propose to introduce two kinds of external potential that tune the two kinds of disorder independently in cold-atom experiments.

2) inhomogeneous hopping amplitude



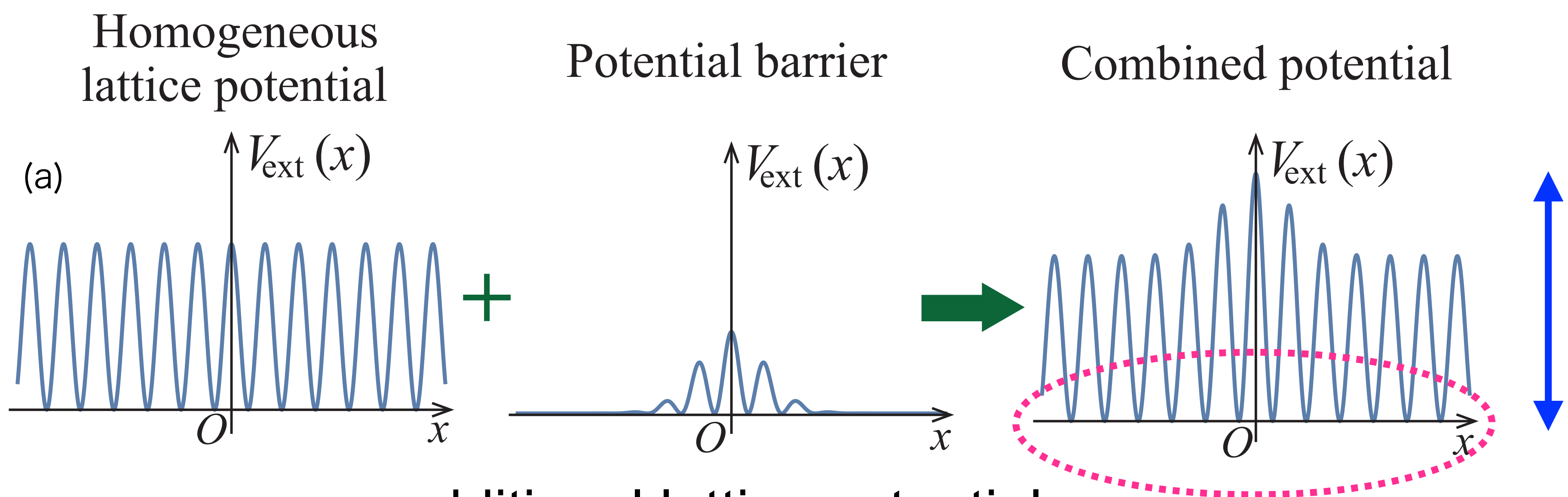
modulation in lattice height

$$J_{ij} = J + J'_{ij}$$

additional lattice potential with Gaussian profile

no change of on-site potential -> fixed μ

2) inhomogeneous hopping amplitude



modulation in lattice height

$J_{ij} = J + J'_{ij}$

additional lattice potential with Gaussian profile

no change of on-site potential -> fixed μ

■ TDGL equation including effects of the potential

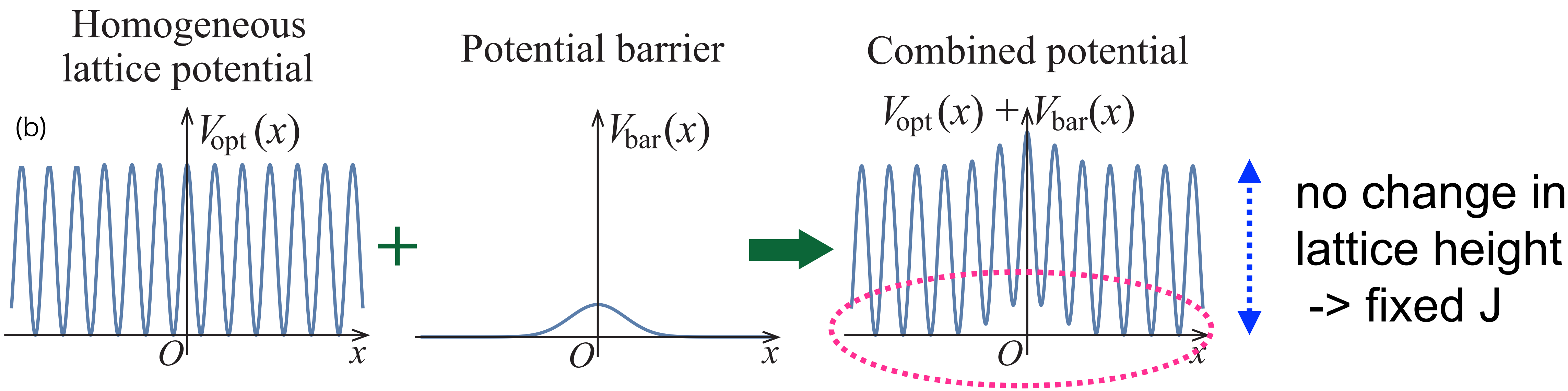
- We assume absence of the 1st order time-derivative term far from the potential barrier: particle-hole symmetry

leading contribution to the linear term $v_r(x) = -2J'(x)$

$$\boxed{-W_0 \frac{\partial^2 \psi}{\partial t^2} = \left(-\frac{\nabla^2}{2m_*} + r_0 + v_r + u_0 |\psi|^2 \right) \psi}$$

$v_r(x)$: a standard potential term that does not break p-h symmetry

1) inhomogeneous on-site potential



$\mu_i = \mu_0$

optical dipole potential

modulation of on-site potential

$$\boxed{\mu_i = \mu_0 - V_i}$$

■ TDGL equation including effects of the potential

leading contribution to the 1st order time-derivative term $v_K(x) = -2W_0 V(x)$

$$\boxed{i v_K \frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left(-\frac{\nabla^2}{2m_*} + r_0 + u_0 |\psi|^2 \right) \psi}$$

$v_K(x)$ breaks p-h symmetry!

