高輝度光子場の冷却過程:
誘導コンプトン散乱

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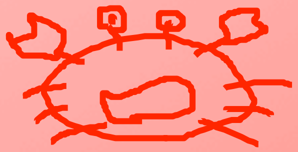


Refs.
1. Tanaka, Asano & Terasawa. 2015 PTEP
2. Tanaka & Takahara 2013 PTEP

Contents

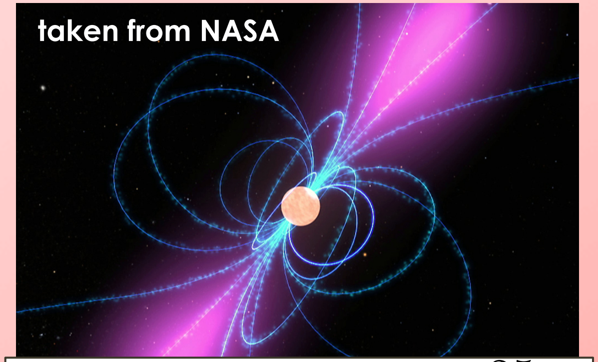
1-1: Situation

Interaction between

- rarefied plasmas ($\omega \gg \omega_{pe}$)

and

- high T_b photons ($k_B T_b \gg m_e c^2$)



$T_{\rm b}(100\ {\rm MHz}) \gtrsim 10^{25} K$

Pulsar radio emissions & surrounding plasmas

Brightness temperature

$k_B T_b(\nu) \equiv h\nu\ n_{ph}(\nu)$ ($P = k_B T_b \Delta\nu$ for $w_0 \sim \lambda$)

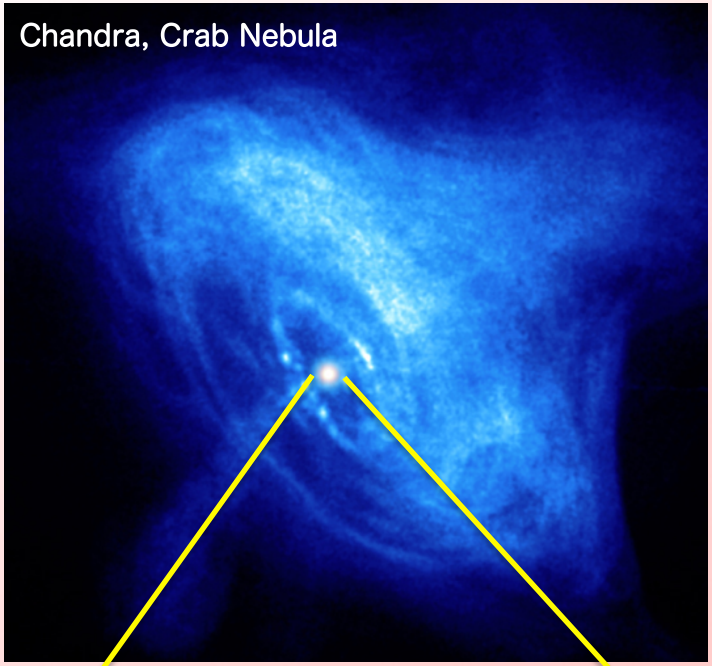
photon occupation number $n_{ph}(k)$ = density in phase-space.

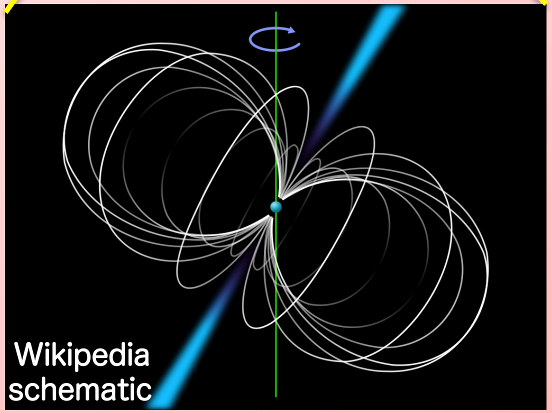
$n_{ph} \gg 2$ is possible because photon is Boson (BEC).

