First oder phase transition in QED_3 with Chern-Simon term

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1 Introduction

problems in Maxwell-Chern-Simon QED

$$L = \overline{\psi}(i\gamma \cdot D - m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{\mu}{4}\epsilon^{\mu\nu\alpha}A_\mu F_{\nu\alpha} - \frac{1}{2a}(\partial_\mu A_\mu)^2 \tag{1}$$

Chern-Simon term breaks parity

$$\frac{\mu}{4}\epsilon^{\mu\nu\alpha}A_\mu F_{\nu\alpha}. \tag{2}$$

if $m = 0$,there is a $U(2)$ symmetry generated by $\{I, \gamma_3, \gamma_5, [\gamma_3, \gamma_5]\}$ for 4-spinor.

Ordinary γ marices $\{\gamma_0, \gamma_1, \gamma_2\}, \{\gamma_3, \gamma\} = 0, \{\gamma_5, \gamma\} = 0, \tau = [\gamma_3, \gamma_5]/2 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$

Chiral like symmetry breaking under $e^{i\gamma_3\theta_3}, e^{i\gamma_5\theta_5}$

$$\overline{\psi}\psi \rightarrow \cos(2\theta)\overline{\psi}\psi + i\sin(2\theta)\overline{\psi}\gamma_3(\gamma_5)\psi \tag{3}$$

$$\overline{\psi}\tau\psi \rightarrow \overline{\psi}\tau\psi \tag{4}$$

Parity $(t, x, y) \rightarrow (t, -x, y)$ under $P\psi P^{-1} = i\gamma^1\gamma^5\psi$

$$\begin{aligned} m_e\overline{\psi}\psi &\rightarrow m_e\overline{\psi}\psi \\ m_o\overline{\psi}\tau\psi &\rightarrow -m_o\overline{\psi}\tau\psi \end{aligned} \tag{5}$$

2 Dyson-Schwinger analysis including C-S term

◯ Kondo-Maris(1994),A.Raya(2010)
Solving Dyson-Schwinger eq.there may be 1-st order phase transition at critical value of topological mass μ_c.
· for small $\mu, m_e \neq 0, m_o \neq 0$,chiral symmetry and Parity symmetry are broken ($\mu \leq \mu_c$),
for large μ,chiral symmetry restored and Parity symmery broken $m_e = 0, m_o \neq 0$ for ($\mu_c \leq \mu$).
· $\langle\overline{\psi}\psi\rangle \neq 0, \langle\overline{\psi}\tau\psi\rangle \neq 0$ at $\mu \leq \mu_c$. $\langle\overline{\psi}\psi\rangle \to 0, \langle\overline{\psi}\tau\psi\rangle \neq 0$ at $\mu \geq \mu_c$.
○topological mass weaken Coulomb force,

$$D_{\mu\nu}(k) = \frac{-(g_{\mu\nu} - k_\mu k_\nu/k^2 - i\mu\epsilon_{\mu\nu\rho}k^\rho/k^2)}{k^2 - \mu^2 + i\epsilon} - a\frac{k_\mu k_\nu}{k^4} \tag{6}$$

○Parity violating mass generation
In Euclid space $\tau = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$,it is simple to separate upper and lower component by$\chi_\pm = (1 \pm \tau)/2$

$$L_F = \overline{\psi}_+(i\gamma \cdot \partial - m_+)\psi_+ + \overline{\psi}_-(i\gamma \cdot \partial - m_-)\psi_- \tag{7}$$

$$S_F(p,a) = -\frac{A_+(p,a)\gamma \cdot p + B_+(p,a)}{A_+^2(p,a)p^2 + B_+^2(p,a)}\chi_+ - \frac{A_-(p,a)\gamma \cdot p + B_-(p,a)}{A_-^2(p,a)p^2 + B_-^2(p,a)}\chi_-.$$

$q = k - p, \alpha = e^2/4\pi, a = 0$

$$A_\pm(p) = 1 + \frac{\alpha}{\pi^2 p^2}\int d^3k \sigma^V_\pm(k)\frac{(k\cdot q)(p\cdot q)}{q^2(q^2+\mu^2)} \mp \frac{\alpha\mu}{\pi^2 p^2}\int d^3k\sigma^S_\pm(k)\frac{(p\cdot q)}{q^2(q^2+\mu^2)},$$
$$B_\pm(p) = \frac{\alpha}{\pi^2}\int d^3k\sigma^S_\pm(k)\frac{1}{q^2+\mu^2} \mp \frac{\alpha\mu}{\pi^2}\int d^3k\sigma^V_\pm(k)\frac{(k\cdot q)}{q^2(q^2+\mu^2)}. \tag{8}$$

