クォーク閉じ込め・非閉じ込め有限温度相転移と磁気的モノポールの役割

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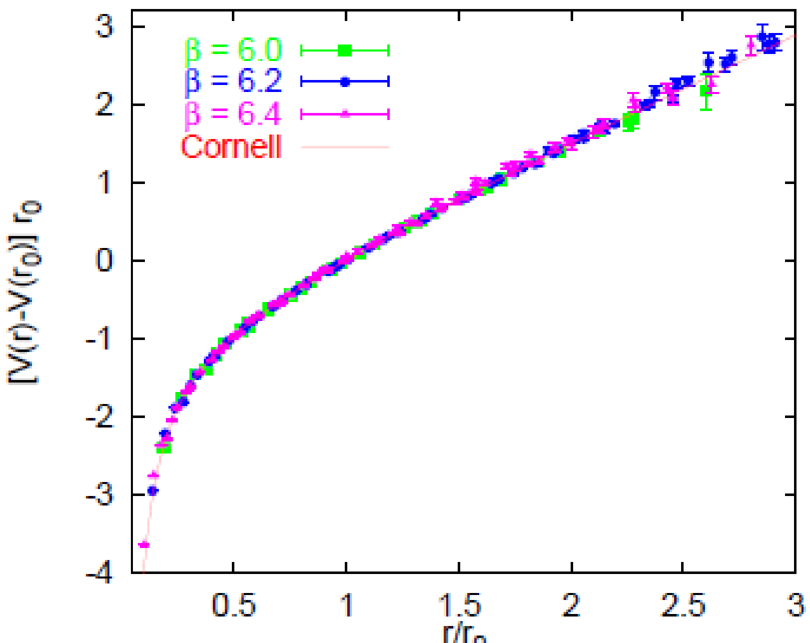
Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

Non-Abelian Wilson loop $\left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\rm YM}^{\rm no\ GF} \sim e^{-\sigma_{NA}|S|}$,



G.S. Bali, [hep-ph/0001312], Phys. Rept. **343, 1–136 (2001)**

$$V(r) = -C\frac{g_{\rm YM}^2(r)}{r} + \sigma r$$

$$F(r) = -\frac{d}{dr}V(r) = -C\frac{g_{\rm YM}^2(r)}{r^2} - \sigma + \cdots \quad (C, \sigma > 0)$$

$$V(r) \to \infty \ \text{ for } r \to \infty$$

What is the mechanism of confinement ?

dual superconductivity

- Dual superconductivity is a promising mechanism for quark confinement. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]

superconductor

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

dual superconductor

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

Evidences for the dual superconductivity

By using Abelian projection

String tension (Linear potential)

- ☐ Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- ☐ Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]

Chromo-flux tube (dual Meissner effect)

- ☐ Measurement of (Abelian) dual Meissner effect
- ◆ Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux
- ◆ Type the super conductor is of order between Type I and Type II [Y.Matsubara, et.al. 1994]

✓ **only obtained in the case of special gauge such as MA gauge**

✓ **gauge fixing breaks the gauge symmetry as well as color symmetry**

The evidence for dual superconductivity

Gauge decomposition method (a new lattice formulation)

- **Extracting the relevant mode *V* for quark confinement** by solving the defining equation in the gauge independent way (gauge-invariant way)

- For SU(2) case, the decomposition is a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.
- For SU(N) case, the formulation is the extension of the SU(2) case.

→ **we have showed in the series of lattice conferences that**

- V-field dominance, magnetic monopole dominance in string tension,
- chromo-flux tube and dual Meissner effect.
- The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect

A new formulation of Yang-Mills theory (on a lattice)

Decomposition of SU(N) gauge links

- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

☐ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

☐ SU(3) Yang-Mills link variables: Two options

maximal option : $U(1) \times U(1) \subset SU(3)$

✓ Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

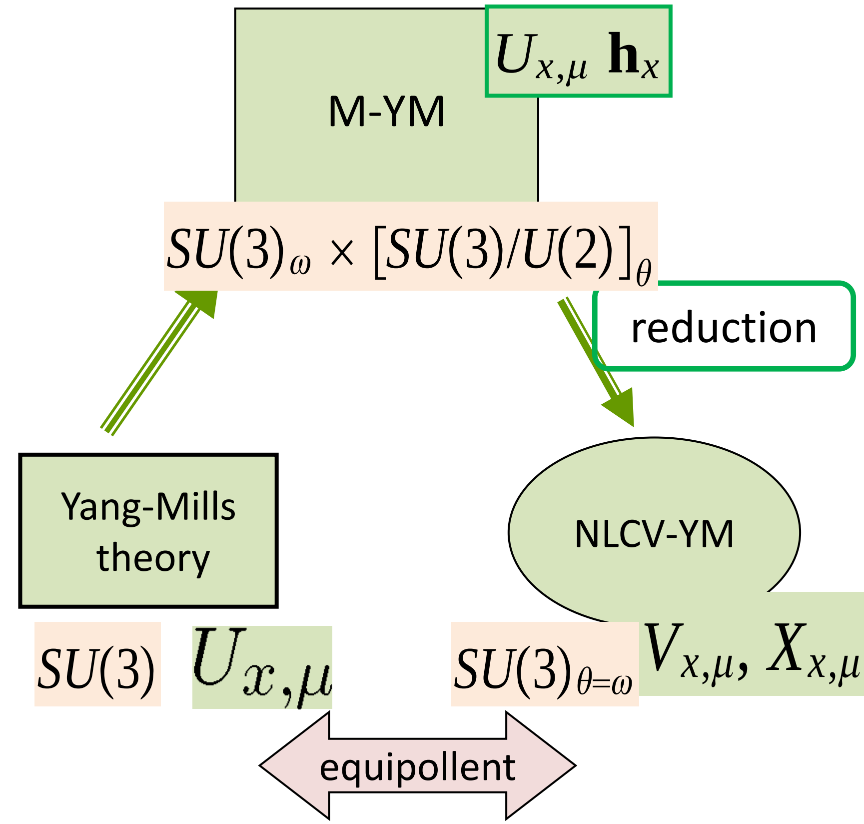
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_x$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.}\, W_C[V] \quad !!$$

■ SU(3) Yang-Mills theory

- In confinement of fundamental quarks, a restricted non-Abelian variable ***V***, and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

gauge independent "Abelian" dominance

$$\frac{\sigma_V}{\sigma_U} = 0.92$$

$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

Gauge independent non-Abalian monople dominance

$$\frac{\sigma_M}{\sigma_U} = 0.85$$

$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

U^* is from the table in R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B517, 377 (1998).

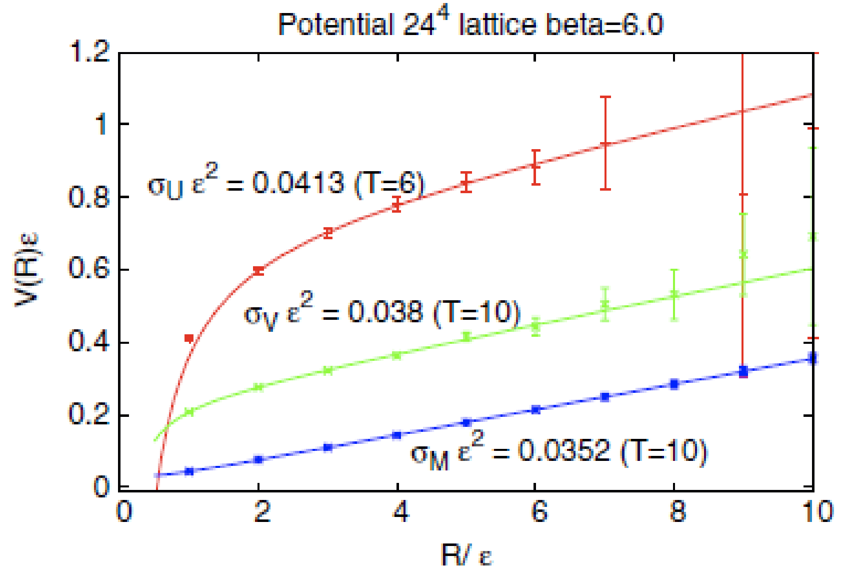


FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance R: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on 24^4 using 500 configurations where ϵ is the lattice spacing.

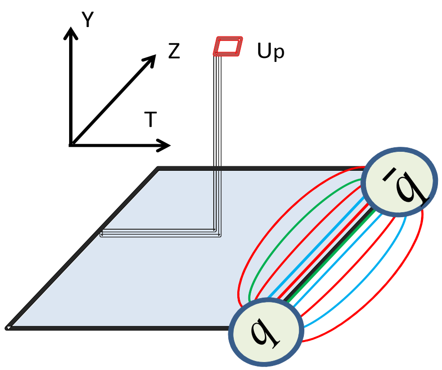
PRD 83, 114016 (2011)

