

基研研究会「熱場の量子論とその応用」

31 August 2015

Yukawa Institute, Kyoto University

冷却原子系による量子シミュレーション

Kyoto University



Yoshiro Takahashi

Outline

I) Preparation of Quantum Gas

Laser cooling and trapping, evaporative cooling, Bose-Einstein condensate, Fermi Degenerate Gas

II) Ultracold Atoms in a Harmonic Trap

Feshbach resonance, Cooper pairing, BEC-BCS crossover, unitary gas, spin-orbit interaction

III) Ultracold Atoms in an Optical Lattice

Superfluid-Mott insulator transition, quantum-gas-microscope, Higgs mode, Non-standard lattices (frustrated magnetism, flat band), Fermi-Hubbard model, Bose-Fermi mixture

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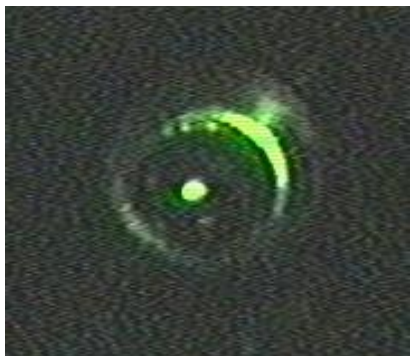
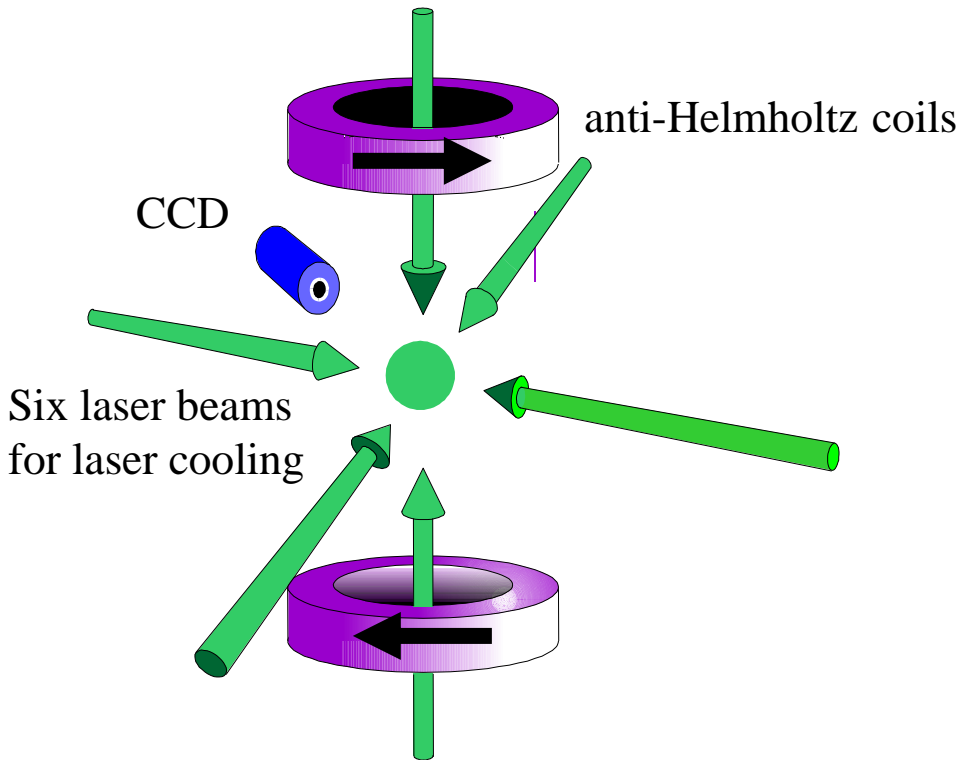
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Laser Cooling and Trapping



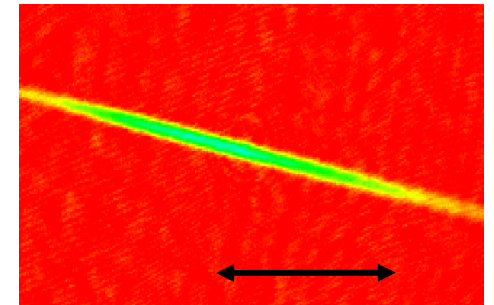
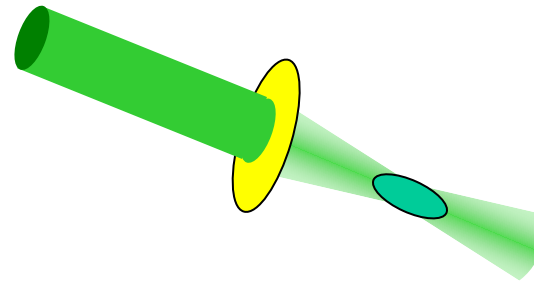
- Number: 10^7
- Density: $10^{11}/\text{cm}^3$
- Temperature: $10\mu\text{K}$

“Magneto-optical Trap”

“optical trap”

$$V_{\text{int}} = -p \cdot E$$

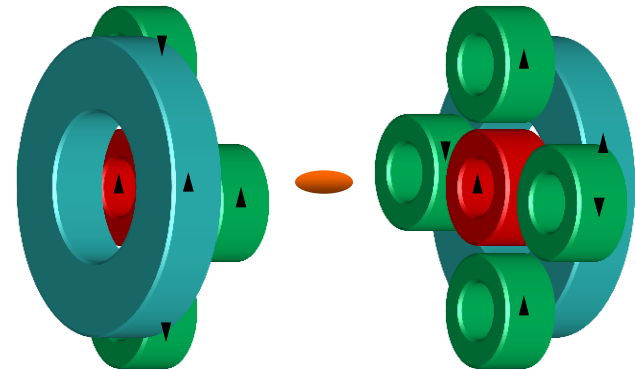
$$U_{\text{pot}}(r) = -\frac{\chi E(r)^2}{2}$$



500 μm

“magnetic trap”

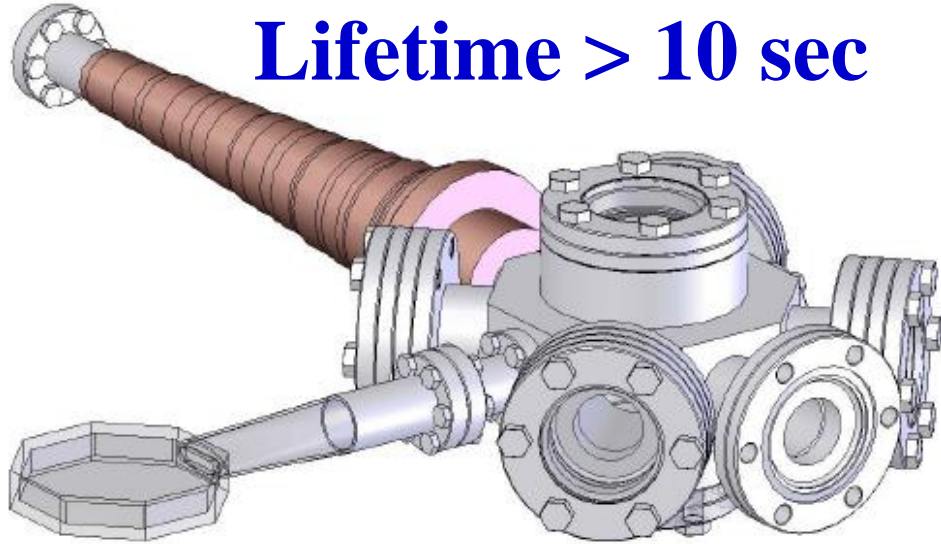
$$V_{\text{int}} = -\mu \cdot B$$



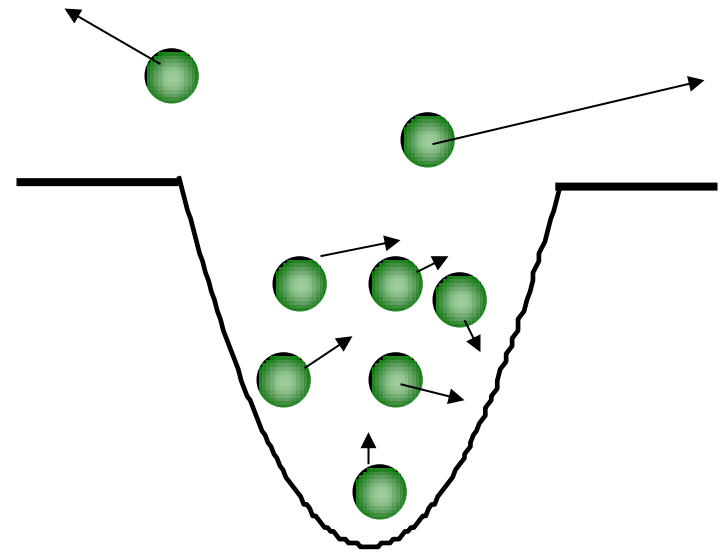
Cooling to Quantum Degeneracy

Pressure $\sim 10^{-12}$ torr

Lifetime > 10 sec

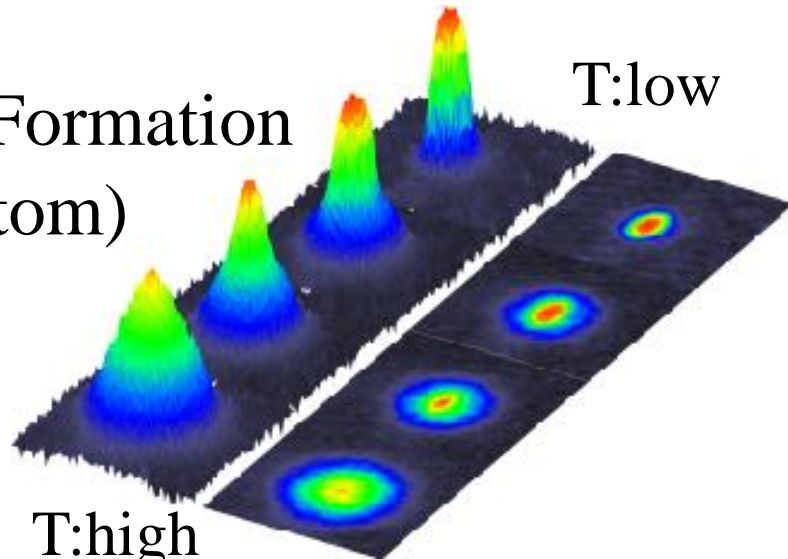


“Evaporative cooling”



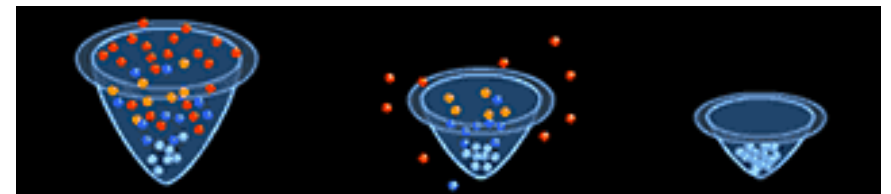
BEC Formation
(Yb atom)

T:low



T:high

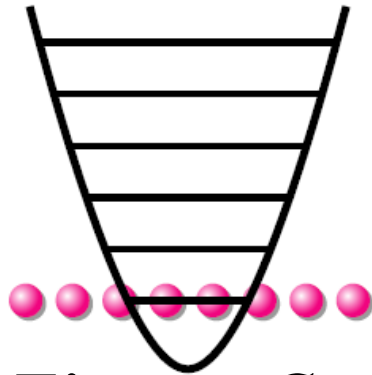
Momentum distribution



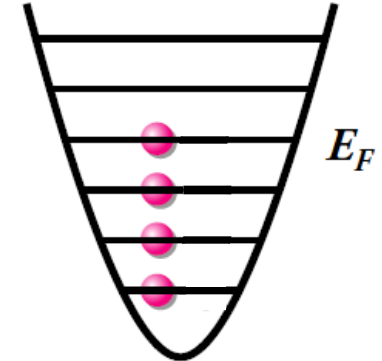
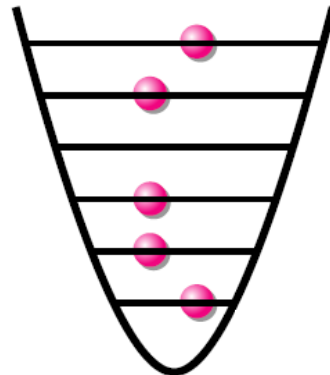
T ~ 100 nK N ~ 10⁵

Cooling to Quantum Degeneracy

“Boson versus Fermion”

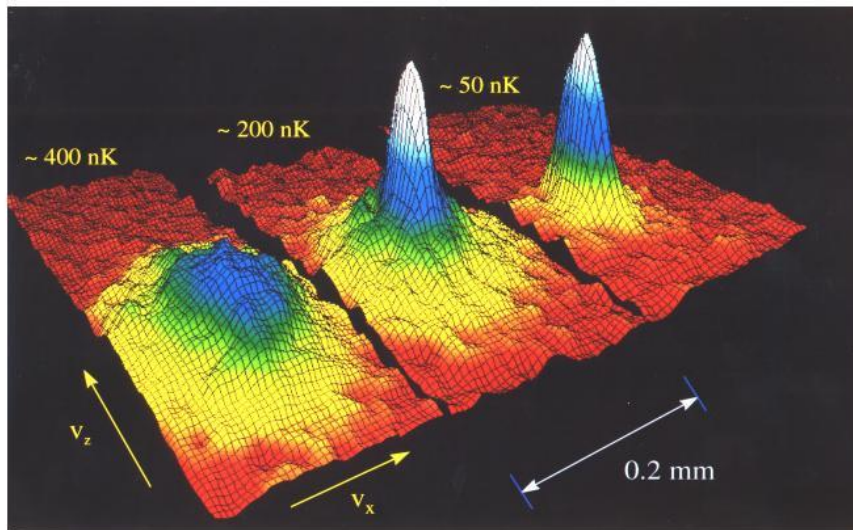


“Bose-Einstein Condensation”



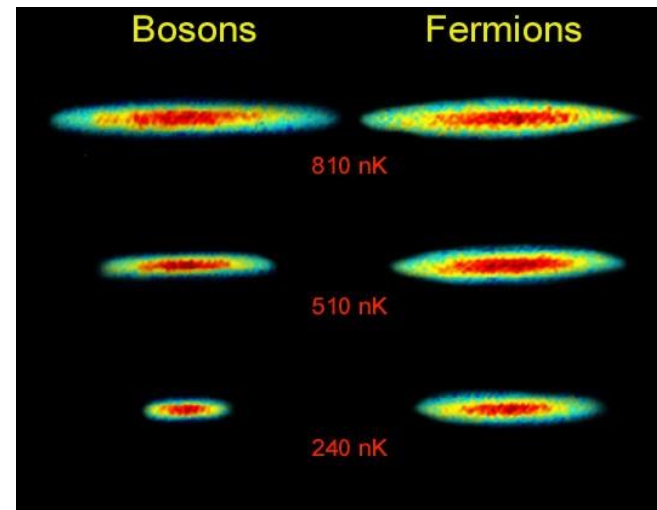
“Fermi Degeneracy”

^{87}Rb



Momentum Distribution

[E. Cornell et al, (1995)]



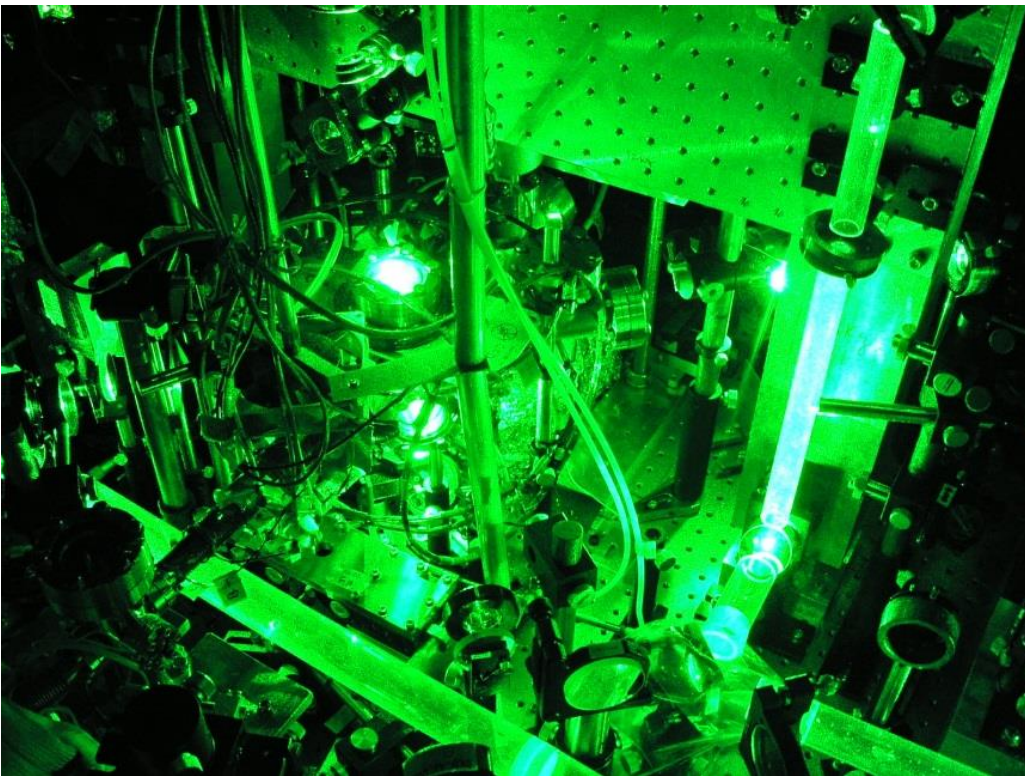
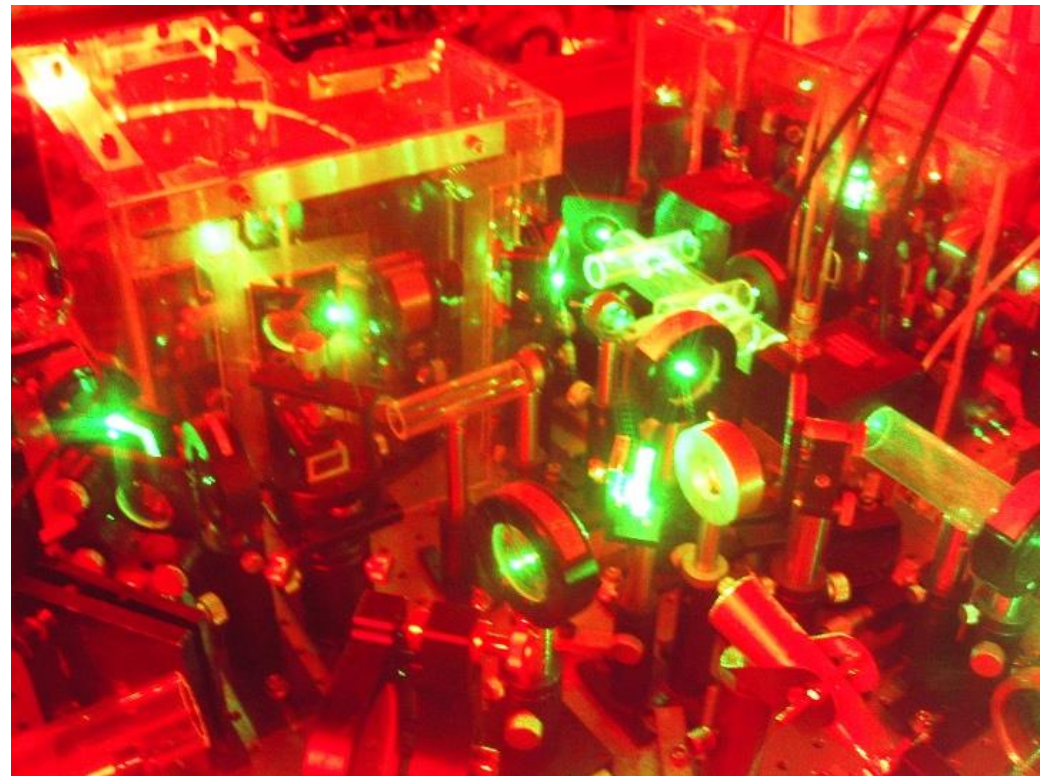
^6Li and ^7Li

Fermi
Pressure

Spatial Distribution

[R. Hulet et al, (2000)]

Experimental Setup for Cold Atom



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Feshbach Resonance:

ability to tune an inter-atomic interaction

Collision is in Quantum Regime

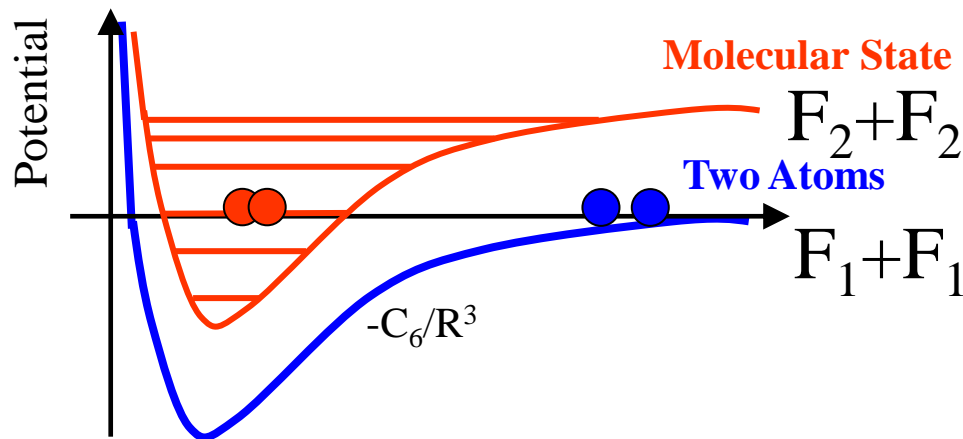
It is described by s-wave scattering length a_s

$$a_s = -\delta_l / k$$

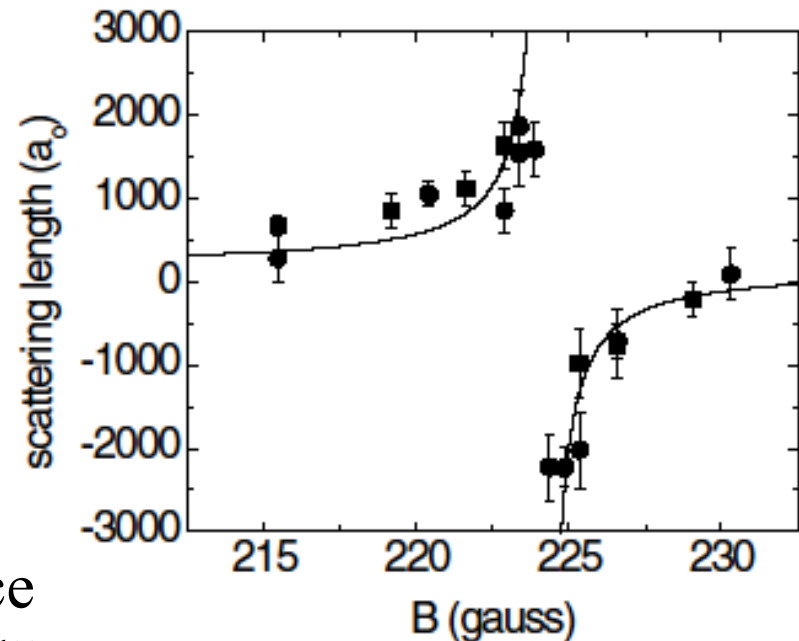
$$\sigma_0 = 4\pi |f_0|^2 = 4\pi |a_s|^2$$

