

カノニカル法による格子QCD 有限密度相転移現象への挑戦

Yusuke Taniguchi (University of Tsukuba)
for
Zn Collaboration



R.Fukuda (Tokyo) A.Nakamura (Hiroshima) S.Oka (Rikkyo) S.Sakai (Kyoto) A.Suzuki (Tsukuba)

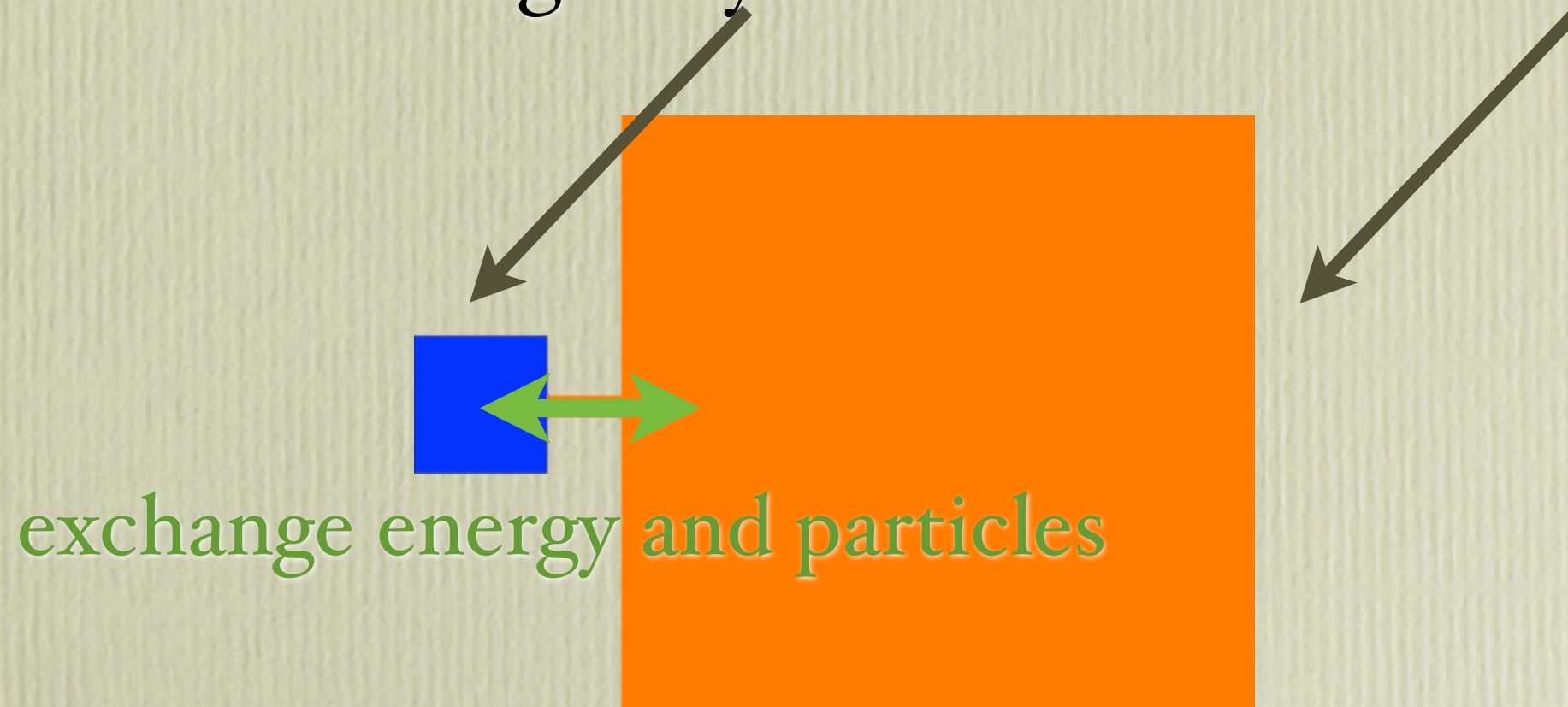
Introduction

Grand canonical ensemble

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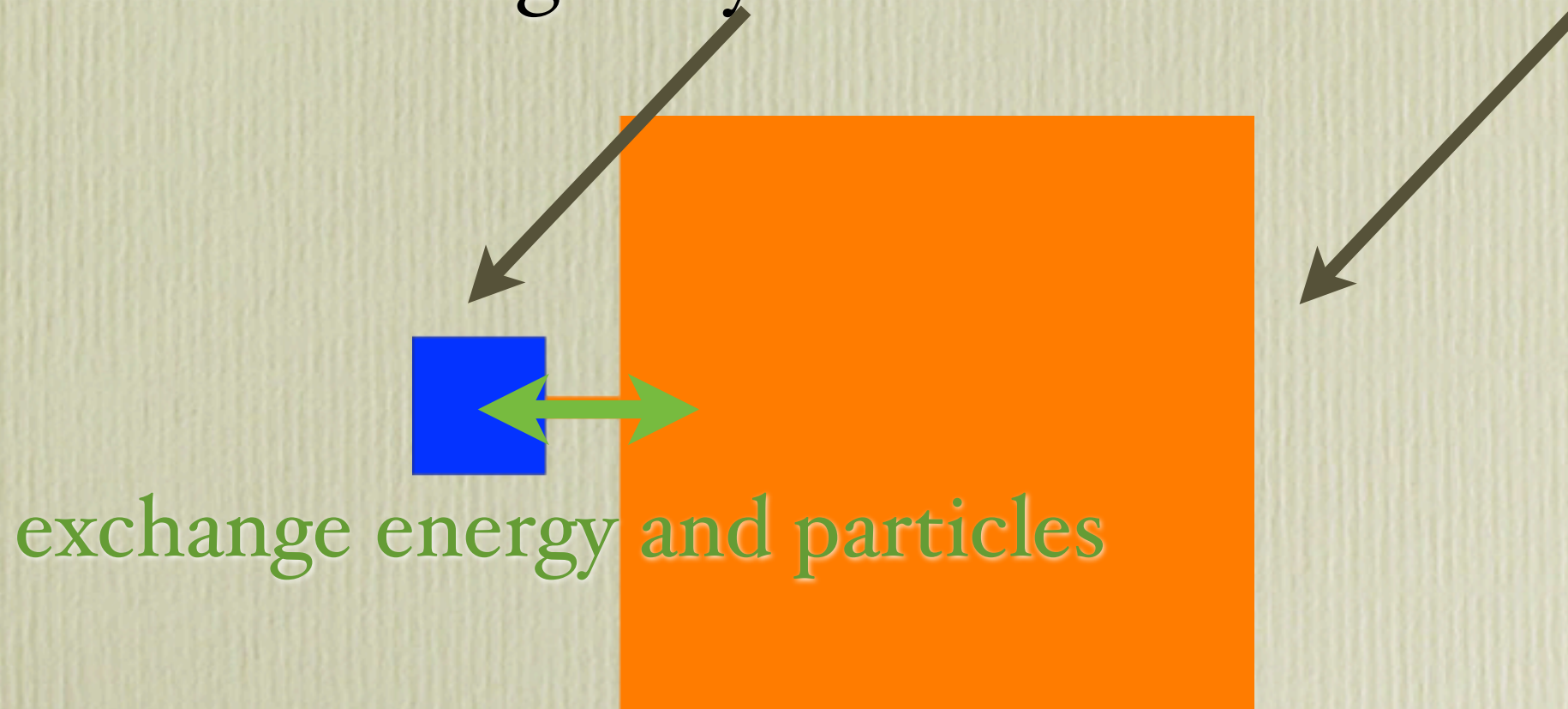
Introduced through a system attached to a reservoir



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Grand canonical ensemble

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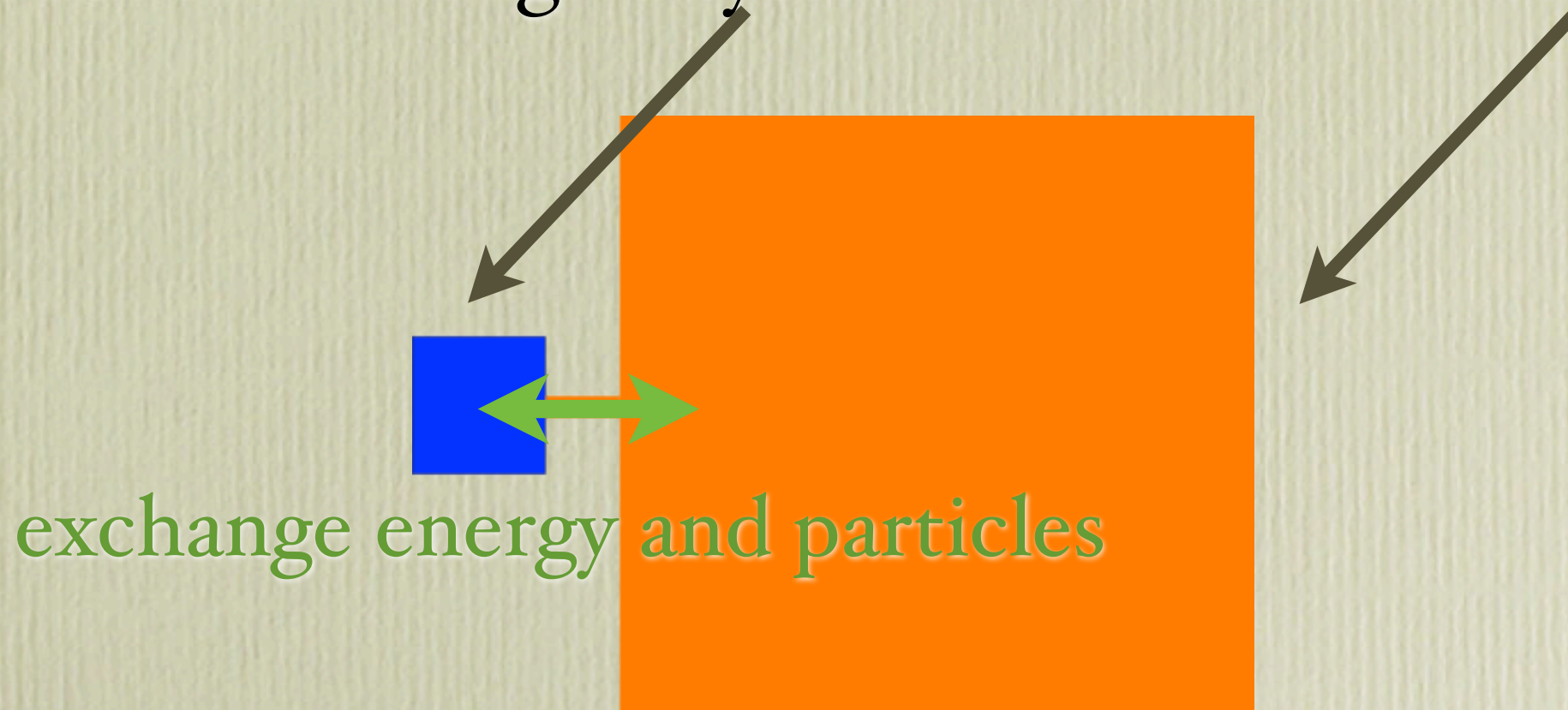


Treat total system as a micro canonical ensemble

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Grand canonical ensemble

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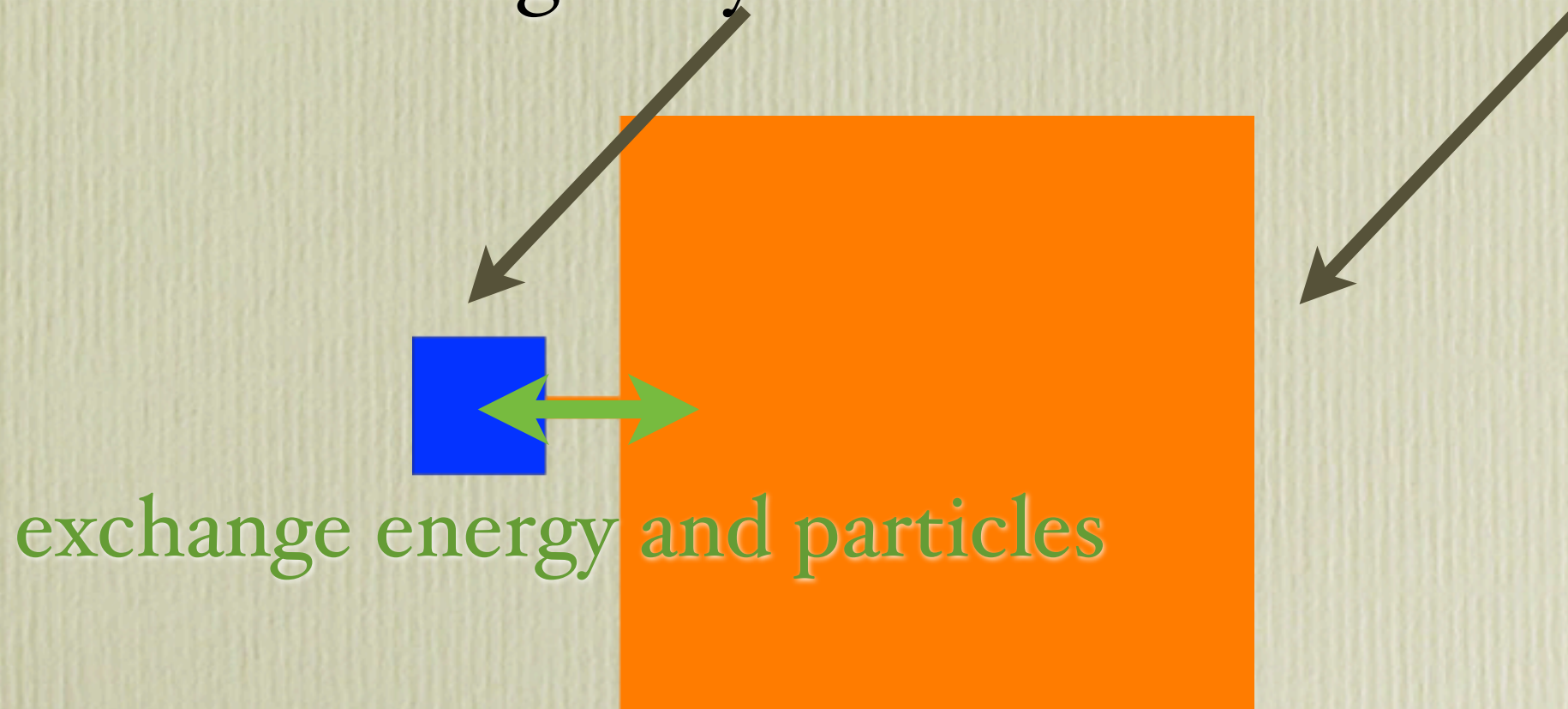


Treat total system as a micro canonical ensemble

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Introduced through a system attached to a reservoir



Treat total system as a micro canonical ensemble

Given in an abstract formulation

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$$Z_G(T, \mu, V) = \text{Tr} \left[\exp \left(-\frac{1}{T} (\hat{H} - \mu \hat{N}) \right) \right]$$

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for every energy and number of particles

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for every energy and number of particles

For QCD $[\hat{H}, \hat{N}] = 0$

Given in an abstract formulation

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$$\begin{aligned} Z_G(T, \mu, V) &= \text{Tr} \left[\exp \left(-\frac{1}{T} \left(\hat{H} - \mu \hat{N} \right) \right) \right] \\ &= \sum_n \sum_E \left\langle E, n \left| \exp \left(-\frac{\hat{H}}{T} + \frac{\mu}{T} n \right) \right| E, n \right\rangle \end{aligned}$$