Parity violating mass is a second term in $B_\pm(p)$.It is proportional to $A(p)$.
In the high energy limit $A(p) \to 1$.High energy limit of $m_o(p)$ is not changed by higher order correction.Topological mass μ dependence of m_e by linear approximation to ladder Dyson-Schwinger equation.
In the Landau gauge with $A(p) = 1$,it is written

$$m(p) = 2e^2\int \frac{d^3p'}{(2\pi)^3}\frac{m(p')}{(p-p')^2+\mu^2}\frac{-\mu(p'\cdot(p-p')/(p-p')^2}{p'^2+m^2}. \tag{9}$$

$$m_o(-p^2)_{p\to\infty} = \frac{e^2\mu}{8\sqrt{-p^2}}, m_o(-p^2)_{p\to 0} = \frac{e^2}{2\pi}\frac{\mu}{m+|\mu|}. \tag{10}$$

In the linear approximation analytic solution has been known for $\mu = 0$ case.

$$m_e(p) = \frac{e^2}{4\pi}\frac{m^2}{p^2+(m+\mu)^2}, m = m(0) = \frac{e^2}{4\pi}, m(x) = \frac{m^2}{8\pi}e^{-mr}. \tag{11}$$

Here we assume this type of mass function $m_e(p)$.

$$\langle\overline{\psi}\psi\rangle = -2\int\frac{d^3k}{(2\pi)^3}\left(\frac{B_+(k)}{A_+^2(k)k^2+B_+^2(k)}+\frac{B_+(k)}{A_+^2(k)k^2+B_+^2(k)}\right), \tag{12}$$

$$\langle\overline{\psi}\tau\psi\rangle = -2\int\frac{d^3k}{(2\pi)^3}\left(\frac{B_+(k)}{A_+^2(k)k^2+B_+^2(k)}-\frac{B_-(k)}{A_-^2(k)k^2+B_-^2(k)}\right). \tag{13}$$

○Absence of screening correction in our approximation for small μ where $B_+, B_- \geq 0$.

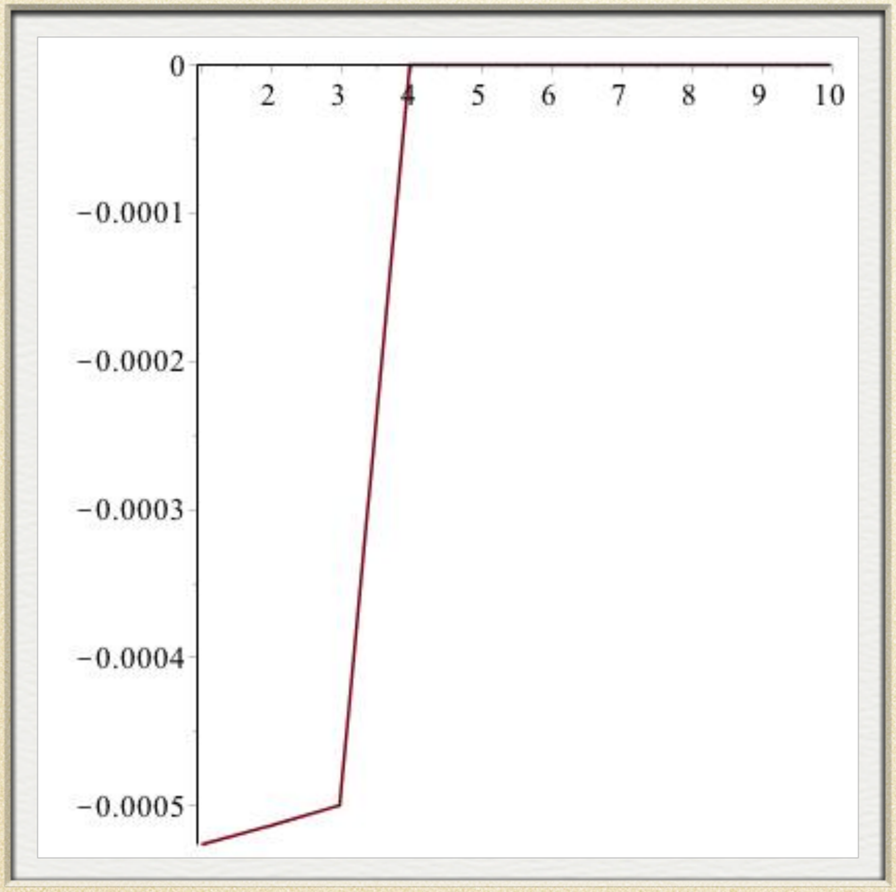
○For large μ where $B_+B_- \leq 0$,$\Pi_{\mu\nu}(k)$ induces Chern-Simon correction $\Delta\mu \geq 0$.

