中性子 $^3P_2$ 超流動体の諸性質

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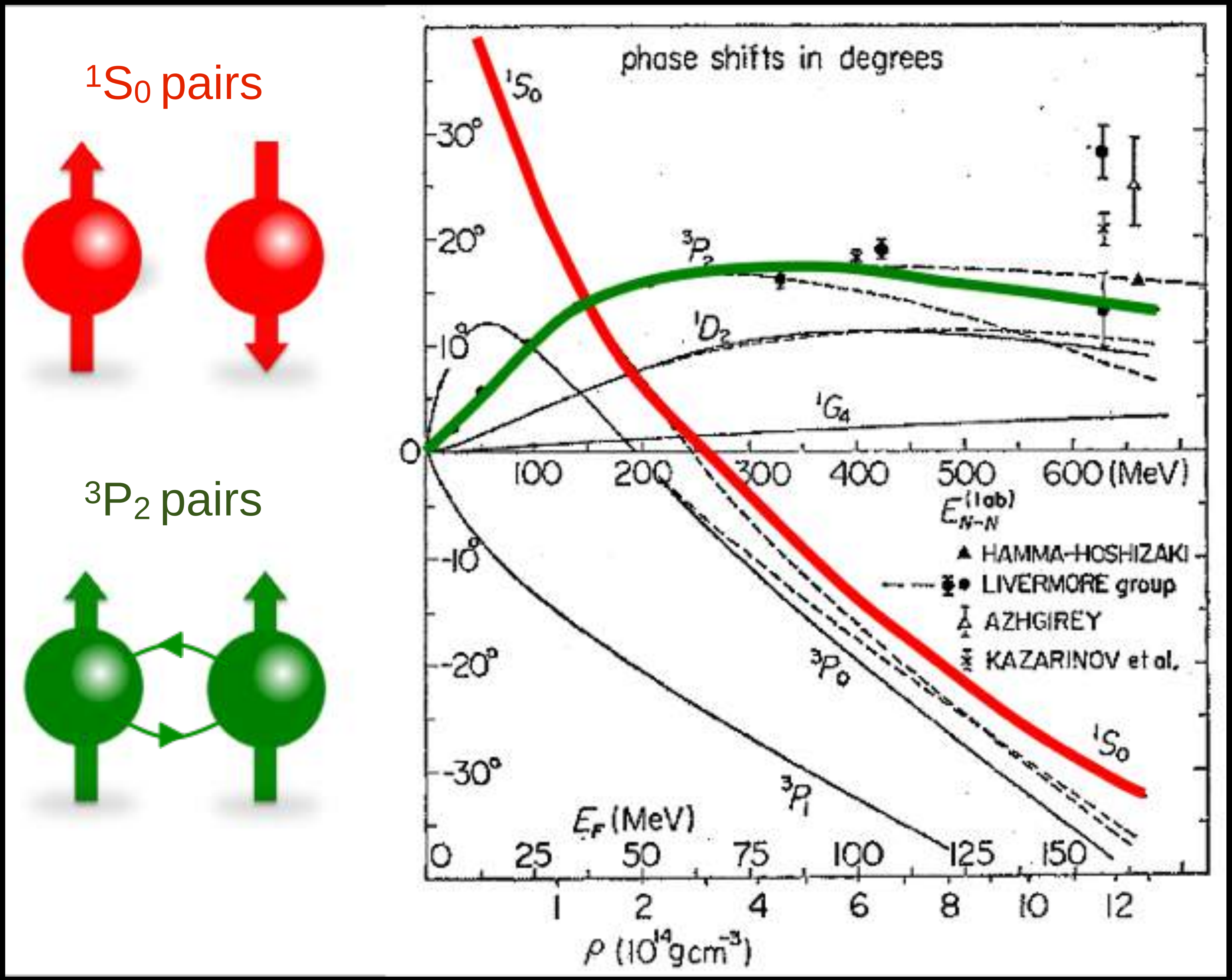
熱場の量子論とその応用 @Kyoto, YITP 2015. 9.1

Nuclear Physics



Tamagaki and Takatsuka (1970,1971,1972)

$\rho \sim 0.7\rho_0$: Transition from $^1S_0$ to $^3P_2$ is predicted to occur.

Order Parameter

The $^3P_2$ order parameter: traceless symmetric tensor

Ginzburg-Landau Free Energy (1)

Fujita and Tsuneto (1972)

Richardson (1972)

$$H = \int d^3r \; \psi^\dagger \left(-\frac{\nabla^2}{2m} - \mu \right) \psi - \frac{1}{2} g T^\dagger_{\alpha\beta} T_{\alpha\beta}$$

$$T_{\alpha\beta} = [t_{\alpha\beta}\psi]\psi$$

$$t_{\alpha\beta} = \frac{1}{2}[S_\alpha \nabla_\beta + S_\beta \nabla_\alpha] - \frac{1}{3}\delta_{\alpha\beta} \vec{S} \cdot \vec{\nabla}$$

$$F = \int d^3r \; f_4 + f_6$$

α	K	β	γ
$\frac{N(0)}{3}\frac{T-T_c}{T}k_F^2$	$\frac{7\xi(3)}{240m^2}\frac{N(0)}{(\pi T_c)^2}k_F^4$	$\frac{7\xi(3)}{60}\frac{N(0)}{(\pi T_c)^2}k_F^4$	$-\frac{31}{16}\frac{\xi(5)}{840}\frac{N(0)}{(\pi T_c)^4}k_F^6$

