Chiral phase transition in a magnetic field

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Motivation

- Magnetic field order of strong interaction

  Magnetars

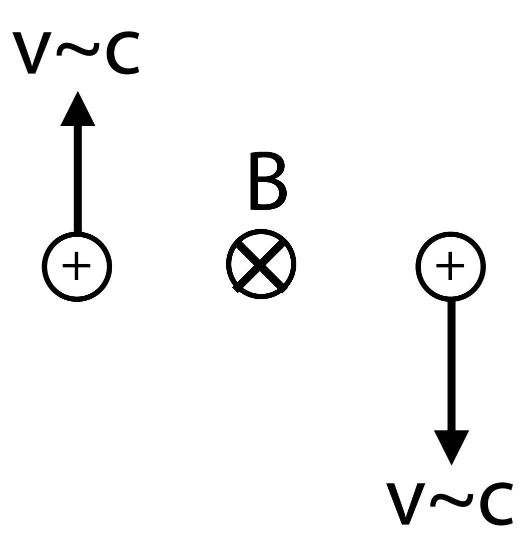
  10^{10} [T] = 10^{6} [G]

  Non central heavy ion collision

  10^{11} [G] ~ $eB = 0.1$ [GeV^2] ~ $5\ m_\pi^2$

  Life time of magnetic field is very short

  0.1fm ~ 10^{-24} s



- In this talk, however, we investigate properties of the strongly interacting matter under static magnetic field.

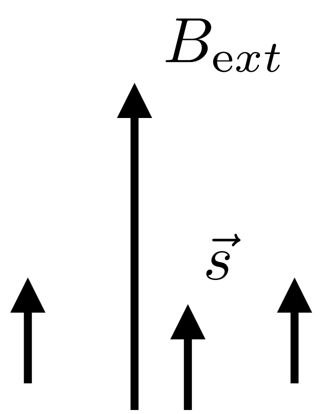
  ・Magnetism of the QCD vacuum

  ・Chiral phase transition under strong magnetic field

Magnetism of electron gas

$$B_{net} = (1+\chi)B_{ext}$$

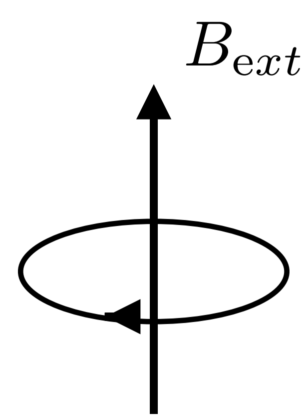
Pauli paramagnetism (Spin)



$$\chi_{\rm spin} = 2\mu_B^2\rho(\epsilon_f) = \frac{\mu_B^2 m k_f}{3\hbar^2\pi^2}$$

Landau diamagnetism (Orbit)

Landau, 1930



$$\chi_{\rm orbit} = -\frac{1}{3}\chi_{\rm spin}$$

$$\chi_{\rm tot} = \chi_{\rm spin} + \chi_{\rm orbit} > 0$$

- Competition of spin and orbital angular momentum
- Spin part provides larger contributions and Fermi gas shows paramagnetism.
- Magnetism of QCD vacuum will be determined by the nature of charged quark and mesons.

Free energy and χ

$$B^{ind} = B^{ext} + M = (1+\chi)B^{ext}$$

Free energy: $\Omega = -P$

Magnetisation: $M = -\dfrac{\partial \Omega}{\partial (eB)} \sim \chi (eB)$

Magnetic susceptibility: χ

$$\Omega \sim \Omega_0 - \frac{\chi}{2}(eB)^2 + O(eB)^4 \qquad \text{or} \qquad P \sim P_0 + \frac{\chi}{2}(eB)^2 + O(eB^4)$$

- Magnetic susceptibility is the second order coefficient of the free energy.

Free quark (vacuum)

$$\Omega_q = \mathrm{Tr}\log[i\not{D} + m_q]$$ Andersen and Khan (2011) $$x_f = \frac{m_q^2}{q_f B}$$

$$= \frac{N_c}{16\pi^2}\sum_f \left(\frac{\Lambda^2}{2|q_f B|}\right)^{\epsilon}\left[\left(\underline{\frac{2(q_f B)^2}{3}} + m_q^4\right)\left(\frac{1}{\epsilon}+1\right) - 8(q_f B)^2\zeta^{(1,0)}(-1,x_f) - 2|q_f B| m_q^2 \log x_f + \mathcal{O}(\epsilon)\right]$$

$$\sim \#\frac{1}{\epsilon}(q_f B)^2 + \text{regular terms}$$

- The B square term has a divergence (ε->0).
- χ must be renormalised by renormalisation of electric charge.
- To avoid the cutoff dependence, the following renormalisation condition is usually imposed in non-perturbative methods. $\chi(T=0)=0$

Normalised pressure *Bonati et.al 2013*

$$\Delta P = (P(T,B) - P(T,0)) - (P(0,B) - P(0,0))$$

$$\sim \frac{\chi(T)}{2}(eB)^2$$

Thermal part

Quark (s = 1/2)

$$P_q^f = N_c \frac{|e_f B| T}{\pi^2} \sum_{n=0}^{\infty} \alpha_n \int_0^\infty dp \, \log\left(1 + \mathrm{e}^{-\beta\sqrt{p^2+m_q^2+2|e_f B| n}}\right)$$

$$\sim P_q^f(T) + \frac{\chi_q}{2}(T)(eB)^2$$

$$\hat{\chi}\big|_q = -\frac{1}{3}\left(\frac{e_f}{e}\right)^2 \mathcal{G}_{\mathrm{q}}{}^{(1)}(0)$$
$$= \frac{N_c}{3\pi^2}\left(\frac{e_f}{e}\right)^2 \int_0^\infty dp \frac{1}{\sqrt{p^2+m_q^2}} \frac{1}{\mathrm{e}^{\beta\sqrt{p^2+m_q^2}}+1} \quad > 0$$

Meson (s = 0)

$$P_{\pi^\pm} = -\frac{|eB| T}{\pi^2} \sum_{n=0}^{\infty} \int_0^\infty dp \, \log\left(1 - \mathrm{e}^{-\beta\sqrt{p^2+m_\pi^2+(2n+1)|eB|}}\right)$$

$$\sim P_{\pi^\pm}(T) + \frac{\chi_\pi}{2}(T)(eB)^2$$

$$\hat{\chi}\big|_{\pi^\pm} = \frac{1}{12}\mathcal{G}_{\pi^\pm}{}^{(1)}(0)$$
$$= -\frac{1}{12\pi^2}\int_0^\infty dp \frac{1}{\sqrt{p^2+m_\pi^2}} \frac{1}{\mathrm{e}^{\beta\sqrt{p^2+m_\pi^2}}-1} \quad < 0$$

- Quarks show paramagnetism ($\chi > 0$), while pions show the diamagnetism ($\chi < 0$).
- This may be understood as a competition of the orbital and spin magnetisation.
- Let's see effects of interaction and phase transition.