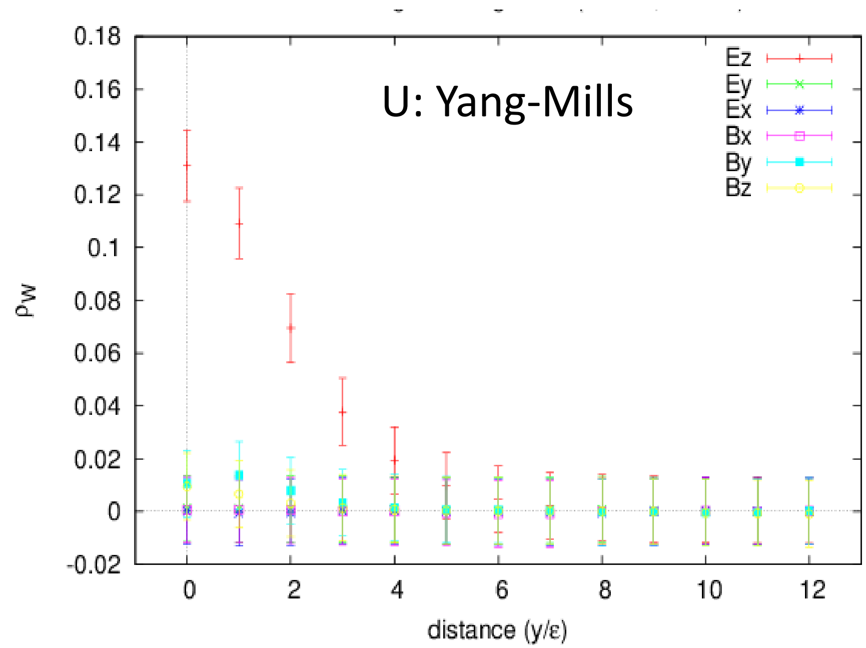
Chromo flux

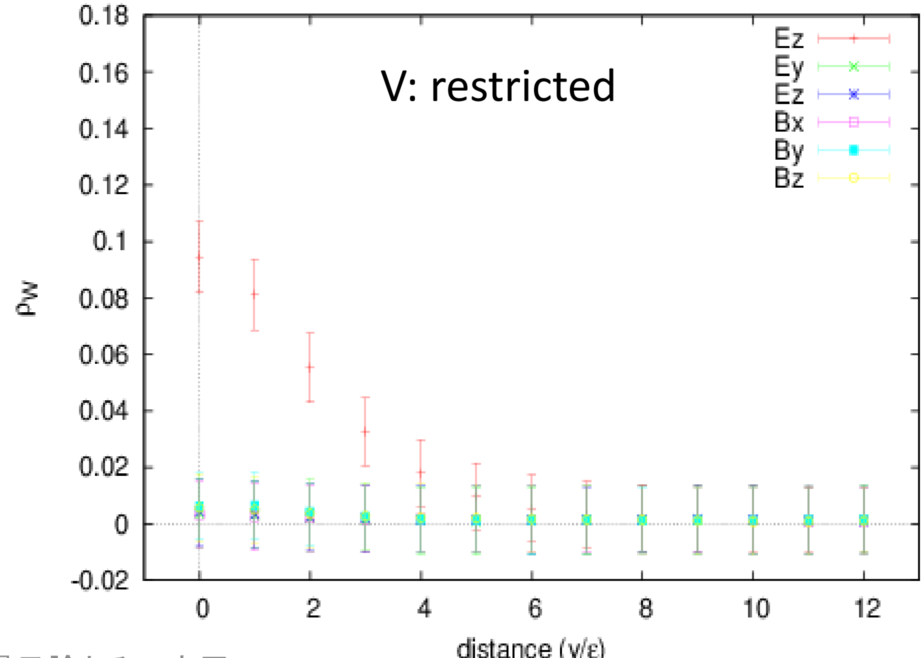
$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger) \rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(W) \mathrm{tr}(U_p) \rangle}{\langle \mathrm{tr}(W) \rangle}$$

Gauge invariant correlation function: This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3) [Adriano Di Giacomo et.al. PLB236:199,1990 NPBB347:441-460,1990]

$\mathrm{tr}(U_p L W L^\dagger)$

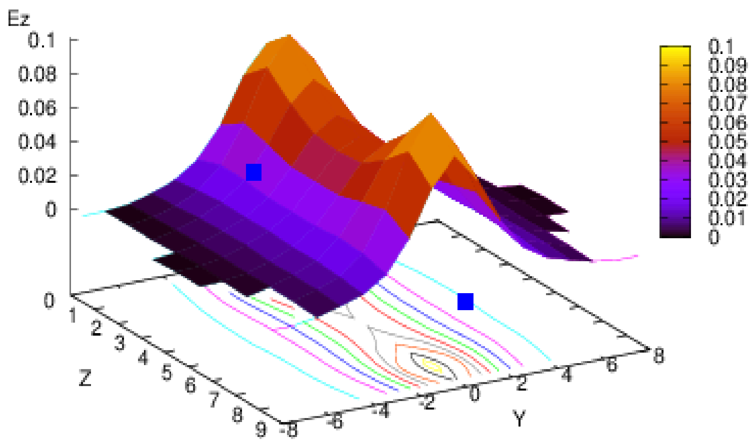




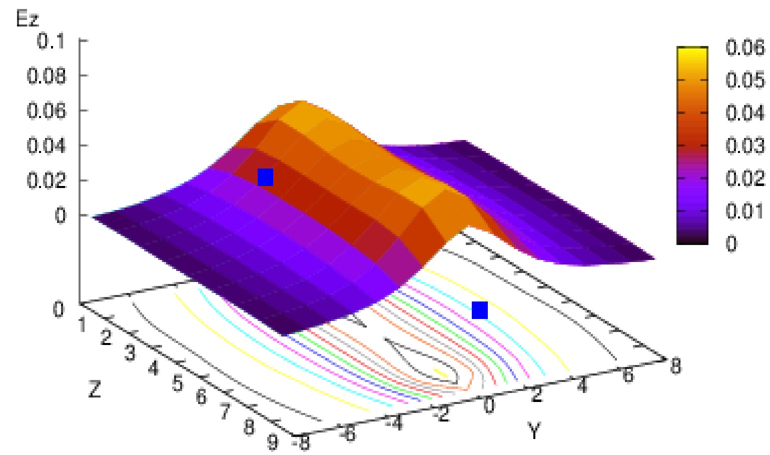


Chromo-electric (color flux) Flux Tube

Original YM filed



Restricted field



A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

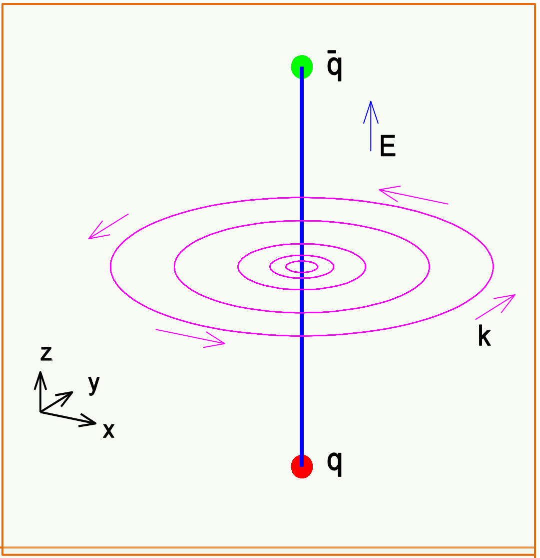
Flux tube is observed for V-field case. :: dual Meissner effect

Magnetic current induced by quark and antiquark pair

Yang–Mills equation (Maxell equation) fo rrestricted field V_μ, the magnetic current (monopole) can be calculated as

$$k = \delta^* F[V] = {}^*dF[V],$$

where $F[V]$ is the field strength of V, d exterior derivative, $*$ the Hodge dual and δ the coderivative $\delta := {}^*d^*$, respectively.



$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$,
and $\mathbf{k} = {}^*dF[\mathbf{V}] = {}^*ddF[\mathbf{V}] = 0$
(Bianchi identity)

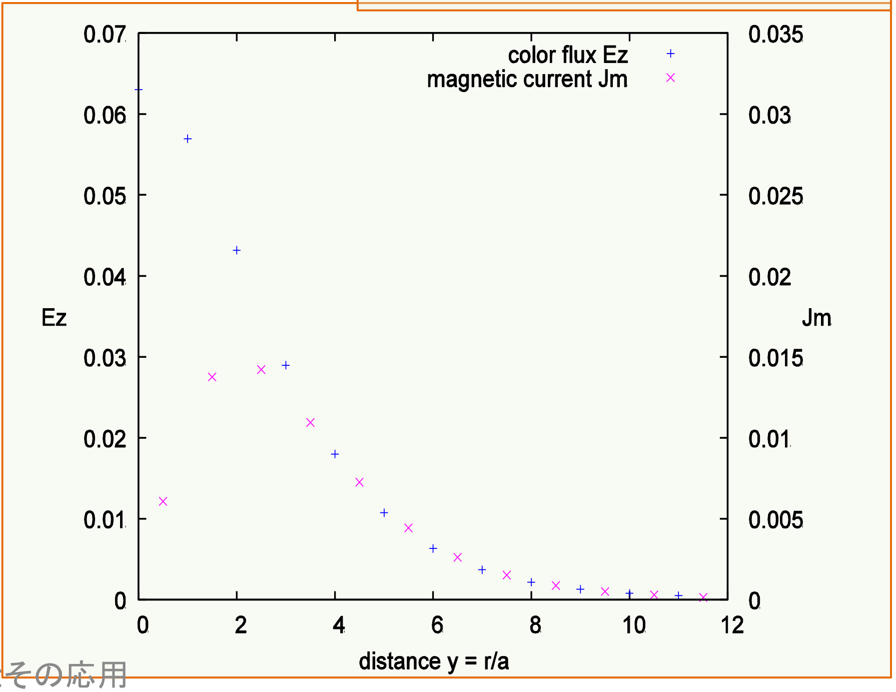


Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current(right scale).

Confinement / deconfinement phase transition in view of the dual Meissner effect.

- We measure the chromo-flux generated by a pair of quark and antiquark at finite temperature applying our new formulation of Yang-Mills theory on the lattice.
- The quark-antiquark source can be given by a pair of Polyakov loops in stead of the Wilson loop.
- Convensionally, average of Ployakov loops <P> is used as order parameter of the phase transition.
- In the view of dual superconductivity

  □ Confinement phase :: dual Meissner effect

  → generation of the chromo-flux tube.

  → Generation of the magnetic current (monopole)

  □ Deconfinement phase :: disappearance of dual Meissner effect.