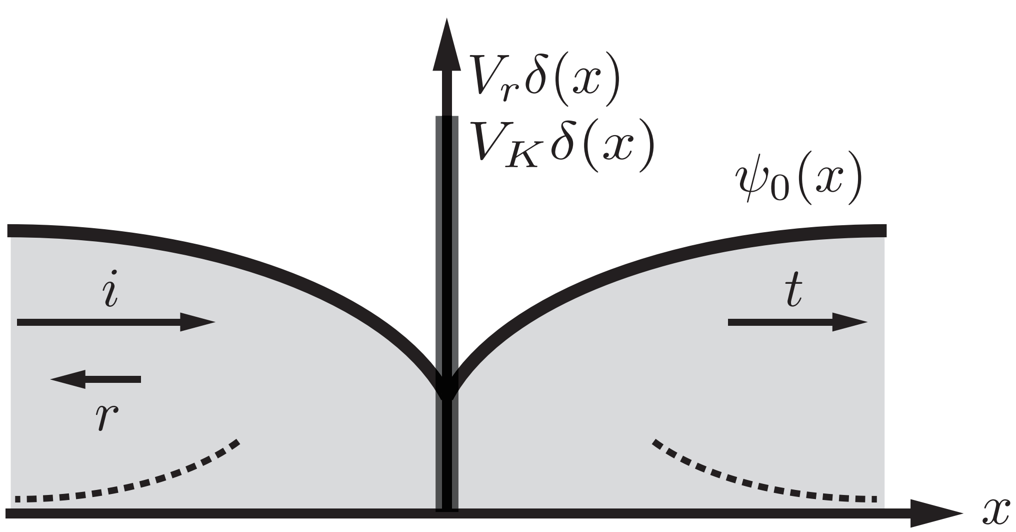
Effective 1D setting

$$iv_K(x)\frac{\partial\psi}{\partial t} - W_0\frac{\partial^2\psi}{\partial t^2} = \left(-\frac{\nabla^2\psi}{2m_*} + r_0 + v_r(x) + u_0|\psi|^2\right)\psi$$

$$\tilde{\psi} = \psi/(-r_0/u_0)^{1/2},\ \tilde{t} = t(-r_0/W_0)^{1/2},$$
$$\tilde{v}_r = -v_r/r_0,\ \tilde{v}_K = v_K/(-r_0W_0)^{1/2}\ \tilde{x} = x/\xi.$$

$\xi \equiv (-m_* r_0)^{-1/2}$: coherence length

$$iv_K(x)\frac{\partial\psi}{\partial t} - \frac{\partial^2\psi}{\partial t^2} = \left(-\frac{\nabla^2}{2} - 1 + v_r(x) + |\psi|^2\right)\psi$$



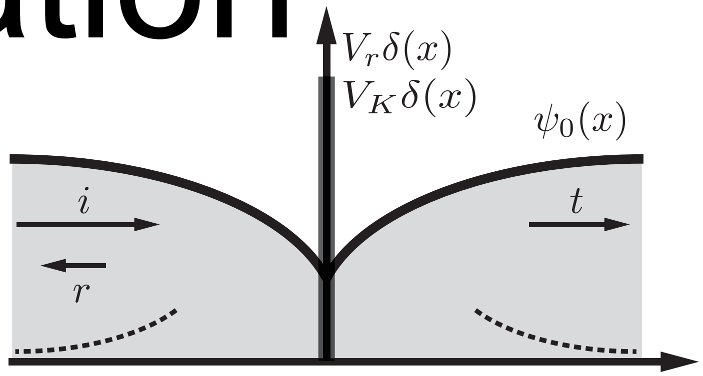
- we assume delta-function potentials :

$$v_K = V_K\delta(x) \qquad v_r = V_r\delta(x)$$

Potentials varying in the order of lattice spacing *d* can be well approximated by the delta-function potentials because of $\xi \gg d$ near the phase boundary

Linearized TDGL equation

$$iv_K \frac{\partial \psi}{\partial t} - \frac{\partial^2 \psi}{\partial t^2} = \left(-\frac{\nabla^2}{2} - 1 + v_r + |\psi|^2 \right) \psi$$



we assume fluctuations of the order parameter only in the x direction

$$\psi(x,t) = \psi_0(x) + \mathcal{U}(x)e^{-i\omega t} + \mathcal{V}^*(x)e^{i\omega^* t}$$

Static eq. (same as the static GP eq.)

$$\left(-\frac{1}{2}\frac{d^2}{dx^2} - 1 + |\psi_0|^2 + v_r(x) \right) \psi_0(x) = 0$$

no effect of $v_K(x)$

$$S(x) = \mathcal{U}(x) - \mathcal{V}(x) \propto \delta\theta(x),\ T(x) = \mathcal{U}(x) + \mathcal{V}(x) \propto \delta n(x)$$

NG: $\left(-\frac{1}{2}\frac{d^2}{dx^2} - 1 + |\psi_0|^2 + v_r(x) \right) S(x) = \omega^2 S(x) - \omega v_K(x) T(x)$

Higgs: $\left(-\frac{1}{2}\frac{d^2}{dx^2} - 1 + 3|\psi_0|^2 + v_r(x) \right) T(x) = \omega^2 T(x) - \omega v_K(x) S(x)$

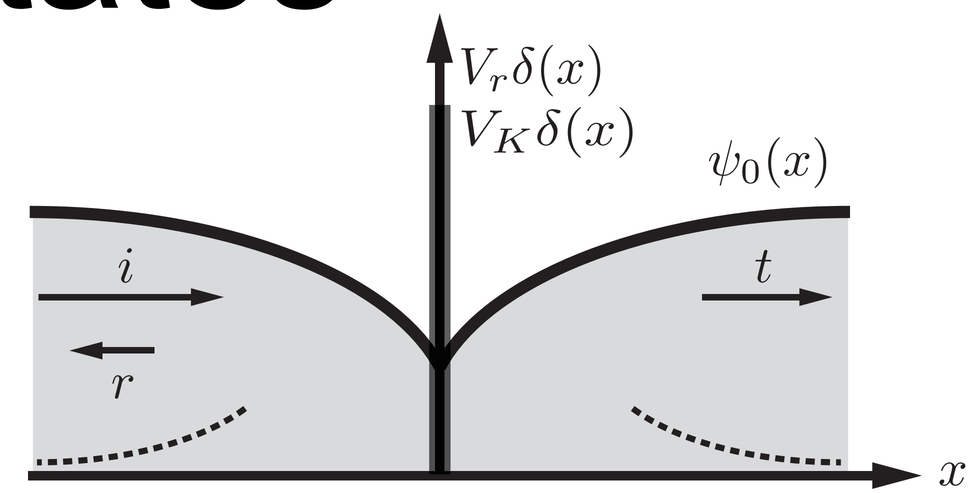
Higgs and NG modes are locally coupled via $v_K(x)$

Higgs bound states

- Static solution for $v_r(x) = V_r\delta(x)$

$$\psi_0(x) = \tanh(|x| + x_0)$$

we set $v_K(x) = 0$



$$\left(-\frac{1}{2}\frac{d^2}{dx^2} - 1 + \underline{|\psi_0|^2} + v_r(x)\right) S(x) = \omega^2 S(x) \cancel{- \omega v_K(x) T(x)}$$

$$\left(-\frac{1}{2}\frac{d^2}{dx^2} - 1 + \underline{3|\psi_0|^2} + v_r(x)\right) T(x) = \omega^2 T(x) \cancel{- \omega v_K(x) S(x)}$$

= Shroeinger eq.