⟶ Analyses on the quark meson model

3-flavor Quark meson model

$$\mathcal{L} = \overline{\psi}\left[\partial\!\!\!/ + g\sum_{a=0}^{8} T_a(\sigma_a + i\gamma_5\pi_a)\right]\psi + \mathrm{tr}[\partial_\mu\Sigma\partial_\mu\Sigma^\dagger] + U(\rho_1,\rho_2) - h_i\sigma_i - c_a\xi$$

$$\xi = \det\Sigma + \det\Sigma^\dagger\,, \qquad U(\rho_1,\rho_2) = a^{(1,0)}\rho_1 + \frac{a^{(2,0)}}{2}\rho_1^2 + a^{(0,1)}\rho_2$$

$$\Sigma = \sum_{a=0}^{8} T_a(\sigma_a + i\pi_a) \qquad \begin{aligned}\rho_1 &= \mathrm{tr}\left[\Sigma\Sigma^\dagger\right] \\ \rho_2 &= \mathrm{tr}\left[\Sigma\Sigma^\dagger - \frac{1}{3}\rho_1\right]^2\end{aligned}$$

- Σ is 3x3 complex matrix i.e., describes 8 scalar and 8 pseudo scalar mesons.
- ρ_1 and ρ_2 are invariants under the U(3) x U(3) flavour-chiral rotation.
- C_a is Kobayashi-Masukawa term which represents the effects of $U_A(1)$ anomaly.
- Inclusion of external magnetic field is achieve by

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$$

Functional RG

$$k\partial_k \Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB}+\Gamma_k^{(0,2)}[\varphi]}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF}+\Gamma_k^{(2,0)}[\varphi]}\right]$$

C. Wetterich (1993)

Γ_k : scale dependent effective action

$$\Gamma_{k=\Lambda}[\phi] = S[\phi] \longrightarrow \Gamma_{k=0}[\phi] = \Gamma[\phi]$$

UV: classical — IR: quantum

- Anzats for effective action (Local potential approximation)

$$\Gamma_k[\psi,\sigma,\pi] = \int_0^\beta dx_4 \int d^3x \left[\bar{\psi}\left[\gamma_\mu D_\mu + g(\Sigma + i\gamma_5\Pi)\right]\psi + U_k(\rho_1,\rho_2) - h_i\sigma_i + \left((D_\mu\Sigma)^2 + (D_\mu\Pi)^2\right)\right]$$

- We solve flow equation for U_k

RG equation for U_k

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left\{ \sum_{\pi,k,a,\phi,\sigma,\sigma',\eta,\eta'} \alpha_{e_b}(k)\frac{\coth\left[\frac{E_b}{2T}\right]}{E_b} - \sum_{u,d,s}\alpha_{e_f}(k)\frac{\tanh\left[\frac{E_f}{2T}\right]}{E_f}\right\}$$

Skokov 2012

$$\alpha_{e_f}(k) = 6N_c\frac{e_f B}{k^2}\left(1+2\sum_{n=1}^{\infty}\sqrt{1-2\frac{e_f B}{k^2}n}\;\theta\left[1-2\frac{e_f B}{k^2}n\right]\right) \to 4N_c\ (e_f B = 0)$$

$$\alpha_{e_b}(k) = 3\frac{e_b B}{k^2}\sum_{n=0}^{\infty}\sqrt{1-\frac{e_b B}{k^2}(2n+1)}\;\theta\left[1-\frac{e_b B}{k^2}(2n+1)\right] \to 1\ (e_b B = 0);.$$

Scale dependent Energies *Mitter et.al 2013*

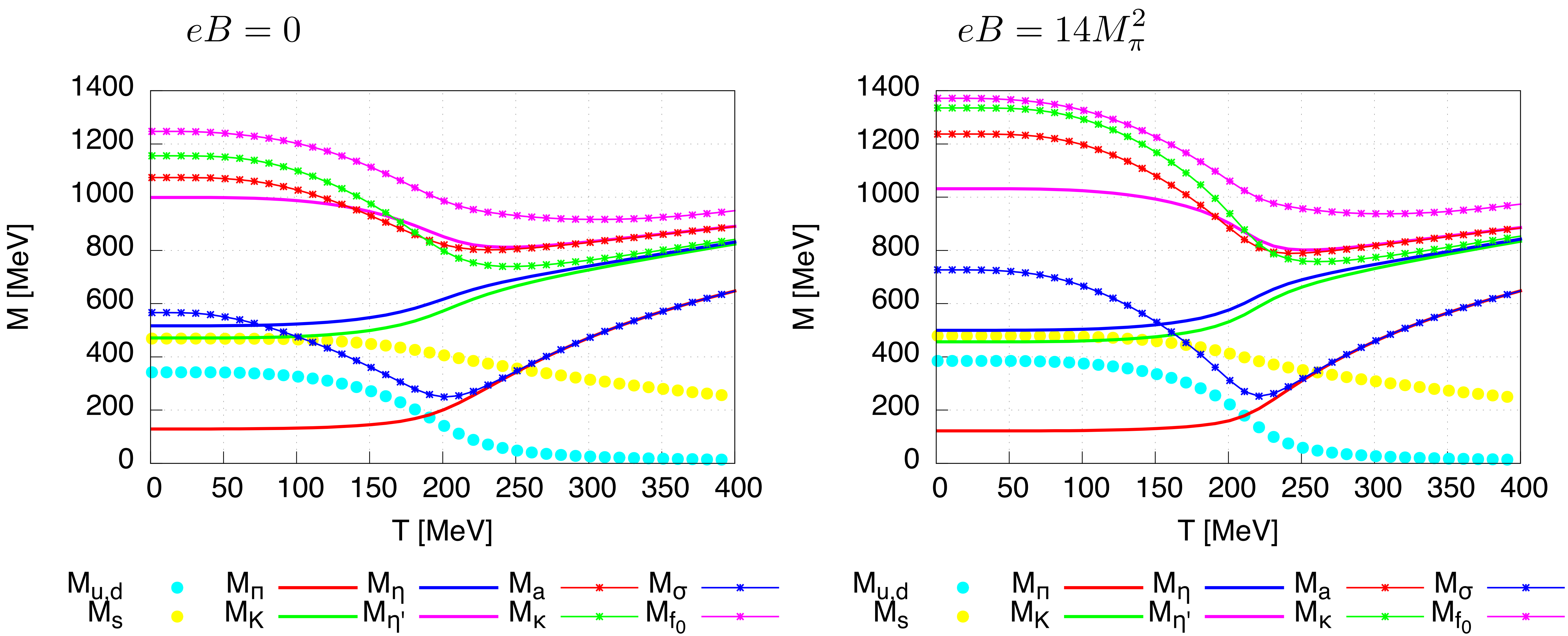
$$E^2_{u,d} = k^2 + \frac{g^2}{4}\sqrt{\frac{4\rho_1 - \sqrt{24\rho_2}}{3}}$$

