É.Cartan's Spinor and Quantum Physics

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Contents

References:
Few Body Syst. DOI 10.1007/s0061-013-0719-9, arXiv:1301.2095 [hep-ph], arXiv:1304.3776[hep-ph]
Int. J .Mod. Phys. A27, 125015(2012) arXiv:1203.5213[hep-ph]
Few Body Syst. **52**(2012), 171, arXiv:1104.1225[hep-ph]
Few Body Syst. **52**(2012), 343, arXiv:1110.3857[hep-ph]
Int. Conf. on Numerical Analysis and Applied Mathematics (ICNAAM2013) AIP Proc. 52-524, 2233-2236
Int. J. of Bifurcation and Chaos, arXiv.1312.3001[nlin.CD], 1406.4346[nlin.CD]

I. Introduction

- In 2007, a possibility of a sector of unparticle, which is scale invariant and very weakly interacting with the rest of the standard model was discussed[Georgi,2007]. The unparticle field that interacts with particles of the standard model will give mass to these particles, which is scale invariant. As the universe expands, the unparticle will produce Higgs particles, and as the universe expands, since the density of the Higgs particles is assumed to be invariant, the total mass of the universe will increase. It violates energy conservation rule. A possible solution is to assume that there are invisible universes which are contracting.

- É. Cartan extended the SU(2) spinor $\phi = {}^t(A, B)$ with elements ξ_{**} which has an even number of indices and spinor $\psi = {}^t(C, D)$ with elements ξ_* which has an odd number of indices[E.Cartan, 1966],

$$
\begin{aligned}
A &= \xi_{14}\sigma_x + \xi_{24}\sigma_y + \xi_{34}\sigma_z + \xi_0\mathbf{I} \\
B &= \xi_{23}\sigma_x + \xi_{31}\sigma_y + \xi_{12}\sigma_z + \xi_{1234}\mathbf{I} \\
C &= \xi_1\sigma_x + \xi_2\sigma_y + \xi_3\sigma_z + \xi_4\mathbf{I} \\
D &= \xi_{234}\sigma_x + \xi_{314}\sigma_y + \xi_{124}\sigma_z + \xi_{123}\mathbf{I},
\end{aligned} \tag{1}
$$

where σ_x, σ_y and σ_z are Pauli spinors, and added vector fields expressed as

$$
\begin{aligned}
E &= x_1\mathbf{i} + x_2\mathbf{j} + x_3\mathbf{k} + x_4\mathbf{I} \\
E' &= x'_1\mathbf{i} + x'_2\mathbf{j} + x'_3\mathbf{k} + x'_4\mathbf{I}.
\end{aligned} \tag{2}
$$

The spinors A, B, C, D and the vectors E, E' are transformed by superspace transformations $G_{23}, G_{12}, G_{13}, G_{123}$ and G_{132}, and there is triality symmetry between the six spaces. The operation on spinors and vectors are summarized in Table 1[S.Furui, 2012].

Table 1a. Transformations of A,B,C,D,E and E' by G_{23}, G_{12}, G_{123}.

G_{23}	G_{12}	G_{123}
$A \to (C_1, C_2, C_3, C_4)$	$A \to (x'_1, x'_2, x'_3, x_4)$	$A \to (x'_1, x'_2, x'_3, x'_4)$
$B \to (D_1, D_2, D_3, D_4)$	$B \to (x_1, x_2, x_3, x'_4)$	$B \to (x_1, x_2, x_3, x_4)$
$C \to (A_1, A_2, A_3, A_4)$	$C \to (C_1, C_2, C_3, D_4)$	$C \to (A_1, A_2, A_3, B_4)$
$D \to (B_1, B_2, B_3, B_4)$	$D \to (D_1, D_2, D_3, C_4)$	$D \to (B_1, B_2, B_3, A_4)$
$E \to (x_1, x_2, x_3, x'_4)$	$E \to (B_1, B_2, B_3, A_4)$	$E \to (D_1, D_2, D_3, C_4)$
$E' \to (x'_1, x'_2, x'_3, x_4)$	$E' \to (A_1, A_2, A_3, B_4)$	$E' \to (C_1, C_2, C_3, D_4)$

Table 1b. Transformations of A,B,C,D,E and E' by G_{13}, G_{132}.

G_{13}	G_{132}	
$A \to (A_1, A_2, A_3, B_4)$	$A \to (C_1, C_2, C_3, D_4)$	$A = (A_1, A_2, A_3, A_4)$
$B \to (B_1, B_2, B_3, A_4)$	$B \to (D_1, D_2, D_3, C_4)$	$B = (B_1, B_2, B_3, B_4)$
$C \to (x'_1, x'_2, x'_3, x_4)$	$C \to (x'_1, x'_2, x_3, 'x_4)$	$C = (C_1, C_2, C_3, C_4)$
$D \to (x_1, x_2, x_3, x'_4)$	$D \to (x_1, x_2, x_3, x'_4)$	$D = (D_1, D_2, D_3, D_4)$
$E \to (D_1, D_2, D_3, D_4)$	$E \to (B_1, B_2, B_3, B_4)$	$E = (x_1, x_2, x_3, x_4)$
$E' \to (C_1, C_2, C_3, C_4)$	$E' \to (A_1, A_2, A_3, A_4)$	$E' = (x'_1, x'_2, x'_3, x'_4)$

- The operators G_{13} and G_{123} produce fermion fields whose 4th component ξ_0 and ξ_{1234} are interchanged and vector fields (photon, ν or $\bar{\nu}$).

$$G_{13}: \begin{pmatrix} A \\ B \end{pmatrix} \to \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} \begin{pmatrix} \tilde{E}'l \\ \tilde{E}l' \end{pmatrix} \to \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} \begin{matrix} W^{\pm} \\ l\bar{\nu}, \bar{l}\nu \end{matrix} .$$

$$G_{123}: \begin{pmatrix} C \\ D \end{pmatrix} \to \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} \begin{pmatrix} E'l \\ El' \end{pmatrix} \to \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} \begin{matrix} W^{\pm} \\ l\bar{\nu}, \bar{l}\nu \end{matrix} .$$

- The operators G_{12} and G_{132} produce fermion fields whose 4th component ξ_4 and ξ_{123} are interchanged and vector fields (photon, ν or $\bar{\nu}$).