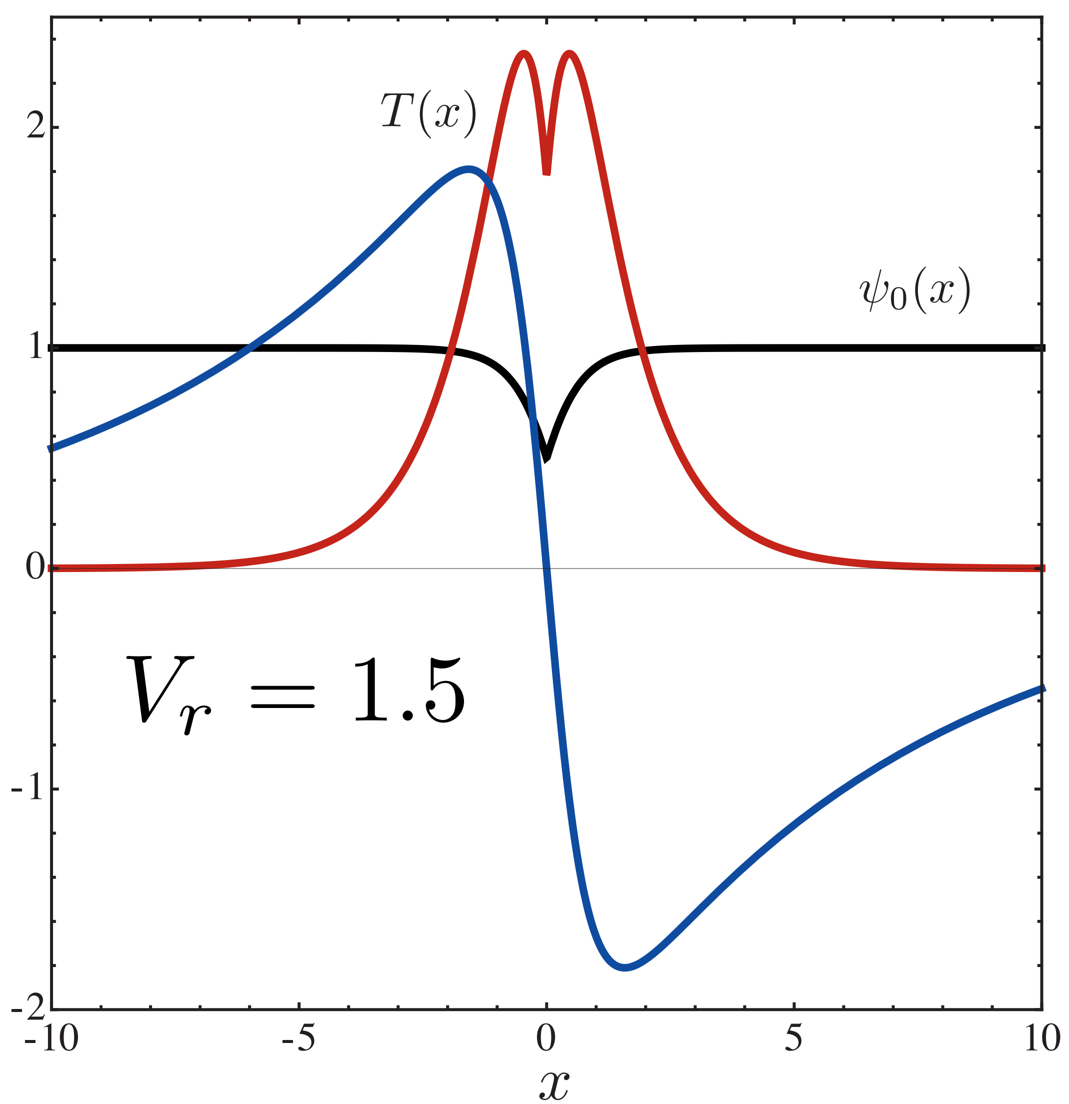
decoupled Higgs and NG mode due to p-h symmetry