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Fugacity expansion **Fugacity** $\xi = e^{\frac{\mu}{T}}$

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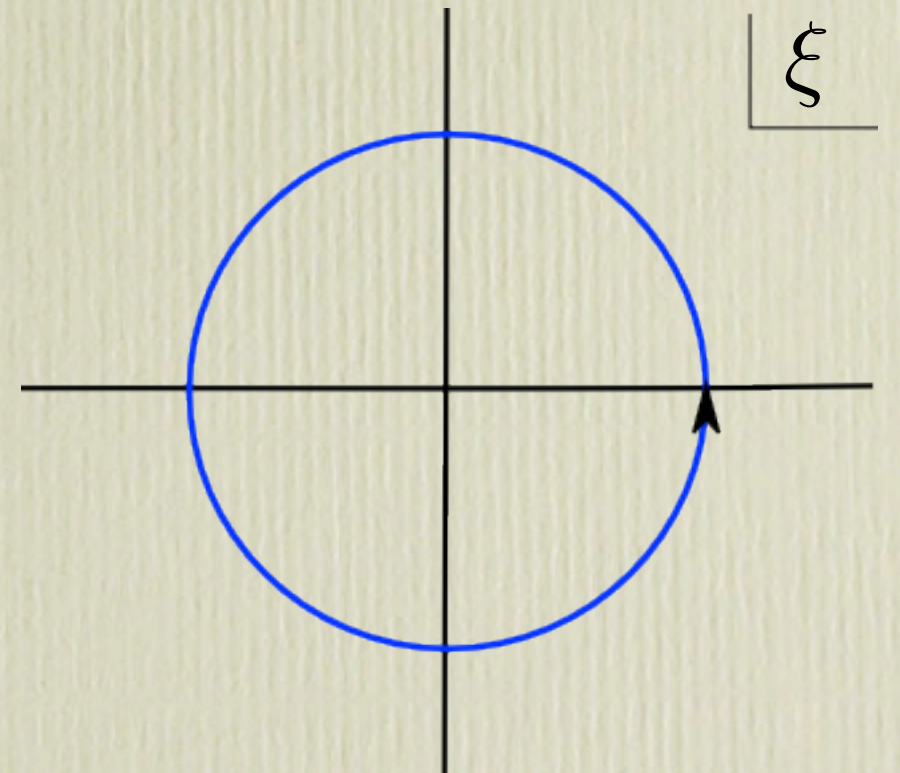
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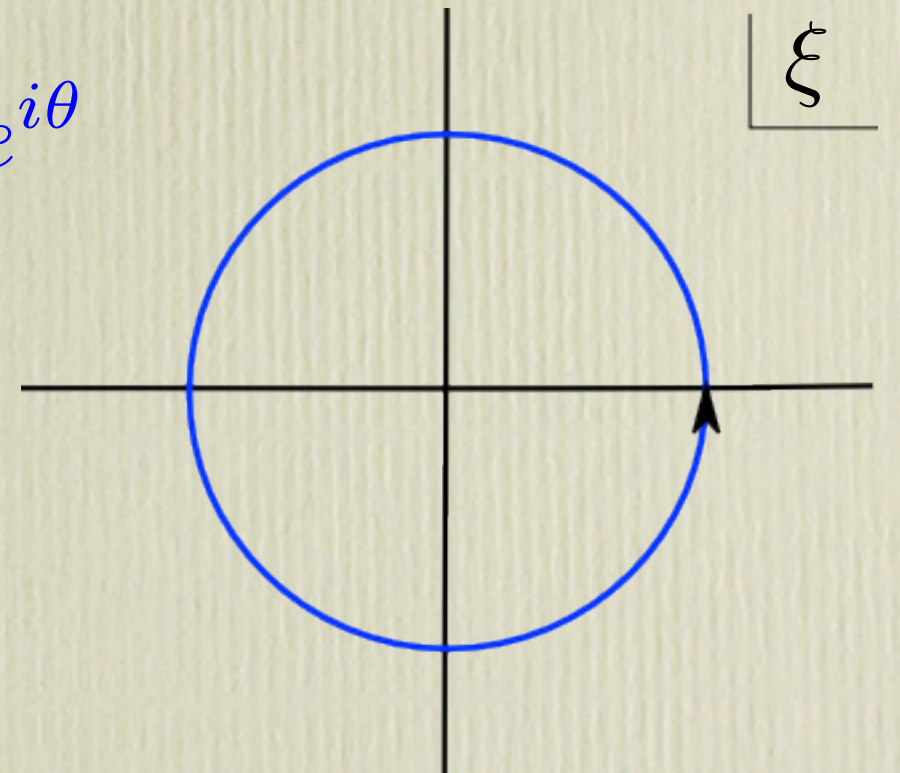
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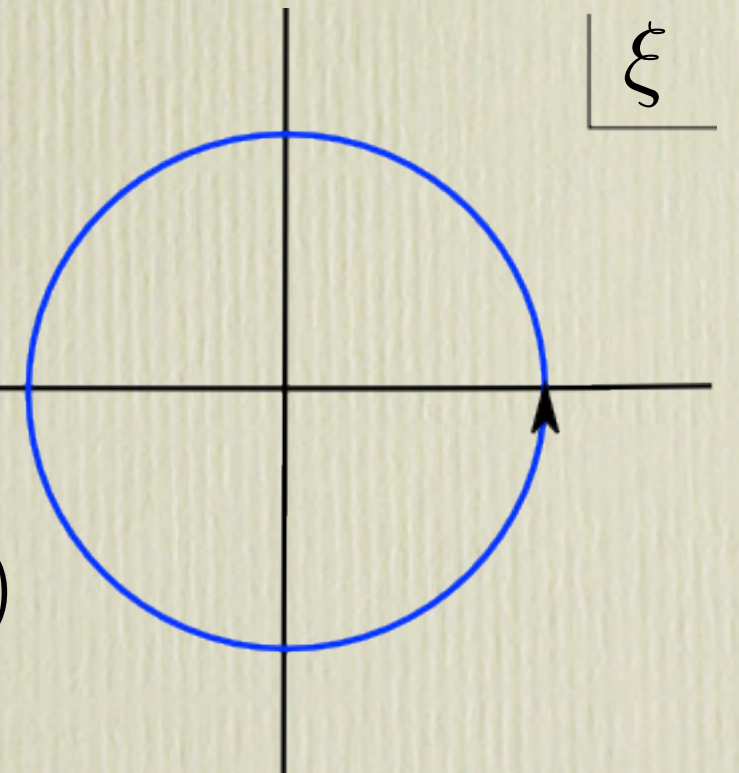
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Fourier tr. imaginary chemical potential!

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A. Hasenfratz and D. Toussaint

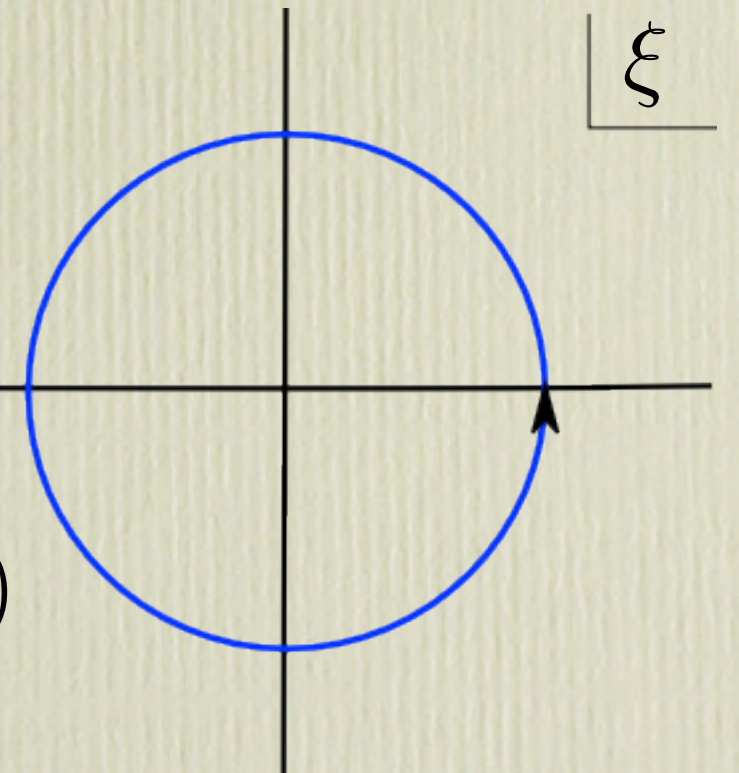
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Partition function $Z_G(T, e^{i\theta}, V)$ is numerically expensive

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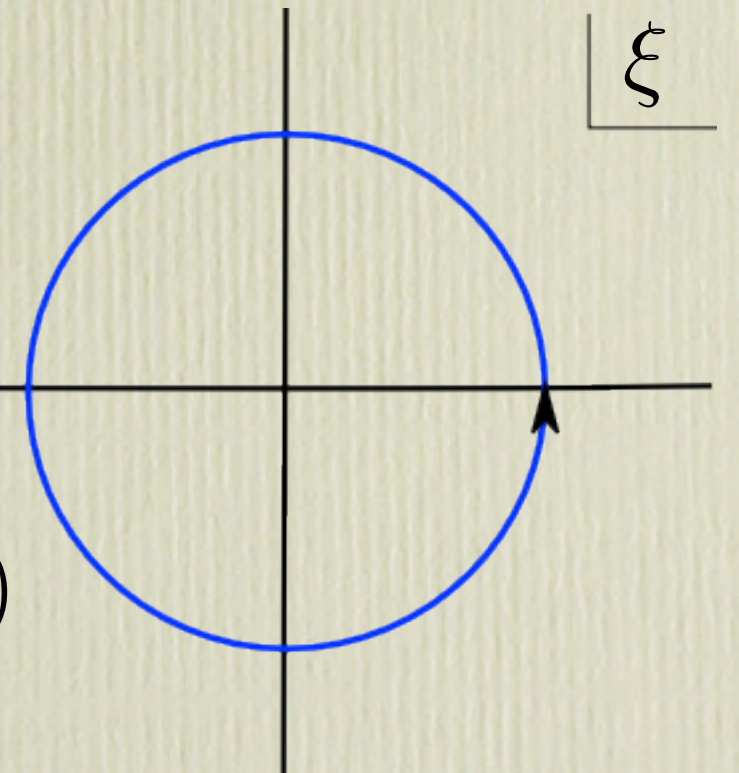
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Numerical Fourier transformation is difficult.

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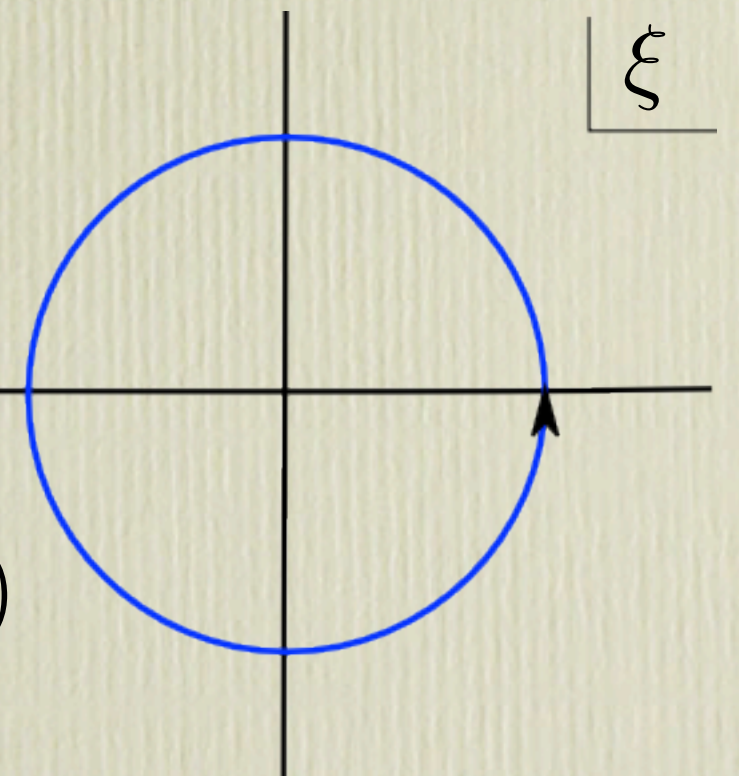
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Plan of the talk

- ✓ 1. Introduction
- 2. Winding number expansion
- 3. Numerical setup
- 4. Numerical results
- 5. Hadronic observables
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Hopping parameter expansion

Everyone need to evaluated $\text{Det } D(\mu)$ to get $Z_G(\mu)$

Direct evaluation is still expensive...

Hopping parameter expansion

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Direct evaluation is still expensive...

We want a cheaper method!

Hopping parameter expansion

Everyone need to evaluated Det $D(\mu)$ to get $Z_G(\mu)$

Chemical potential appears in temporal hopping

$$D_W(\mu) = 1 - \kappa Q_s + \kappa e^{\mu a} Q_4^+ + \kappa e^{-\mu a} Q_4^-$$

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$$\text{Log} (I - \kappa Q) = - \sum_n \frac{\kappa^n Q^n}{n} \quad \text{easy to expand}$$

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numerically stable Meng et al. (Kentucky)

Evaluation of $Z_c(n)$

Kentucky 'o8

Grand partition function $Z_G(\mu)$ ← re-weighting technique

Evaluation of $Z_c(n)$

Kentucky '08

Grand partition function $Z_G(\mu)$ ← re-weighting technique

$$Z_G(\mu) = \int DU \frac{\text{Det} D_W(\mu)}{\text{Det} D_W(\mu_0)} \text{Det} D_W(\mu_0) e^{-S_G}$$

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set to 0 or imaginary

$$= \left\langle \frac{\exp \left(\sum_{k=-\infty}^{\infty} W_k \xi^k \right)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

winding number exp.