$$f_4 = \alpha \mathrm{Tr} AA^\dagger + K(\partial_i A_{\mu\nu}\partial_i A^\dagger_{\mu\nu} + \partial_i A_{\mu i}\partial_j A^\dagger_{\mu j} + \partial_i A_{\mu j}\partial_j A^\dagger_{\mu i}) + \beta[(\mathrm{Tr} AA^\dagger)^2 - \mathrm{Tr} A^2 A^{\dagger 2}],$$

$$f_6 = \gamma[-3(\mathrm{Tr} AA^\dagger)|\mathrm{Tr} AA|^2 + 4(\mathrm{Tr} AA^\dagger)^3 + 12(\mathrm{Tr} AA^\dagger)(\mathrm{Tr} AA^\dagger)^2 + 6(\mathrm{Tr} AA^\dagger)\mathrm{Tr}(A^2 A^{\dagger 2}) \\ + 8\mathrm{Tr}(AA^\dagger)^3 + 12\mathrm{Tr}[(AA^\dagger)^2 A^\dagger A] - 12\mathrm{Tr}[AA^\dagger A^\dagger A^\dagger AA] - 12\mathrm{Tr} AA(\mathrm{Tr} AA^\dagger AA)^*]$$

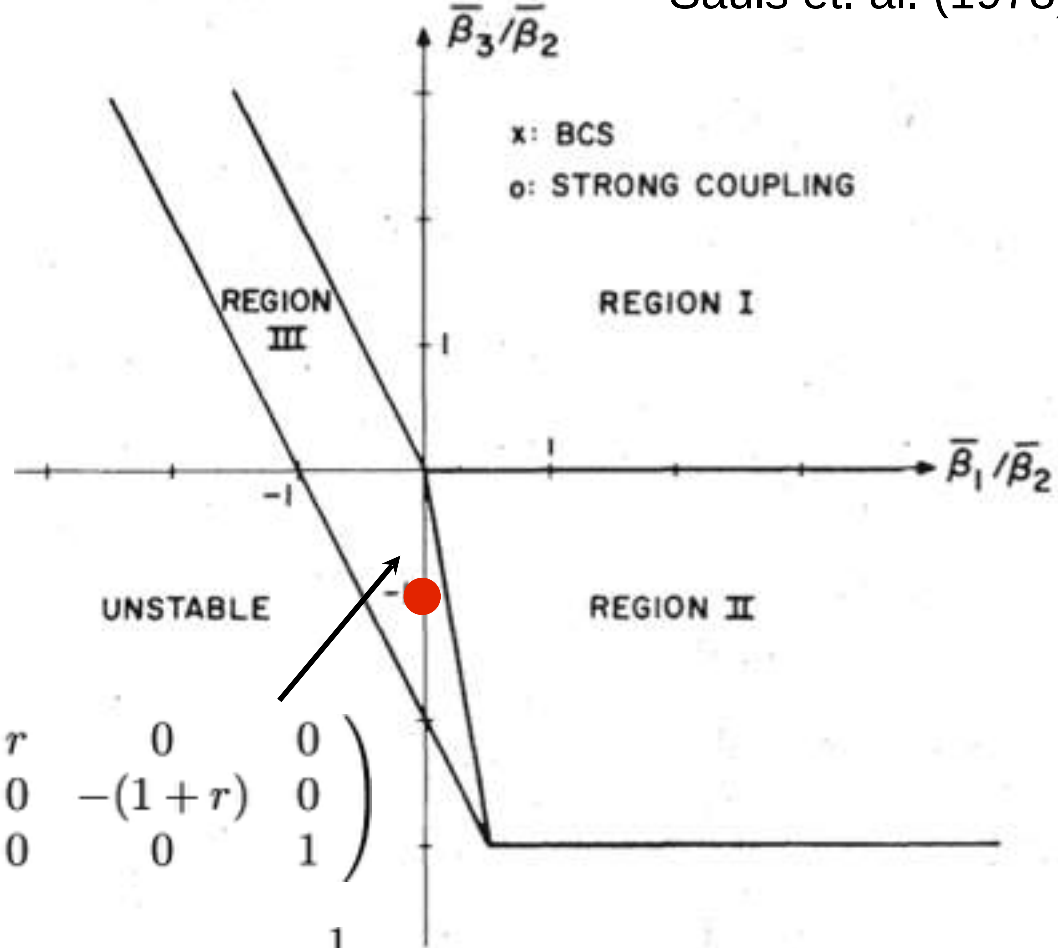
Ginzburg-Landau Free Energy (2)

Ginzburg-Landau Free Energy Density (4th order): $G = SO(3)_{L+S} \times U(1)$

$$\Omega_{4th} = \frac{1}{3}\alpha(T)\mathrm{Tr}AA^* + \bar{\beta}_1(\mathrm{Tr}AA^*)^2 + \bar{\beta}_2\mathrm{Tr}(AA^*)^2 + \bar{\beta}_3\mathrm{Tr}(A^2A^{*2})$$

$^3P_2$ Phase Diagram

Sauls et. al. (1978)



