SU(3)ゲージ理論における
エントロピックC関数の測定

伊藤　悦子 (KEK)

collaboration with K. Nagata (KEK), Y.Nakagawa, A. Nakamura (Hiroshima U., RCNP) and V.I.Zakharov (Max Planck Inst.)

cf. arXiv:0911.2596 and 1104.1011:
Y.Nakagawa, A.Nakamura, S.Motoki and V.I.Zakharov

基研研究会「熱場の量子論とその応用」@ YITP, Kyoto University 2015/8/31

メインテーマ

エンタングルメント・エントロピー(エントロピックC関数)から
閉じ込めに対する新しい知見を得る

４次元ゲージ理論における
量子エンタングルメントの非摂動論的な振る舞いは？

cf) arXiv:1508.07132[hep-lat] **today!**
Lattice study on QCD-like theory with exact center symmetry
T.Iritani, E.I and T.Misumi

⇒ 関連した研究: ９月2日（水）河野さん

Basic properties of E.E.

entanglement entropy for quantum system

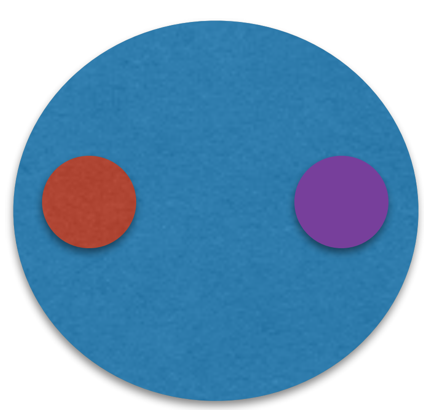
- how much a quantum state is entangled quantum mechanically
- d.o.f of the system
- quantum properties of the ground state for the system
- in finite T system, it gives a thermal entropy

QCD theory (T=0)

A color confinement changes the d.o.f of the system

microscopically | $\Lambda_{\rm QCD}$ | macroscopically

colorful (gluons) $\sim O(N_c^2)$



colorless (singlet) $\sim O(1)$

Definition of the entanglement entropy

Entanglement entropy (E.E.)

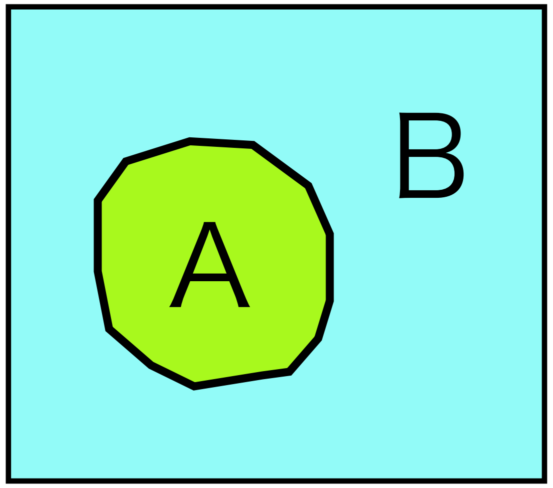
von Neumann entropy $$S_{tot} = -\mathrm{Tr}\rho_{tot} \log \rho_{tot}$$

density matrix $$\rho_{tot} = |\Psi\rangle\langle\Psi|$$ $|\Psi\rangle$:pure ground state

decompose total Hilbert space into two subsystems $$\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$$

reduced density matrix $$\rho_A = -\mathrm{Tr}_{\mathcal{H}_B} \rho_{tot}$$

entanglement entropy $$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

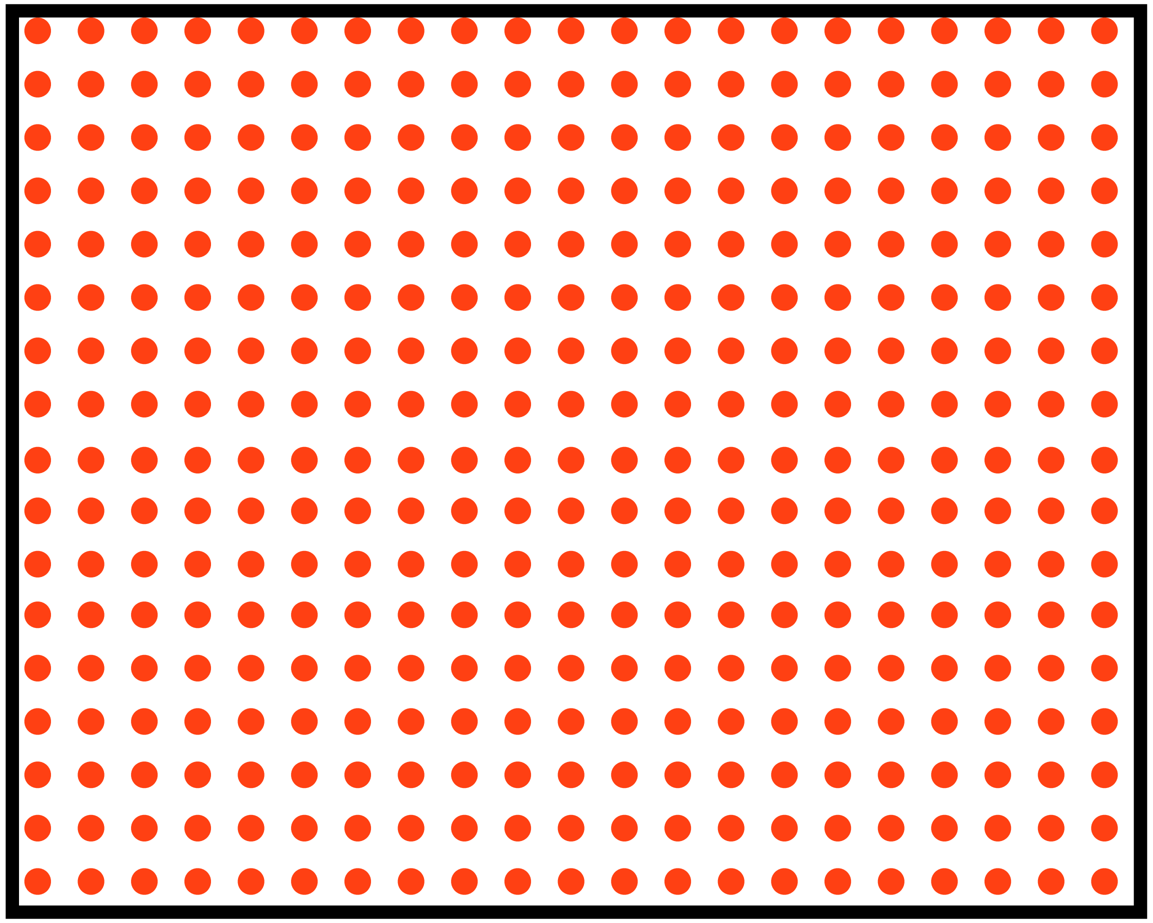


At finite T, it is equivalent to the thermal entropy.

Schematic picture of E.E.