Evaluation of $Z_C(n)$

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Grand partition function $Z_G(\mu)$ ← re-weighting technique

$$Z_G(\mu) = \int DU \frac{\text{Det} D_W(\mu)}{\text{Det} D_W(\mu_0)} \text{Det} D_W(\mu_0) e^{-S_G}$$

fugacity exp.

$$= \left\langle \frac{\text{Det} D_W(\mu)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

set to 0 or imaginary

winding number exp.

$$\sum_{n=-\infty}^{\infty} Z_C(n) \xi^n = \left\langle \frac{\exp \left(\sum_{k=-\infty}^{\infty} W_k \xi^k \right)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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Cauchy's integral theorem

$$Z_C(n) = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle \frac{\exp \left(\sum_{k=-\infty}^{\infty} W_k \xi^k \right)}{\text{Det} D_W(\mu_0)} \right\rangle_0$$

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 \end{aligned}$$

fugacity exp. (arrow from $Z_G(\mu)$ to $Z_C(n)$)
 set to 0 or imaginary (arrow from μ_0 to ξ)
 winding number exp. (arrow from $\sum_{k=-\infty}^{\infty} W_k \xi^k$ to ξ^n)

Fourier transformation

$$Z_C(n) = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-in\theta} \left\langle \frac{\exp \left(\sum_{k=-\infty}^{\infty} W_k e^{ik\theta} \right)}{\text{Det} D_W(\mu_0)} \right\rangle_0$$

Numerical setup

★ Iwasaki gauge action

★ Clover fermion $N_f=2$

• APE stout smeared gauge link $c_{SW} = 1.1$

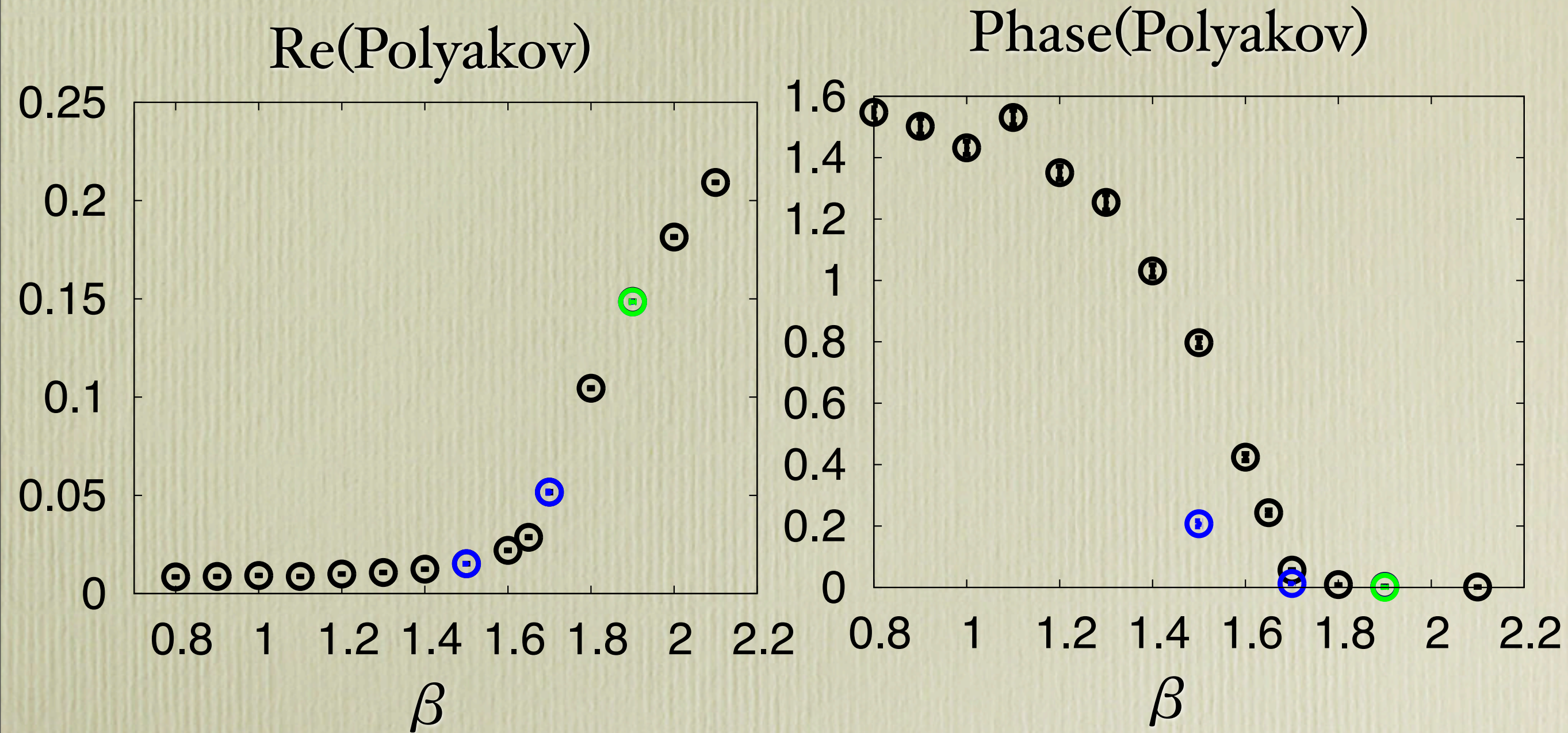
★ Box sizes $8^3 \times 4$ $12^3 \times 4$ $16^3 \times 4$

β	κ	PCAC	m_π/m_ρ
0.9	0.137	0.17(13)	0.8978(55)
1.1	0.133	0.18(19)	0.9038(56)
1.3	0.133	0.088(53)	0.8656(72)
1.5	0.131	0.116(39)	0.8482(57)
1.7	0.129	0.168(21)	1.0429(46)
1.9	0.125	0.1076(68)	0.7310(95)
2.1	0.122	0.1259(11)	0.833(12)

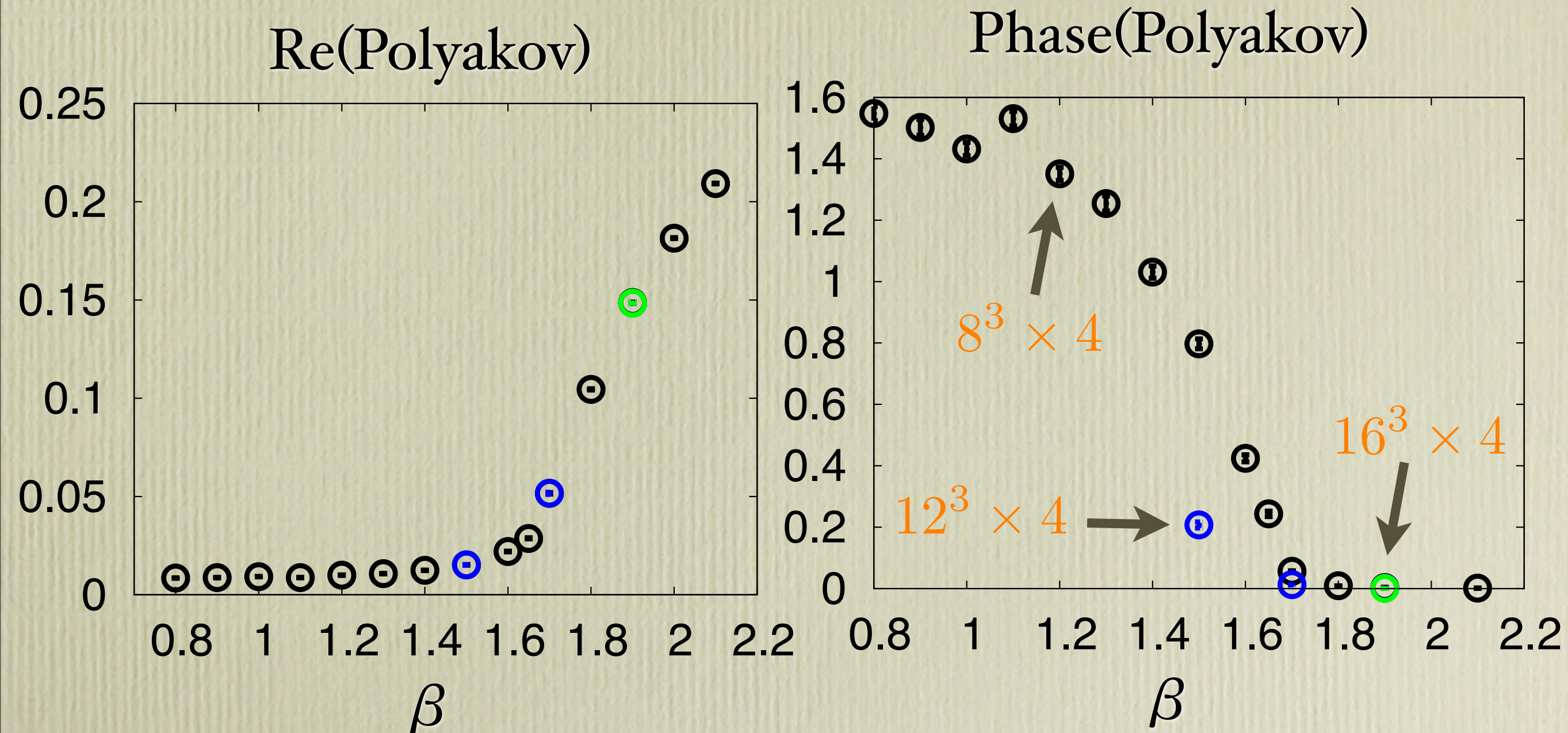
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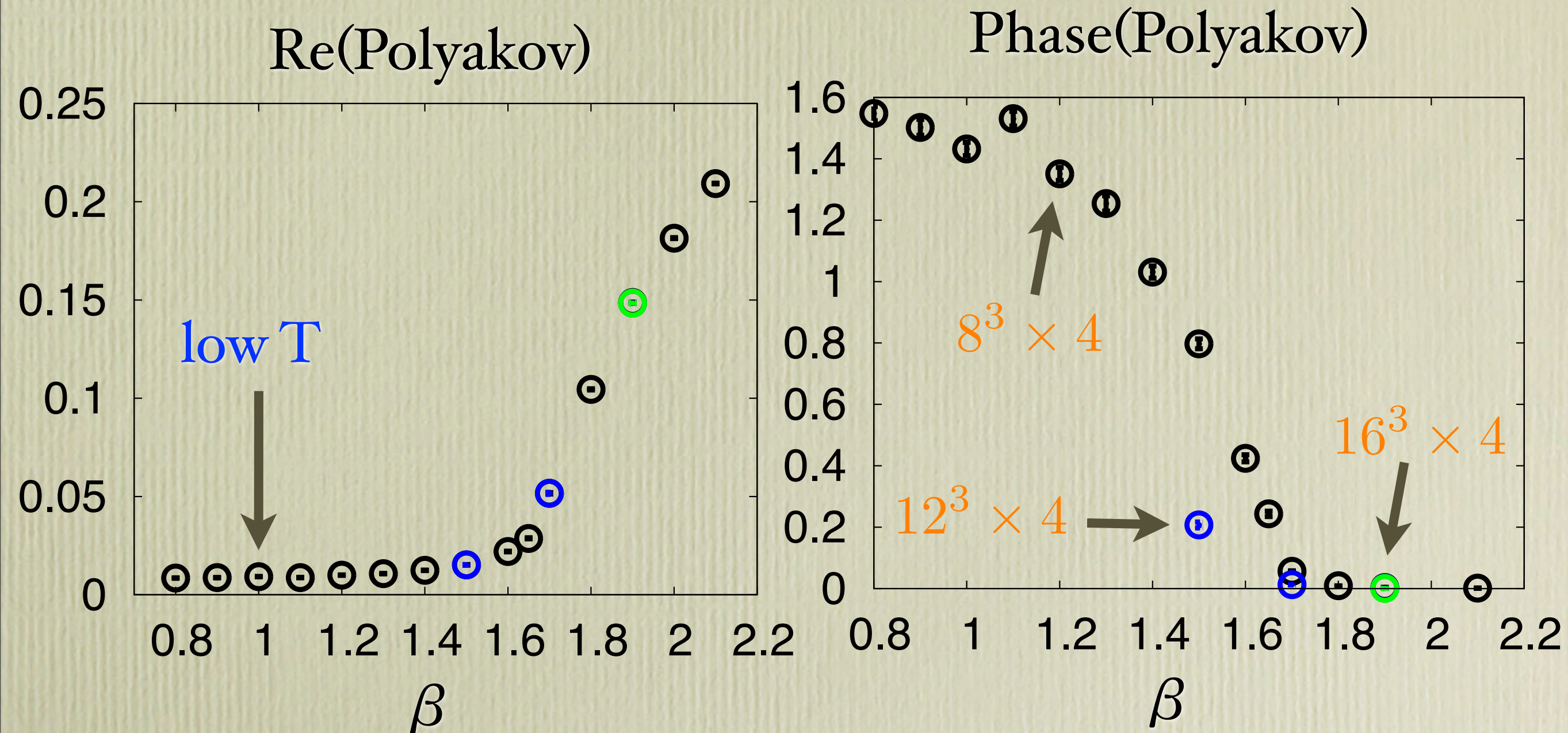
Numerical results (Polyakov loop)



