Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

# Heavy quark master equations in the Lindblad form

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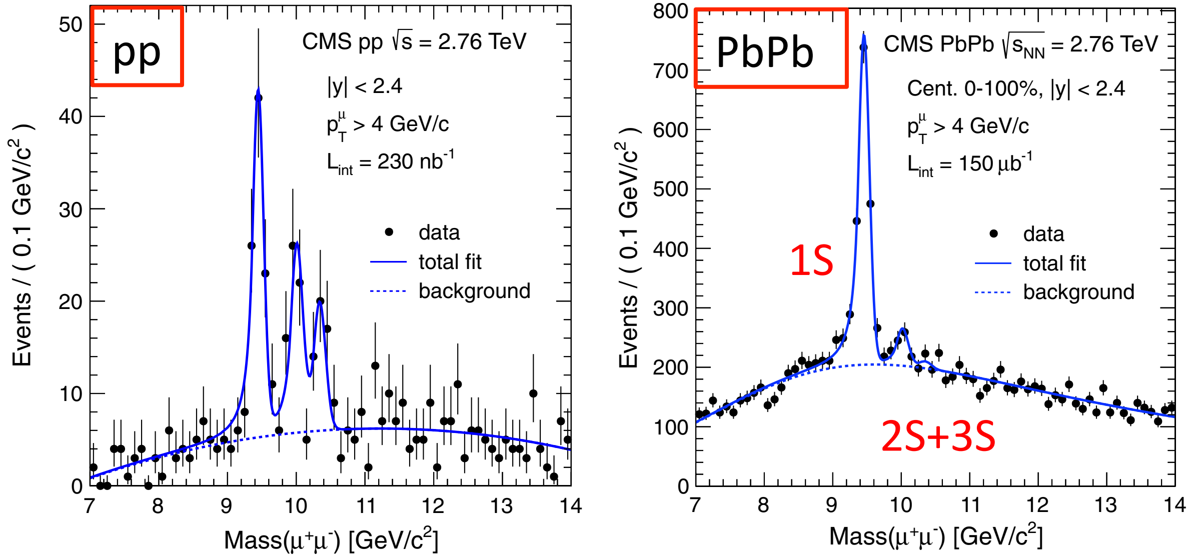
Refs:
Y.A., arXiv:1403.5783 [hep-ph],
Y.A., PRD87(2013),045016 [arXiv:1209.5068 [hep-ph]]
Y.A. and A.Rothkopf, PRD85(2012),105011 [arXiv:1110.1203 [hep-ph]]

# Contents

# 1. Introduction

- 重イオン衝突でQGPが生成されたら？
  – カラー荷が遮蔽されJ/ΨやΥの収量が減少　Matsui, Satz (86)

CMS@LHC




  – Open HQが多くなると逆に増加も(regeneration)

# 1. Introduction

- 環境(QGP)と相互作用するクォーコニウム
  - 量子開放系 $|\mathcal{H}\rangle = |qA\rangle \otimes |Q\rangle$

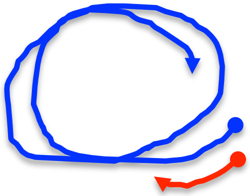
    $\rho_Q(t) \equiv \mathrm{Tr}_{qA}\left[\rho_{\mathrm{tot}}(t)\right]$ $\qquad \dot{\rho}_Q(t) = \mathcal{L}\left[\rho_Q(t)\right]$ $\qquad$ Markov極限

    - Lindblad形式:密度行列の正値性を保つ形 $\qquad$ Lindblad (76)

$$\dot{\rho}_Q(t) = -i\left[H, \rho_Q\right] + \sum_i \gamma_i \left( L_i \rho_Q L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho_Q - \frac{1}{2} \rho_Q L_i^\dagger L_i \right) \qquad (\gamma_i > 0)$$

