クォーク閉じ込め・非閉じ込め有限温度相転移と磁気的モノポールの役割

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Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson, 1974]

\[
\langle \text{tr} \left[ \mathcal{P} \exp \left\{ ig \int_C dx^\mu A_\mu(x) \right\} \right] \rangle_{\text{YM}}^{\text{no GF}} \sim e^{-\sigma_N A|S|}
\]

\[
V(r) = -C \frac{g_{YM}^2(r)}{r} + \sigma r
\]

\[
F(r) = -\frac{d}{dr} V(r) = -C \frac{g_{YM}^2(r)}{r^2} - \sigma + \cdots \quad (C, \sigma > 0)
\]

\[
V(r) \to \infty \quad \text{for} \quad r \to \infty
\]

What is the mechanism of confinement?


**superconductor**
- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

**dual superconductor**
- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

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Electro- magnetic duality
Evidences for the dual superconductivity

By using Abelian projection

String tension (Linear potential)
- Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley, 1994][Shiba & Suzuki, 1994]

Chromo-flux tube (dual Meissner effect)
- Measurement of (Abelian) dual Meissner effect
  - Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux
  - Type the super conductor is of order between Type I and Type II [Y.Matsubara, et.al. 1994]

✓ only obtained in the case of special gauge such as MA gauge
✓ gauge fixing breaks the gauge symmetry as well as color symmetry
The evidence for dual superconductivity

Gauge decomposition method (a new lattice formulation)

- Extracting the relevant mode $V$ for quark confinement by solving the defining equation in the gauge independent way (gauge-invariant way)

  - For SU(2) case, the decomposition is a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.
  - For SU(N) case, the formulation is the extension of the SU(2) case.

   we have showed in the series of lattice conferences that

   - $V$-field dominance, magnetic monopole dominance in string tension,
   - chromo-flux tube and dual Meissner effect.
   - The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect
A new formulation of Yang-Mills theory (on a lattice)

**Decomposition of SU(N) gauge links**

- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:
  - SU(2) Yang-Mills link variables: unique \( U(1) \subset SU(2) \)
  - SU(3) Yang-Mills link variables: **Two options**
    - **maximal option**: \( U(1) \times U(1) \subset SU(3) \)
      - Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)
    - **minimal option**: \( U(2) \cong SU(2) \times U(1) \subset SU(3) \)
      - Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes’ theorem
The decomposition of SU(3) link variable: minimal option

\[ W_C[U] := \frac{\mathrm{Tr} \left[ P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right]}{\mathrm{Tr}(1)} \]

\[ U_{x,\mu} = X_{x,\mu} V_{x,\mu} \]

\[ U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger \]
\[ V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger \]
\[ X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger \]

\[ \Omega_x \in G = SU(N) \]

\[ W_C[V] := \frac{\mathrm{Tr} \left[ P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right]}{\mathrm{Tr}(1)} \]

\[ W_C[U] = \text{const.} W_C[V] \]
SU(3) Yang-Mills theory

- In confinement of fundamental quarks, a restricted non-Abelian variable $V$, and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

**gauge independent “Abelian” dominance**

\[
\frac{\sigma_V}{\sigma_U} = 0.92
\]

\[
\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82
\]

**Gauge independent non-Abelian monopole dominance**

\[
\frac{\sigma_M}{\sigma_U} = 0.85
\]

\[
\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76
\]


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Chromo flux

\[ \rho_W = \frac{\langle \text{tr}(WLU_pL^\dagger) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(W)\text{tr}(U_p) \rangle}{\langle \text{tr}(W) \rangle} \]

**Gauge invariant correlation function:** This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3) [Adriano Di Giacomo et al. PLB236:199,1990 NPBB347:441-460,1990]
A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

**Flux tube is observed for V-field case. :: dual Meissner effect**
Magnetic current induced by quark and antiquark pair

Yang–Mills equation (Maxell equation) for restricted field $V_\mu$, the magnetic current (monopole) can be calculated as

$$ k = \delta^* F[V] = *dF[V], $$

where $F[V]$ is the field strength of $V$, $d$ exterior derivative, $*$ the Hodge dual and $\delta$ the coderivative $\delta := *d^*$, respectively.

$k \neq 0 \implies$ signal of monopole condensation. Since field strength is given by $F[V] = dV$, and $k = *dF[V] = *ddF[V] = 0$ (Bianchi identity)

Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current (right scale).
Confinement / deconfinement phase transition in view of the dual Meissner effect.