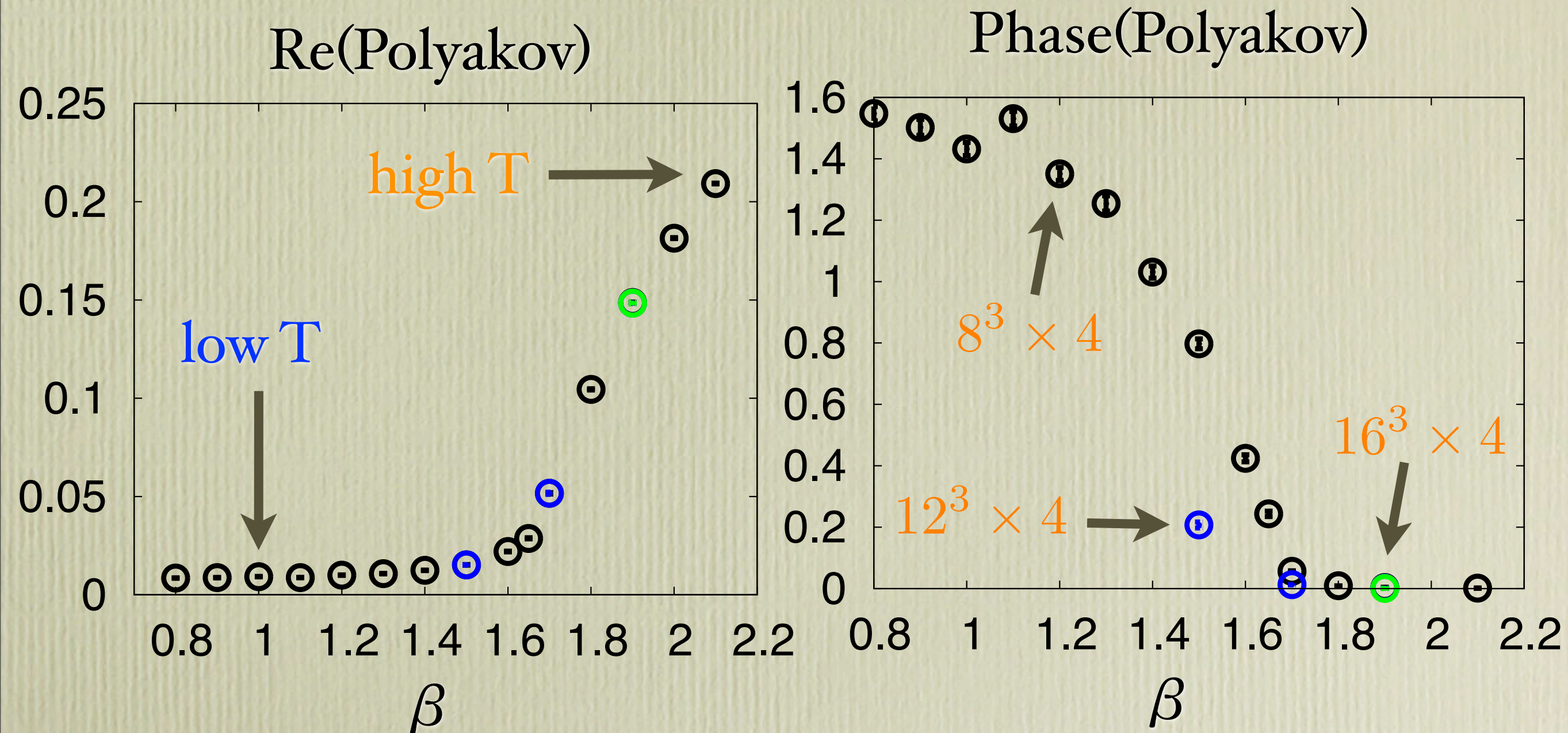
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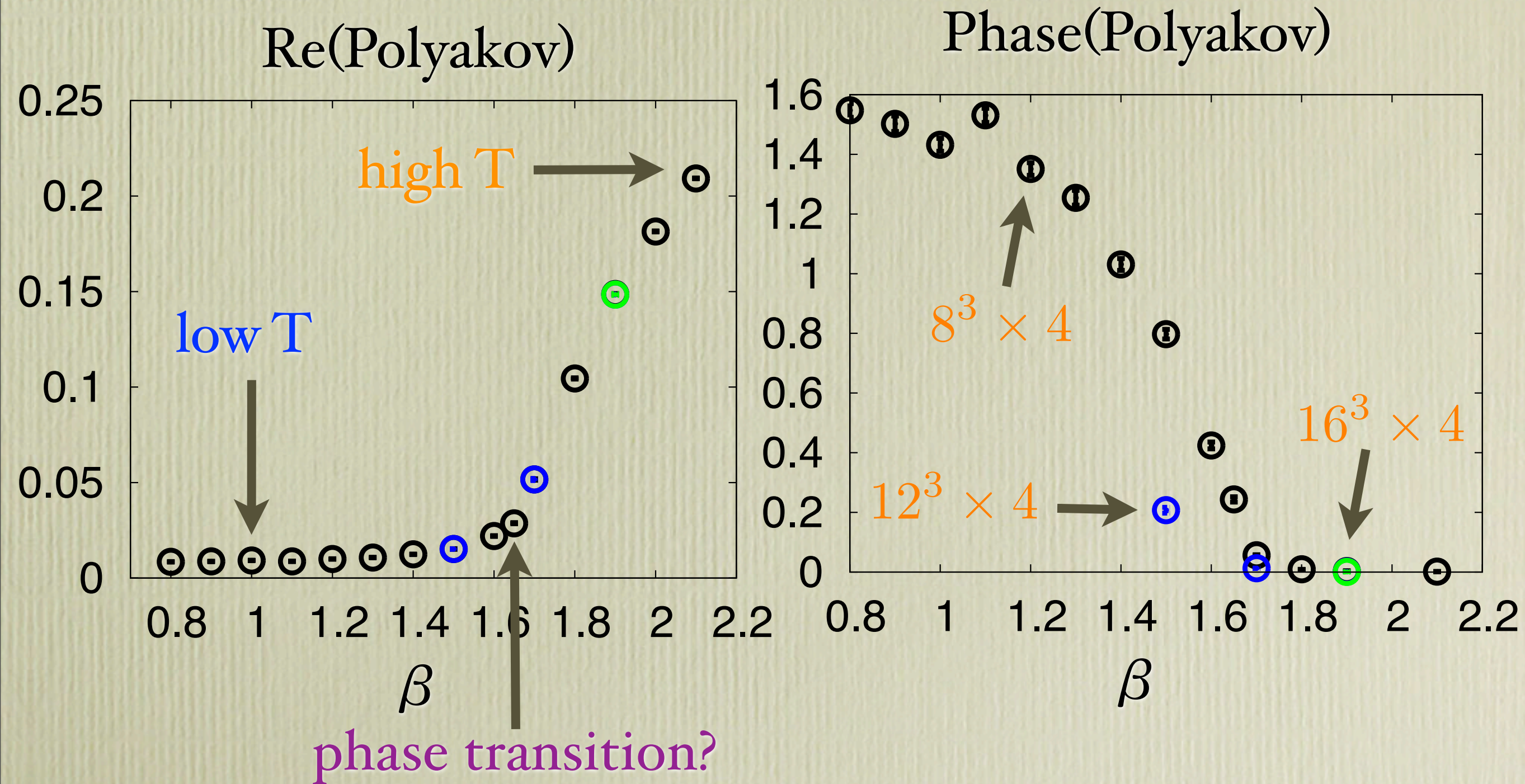
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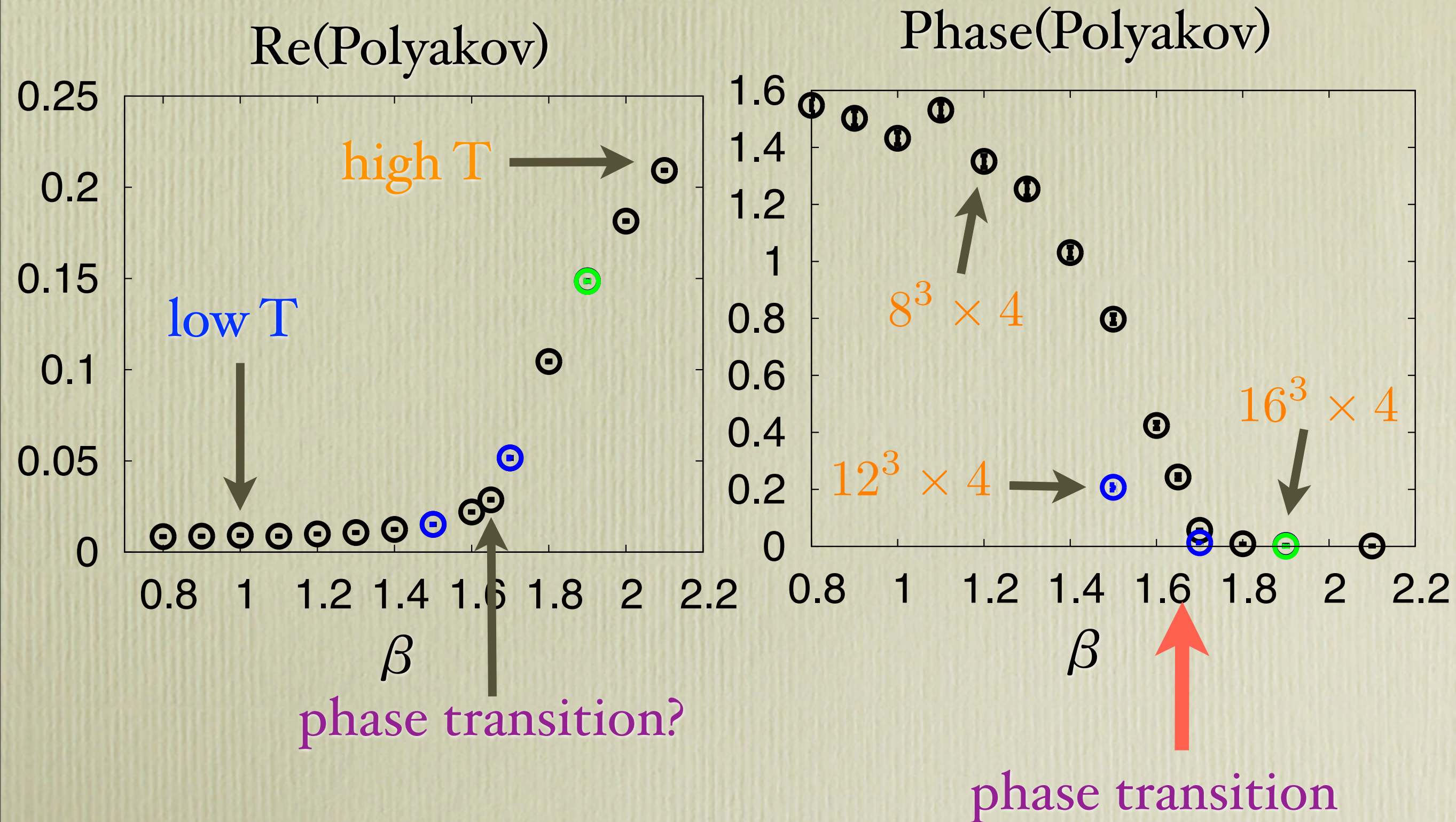
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Numerical results $Z_c(n)$

Before the main dish...

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Test of the hopping parameter expansion

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Numerical results $Z_c(n)$

Test of the hopping parameter expansion

- Where to truncate the expansion?

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We have a good benchmark!

Fugacity expansion by reduction formula

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for staggered

A. Hasenfratz and D. Toussaint

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A. Hasenfratz and D. Toussaint

K. Nagata and A. Nakamura

for Wilson

A. Alexandru and U. Wenger

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$$\text{Det} D_W(\mu)$$

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$$\text{Det} D_W(\mu) = C_0 \xi^{-N_R/2} \text{Det} (\xi + Q)$$

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Exact!

Computational cost is heavy...

Numerical results $Z_c(n)$

Canonical partition function measured on a single conf.

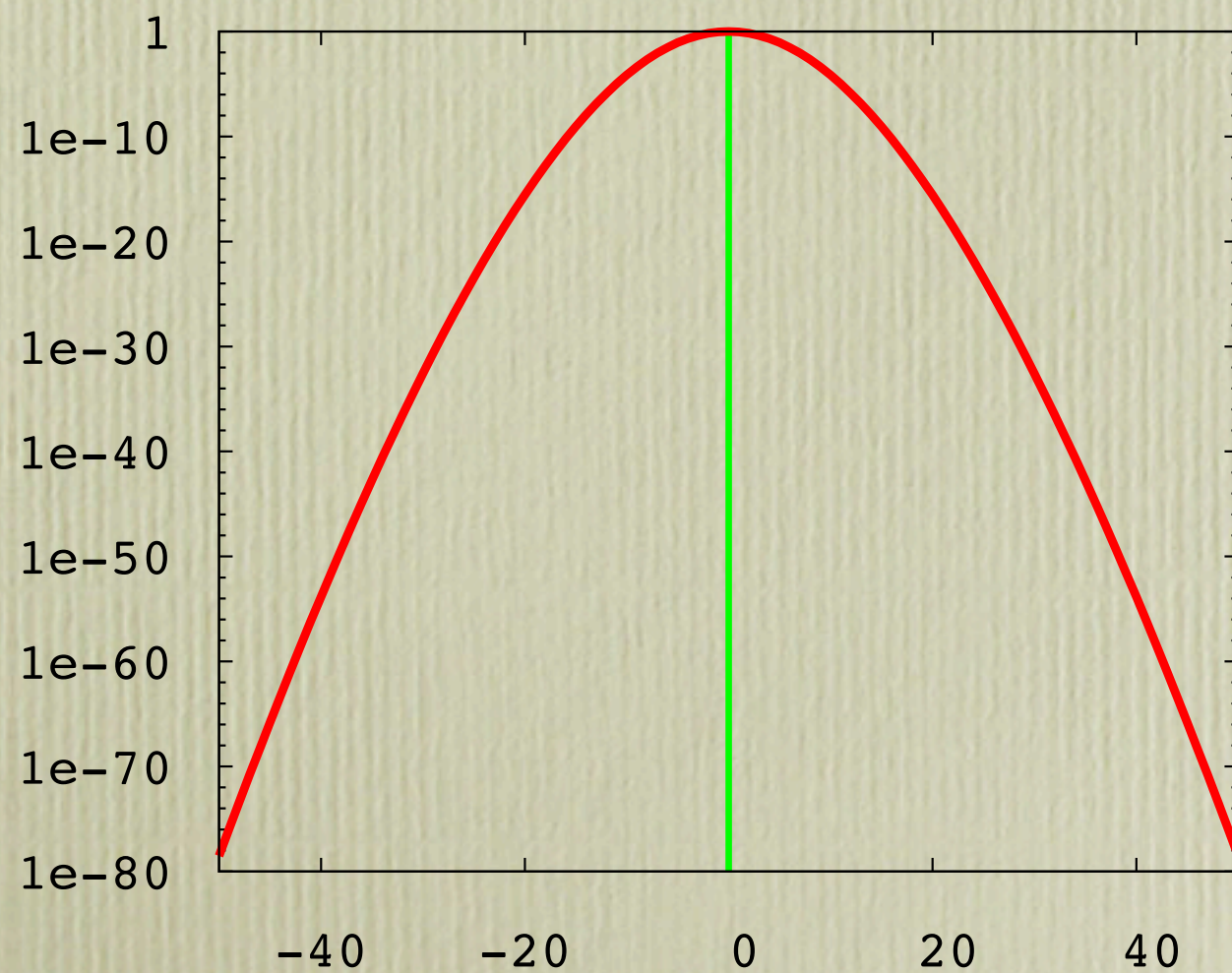
$$8^3 \times 4 \quad \beta = 1.9 \quad \kappa = 0.1250 \quad am_{\text{PCAC}} = 0.1076(68) \quad \mu = 0$$

Numerical results $Z_C(n)$

Canonical partition function measured on a single conf.

$$8^3 \times 4 \quad \beta = 1.9 \quad \kappa = 0.1250 \quad am_{\text{PCAC}} = 0.1076(68) \quad \mu = 0$$

$\log |Z_C(n)|$



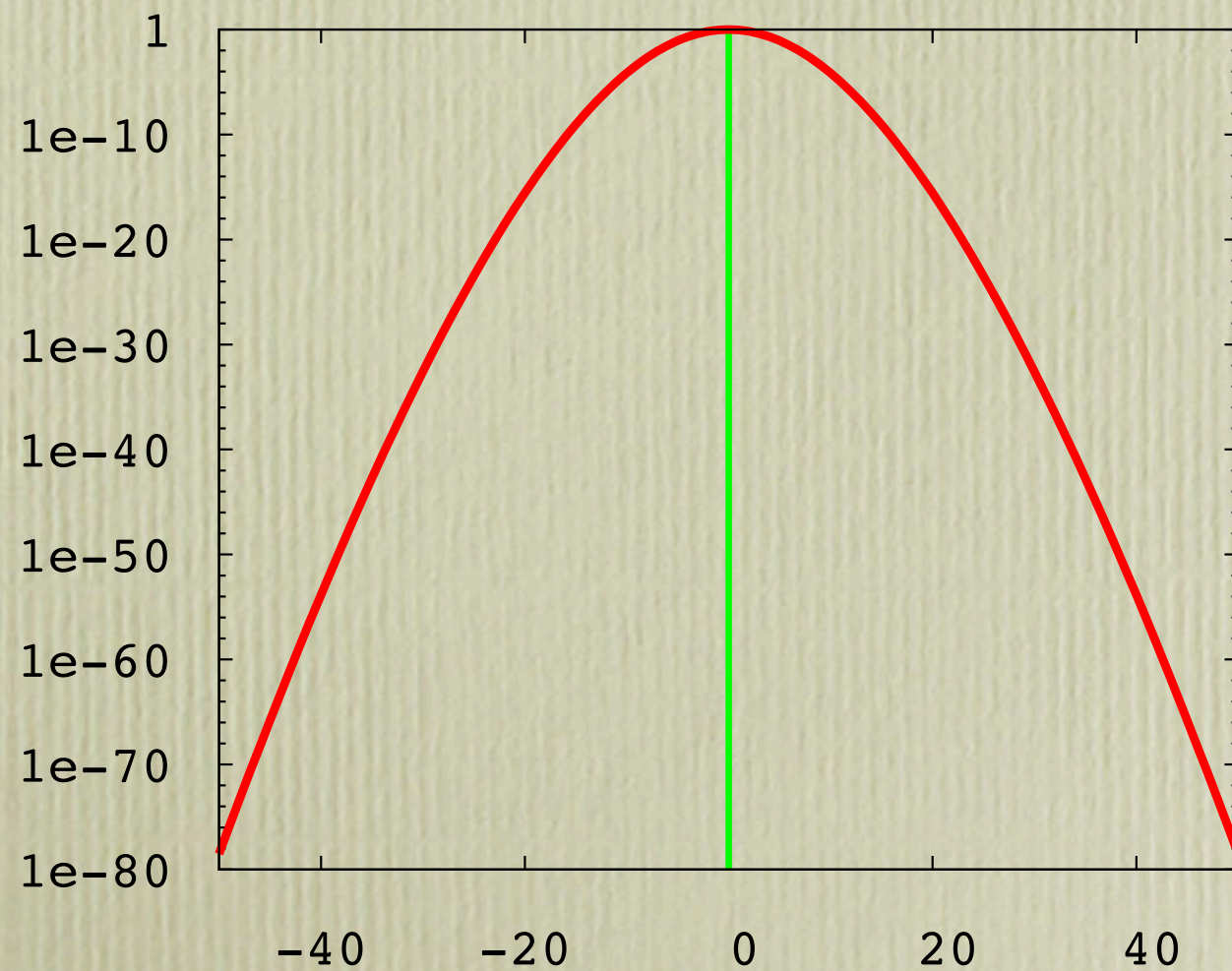
baryon number

Numerical results $Z_C(n)$

Canonical partition function measured on a single conf.