$$G_{12}: \begin{pmatrix} C \\ D \end{pmatrix} \to \begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} \begin{pmatrix} \tilde{E}'l \\ \tilde{E}l' \end{pmatrix} \to \begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} \begin{matrix} W^{\pm} \\ l\bar{\nu}, \bar{l}\nu \end{matrix} .$$

$$G_{132}: \begin{pmatrix} A \\ B \end{pmatrix} \to \begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} \begin{pmatrix} \tilde{E}'l \\ \tilde{E}l' \end{pmatrix} \to \begin{pmatrix} \tilde{C} \\ \tilde{D} \end{pmatrix} \begin{matrix} W^{\pm} \\ l\bar{\nu}, \bar{l}\nu \end{matrix}$$

Table 1c. The coupling of the spinors and the vectors..

$$\begin{aligned}
\mathcal{F} &= {}^t\phi C X \psi \\
&= x^1(\xi_{12}\xi_{314} - \xi_{31}\xi_{124} - \xi_{14}\xi_{123} + \xi_{1234}\xi_1) \\
&+ x^2(\xi_{23}\xi_{124} - \xi_{12}\xi_{234} - \xi_{24}\xi_{123} + \xi_{1234}\xi_2) \\
&+ x^3(\xi_{31}\xi_{234} - \xi_{23}\xi_{314} - \xi_{34}\xi_{123} + \xi_{1234}\xi_3) \\
&+ x^4(-\xi_{14}\xi_{234} - \xi_{24}\xi_{314} - \xi_{34}\xi_{124} + \xi_{1234}\xi_4) \\
&+ x^{1'}(-\xi_0\xi_{234} + \xi_{23}\xi_4 - \xi_{24}\xi_3 + \xi_{34}\xi_2) \\
&+ x^{2'}(-\xi_0\xi_{314} + \xi_{31}\xi_4 - \xi_{34}\xi_1 + \xi_{14}\xi_3) \\
&+ x^{3'}(-\xi_0\xi_{124} + \xi_{12}\xi_4 - \xi_{14}\xi_2 + \xi_{24}\xi_1) \\
&+ x^{4'}(\xi_0\xi_{123} - \xi_{23}\xi_1 - \xi_{31}\xi_2 - \xi_{12}\xi_3)
\end{aligned}$$

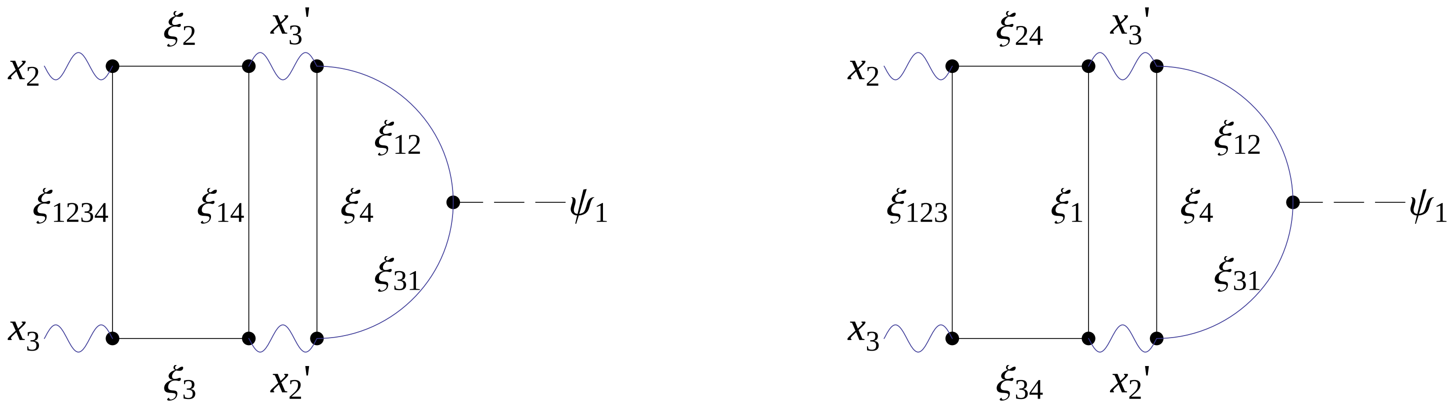
II. É.Cartan's spinor in pseudoscalar meson decays into $\gamma\gamma$

- In the universe, there are fermions which are transformed by quaternions and expressed by two-component spinors, and which are transformed by octonions and expressed by four component spinors. Three kinds of fermions (e, μ and τ) that interact electromagnetically and neutrinos which interact weakly are called leptons; and protons, neutrons etc. which interact strongly are called hadrons.

- In Quantum Chromo Dynamics(QCD), complex numbers and quaternions are used. A quaternion operates on a two-component spinor, i.e. Pauli spinor. The Dirac spinor is a four component spinor, but the octonion operates on four component spinors, which have the triality symmetry.

- Phenomenologically, the decay of a pion into two gamma rays is well described by a divergence of the axial current, since a pion can be regarded as a Nambu-Goldstone boson. The Adler-Bardeen's theorem says that higher order effects in the triangular diagram can be incorporated in the renormalization, represented by a rescattering diagram.

Fig.1 Half circle diagrams of axial anomaly with rescattering. $\pi(\psi_1)x'_3x'_2$ type and $\pi(\psi_1)x_3x_2$ type.



- The theoretical decay width of π^0 and that of η into two γ are

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2}{32\pi^3} \frac{{m_\pi}^3}{{f_\pi}^2} = 7.7\text{eV}$$

and

$$\Gamma(\eta \to \gamma\gamma) = \frac{\alpha^2}{32\pi^3} \frac{1}{3} \frac{{m_\eta}^3}{{f_\eta}^2} = 0.13\text{keV}.$$

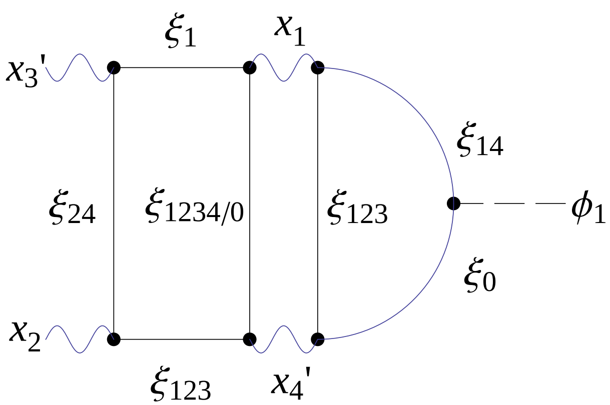
- The theoretical decay width of π^0 is consistent with experiment, but that of η is 4 times smaller [Exp. 0.510$\pm$0.026keV].