• We measure the chromo-flux generated by a pair of quark and antiquark at finite temperature applying our new formulation of Yang-Mills theory on the lattice.

• The quark-antiquark source can be given by a pair of Polyakov loops in stead of the Wilson loop.

• Conventionally, average of Ployakov loops $<P>$ is used as order parameter of the phase transition.

• In the view of dual superconductivity
  - Confinement phase :: dual Meissner effect
    - Generation of the chromo-flux tube.
    - Generation of the magnetic current (monopole)
  - Deconfinement phase :: disappearance of dual Meissner effect.
The decomposition of SU(3) link variable: minimal option

\[ W_C[U] := \text{Tr} \left[ P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \text{Tr}(1) \]

\[ U_{x,\mu} = X_{x,\mu} V_{x,\mu} \]

\[ U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu} \]

\[ V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu} \]

\[ X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_x \]

\[ \Omega_x \in G = SU(N) \]

\[ W_C[V] := \text{Tr} \left[ P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \text{Tr}(1) \]

\[ W_C[U] = \text{const.} W_C[V] !! \]
Introducing a color field $h_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D^\varepsilon[V]h_x = \frac{1}{\varepsilon}(V_{x,\mu}h_{x+\mu} - h_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N} \exp(-a_x^{(0)} h_x - i \sum_{i=1}^3 a_x^{(i)} u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $A_\mu(x) = V_\mu(x) + X_\mu(x)$,

$$D_\mu[V_\mu(x)]h(x) = 0, \quad \text{tr}(X_\mu(x)h(x)) = 0.$$

**Exact solution (N=3)**

$$X_{x,\mu} = \hat{L}^\dagger_{x,\mu}(\det \hat{L}_{x,\mu})^{1/N} g_x^{-1} V_{x,\mu} = X^\dagger_{x,\mu} U_x, = g_x \hat{L}_{x,\mu} U_x, (\det \hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu} L^\dagger_{x,\mu}}\right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2 - 2N + 2}{N} \mathbf{1} + (N - 2) \sqrt{\frac{2(N-2)}{N}} \left( h_x + U_{x,\mu} h_{x+\mu} U_{x,\mu}^{-1} \right)$$

$$+ 4(N - 1) h_x U_{x,\mu} h_{x+\mu} U_{x,\mu}^{-1}$$

**continuum version by continuum limit**

$$V_\mu(x) = A_\mu(x) - \frac{2(N-1)}{N} [h(x), [h(x), A_\mu(x)]] - ig^{-1} \frac{2(N-1)}{N} [\partial_\mu h(x), h(x)].$$

$$X_\mu(x) = \frac{2(N-1)}{N} [h(x), [h(x), A_\mu(x)]] + ig^{-1} \frac{2(N-1)}{N} [\partial_\mu h(x), h(x)].$$
Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory.
- The configuration of the color fields $h_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$

$$F_{\text{red}}[h_x; U_{x,\mu}] = \sum_{x,\mu} \text{tr}\left\{ (D_{\mu}[U]h_x)^\dagger (D_{\mu}[U]h_x) \right\}$$

$SU(3)_\omega \times [SU(3)/U(2)]_\theta \to SU(3)_{\omega=\theta}$

- This is invariant under the gauge transformation $\theta=\omega$
- The extended gauge symmetry is reduced to the same symmetry as the original YM theory.
- We choose a reduction condition of the same type as the SU(2) case
Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection.

\[ W_C[A] = \int [d\mu(\xi)]_\Sigma \exp \left( -ig \int_{SC=\partial \Sigma} dS^{\mu \nu} \sqrt{\frac{N-1}{2N}} \text{tr}(2h(x)F_{\mu \nu}[V](x)) \right) \]

\[ = \int [d\mu(\xi)]_\Sigma \exp \left( ig \sqrt{\frac{N-1}{2N}} (k, \Xi) + ig \sqrt{\frac{N-1}{2N}} (j, N) \right) \]

magnetic current \( k := \delta^* F = *dF, \quad \Xi \Sigma := \delta^* \Theta \Sigma \Delta^{-1} \)
electric current \( j := \delta F, \quad N \Sigma := \delta \Theta \Sigma \Delta^{-1} \)

\[ \Delta = d\delta + \delta d, \quad \Theta \Sigma := \int_\Sigma d^2S^{\mu \nu}(\sigma(x))\delta^D(x-x(\sigma)) \]