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

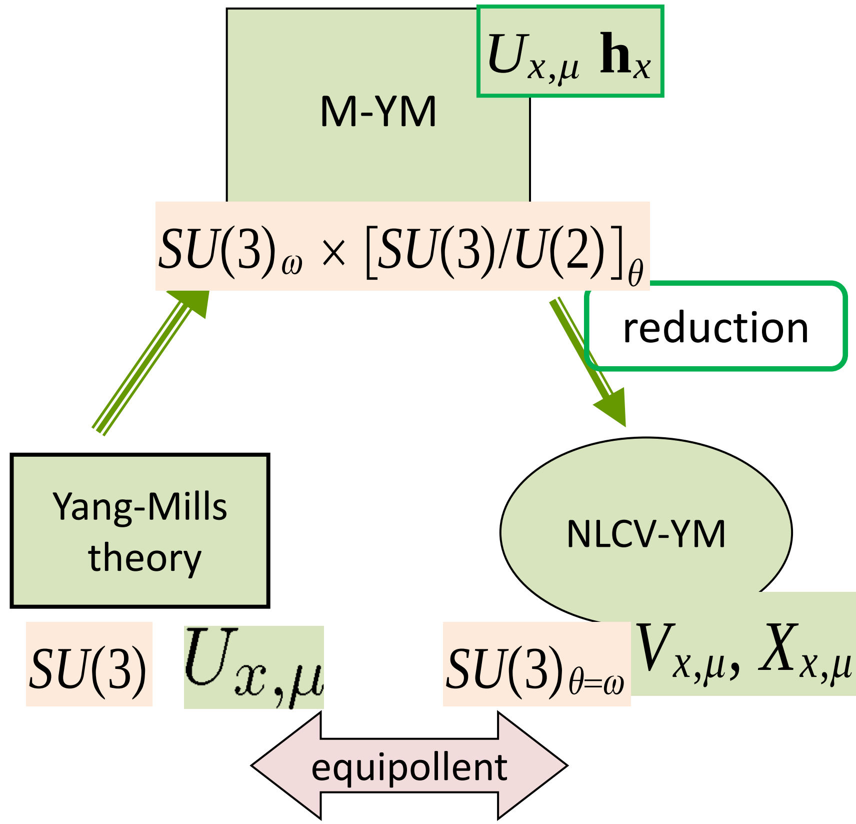
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^{\dagger}_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^{\dagger}_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^{\dagger}_{x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Defining equation for the decomposition

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger(\det\hat{L}_{x,\mu})^{1/N}g_x^{-1} \quad V_{x,\mu} = X_{x,\mu}^\dagger U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det\hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L_{x,\mu}^\dagger}\right)^{-1}L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}$$

continuum version by continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- The configuration of the color fields $\mathbf{h}_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$

$$F_{\mathrm{red}}[\mathbf{h}_x; U_{x,\mu}] = \sum_{x,\mu} \mathrm{tr}\left\{ \left(D_\mu^\epsilon[U]\mathbf{h}_x \right)^\dagger \left(D_\mu^\epsilon[U]\mathbf{h}_x \right) \right\}$$

$$SU(3)_\omega \times [SU(3)/U(2)]_\theta \to SU(3)_{\omega=\theta}$$

- This is invariant under the gauge transformation θ=ω
- The extended gauge symmetry is reduced to the same symmetry as the original YM theory.
- We choose a reduction condition of the same type as the SU(2) case

Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int [d\mu(\xi)]_\Sigma \exp\left(-ig\int_{S:C=\partial\Sigma} dS^{\mu\nu}\sqrt{\frac{N-1}{2N}}\,\mathrm{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x))\right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left(ig\sqrt{\frac{N-1}{2N}}(k,\Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j,N_\Sigma)\right)$$

magnetic current $k := \delta^* F = {}^* dF,\quad \Xi_\Sigma := \delta^*\Theta_\Sigma\Delta^{-1}$

electric current $j := \delta F,\quad N_\Sigma := \delta\Theta_\Sigma\Delta^{-1}$

$\Delta = d\delta + \delta d,\quad \Theta_\Sigma := \int_\Sigma d^2S^{\mu\nu}(\sigma(x))\delta^D(x - x(\sigma))$

k and j are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

K.-I. Kondo PRD77 085929(2008)

The lattice version is defined by using plaquette:

$$\Theta^8_{\mu\nu} := -\arg \mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x\right)V_{x,\mu}V_{x+\mu,\mu}V^\dagger_{x+\nu,\mu}V^\dagger_{x,\nu}\right],$$

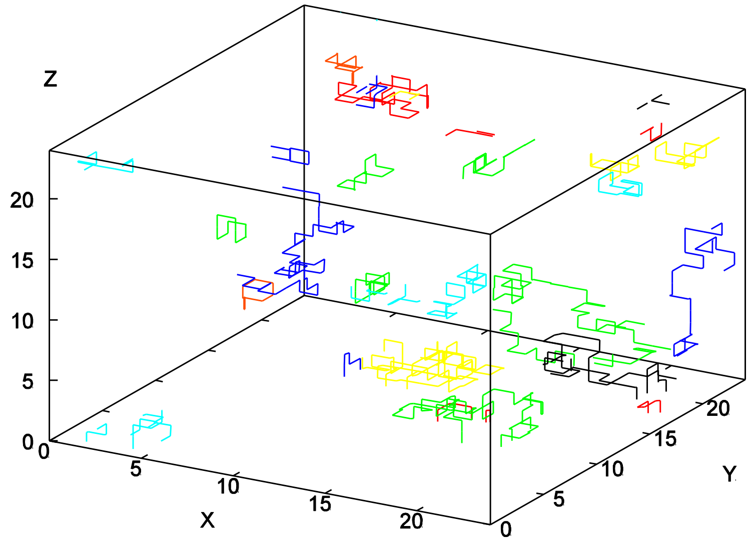
$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\Theta^8_{\alpha\beta},$$

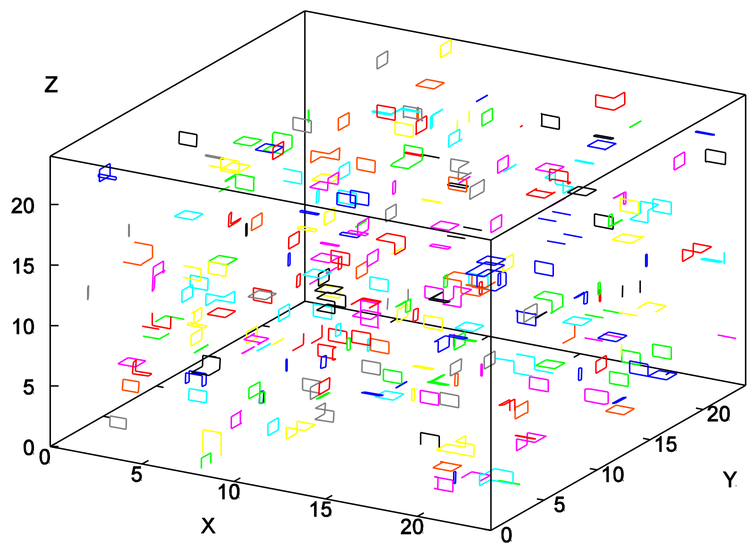
Non-Abelian magnetic monopole loops: 24^3 x8 lattice b=6.0 (T≠0)

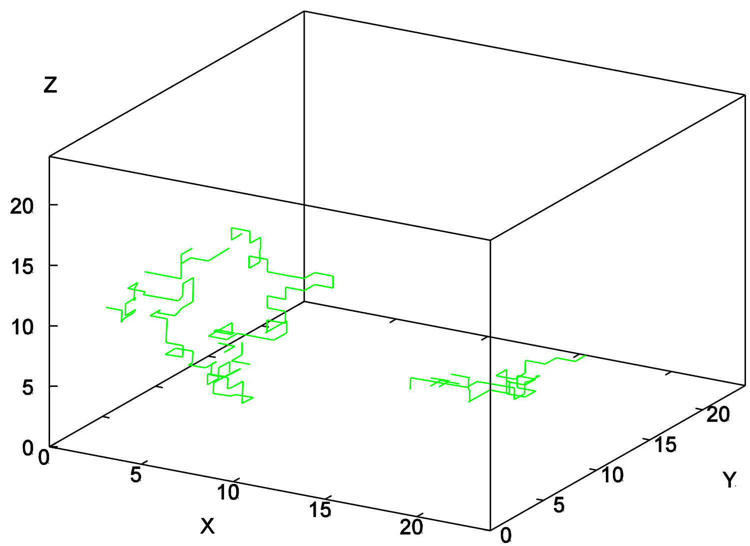
Projected view (x,y,z,t) ➔(x,y,z)

(left lower) loop length 1-10
(right upper) loop length 10 -- 100
(right lower) loop length 100 -- 1000

Monopole loop is winding to T direction.