Coupling between “Open Channel” and “Closed Channel”

Control of Interaction(a_s)



$$a_s(B) = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right)$$

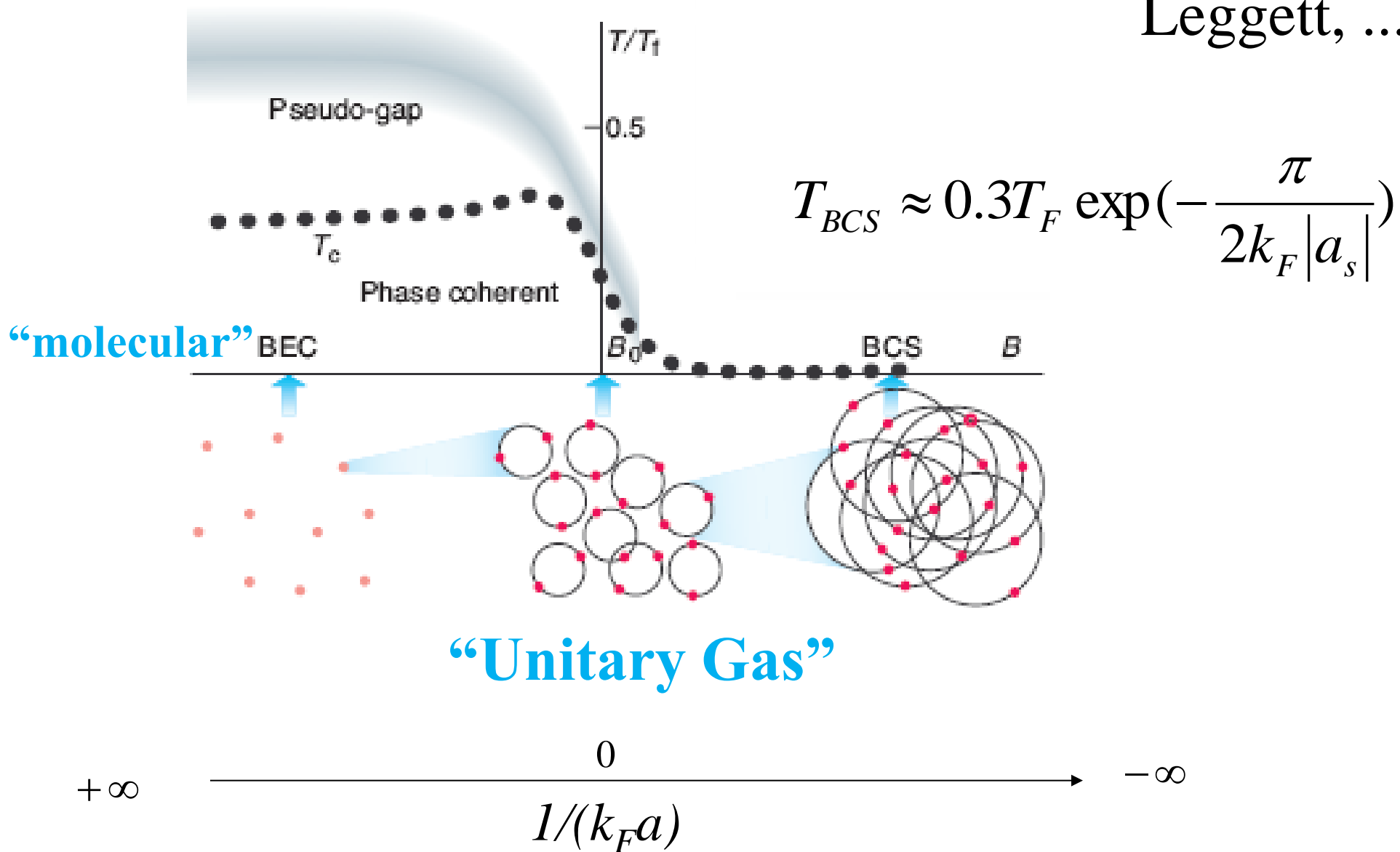


“Magnetic field tunes the energy difference between closed channel and open channel”

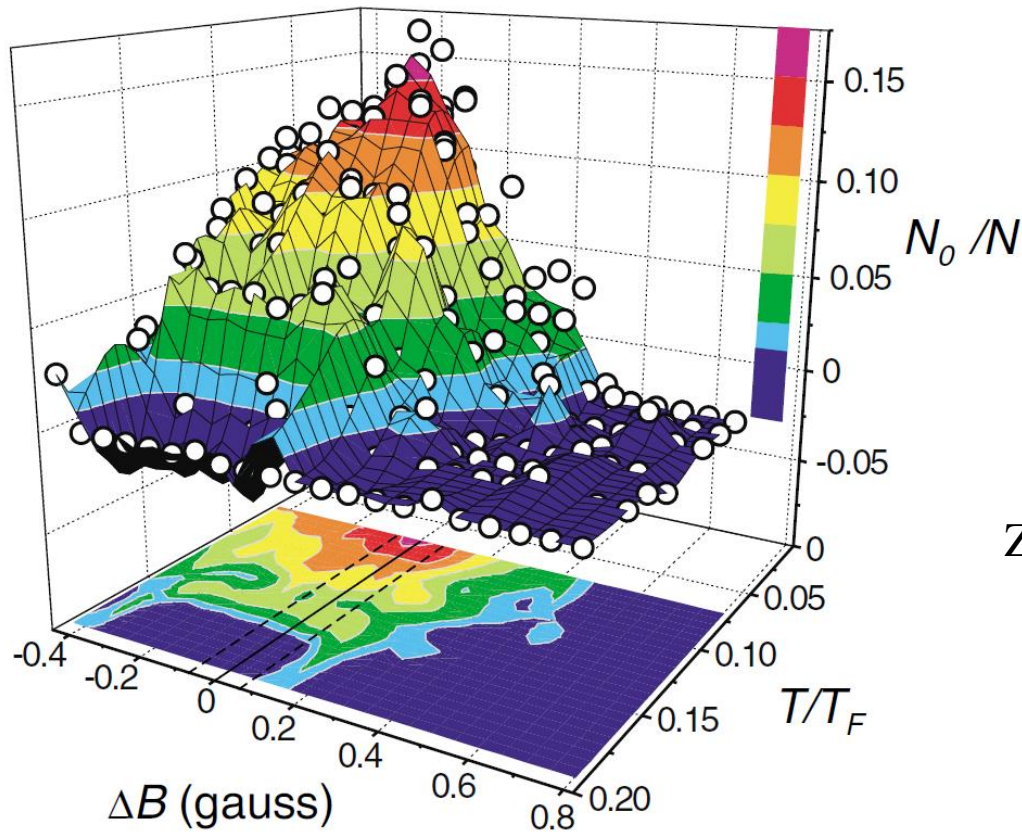
[C. Regal and D. Jin, PRL90, 230404(2003)]

BEC – BCS Crossover

Leggett, ...

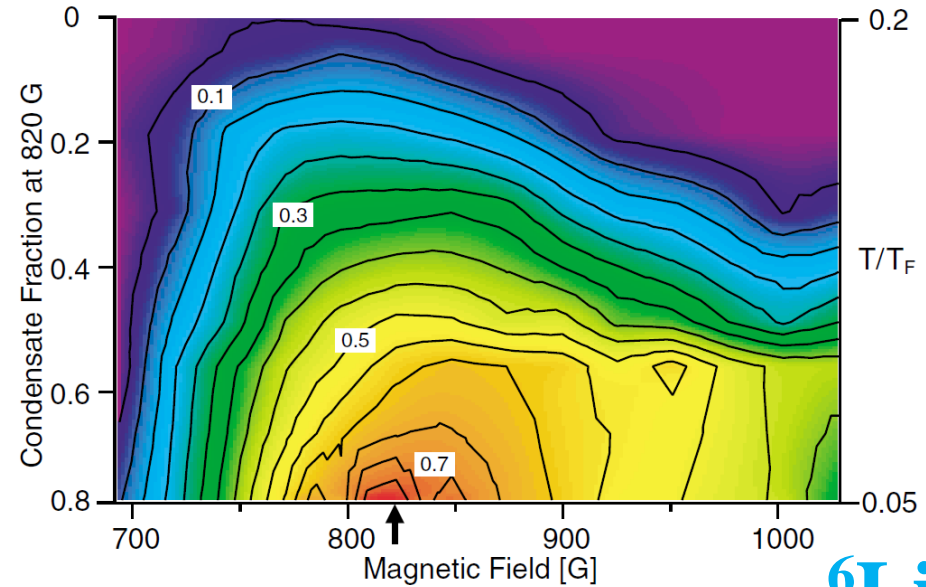


BEC – BCS Crossover: experiments



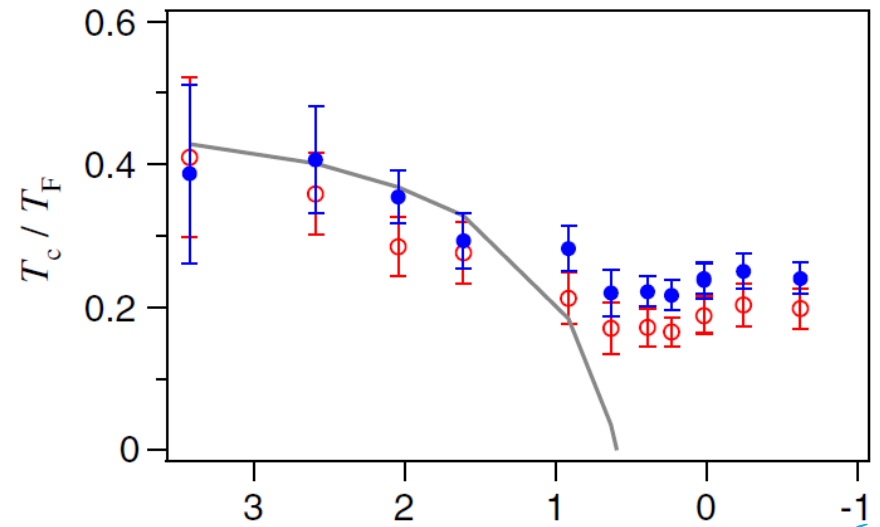
Regal et al., PRL(2004)

^{40}K



Zwierlein et al., PRL(2004)

^6Li



Inada et al., PRL(2008)

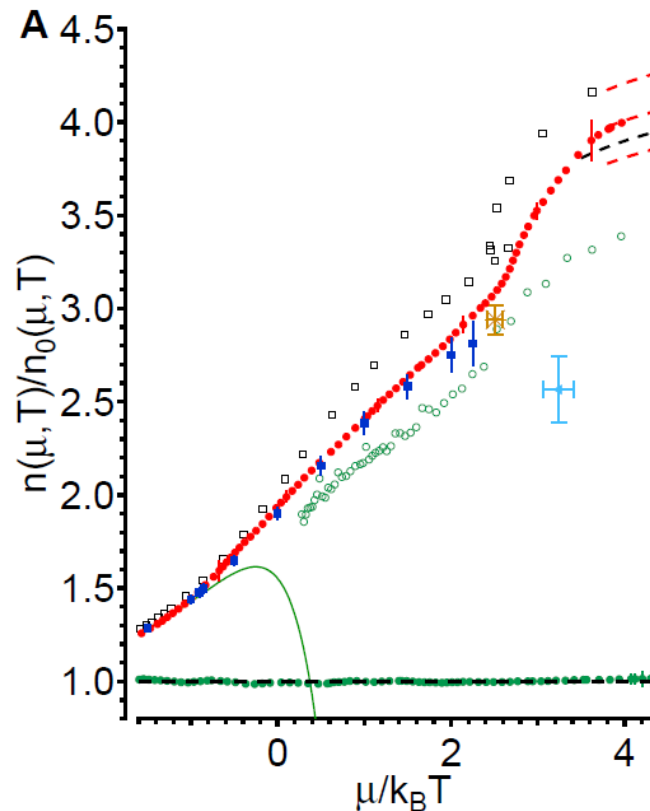
^6Li

Equation of State for Unitary Gas

M. J. H. Ku *et al.*, (2011): MIT Zwierlein Group

$$\text{EoS} \quad n(\mu, T) \equiv \frac{1}{\lambda^3} f_n(\beta\mu)$$

Density



- : experimental EoS
- : Monte Carlo
- : 3rd order Virial expansion
- : self-consistent T-matrix
- ◆ : ENS experiment
- ◇ : Tokyo experiment

^6Li


Spin-Orbit Interaction in Cold Atoms:

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} - \frac{g\mu_B}{\hbar} \mathbf{S} \cdot (\mathbf{B}^{(D)} + \mathbf{B}^{(R)} + \mathbf{B}^{(Z)}), \quad \begin{aligned} \mathbf{B}^{(R)} &= \alpha(-k_y, k_x, 0) \\ \mathbf{B}^{(D)} &= \beta(k_y, k_x, 0) \end{aligned}$$


$$\alpha = \beta: \text{SOI} \propto \sigma_y k_x$$

Y. -J. Lin, et al., Nature 471, 83(2011)

“lasers for Raman Transition”



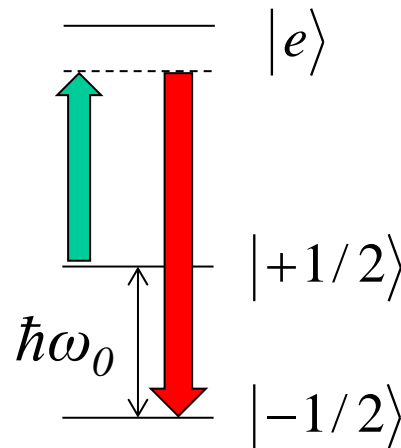
$$E = E_2 \cos(k_2 x - \omega_2 t)$$



$$E = E_1 \cos(k_1 x - \omega_1 t)$$

$$\delta = \omega_0 - (\omega_1 - \omega_2) : \text{detuning}$$

$$Q = k_1 - k_2 : \text{momentum transfer}$$



“pseudo-spin +1/2”:

$$\frac{\hbar\omega_0}{2} \hat{\sigma}_z = \frac{\hbar\omega_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ after the local pseudo-spin rotation with $\theta = Qx$ about z-axis $\hat{U} = \exp(i \frac{Qx}{2} \hat{\sigma}_z)$

$$\hat{H} = \frac{\hbar^2 k^2}{2m} \hat{1} + \underbrace{\frac{\hbar\delta}{2} \hat{\sigma}_y}_{\mathbf{B}_z} + \frac{\hbar\Omega}{2} \hat{\sigma}_z + \underbrace{\hbar Q \frac{\hbar k_x}{2m} \hat{\sigma}_y}_{\text{SOI}} + \frac{E_R}{4} \hat{1}$$

$$E_R = \frac{\hbar^2 Q^2}{2m}$$

:Recoil Energy

\mathbf{B}_z

SOI

Spin-Orbit Interaction in Cold Atoms:

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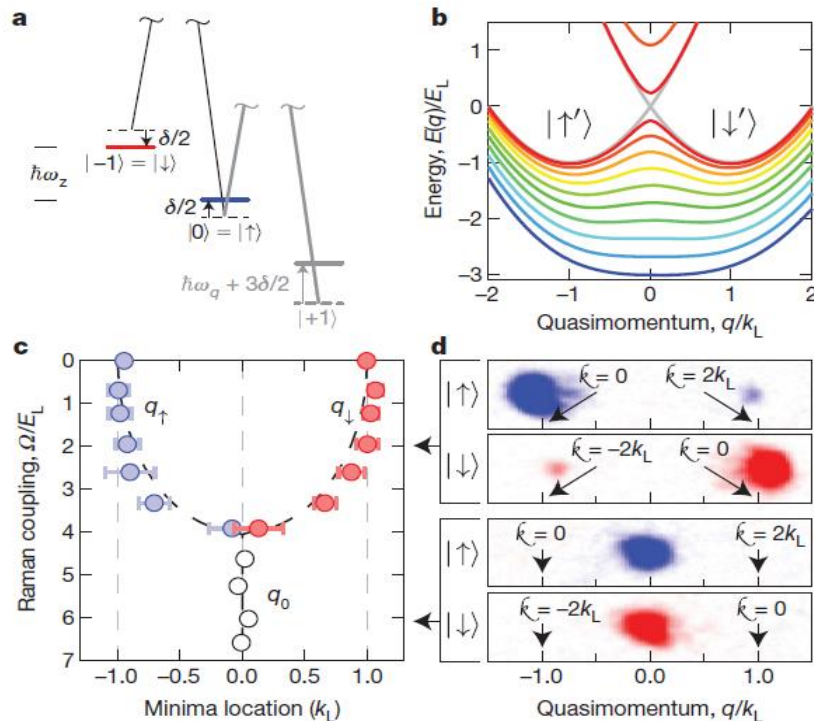
$$\mathbf{B}^{(D)} = \beta(k_y, k_x, 0)$$

$$\alpha = \beta: \text{SOI} \propto \sigma_y k_x$$

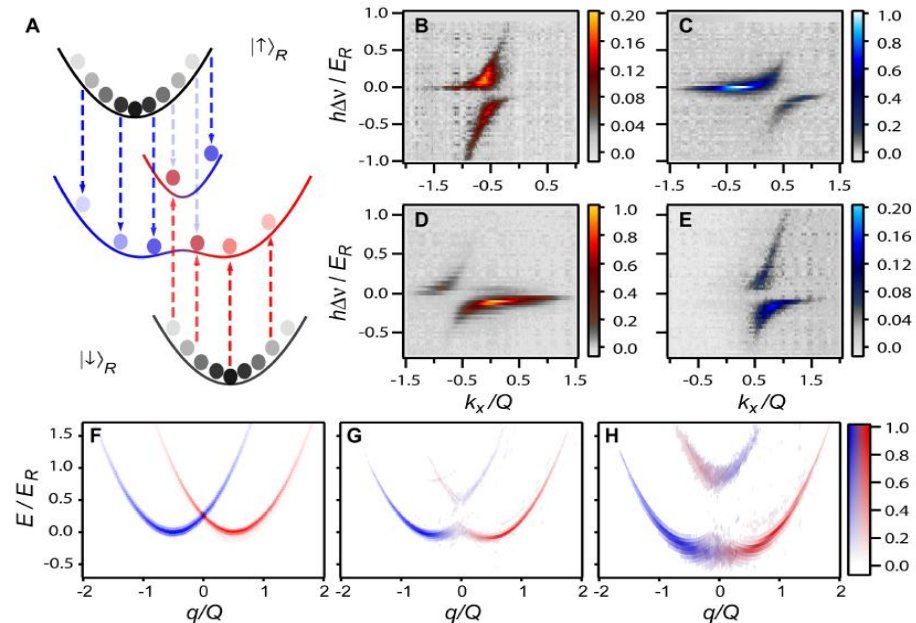
Y. -J. Lin, et al., Nature 471, 83(2011)

P. Wang et al., (2012)

L. W. Cheuk et al., (2012)



“Boson: ^{87}Rb ”



“Fermion: ^6Li , ^{40}K ”

Topological Superfluids, Majorana Edge state

by Spin-Orbit Interaction + Strong s-Wave Interaction



[M. Sato *et al.*, PRL103,020401(2009), PRB82, 134521(2010)]



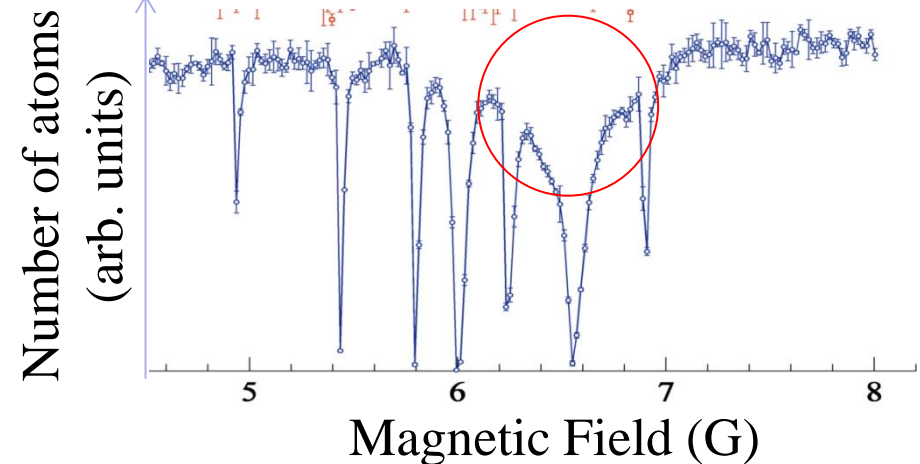
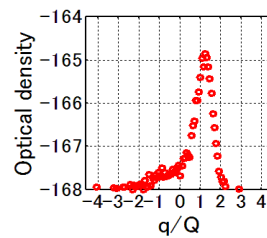
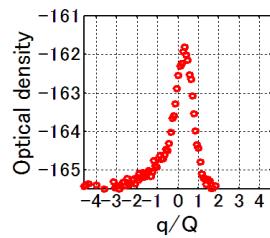
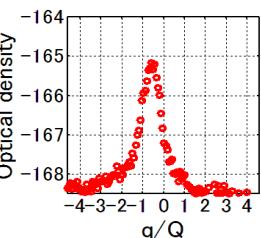
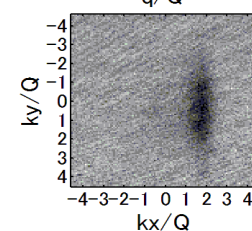
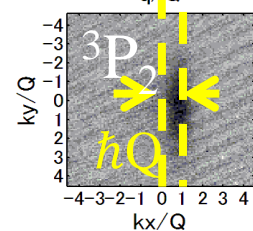
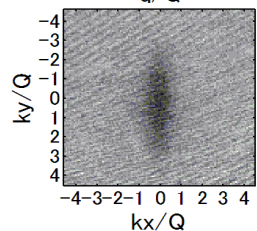
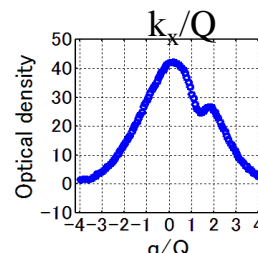
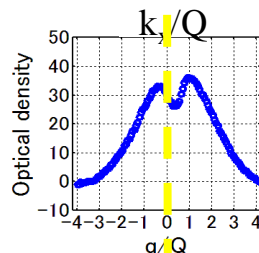
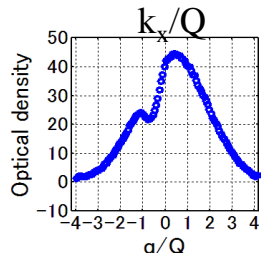
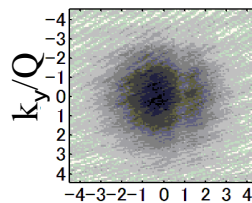
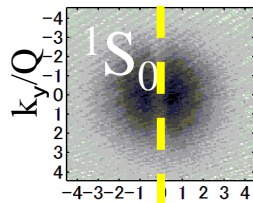
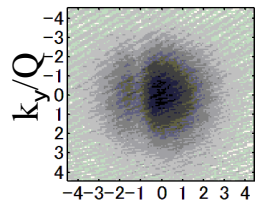
SOI between $^1S_0 + ^3P_2$ (^{174}Yb)