\( k \) and \( j \) are gauge invariant and conserved currents; \( \delta k = \delta j = 0. \)

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The lattice version is defined by using plaquette:

\[ \Theta^8_{\mu \nu} := -\text{arg} \ \text{Tr} \left[ \left( \frac{1}{3} \mathbf{1} - \frac{2}{\sqrt{3}} h_x \right) V_{x,\mu} V_{x+\mu,\mu} V_{x+v,\mu}^+ V_{x,v}^+ \right], \]

\[ k_\mu = 2\pi n_\mu := \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} \partial_\nu \Theta^8_{\alpha \beta}. \]
Non-Abelian magnetic monopole loops: $24^3 \times 8$ lattice $b=6.0 \ (T \neq 0)$

Projected view $(x,y,z,t) \rightarrow (x,y,z)$

(left lower) loop length 1-10
(right upper) loop length 10 -- 100
(right lower) loop length 100 -- 1000

Monopole loop is winding to T direction.
Lattice set up

• Standard Wilson action
• $24^3 \times 6$ lattice
• Temperature is controlled by using $\beta (=6/g^2)$;
  $\beta=5.8, \ 5.9, \ 6.0, \ 6.1, \ 6.2, \ 6.3$
• Measurement by 1000 configurations
Distribution of Polyakov loop

\[ P_U(x) = \text{tr} \left( \prod_{t=1}^{Nt} U_{(x,t),4} \right) \] for original Yang–Mills field

\[ P_V(x) = \text{tr} \left( \prod_{t=1}^{Nt} V_{(x,t),4} \right) \] for restricted field
Polyakov loop average YM-field v.s. V-field
Chromo-electric flux at finite temperature

\[ \rho_W = \frac{\langle \text{tr}(WLU_p L^+) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(W) \text{tr}(U_p) \rangle}{\langle \text{tr}(W) \rangle} \]

\[ F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_W(x) \]

Size of Wilson loop T-direction = Nt

The quark and antiquark sources are given by Plyakov loops.
Chromo-flux $\beta=5.8$

YM field

V field

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Chromo-flux $\beta=5.9$

YM field

V field
Chromo-flux $\beta=6.0$

YM field  V field
Chromo-flux $\beta=6.1$

YM field

V field
Chromo-flux $\beta=6.2$

YM field

V field
Chromo-flux $\beta=6.3$
Chromo-electric flux in deconfinement phase

- $E_y \neq 0$ for deconfinement phase i.e., No sharp chromo-flux tube
  $\Rightarrow$ Disappearance of dual superconductivity.
Chromo-magnetic current (monopole current)

• To know relation to the monopole condensation, we further need the measurement of magnetic current in Maxwell equation for V field.

\[ k = \delta * F[V] = {^*dF[V]} \]

\( k \neq 0 \Rightarrow \) signal of monopole condensation. Since field strength is given by \( F[V] = dV \), and \( k = {^*dF[V]} = {^*ddF[V]} = 0 \) (Bianchi identity)
Chromo-magnetic (monopole) current $\beta = 5.8$

Confinement phase

Chromo-magnetic current $k_x$

Chromo-flux

$V_{b} = 5.80$

$E_x$, $E_y$, $E_z$, $B_x$, $B_y$, $B_z$
Chromo-magnetic (monopole) current $\beta=6.3$

deconfinement phase

chromo-magnetic current $k_x$

Chromo-flux

$\beta=6.3$

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Chromo-magnetic current $k_x ::$ (combined plot)
Summary

- **We investigate non-Abelian dual Meissner effects at finite temperature**, applying our new formulation of Yang-Mills theory on the lattice.
- We measure chromo-flux created by a pair of quark and antiquark and the induced chromo-magnetic current (magnetic monopole) due to dual-Meissner effect.
  - In confinement phase, **observation of the chromo-electric flux tube and induced magnetic monopole**
  - deconfinement phase, **disappearance of the chromo-electronic flux tube and vanishing the magnetic monopole**

⇒ The magnetic monopole plays the dominant role in confinement/deconfinement phase transition.

Outlook
- Distribution of chromo-flux and magnetic monopole in 2D (3D) space
- Measurement by Magnetic monopole operator \( k_\mu(x) = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \partial_\nu \Theta_{\alpha\beta}(x) \)