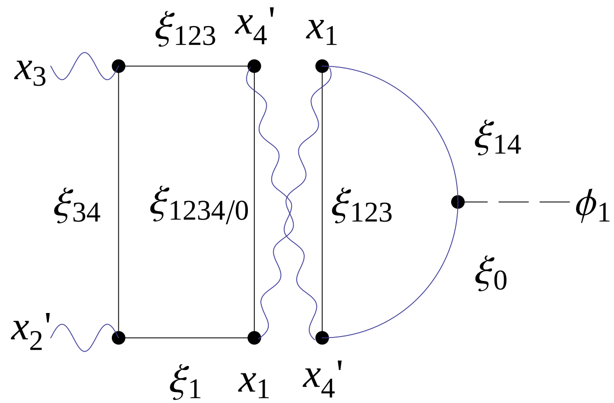
- Two γ in π^0 decay belong to the same sector of vector particles of Cartan E or E'.

- $\eta(\eta')$ can decay into two γ, in different sectors E and E', which could enhance the decay width.

- The η or η' decay into two photons given in Fig.2a, contains ξ_0 and ξ_{1234} in the intermediate state. The coherent sum of the two amplitude enhances the η or η' decay amplitude by a factor 2, and both π^0 and η, η' decay width become consistent.

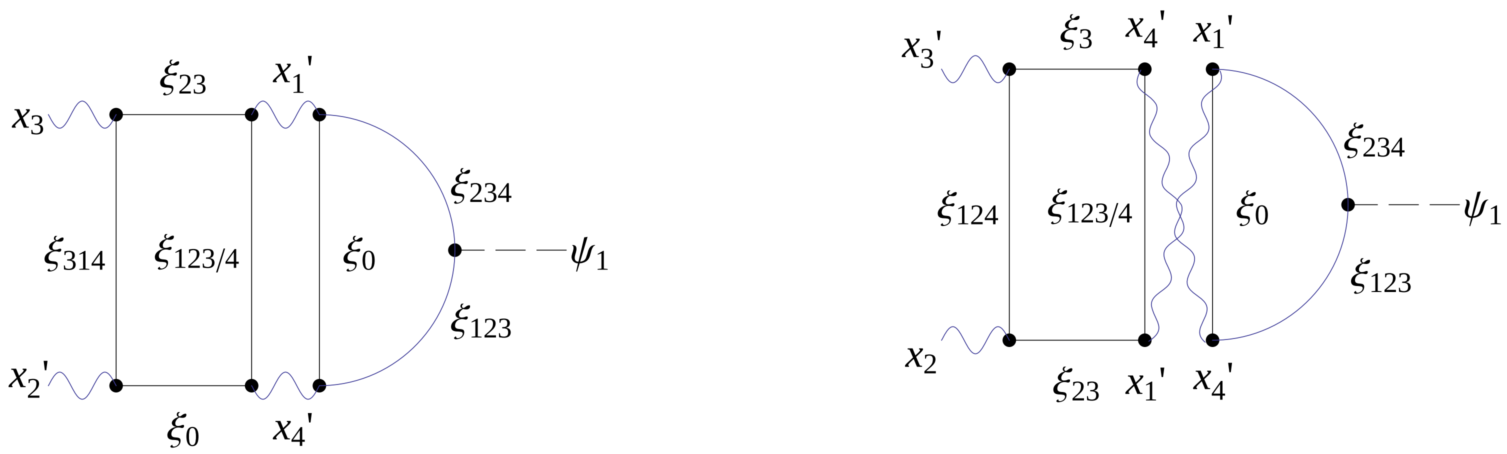
Fig.2a Half circle diagrams of $\eta(\eta')$ decay with rescattering. The $\eta(\eta') \to x'_3 x_2$ type and a twisted $\eta(\eta') \to x_3 x'_2$ type.





- Mixing of ξ_{123} and ξ_4 does not occur in the $\eta(\eta')$ decay, since intermediate x_1', x_4' state is ψ but the final $x_2'x_3$ or x_2, x_3' state is ϕ.

Fig.2b Improper diagrams of $\eta(\eta')$ production after rescattering. The $\eta(\eta')$ of x_3x_2' type and $x_3'x_2$ type.



III. É.Cartan's spinor in scalar boson decays into $\gamma\gamma\gamma\gamma$

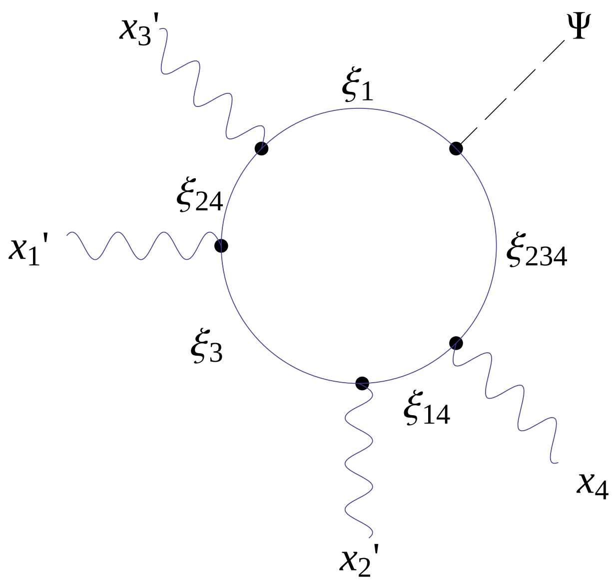
- É.Cartan's spinor predicts two scalar bosons Φ and Ψ.