Feshbach Resonance: $^1S_0 + ^3P_2$ (^{171}Yb)

$\delta/2\pi = -8\text{kHz}$

$\delta/2\pi = 0\text{kHz}$

$\delta/2\pi = 8\text{kHz}$

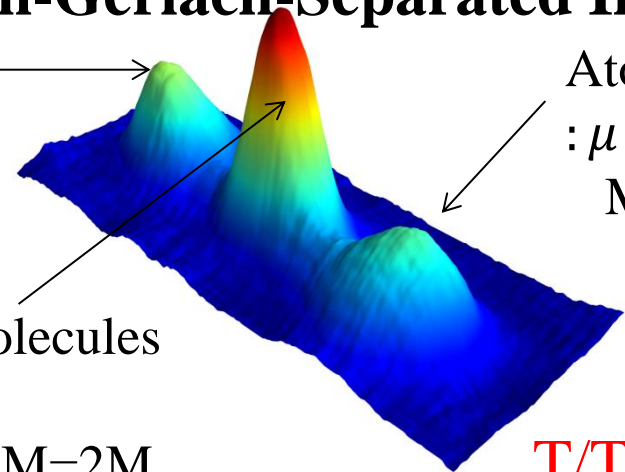


Stern-Gerlach-Separated Images

Atoms(1S_0)
: $\mu = 0$,
 $M = M_a$

Atoms(3P_2)
: $\mu = \mu_a(^3P_2)$,
 $M = M_a$

Feshbach Molecules
($^1S_0 + ^3P_2$):
 $\mu \approx \mu_a(^3P_2)$, $M = 2M_a$



$T/T_F \sim 0.25$

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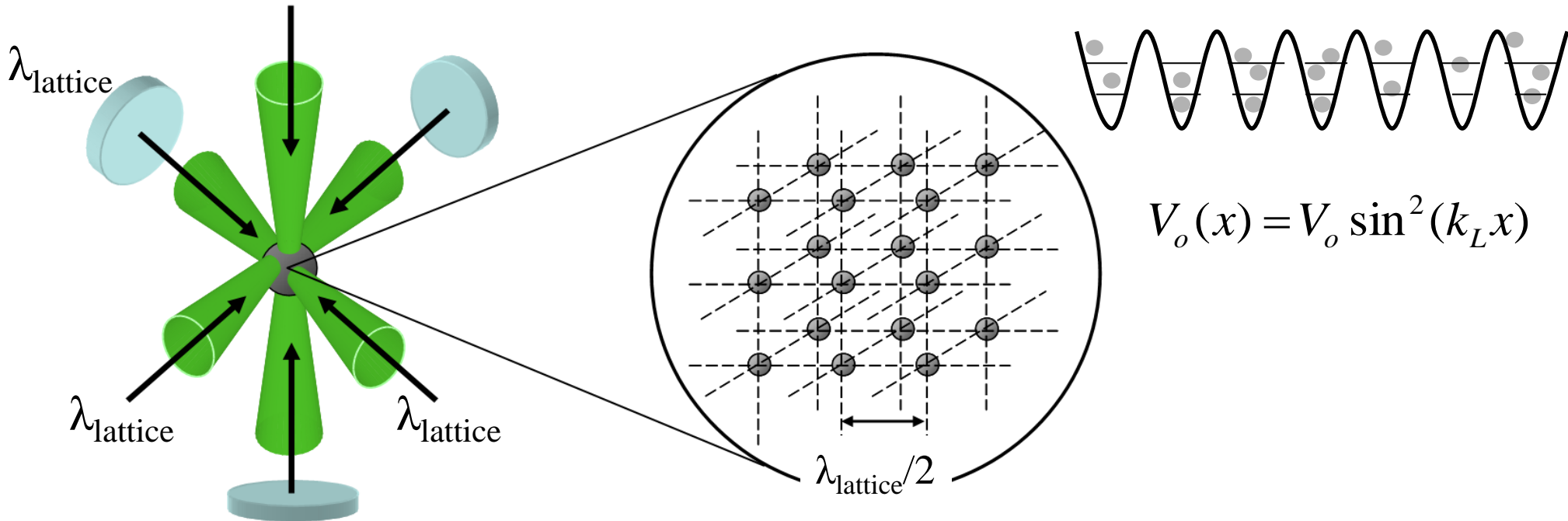
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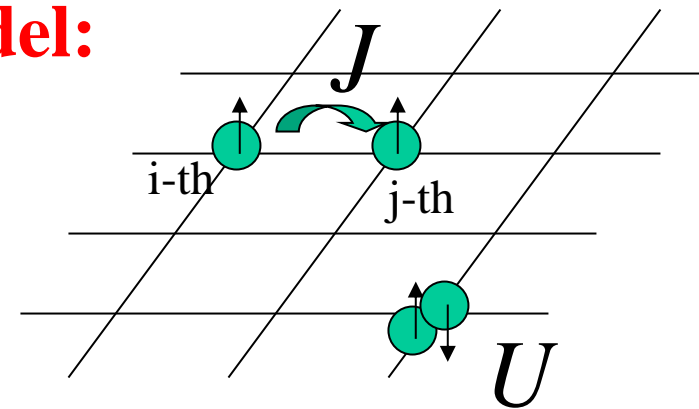
Quantum Simulation of Strongly Correlated Electron System



“ultracold atoms in an optical lattice”

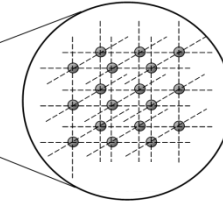
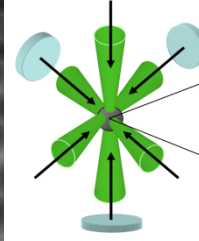
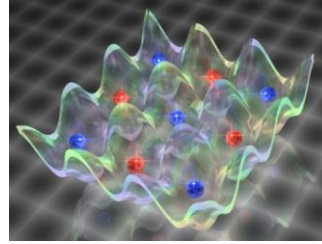
ideal Quantum Simulator of **Hubbard Model**:

$$H = -J \sum_{\langle i, j \rangle} c_i^+ c_j + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



Nice Features of ultracold Atoms in an Optical lattice

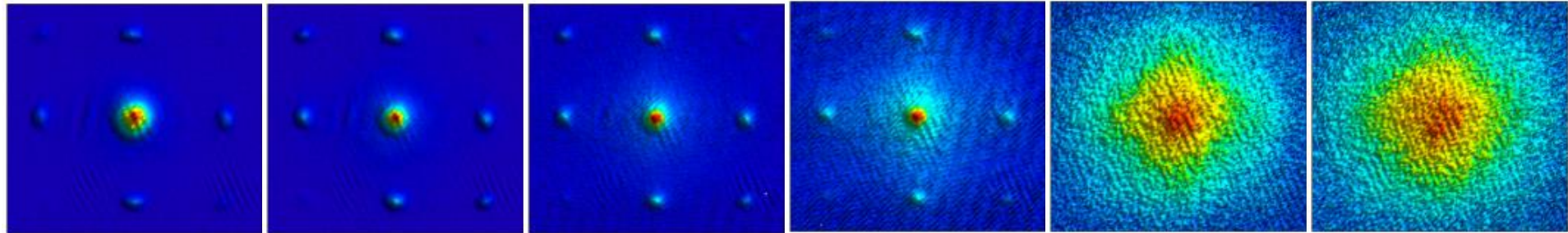
$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$



$$V = V_o \sin^2(kx)$$

- 1) Macroscopic Quantum System: typically $\sim 10^5$
- 2) Clean (no impurity, no lattice defects)
- 3) High controllability of Hubbard parameters

Small \leftarrow U/J \rightarrow Large

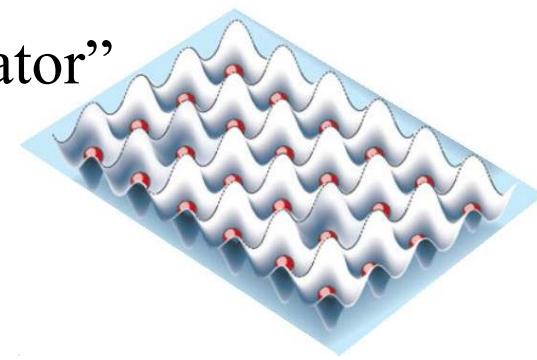
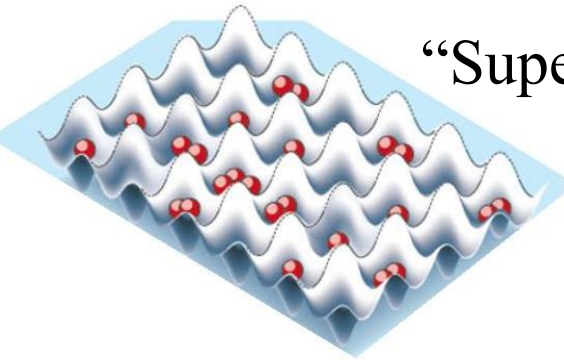


Weakly-interacting

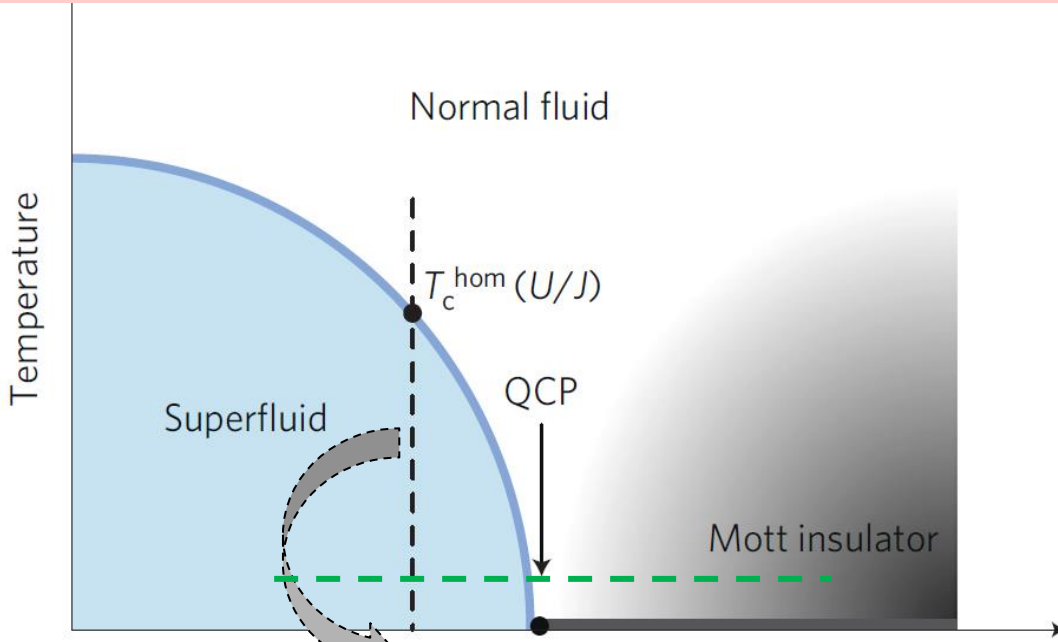
Strongly-correlated

“Superfluid”

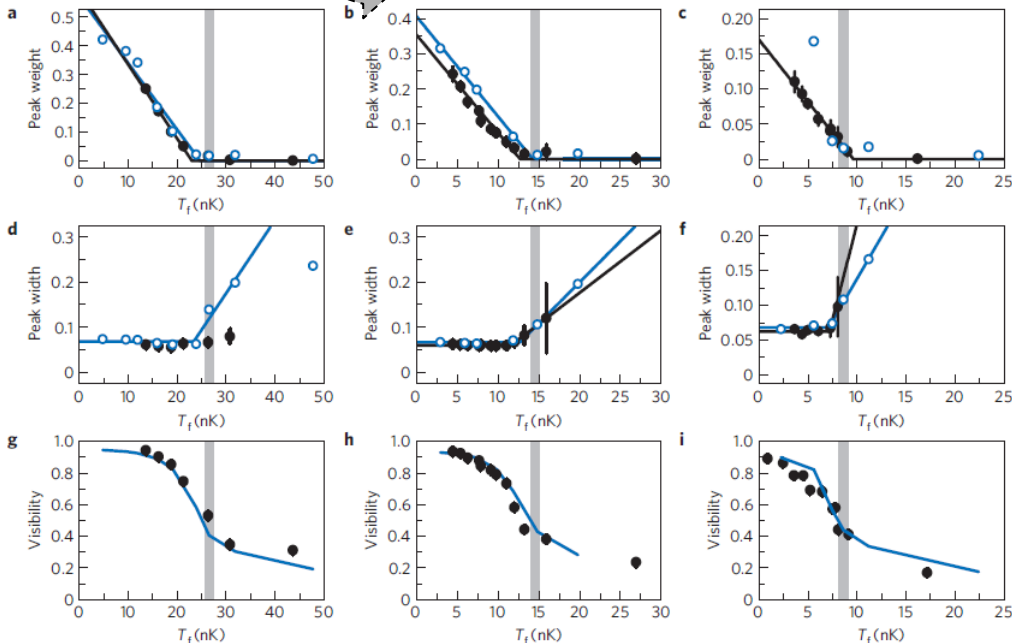
“Mott Insulator”



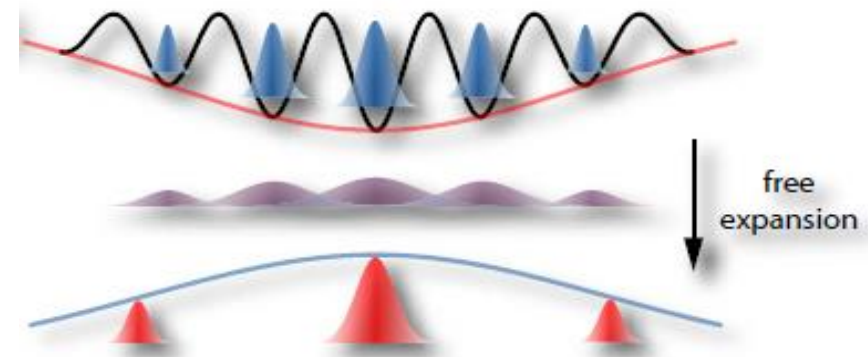
Phase Diagram of Bose-Hubbard Model ($T > 0$)



$$\begin{aligned}
 H = & -J \sum_{\langle i,j \rangle} a_i^+ a_j \\
 & + \frac{U}{2} \sum_i n_i (n_i - 1) \\
 & + \sum_i \epsilon_i n_i
 \end{aligned}$$

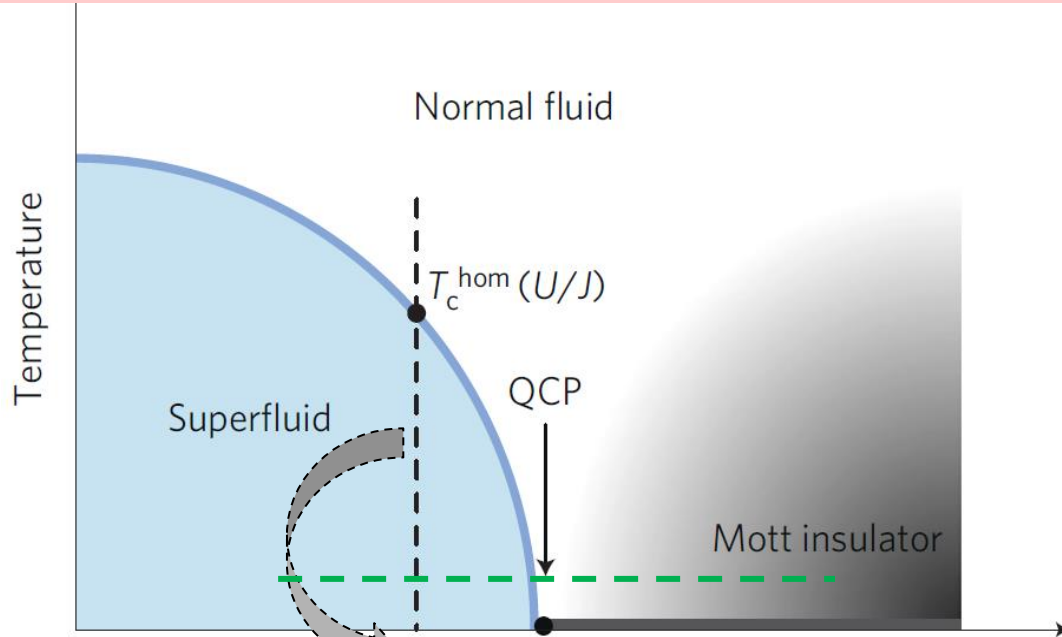


“An interference fringe is the direct signature of the phase coherence”

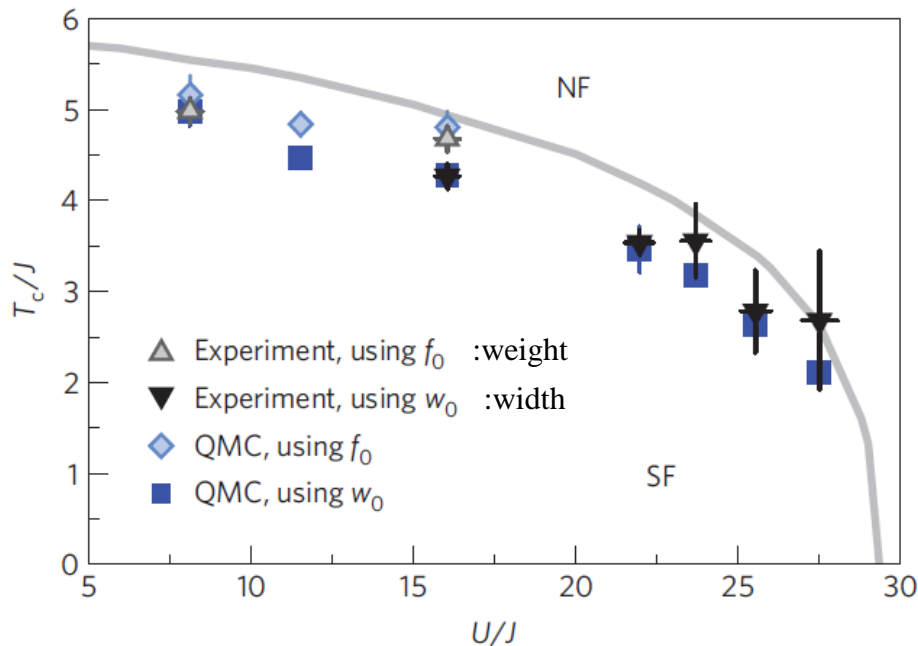


S. Trotzky, et al, Nature Physics 6, 998(2010)

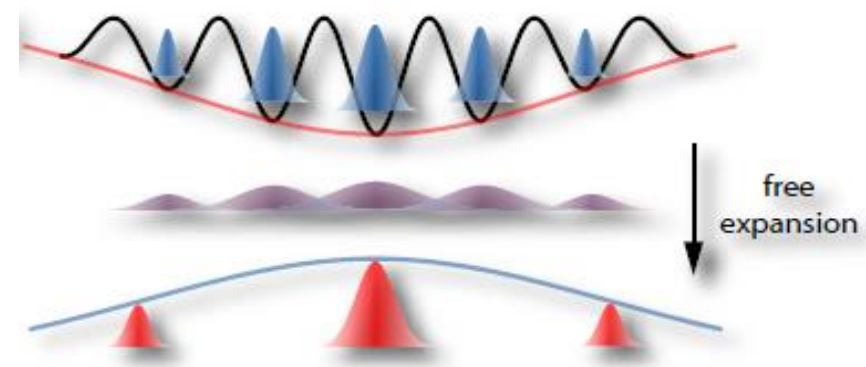
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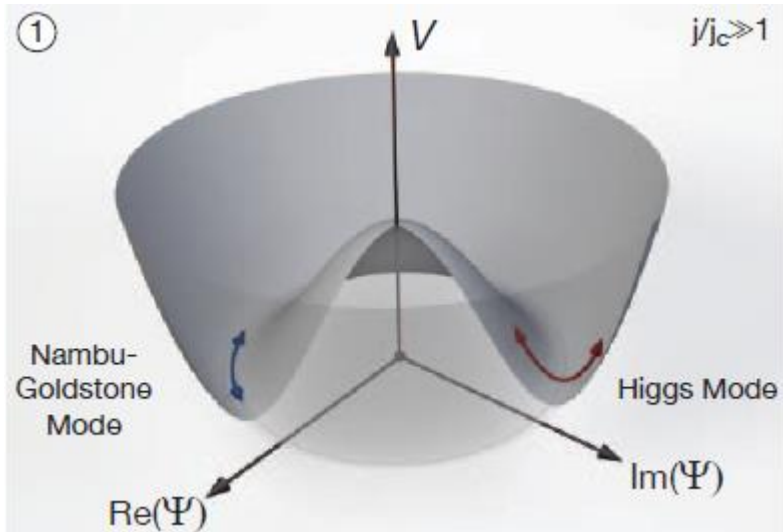
$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1) + \sum_i \varepsilon_i n_i$$



“An interference fringe is the direct signature of the phase coherence”

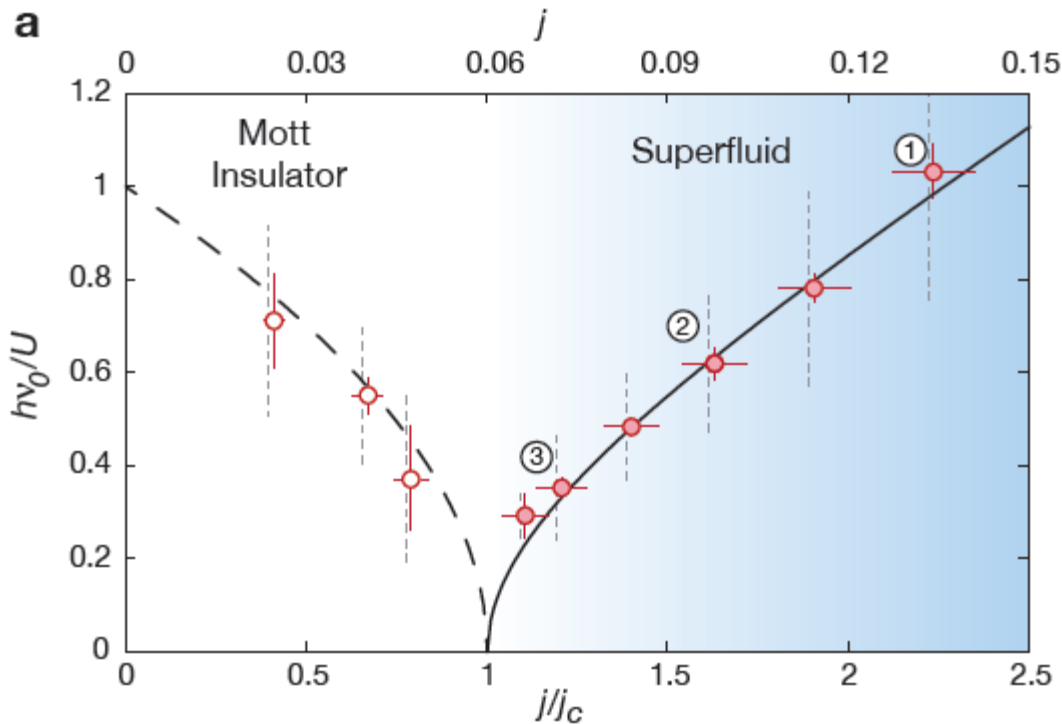


“amplitude-(Higgs-)mode”