$$E^2_{k,\pi} = k^2 + \partial_{\rho_1}U_k + \sqrt{24\rho_2}\partial_{\rho_2}U_k - \frac{c_a}{2}\sqrt{\frac{4\rho_1 + 2\sqrt{24\rho_2}}{3}}$$, etc

- Vacuum is determined by $\partial_{\rho_1}U_{k=0} = \partial_{\rho_2}U_{k=0} = 0$
- Pressure is given as $P = -(U_{k=0}(\rho_1,\rho_2) - c\sigma_0)$
- Meson masses are given as a function of $U_{k=0}$ or derivatives of $U_{k=0}$

  ex: $M^2_\pi = \partial_{\rho_1}U_k + \sqrt{24\rho_2}\partial_{\rho_2}U_k - \frac{c_a}{2}\sqrt{\frac{4\rho_1 + 2\sqrt{24\rho_2}}{3}}$

Meson screening masses

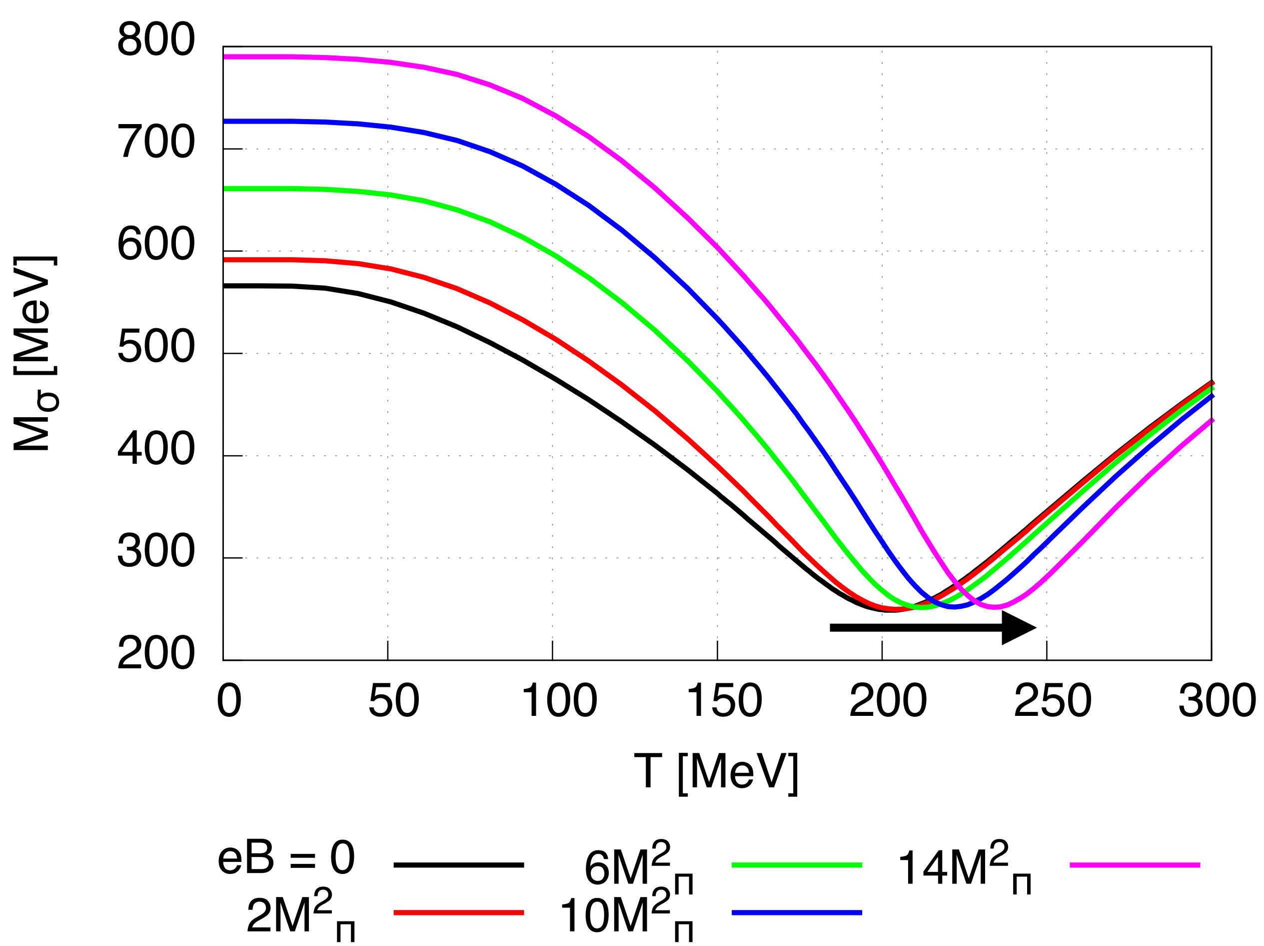


- Chiral phase transition occurs.
- Lowest Landau level of charged mesons are given