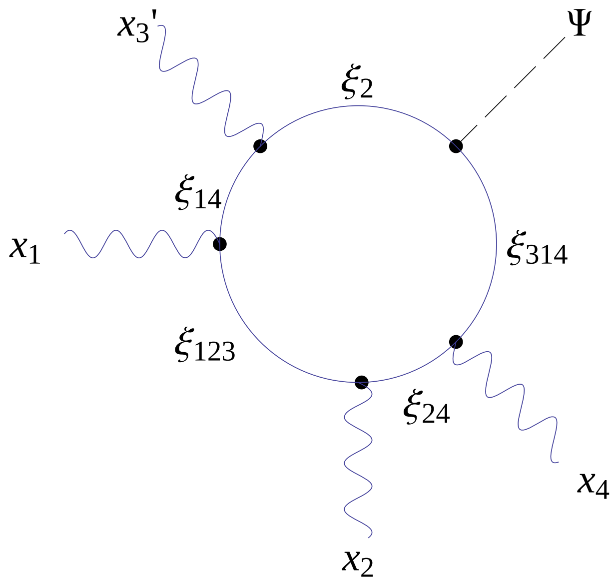
$$\begin{aligned} \Psi &= {}^t\psi C\psi = -\xi_1\xi_{234} - \xi_2\xi_{314} - \xi_3\xi_{124} + \xi_4\xi_{123} \\ \Phi &= {}^t\phi C\phi = \xi_0\xi_{1234} - \xi_{23}\xi_{14} - \xi_{31}\xi_{24} - \xi_{12}\xi_{34} \end{aligned}$$

- In one loop order, we can derive decays into 4γ particles are possible.

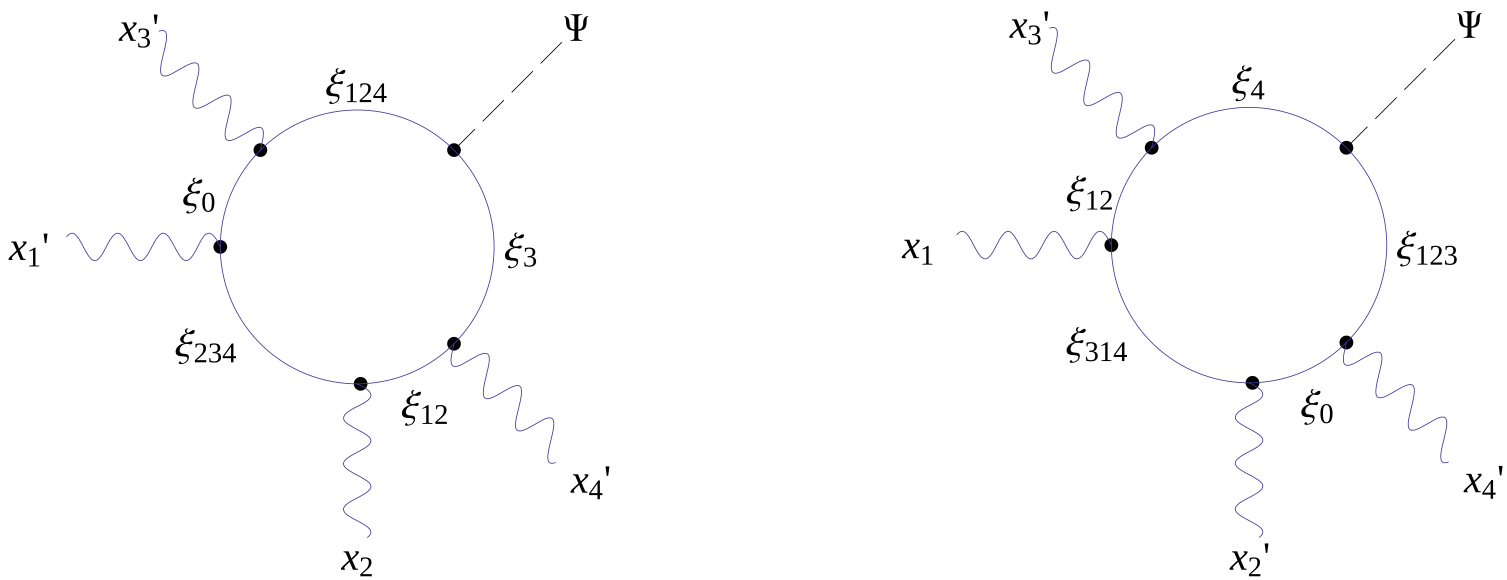
- The order of 4 polarizations of γ' s can be interchanged.

- Typical diagrams of Ψ decay into 4γ. ($\bar{\psi}$ polarized in $\boldsymbol{i}$ and $\boldsymbol{j}$ direction, respectively.)

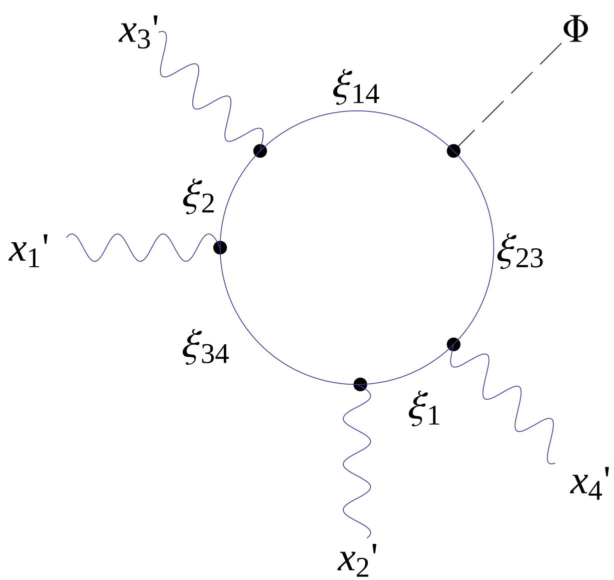


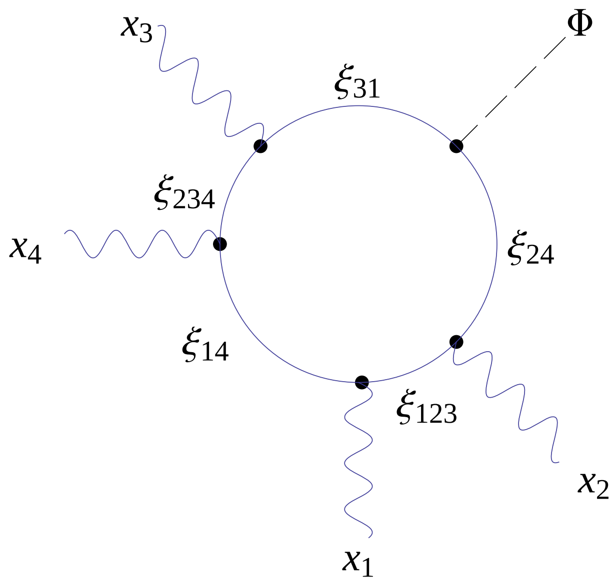


- Typical diagrams of Ψ decay into 4γ. ($\bar{\psi}$ polarized in $\boldsymbol{k}$ direction and time direction, respectively.)