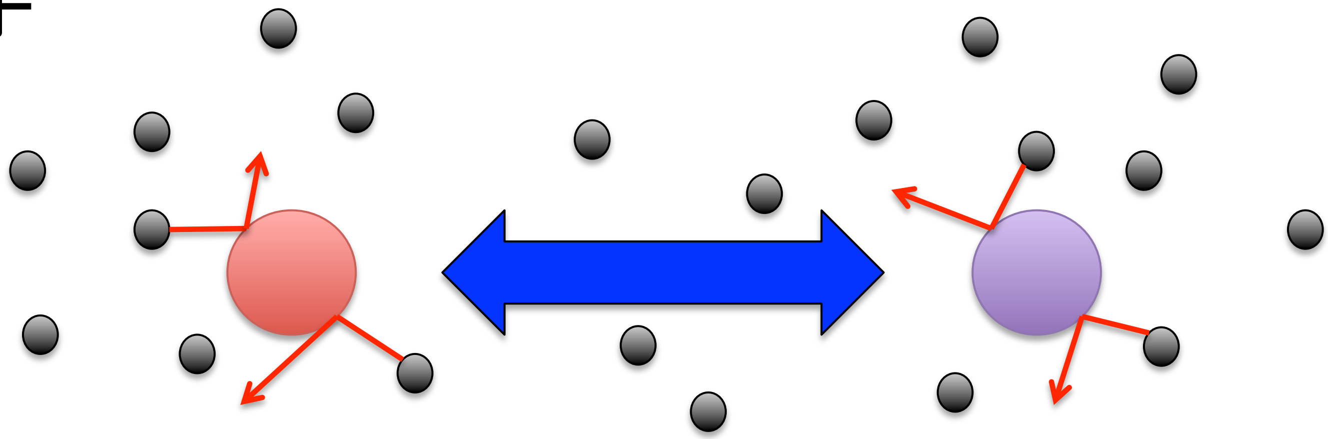
  - 時間スケールの階層性
    - 量子光学系:束縛状態|*n*>で記述
    - 量子Brown運動:波動関数*Ψ*(*x*)で記述

システムの緩和時間の間の
束縛状態の古典的軌道



# 1. Introduction

- 古典的描像
  - 遮蔽ポテンシャルで相互作用する2つのBrown粒子



→対応する量子論的記述

# 2. Influence functional for the Lindblad form

- 影響汎関数 $F[x,y]$
  - 経路積分形式の量子開放系の記述　Feynman and Vernon (63)

$$S[x,R]=\int_0^t d\tau L(x,R)=S_{\mathrm{A}}[x]+S_{\mathrm{I}}[x,R]+S_{\mathrm{B}}[R]$$

A:システム、B:環境

密度行列

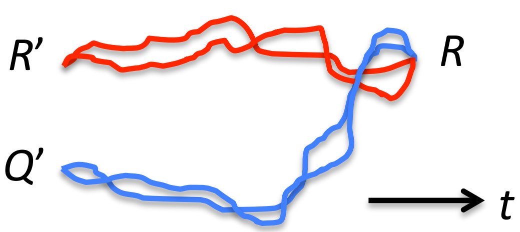
$$\rho_{\mathrm{red}}(t,x,y)=\int dx'dy'J(t,x,y;0,x',y')\rho_{\mathrm{sys}}(0,x',y')$$

システムの初期条件

プロパゲータ

$$J(t,x,y;0,x',y')=\int_{x',y'}^{x,y} D\tilde{x}D\tilde{y}\exp\left[\frac{i}{\hbar}\left(S_{\mathrm{A}}[\tilde{x}]-S_{\mathrm{A}}[\tilde{y}]\right)\right]F[\tilde{x},\tilde{y}]$$

x,yのそれぞれの軌道について




$$F[x,y]=\int dR'dQ'dR\rho_{\mathrm{B}}(0,R',Q')$$

環境の初期条件

$$\times\int_{R',Q'}^{R,R} D[\tilde{R},\tilde{Q}]\exp\left[\frac{i}{\hbar}\left(S_{\mathrm{B}}[x]+S_{\mathrm{I}}[x,\tilde{R}]-S_{\mathrm{B}}[y]-S_{\mathrm{I}}[y,\tilde{Q}]\right)\right]$$