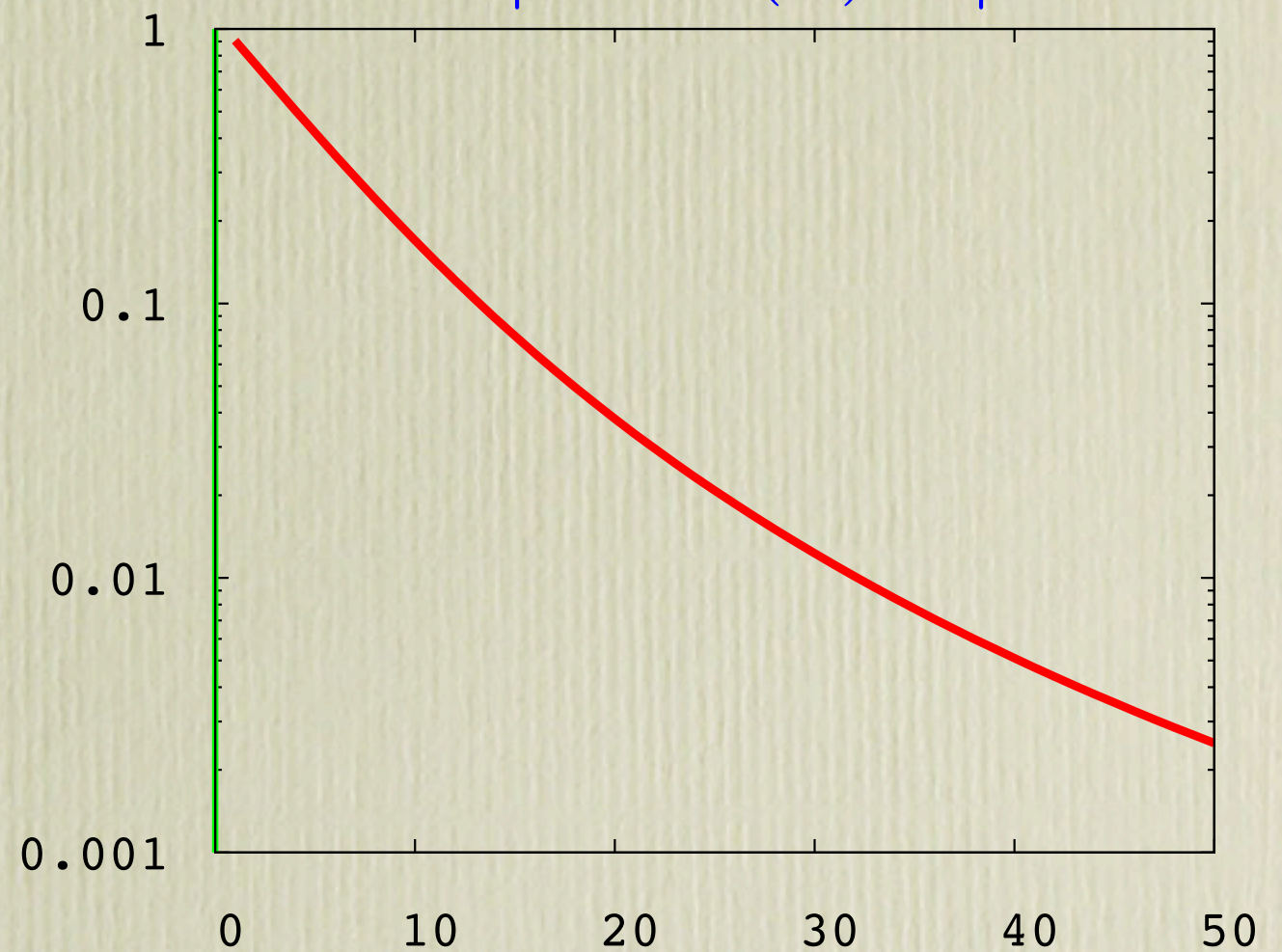
$$8^3 \times 4 \quad \beta = 1.9 \quad \kappa = 0.1250 \quad am_{\text{PCAC}} = 0.1076(68) \quad \mu = 0$$

$$\log |Z_C(n)|$$



baryon number

$$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$$



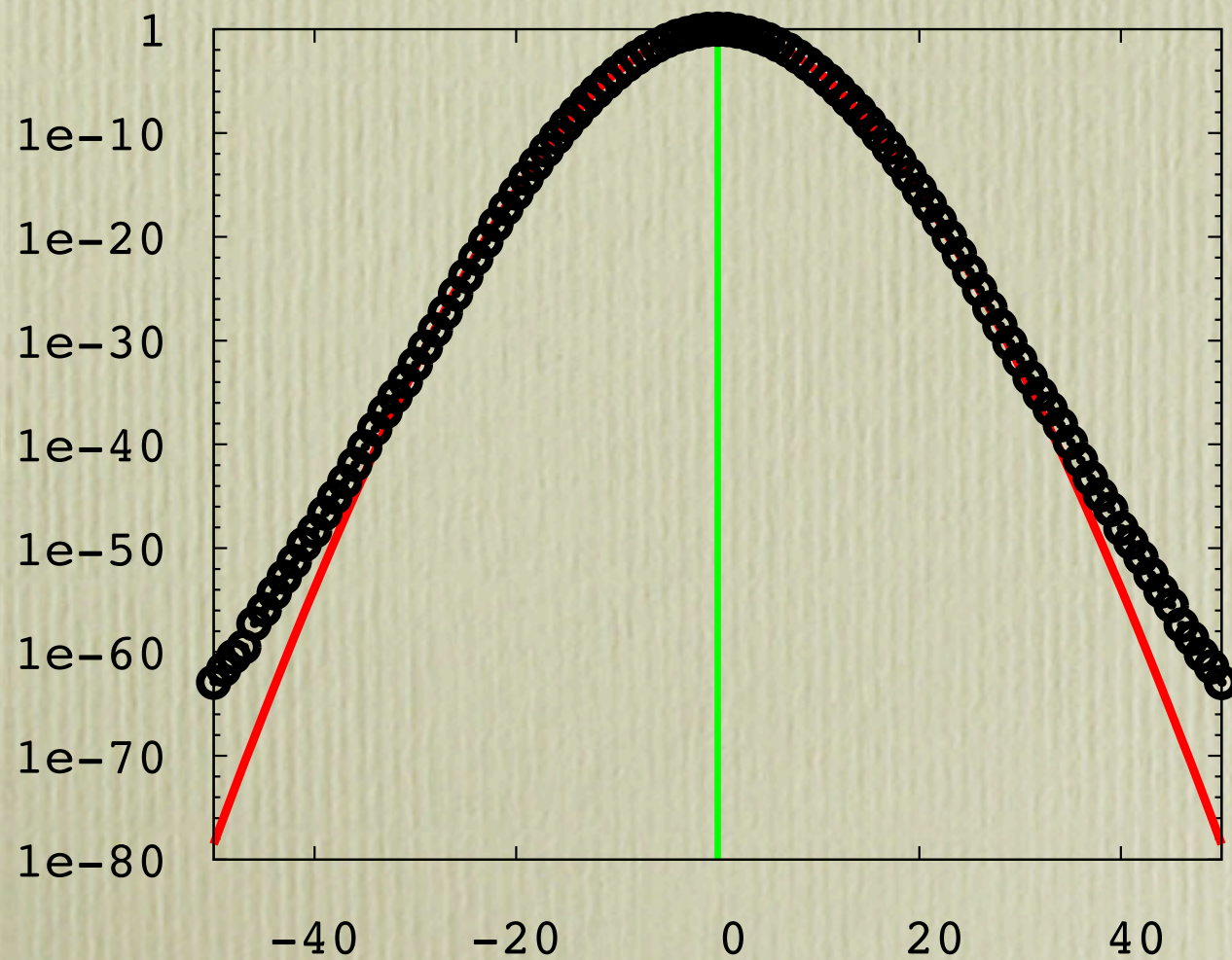
baryon number

Numerical results $Z_C(n)$

Multi precision for Fourier transformation

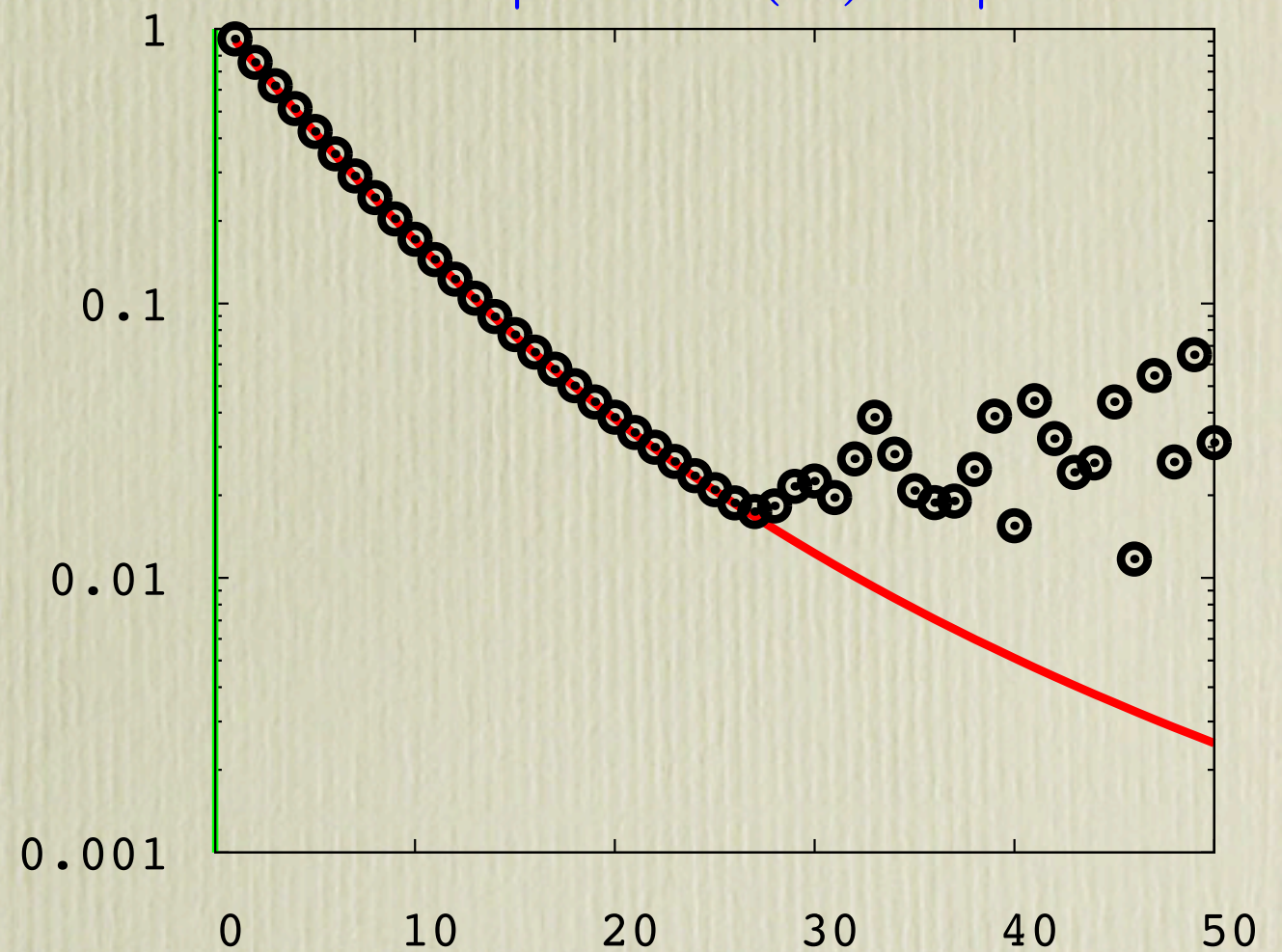
winding number=120 (number of hop =480)