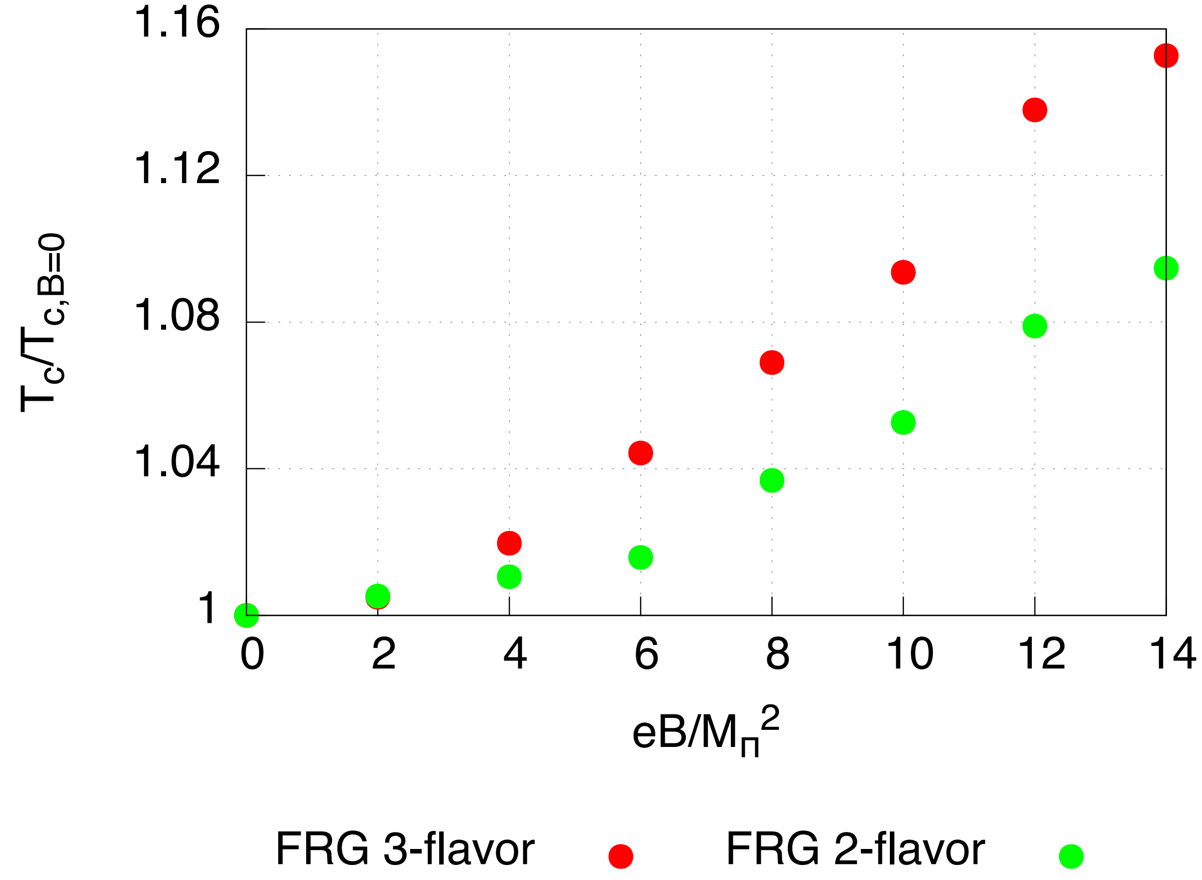
$$M_{LL}^2 = M_{\rm scr}^2 + 2eB$$

Phase diagram

σ mass at varying eB



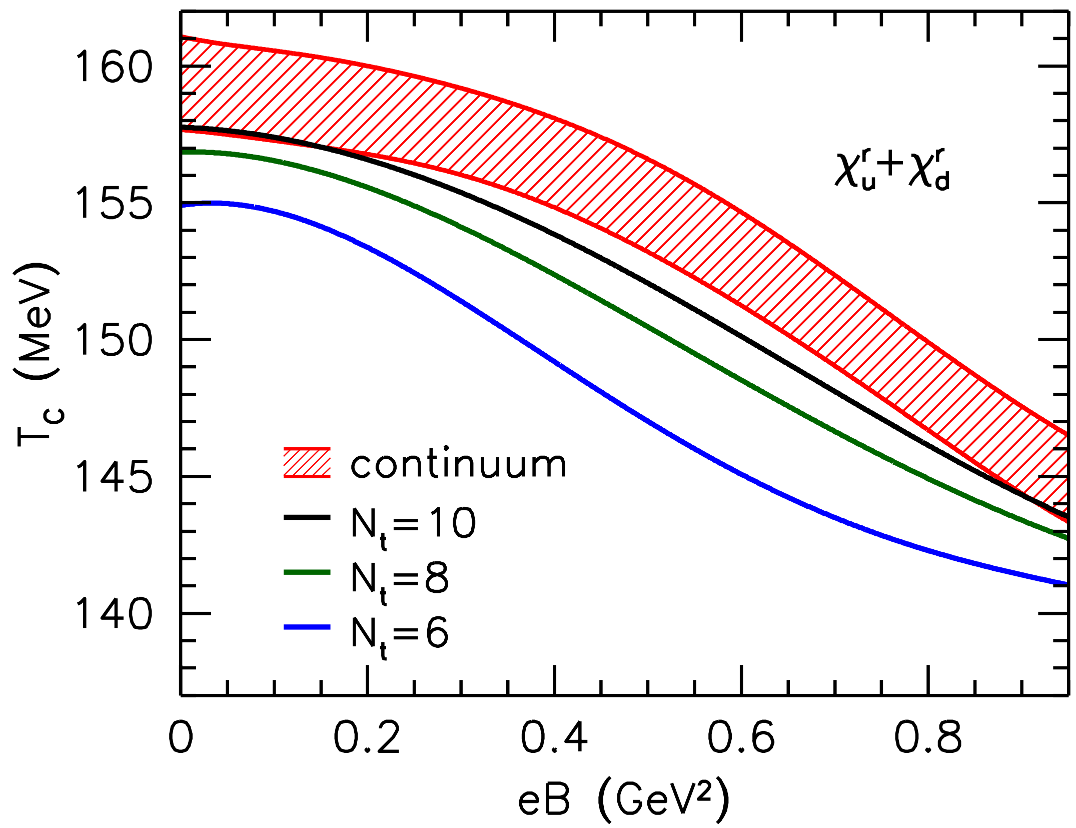
T_c at finite eB



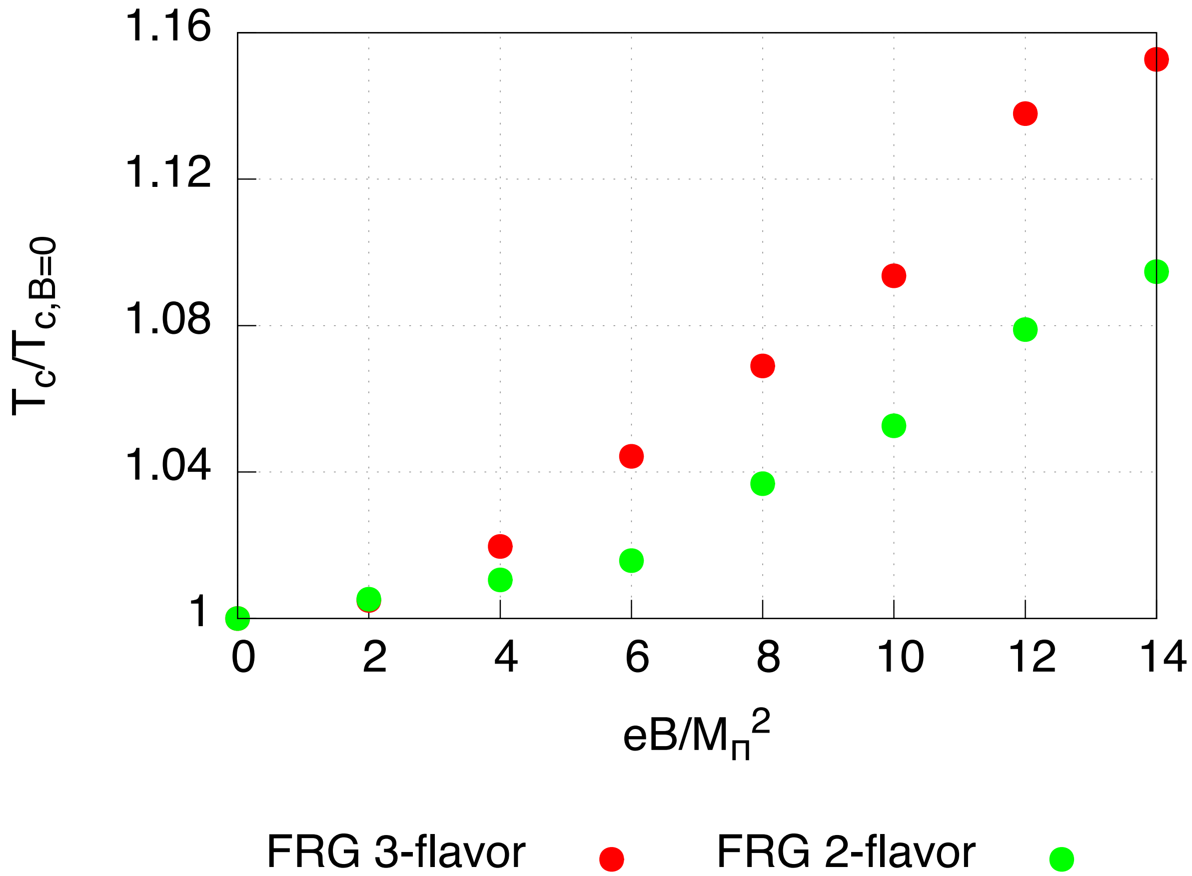