Holzhey,Larsen and Wilczek: NPB424 (1994) 443
Calabrese and Cardy: J.S.M.0406(2004)P06002
Calabrese and Cardy, arXiv:0905.4013

(1+1)-dim. model



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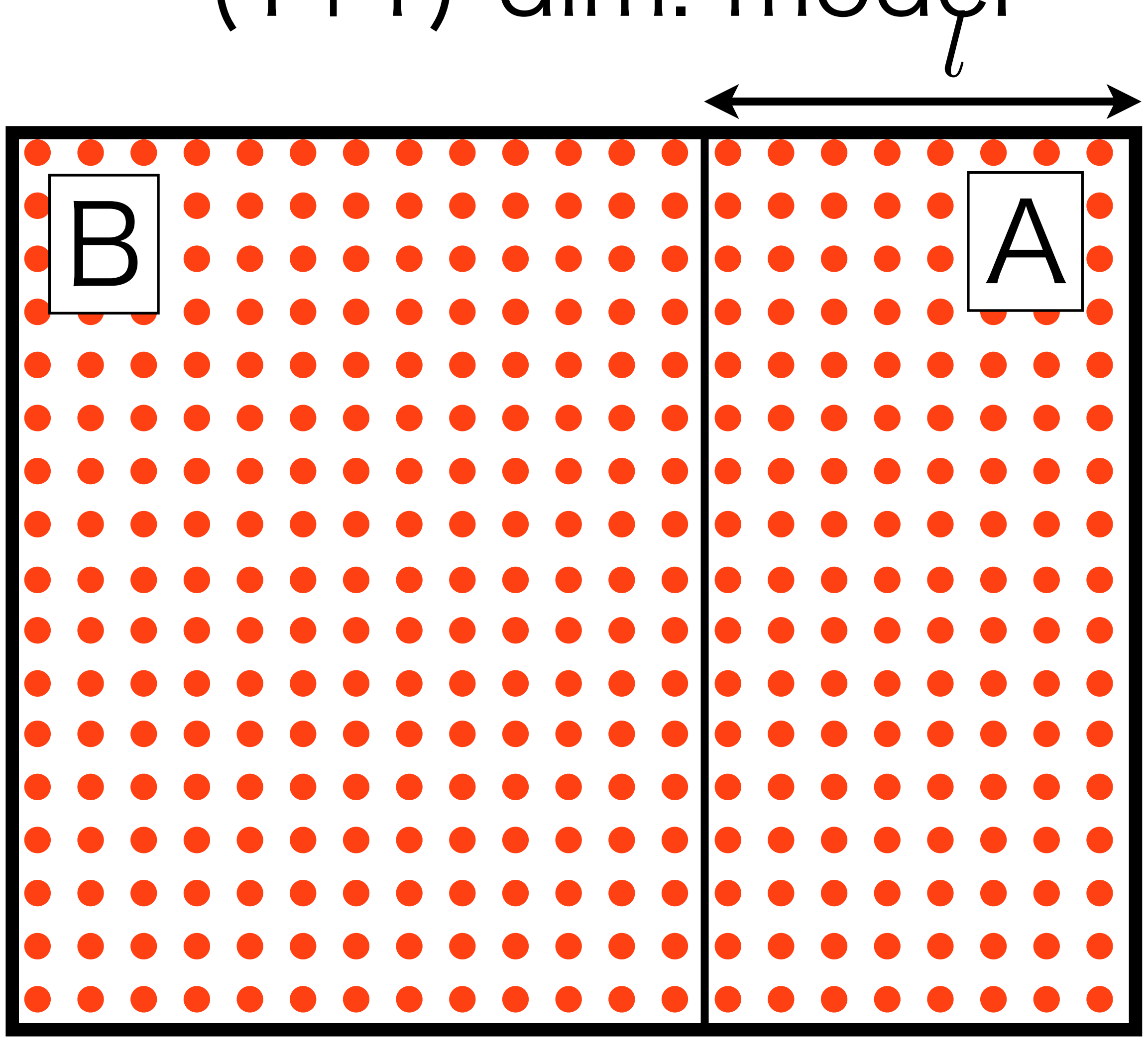

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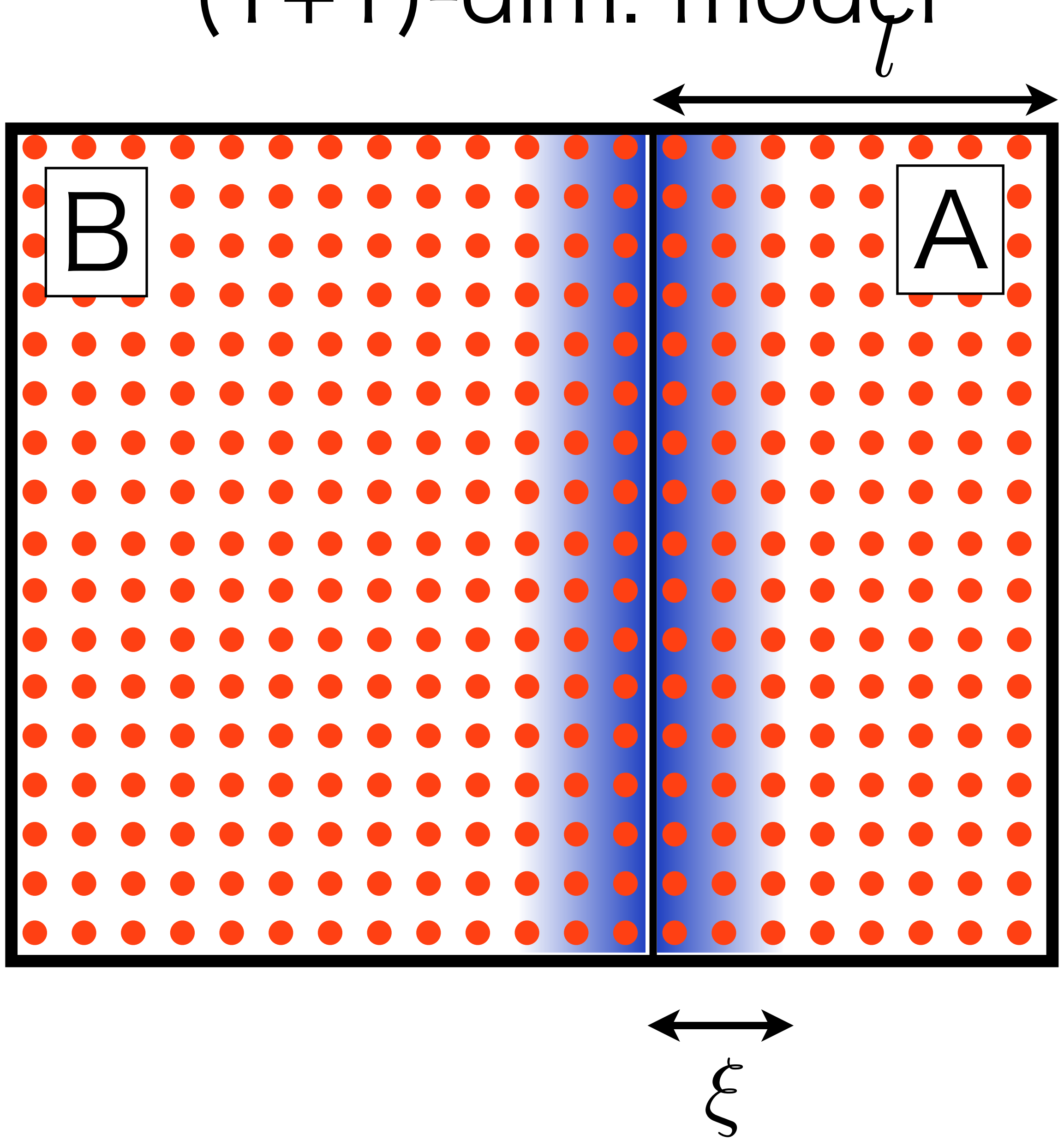