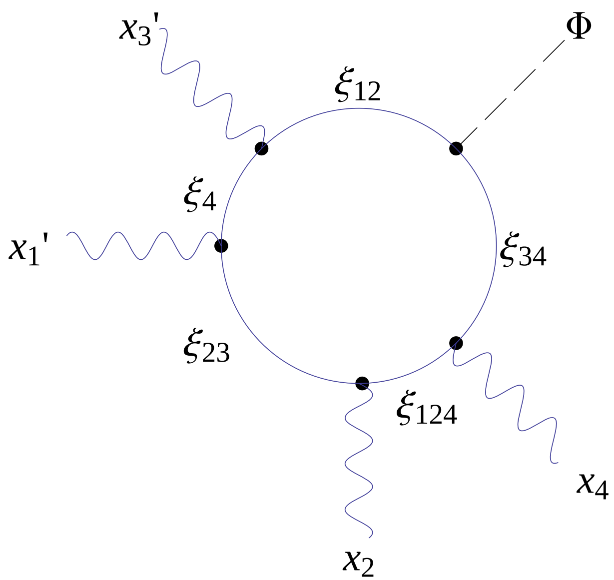


- Typical diagrams of Φ decay into 4γ. ($\bar{\phi}$ polarized in $\boldsymbol{i}$ and $\boldsymbol{j}$ direction, respectively.)





- Typical diagrams of Φ decay into 4γ. ($\bar{\phi}$ polarized in $\boldsymbol{k}$ direction and time direction, respectively.)



IV. The Dark Matter and the triality symmetry

- In the É. Cartan's spinor theory based on Clifford algebra, a lepton has superpartners, but they cannot be detected by our electromagnetic probes. Leptons (A, B, C, D) and vector field (E, E') are transformed by three transformations G_{ij} and two transformations G_{ijk}. Electromagnetic probes will not detect electromagnetic waves (E, E') transformed by G_{12}, G_{13}, G_{123} and G_{132}, and 4/6 of the electromagnetic waves will appear as dark energy.

- The transformation G_{23} interchanges two components of the four component spinors, or it transforms matter to anti-matter. Then 5/6 of the matter in the universe appears different from actuality and rather appears as the dark matter. The Wilkinson Microwave Anisotropy Probe(WMAP) space craft confirms that almost 5 times more dark matter (24%) than normal matter (4.6%) are observed.

- Photons emitted from matter made of quarks that belong to a triality sector different from that of electromagnetic probes, will not be detected, and the matter will be assigned as a dark matter.

- We considered three massless neutrinos in different triality sectors interacting with each other and proposed a theory with a heavy and two degenerate light neutrinos. ν_e, ν_μ and ν_τ have their lepton partners. We expect e, μ and τ are sensitive to flavors, but blind to the triality of neutrinos, quarks and gluons, and that they are sensitive to the triality of electromagnetic waves.

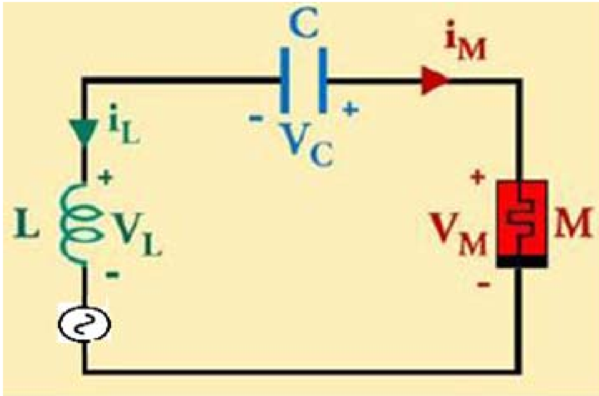
- If electromagnetic waves from different triality sectors cannot be detected by electromagnetic probes in our universe, we can understand the presence of the dark matter.

IV. É.Cartan's spinor in the memristor circuit

- In 2010, chaotic oscillation was observed in an electronic system containing inductor, capacitor and non-linear memristors [B.Muthuswamy and L.O. Chua, 2010]. The three-element circuit with the voltage across the capacitor $x(t) = v_C(t)$, the current through the inductor $y(t) = i_L(t)$ and the internal state of the memristor $z(t)$ satisfy a coupled equation[J-M.Ginoux, Ch.Letellier and L.O. Chua, 2010].

- Memristor is the 4th element of circuits. It is used also in Non Destructive Testing (NDT) of complex structures, like electronic systems and biological systems.