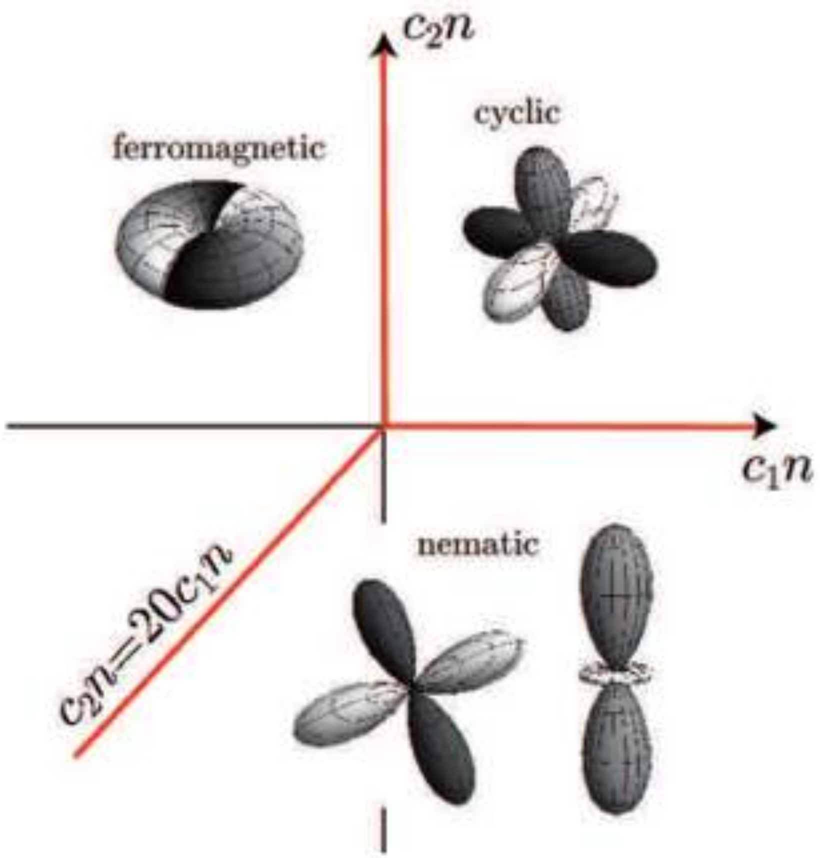
$$\frac{1}{N}\begin{pmatrix} r & 0 & 0 \\ 0 & -(1+r) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$-1 \le r \le -\frac{1}{2}$$

Ground state is degenerate

Spin2-BEC Phase Diagram

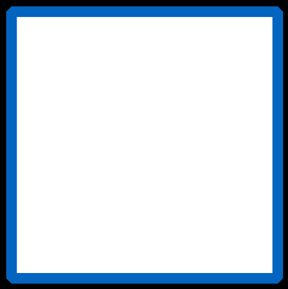
Kawaguchi and Ueda (2010)



$$\Delta = \sum_{\mu\nu} i\sigma_\mu\sigma_2 A_{\mu\nu}\hat{k}_\nu$$

Ground State (4th)

$$\Omega = \Omega_{4\text{th}}$$

Phase	G/H		H
Uniaxial (r=-1/2)	$U(1) \times S^2/\mathbb{Z}_2$		$O(2)$
Dihedral-2	$U(1) \times SO(3)/D_2$		D_2
Biaxial (r=-1)	$[U(1) \times SO(3)]/D_4$		D_4

$$A = N \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & -f_1 - f_2 \end{pmatrix}$$

Ground State (6th)

$$\Omega = \Omega_{4\mathrm{th}} + \Omega_{6\mathrm{th}}$$

Phase	G/H	6th
Uniaxial (r=-1/2)	$U(1) \times S^2/\mathbb{Z}_2$	✔
Dihedral-2	$U(1) \times SO(3)/D_2$	
Biaxial (r=-1)	$[U(1) \times SO(3)]/D_4$	

$$A = N \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & -f_1 - f_2 \end{pmatrix}$$

Ground State (B-field)

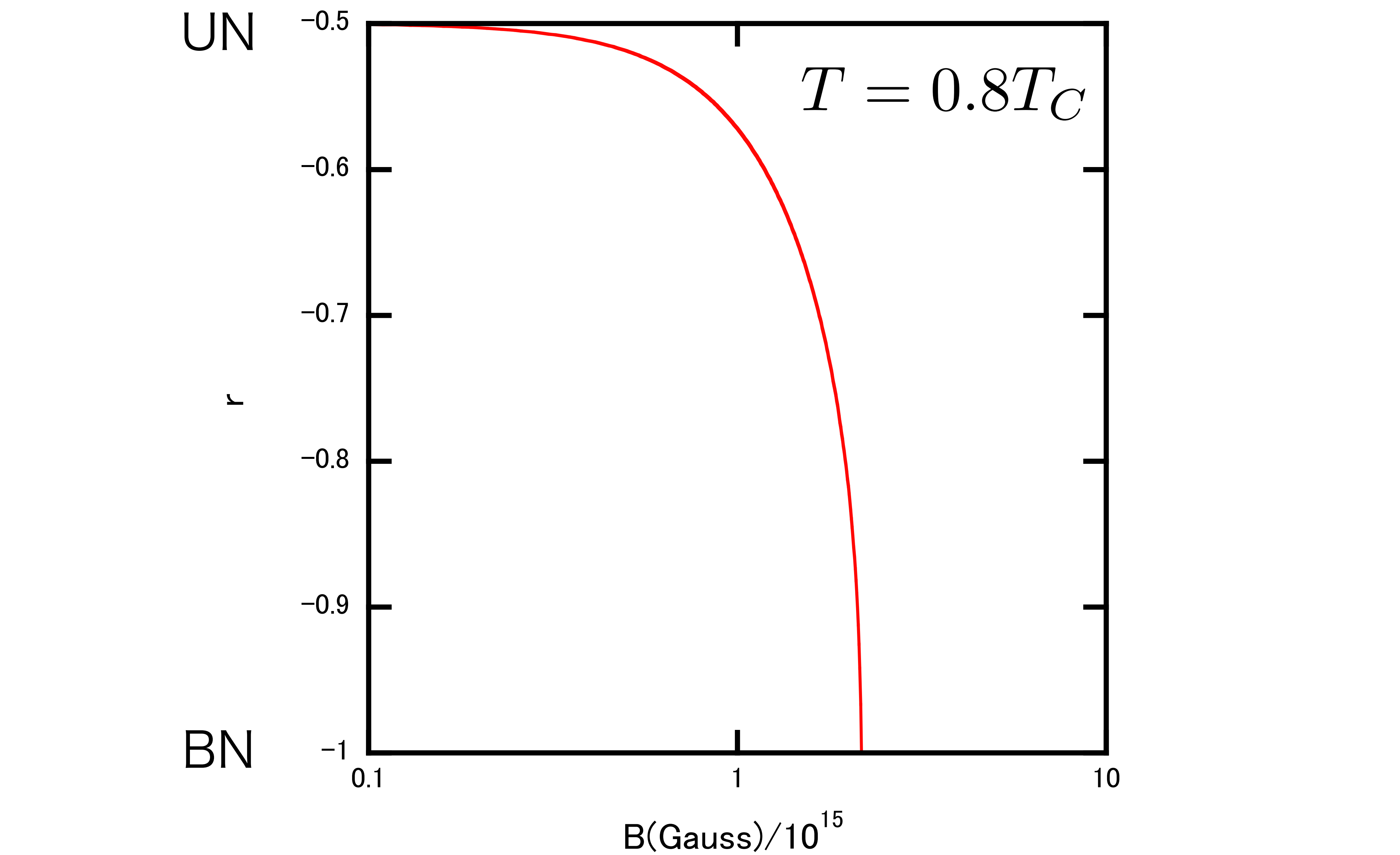
$$\Omega = \Omega_{\mathrm{4th}} \qquad + gH_i(AA^*)_{ij}H_j$$