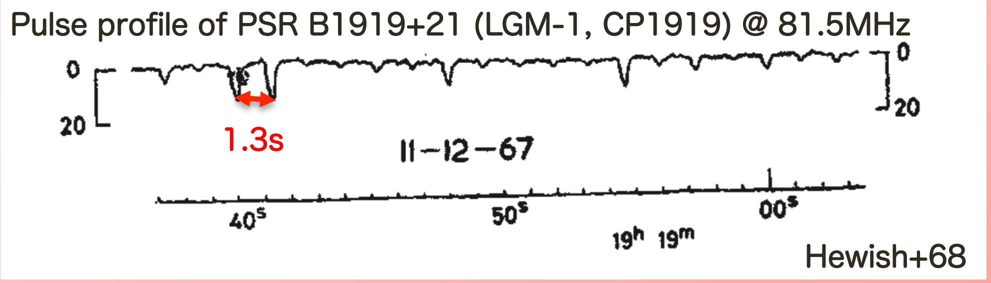
($n_{ph} \sim 10^{27}$!! for pulsar emission)

$T_{\rm b}(100\ {\rm THz}) \gtrsim 10^{20} K$

Laser experiments







1-2: Pulsar

中性子星周りの
電磁場が卓越した領域
> 10^{16}V の電池!

粒子加速と電磁カスケード
による$e^{\pm}$プラズマ生成

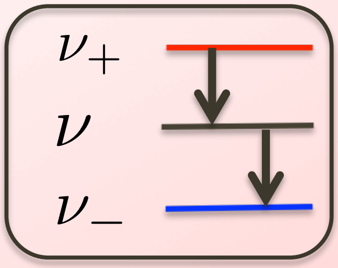
極限プラズマ現象の宝庫
・パルス放射の生成
・相対論的Outflow

Contents

2-1: Induced Compton Scattering

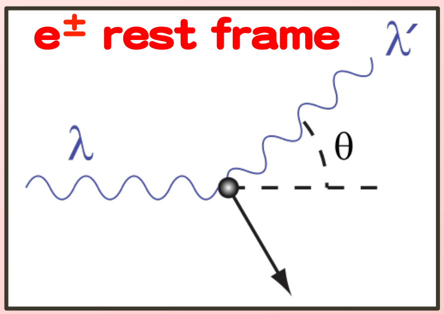
Induced process for Compton scattering

Photons lose energy in e$^\pm$ rest frame.



$$\frac{dn_{\rm ph}(\nu)}{dt} \propto n_{\rm ph}(\nu_+)\underline{(1+n_{\rm ph}(\nu))} - n_{\rm ph}(\nu)\underline{(1+n_{\rm ph}(\nu_-))}$$

spontaneous + induced terms



$\lambda_1 - \lambda = \lambda_c(1-\cos\theta)$

$$\tau_{\rm ind} \approx \sigma_{\rm T} r_{\rm LC} n_{\rm pl} \times \frac{k_{\rm B} T_{\rm b}(\nu)}{m_{\rm e} c^2} \times \Delta\Omega^2 \gg 1$$