ξ: correlation length of the system

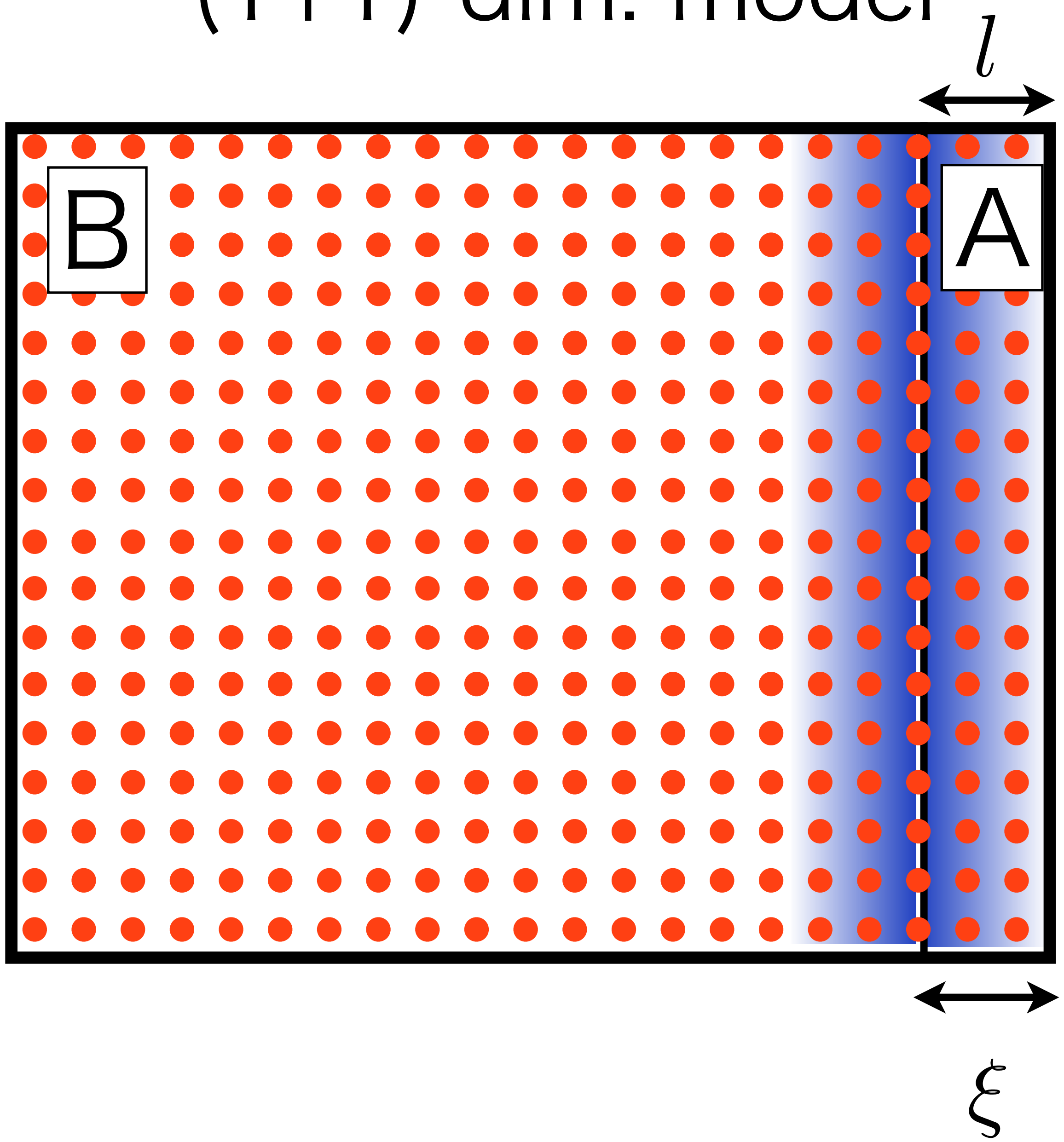
At the critical point,

$$S_A(l) = \frac{c}{3}\log\frac{l}{a} + c_1$$

c is the central charge in 2d CFT.

Schematic picture of E.E.

(1+1)-dim. model



ξ: correlation length of the system

In the non critical system,

$l \ll \xi$

$$S_A(l) \sim \frac{c}{3} \log \frac{l}{a}$$

Schematic picture of E.E.

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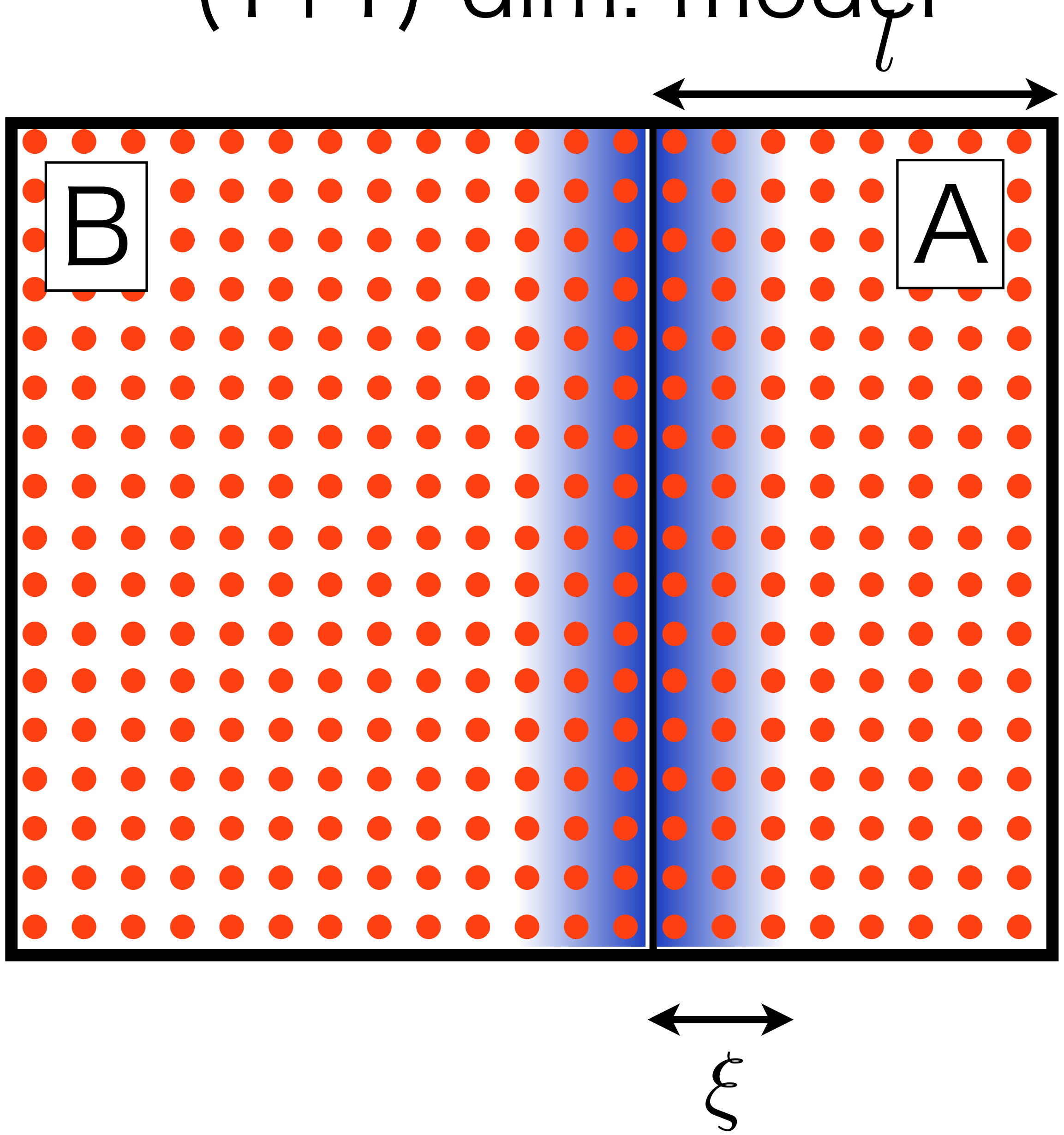