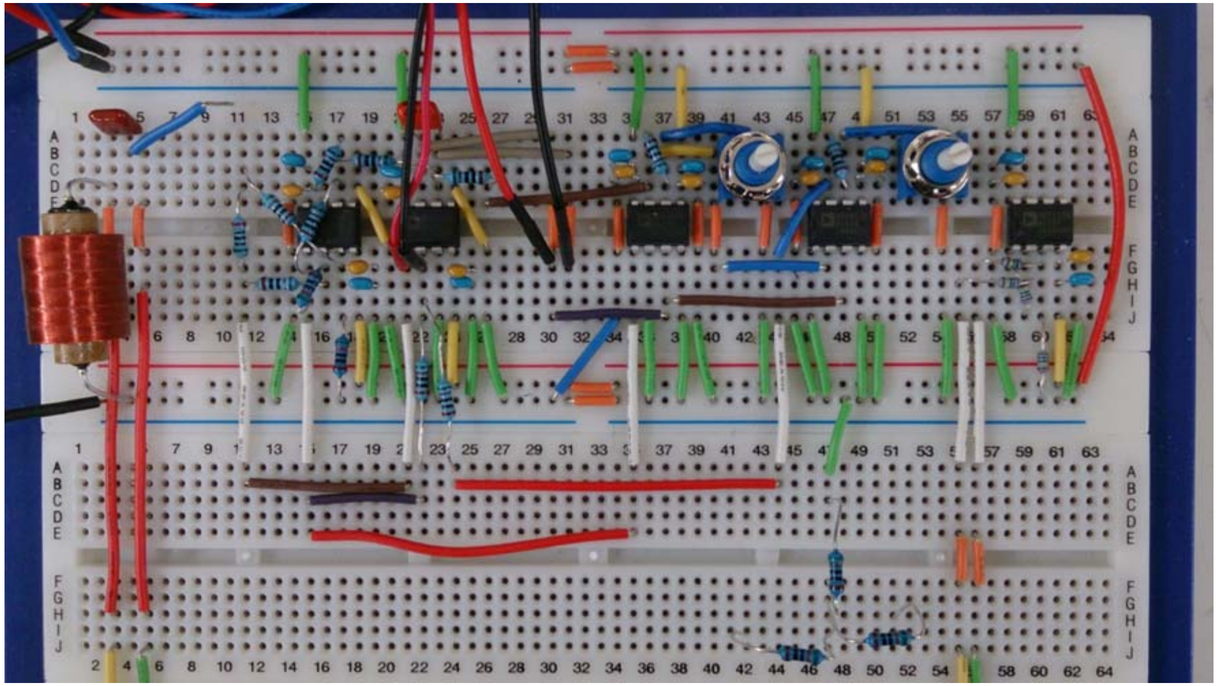
Fig.3a The memristor circuit of Muthusmamy and Chua.



- We added an oscillation $\gamma \sin \omega t$ to the memristor. The coupled differential equation was studied, with $\gamma = 0.2, \alpha = 0.1$ fixed and ω changed.

$$\dot{x} = \frac{y}{C}, \quad \dot{y} = \frac{-1}{L}[x + \beta(z^2 - 1)y] + \gamma \sin \omega t, \quad \dot{z} = -y - \alpha z + y\,z.$$

Fig.3b The memristor circuit represented on a breadboard by Mr.Takano.



- We observed bifurcation diagrams and the frequency of the bifurcation. The response frequency of the oscillation ω as well as ω/n were fitted as a function of n, in the case of $C = 1.2$ and $C = 1.0$

- The frequency of the single oscillation divided by 8 ($\omega_1/8$) is close to the frequency of the 7 oscillations divided by 7 ($\omega_7/7$). The data suggest 8 fold degeneracy in the single mode.

- The 4component spinors $(A, B), (C, D)$ and input vectors x and x' belong to a definite triality sector, and since the x and x' belong to a different triality sector, the spinor (A, B) produced by an operation of G_{12} on (E', E) has diffferent triality sectors, and (A, B) cannot be a Dirac fermion.

Fig.4 The bifurcation diagram of the driven memristor.
$C = 1.2, L = 3.3, 0.24 \leq \omega \leq 0.36$(left) and $0.36 \leq \omega \leq 0.46$(right).

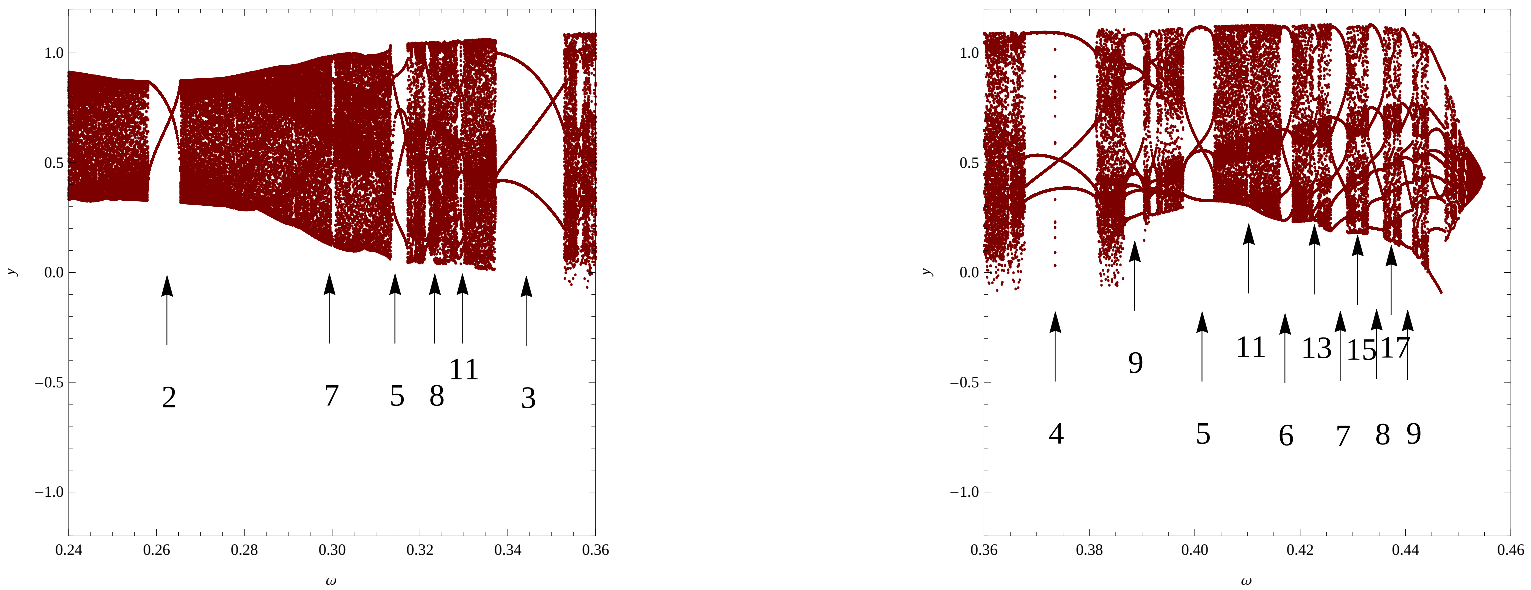
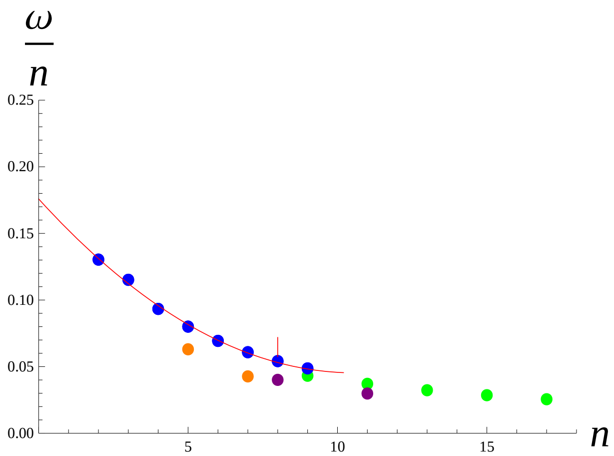
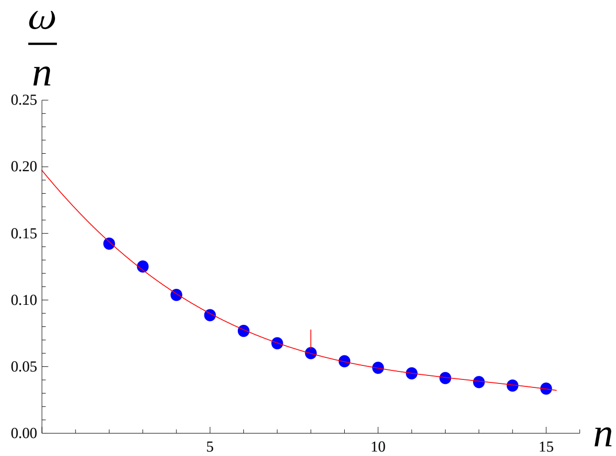