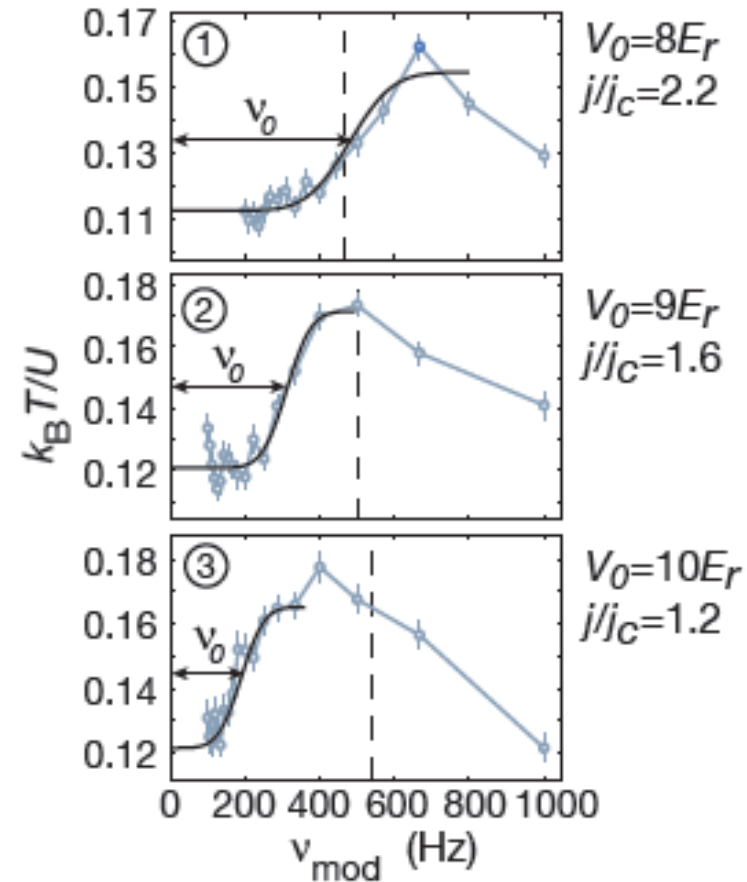


The ‘Higgs’ Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator Transition

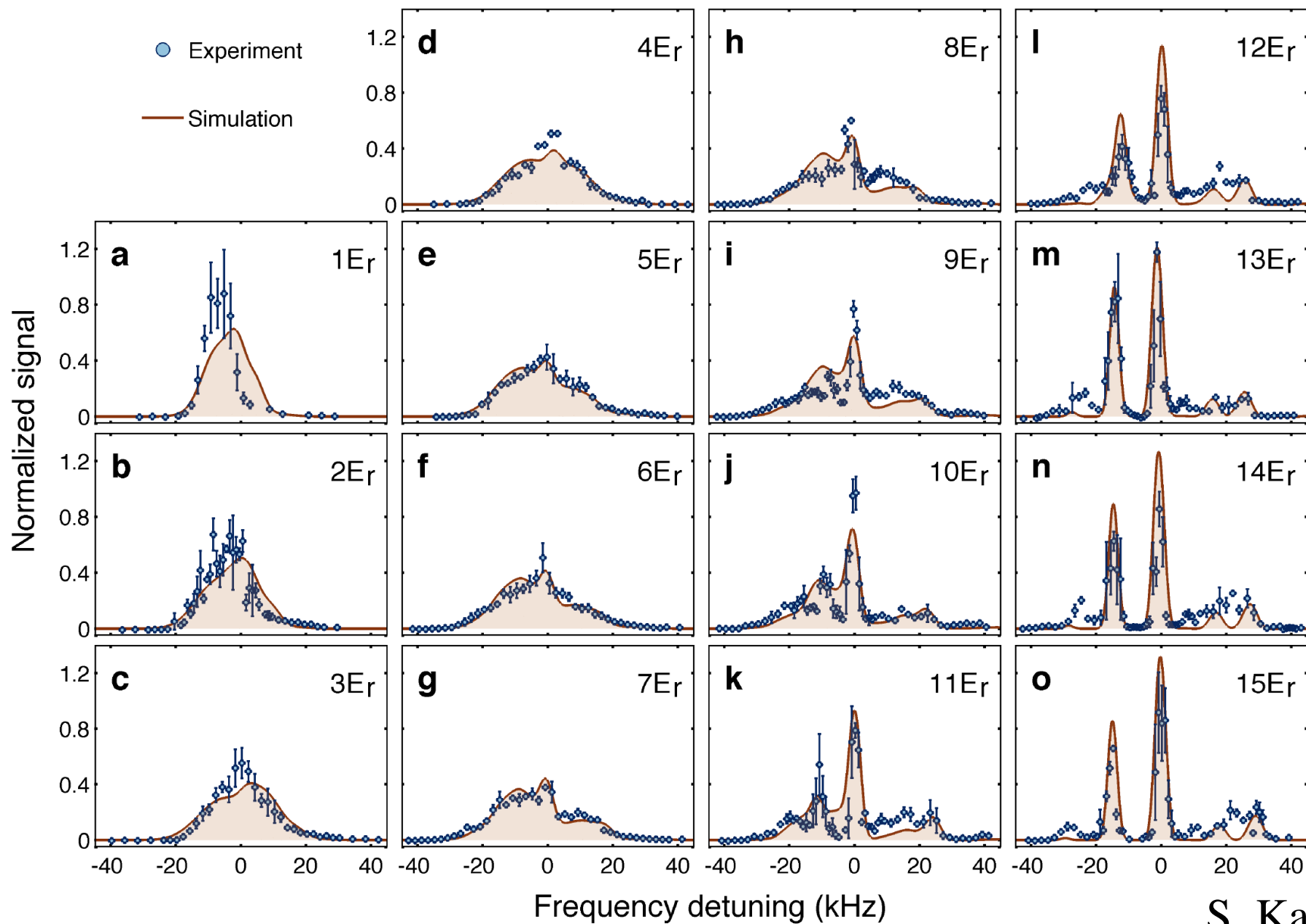
M. Endres *et al* (2012)



Lattice modulation spectra



Spectroscopy of Superfluid-Mott Insulator Transition



quantum simulation of antiferromagnetic spin chains in an optical lattice

[J. Simon, *et al.*, Nature, 472, 307(2011)]

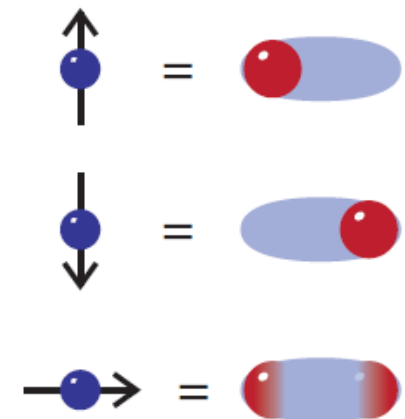
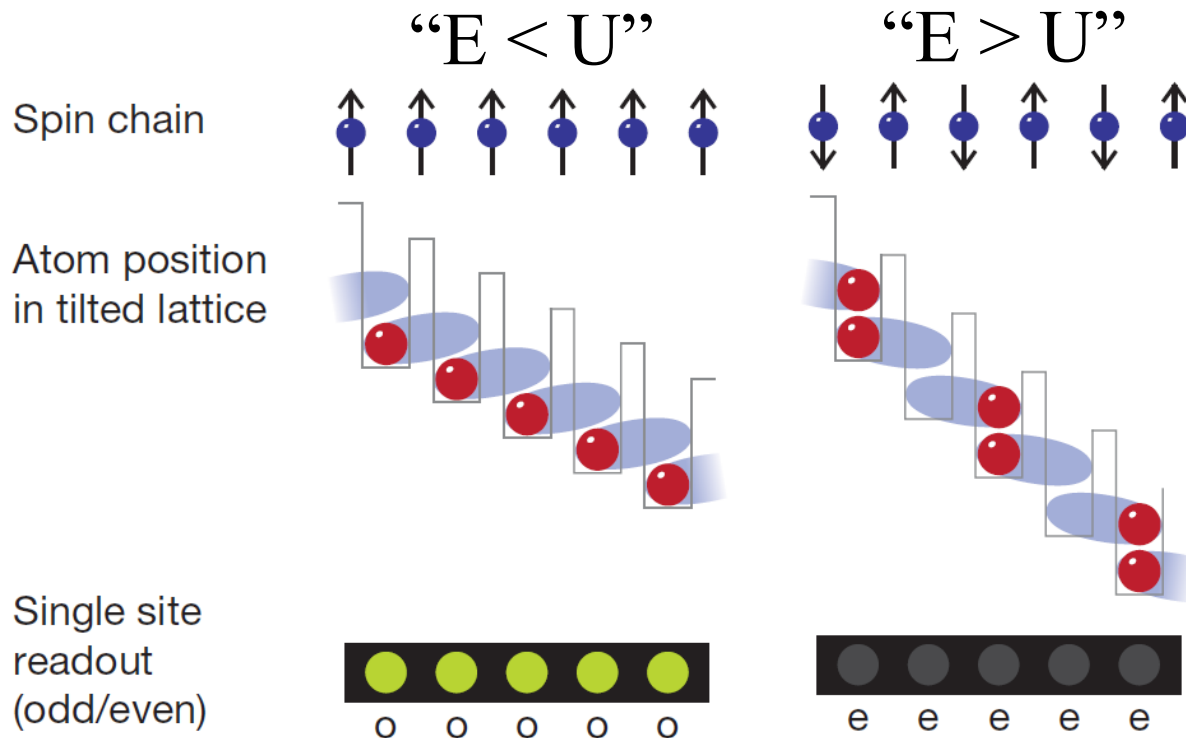
1D Bose-Hubbard Model: $\longrightarrow H = J \sum_i (S_z^i S_z^{i+1} - h_z^i S_z^i - h_x^i S_x^i)$

$$H = -t \sum_j (a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger) + \frac{U}{2} \sum_j n_j (n_j - 1) - E \sum_j j n_j$$

$$(h_z, h_x) = (1 - \tilde{\Delta}, 2^{3/2} \tilde{t})$$

$$\tilde{t} = t/J$$

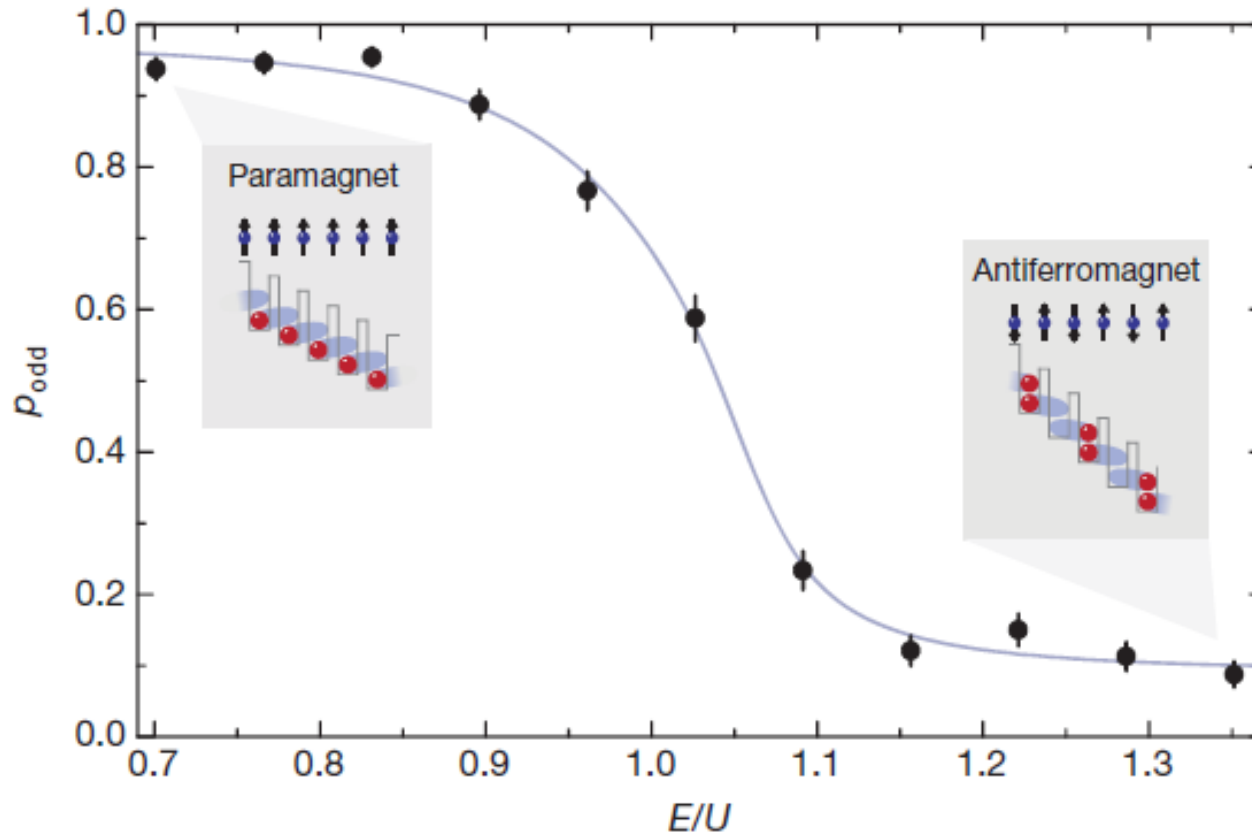
$$\tilde{\Delta} = \Delta/J = (E - U)/J$$



quantum simulation of antiferromagnetic spin chains in an optical lattice

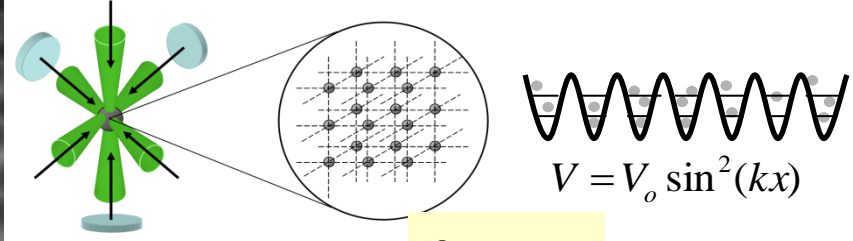
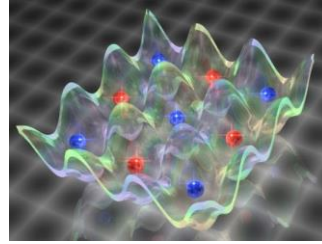
[J. Simon, *et al.*, Nature, 472, 307(2011)]

They successfully observe the transition from paramagnetic spin state to AF ordered state.



Nice Features of ultracold Atoms in an Optical lattice

$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$



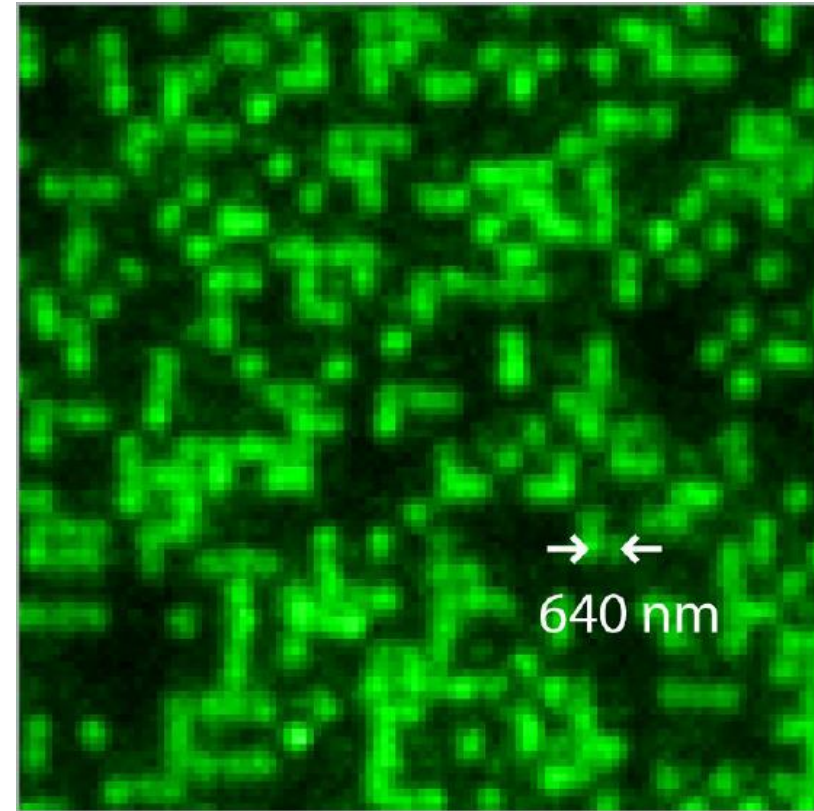
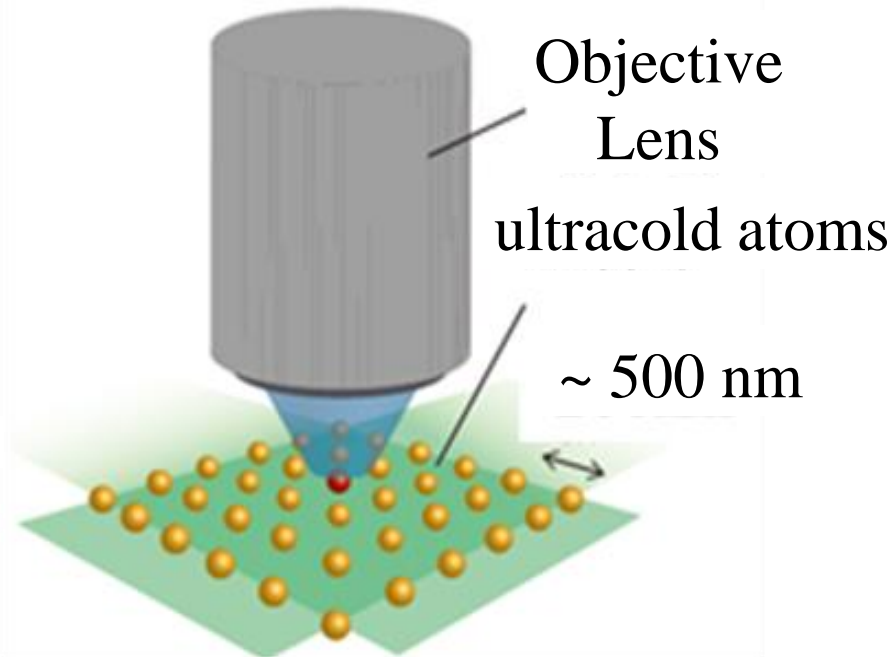
^{87}Rb

Fluorescence Imaging

4) Powerful Measurement Methods:

in situ imaging

by Quantum Gas Microscope



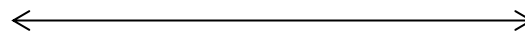
[WS. Bakr, et al Nature 462,5 (2009)]

Single Site Resolved Detection of SF-MI Transition

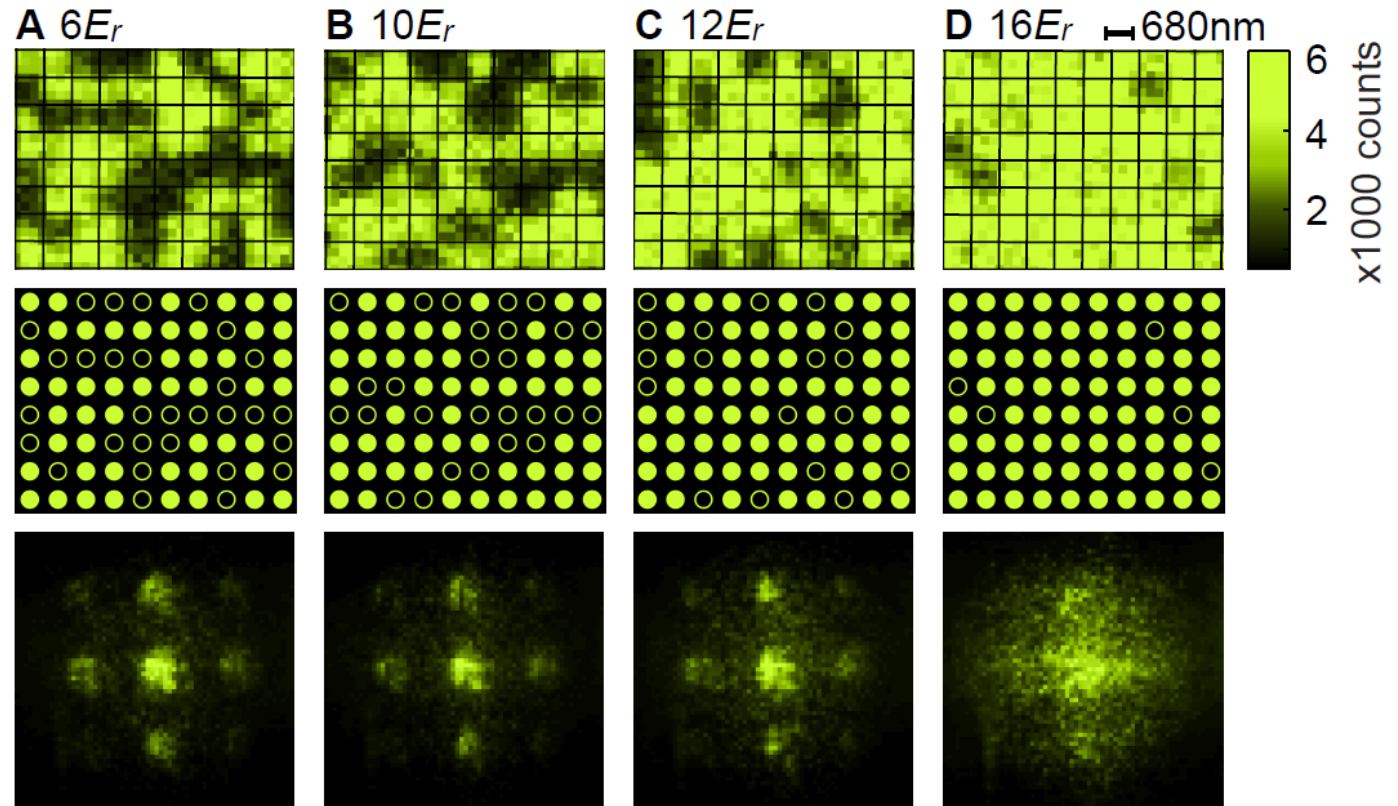
[WS Bakr, et al., Science 329, 547(2010)]