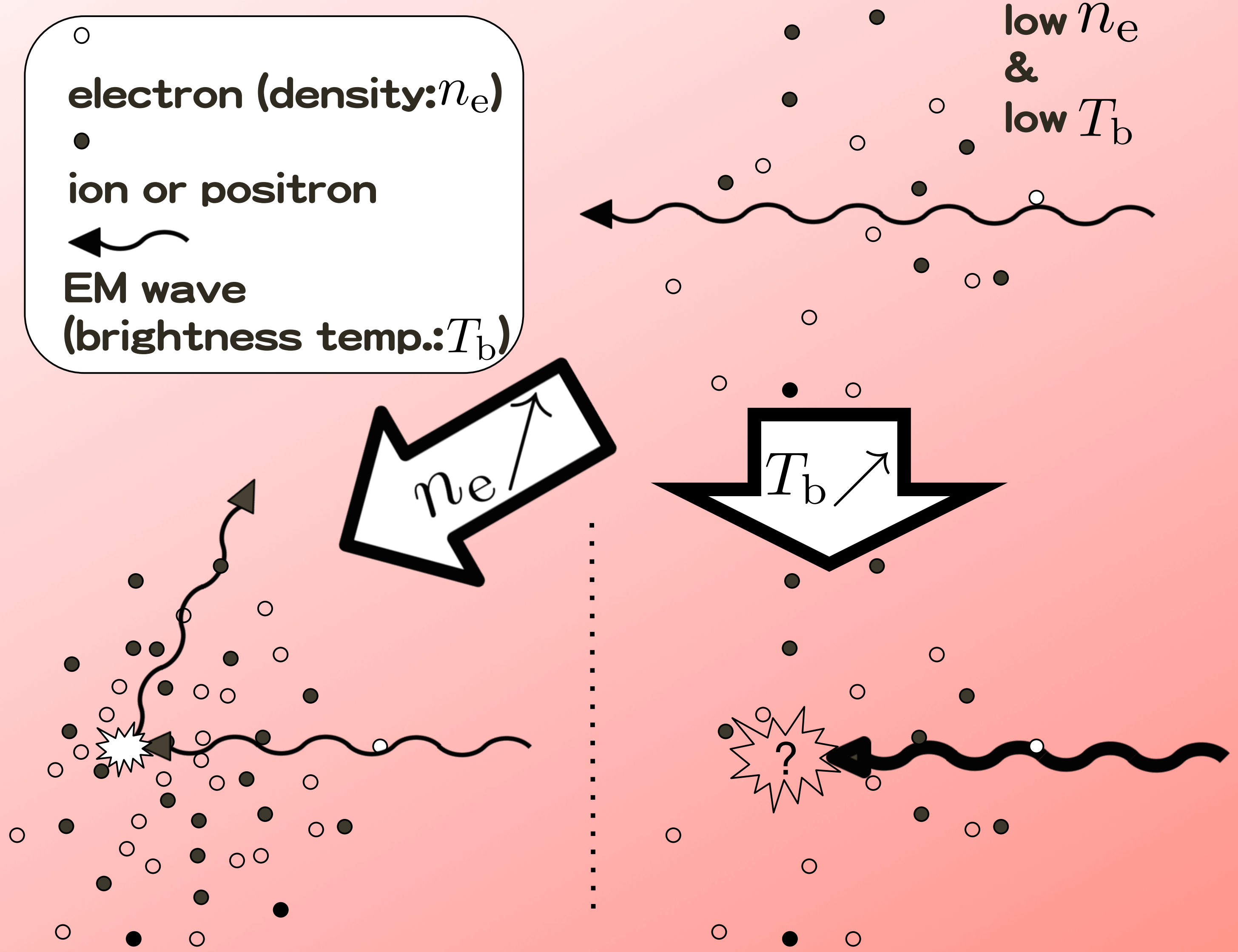
Optical depth for Spontaneous scattering
$\tau_{\rm spon}$ < 1 @ light cylinder

Correction for induced process
>10^{15} @10^8Hz for Crab
T_b: brightness temperature

Opening angle of radio beam. Can be estimated from pulse width.

This is when e$^\pm$s are at rest.

2-2: Schematic



Thomson scattering

Induced Compton scattering

2-3: Kinetic Equation for Photons

Compton scattering of photons $n_{ph}(\nu)$ by plasmas f(p).

$$\left(\frac{\partial}{\partial t} + c\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\right) n(\boldsymbol{k}) = c n_{\mathrm{pl}} \int d^3\boldsymbol{p} f(\boldsymbol{p}) \int \frac{d^3\boldsymbol{k}_1}{k_1^2} \sigma_{\mathrm{KN}}([\boldsymbol{k},\boldsymbol{k}_1],\boldsymbol{p})$$

$$\times \left[n(\boldsymbol{k}_1)(1+\underbrace{n(\boldsymbol{k})}_{\text{induced term}})\left(\frac{k_1}{k}\right)^2 - n(\boldsymbol{k})(1+\underbrace{n(\boldsymbol{k}_1)}_{\text{induced term}})\right]$$

Cross-section for Compton scattering
(Klein-Nishina formula)

$$\sigma_{\mathrm{KN}}(\boldsymbol{k}_{\mathrm{i}},\boldsymbol{k}_{\mathrm{f}},\boldsymbol{p}) = \frac{3\sigma_{\mathrm{T}}}{16\pi}\frac{1}{\gamma^2 D_{\mathrm{i}}^2}\left(\frac{k_{\mathrm{f}}}{k_{\mathrm{i}}}\right)^2 \boxed{\delta\left(k_{\mathrm{f}} - \frac{\gamma D_{\mathrm{i}} k_{\mathrm{i}}}{\gamma D_{\mathrm{f}} + k_{\mathrm{i}}(1-\mu)}\right)}$$

$$\times\left[1 + \left(1 - \frac{1-\mu}{\gamma^2 D_{\mathrm{i}} D_{\mathrm{f}}}\right)^2 + \frac{k_{\mathrm{i}} k_{\mathrm{f}}(1-\mu)^2}{\gamma^2 D_{\mathrm{i}} D_{\mathrm{f}}}\right]$$