○In weak coupling $\mu_c \simeq .01e^2$ for quenched case.Chiral symmetry is restored for the same region of μ in unquenched case with screening correction.$\mu_c$may be very small.for unqenched case as $10^{-6}e^2$. Infrared mass vanishes at μ_c.zero momentum mass is sensitive by solving Dyson equation.

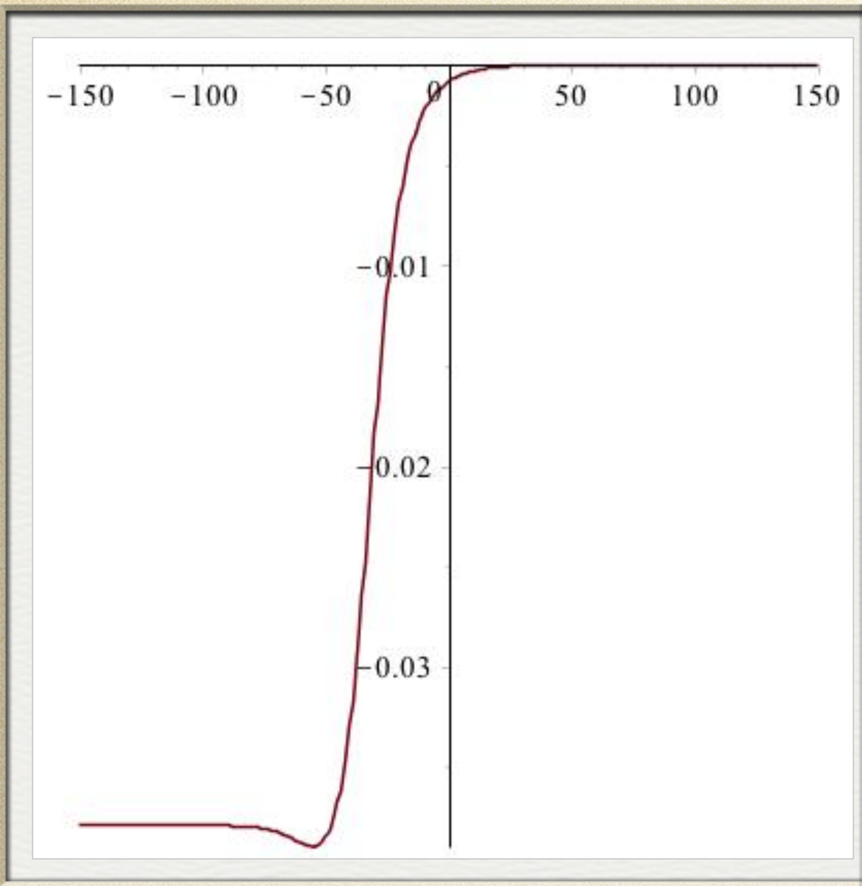
○if $\mu/m_e \simeq 0.1$,$m_e \to 0$ by iteration.or (effective potential for m_e and m_o)