影響汎関数

# 2. Influence functional for the Lindblad form

- 量子Brown運動：Caldeira-Leggett模型

Caldeira and Leggett (83)

$\alpha$：環境系の2点関数

$$F[x,y]=\exp\left[\begin{array}{l}-\dfrac{i}{\hbar}\displaystyle\int_0^t d\tau\int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{I}}(\tau-s)\left[x(s)+y(s)\right]\\ -\dfrac{1}{\hbar}\displaystyle\int_0^t d\tau\int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\mathrm{R}}(\tau-s)\left[x(s)-y(s)\right]\end{array}\right]$$

時間の粗視化

$$\cong\exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau(x\dot{x}-y\dot{y}+x\dot{y}-y\dot{x})-\frac{\eta k_{\mathrm{B}}T}{\hbar^2}\int_0^t d\tau(x-y)^2\right]$$

- Lindblad形式にするには？

時間微分展開の2次まで

$$F[x,y]=\exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau(x\dot{x}-y\dot{y}+x\dot{y}-y\dot{x})-\frac{\eta k_{\mathrm{B}}T}{\hbar^2}\int_0^t d\tau(x-y)^2+\frac{\hbar^2(\dot{x}-\dot{y})^2}{12(k_{\mathrm{B}}T)^2}\right]$$

Diosi (93)

# 2. Influence functional for the Lindblad form

- QGP中の重クォーク系の場合
  - CL模型の$x$⇔カラー密度$\rho^a$

時間スケールの階層性
$1/gT << 1/M\alpha^2$

$$S^{\mathrm{IF}}[\rho_1,\rho_2] \cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} V+iD & -iD \\ -iD & -V+iD \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \rho_1^a \\ \rho_2^a \end{pmatrix}_{(t,\vec{y})}$$

遮蔽ポテンシャルと熱揺らぎ（デコヒーレンス）

Lindblad form

近似
- 摂動展開
- Velocity展開
- 時間粗視化

$$-\frac{1}{4T}\int_{t,\vec{x},\vec{y}} \left(\rho_1^a, \rho_2^a\right)_{(t,\vec{x})} \begin{bmatrix} -D & -D \\ D & D \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})}$$

運動量の拡散

$$+\frac{i}{24T^2}\int_{t,\vec{x},\vec{y}} \left(\dot{\rho}_1^a, \dot{\rho}_2^a\right)_{(t,\vec{x})} \begin{bmatrix} -D & D \\ D & -D \end{bmatrix}_{(\vec{x}-\vec{y})} \begin{pmatrix} \dot{\rho}_1^a \\ \dot{\rho}_2^a \end{pmatrix}_{(t,\vec{y})}$$

運動量の拡散の補正

Lindblad form

# 3. Stochastic potential with color

- 遮蔽ポテンシャルと熱揺らぎ（Recoilless limit）

$$S^{\mathrm{IF}}[\rho_1,\rho_2]\cong -\frac{1}{2}\int_{t,\vec{x},\vec{y}}\left(\rho_1^a,\rho_2^a\right)_{(t,\vec{x})}\begin{bmatrix} V+iD & -iD \\ -iD & -V+iD \end{bmatrix}_{(\vec{x}-\vec{y})}\begin{pmatrix}\rho_1^a \\ \rho_2^a\end{pmatrix}_{(t,\vec{y})}$$

↓ 確率的変数（熱揺らぎ）を使って書き換える

$$e^{iS_{\mathrm{IF}}} = \exp\left[-\frac{i}{2}\int_{t,\vec{x},\vec{y}} \boxed{V(\vec{x}-\vec{y})}\,\rho_1^a(t,\vec{x})\rho_2^a(t,\vec{y})\right]$$