^{87}Rb

SF



MI



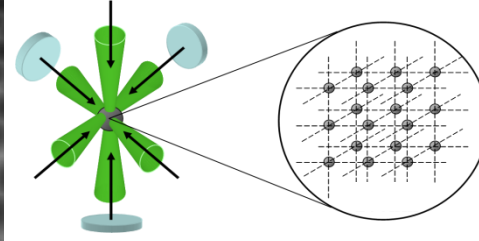
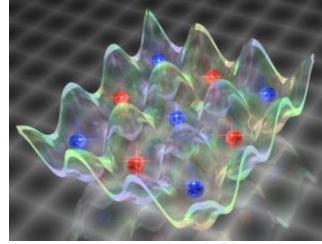
In Situ-image

after analysis

TOF-image

Nice Features of ultracold Atoms in an Optical lattice

$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$

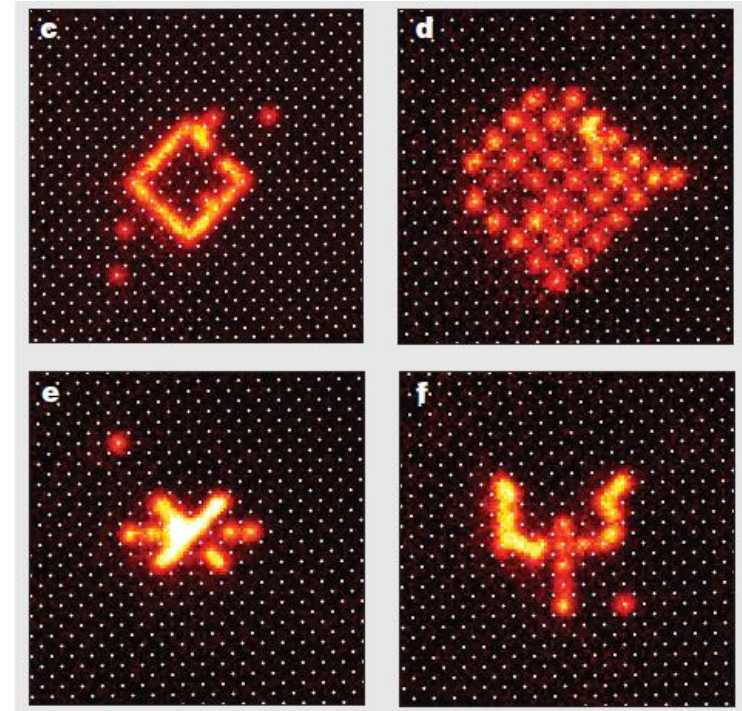
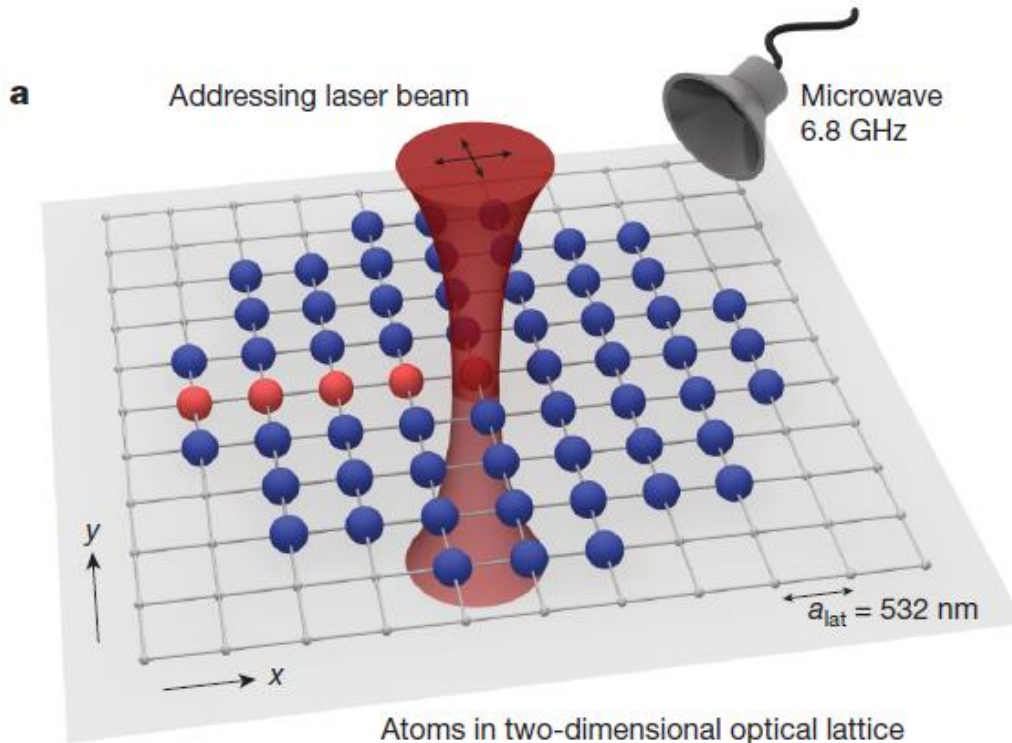


$$V = V_o \sin^2(kx)$$

4) Powerful Measurement Methods:

Single-site manipulation by Quantum Gas Microscope

⁸⁷Rb



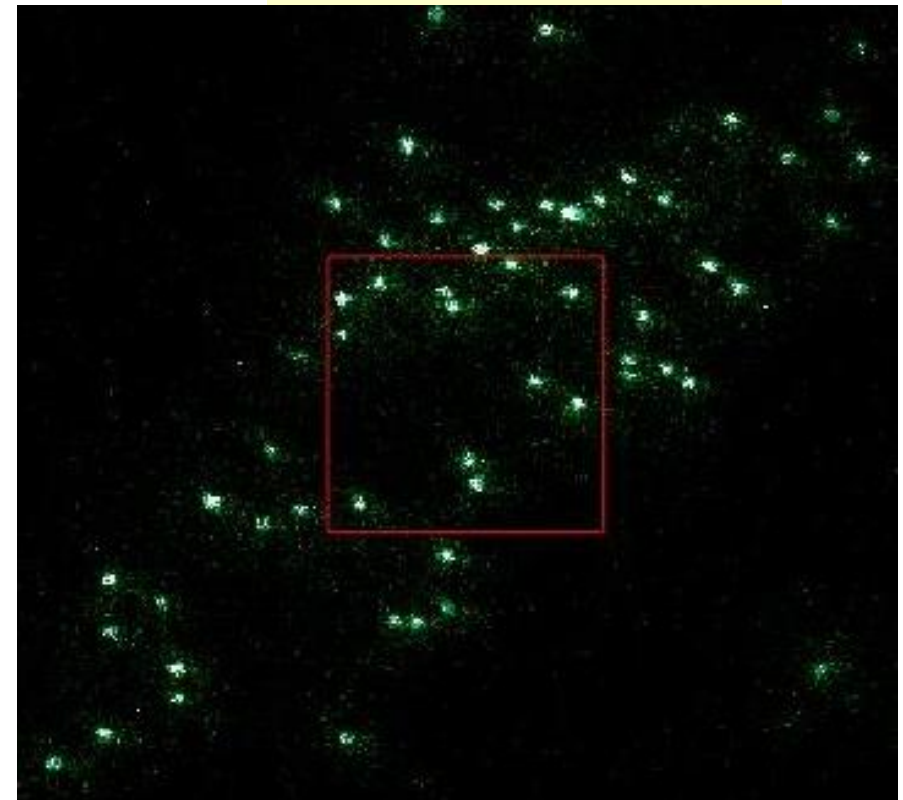
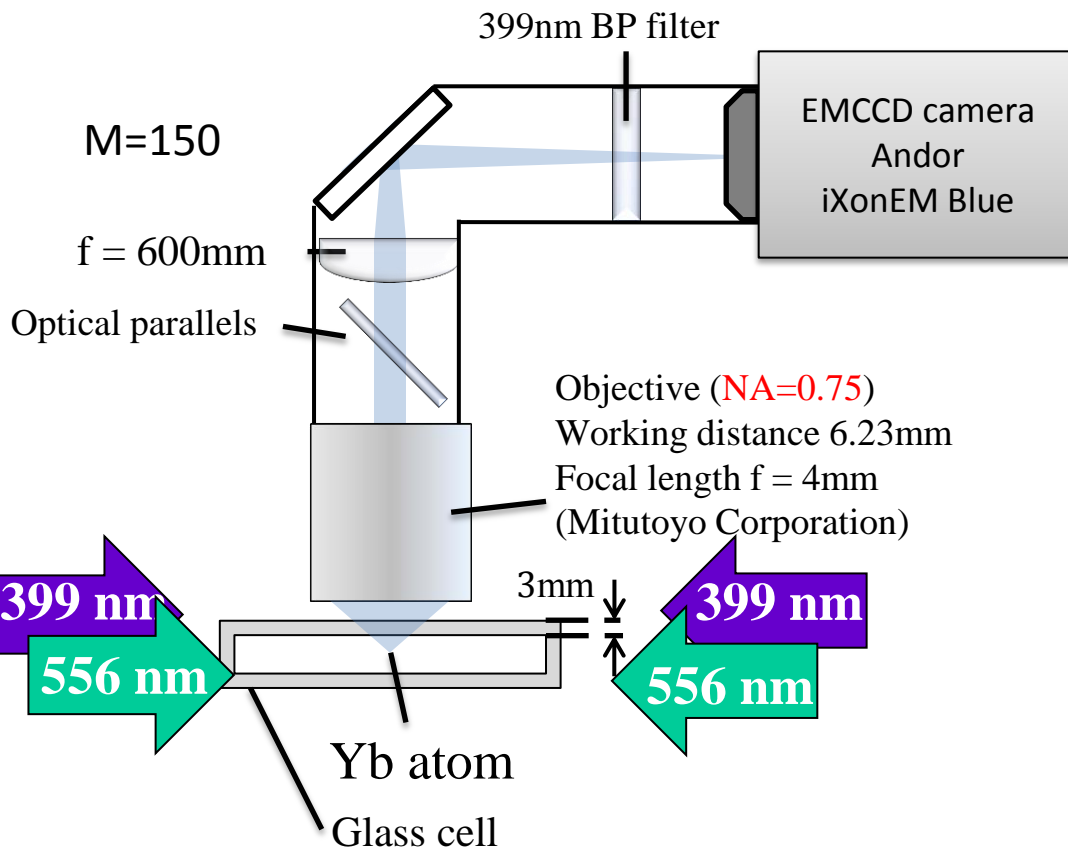
[C. Ewitenberg *et al*, Nature 471, 319(2011)]

Ytterbium Quantum Gas Microscope

“Observation of Single Yb Atoms in an Optical Lattice with *Hubbard Regime*”

“dual molasses”

^{174}Yb (boson)



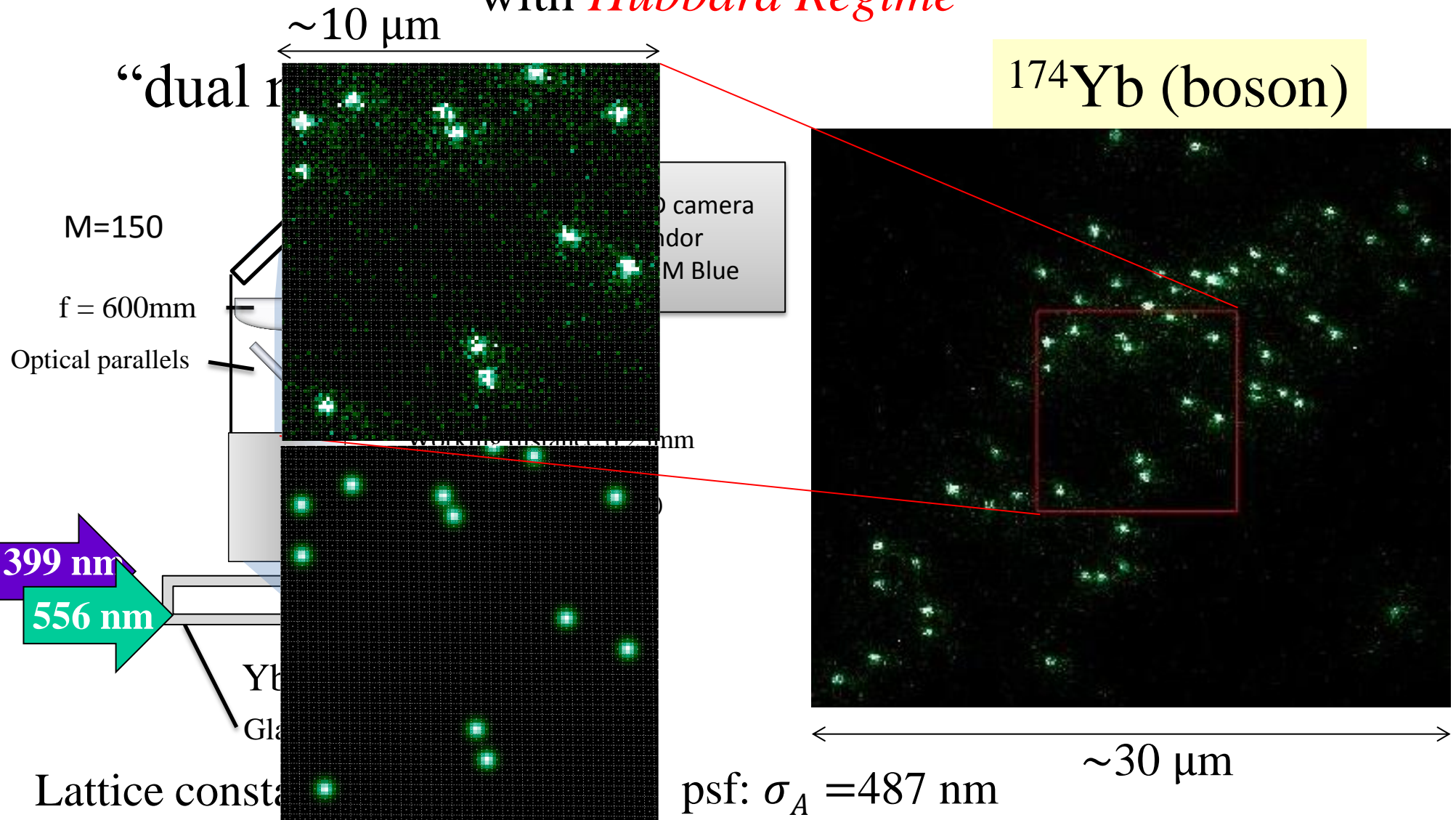
Lattice constant: $d=266$ nm

psf: $\sigma_A = 487$ nm

~ 30 μm

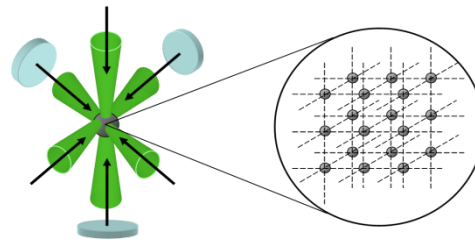
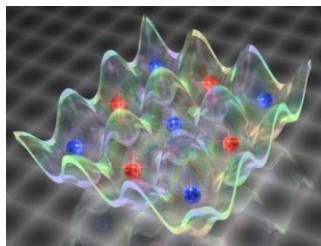
Ytterbium Quantum Gas Microscope

“Observation of Single Yb Atoms in an Optical Lattice with *Hubbard Regime*”



Nice Features of ultracold Atoms in an Optical lattice

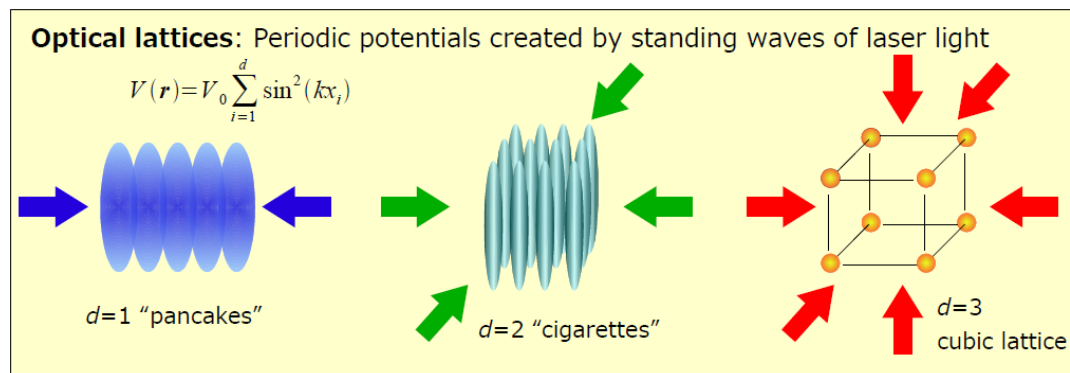
$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1)$$



$$V = V_o \sin^2(kx)$$

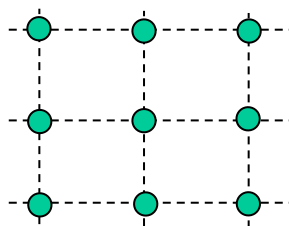
5) Various Lattice Geometries :

Dimensionality:

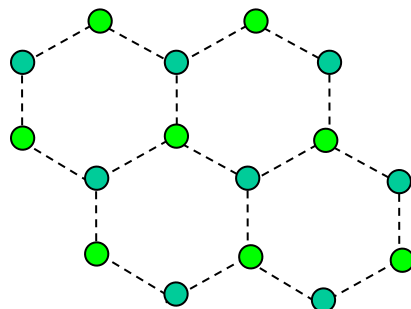


Standard and Non-standard Lattices:

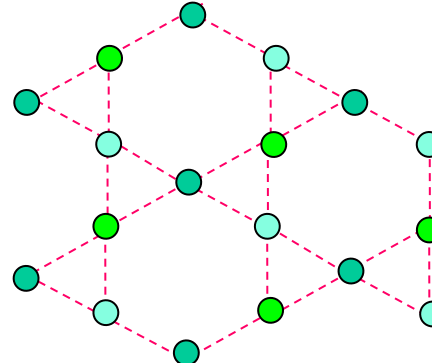
Cubic(Square)



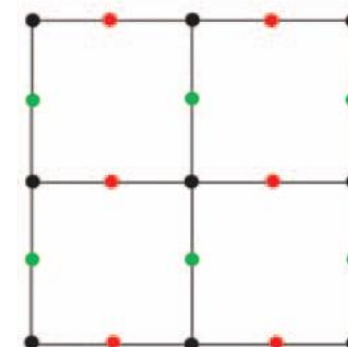
**Honeycomb
(hexagonal)**

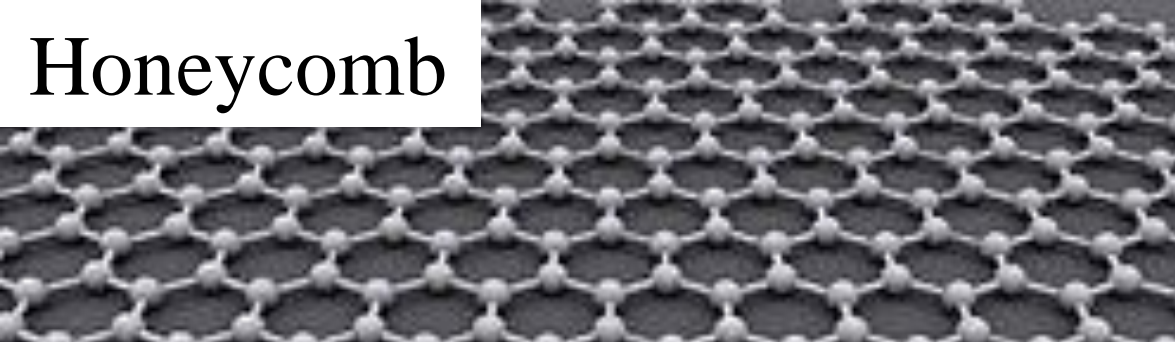


Kagome

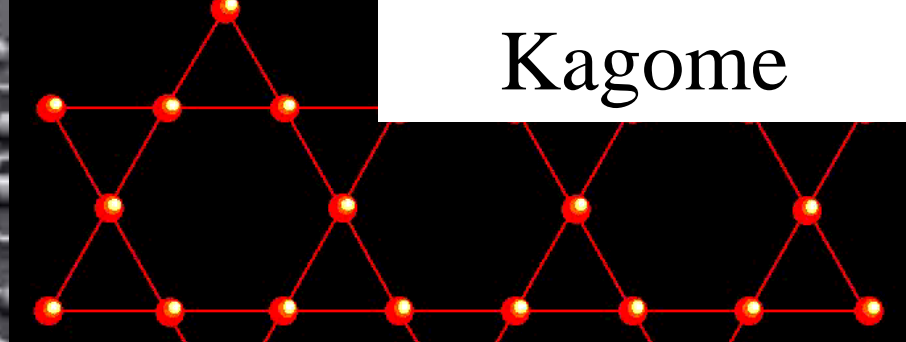


Lieb





Honeycomb



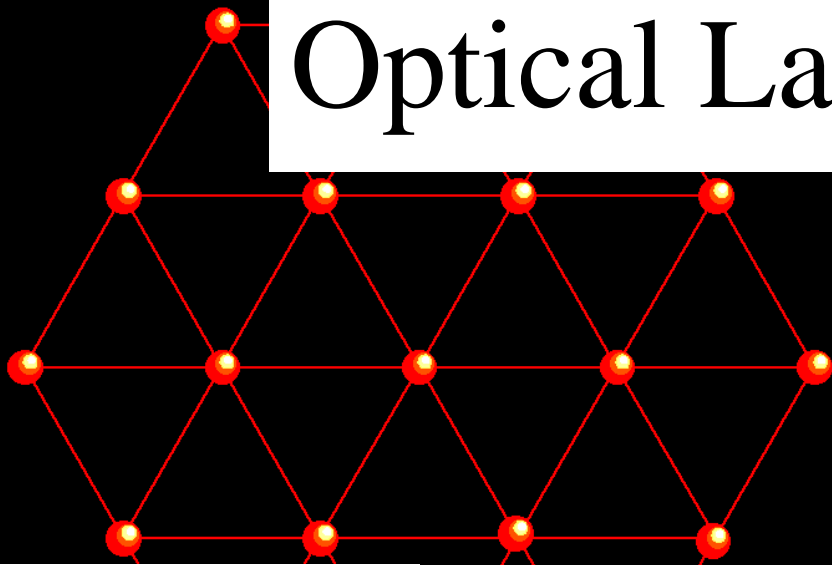
Kagome

Many interesting materials (high- T_c Cuprate, graphene, ..) have non-standard lattice structures