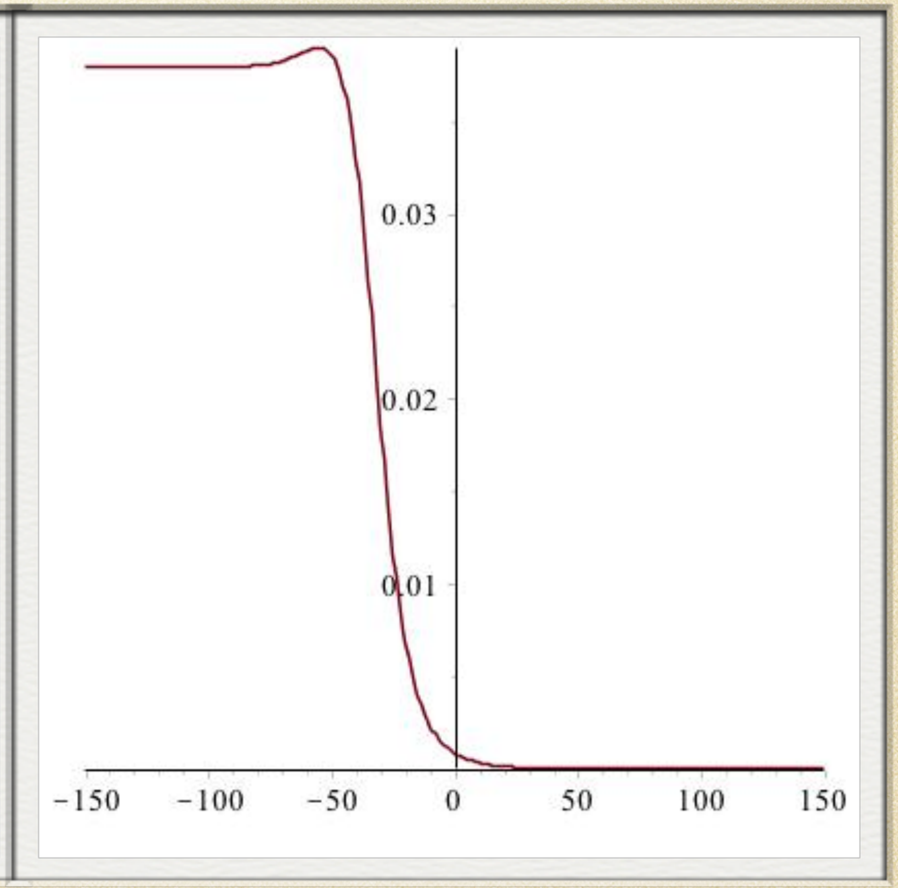
Shortdistance origin :zero anomalous dimension for $\mu \geq \mu_c$.
Anomalous dimension $D = e^2/8\pi m - e^2/32\pi\mu = 0$ for 2-component fermion.by deriving electron spectral function.$e^2/8\pi m = 1, e^2/32\pi\mu = 1 \to \mu_c = e^2/32\pi = .01e^2$.

vev..008+(h-1).0005



B(+) at critical myu B(-)





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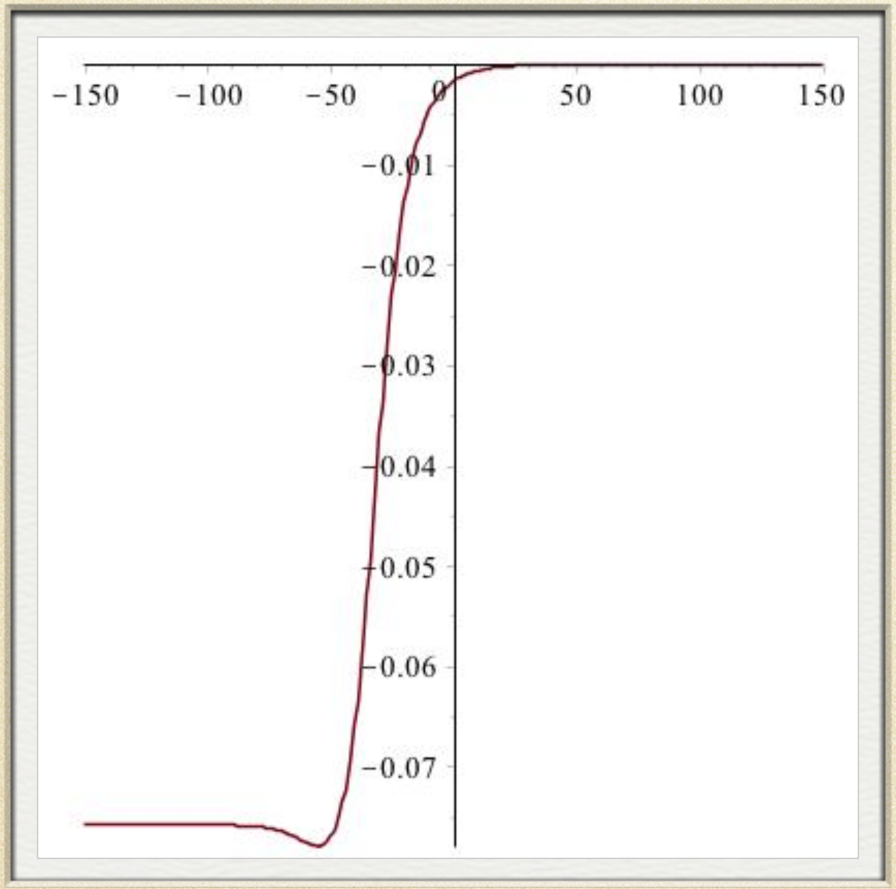