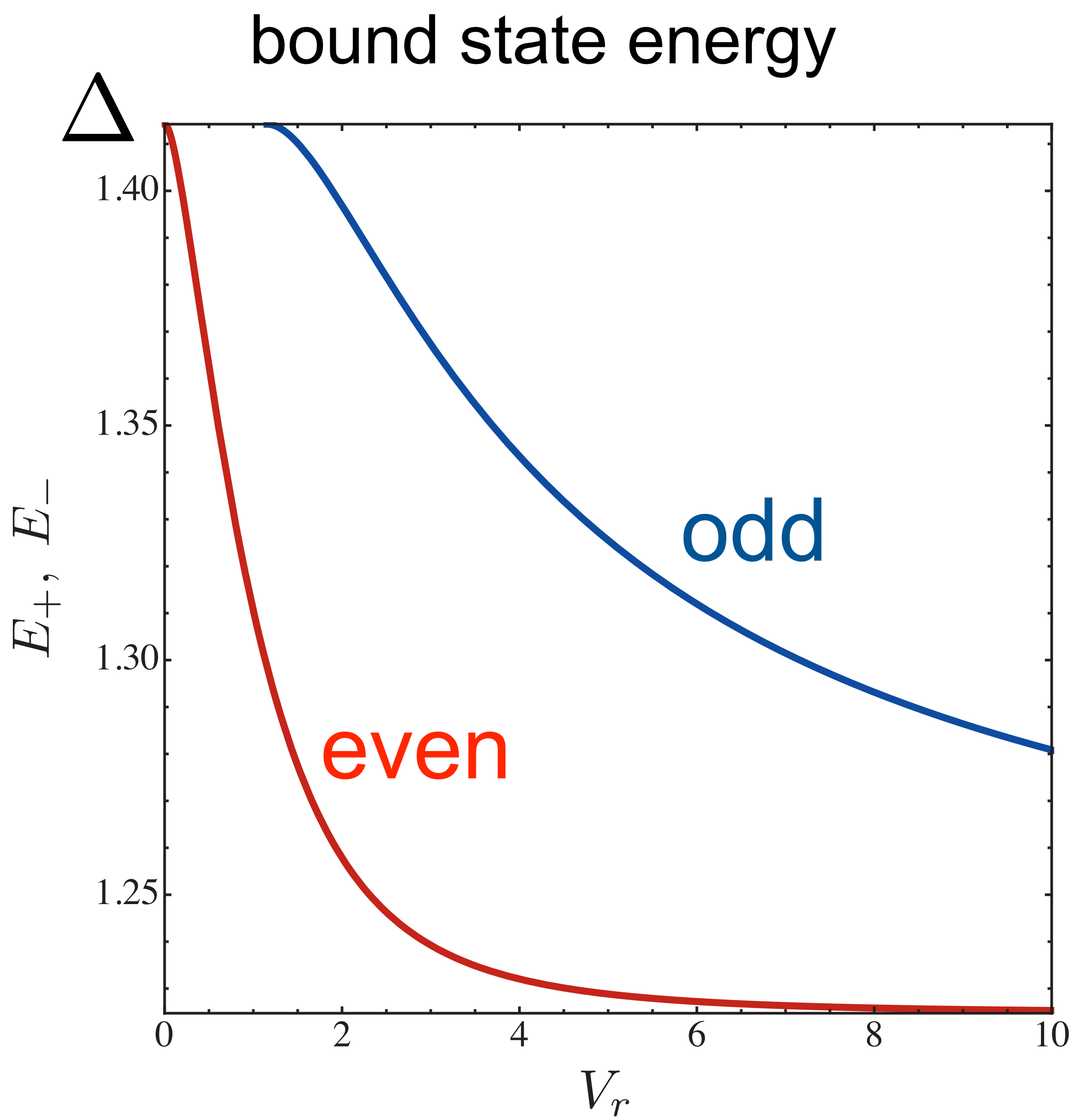
- Bound-state solutions of amplitude fluctuation T(x) **below** the bulk Higgs gap due to the deep condensate potential

Higgs bound states

$$\left(-\frac{1}{2}\frac{d^2}{dx^2} - 1 + 3|\psi_0|^2 + V_r\delta(x)\right)T(x) = \omega^2 T(x)$$

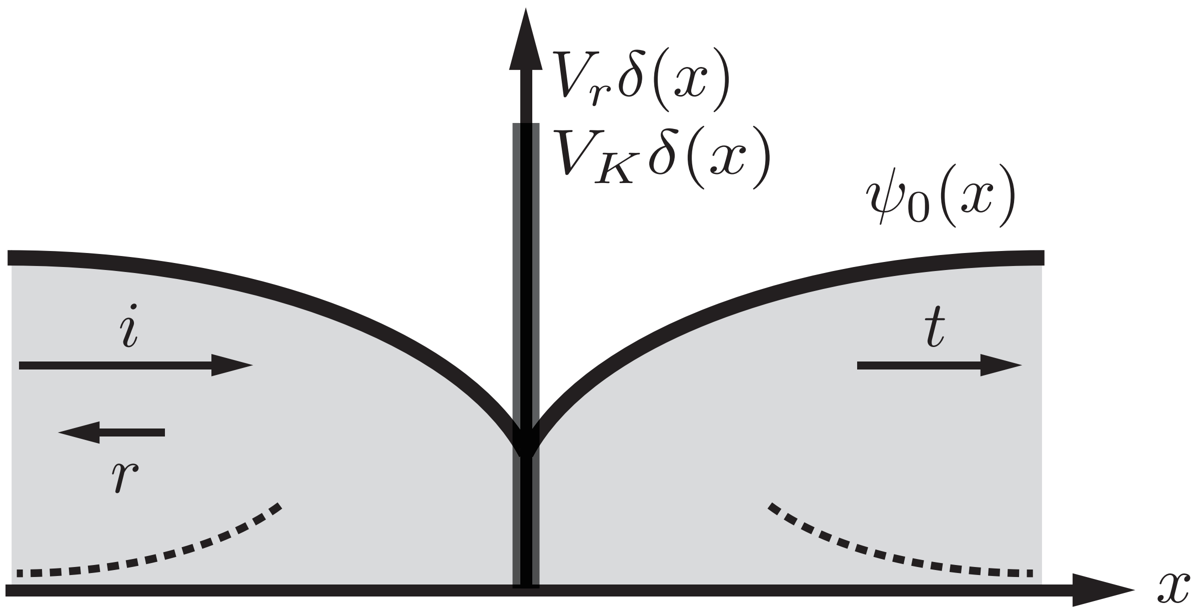
$\psi_0(x) = \tanh(|x| + x_0)$





Δ : bulk Higgs gap

Scattering of collective modes



Scattering of NG modes with energy below the bulk Higgs gap $E < \Delta = \sqrt{2}$ incident to the potential barriers

$$v_K = V_K\delta(x) \quad v_r = V_r\delta(x)$$

NG: $$S(x) = \begin{cases} (\gamma(x) + ik_s)e^{ik_s x} + r_{\rm ng}(\gamma(x) - ik_s)e^{-ik_s x} & (x<0) \\ \text{Incident} \qquad\qquad \text{Reflected} & \\ t_{\rm ng}(\gamma(x) - ik_s)e^{ik_s x} & (x>0) \\ \text{Transmitted} & \end{cases}$$

$$\gamma(x) = \tanh(|x| + x_0) \quad k_s = \sqrt{2}E$$

Higgs: $$T(x) = \begin{cases} r_{\rm h}(3\gamma(x)^2 + 3\kappa_t\gamma(x) + \kappa_t^2 - 1)e^{\kappa_t x} & (x<0) \\ t_{\rm h}(3\gamma(x)^2 + 3\kappa_t\gamma(x) + \kappa_t^2 - 1)e^{-\kappa_t x} & (x>0) \end{cases}$$