- Tc increases with increasing eB. (Magnetic catalyses)

Anti-Magnetic catalyses

2+1 flavor Lattice [G.S. Bali et al. (2012)]

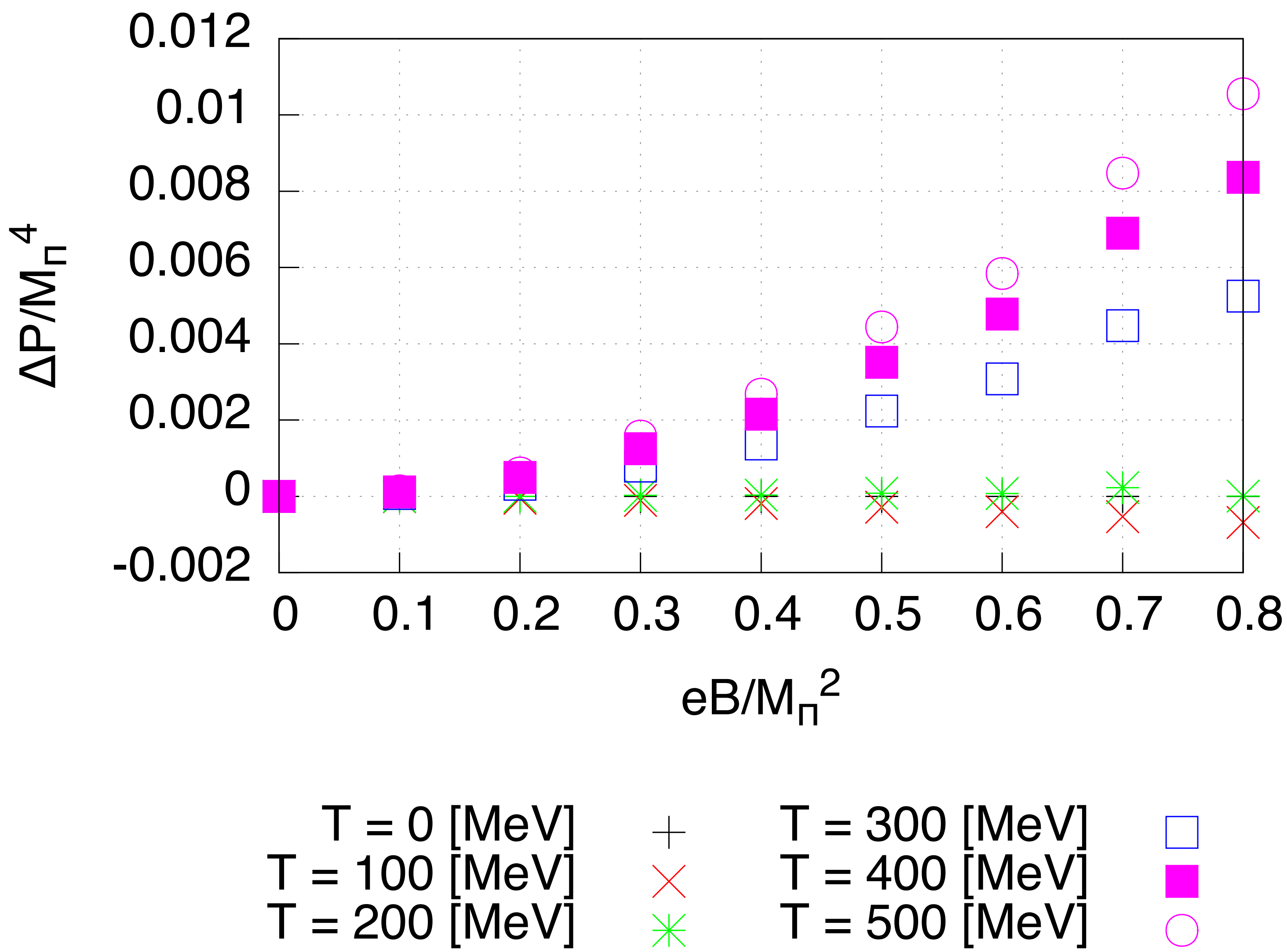


Our results



- As far as I know, no chiral effective model explains the Anti-Magnetic catalyses.