Compton effect
(not symmetric!!)

$$D_{\mathrm{i,f}} = 1 - \boldsymbol{\beta}\cdot\boldsymbol{\Omega}_{\mathrm{i,f}} \quad \mu = \boldsymbol{\Omega}_{\mathrm{i}}\cdot\boldsymbol{\Omega}_{\mathrm{f}}$$

2-4: Kompaneets equation

uniform + isotropic + 1st order in $h\nu \ll m_e c^2$, $k_B T_e \ll m_e c^2$

Kompaneets 1957

$$\boxed{\frac{\partial n(x)}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}x^4\left(n(x) + n^2(x) + \frac{\partial n(x)}{\partial x}\right)}$$

$$\boxed{x \equiv \frac{h\nu}{k_\mathrm{B}T_\mathrm{pl}}, y \equiv \frac{k_\mathrm{B}T_\mathrm{pl}}{m_\mathrm{e}c^2}n_\mathrm{pl}\sigma_\mathrm{T}ct}$$

- Photon number conservation
- Bose-Einstein distribution as equilibrium solution
- No Thomson scatt. (0^{th} order) because of isotropy.
- 1st term = Compton effect (energy loss for photon)

$$\tau_\mathrm{Comp} \approx \sigma_\mathrm{T} l n_\mathrm{pl} \times \frac{h\nu}{m_\mathrm{e}c^2}$$

- 2nd term = Induced Compton (energy loss for photon)

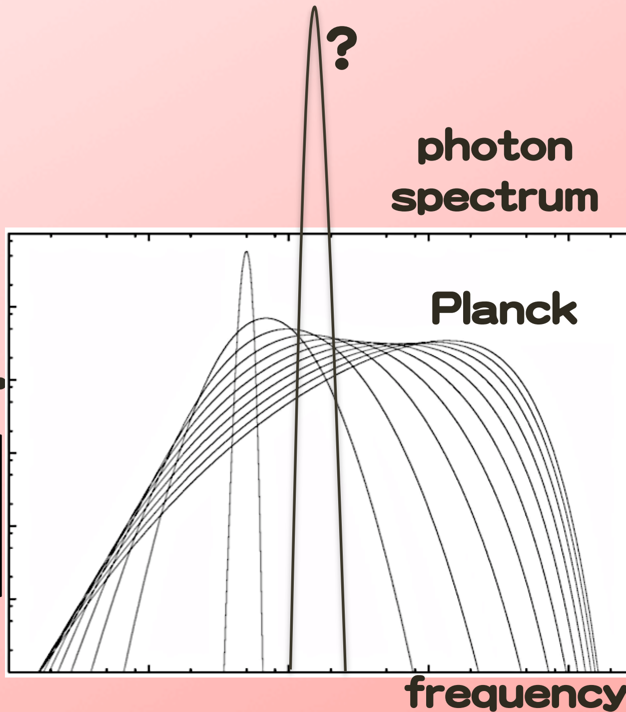
$$\tau_\mathrm{ind} \approx \sigma_\mathrm{T} l n_\mathrm{pl} \times \frac{k_\mathrm{B}T_\mathrm{b}(\nu)}{m_\mathrm{e}c^2}$$

- 3rd term = Inverse Compton (energy gain for photon)

$$\tau_\mathrm{IC} \approx \sigma_\mathrm{T} l n_\mathrm{pl} \times \frac{k_\mathrm{B}T_\mathrm{e}}{m_\mathrm{e}c^2} \approx y$$

No induced inverse Compton term.

2-5: Difficulty of ICS



A well-known difficulty to describe evolution of n_{ph} for $n_{ph} \gg 1$ with Kompaneets equation.