decay at infinity

$$\kappa_t = \sqrt{4 - 2E^2}$$

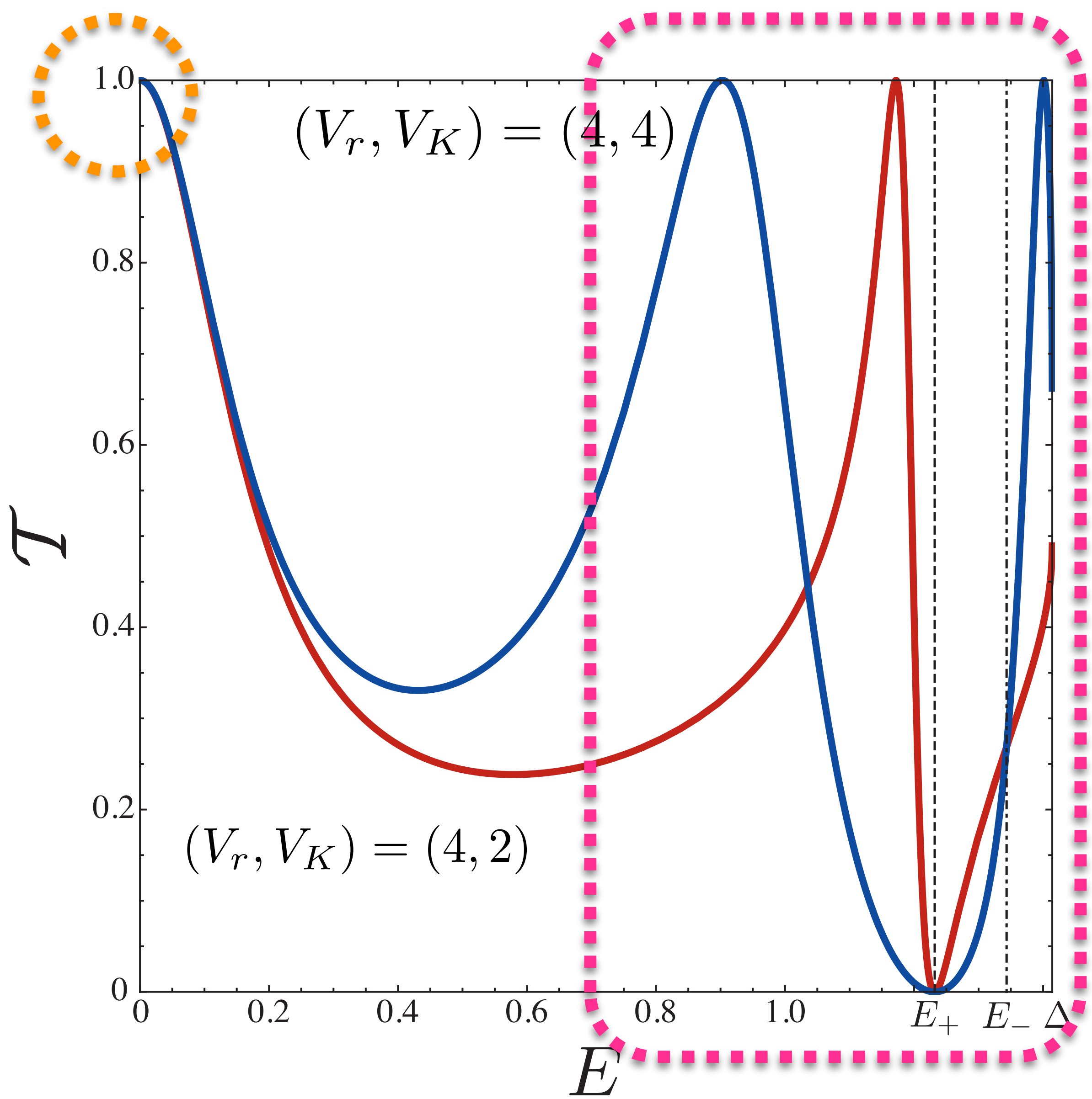
Boundary condition at x=0

Tunneling property of NG modes

Transmission probability: $$\mathcal{T}(E) = \frac{1}{1 + \frac{2E^2}{(2E^2+1)^2} V_{\rm eff}(E)^2}$$

$E < \Delta$

Effective potential: $$V_{\rm eff}(E) = (1 - V_K^2 f(E)) V_r$$



- Perfect transmission of NG mode in the low energy limit

anomalous tunneling

Kovrizhin, Phys. Lett. A, 287, 392 (2001)

Kagan, et al., PRL 90, 130402 (2003)

- Characteristic *asymmetric peak* near the energy of the Higgs bound state E_+

Fano resonance

Tunneling property of NG modes

Transmission probability: $\mathcal{T}(E) = \dfrac{1}{1 + \frac{2E^2}{(2E^2+1)^2} V_{\text{eff}}(E)^2}$

Effective potential:

$$V_{\text{eff}}(E) = (1 - V_K^2 f(E))V_r \quad \simeq \boxed{V_r} - \boxed{\frac{\alpha V_K^2}{E - E_+} V_r} \qquad E \simeq E_+$$