$$\log |Z_C(n)|$$



baryon number

$$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$$



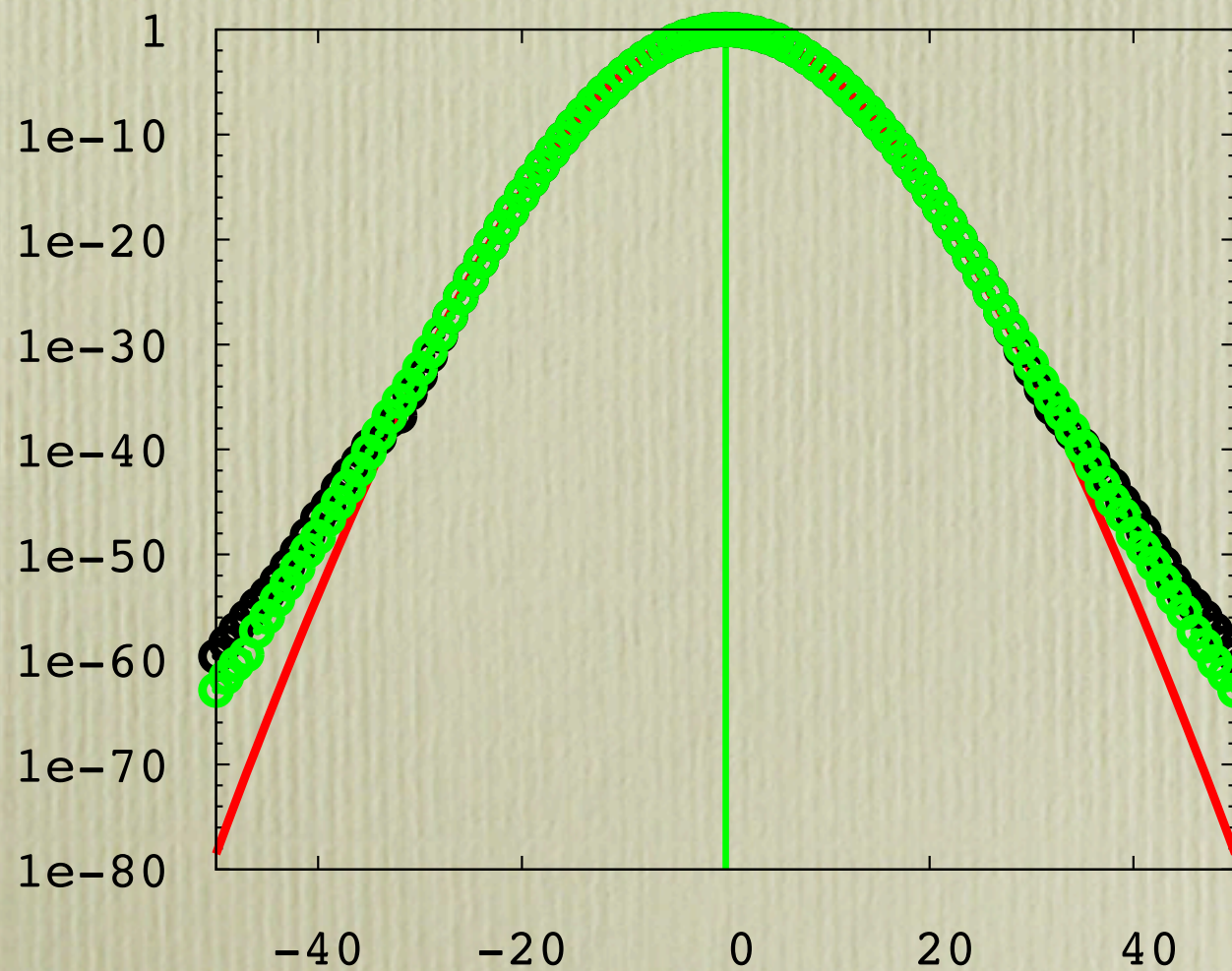
baryon number

Numerical results $Z_C(n)$

Multi precision for Fourier transformation

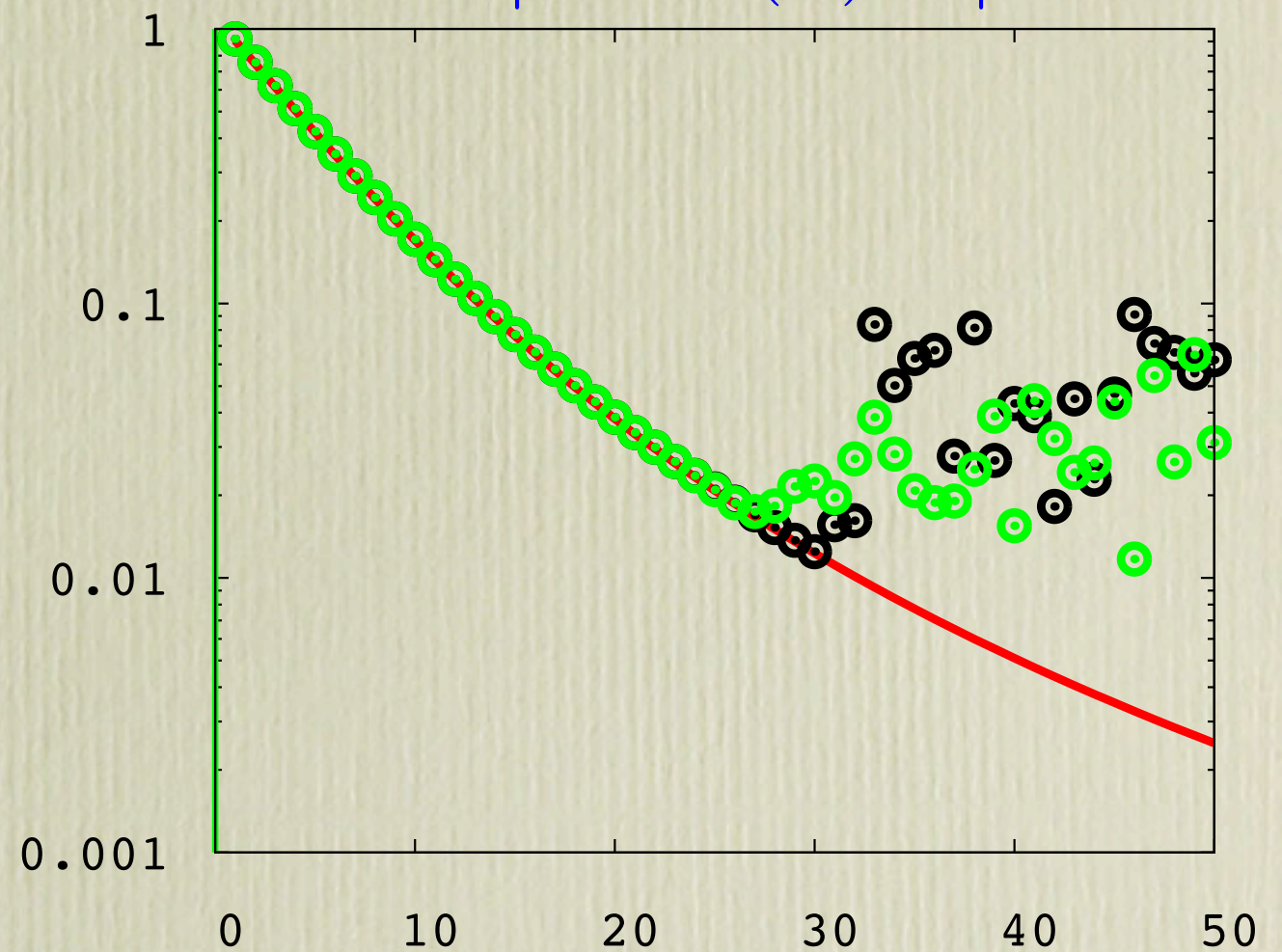
winding number=120 vs 240

$$\log |Z_C(n)|$$



baryon number

$$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$$



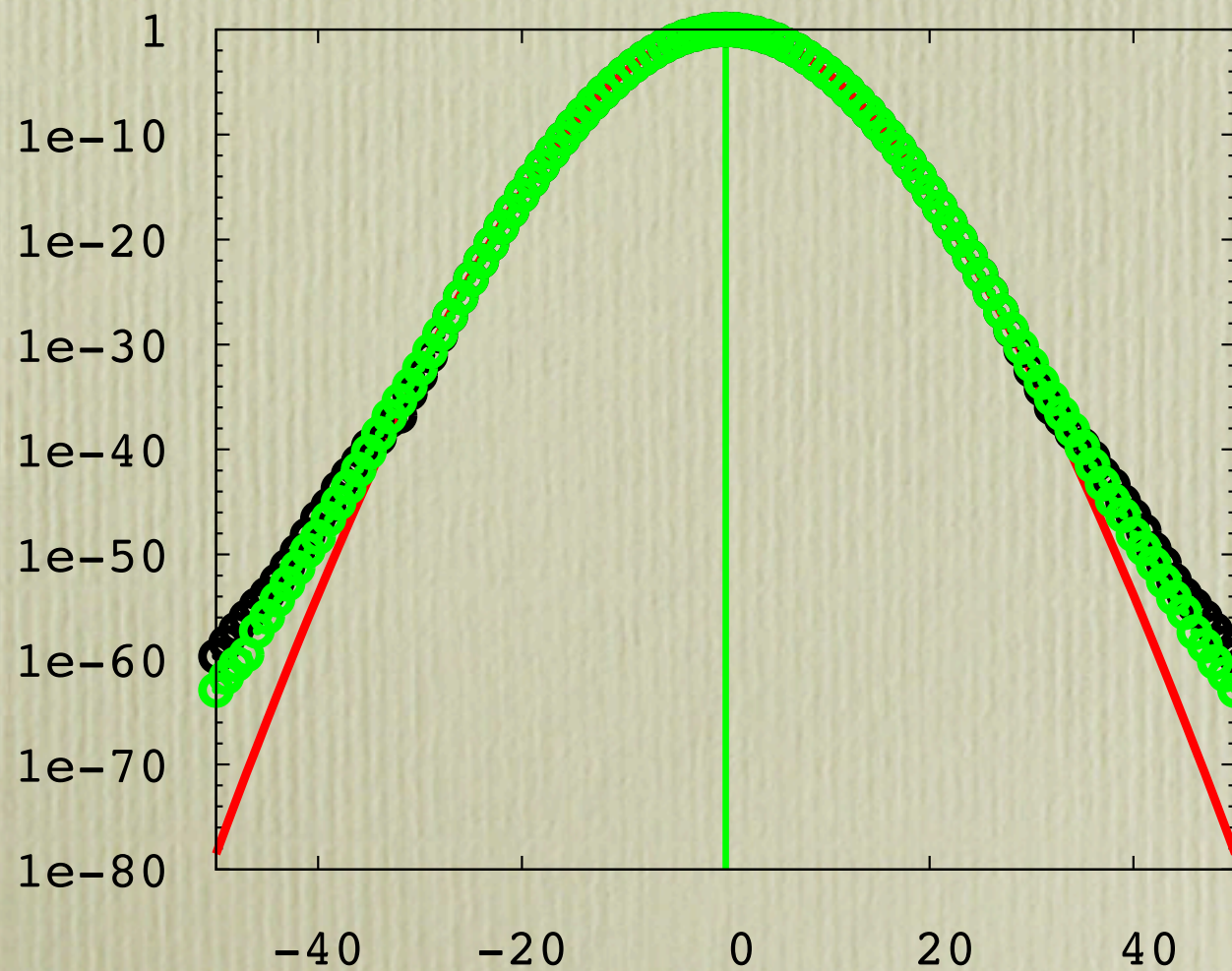
baryon number

Numerical results $Z_C(n)$

Multi precision for Fourier transformation

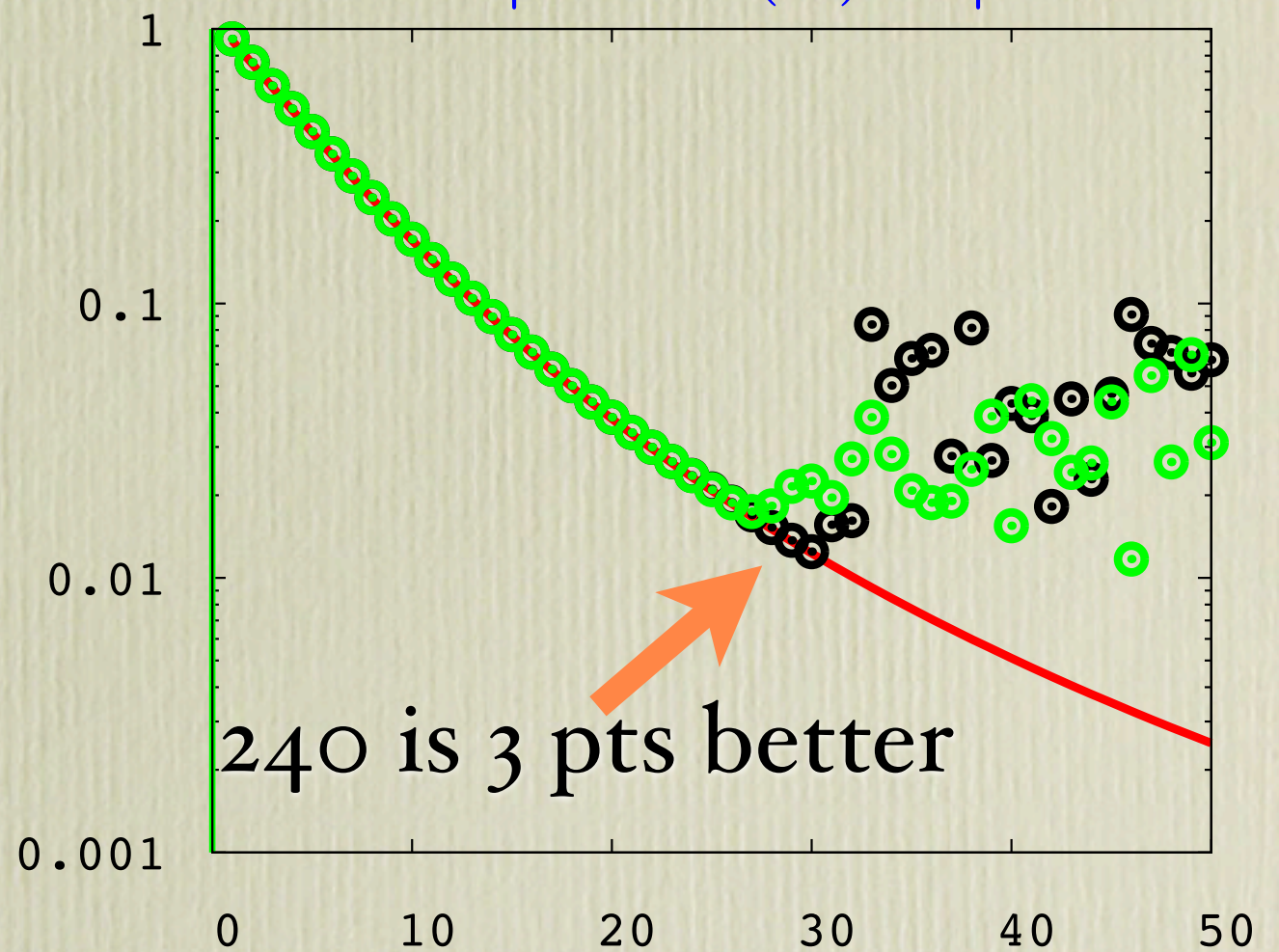
winding number=120 vs 240

$$\log |Z_C(n)|$$



baryon number

$$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$$



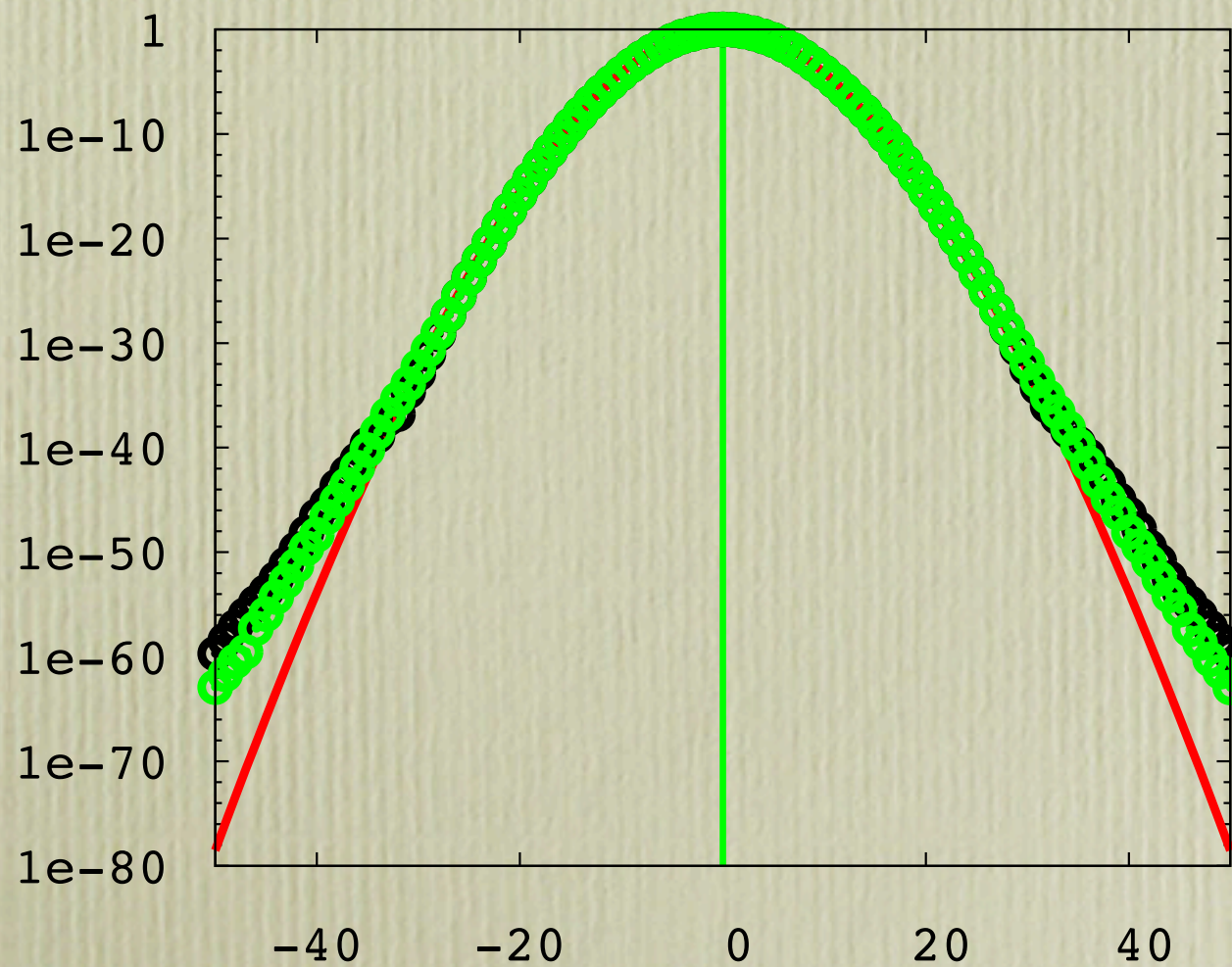
baryon number

Numerical results $Z_C(n)$

Multi precision for Fourier transformation

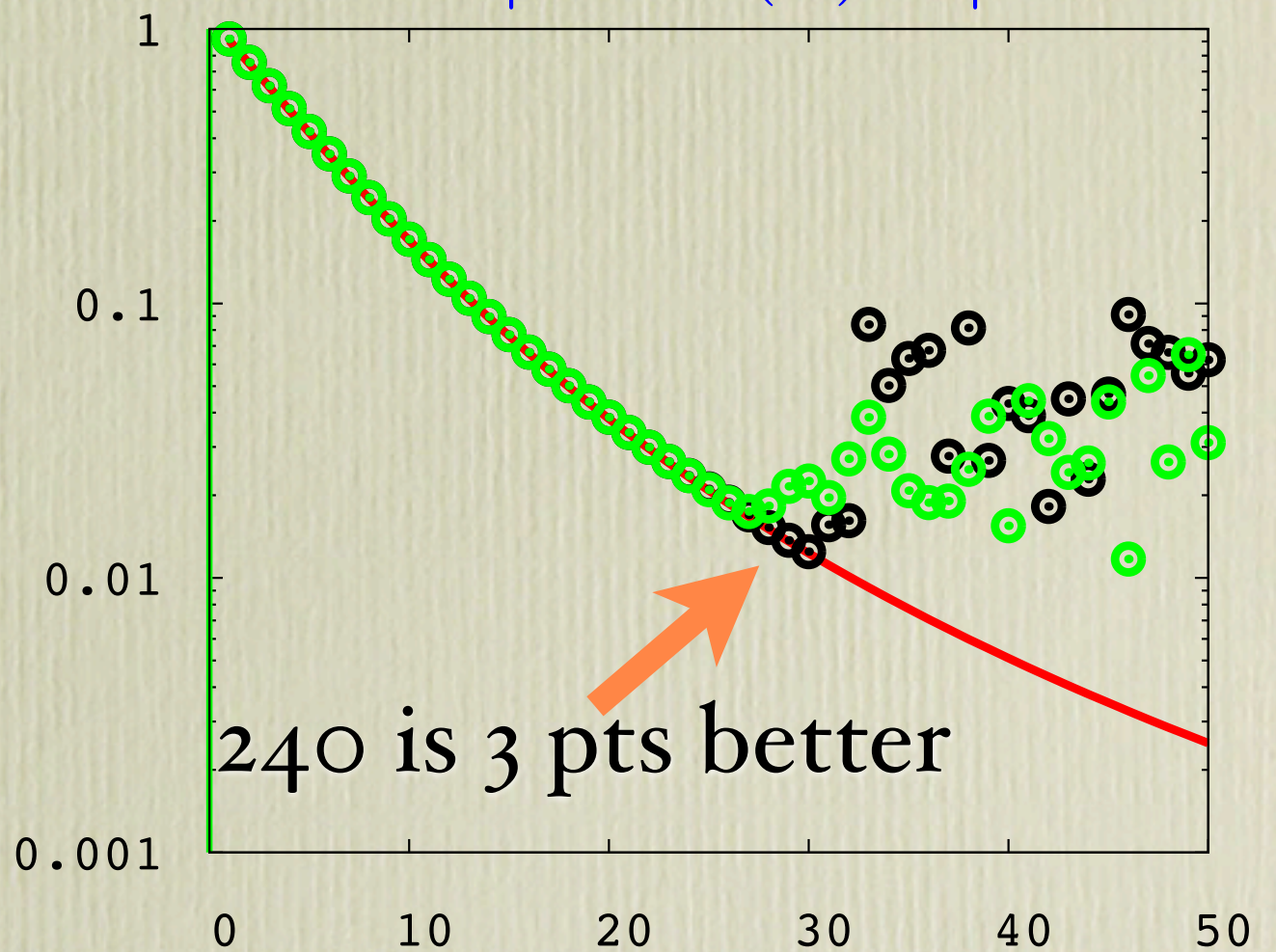