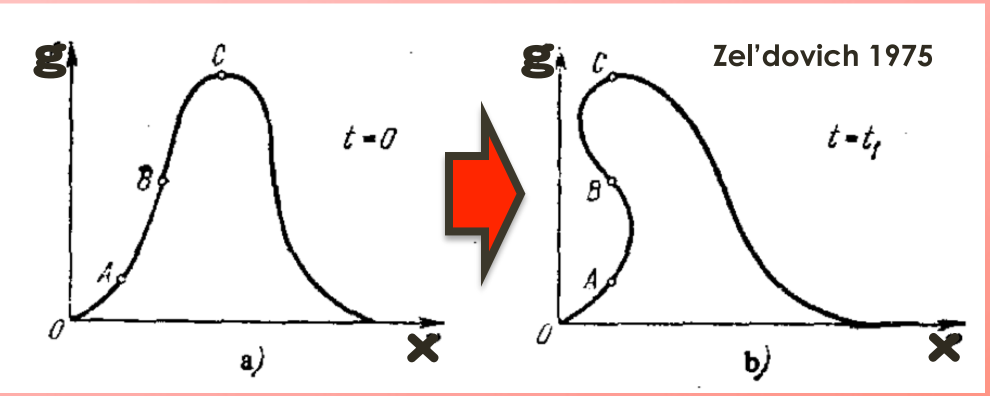
$$\boxed{\frac{\partial n(x)}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}x^4\left(\cancel{n(x)} + n^2(x) + \cancel{\frac{\partial n(x)}{\partial x}}\right)}$$

if $n_{ph} \gg 1$

$(g = x^2 n)$

$$\boxed{\frac{\partial g}{\partial y} - 2g\frac{\partial g}{\partial x} = 0}$$

This is a nonlinear convection equation, which has an unphysical solution!



There is a similar problem in Euler equation of hydrodynamics

Contents

1. Situations

2. Semi-classical formulation of
   induced Compton scattering (ICS)

3. Results & summary

3-1: Higher-order Kompaneets equation

$^{=3}$

1st - 2nd order in $(k, T_*) \equiv \left(\frac{h\nu}{m_\mathrm{e} c^2}, \frac{k_\mathrm{B} T_\mathrm{pl}}{m_\mathrm{e} c^2} \right)$

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x} x^4 (n(x) + n^{(1)}(x) + n^2(x))$$

$$\boxed{\begin{aligned} &+ \frac{T_*}{10}\frac{1}{x^2}\frac{\partial}{\partial x}[7x^6(n^{(3)} + 2n^{(2)} + n^{(1)} + 2(n^2)^{(2)} - 6(n^{(1)})^2) \\ &+ 42x^5(n^{(2)} + n^{(1)} + (n^2)^{(1)}) + 25x^4(n^{(1)} + n + n^2)] \end{aligned}}$$

Number conservation => OK, Bose-Einstein distribution => OK.

(the result is consistent with Challinor&Lasenby98)

For $n_{ph} \gg 1$, how this equation reduced?

3-2: The case for $n_{ph} \gg 1$

$$\boxed{n\Theta \times \min(1, x) \gg 1}$$

$$\boxed{\frac{\partial g}{\partial y} - 2g\frac{\partial g}{\partial x} = 0} \xrightarrow{\text{2nd order}} \boxed{\frac{\partial g}{\partial y} - 2g\frac{\partial g}{\partial x} + \frac{17\Theta}{5}g\frac{\partial}{\partial x}g = \frac{14\Theta}{5}(xg)\frac{\partial^3}{\partial x^3}(xg)}$$

$$\frac{\partial g}{\partial y} - 2g\frac{\partial g}{\partial x} \approx \underbrace{\frac{14\Theta}{5}(xg)\frac{\partial^3}{\partial x^3}(xg)}_{\text{order of } (h\nu)\Theta/(m_e c^2)^2}$$

$$x = \frac{h\nu}{k_B T_e}, \quad \Theta = \frac{k_B T_e}{m_e c^2}, \quad g(x) = x^2 n(x)$$

Dispersive term! => soliton formation?
However, coefficient depends on g(x)

3-3: Steady State Solutions

$$\frac{1}{x^2}\frac{\partial}{\partial x}x^4\left(n(x)+n^2(x)+\frac{\partial n(x)}{\partial x}\right)=0 \qquad n(x)=(1-e^{x-\mu_c})^{-1}$$