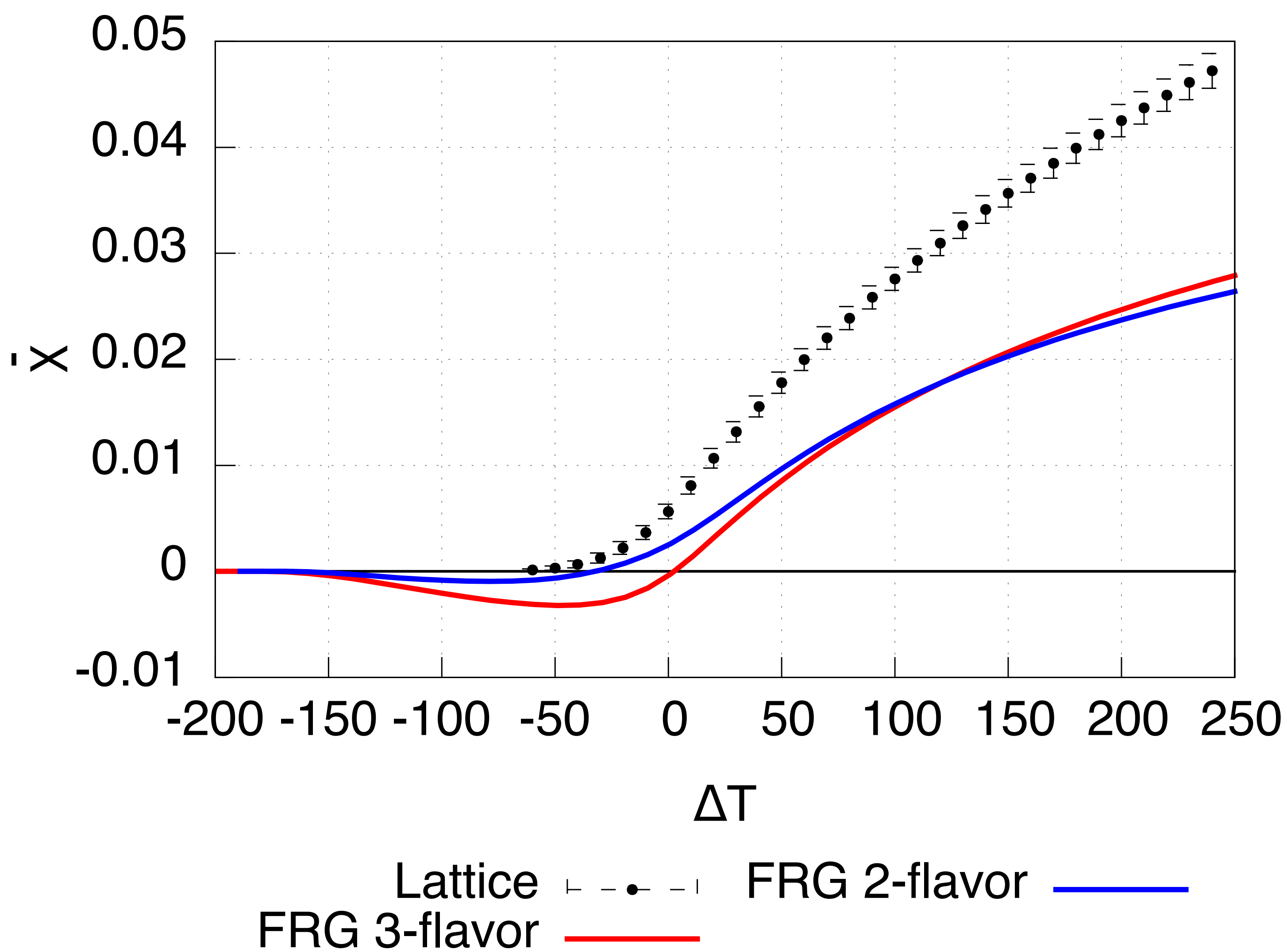
Pressure



$$\Delta P(eB) \equiv (P(T,B) - P(T,0)) - (P(B,0) - P(0,0))$$

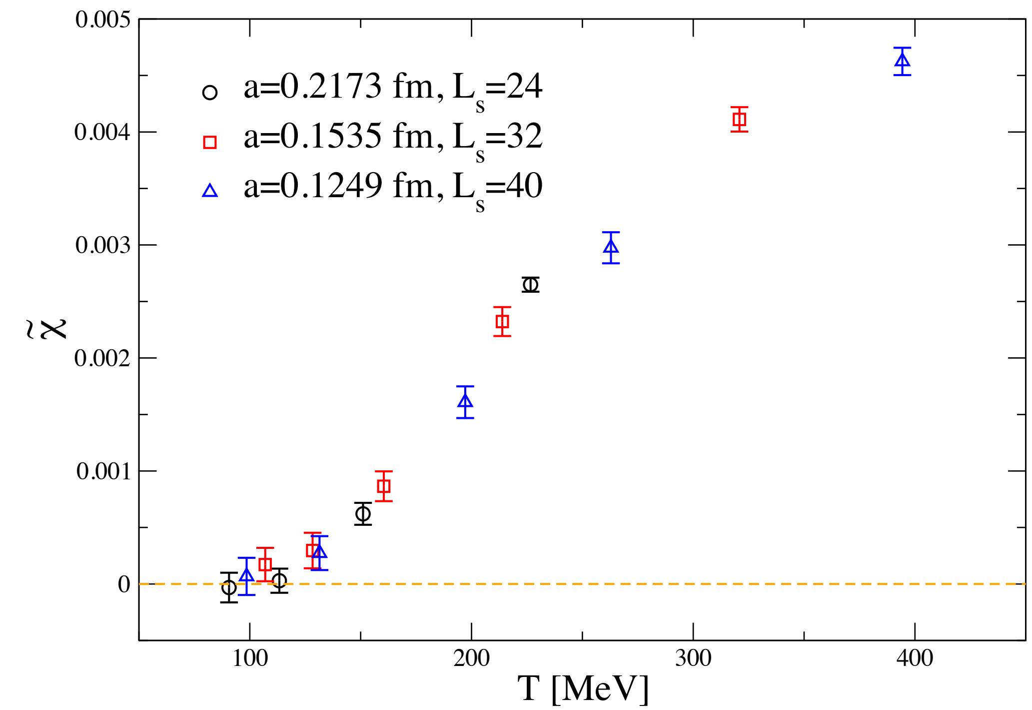
- Pressure vs eB for ($0 < T < 500$ [MeV] ~ 2.5 T_c)
- We fit the pressure with trial function $f(eB) = \frac{\chi}{2}(eB)^2$ using Gnuplot