We adopt winding number=120

$$\log |Z_C(n)|$$



baryon number

$$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$$



baryon number

Numerical results $|Z_C(n)|$

Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

Numerical results $|Z_C(n)|$

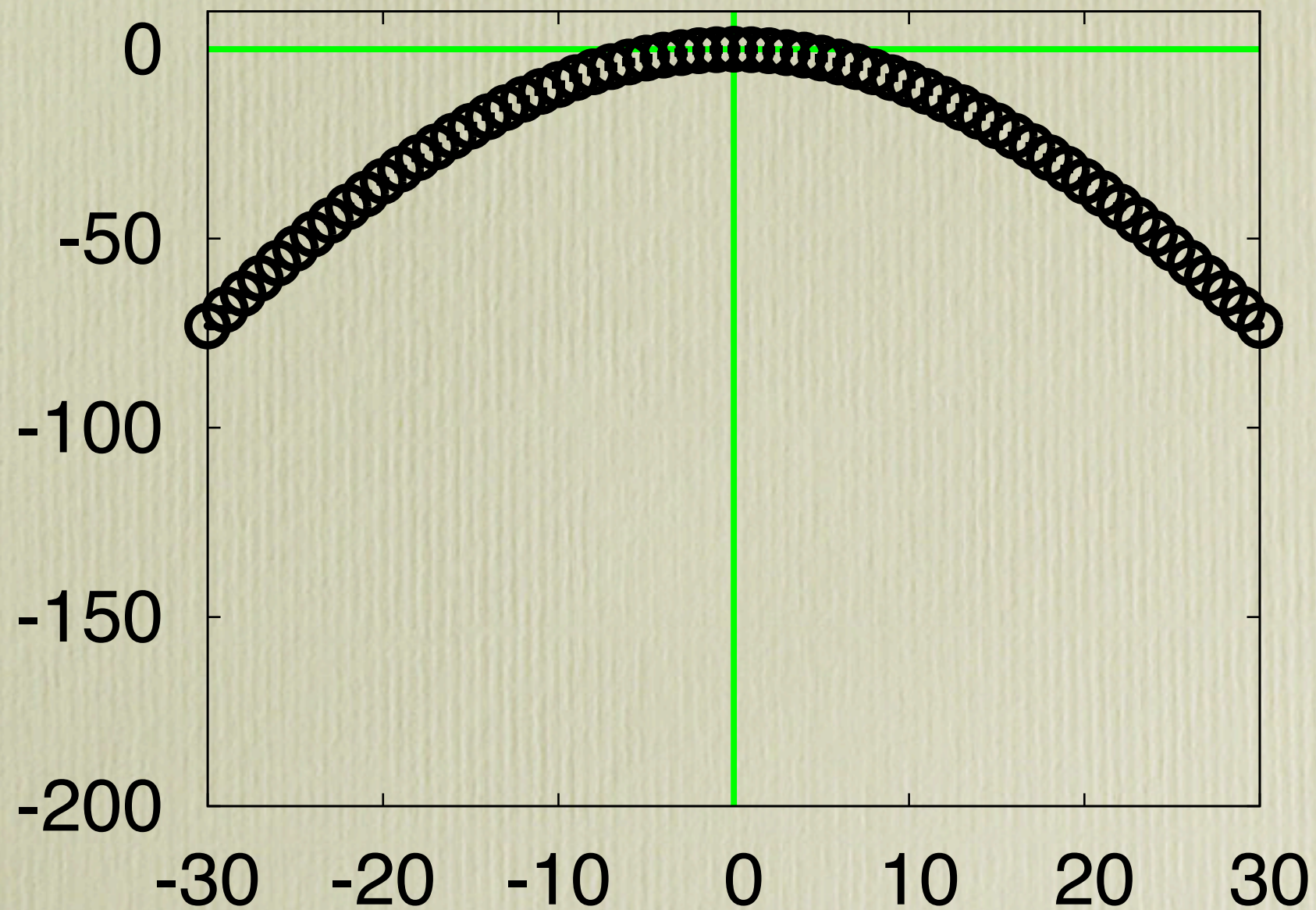
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 2.1$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$



baryon number

Numerical results $|Z_C(n)|$

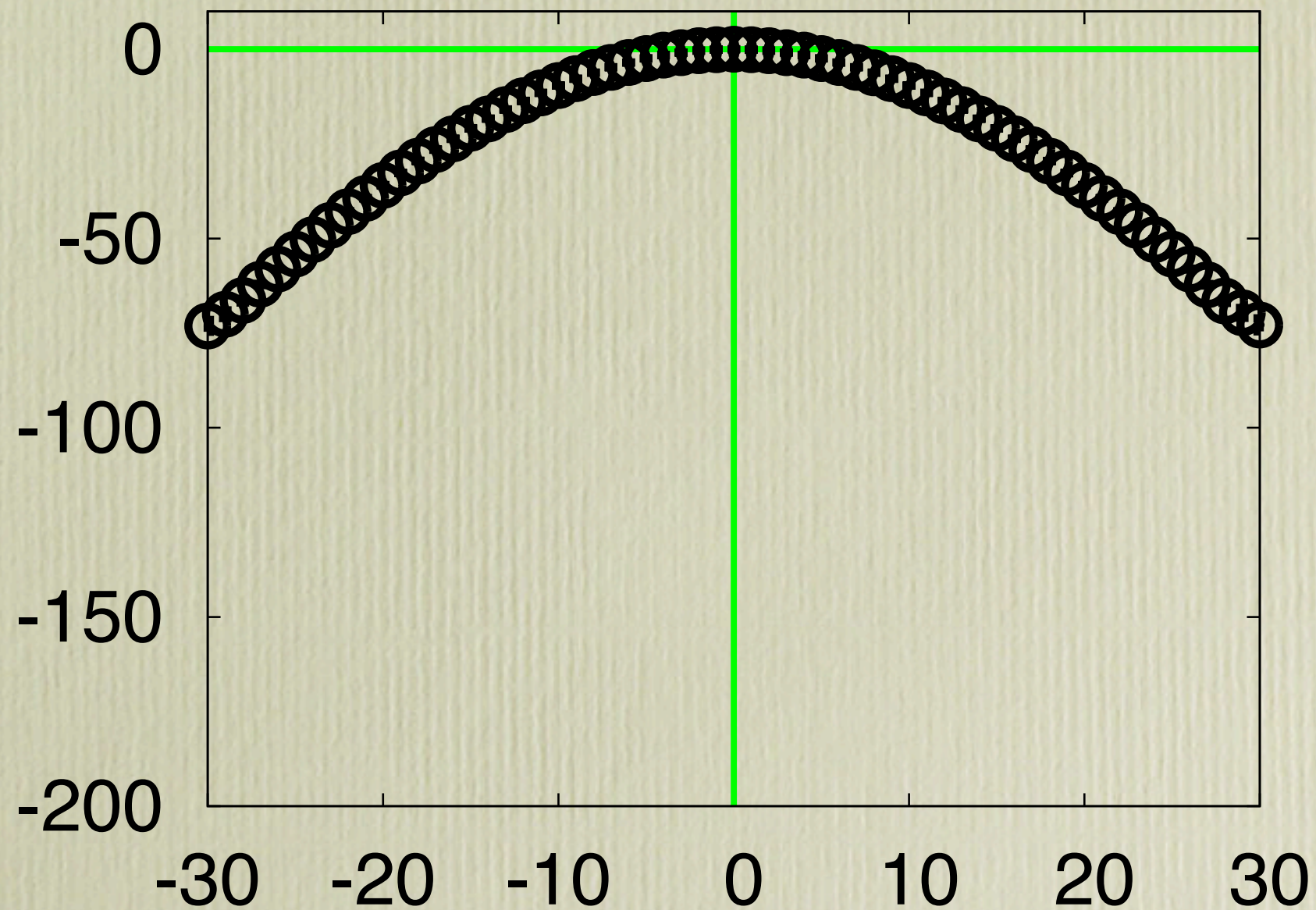
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.9$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$



baryon number

Numerical results $|Z_C(n)|$

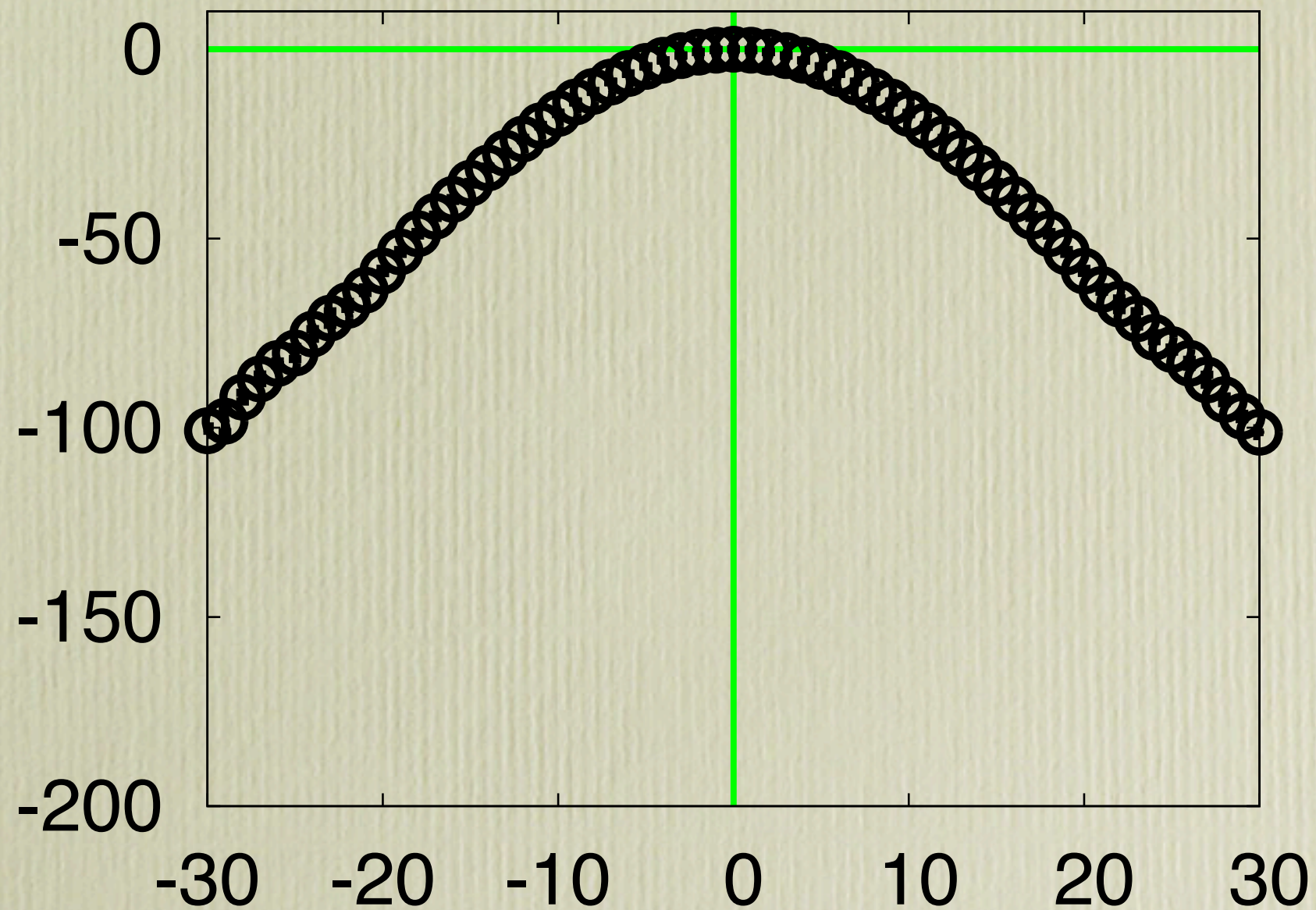
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.7$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$