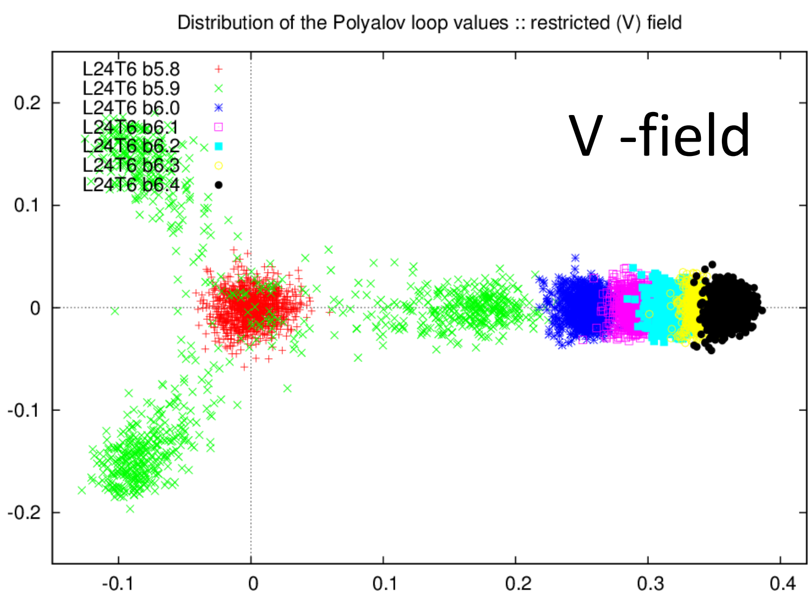
Lattice set up

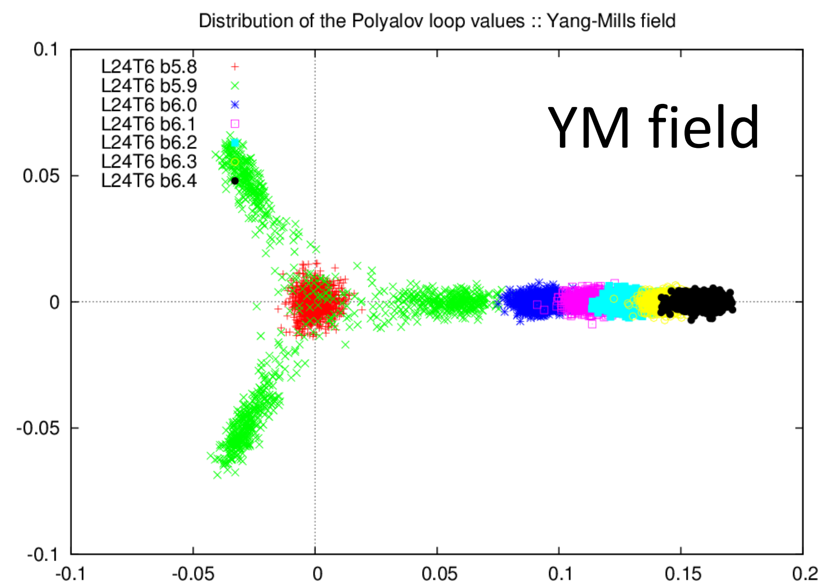
- Standard Wilson action
- 24^3 x 6 lattice
- Temperature is controlled by using β (=6/g^2);

  β=5.8, 5.9, 6.0, 6.1, 6.2, 6.3
- Measurement by 1000 configurations

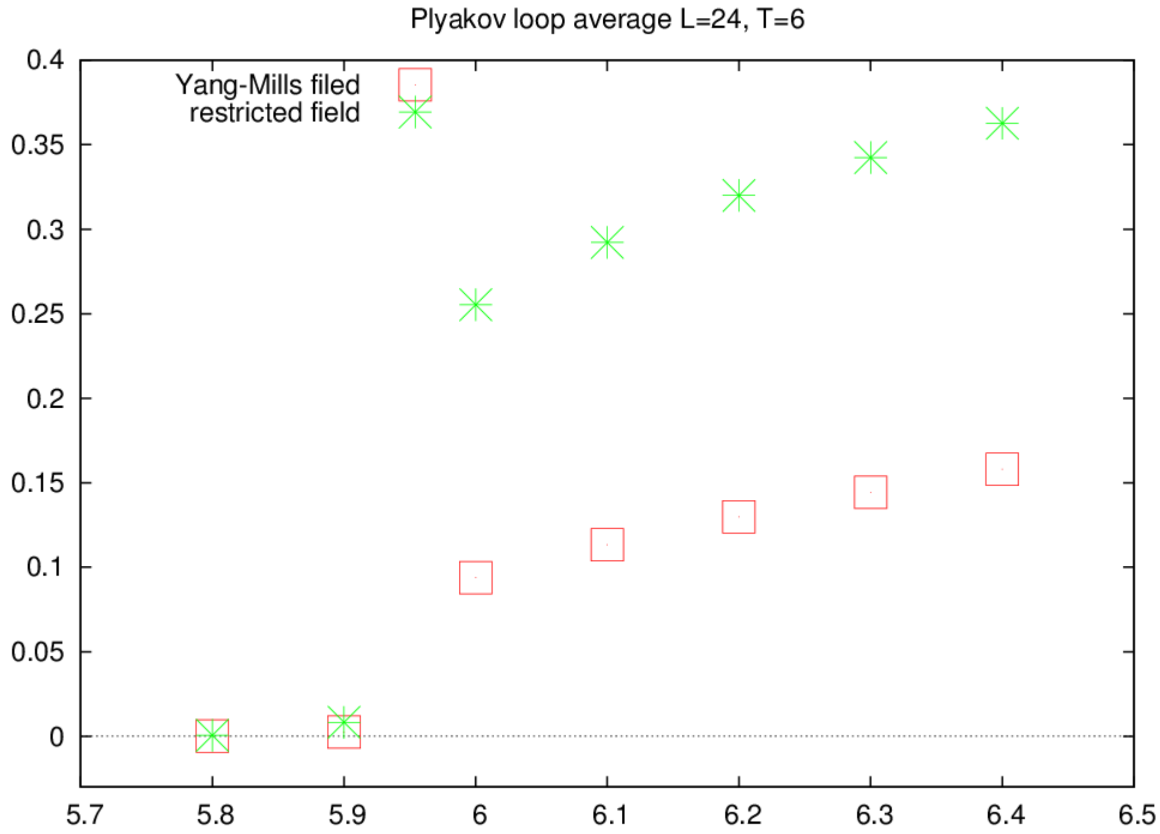
Distribution of Polyakov loop

$$P_U(x) \; = \; \mathrm{tr}\left(\prod_{t=1}^{Nt} U_{(x,t),4}\right) \text{ for original Yang–Mills filed}$$
$$P_V(x) \; = \; \mathrm{tr}\left(\prod_{t=1}^{Nt} V_{(x,t),4}\right) \text{ for restricted field}$$

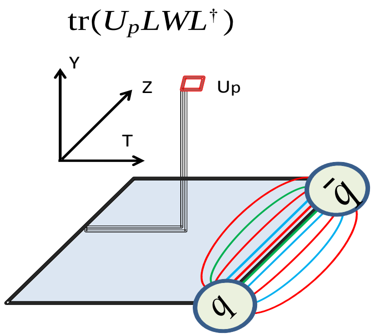


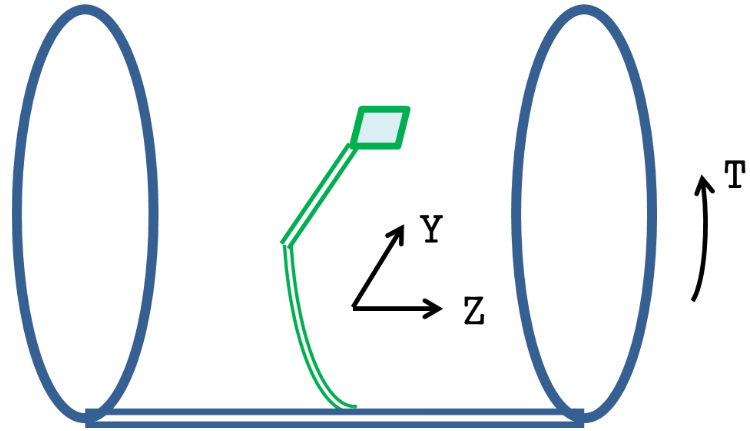


Polyakov loop average YM-field v.s. V - field



Chromo-electric flux at finite temperature

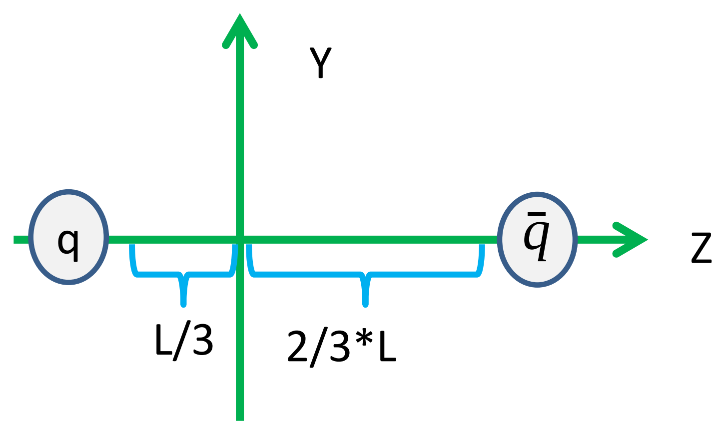




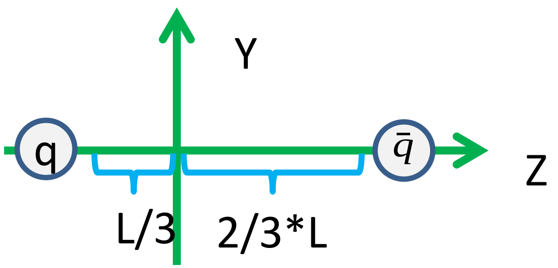
Size of Wilson loop T-direction = Nt
➔The quark and antiquark sources are given by **Plyakov loops**.