Phase	G/H	B-field
Uniaxial (r=-1/2)	$U(1)\times S^2/\mathbb{Z}_2$	
Dihedral-2	$U(1)\times SO(3)/D_2$	
Biaxial (r=-1)	$[U(1)\times SO(3)]/D_4$	✔

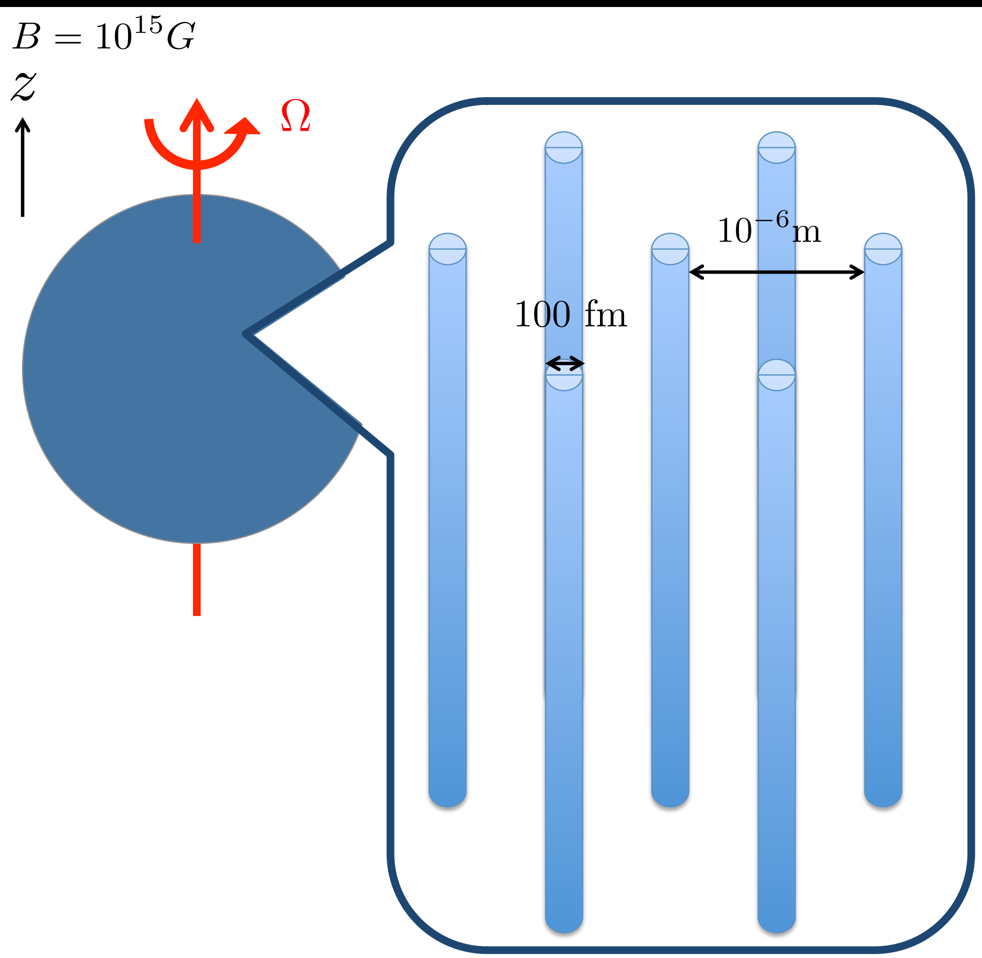
$$A = N\begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & -f_1-f_2 \end{pmatrix}$$

Ground State (6th + B)

$$\Omega = \Omega_{4\mathrm{th}} + \Omega_{6\mathrm{th}} + \underline{gH_i(AA^*)_{ij}H_j}$$