In the non critical case,

$l \ll \xi$

$$S_A(l) \sim \frac{c}{3} \log \frac{l}{a}$$

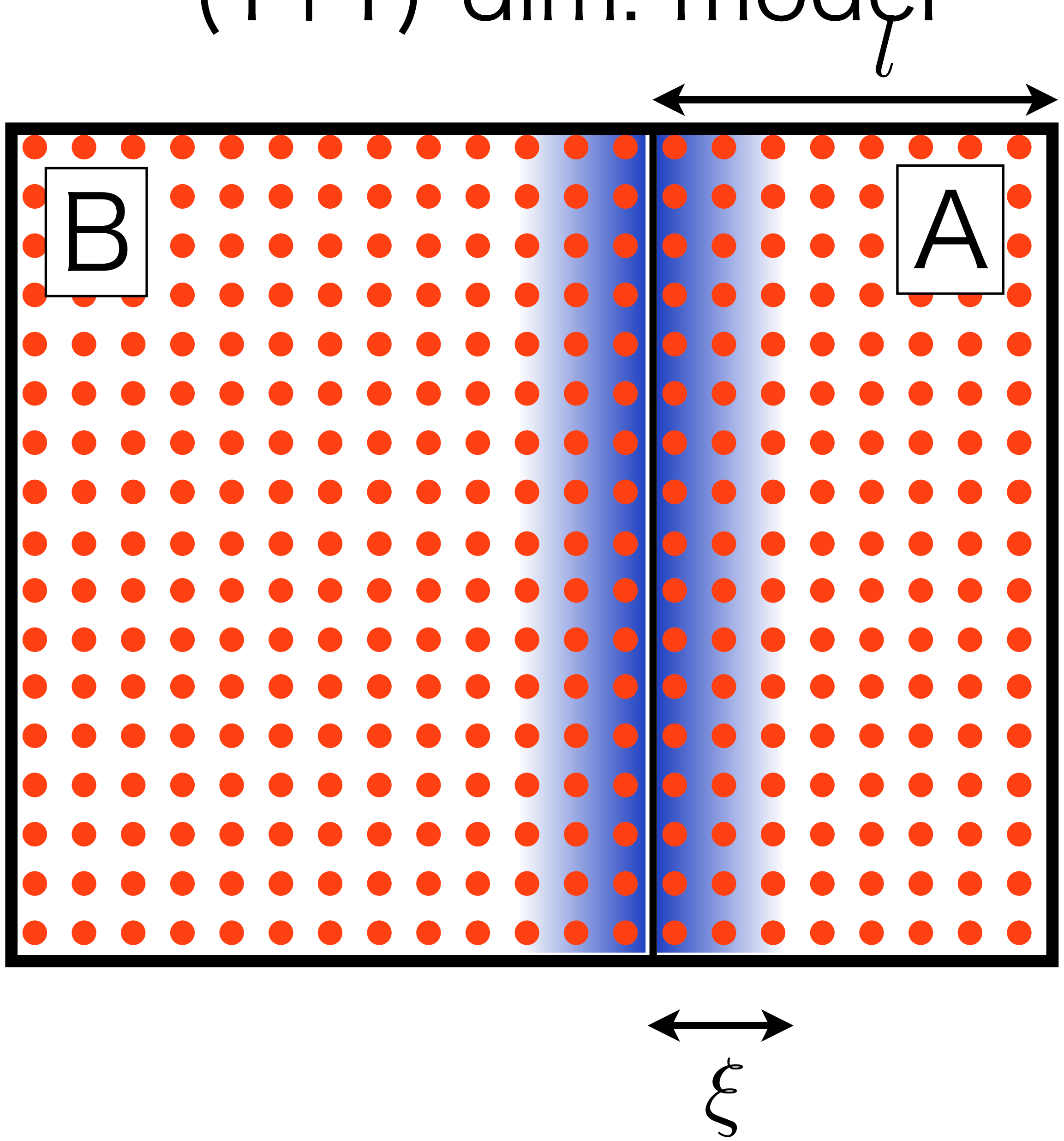
$l \gg \xi$

$$S_A(l) \to \frac{c}{3} \log \frac{\xi}{a}$$

ξ: correlation length of the system

Schematic picture of E.E.

(1+1)-dim. model



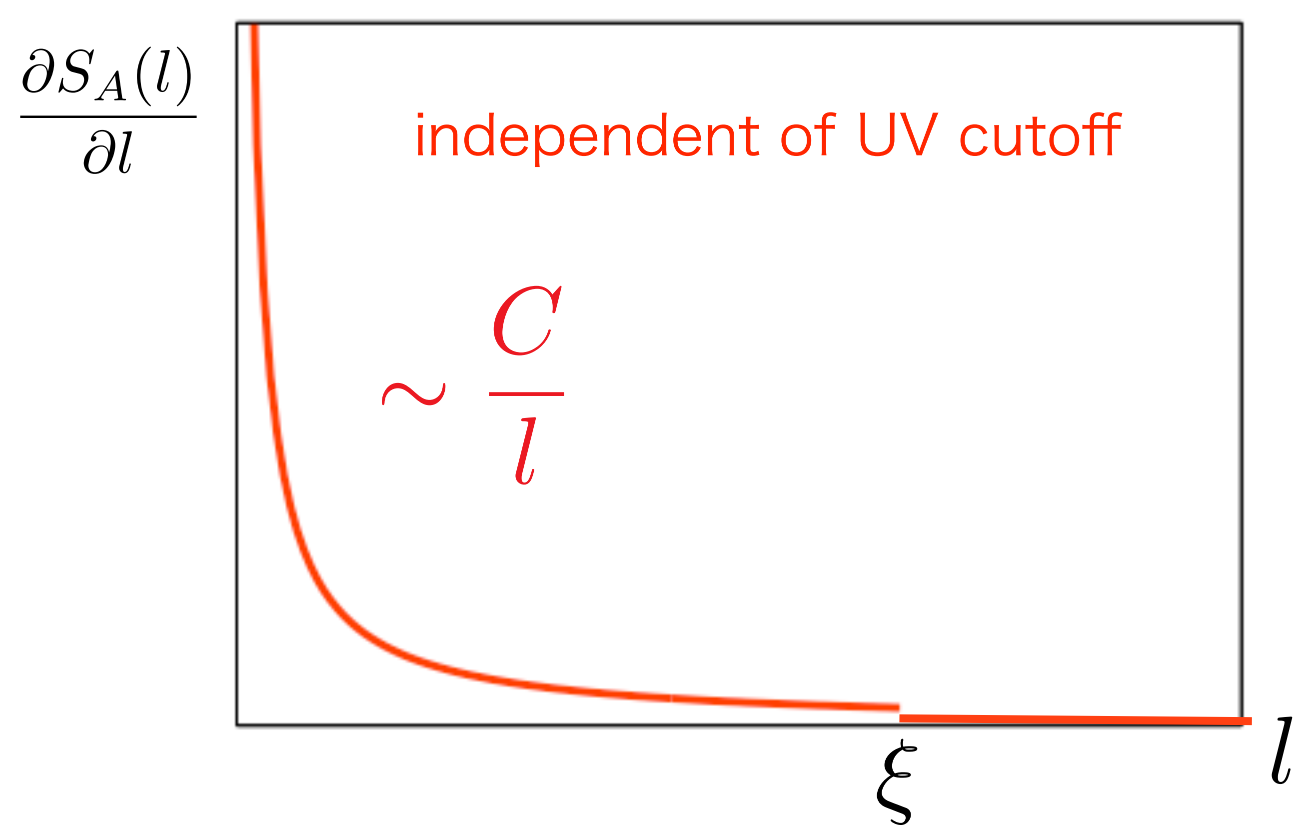
ξ: correlation length of the system

At the critical point or $l \ll \xi$ in the noncritical system

$$S_A(l) = \frac{c}{3}\log\frac{l}{a} + c_1$$

In the non-critical system,

$$S_A(l) \underset{l \gg \xi}{\longrightarrow} \frac{c}{3}\log\frac{\xi}{a}$$



Difficulties to obtain E.E. in 4d gauge theory

- UV cutoff dependence of 4d E.E.