direct scattering

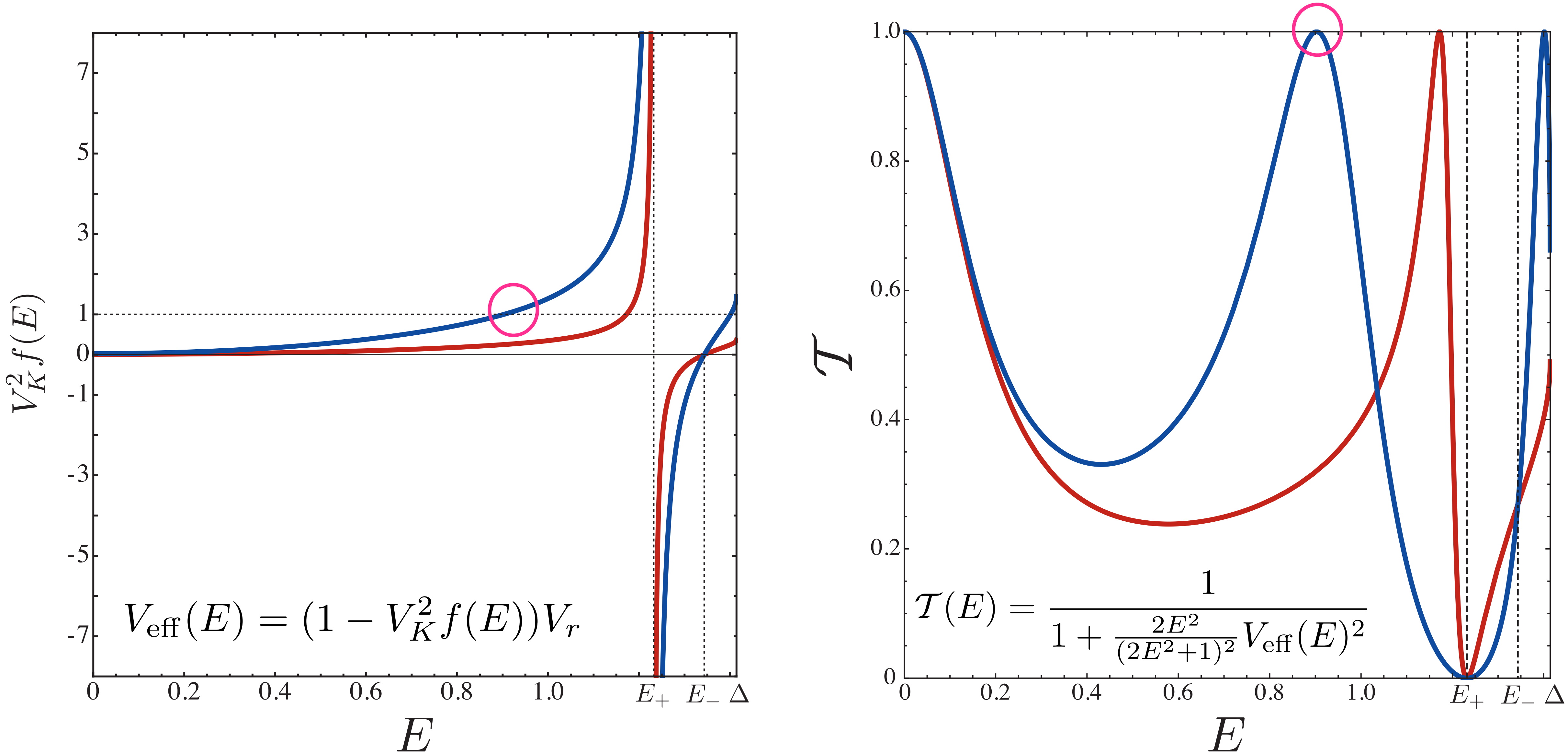
resonant scattering involving excitation of the even Higgs bound state

$$f(E \to E_+) \to \infty \quad \Rightarrow \quad V_{\text{eff}} \to \infty \quad \Rightarrow \quad \mathcal{T} \to 0$$

$$1 - V_K^2 f(E) = 0 \quad \Rightarrow \quad V_{\text{eff}} = 0 \quad \Rightarrow \quad \mathcal{T} = 1$$

destructive interference

- Interference between directly scattered waves within continuum and resonantly scattered waves mediated by discrete states
  - **Fano resonance**



$$f(E \to E_+) \to \infty \quad \Rightarrow \quad V_{\rm eff} \to \infty \quad \Rightarrow \quad \mathcal{T} \to 0$$

$$1 - V_K^2 f(E) = 0 \quad \Rightarrow \quad V_{\rm eff} = 0 \quad \Rightarrow \quad \mathcal{T} = 1$$

The asymmetric peak is manifestation of the Fano resonance of the NG mode mediated by the even Higgs bound state.

Summary

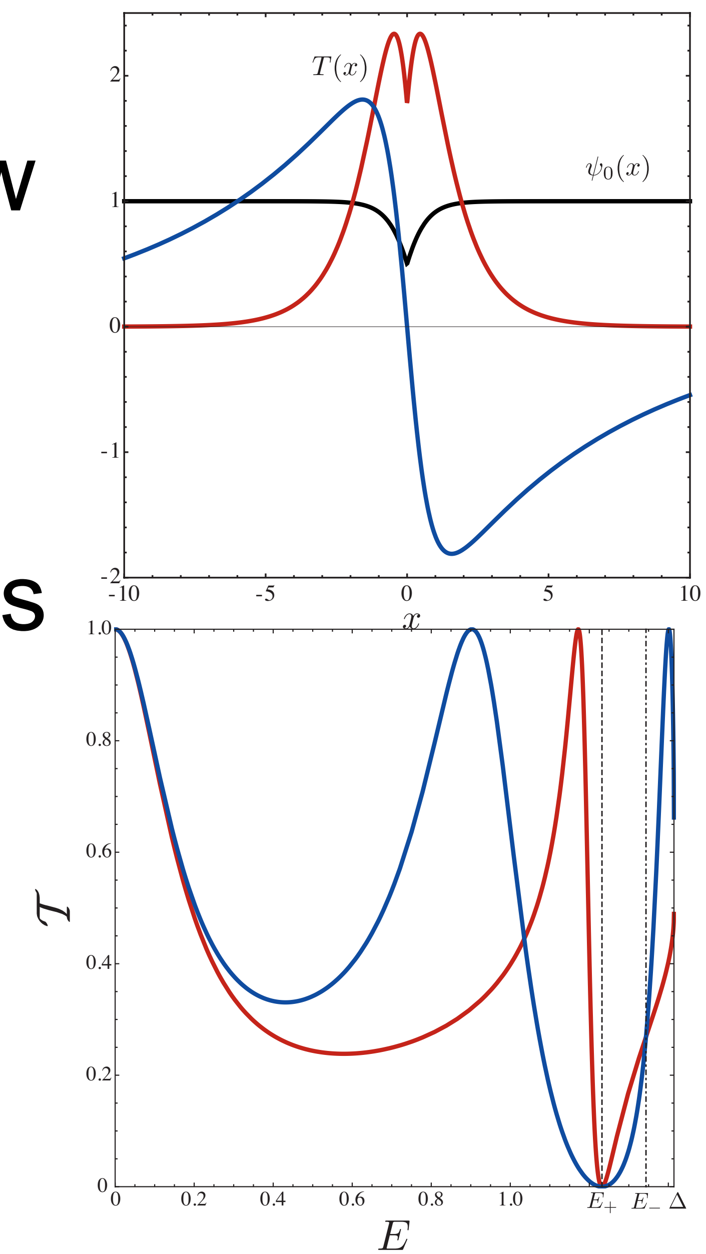
- TDGL eq. including effects of two kinds of potential barriers - inhomogeneous on-site potential and hopping amplitude.

$$i v_K(x)\frac{\partial \psi}{\partial t} - W_0 \frac{\partial^2 \psi}{\partial t^2} = \left(-\frac{\nabla^2 \psi}{2m_*} + r_0 + v_r(x) + u_0 |\psi|^2 \right) \psi$$

- Localized Higgs bound states below the bulk Higgs gap

- Fano resonance of NG mode mediated by the Higgs bound states

Outlook Higgs bound states in other condensed matter systems, e.g. disordered SCs



Sherman et al., Nat. Phys. (2015)

Higgs modes in condensed matter physics

- Raman spectroscopy in $NbSe_2$

  - First observation of Higgs mode!