$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

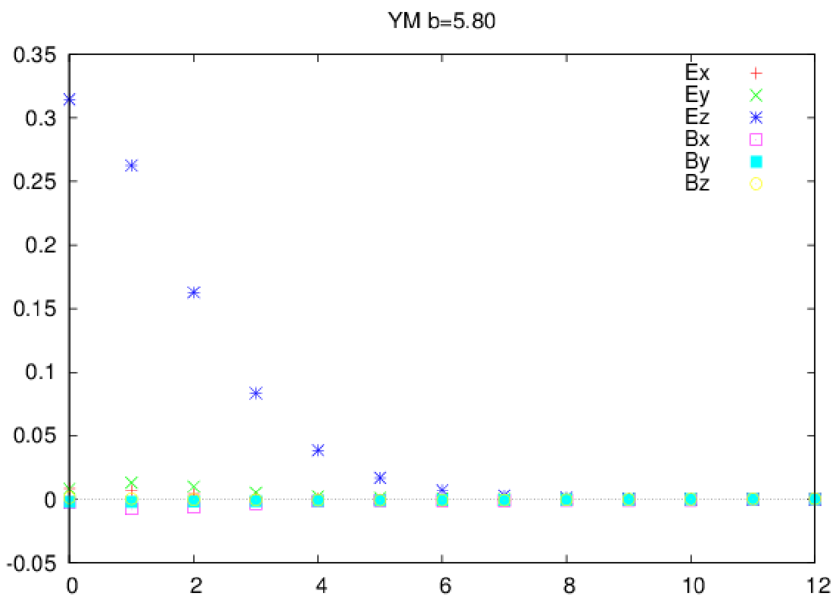
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W(x)$$