Rotation



U(1) winding number k

$$E \sim k^2 \log L$$

The vortex of winding number 2

↓

two vortices each of unit winding number

Number of Vortices

$$N_v = 1.9 \times 10^{19} \left(\frac{1\text{ms}}{P}\right)\left(\frac{M}{900\text{MeV}}\right)\left(\frac{R}{10\text{km}}\right)$$

Gradient Terms (1)

$$\Omega = \Omega_{4\mathrm{th}} + \underline{K_1 A_{\mu i,j} A_{\mu i,j}} + \underline{K_2 A_{\mu i,i} A_{\mu j,j} + K_3 A_{\mu i,j} A_{\mu j,i}}$$

Gradient (internal) Gradient (external)

$$\to c_1 \log L \ (\rho \to \infty)$$

$$A^{(x,y,z)} = N R(n\theta) \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & -f_1 - f_2 \end{pmatrix} R^T(n\theta) e^{i\theta} \quad R(n\theta) = \begin{pmatrix} \cos n\theta & -\sin n\theta & 0 \\ \sin n\theta & \cos n\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

n=1 ρ θ z

To be the single-valued function, $n \in \mathbb{Z}$

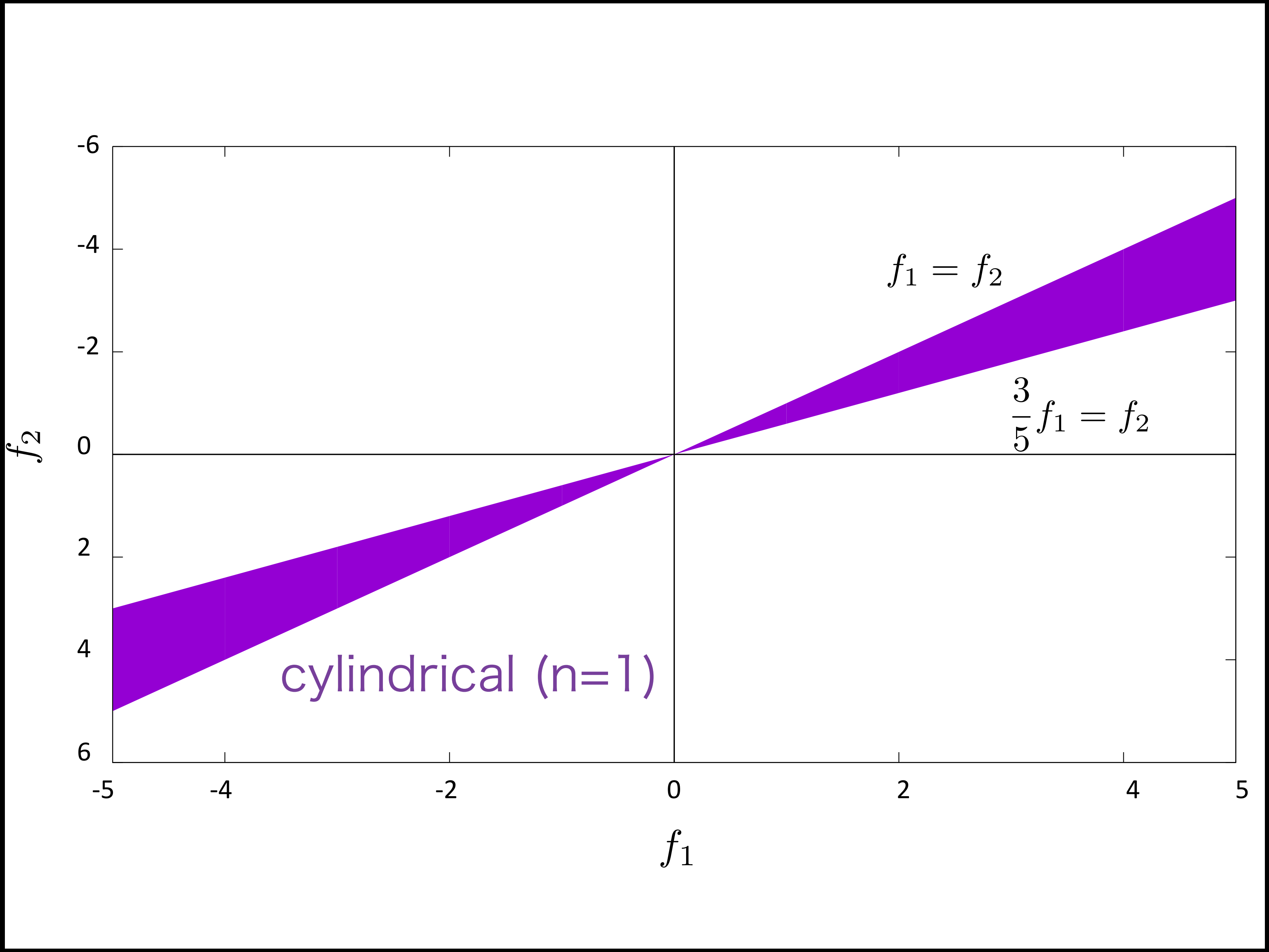
$n = 1$	$n \neq 1$
$2(f_1^2 + f_2^2 + f_1 f_2)$	$2(f_1^2 + f_2^2 + f_1 f_2)$
$+4(f_1 - f_2)^2 + 2f_2^2$	$+4\,n^2\,(f_1 - f_2)^2 + f_1^2 + f_2^2$