確率ポテンシャル

$$\times\left\langle \exp\left[-i\int_{t,\vec{x}} \boxed{\xi^a(t,\vec{x})}\left(\rho_1^a(t,\vec{x})-\rho_2^a(t,\vec{x})\right)\right]\right\rangle_{\xi}$$

$$\left\langle \xi^a(t,\vec{x})\xi^b(s,\vec{y})\right\rangle = -D(\vec{x}-\vec{y})\delta^{ab}\delta(t-s)$$

（$D$:負定値）

# 3. Stochastic potential with color

- 確率ポテンシャル中の量子力学
  - 確率的Schrödinger方程式

$$i\frac{\partial}{\partial t}\Psi(t,\vec{r})=\begin{bmatrix}-\frac{\nabla_r^2}{M}+iC_{\mathrm{F}}D(0)+\left(V(r)+iD(r)/2\right)\left[t^a\otimes(-t^{*a})\right]\\+\xi^a(t,\vec{r}/2)\left[t^a\otimes 1\right]+\xi^a(t,-\vec{r}/2)\left[1\otimes(-t^{*a})\right]\end{bmatrix}\Psi(t,\vec{r})$$

$(\Psi: 3\otimes 3^*,\ \ \vec{r}=\vec{x}-\vec{y})$

- 熱揺らぎによるカラー回転 : singlet ⇄ octet
- 典型的な熱揺らぎの空間スケール : $l_{\mathrm{fluct}}\sim 1/gT$
- 束縛状態のサイズ : $l_{\mathrm{coh}}$ (コヒーレンス長)

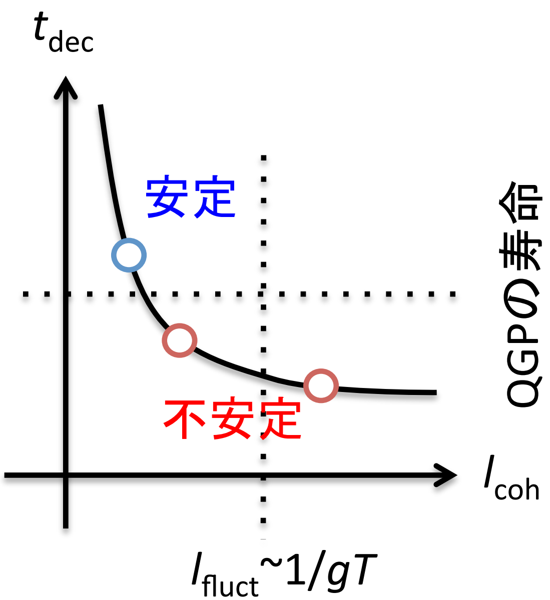
# 3. Stochastic potential with color

- カラー射影密度行列

$$\rho_{1,8}(t,\vec{r},\vec{s}) \equiv \mathrm{Tr}_{\mathrm{color}}\left[ P_{\substack{\mathrm{singlet,}\\ \mathrm{octet}}} \left\langle \Psi(t,\vec{r})\Psi^*(t,\vec{s}) \right\rangle_\xi \right]$$

$$\frac{\partial}{\partial t}\rho_{1,8}(t,\vec{r},\vec{s}) = \cdots$$

  – デコヒーレンスの時間スケール
    - $l_{\mathrm{coh}} << l_{\mathrm{fluct}}$のとき、$t_{\mathrm{dec}} \sim 1/g^4T^3l_{\mathrm{coh}}{}^2$
    - $l_{\mathrm{coh}} >> l_{\mathrm{fluct}}$のとき、$t_{\mathrm{dec}} \sim 1/g^2T$
      （Soft scatteringの時間スケール）