2d

$$S_A(l) = c_0 \log(l/a)$$

Entropic C-function $C(l) = l\frac{dS}{dl}$

4d

$$S_A(l) = c_0 \frac{\text{Area}}{a^2} - c_0' \frac{\text{Area}}{l^2} + c_1 \log(l/a) + (\text{regular terms})$$

Entropic C-function $C(l) = l^3 \frac{1}{|\partial A|}\frac{dS}{dl}$

Ryu and Takayanagi:PRL96(2006)181602
JHEP 0608(2006)045

Nishioka and Takayanagi JHEP 0701(2007) 090

cf) Holographic approach　　C is obtained by AdS and QFT

- (local) gauge invariance

E.E. for gauge theory

- P.V.Buividovich and M.I.Polikarpov PLB670(2008)141

  extended Hilbert space

- H.Casini, M.Muerta and J.A.Rosabal arXiv:1312.1183

  electric b.c.(electric center), magnetic center, trivial center

- D.Radicevic arXiv:1404.1391

  magnetic center

- W.Donnelly PRD85 (2012) 085004

  extended lattice construction

- S.Ghosh, R.M.Soni,S.P.Trivedi arXiv:1501.02593

- S.Aoki, T.Iritani, M.Nozaki et.al. arXiv:1502.04267

  maximally gauge invariant reduced density matrix

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red definitions are inadequate for E.E. or rhoA

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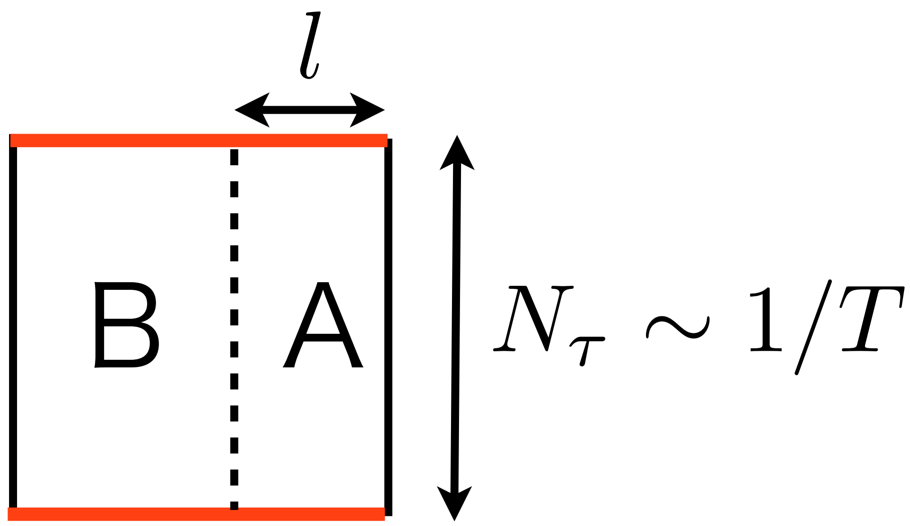
Replica method

Calabrese and Cardy: J.S.M.0406(2004)P06002

Replica method

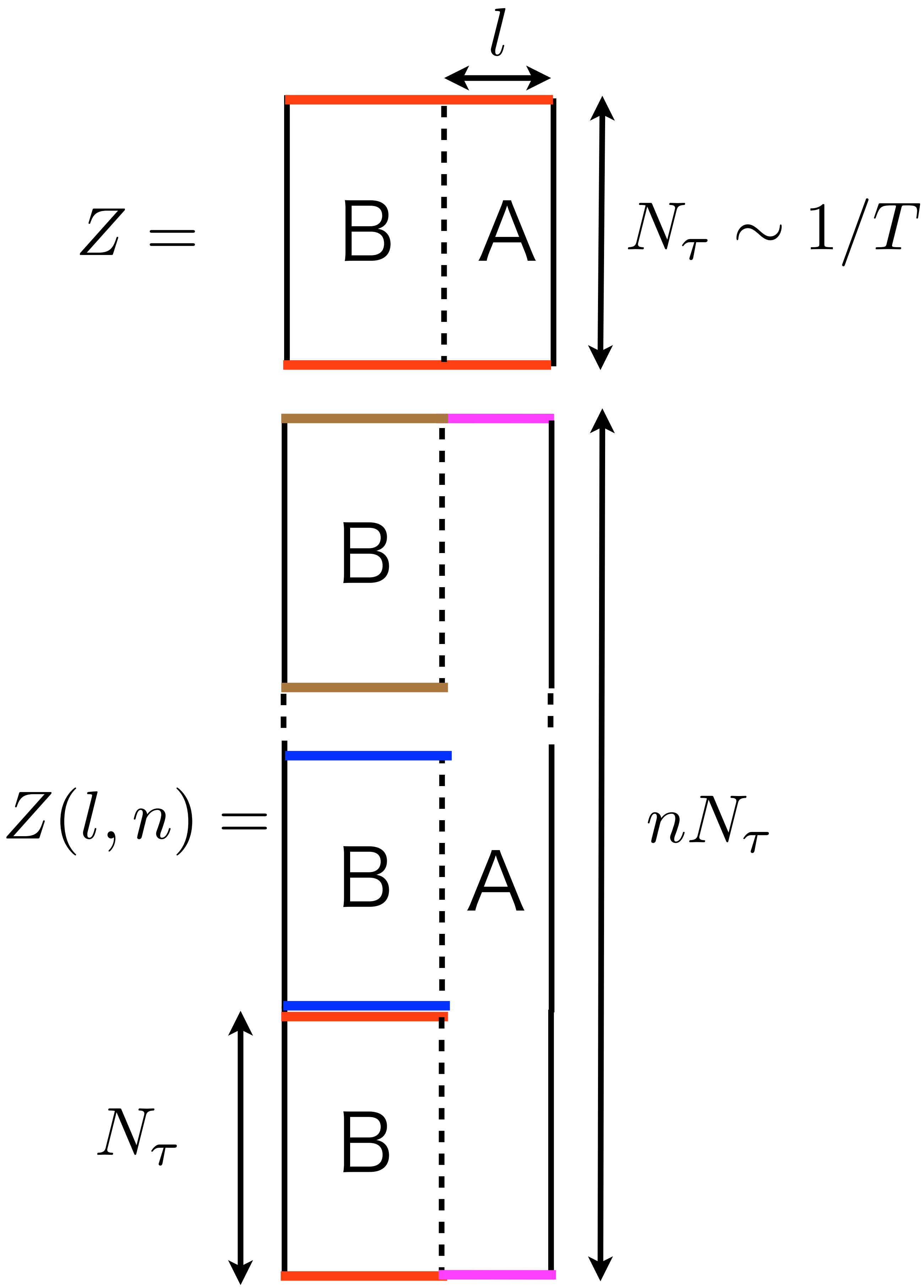
$Z =$ 

entanglement entropy:

$$S_A = -\lim_{n\to 1}\frac{\partial}{\partial n}\ln \mathrm{Tr}_A \rho_A^n$$

$$S_A(l) = -\lim_{n\to 1}\frac{\partial}{\partial n}\ln\left(\frac{Z(l,n)}{Z^n}\right)$$