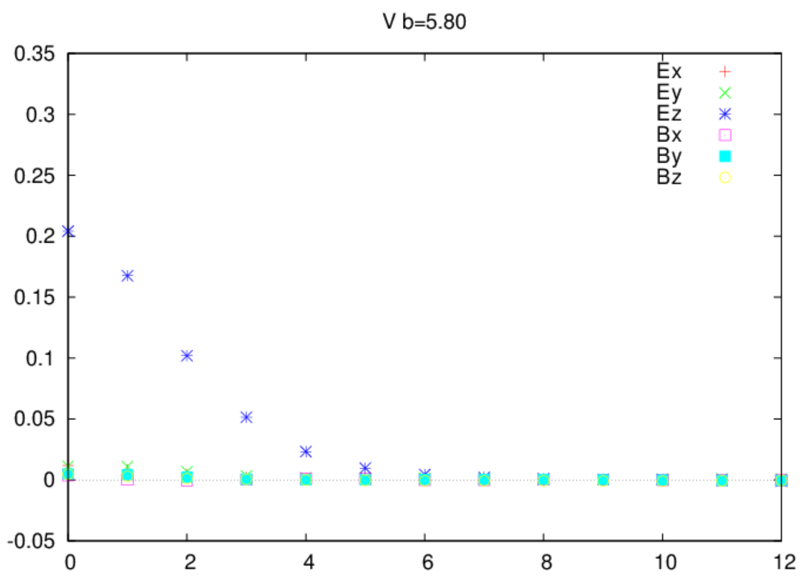
Chromo-flux β=5.8



YM field

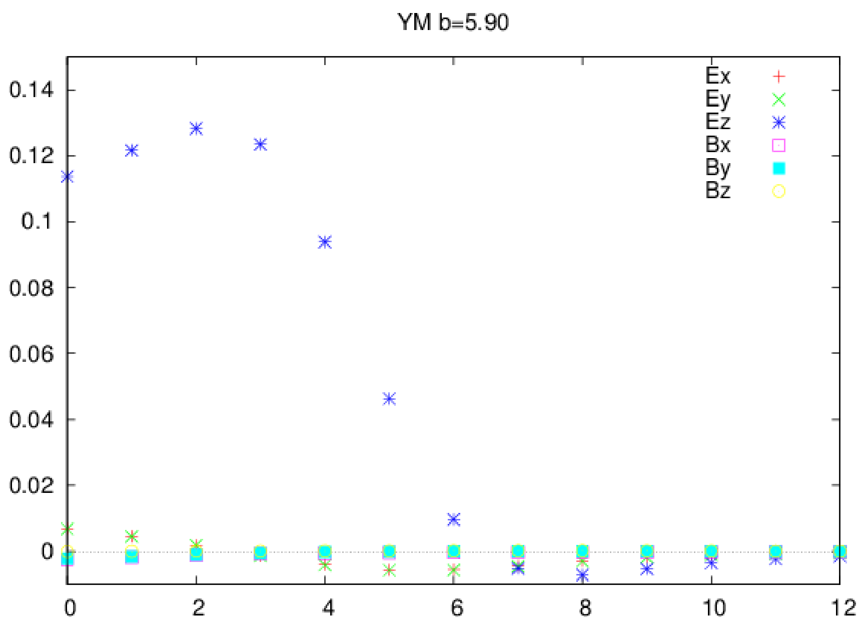


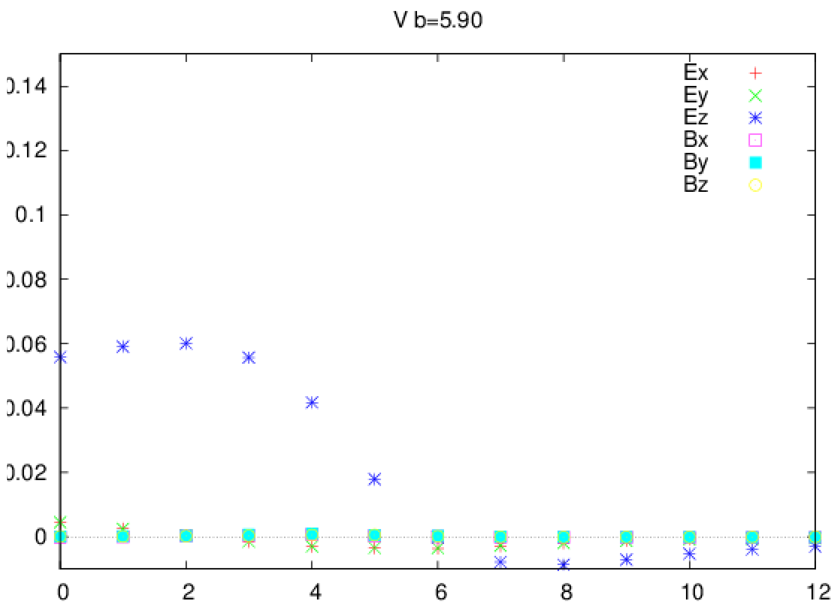
V field



Chromo-flux β=5.9

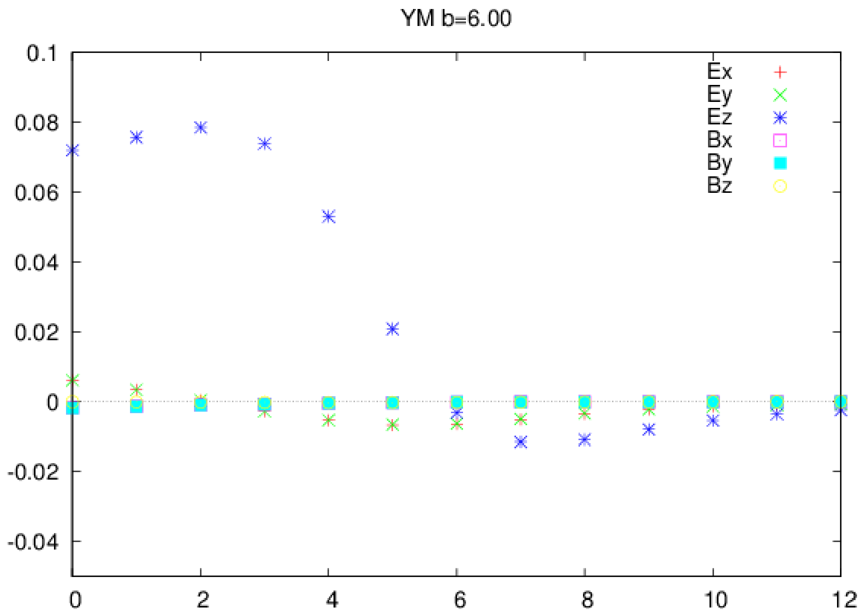
YM field　　　　V field

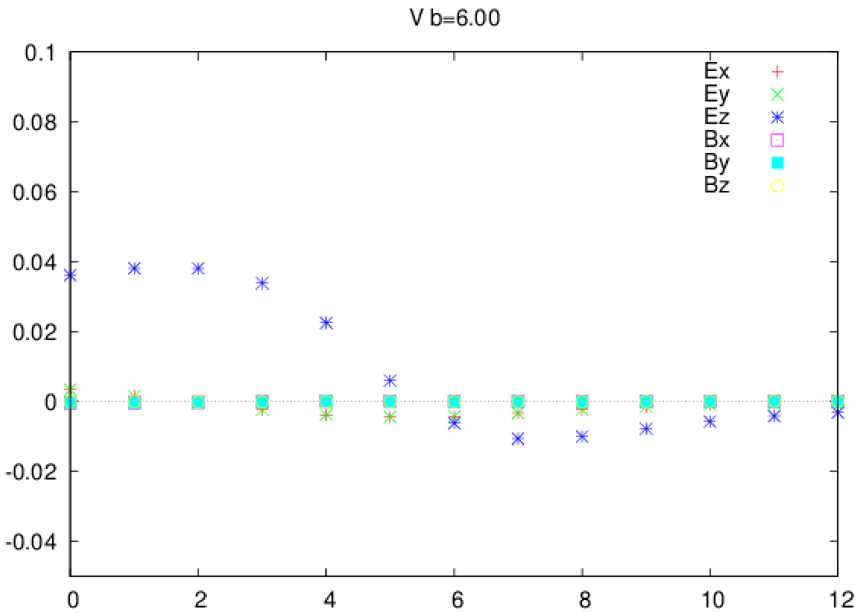




Chromo-flux β=6.0

YM field　　　　　V field

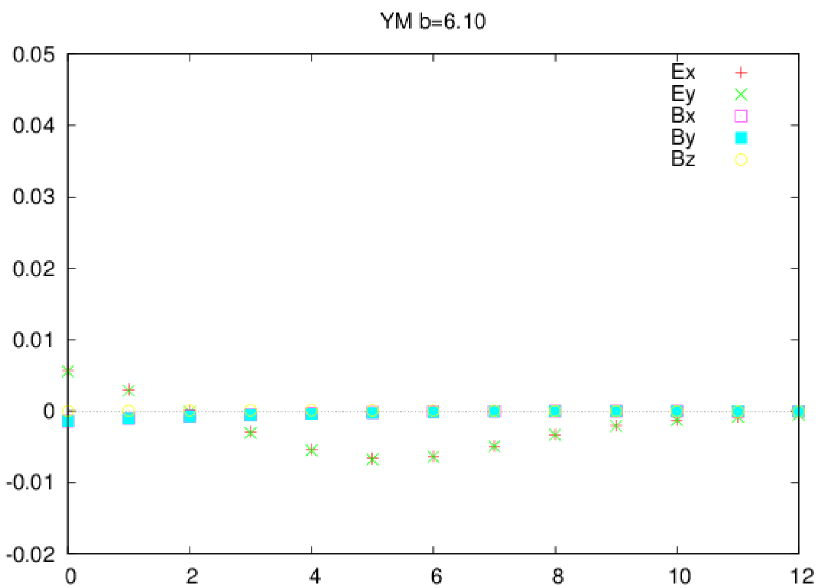


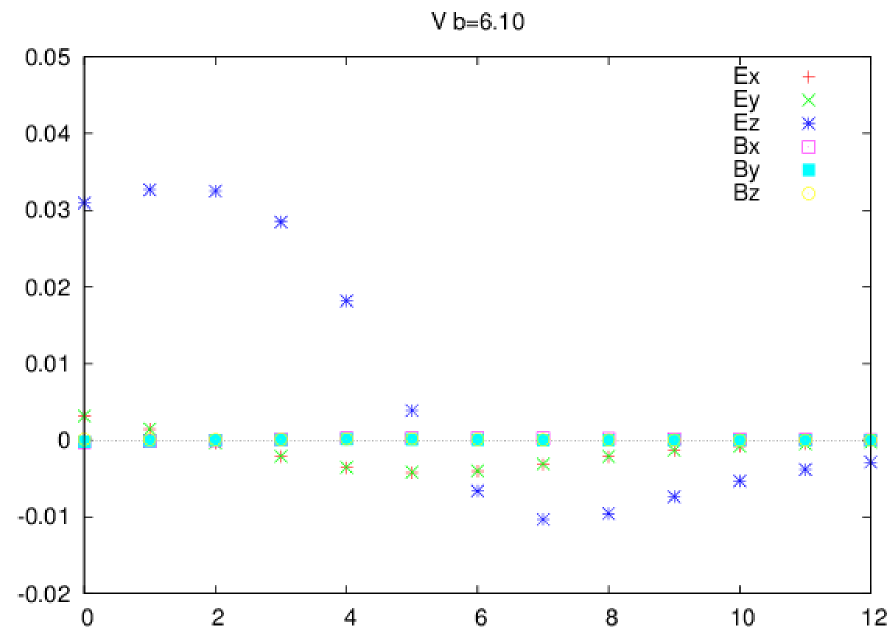


Chromo-flux β=6.1

YM field

V field

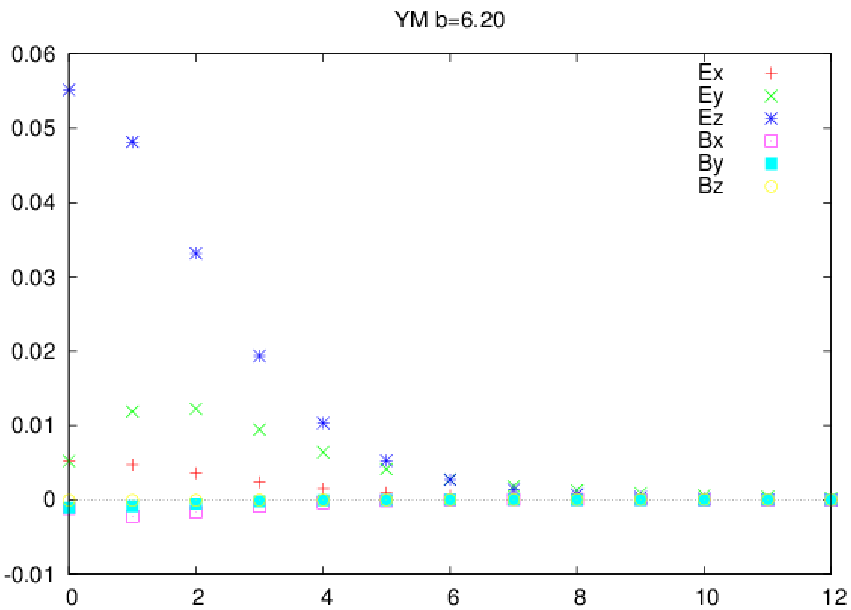


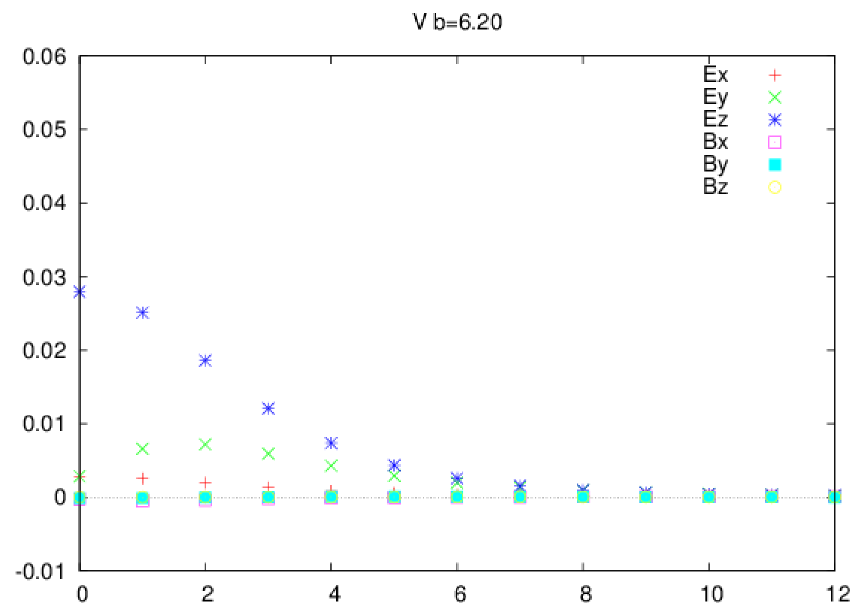


Chromo-flux β=6.2

YM field

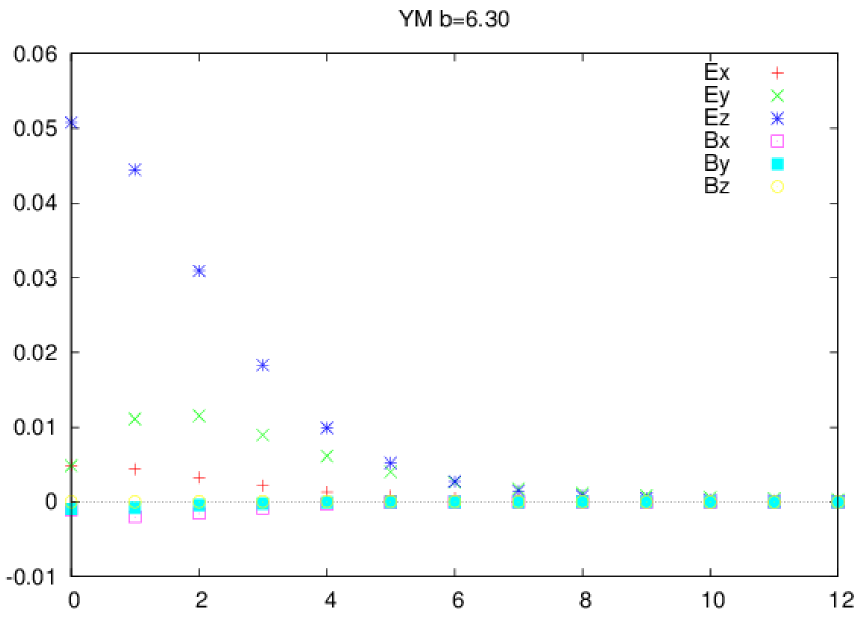
V field

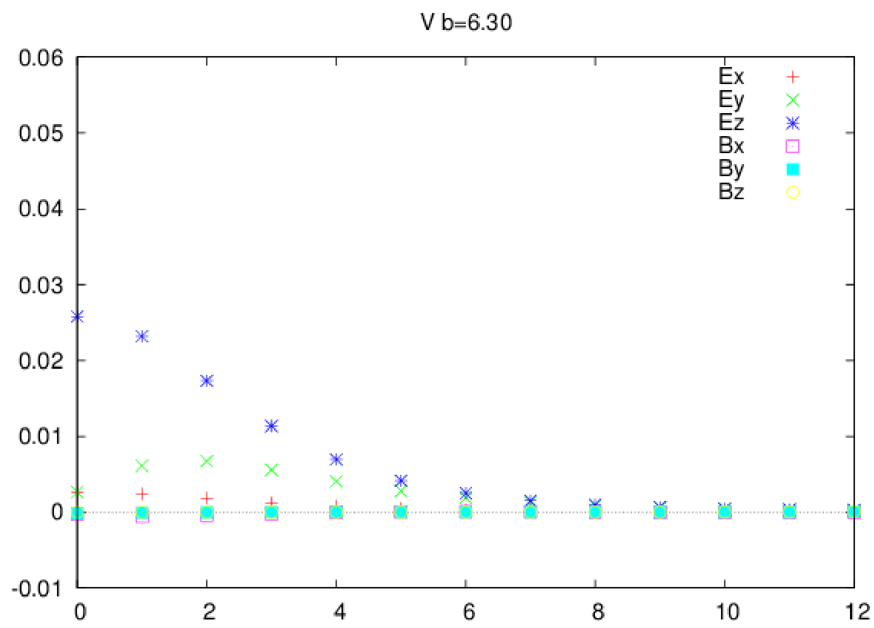




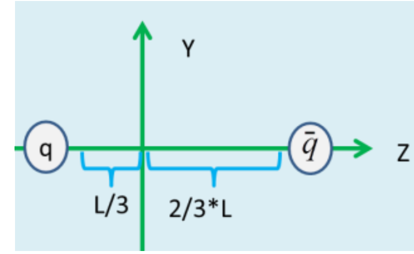
Chromo-flux β=6.3

YM field V field