$NbSe_2$: CDW transition at 40K and SC transition at 7.2 K

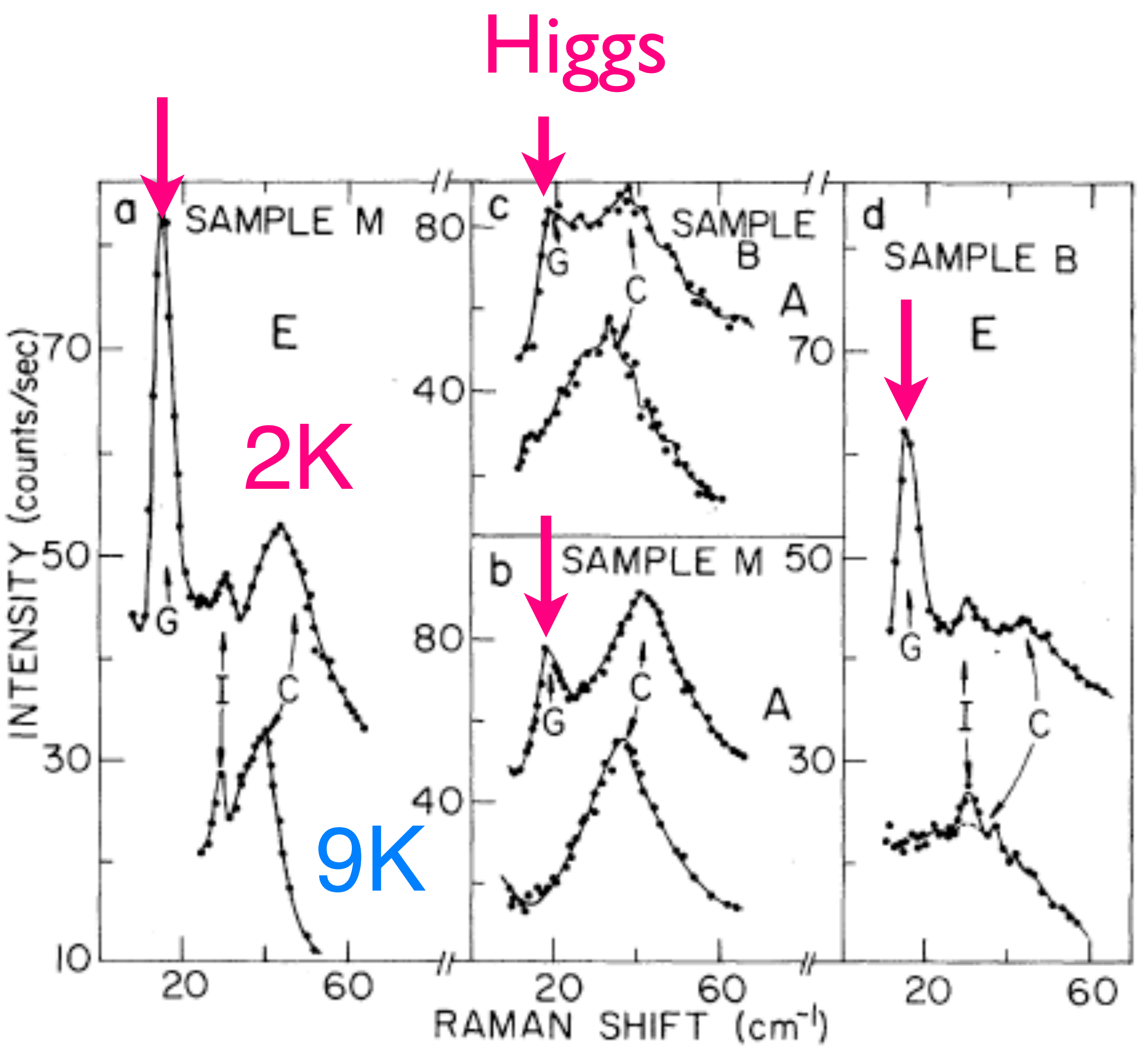


FIG. 1. Raman spectrum of samples *M* and *B*. The lower curve of each pair [(a)–(d)] is at 9 K and the upper at 2 K. Raman symmetries [polarizations] are *E* [(*xy*)] and *A* [(*xx*) – (*xy*)]. *C* labels CDW modes; *G*, gap excitations; and *I*, the interlayer mode characteristic of the 2*H* polytype. Incident laser beam at

Sooryakumar and Klein, PRL 45, 660 (1980)

- A new peak at the frequency twice of the SC gap arises below SC Tc.
- Littlewood and Varma developed a microscopic theory by extending the BCS-Nambu theory. They found the peak due to amplitude oscillations of superconducting gap - Higgs mode.

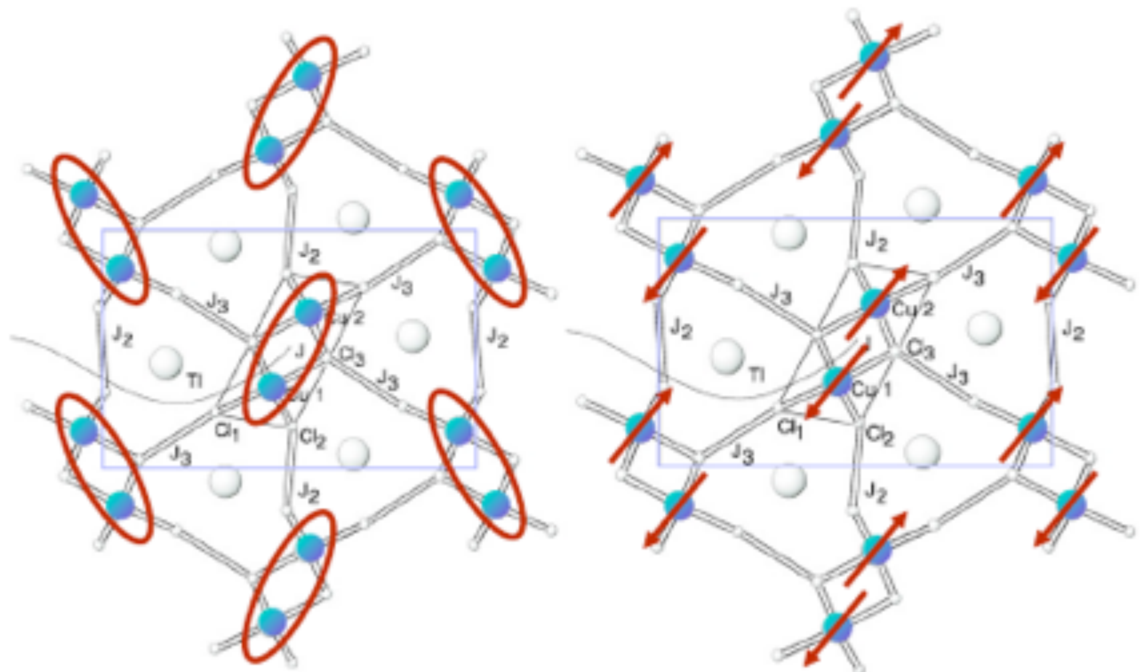
P. Littlewood and C. Varma, PRL 47, 811 (1980)