Replica method



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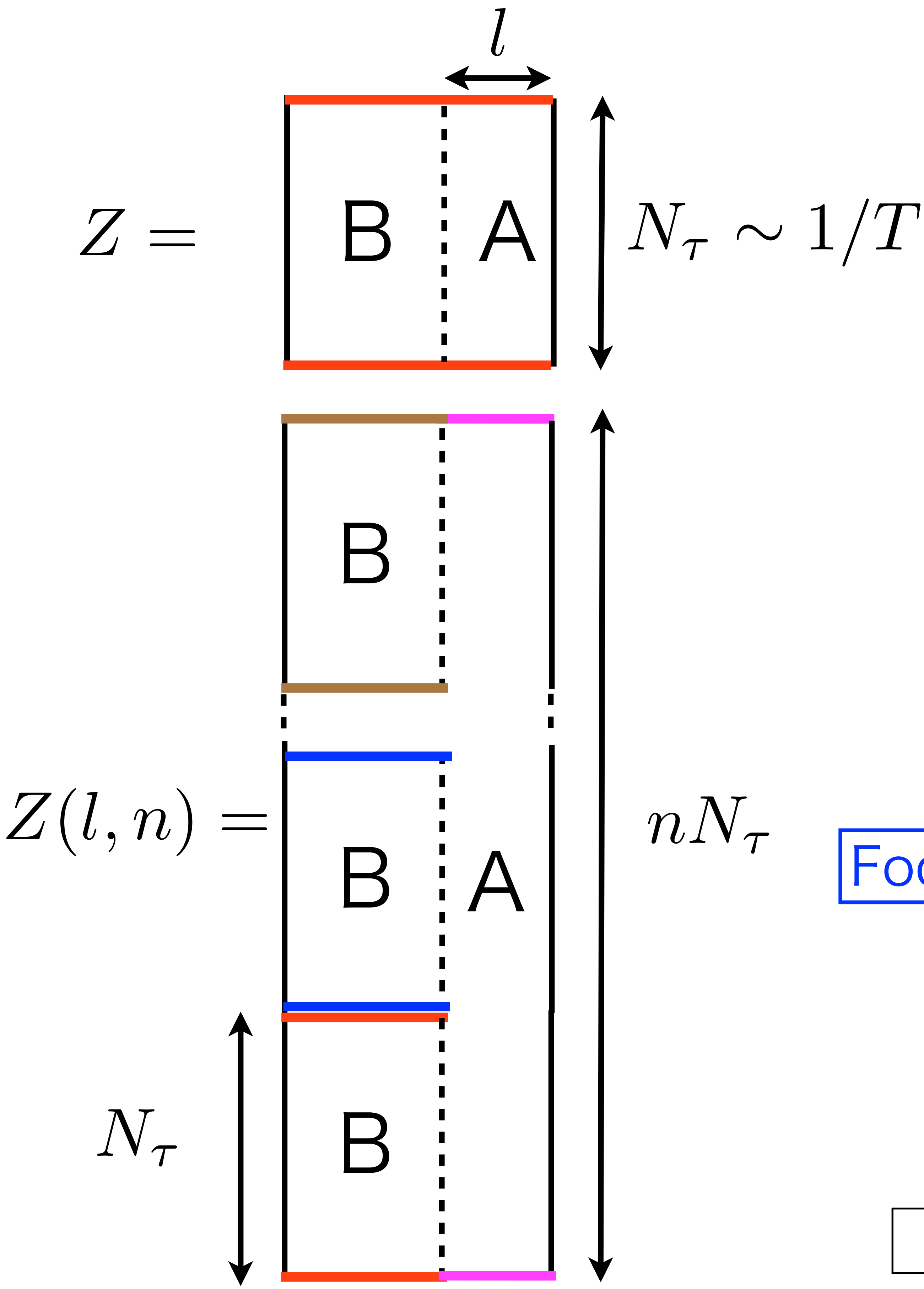

entanglement entropy:

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$$S_A(l) = -\lim_{n\to 1}\frac{\partial}{\partial n}\ln\left(\frac{Z(l,n)}{Z^n}\right)$$

observable:

$$\frac{\partial S_A(l)}{\partial l} = \lim_{n\to 1}\frac{\partial}{\partial l}\frac{\partial}{\partial n}F[l,n] \quad \text{:free energy}$$

$$\to \frac{F[l+a,n=2]-F[l,n=2]}{a}$$

Fodor (2009)

$$= \int_0^1 d\alpha \langle S_{l+a}[U]-S_l[U]\rangle_\alpha$$

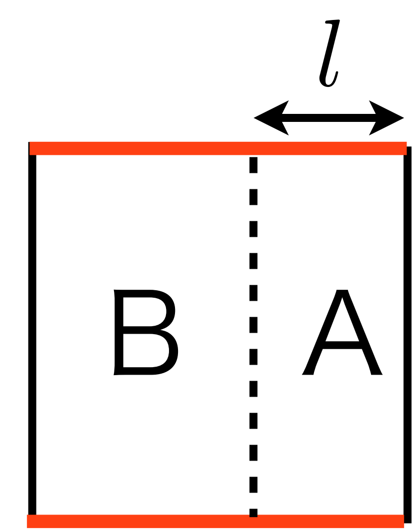
using the interpolation action

$$S_{int} = (1-\alpha)S_l[U] + \alpha S_{l+a}[U]$$

we measure the diff. of the action density

Simulation results

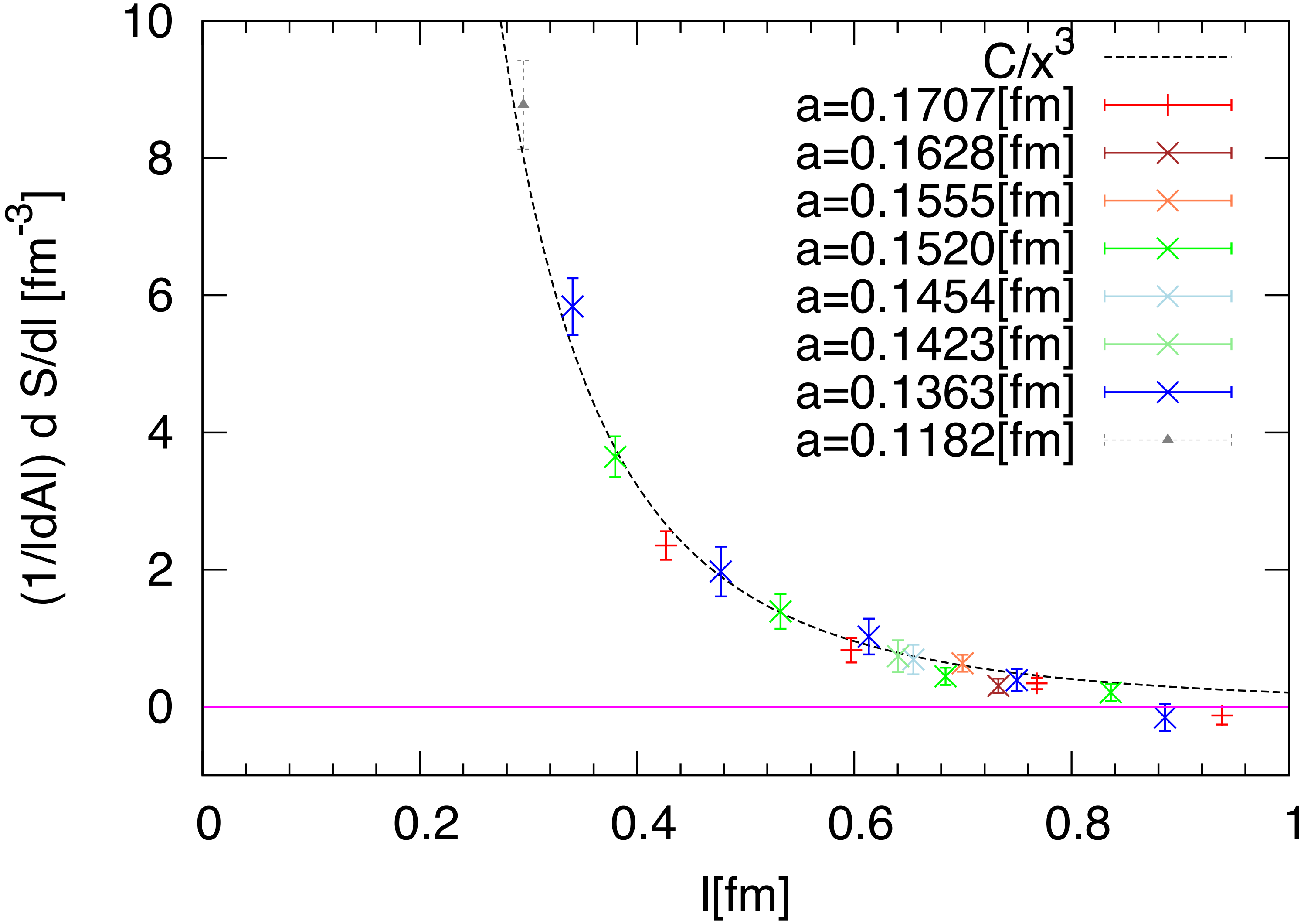
Simulation setup



- Wilson plaquette gauge action
- Ns=Nt=16, 32
- l/a=2,3,4,5,(6)
- beta=5.70 - 5.87
- # of configuration 12,000~84,000
- scale setting r_0 =0.5 fm and ALPHA coll.

Lattice results for quenched SU(3)

T=0, quenched QCD



We measure ~84,000 configs.