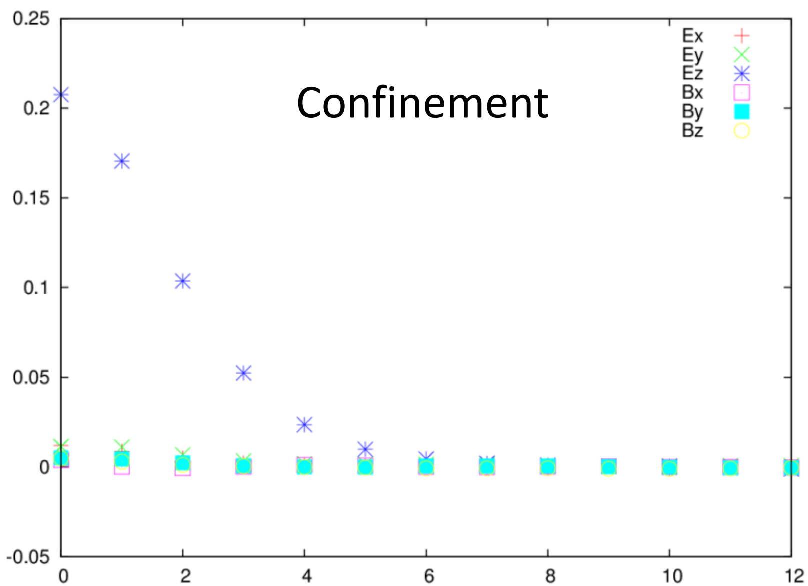


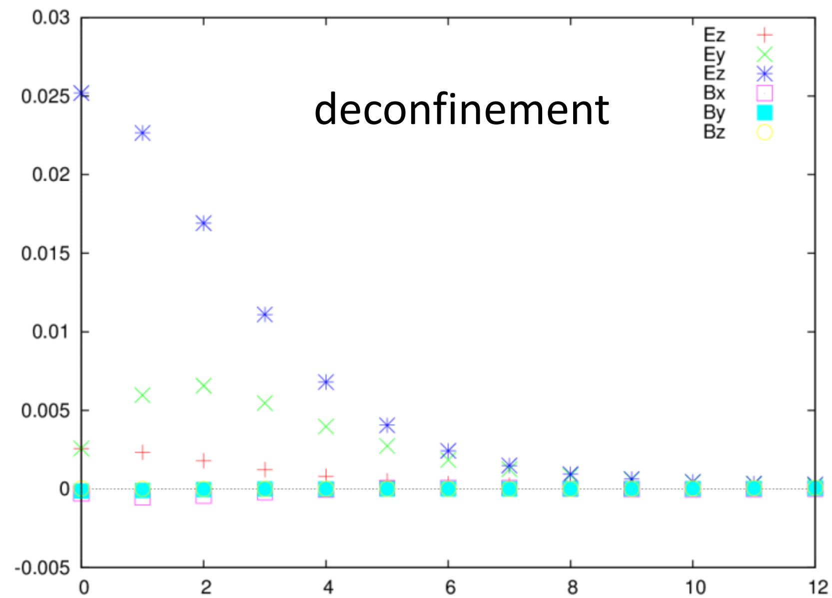
Chromo-electric flux in deconfinement phase



- $E_y \neq 0$ for deconfinemnte phase i.e., No sharp chromo-flux tube

➔ Disappearance of dual superconductivity.

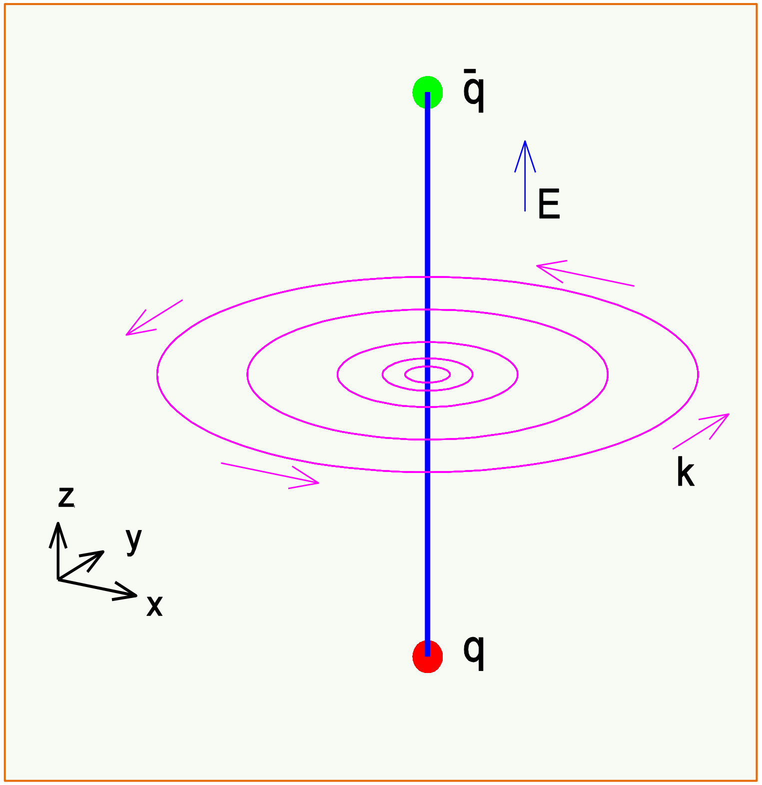




Chromo-magnetic current (monopole current)

- To know relation to the monopole condensation, we further need the measurement of magnetic current in Maxell equation for V field.

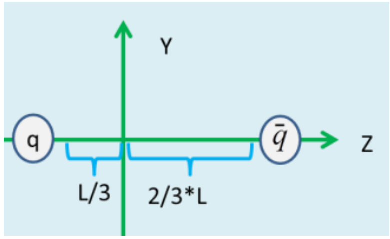
$$k = \delta^* F[V] = {}^* dF[V]$$



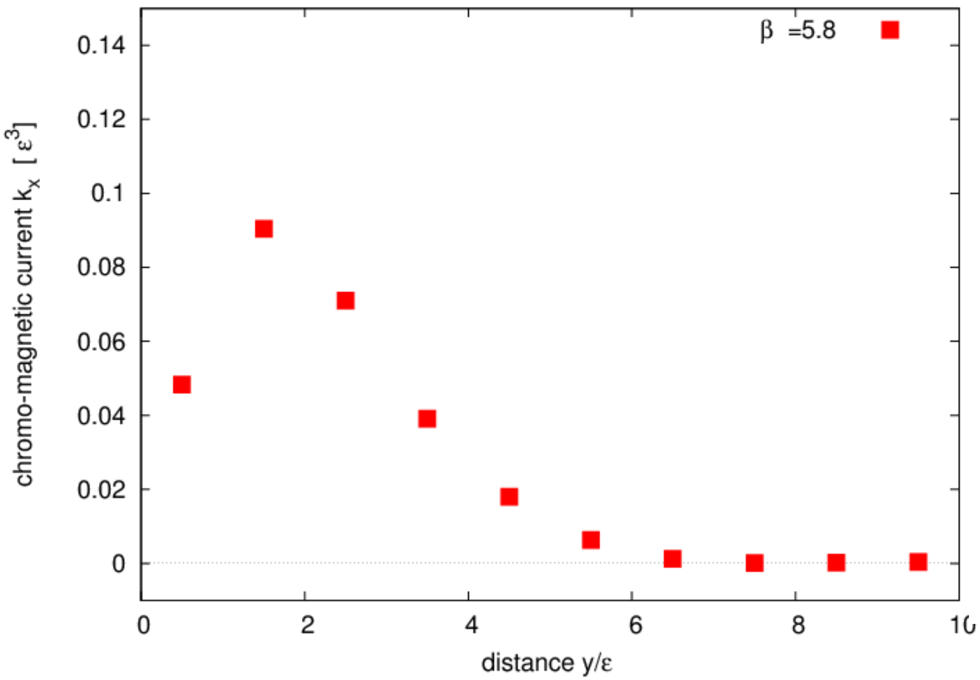
$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$,
and $\mathbf{k} = {}^*dF[\mathbf{V}] = {}^*ddF[\mathbf{V}] = 0$
(Bianchi identity)