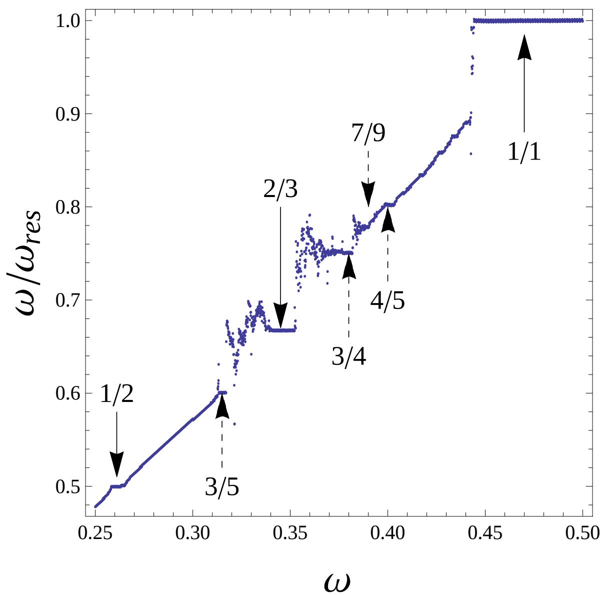
Fig.5 The fit of ω/n as a function of the torus loop n.
$L = 3.3, \alpha = 0.2, \beta = 0.5,\ C = 1.2$(left) and $C = 1.0$(right)

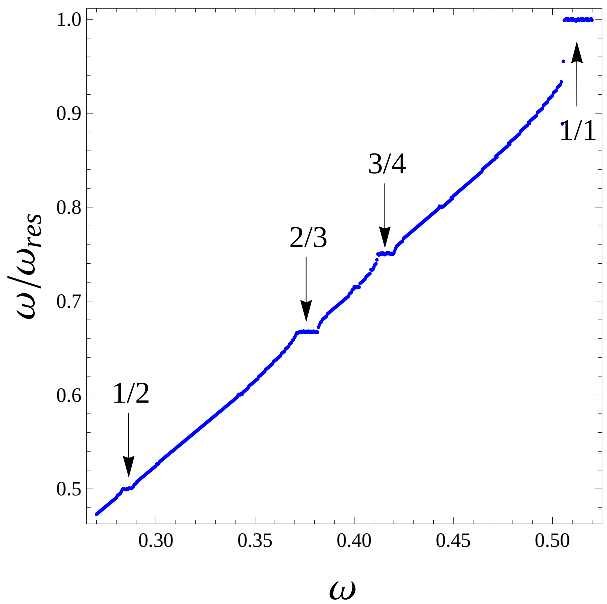




- The Devil's staircase structures in the ratio of the frequency of the driving oscillation and that of the response oscillation were observed[S. Dos Santos 1998].

Fig.6 The Devil's staircase of memristic circuit. $C = 1.2$ (left) and $C = 1$ (right).





- The systems of $C = 1.2$ is unstable, when $\omega < \omega_{res}$, but the system of $C = 1$ is stable. Near $\omega \geq \omega_{res}$, there is no qualitative difference.

- The perturbation $\dot{y} \simeq \frac{\gamma}{L} \sin \omega t$ on $\dot{x} = y/C$ is less serious in the case of $C = 1$ as compared to the case of $C = 1.2$.

- The Hölder exponent of the cycle limit of the rotation number $\nu(x)$, where x is given by the ratio of the frequency of the perturbation f_s and that of response f_d defined as $x = \frac{f_s}{f_d}$, checks the stability of the system[M. Planat and P. Koch, 1993],

$$||\nu(x) - \nu(y)|| = ||x - y||^{\alpha}.$$

  We found $x = \frac{2}{3}, y = \frac{3}{4}$ of $C = 1.2$ yields $\alpha = 0.695 < 1$, and $x = \frac{2}{3}, y = \frac{1}{2}$ of $C = 1$ yields $\alpha = 1.211 > 1$. It indicates the system of $C = 1.2$ is unstable or chaotic.

V. Discussion and Conclusion

- Clifford Algebra Cl_3^+ which has bases $\boldsymbol{R} \oplus \boldsymbol{R}^3 \oplus \wedge^2 \boldsymbol{R}^3 \oplus \wedge^3 \boldsymbol{R}^3$ can produce $Spin(3)$ which is expressed by quaternion $\boldsymbol{H} = \{1, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$, and an extension $\boldsymbol{H} \oplus \boldsymbol{H}\boldsymbol{l}$ with a new imaginary unit $\boldsymbol{l}$ makes an octonion $\boldsymbol{O}$.