Gradient Terms (2)

$$\Omega = \Omega_{4\mathrm{th}} + \underline{K_1 A_{\mu i,j} A_{\mu i,j}} + \underline{K_2 A_{\mu i,i} A_{\mu j,j} + K_3 A_{\mu i,j} A_{\mu j,i}}$$

Gradient (internal)

Gradient (external)



Set Up (6th+B(z) 10^{15} Gauss)

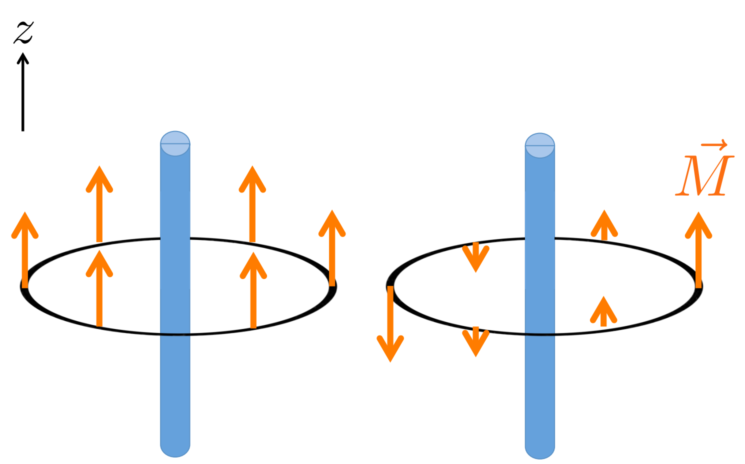
	$\rho = 0\ (xyz)$	$0 < \rho < \infty$	$\rho \to \infty$
$m \neq -1$	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} f_1 & ige^{im\theta} & 0 \\ ige^{im\theta} & f_2 & 0 \\ 0 & 0 & -f_1 - f_2 \end{pmatrix} e^{i\theta}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r_{\rm tot} & 0 \\ 0 & 0 & -1 - r_{\rm tot} \end{pmatrix} e^{i\theta}$ **$r_{tot} \sim$ -0.572**
$m = -1$	**Neumann** $g'(0) = 0$	$\begin{pmatrix} f_1 & ige^{im\theta} & 0 \\ ige^{im\theta} & f_2 & 0 \\ 0 & 0 & -f_1 - f_2 \end{pmatrix} e^{i\theta}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & r_{\rm tot} & 0 \\ 0 & 0 & -1 - r_{\rm tot} \end{pmatrix} e^{i\theta}$

Spontaneous Magnetization

$$A=\begin{bmatrix} -\frac{1}{\sqrt{2}}a_0+\frac{1}{2\sqrt{2}}(a_2+a_{-2}) & \frac{i}{2\sqrt{2}}(a_2-a_{-2}) & -\frac{1}{\sqrt{2}}(a_1-a_{-1}) \\ \frac{i}{2\sqrt{2}}(a_2-a_{-2}) & -\frac{1}{\sqrt{2}}a_0-\frac{1}{2\sqrt{2}}(a_2+a_{-2}) & \frac{-i}{\sqrt{2}}(a_1+a_{-1}) \\ -\frac{1}{\sqrt{2}}(a_1-a_{-1}) & \frac{-i}{\sqrt{2}}(a_1+a_{-1}) & \sqrt{2}\,a_0 \end{bmatrix}$$

$$A^{(x,y,z)}=NR(n\theta)\begin{pmatrix} f_1 & ige^{im\theta+i\alpha} & 0 \\ ige^{im\theta+i\alpha} & f_2 & 0 \\ 0 & 0 & -f_1-f_2 \end{pmatrix}R^T(n\theta)e^{i\theta}$$

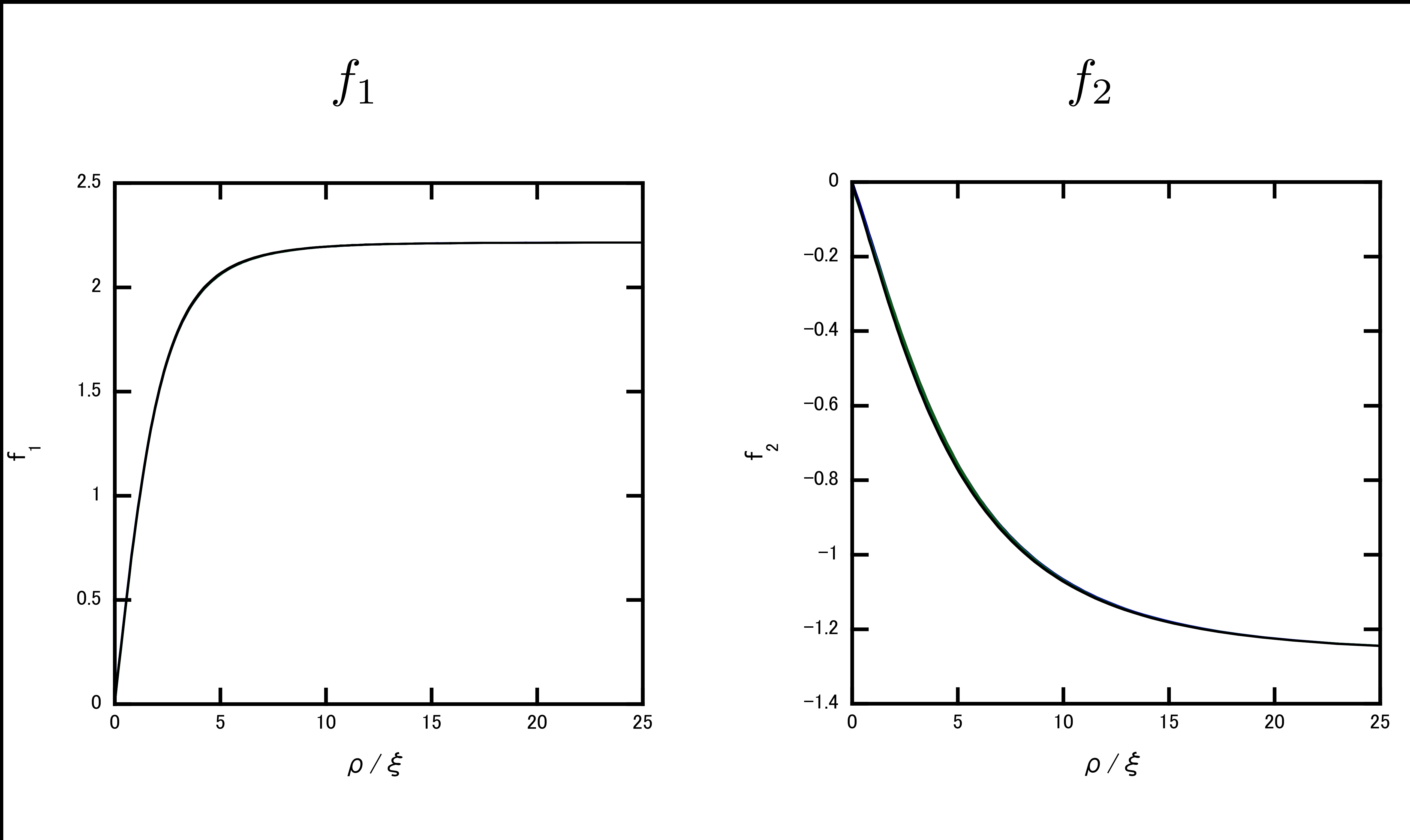
$$|a_2|^2-|a_{-2}|^2\propto(f_1-f_2)g\cos m\theta$$