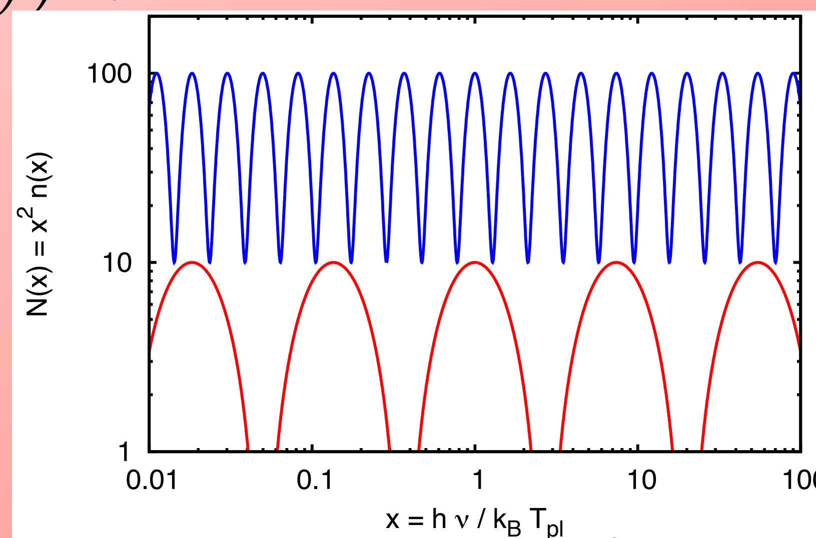
$$\frac{1}{x^2}\frac{\partial}{\partial x}x^4\left(n(x)+\frac{\partial n(x)}{\partial x}\right)=0 \;\; n(x)=e^{-x+\mu_c} \qquad \frac{1}{x^2}\frac{\partial}{\partial x}x^4 n^2(x)=0 \;\; n(x)\propto x^{-2}$$

$$\frac{1+k_\Theta^2}{x^2}\frac{\partial}{\partial x}x^4 n^2(x)+(x^3 n(x))\frac{\partial^3}{\partial x^3}x^3 n(x)=0$$

$$g(x)=A\cos(k_\Theta \ln x+\phi))+B$$

$$k_\Theta^2 \equiv \frac{5}{7\Theta}-\frac{31}{14}\gg 1$$

Integration constants,
A : Amplitude of wave
B : DC component
ϕ: phase of wave

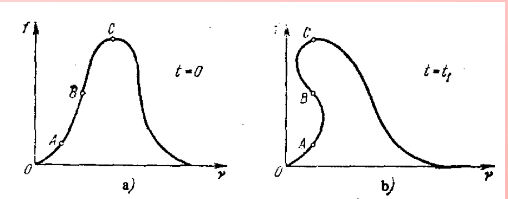


SJT+15 PTEP

$$\lambda_\Theta \equiv \frac{2\pi}{k_\Theta}$$

3-4: Numerical Calculation

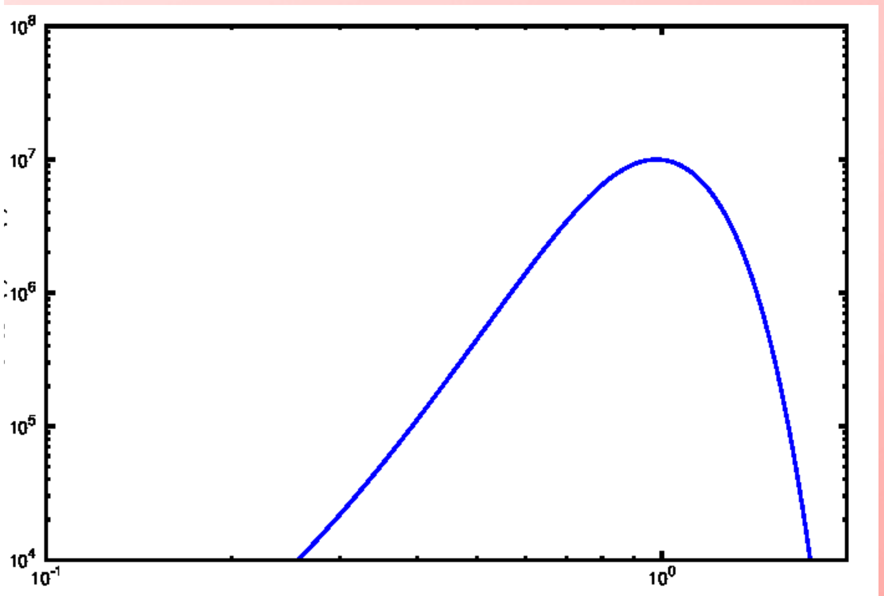
$$\frac{\partial g}{\partial y} = \frac{14\Theta}{5} g \left(x \frac{\partial^3}{\partial x^3}(xg) + (1 + k_\Theta^2)\frac{\partial g}{\partial x} \right)$$