- in short range, 1/l^3 scaling
- the coefficient is C=0.2064(73)

cf.) C~0.09 in SU(2)

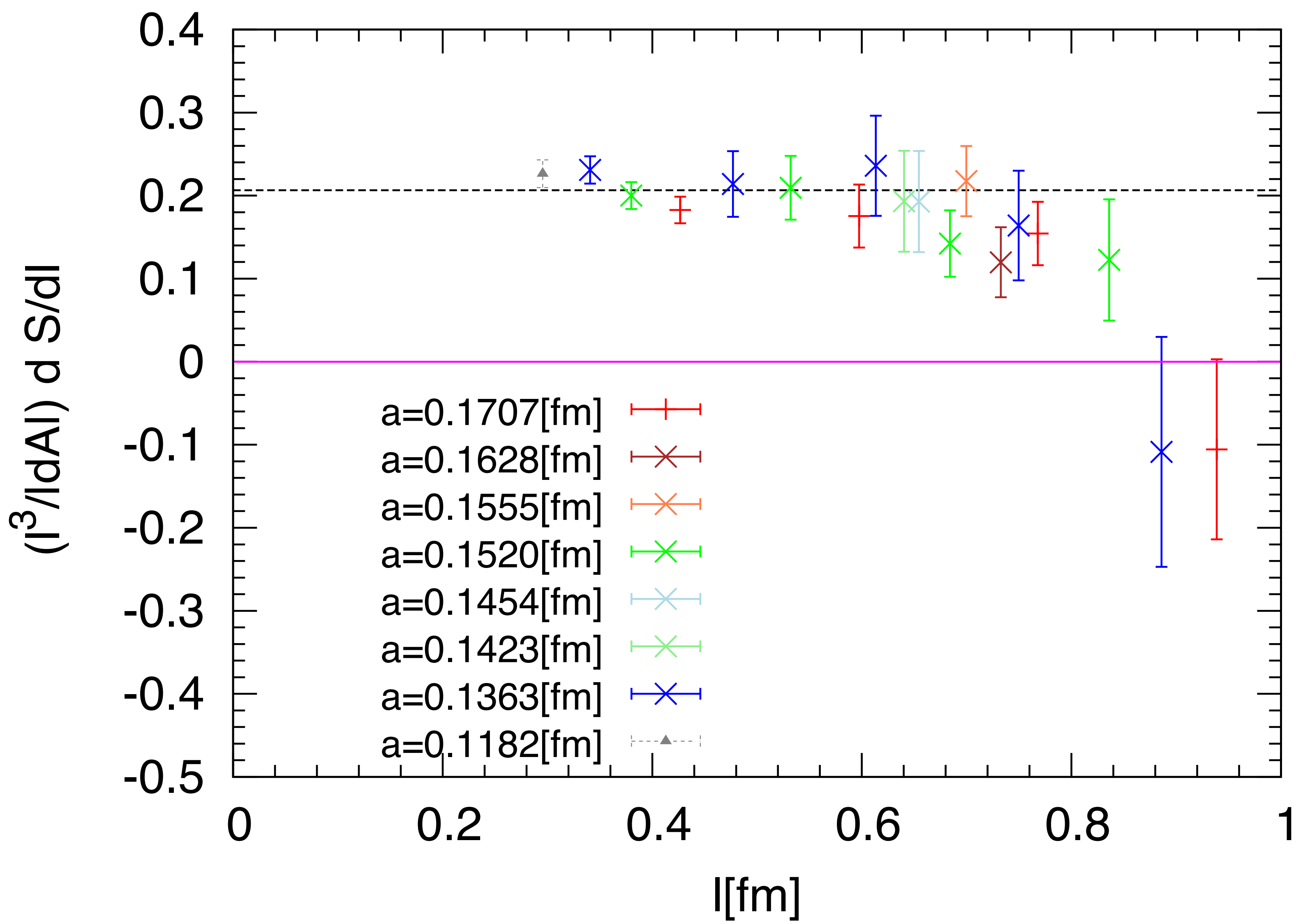
The Nc dependence is

$$\frac{0.2064}{0.09} = 2.29 \sim \frac{3^2}{2^2}$$

Entropic C-function

independent of UV cutoff

$$C(l) = l^3 \frac{1}{|\partial A|} \frac{dS}{dl}$$



In short l region, C is constant.

The discontinuity is not clear.

cf.) $\frac{1}{\Lambda_{QCD}} \sim 0.7[\mathrm{fm}]$

Comparison with Ryu-Takayanagi results

Ryu and Takayanagi:PRL96(2006)181602
JHEP 0608(2006)045

Holographic (or field theoretical) approach

(3+1)-dim. CFT $$\frac{1}{|\partial A|} S_A(l) = c\frac{N_c^2}{a^2} - c'\frac{N_c^2}{l^2}$$

c' is obtained by AdS and QFT

$c' \sim 0.0049$ for free real scalar theory

Estimation for non-abelian gauge theory

$$A_\mu^a \quad \begin{cases} a = 1, \cdots, 8 \\ \mu = 1, \cdots, 4 \end{cases}$$

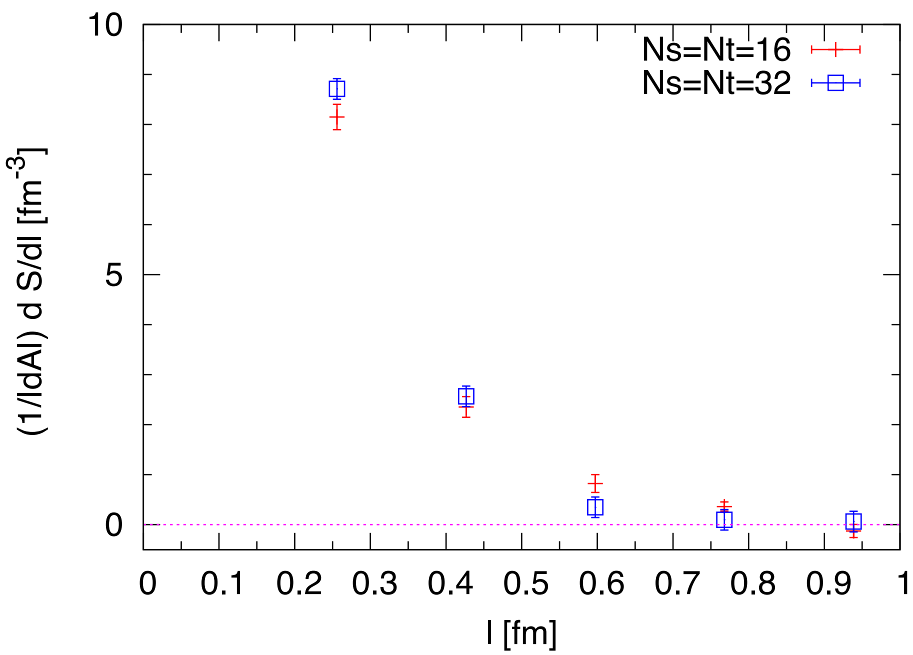
$$C_{\text{gauge}} \sim 2c' \cdot 2 \cdot 8 \sim 0.1568$$