- In the Clifford product $\mathcal{C}l_{0,7}$ of $\mathbf{R}^{\mathbf{0,7}}$, the volume element $\boldsymbol{v} \in \bigwedge^3 \mathbf{R}^{0,7}$ is expressed as

$$\boldsymbol{v} = \boldsymbol{e}_{124} + \boldsymbol{e}_{235} + \boldsymbol{e}_{346} + \boldsymbol{e}_{457} + \boldsymbol{e}_{561} + \boldsymbol{e}_{672} + \boldsymbol{e}_{713}$$

  and we define $\boldsymbol{w} = \boldsymbol{v}\mathbf{e}_{12\cdots 7}^{-1} \in \bigwedge^4 \mathbf{R}^{0,7}$ and include the unit vector $\mathbf{e}_8$. Clifford Algebra Cl_8 of $\boldsymbol{R}^8$ is represented by $\boldsymbol{O}$, and there is an automorphysm $\boldsymbol{Spin}(8)$.

- All automorphisms of $\boldsymbol{Spin}(n), n \neq 8$ are of the form $u \to sus^{-1}$ where $s \in \boldsymbol{Pin}(n)$, but the group $\boldsymbol{Spin}(8)$ has exceptional automorphisms, which permute the non-identity elements -1,$\mathbf{e}_{12\cdots 8}, -\mathbf{e}_{12\cdots 8}$ in the center of $\boldsymbol{Spin}$:

$$-1 \to \mathbf{e}_{12\cdots 8} \to -\mathbf{e}_{12\cdots 8} \to -1.$$

- This automorphism of order three is called triality automorphism. One can assign 3 physical states like (e, μ, τ) leptons, (ν_1, ν_2, ν_3) neutrinos on definite triality states.

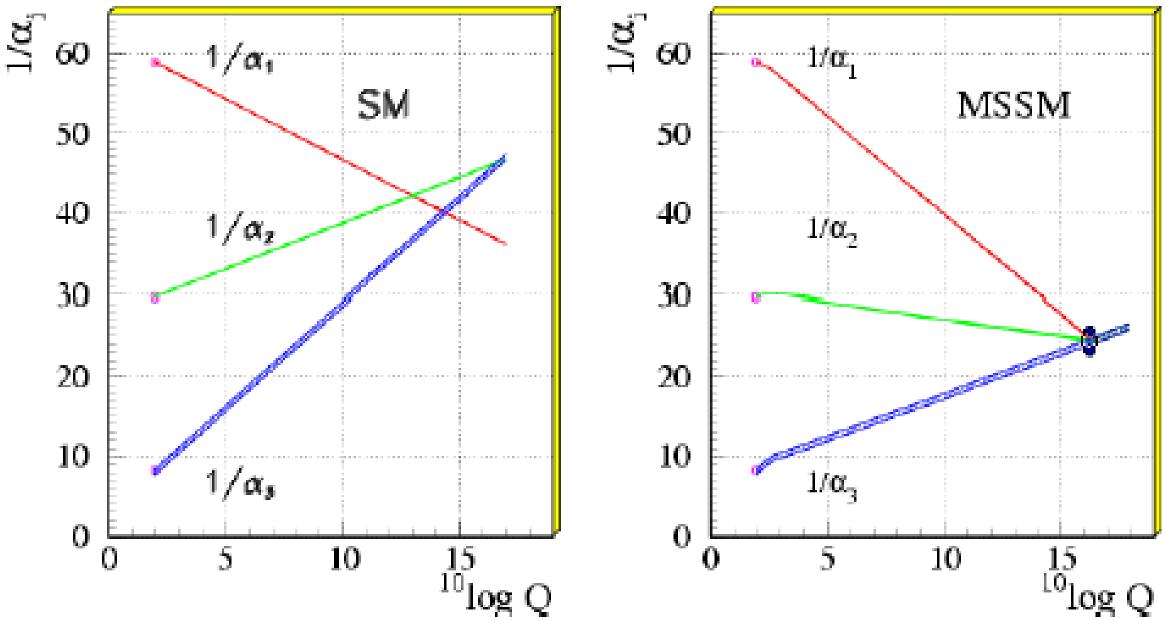
- Quarks are separated in three sectors.

$$\begin{pmatrix} d & s & b \\ u & c & t \end{pmatrix}$$

- Fermions in our universe are transformed by G_{12}, G_{13}, G_{123} and G_{132} to vectors, but the vectors produced by these transformations cannot be detected by electromagnetic probes in our universe. Invisible universe due to triality selection rules is expected.

- É. Cartan's spinor presents triality symmetry, which plays an important role in the memristic circuit.

Fig.7 The GUT running coupling constants. Should electroweak, strong and gravitational strengths agree at a point ?.



幾何学を参照することなく物理法則を述べることは、言葉を使わずに考えを述べるようなものです。

(アインシュタイン　1922年　京都で行った講演)

The New Quotable Einstein, A. Calaprice, Princeton Univ. Press (2005), p.230

References

1. H. Georgi, Phys. Rev. Lett.**98**, 221601(2007).

2. É.Cartan, *The theory of Spinors*, Dover Pub. (1966) p.118.

3. P. Lounesto, *Clifford Algebras and Spinors* Second Edition Cambridge Univ. Press (2006)

4. S. Furui, Few Body Syst. **52**, 171(2012); **53**, 343 (2012).

5. S. Furui, Few Body Syst. **54** , 2097 (2013), arXiv:1301.2095.

6. S.Furui, Int. J. Mod. Phys. **A27**, 1203.5213(2013) ; arXiv:1304.3776.

7. B. Muthuswamy and L.O. Chua, International Journal of Bifurcation and Chaos, **20** 1567 (2010).

8. J-M.Ginoux, Ch.Letellier and L.O. Chua, International Journal of Bifurcation and Chaos, **20** 3819 (2010).

9. S. Dos Santos, Thèse (1998)

10. S. Furui and T. Takano, Int. J. of Bifurcation and Chaos(2014), arXiv:1312.3001, Int. J. of Bifurcation and Chaos(2014) arXiv:1406.4346[nlinCD].

11. S. Roberts, *King of Infinite Space, Donald Coxeter, the Man Who Saved Geometry*, Walker Pub. (2006); 糸川 洋訳 "多面体と宇宙の謎に迫った幾何学者" 日経BP社(2009)