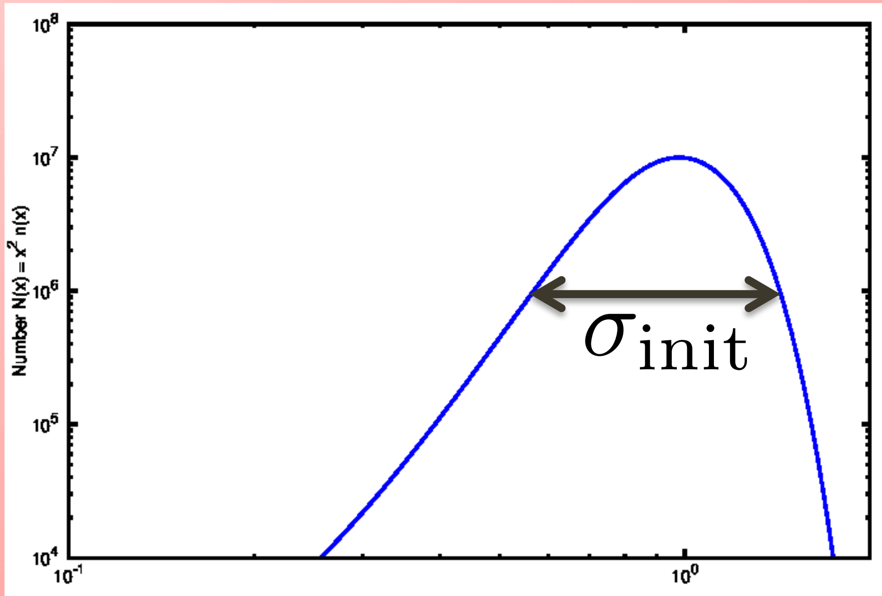
$\Theta = 4 \times 10^{-5}$ (20eV, $\lambda_\Theta = 0.1$)

$\Theta = 2 \times 10^{-4}$ (10^2eV, $\lambda_\Theta = 0.05$)

g(x): Photon Number



x: Normalized Freq.