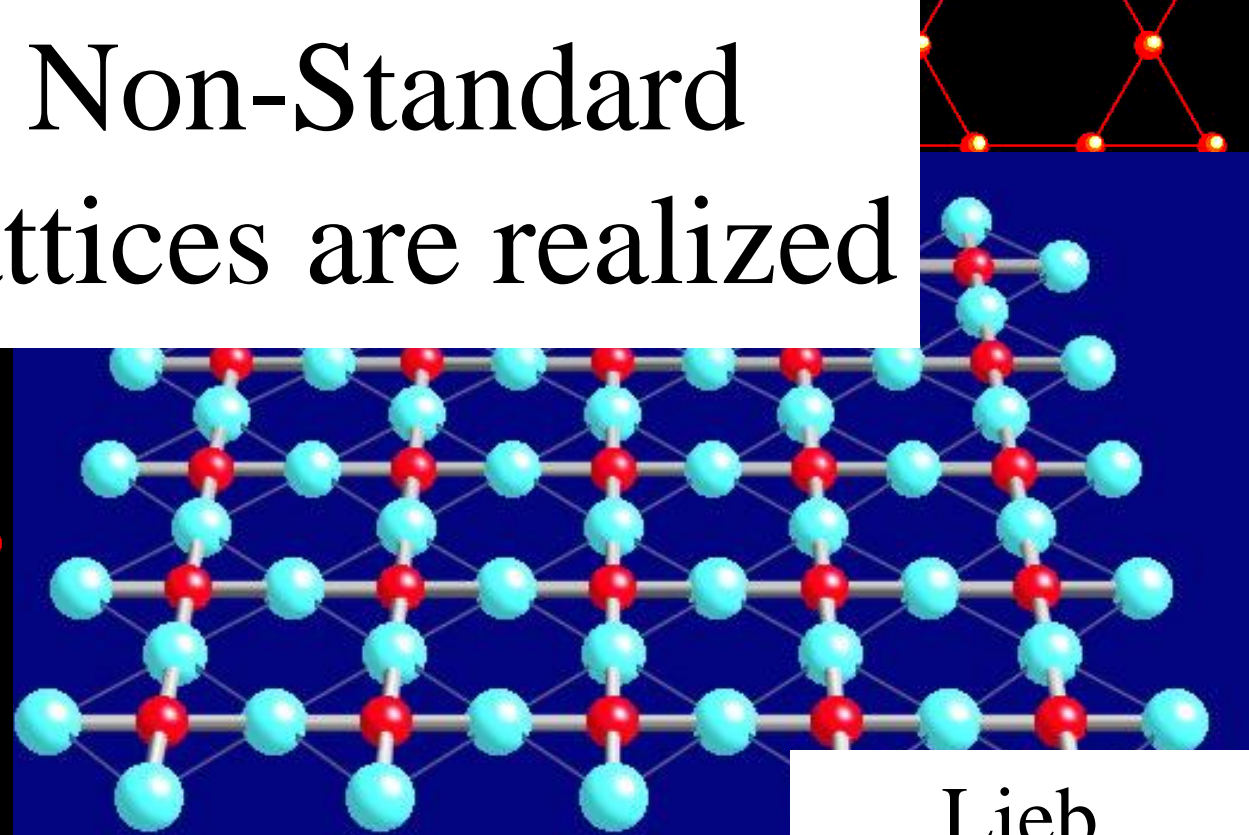
Several Non-Standard Optical Lattices are realized



<http://en.wikipedia.org/wiki/>



Triangular



Lieb

http://hiro.iissp.u-tokyo.ac.jp/data/crystal_gallery/crystal_gallery-Pages/Image31.html

Quantum Simulation of Frustrated Magnetism in a Triangular Lattice

[J. Struck et al, Science (2011)]

Phase Modulation of Optical Lattice:



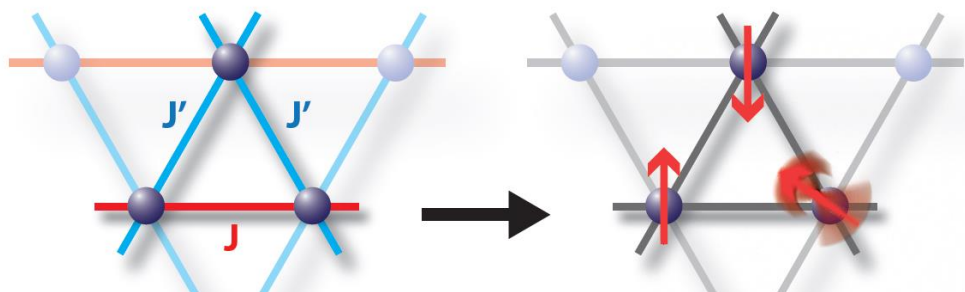
$$K \cos(\omega t)$$



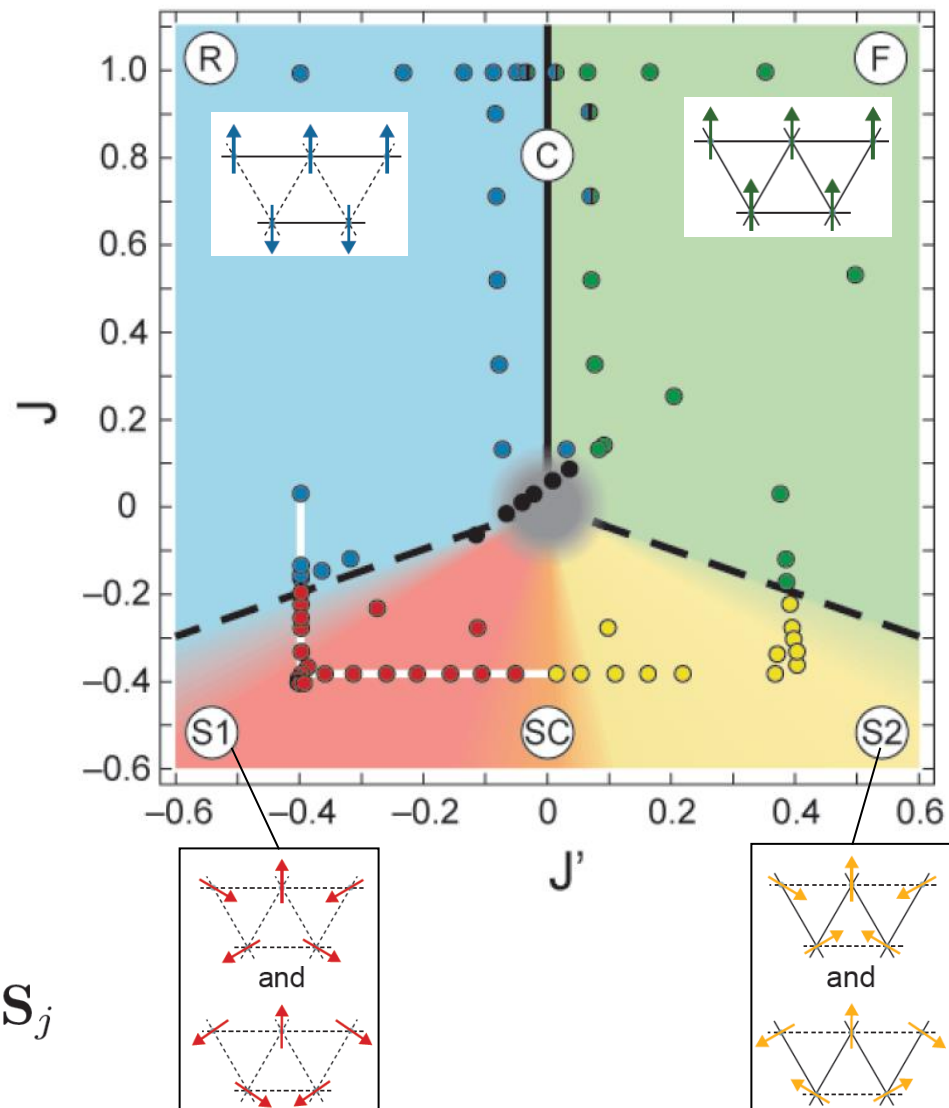
$$J \rightarrow J \times J_0(\beta)$$

:Zero-th order
Bessel Function

$$\beta = K/\omega$$



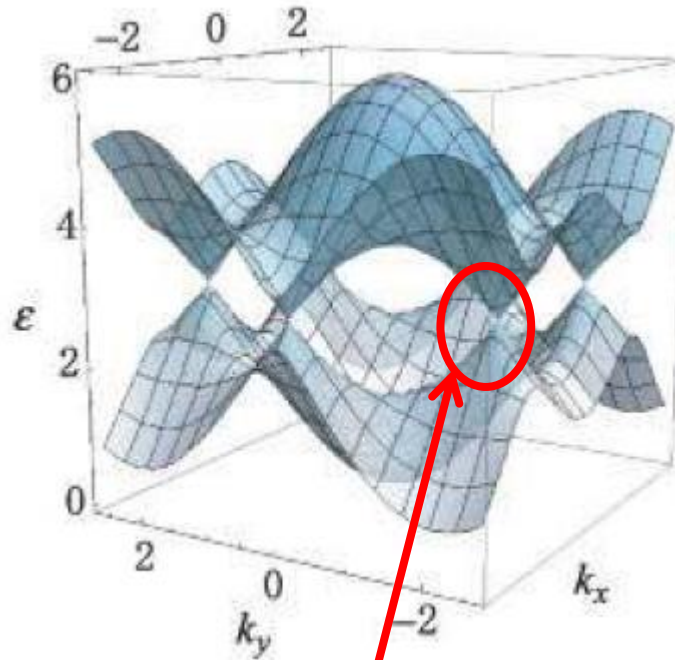
$$E(\theta_i) = - \sum_{\langle i,j \rangle} J_{ij} \cos(\theta_i - \theta_j) = - \sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



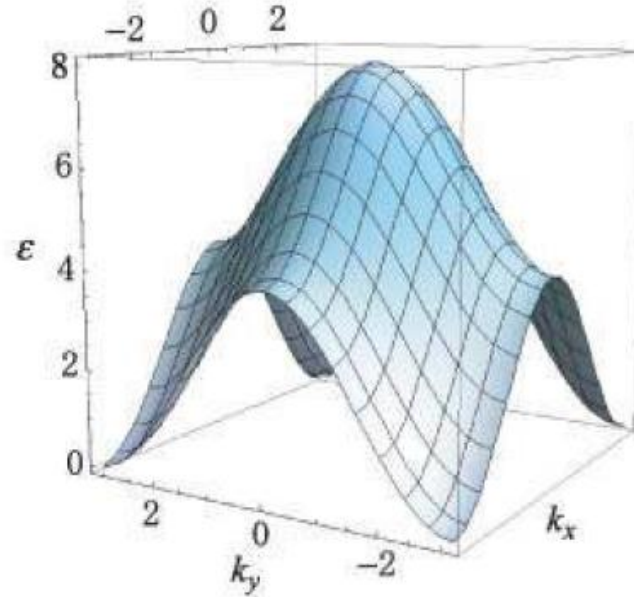
Novel Band Structures

[from Katsura & Maruyama(2014)]

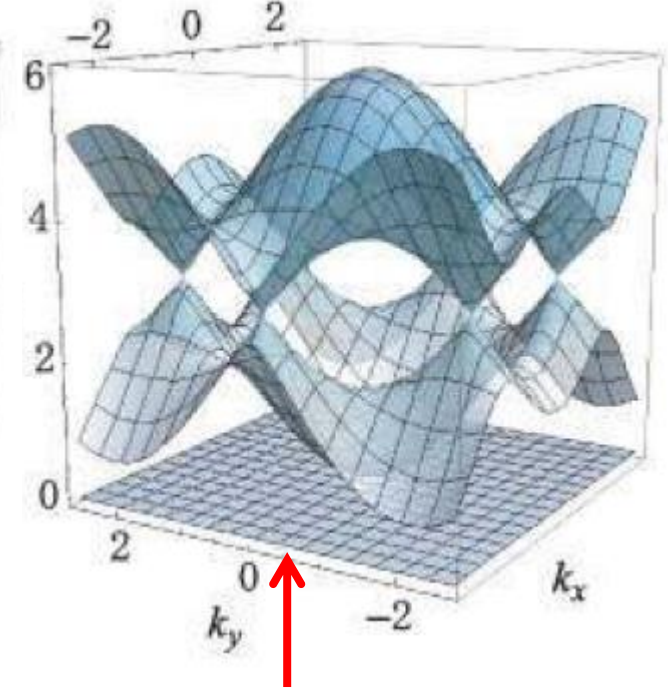
Honeycomb



Square (standard)



Kagome



Dirac Cone:
linear dispersion

$$E(k) = c k$$

: massless Dirac particle

Cosine Band:

$$E(k) = J \cos(kd)$$

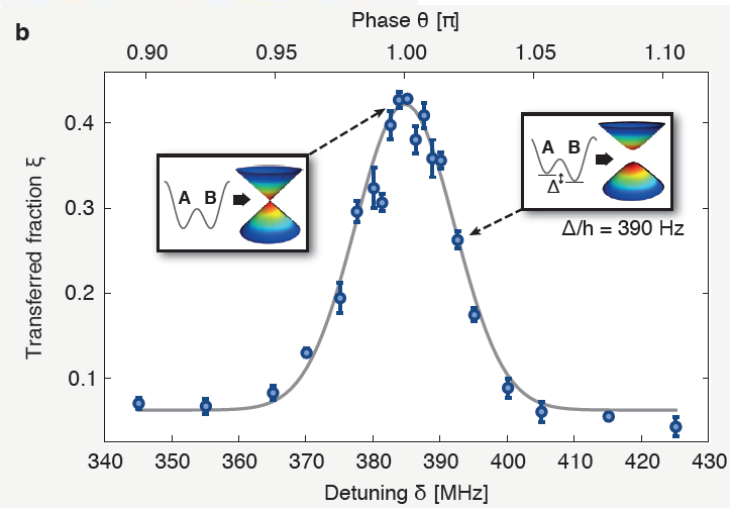
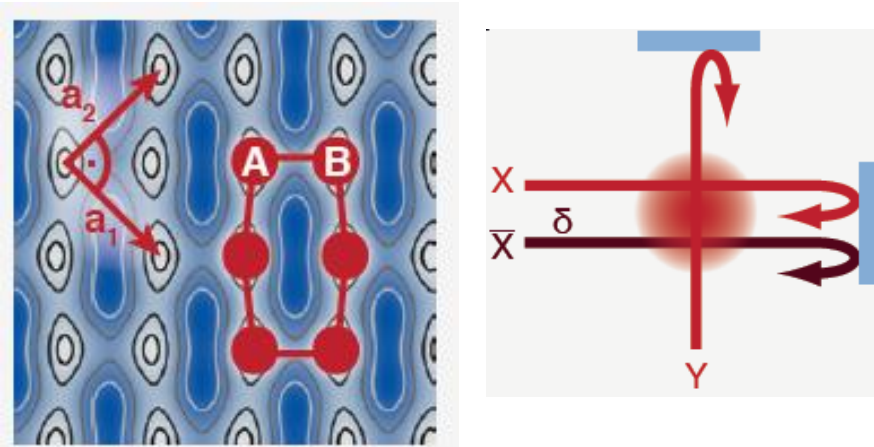
Flat Band:

$$E(k) = \text{const.}$$

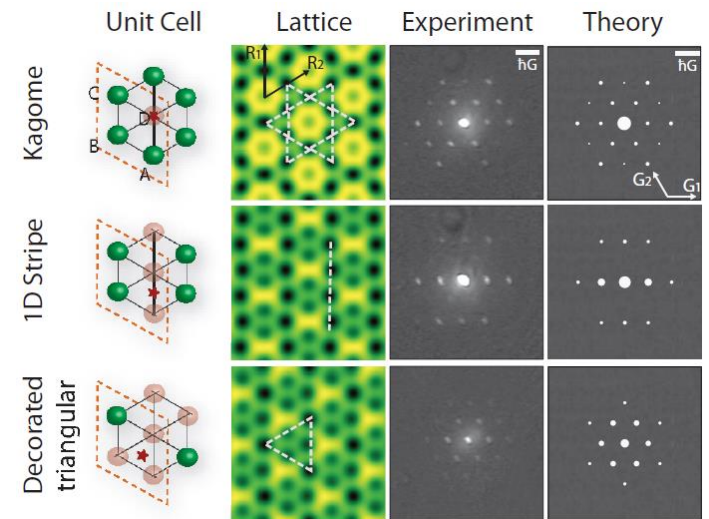
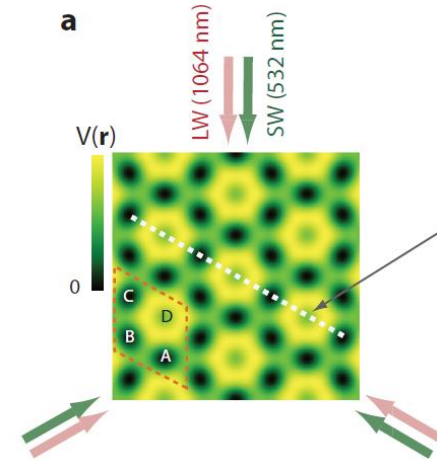
: infinite mass/localization

Novel Band Structures

Honeycomb



Kagome



“Creating, moving, and merging Dirac points with a Fermi gas in a tunable honeycomb lattice”, Tarruell, *et al Nature* (2011)

“Ultracold atoms in a Tunable Optical Kagome Lattice”
Gyu-Boong Jo, *et al, PRL* (2012)

“Ultracold atoms in a flat band”

(case of Kagome lattice)

resonating hexagon:
“localized eigenstate”

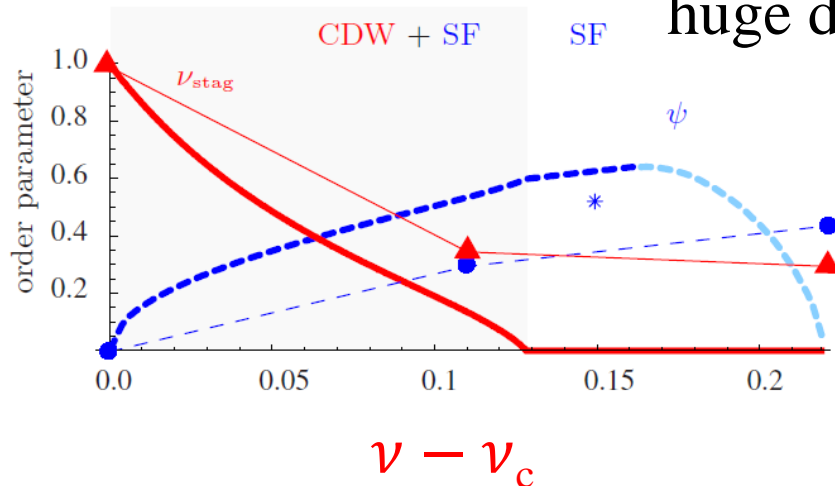
closed packing
at $\nu_c = 1/9$

$$\nu > \nu_c$$

Spatial overlap
between hexagons

Exotic strongly
correlated state:
Super-Solid

huge degeneracy !



**Prediction of super-solid
for bosons in Kagome lattice**

Huber & Altman PRB 82, 184502 (2010)

[Discussion with S. Furukawa]

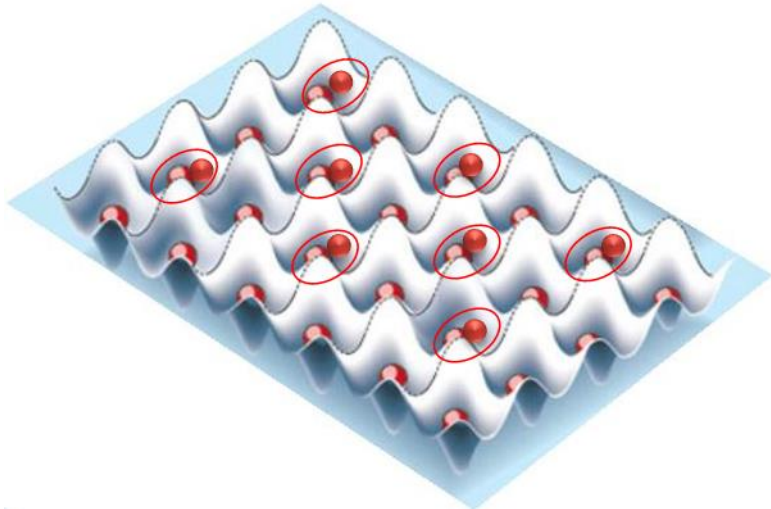
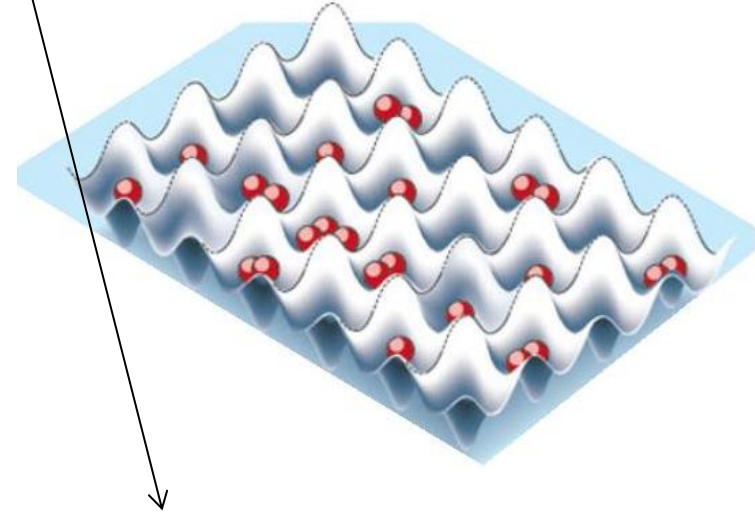
What is Super-Solid ?