# 4. Summary & outlook

- 量子開放系としてのQGP中のクォーコニウム：影響汎関数、Lindblad形式
- 有限温度のポテンシャル描像には、遮蔽以外に熱揺らぎも必要（確率ポテンシャル）
- デコヒーレンスの時間スケールからクォーコニウムの安定性を議論

- 現象論的応用：流体発展＋クォーコニウム

*Backup slides*

# Subtle issues (personal views)

- Time ordering in coarse graining

Later Earlier

$$-\frac{i}{\hbar}\int_0^t d\tau\int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\rm I}(\tau-s)\left[x(s)+y(s)\right]\cong-\frac{i\eta}{2\hbar}\int_0^t d\tau(x\dot{x}-y\dot{y}+x\dot{y}-y\dot{x})$$

Later Earlier

$$-\frac{1}{\hbar}\int_0^t d\tau\int_0^\tau ds\left[x(\tau)-y(\tau)\right]\alpha_{\rm R}(\tau-s)\left[x(s)-y(s)\right]\cong-\frac{\eta k_{\rm B}T}{\hbar^2}\int_0^t d\tau\left[(x-y)^2+\frac{\hbar^2(\dot{x}-\dot{y})^2}{12(k_{\rm B}T)^2}\right]$$

CL model Diosi's term

- Time differentiation in coarse graining
  - Should not be treated as one of the kinetic terms
  - If treated so, master equation does not change in CL model while it becomes different with Diosi's term.

# Subtle issues (personal views)

- Integration by parts?

$$F[x,y] = \exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau (x\dot{x} - y\dot{y} + x\dot{y} - y\dot{x}) - \frac{\eta k_B T}{\hbar^2}\int_0^t d\tau (x-y)^2\right]$$

$$\text{or}\quad \tilde{F}[x,y] = \exp\left[-\frac{i\eta}{2\hbar}\int_0^t d\tau (-\dot{x}x + \dot{y}y - \dot{x}y + \dot{y}x) - \frac{\eta k_B T}{\hbar^2}\int_0^t d\tau (x-y)^2\right]$$

$$\Rightarrow \tilde{\rho}_{\text{red}} = \exp\left[i\eta \hat{x}^2/2\right]\rho_{\text{red}}\exp\left[-i\eta\hat{x}^2/2\right]$$

  – Influence functional should be determined without ambiguity with total derivative terms.

# (partial) gauge invariance

- Gauge transformation (local only in time)

$$\because S_{\text{int}} = g\int d^4 x \rho_a(x) A_a^0(x)$$

  - Density matrix

$$\rho_Q(t,\vec{x},\vec{y}) \to \tilde{\rho}_Q(t,\vec{x},\vec{y}) = U(t)\rho_Q(t,\vec{x},\vec{y})U^{-1}(t)$$

$$\dot{\rho}_Q = \mathcal{L}\left[\rho_Q\right] \to \dot{\tilde{\rho}}_Q = U\mathcal{L}\left[\rho_Q\right]U^{-1} + \left(\dot{U}U^{-1}(t)\tilde{\rho}_Q + \tilde{\rho}_Q U\dot{U}^{-1}\right)$$

  - Physical observable (singlet)

$$\left\langle O_{\text{singlet}}\right\rangle_\rho (t) = \int d^3x d^3y \mathrm{Tr}_{\text{color}}\left[\rho_Q(t,\vec{x},\vec{y})\left\langle \vec{y}\left|O_{\text{singlet}}\right|\vec{x}\right\rangle\right]$$

$$\frac{d}{dt}\left[\left\langle O_{\text{singlet}}\right\rangle_\rho (t) - \left\langle O_{\text{singlet}}\right\rangle_{\tilde{\rho}} (t)\right] = 0 \quad \because \dot{U}(t)U^{-1}(t) + U(t)\dot{U}^{-1}(t) = 0$$

*U* and singlet observable *O* commute