$m=0$ $m=1$

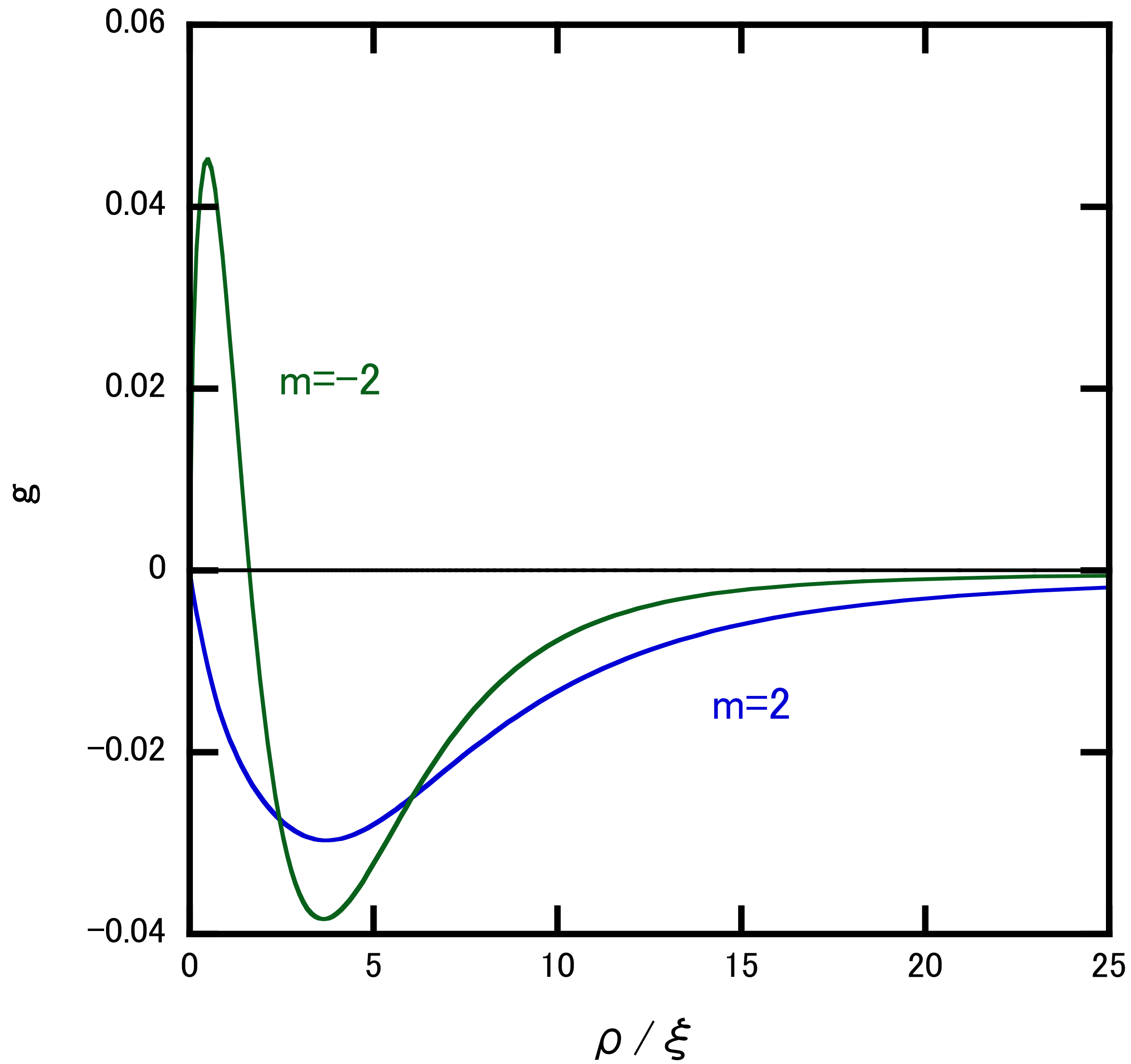
$$\begin{aligned} \boldsymbol{M} &= \frac{\gamma\hbar}{2}\boldsymbol{\sigma} \\ &= \frac{\gamma\hbar}{2}T\sum_n\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}(\boldsymbol{\sigma}G(k,\omega_n)) \\ &= \int\frac{d\Omega}{4\pi}\mathrm{Tr}(\boldsymbol{\sigma}\Delta\Delta^\dagger)T\sum_n\int d\xi N(0)\frac{i\omega_n+\xi}{(\omega_n^2+\xi^2)^2} \\ &= \frac{4}{9}N'(0)\frac{|\alpha|}{6\beta}g(\rho)(f_1(\rho)-f_2(\rho))\cos m\theta\,\hat{\boldsymbol{z}}. \end{aligned}$$