Comparison with Lattice QCD



Lattice results on 2+1 flavor

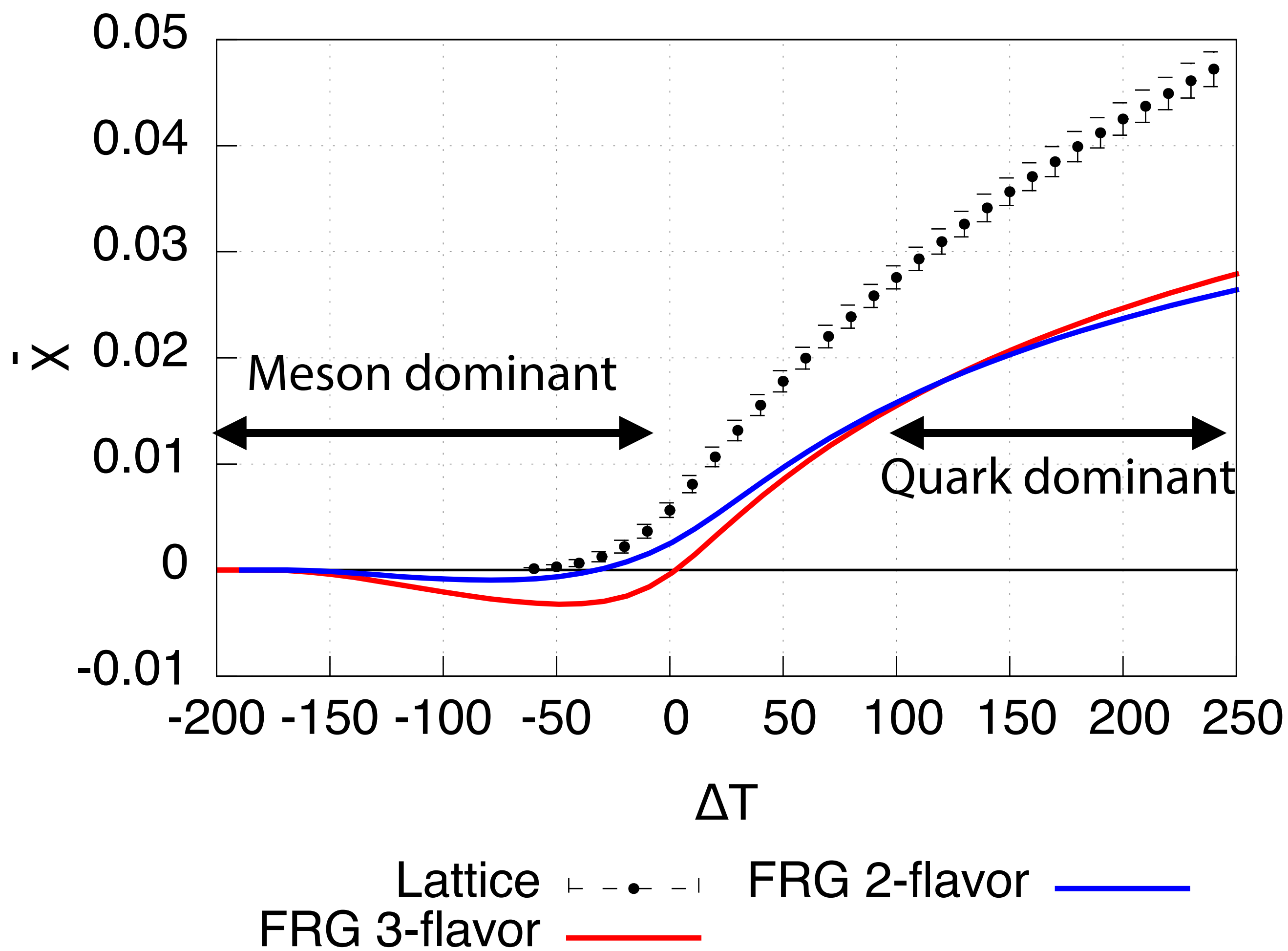
Bonati et.al 2013



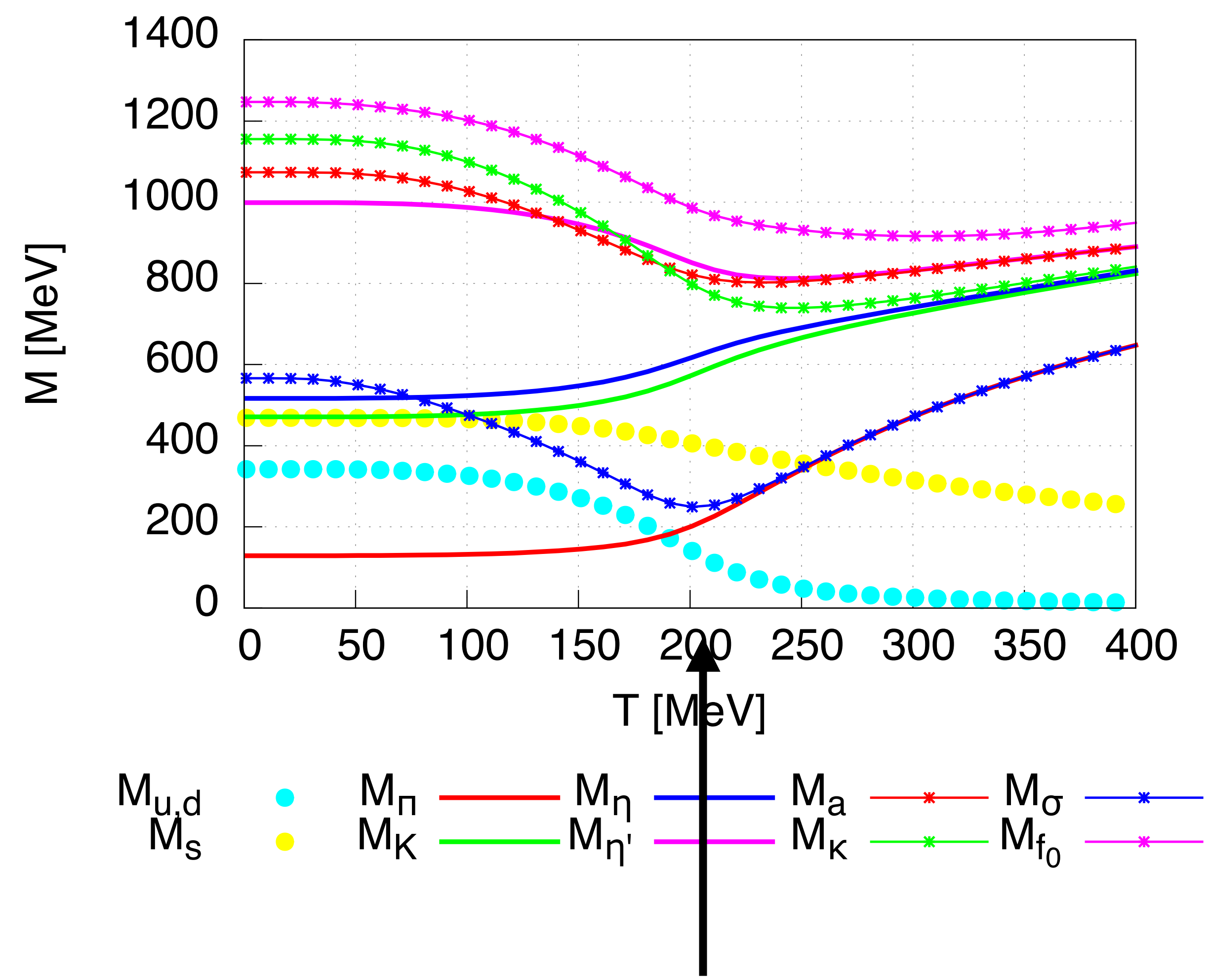
$$\Delta T \equiv T - T_c$$

- At low temperature, the matter has diamagnetism.
- Beyond Tc, χ changes the sign and the matter has paramagnetism.

Comparison with Lattice QCD



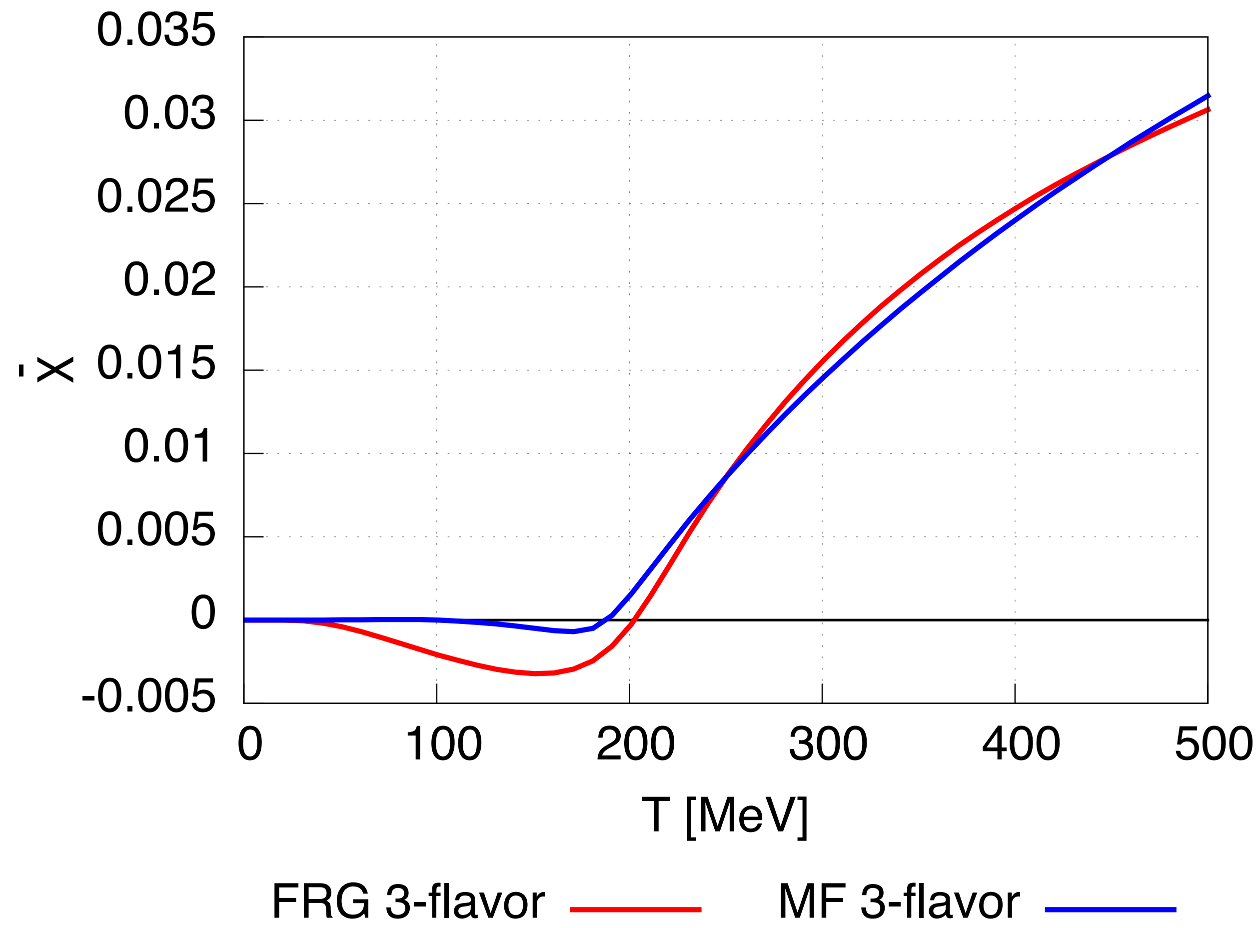
$$\Delta T \equiv T - T_c$$



$$\Delta T = 0$$

- At Hadron phase, charged mesons (especially pion) are dominant
- While QGP phase, u,d quarks are dominant.

Mean field



$$k\partial_k\Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\left[\cancel{\frac{k\partial_k R_{kB}}{R_{kB}+\Gamma_k^{(0,2)}[\varphi]}}\right] - \mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF}+\Gamma_k^{(2,0)}[\varphi]}\right]$$

MF: no meson loop contributions

- If we neglect meson loop contributions (mean field approximation), the matter is paramagnetic for almost all region.
- The origin of diamagnetism is charged mesons.

Summary

- We solve the 3-flavor quark-meson model under strong magnetic field with Functional-RG.
- We have calculated magnetisation of the QCD matter at zero chemical potential.
- At the hadron phase, QCD vacuum shows diamagnetism, due to light charged pions.
- At the QGP phase, the matter shows paramagnetism, due to almost bare quarks.