“superfluid”

(Off-Diagonal Long-Range Order)

$$G^{(1)}(\mathbf{x}, \mathbf{x}') = \langle \mathbf{x}' | \hat{\rho}_1 | \mathbf{x} \rangle = \langle \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}(\mathbf{x}') \rangle$$

$\rightarrow n_0$



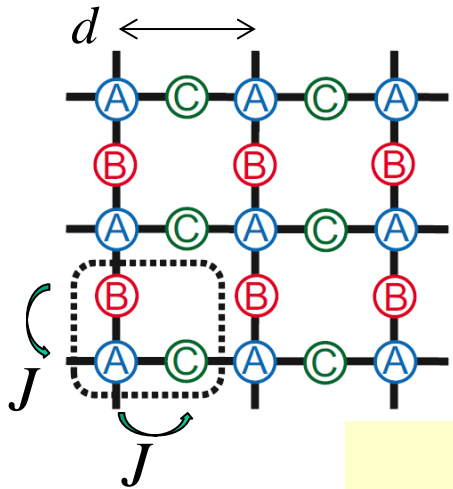
“Solid order”

**(Density Long-Range Order
/ density wave)**

$$\delta n(\mathbf{x}) = \delta n(\mathbf{x} + d)$$

Absence of Supersolidity in Solid Helium in Porous Vycor Glass:2013

Band Structure of Lieb Lattice



(Tight-Binding Approximation)

$$\hat{H}_{TB} = \sum_{\mathbf{k}} \begin{pmatrix} \hat{a}_{\mathbf{k},A}^\dagger & \hat{a}_{\mathbf{k},B}^\dagger & \hat{a}_{\mathbf{k},C}^\dagger \end{pmatrix} T_{Lieb} \begin{pmatrix} \hat{a}_{\mathbf{k},A} \\ \hat{a}_{\mathbf{k},B} \\ \hat{a}_{\mathbf{k},C} \end{pmatrix}$$

Operators in k-space:

$$\hat{a}_{i,S} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{x}_{i,S}} \hat{a}_{\mathbf{k},S} \quad S=A,B,C$$

$$T_{Lieb} = \begin{pmatrix} 0 & -2J \cos(k_x d / 2) & -2J \cos(k_z d / 2) \\ -2J \cos(k_x d / 2) & 0 & 0 \\ -2J \cos(k_z d / 2) & 0 & 0 \end{pmatrix}$$

By diagonalization, we obtain 3 eigenvalues;

$$E_{\pm} = \pm 2J \sqrt{\cos^2(k_x d / 2) + \cos^2(k_z d / 2)},$$

$$E_0 = 0,$$

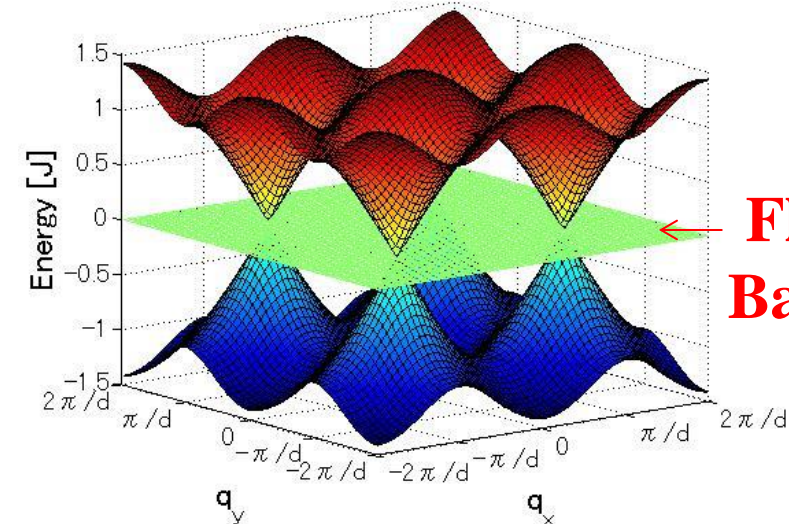
$$(\tan \theta_k = \cos(k_x d / 2) / \cos(k_z d / 2))$$

Flat Band

$$|\mathbf{k}, 1st\rangle = \frac{1}{\sqrt{2}} (|\mathbf{k}, A\rangle + \sin \theta_k |\mathbf{k}, B\rangle + \cos \theta_k |\mathbf{k}, C\rangle),$$

$$|\mathbf{k}, 2nd\rangle = \cos \theta_k |\mathbf{k}, B\rangle - \sin \theta_k |\mathbf{k}, C\rangle,$$

$$|\mathbf{k}, 3rd\rangle = \frac{1}{\sqrt{2}} (|\mathbf{k}, A\rangle - \sin \theta_k |\mathbf{k}, B\rangle - \cos \theta_k |\mathbf{k}, C\rangle),$$



Ultracold Atoms in a Flat Band

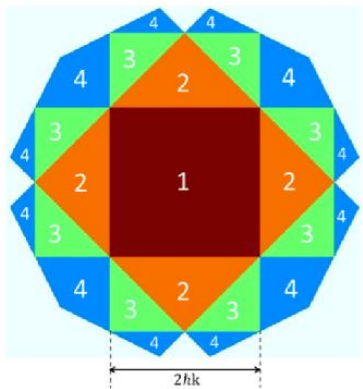
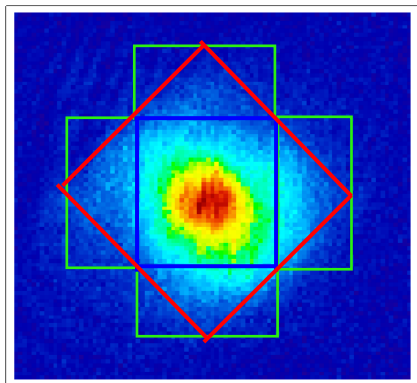
In order to study interesting physics of flat band, we need to load ultracold atoms into flat band.

flat band: $|\mathbf{B}\rangle - |\mathbf{C}\rangle$ ($\mathbf{q}=\mathbf{0}$)

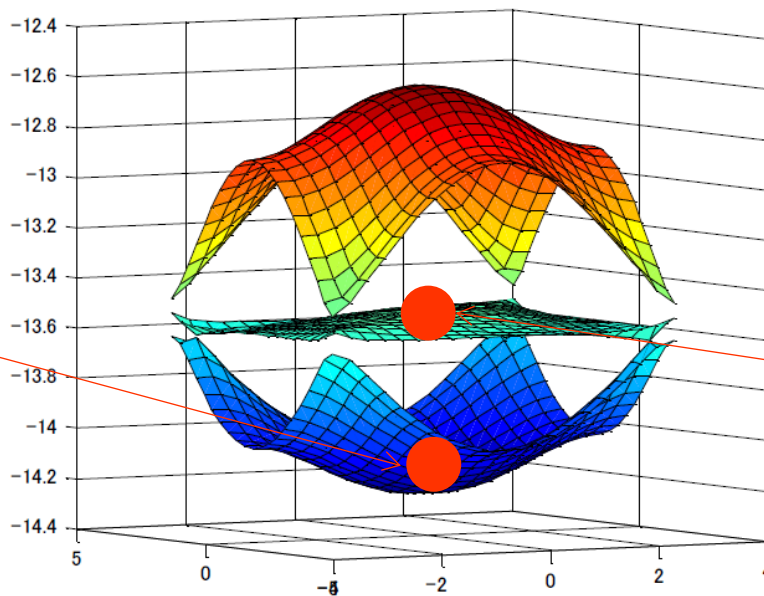
$$|\mathbf{q}, X\rangle = \frac{1}{\sqrt{N}} \sum_{i \in X} e^{i\mathbf{q} \cdot \mathbf{x}_i} c_i^\dagger |0\rangle \quad X=A, B, C$$

initially

$|\mathbf{B}\rangle + |\mathbf{C}\rangle$ ($\mathbf{q}=\mathbf{0}$)

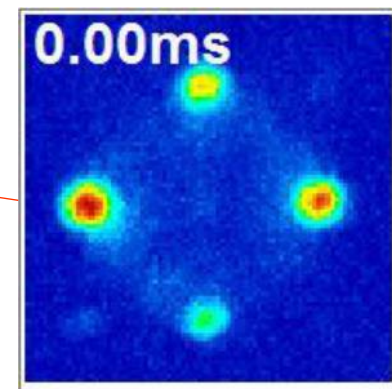


Brillouin Zones

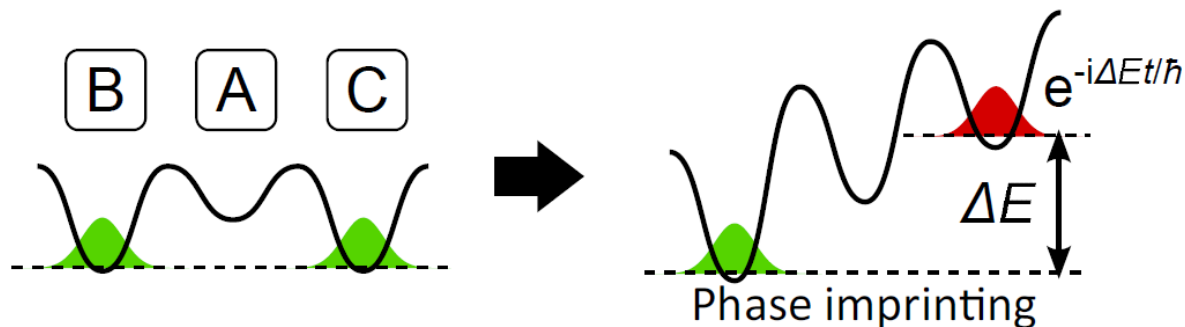


after Phase Imprinting

$|\mathbf{B}\rangle - |\mathbf{C}\rangle$ ($\mathbf{q}=\mathbf{0}$)

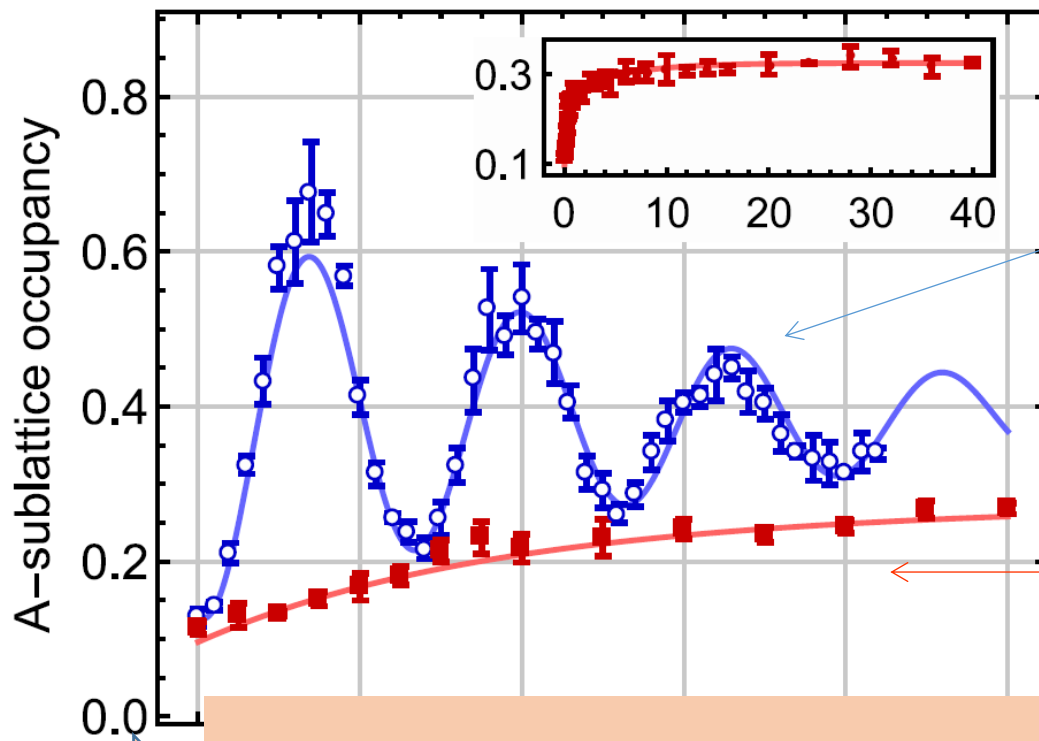


“Phase Imprinting Method”



Ultracold Atoms in a Flat Band

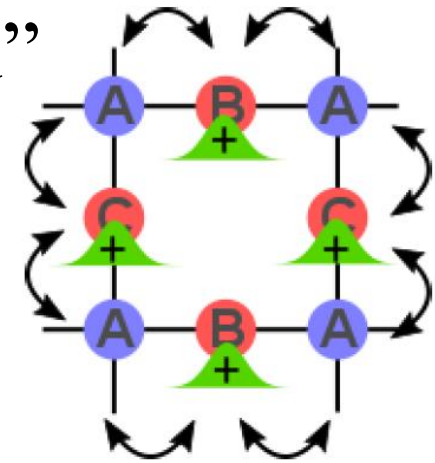
We could observe **Unique Dynamics** in **Flat band** by measuring A-site population.



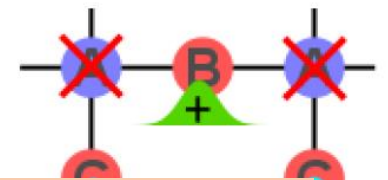
“Usual band”

$$|B\rangle + |C\rangle$$

($q=0$)



“Flat band”

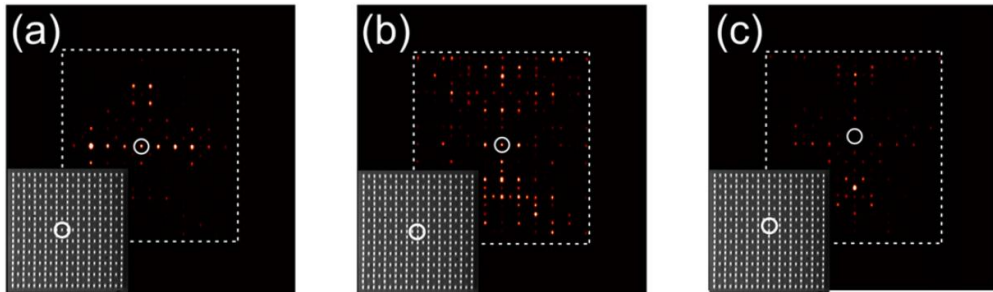
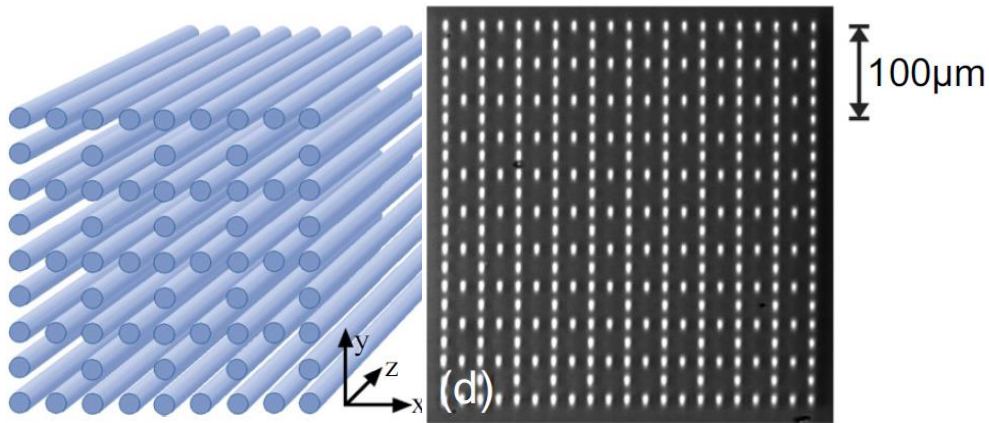


This is a direct confirmation of “localized state” in a flat band

Other Bosons in a Lieb lattice

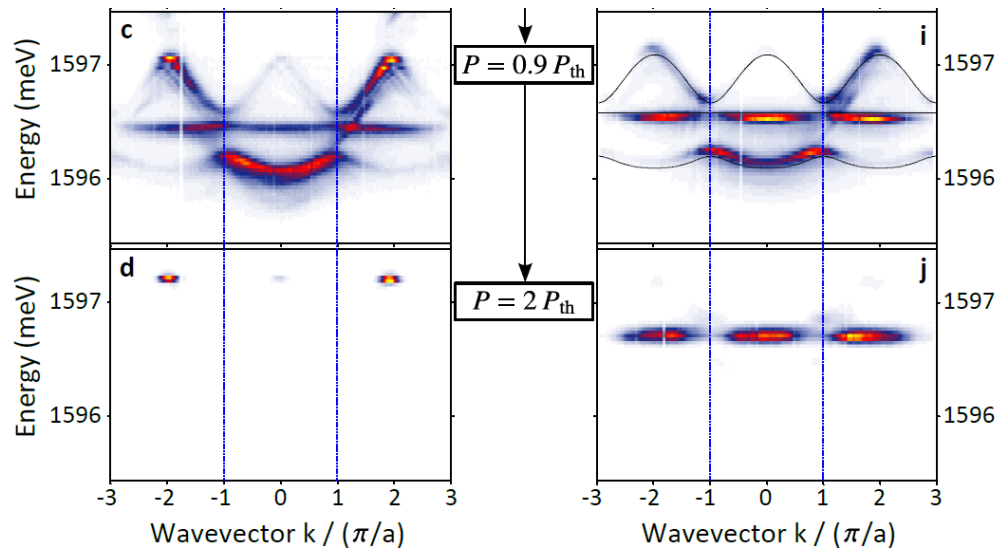
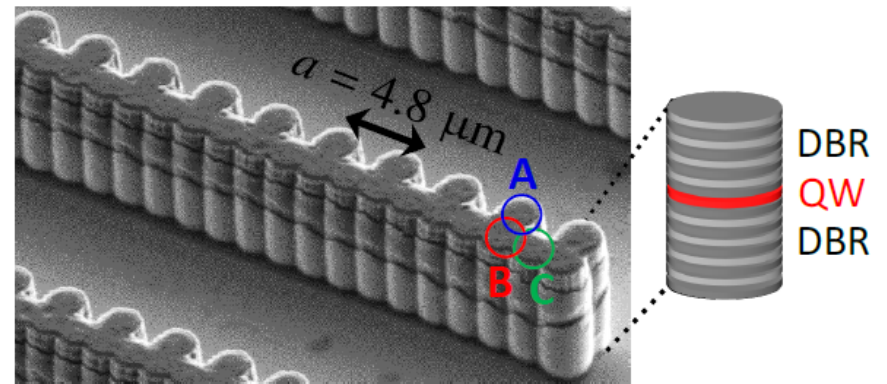
Photonic Waveguide

D. Guzman-Silva *et al.*,
NJP16, 063061(2014)



Exciton-Polariton Condensate

arXiv:1505.05652, F. Baboux *et al.*,

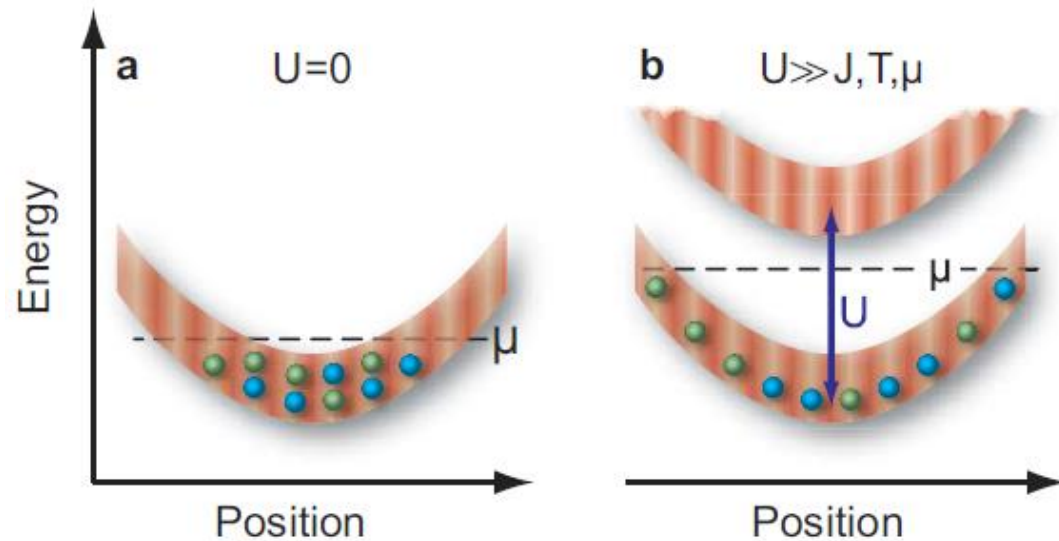
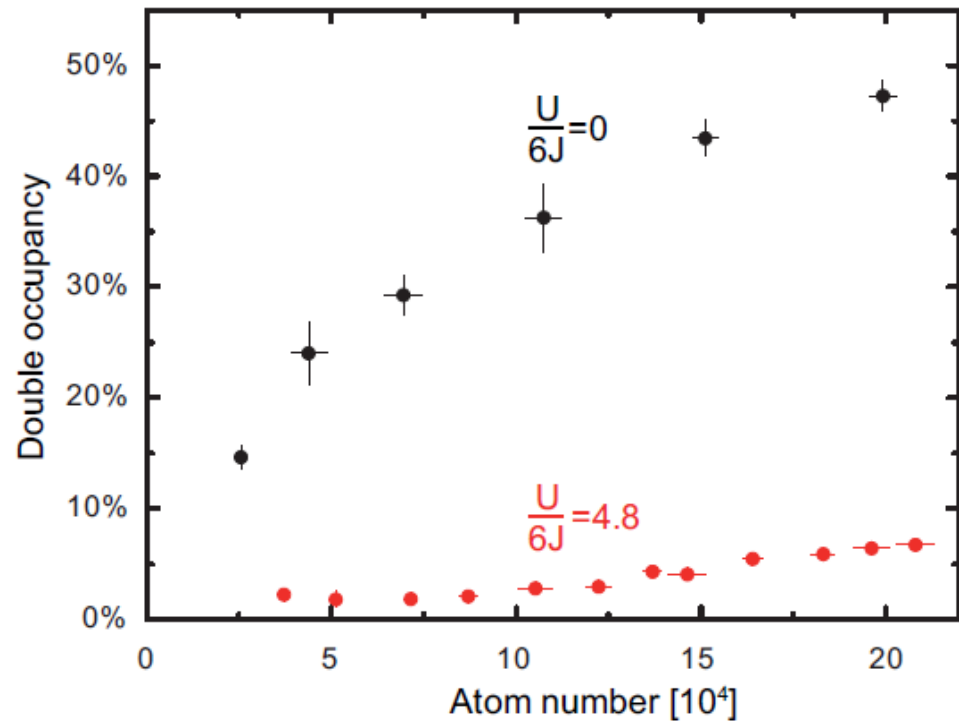


Fermions in an Optical Lattice

Fermi-Hubbard Model:

^{40}K “A Mott insulator of ^{40}K atoms in an optical lattice”

[R. Jördens *et al.*, Nature **455**, 204 (2008)] [U. Schneider, *et al.*, Science **322**,1520(2008)]