Higgs gap (mass) = 2Δ in BCS (s-wave)

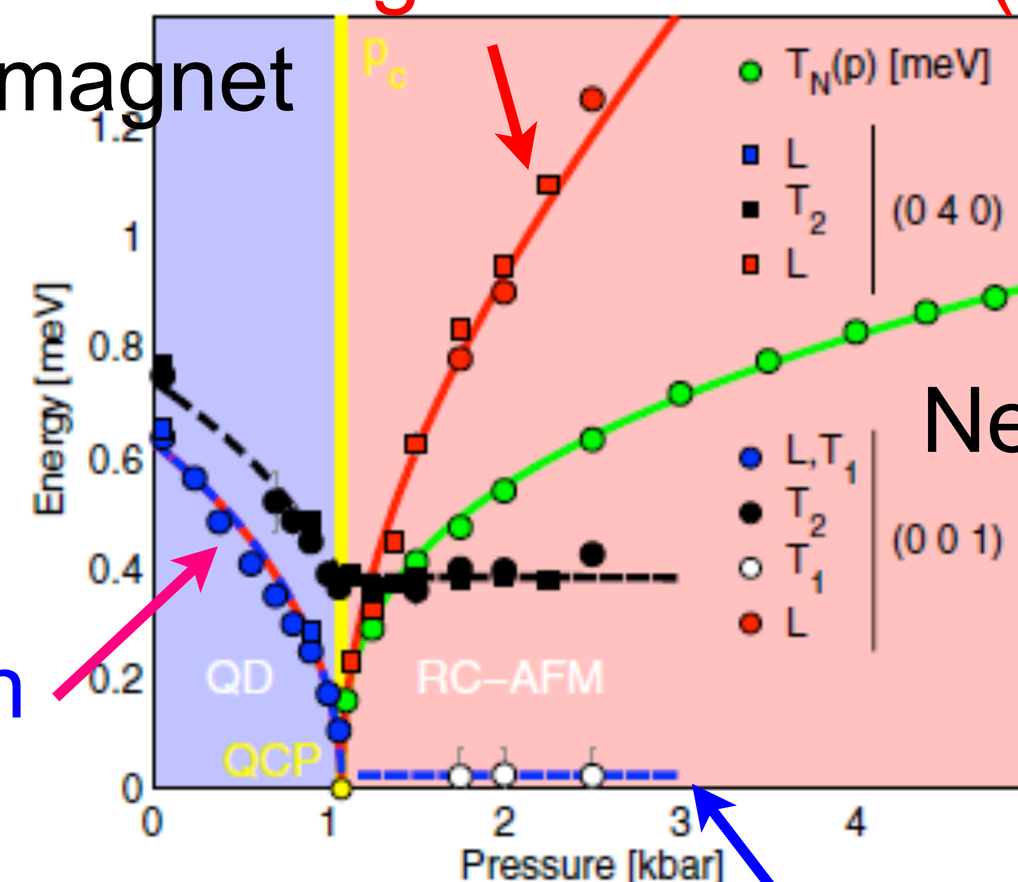
PRL 100, 205701 (2008) PHYSICAL REVIEW LETTERS week ending 23 MAY 2008

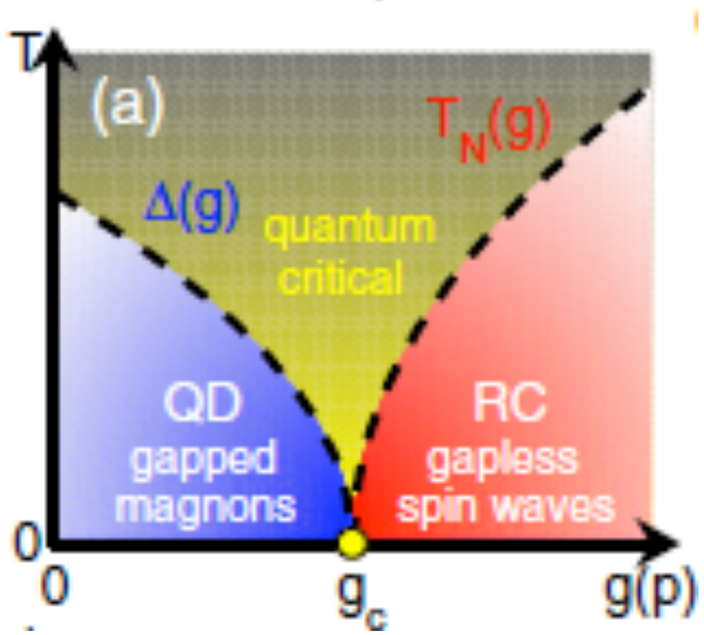
Quantum Magnets under Pressure: Controlling Elementary Excitations in $TlCuCl_3$

Ch. Rüegg,[1] B. Normand,[2,3] M. Matsumoto,[4] A. Furrer,[5] D. F. McMorrow,[1] K. W. Krämer,[6] H.-U. Güdel,[6] S. N. Gvasaliya,[5] H. Mutka,[7] and M. Boehm[7]



$$\mathcal{H} = \sum_i J(p)\mathbf{S}_{i,l}\cdot\mathbf{S}_{i,r} + \sum_{ij;m,m'=l,r} J_{ij}(p)\mathbf{S}_{i,m}\cdot\mathbf{S}_{j,m'}$$





■ triplon (broken valence bond excitations)

condensation → Higgs + NG mode