3 Phase fluctuation by massless field(Goldstone)

uniform vev (order parameter)→distribution by superfluidty

$$\begin{aligned} \psi(r) &= \psi_0 e^{i\phi(r)}, \quad (14) \\ F &= \frac{1}{2}\int d^2 r K_0 |\nabla\phi|^2, \\ v_s &\propto \nabla\phi \quad (15) \end{aligned}$$

$$\begin{aligned} S_{ik} &\to -i\left\langle 0|T\psi_i(x)e^{i\phi(x)}e^{-i\phi(y)}\overline{\psi}_k(y)|0\right\rangle \\ &= S_{ik}\left\langle 0|Te^{i\phi(x)}e^{-i\phi(y)}|0\right\rangle \quad (16) \end{aligned}$$

We expand the exponentail factor in powers of ϕ as far as quadratic terms:

$$\begin{aligned} &\left\langle 0|Te^{i\delta\phi(x)}e^{-i\delta\phi(y)}|0\right\rangle \\ &\simeq -\frac{1}{2}\left\langle 0|\delta\phi(x)^2 + \delta\phi(y)^2 - 2T\delta\phi(x)\delta\phi(y)|0\right\rangle \quad (17) \end{aligned}$$

Propagator changes as the gauge transformation

$$
\begin{aligned}
S &\rightarrow S + \delta S, \delta S = S(x-y)(D(x-y) - D(0)) \\
S(x-y)' &= S(x-y)_0 \exp(D(x-y) - D(0)), \\
\Box D(x) &= 0.
\end{aligned} \tag{18}
$$

$$
\rho(r) = |\psi_0|^2 \left\langle e^{i(\phi(x)-\phi(0))} \right\rangle \tag{19}
$$

$$
= |\psi_0|^2 \left\langle \exp(-i \sum_k (e^{ik\cdot r} - 1)\phi_k \right\rangle \tag{20}
$$

$$
= |\psi_0|^2 \exp[-\sum_k \frac{k_B T}{K_0 k^2}\{1 - \cos(k \cdot r)\}] \tag{21}
$$

XY model:

$$
\begin{aligned}
\rho(x) &= |\psi_0|^2 \exp(-\frac{k_B T}{2\pi K_0} \ln(\frac{r}{a})) \\
&\propto (\frac{r}{a})^{-\eta}
\end{aligned} \tag{22}
$$

3.1 T dependence of vev

2-dimension: vanishing vev by infrared divergence

$$|\psi_0|^2 \rightarrow |\psi_0|^2 e^{-D(0)/c^2} \tag{23}$$

$D(x)$ is a massless propagator

$$vev = \psi_0 \left\langle 0|e^{i\phi(x)/c}|0\right\rangle = \psi_0 \exp(-\frac{1}{c^2}D(0)), \tag{24}$$

$$\begin{aligned} D(0) &= \left\langle 0|\phi(x)^2|0\right\rangle \\ &= \frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega_k}(on - shell \exp) \end{aligned} \tag{25}$$

$$D(0) = \int \frac{d^{n-1}k}{(2\pi)^{n-1}2\omega_k}\frac{1}{\exp(\beta\omega_k) - 1} \tag{26}$$

At high T

$$D(0) = \int_\mu \frac{d^2k}{(2\pi)^2 2k^2}k_BT = \frac{k_BT}{8\pi}\ln(\frac{k^2}{\mu^2}) \tag{27}$$

$$vev = \psi_0 \exp(-\frac{k_BT}{8\pi c^2}\ln(\frac{k^2}{\mu^2})) \rightarrow 0 \tag{28}$$

At low T

$$D(0) = \int \frac{d^2k}{(2\pi)^2 2k} \exp(-\frac{k}{k_B T}) = \frac{k_B T}{8\pi} \quad (29)$$

$$vev = \psi_0 \exp(-\frac{k_B T}{8\pi c^2}) \quad (30)$$

4 Finite temperature superconductivity

At finite temperature m_e disappear without chiral symmetry breaking.
$\circ m_o$ may survives at finite temperature.Phonon attraction and weak Coulomb repulsive force may be attractive for two electron system.

5 Summary

For one particle problem as mass and condensation effects of Chern-Simon term was examined.
There is a first order phase transion at critical value of topological mass μ_c above which chiral order parameter vanishes and parity odd mass survives.At high temperature chiral order parameter vanishes by phase flucutuation.
Future problem:Parity odd mass may survive at high temperature and candidate for super conductor.