Chromo-magnetic (monopole) current β=5.8

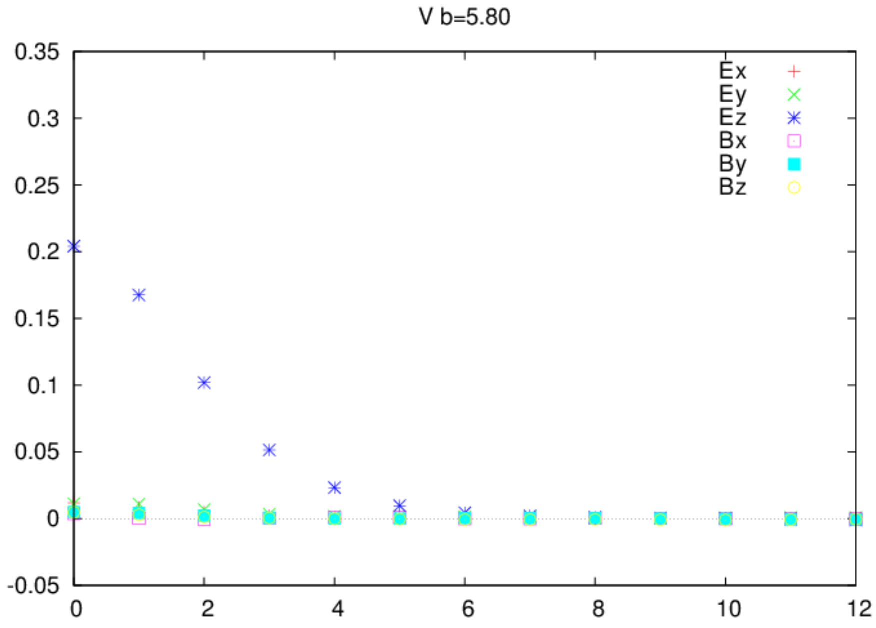
Confinement phase



chromo-magnetic current k_x

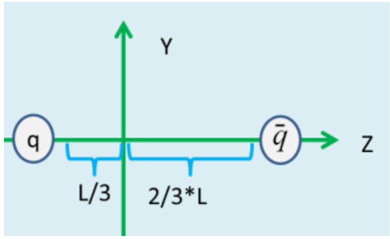


Chromo-flux

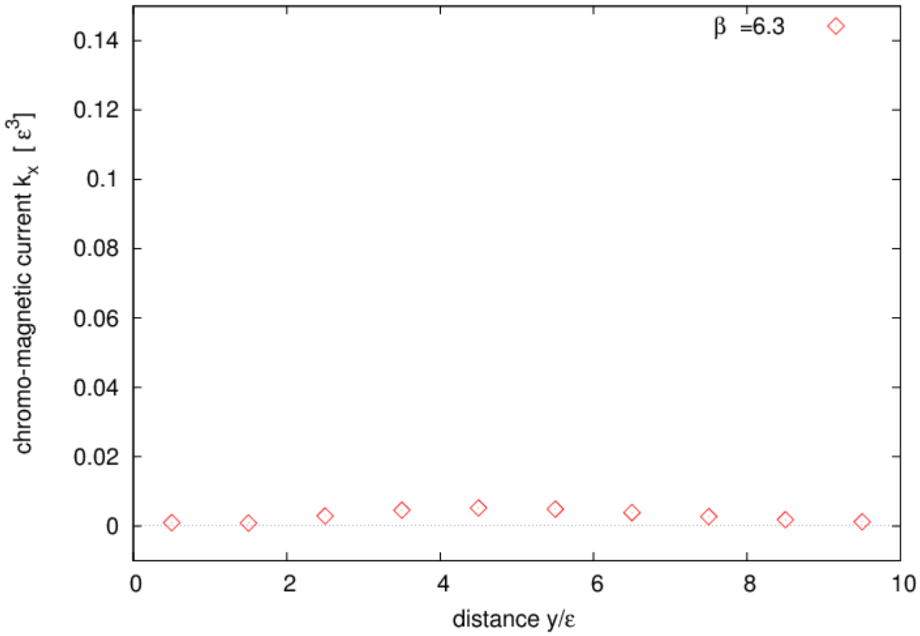


Chromo-magnetic (monopole) current β=6.3

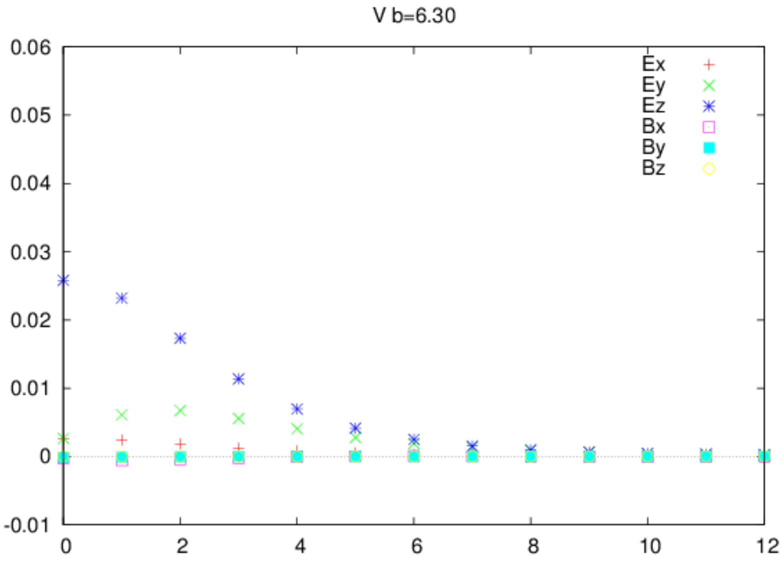
deconfinement phase



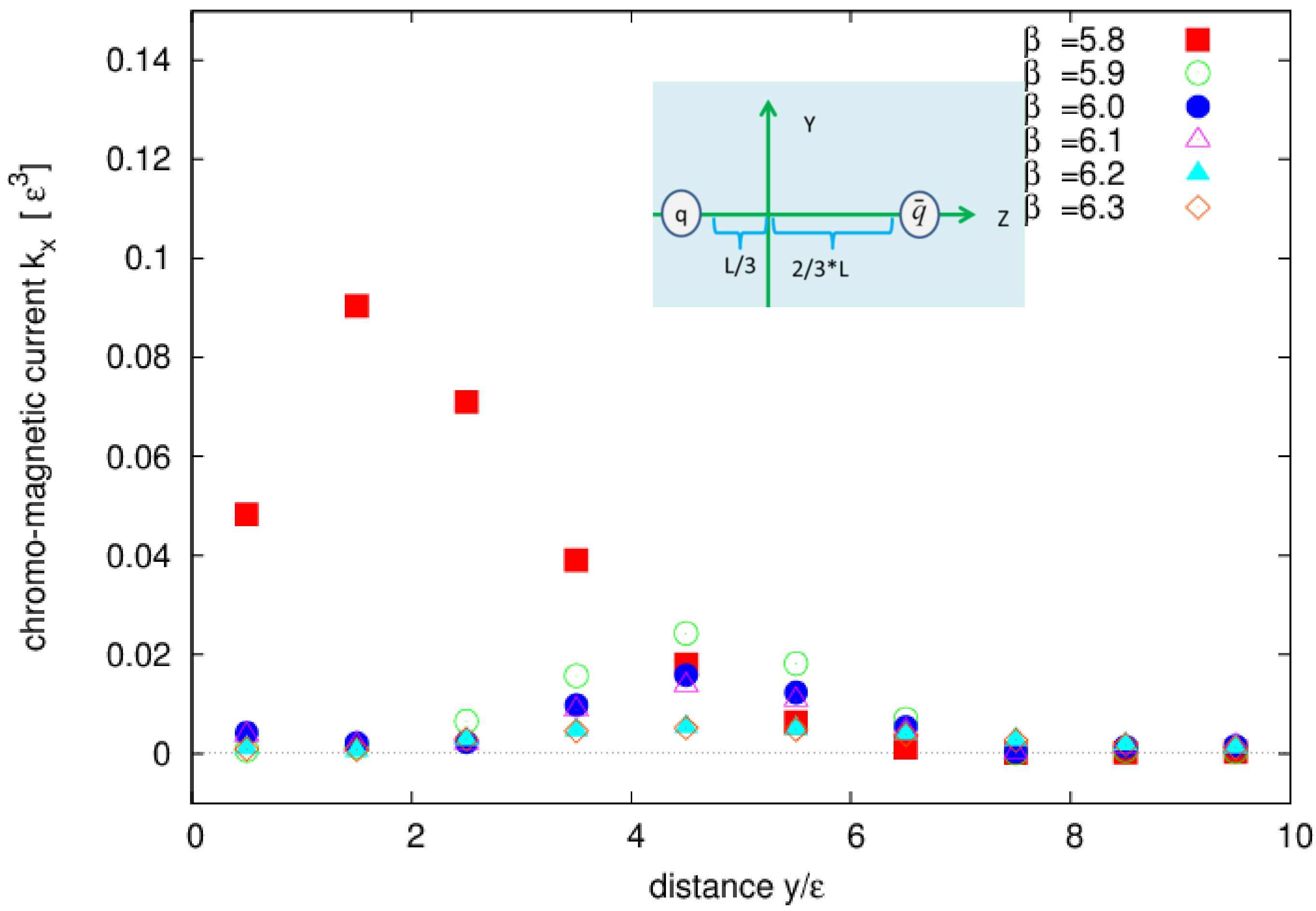
chromo-magnetic current k_x



Chromo-flux



Chromo-magnetic current k_x :: (conbied plot)



Summary

- **We investigate non-Abelian dual Meissner effects at finite temperature,** applying our new formulation of Yang-Mills theory on the lattice.
- We measure chromo-flux created by a pair of quark and antiquark and the induced chromo-magnetic current (magnetic monopole) due to dua-Meissner effect.

➢ In confinement phase, **observation of the chromo-electric flux tube and induced magnetic monopole**

➢ deconfiment phase, **disappearance of the the chromo-electronic flux tube and vanishing the magnetic monopole**

➔The magnetic monopole plays the dominant role in confinement/ deconfinement phase transition.

Outlook

☐ Distribution of chromo-flux and magnetic monopole in 2D (3D) space

☐ Measurment by Magnetic monopole operator $\mathbf{k}_\mu(x) = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\Theta_{\alpha\beta}(x)$