Fermions in an Optical Lattice

onset of anti-ferromagnetic correlation

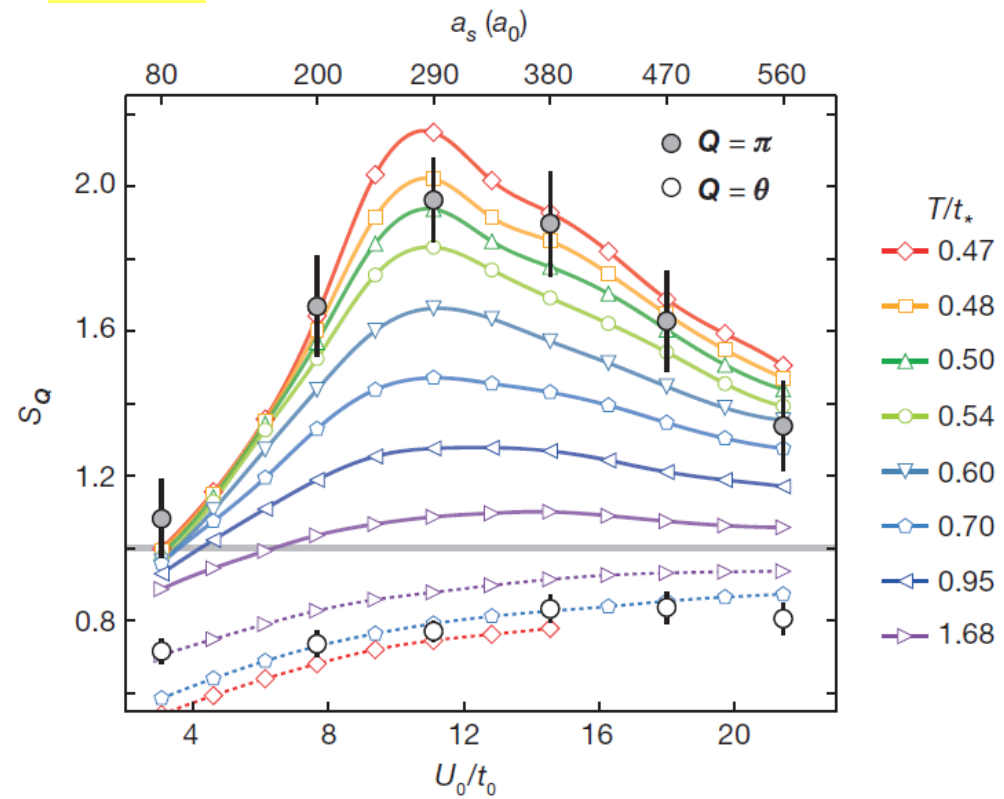
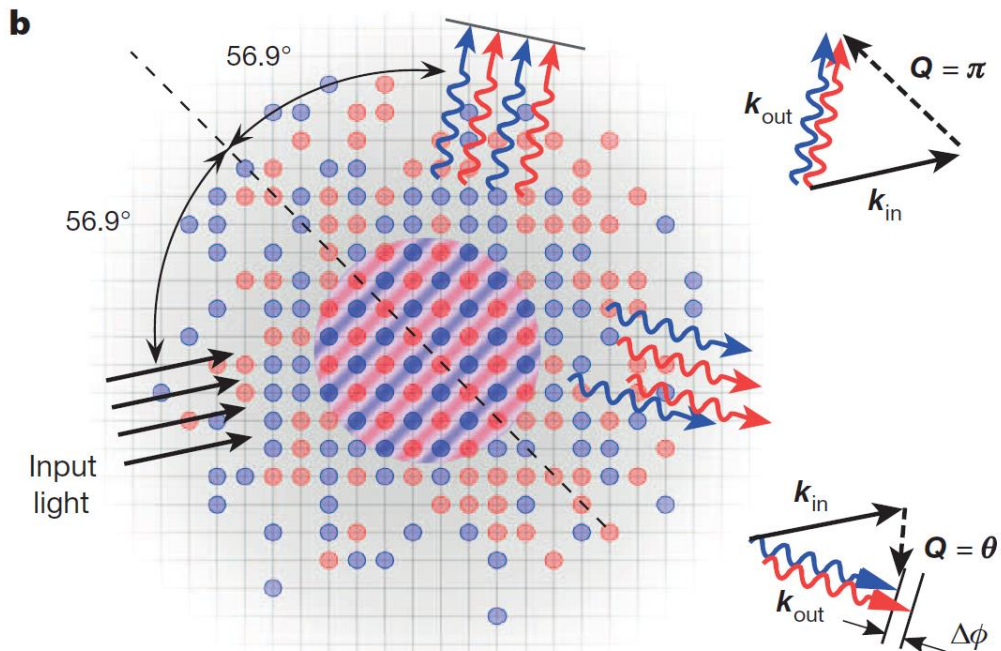
[R. A. Hart *et al.*, Nature **519**, 211 (2015)]

Spin-structure factor:

$$S_Q \equiv \frac{4}{N} \sum_{i,j} e^{iQ \cdot (R_i - R_j)} \langle \sigma_{zi} \sigma_{zj} \rangle$$

${}^6\text{Li}$ $T \sim 1.4 T_N (=0.5t)$

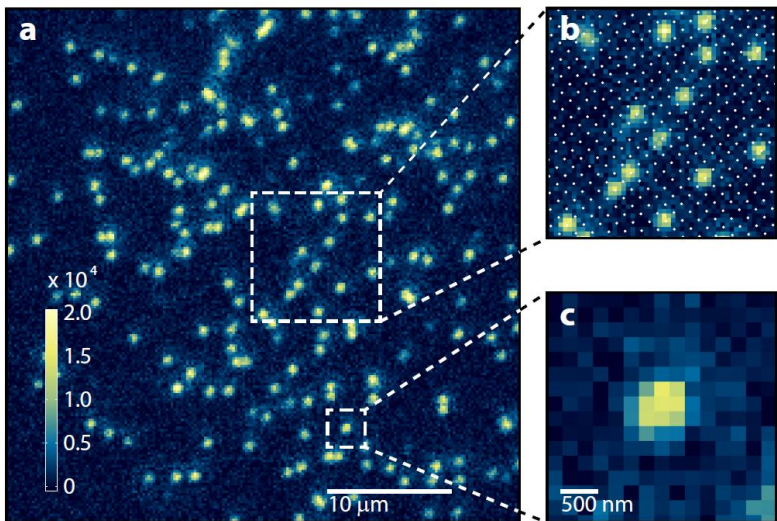
optical Bragg scattering



Fermionic quantum gas microscope

^{40}K

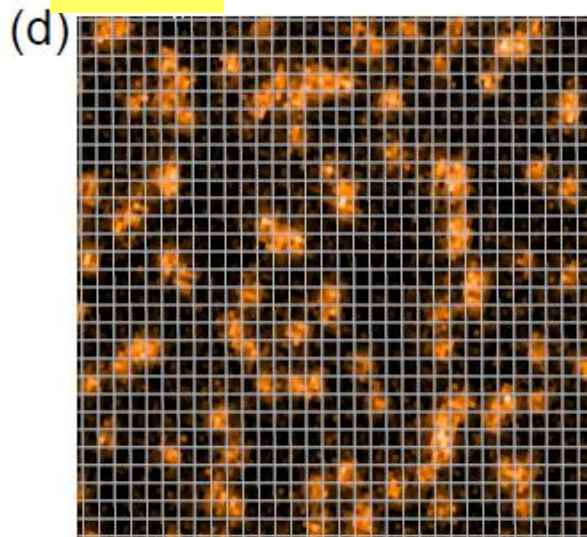
EIT-cooling



E. Haller et al., arXiv:1503.02005v2

^{40}K

Raman-sideband cooling

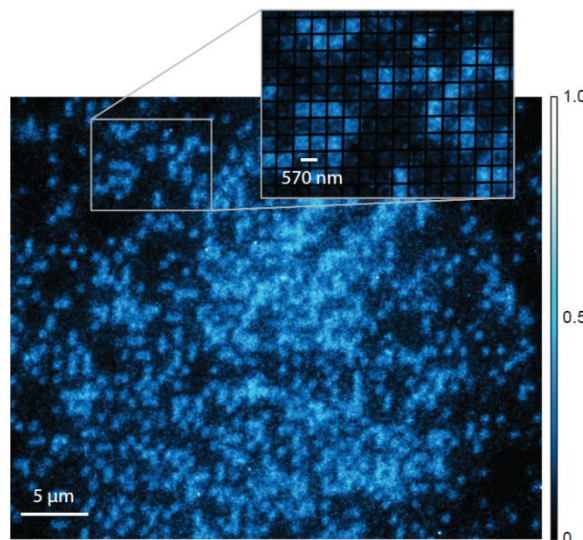


L. W. Cheuk et al.,
arXiv:1503.02648v1,
PRL(2015)

^6Li

Raman-sideband cooling

M. F. Parsons, et al, PRL(2015)



SU(N=6) Fermions in an Optical Lattice

^{173}Yb
($I=5/2$) **SU(N=6) system**



SU(N) Heisenberg model:

Nuclear spin permutation operators:

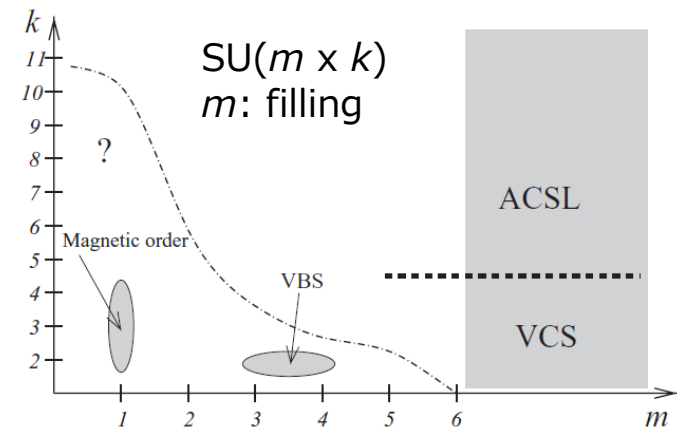
$$H = \frac{2t^2}{U} \sum_{\langle i,j \rangle} S_n^m(i) S_m^n(j)$$

$$S_n^m \equiv c_n^+ c_m = |n\rangle\langle m|$$

*SU(N=6) Mott Insulator is already created [Taie *et al*, NP(2012)]

Novel magnetic phases:

Neel order, dimerization,
Valence-Bond-Solid,
Chiral Spin Liquid, ...

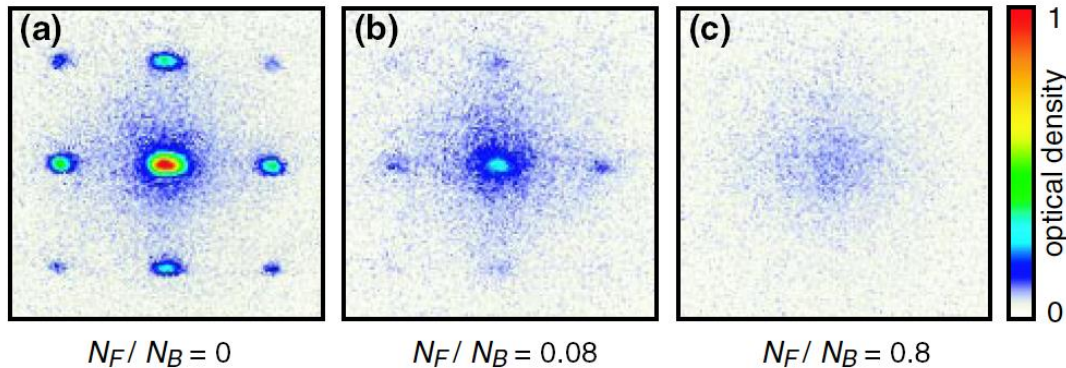


*Two-orbital SU(N) fermions is successfully created [MPQ, LENS, ...]

Bose-Fermi Mixture in a 3D optical lattice

$$H = -t_B \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U_{BB}}{2} \sum_i n_{Bi} (n_{Bi} - 1) - t_F \sum_{\langle i,j \rangle} c_i^\dagger c_j + U_{BF} \sum_i n_{Bi} n_{Fi}$$

“ ^{40}K (Fermion)- ^{87}Rb (Boson)” $a_{BF} = -10.9 \text{ nm}$

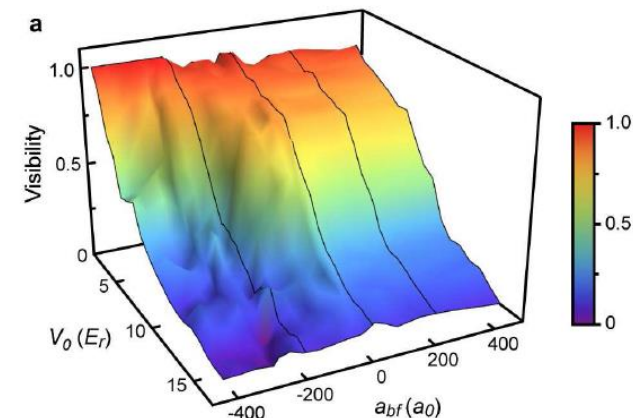


[K. Günter, et al, PRL96, 180402 (2006)]

[S. Ospelkaus, et al, PRL96, 180403 (2006)]

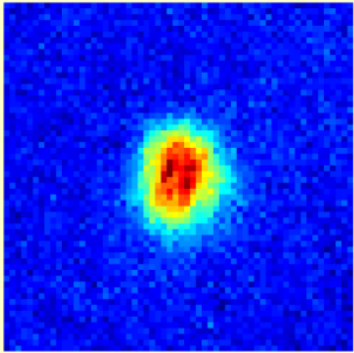
“Role of interactions in Rb-K Bose-Fermi mixtures in a 3D optical lattice”

[Th. Best, *et al*, PRL102, 030408 (2008)]

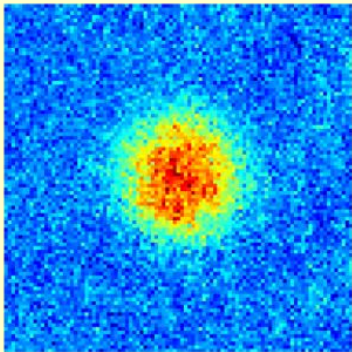


Bose-Fermi Mixture in a 3D Optical Lattice

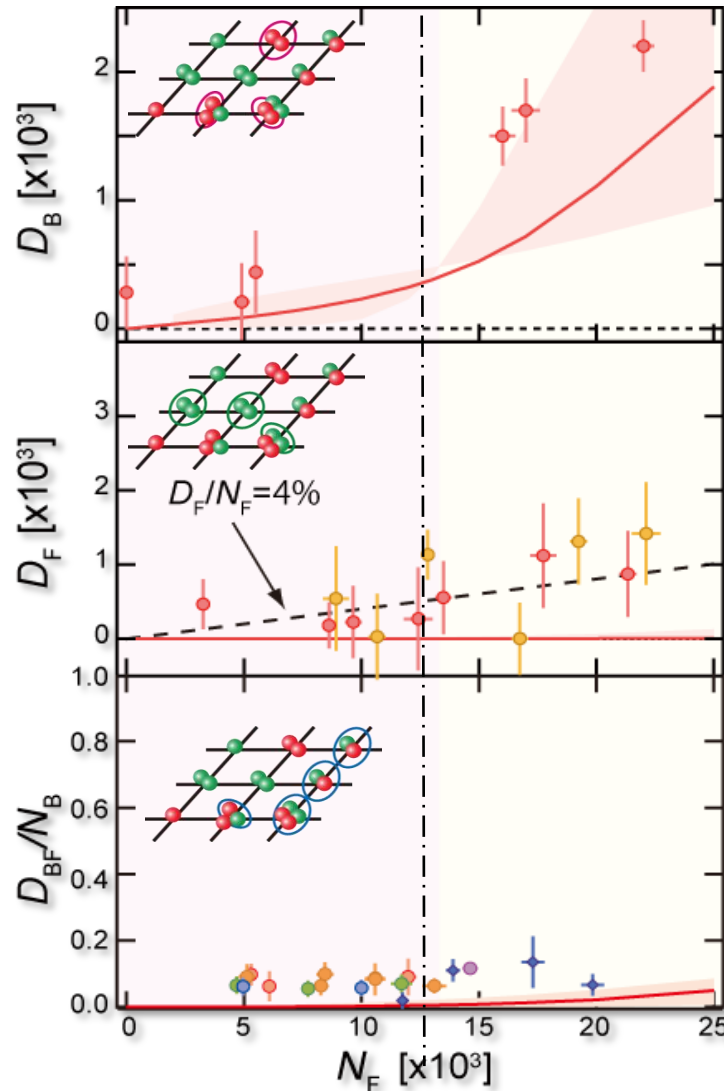
Boson
: ^{174}Yb



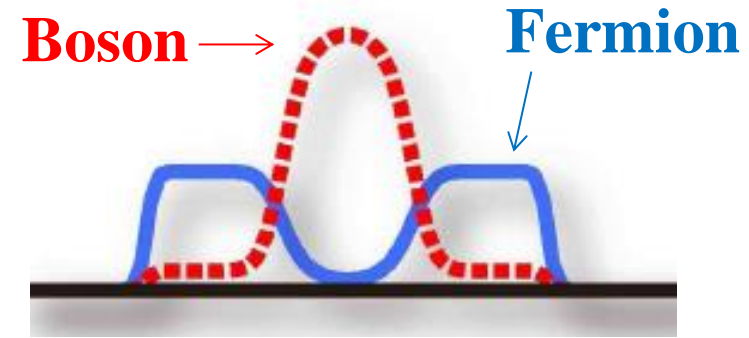
Fermion
: ^{173}Yb



$T/T_F = 0.17$



“Phase Separation”

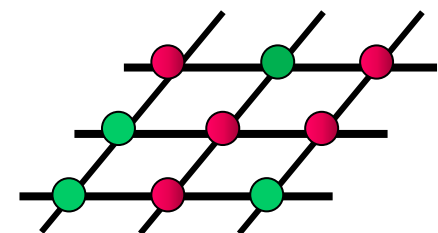


$N_F = 2 \times 10^4$

“Mixed Mott Insulator”



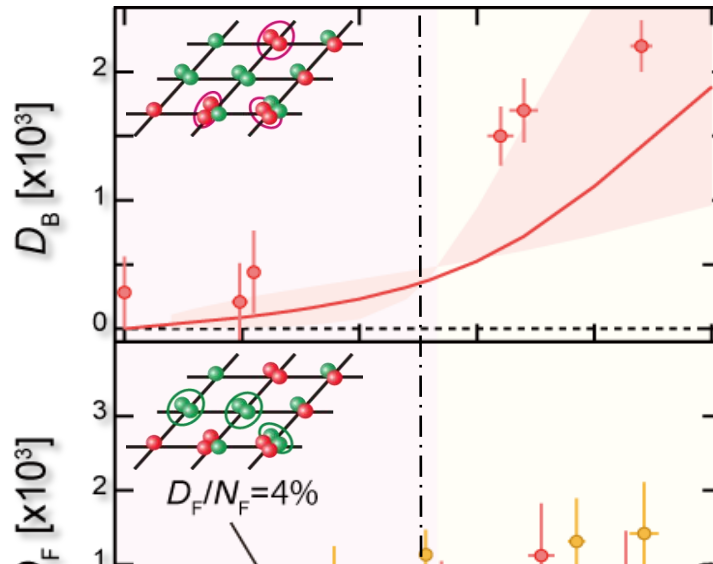
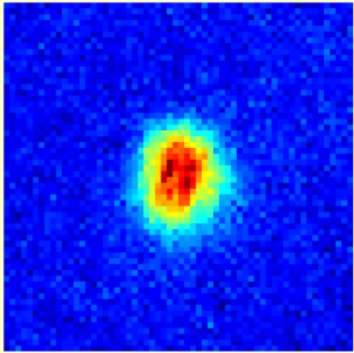
$N_F = 1 \times 10^4$



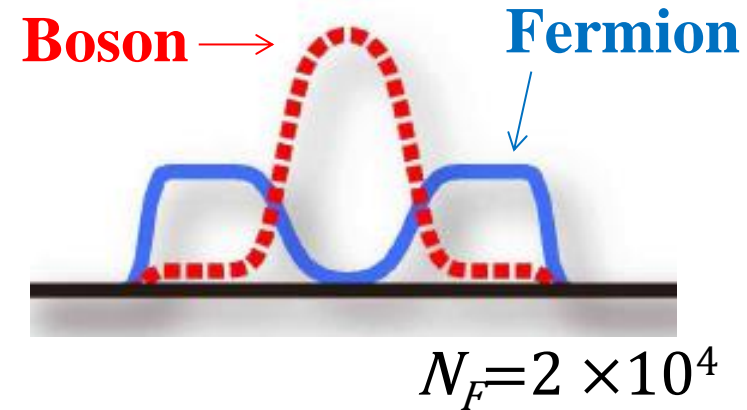
[S. Sugawa, *et al.*, NP.7, 642(2011)]

Bose-Fermi Mixture in a 3D Optical Lattice

Boson
: ^{174}Yb



“Phase Separation”



Fermion
: ^{171}Yb

Theory:

Ehud Altman, Eugene Demler, Achim Rosch

“Mott criticality and pseudogap in Bose-Fermi mixtures” PRL(2013)

I. Danshita and L. Mathey

“Counterflow superfluid of polaron pairs in Bose-Fermi mixtures in optical lattices” PRA(2013)

[S. Sugawa, *et al.*, NPJ, 642(2011)]

Quantum Simulation of Lattice-Gauge-Higgs-Model

Kasamatsu *et al*, PRL(2013), Kuno et al, NJP(2015)

Bose-Hubbard Model with Off-Site Interaction:

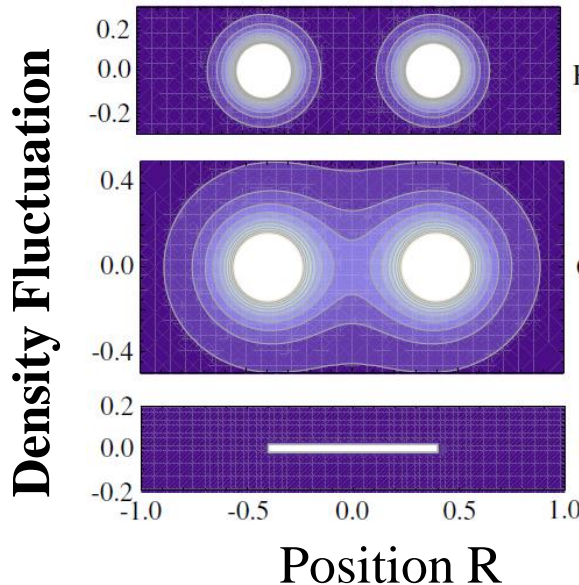
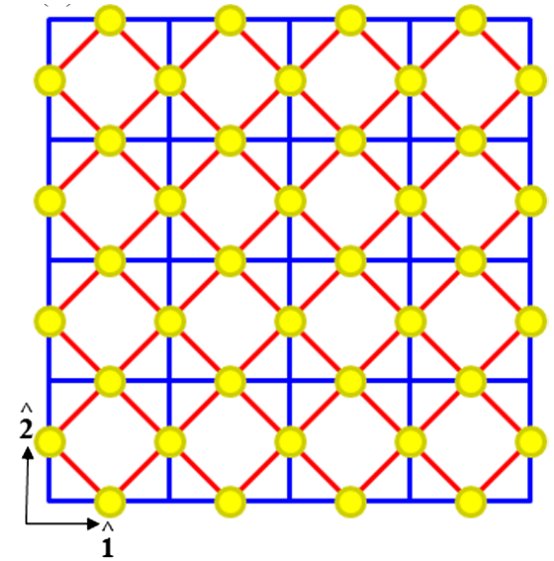
$$H = -J \sum_{\langle i,j \rangle} a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1) + \frac{V}{2} \sum_{\{i,j\}} n_i n_j$$

hopping
(red + blue lines)

On-site interaction
(yellow circles)

Off-site Interaction
(red + blue lines)

Mielke Lattice



Higgs Phase:
 $\exp(-mR)/R$

Coulomb Phase:
 $1/R$

Confinement Phase:
 R

Summary

I) Preparation of Quantum Gas

Laser cooling and trapping, evaporative cooling, Bose-Einstein condensate, Fermi Degenerate Gas

II) Ultracold Atoms in a Harmonic Trap

Feshbach resonance, Cooper pairing, BEC-BCS crossover, unitary gas, spin-orbit interaction

III) Ultracold Atoms in an Optical Lattice

Superfluid-Mott insulator transition, quantum-gas-microscope, Higgs mode, Non-standard lattices (frustrated magnetism, flat band), Fermi-Hubbard model, Bose-Fermi mixture

Group Members



Thank you very much for attention



16 August Mount Daimonji at Kyoto