Result1: 6th+B(z) 10^{15} Gauss

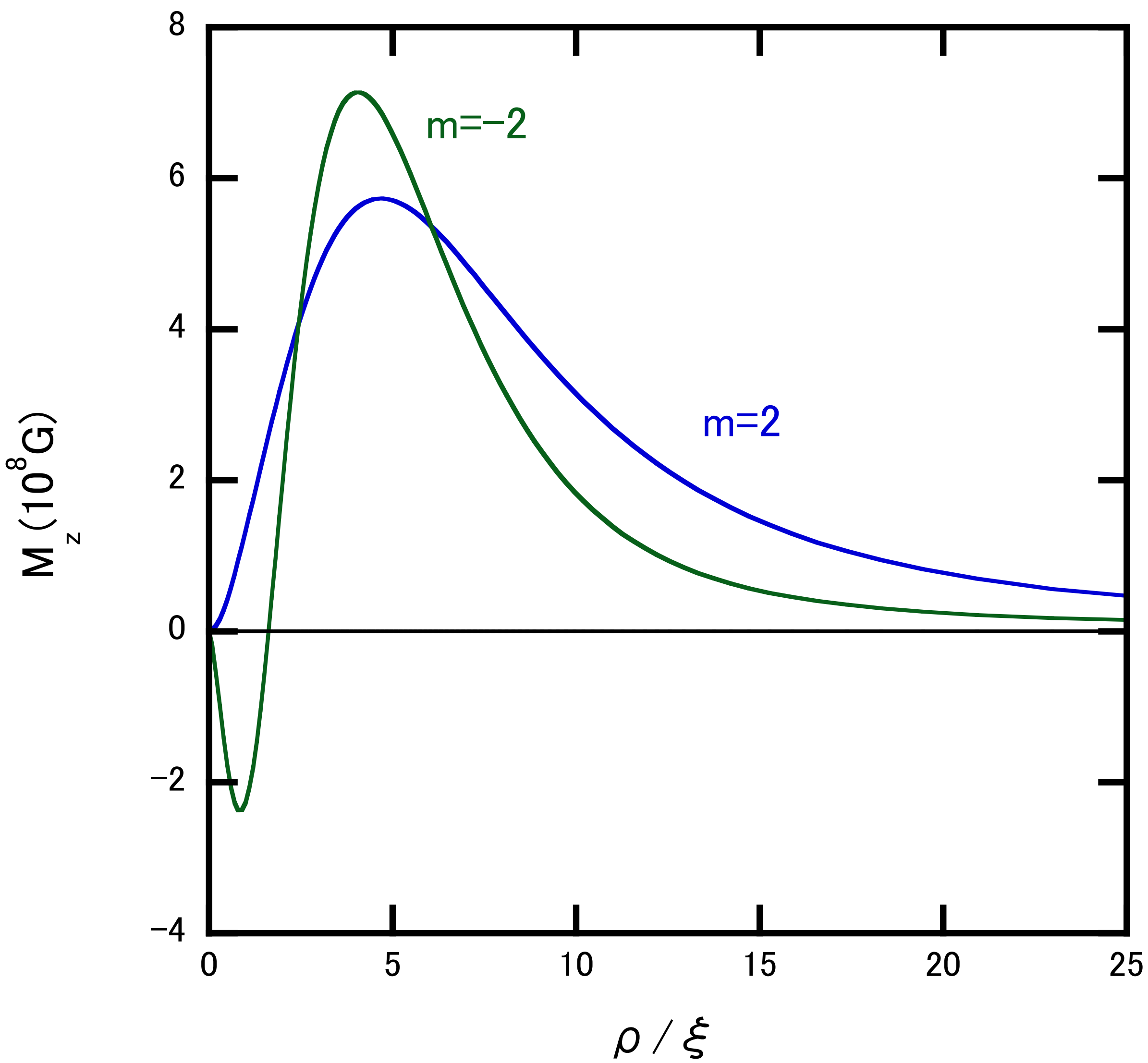


Result2: 6th+B(z) 10^{15} Gauss

g



Spontaneous Magnetization



Summary

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- BN phase can be realized in NSs
  if magnetic field is very strong or if T is near T_c.

- Vortex core has spontaneous magnetization.

  $\longleftrightarrow$ $^1S_0$ phase has no spontaneous magnetization.

Future Works

- Effects on NS observables.
- Treat $^3P_2$ by using Bogoliubov-de-Gennes equation
  (zero modes, topological superfluidity, …)

Thank you !