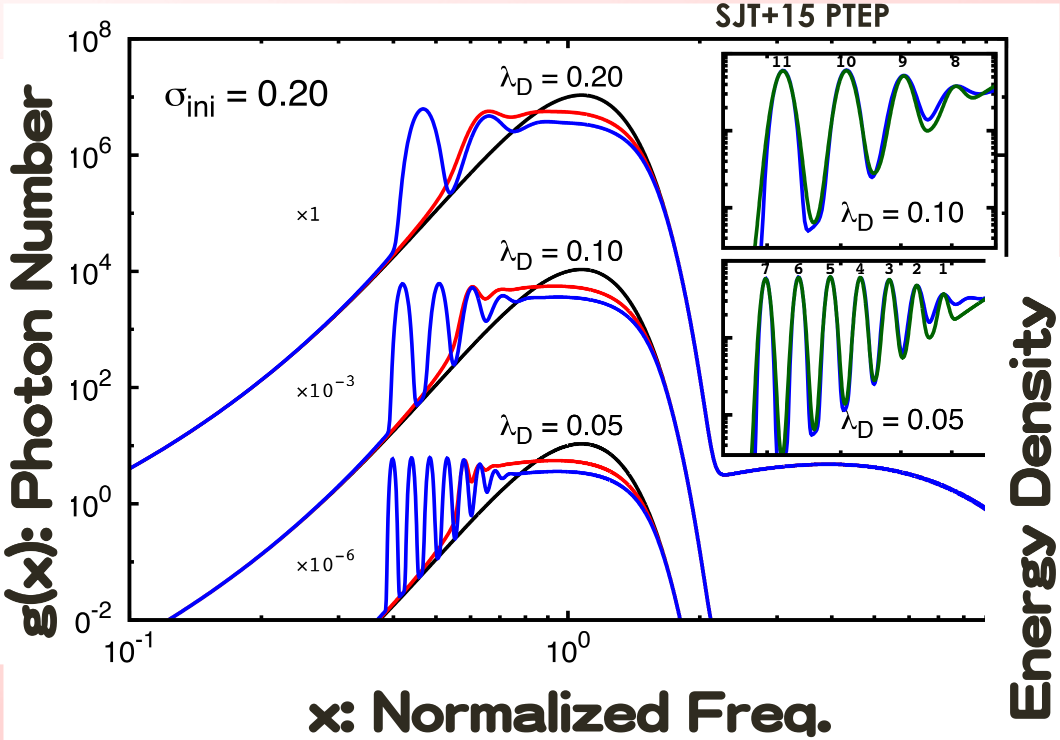


x: Normalized Freq.

3-5: Results

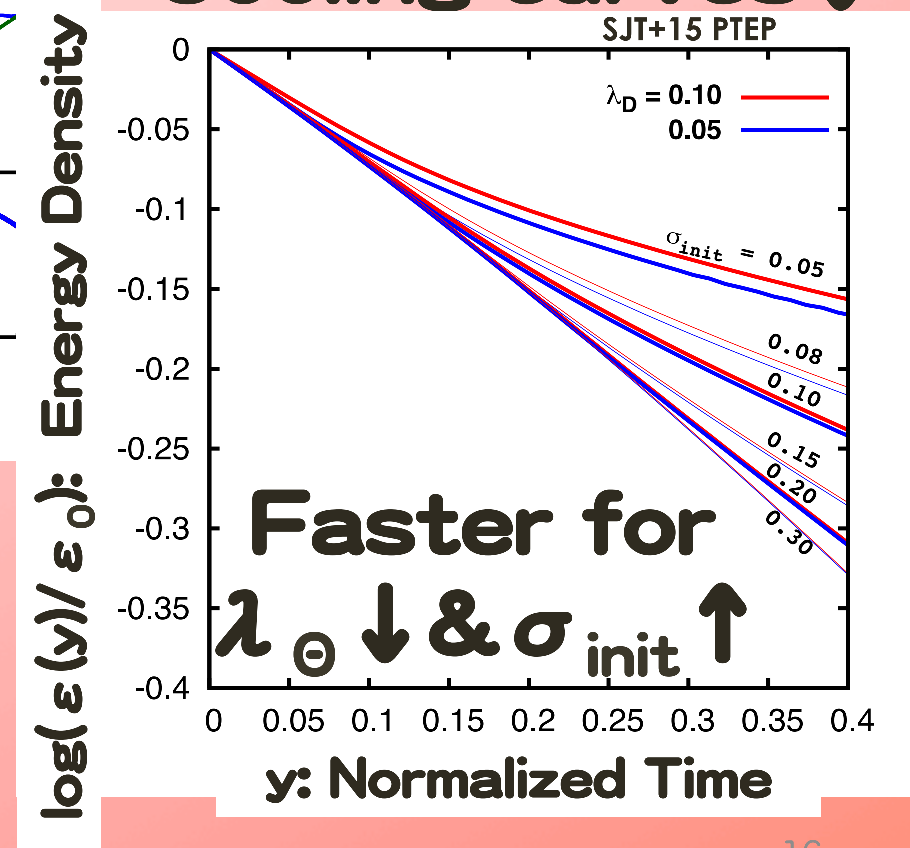
$$x \equiv \frac{h\nu}{k_B T_{pl}}, y \equiv \frac{k_B T_{pl}}{m_e c^2} n_{pl} \sigma_T c t$$

$$\epsilon(y) \equiv \int x^3 n(x, y) dx$$



↑Solitary structures of logarithmic width

Cooling curves↓



Faster for λ_Θ↓&σ_{init}↑

まとめ

- パルサーは極限プラズマ現象の宝庫.
  - パルサーからの放射などで, 自発より誘導コンプトン散乱が効く現象が存在しうる.
  - パルサー風プラズマ密度の制限などが可能

e.g., Wilson & Rees 78, ST & Takahara 13

- 誘導散乱が卓越する場合の光子スペクトルの変化.
  - Kompaneets 方程式の高次展開の式を導出した.
- 非線形分散項による, 不連続の回避が起こるはず.
- レーザー実験による検証が野望.