Our numerical result

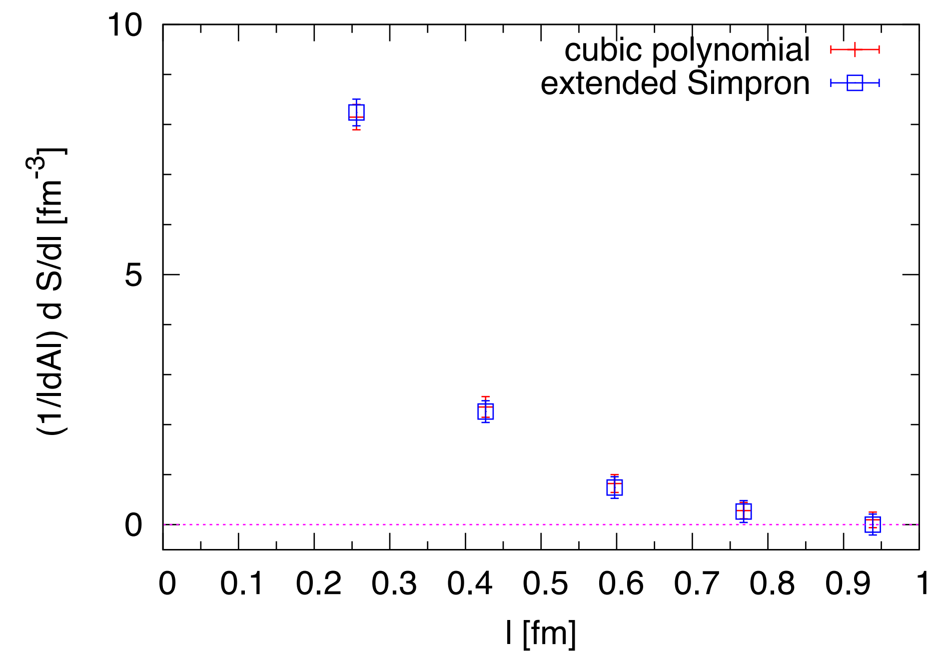
$$C_{\text{gauge}} \sim 0.2064$$

Detail analyses

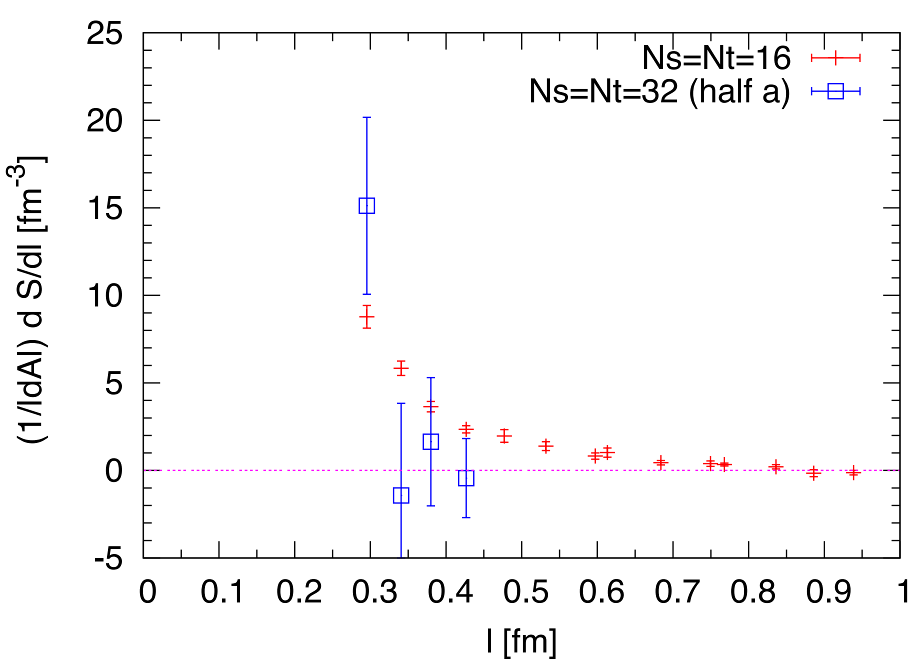
finite vol. effect



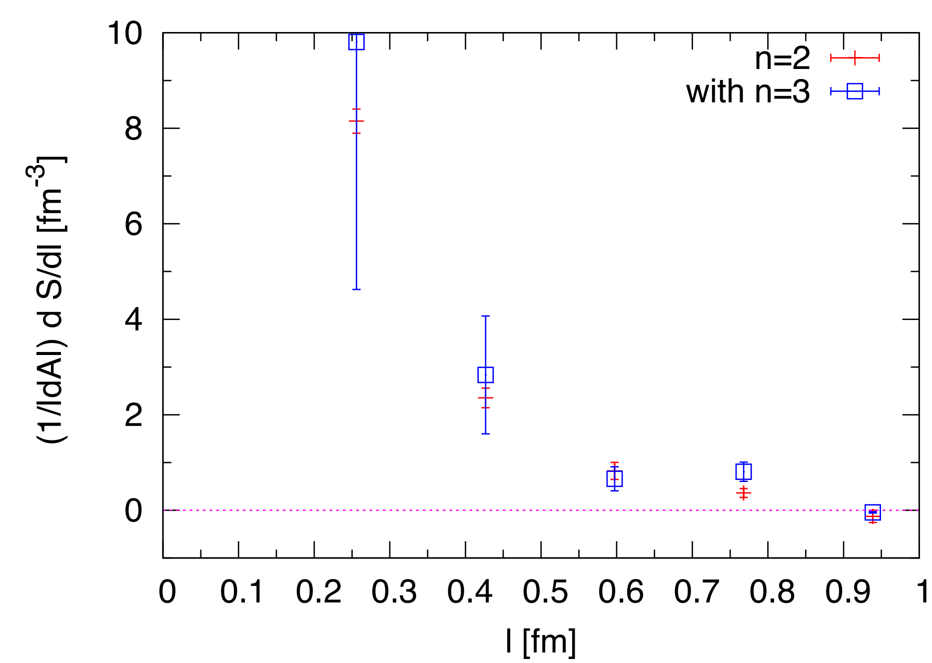
algorithm dependence of the numerical integration



UV cutoff dependence



replica number dependence



Summary

- This is the first precise determination of E.E. for quenched QCD
- Nc dependence in the short l region is Nc^2 as expected by AdS/CFT and field theoretical insights
- No discontinuity exists as contrast with SU(2) results
- Entropic C-function shows UV cutoff independence
- Value of C-function agrees with Ryu-Takayanagi work
- replica number (n ->1) dependence

Future directions for E.E. using the lattice

• QCD at zero T

- give a novel observation for confinement
- even in full QCD case

• QCD at finite T

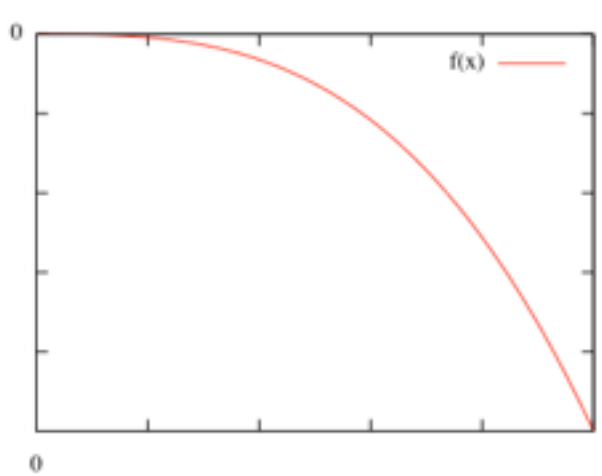
- gives the thermal entropy and the correlation length in QGP phase

• conformal window in 4dim Nf flavor QCD

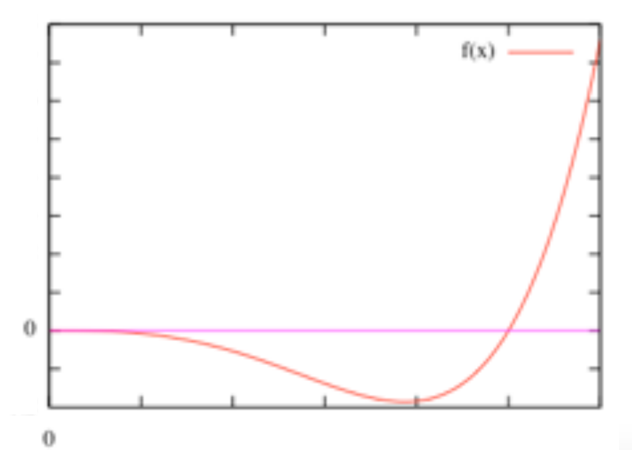
- would give the a-function and central charge

beta fn.

Nf<9



9<Nf<17



nontrivial IR fixed point is found by lattice simulation

cf) E.I. PTEP(2013)083B01