baryon number

Numerical results $|Z_C(n)|$

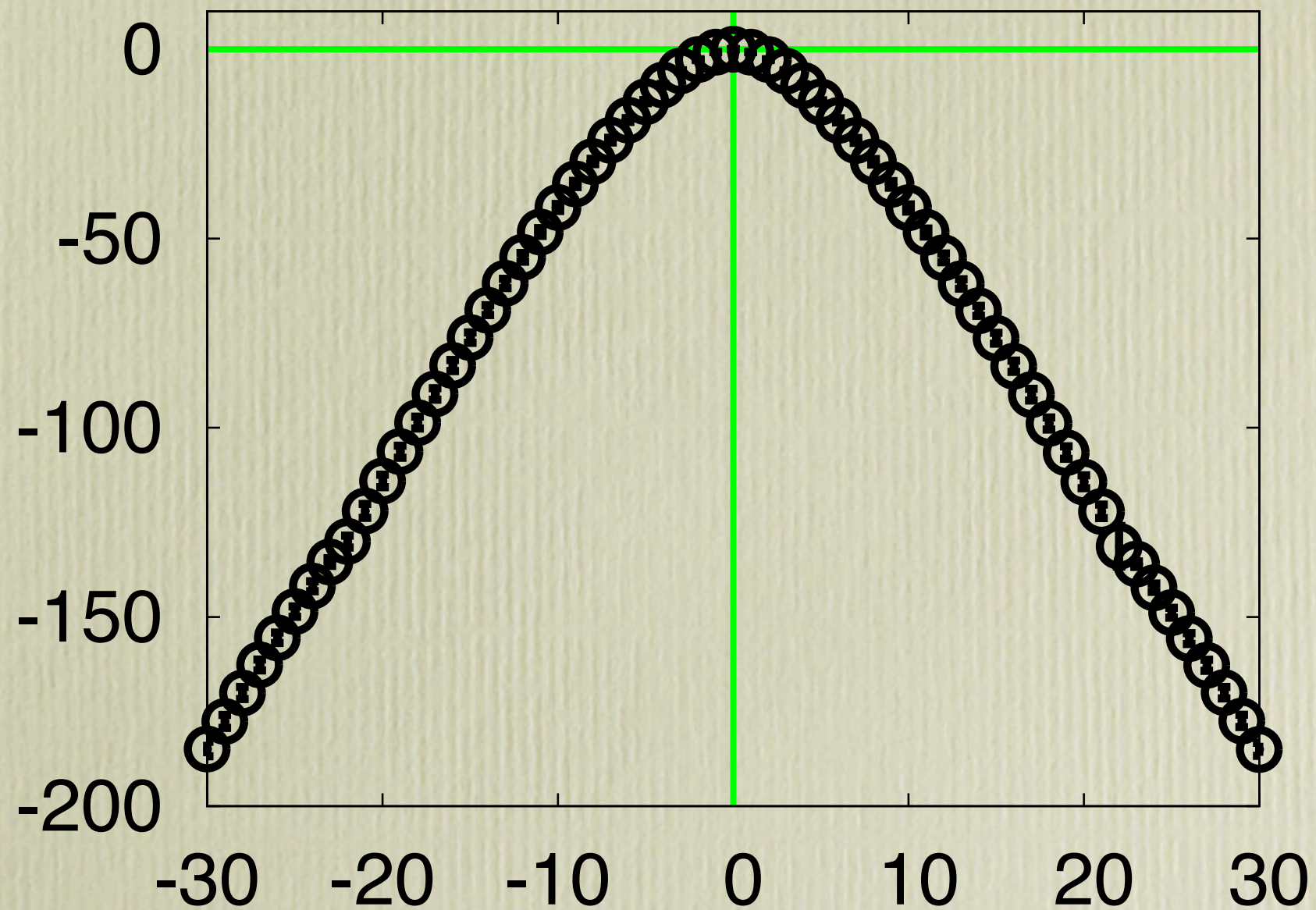
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.5$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$



baryon number

Numerical results $|Z_C(n)|$

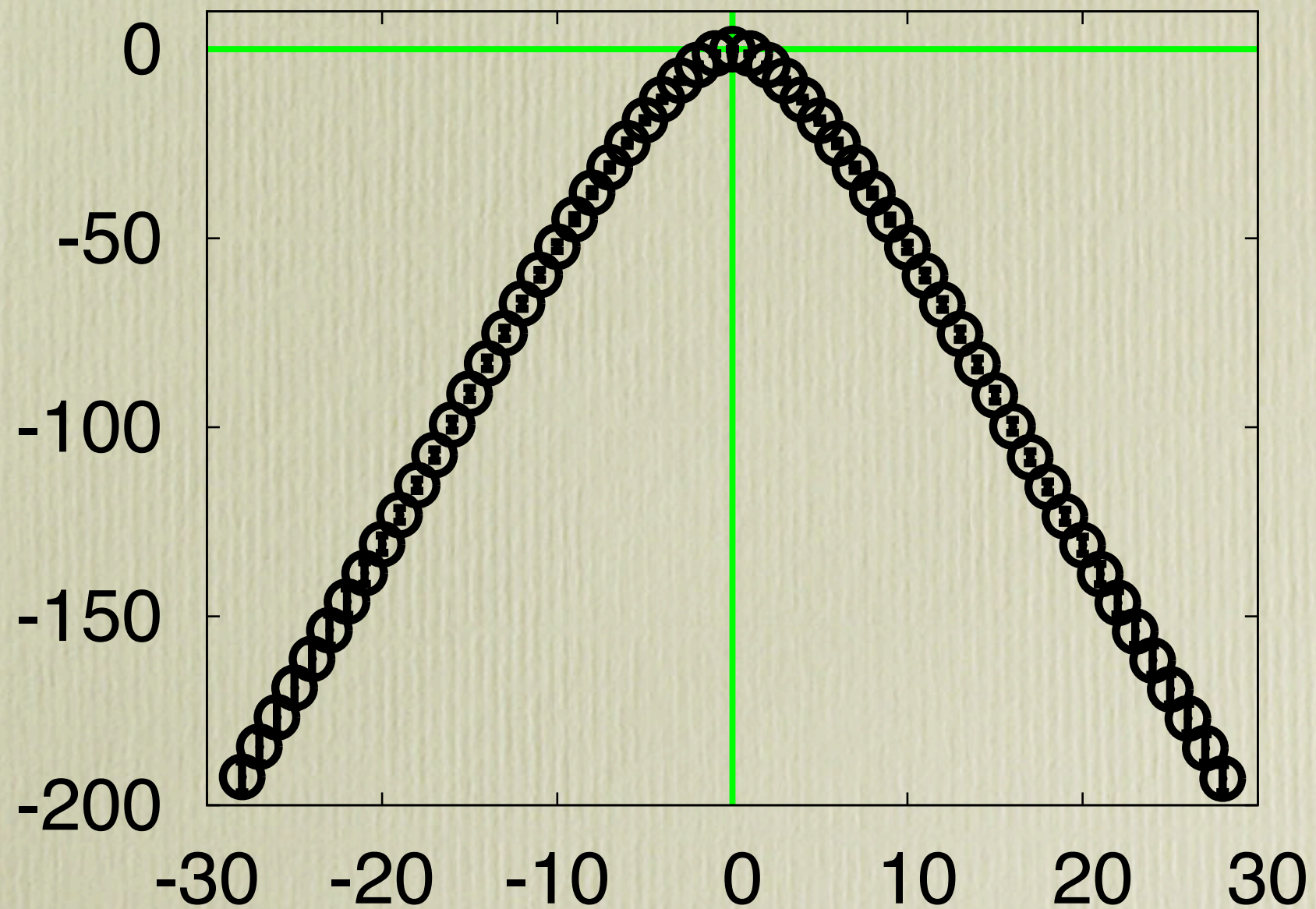
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.3$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$



baryon number

Numerical results $|Z_C(n)|$

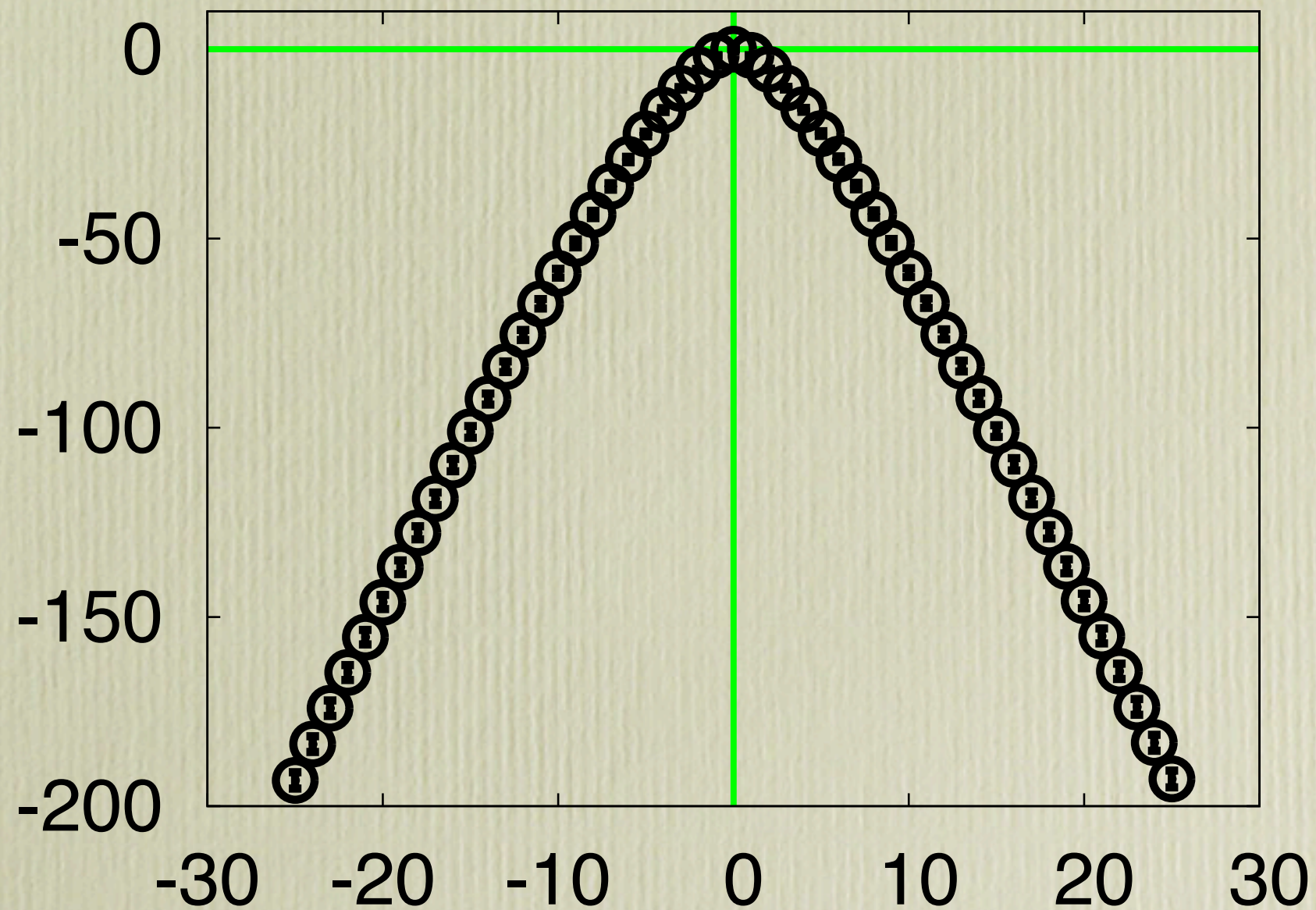
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.1$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$



baryon number

Numerical results $|Z_C(n)|$

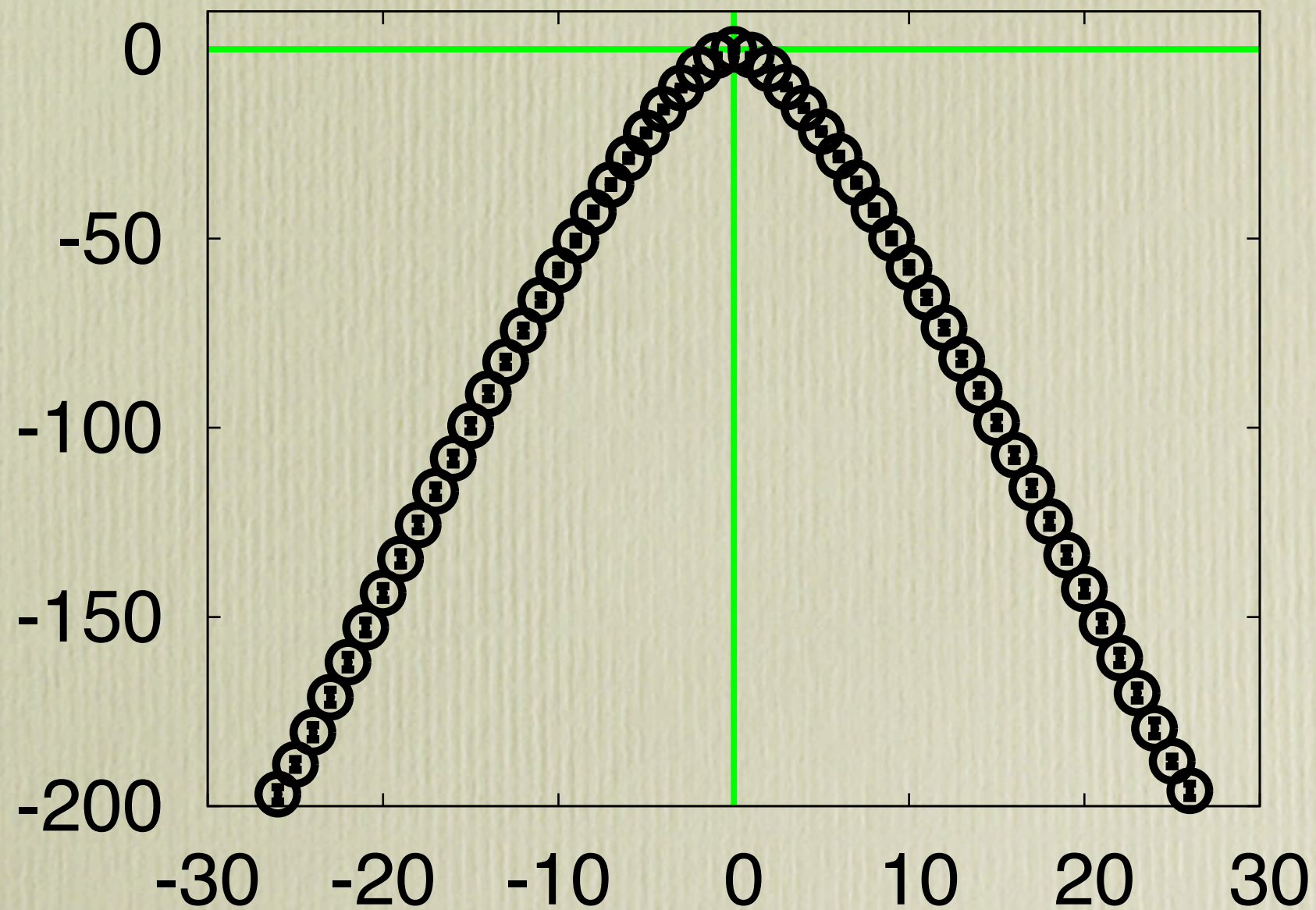
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 0.9$$

$$\mu = 0$$

$$\log |Z_C(\beta, n)|$$

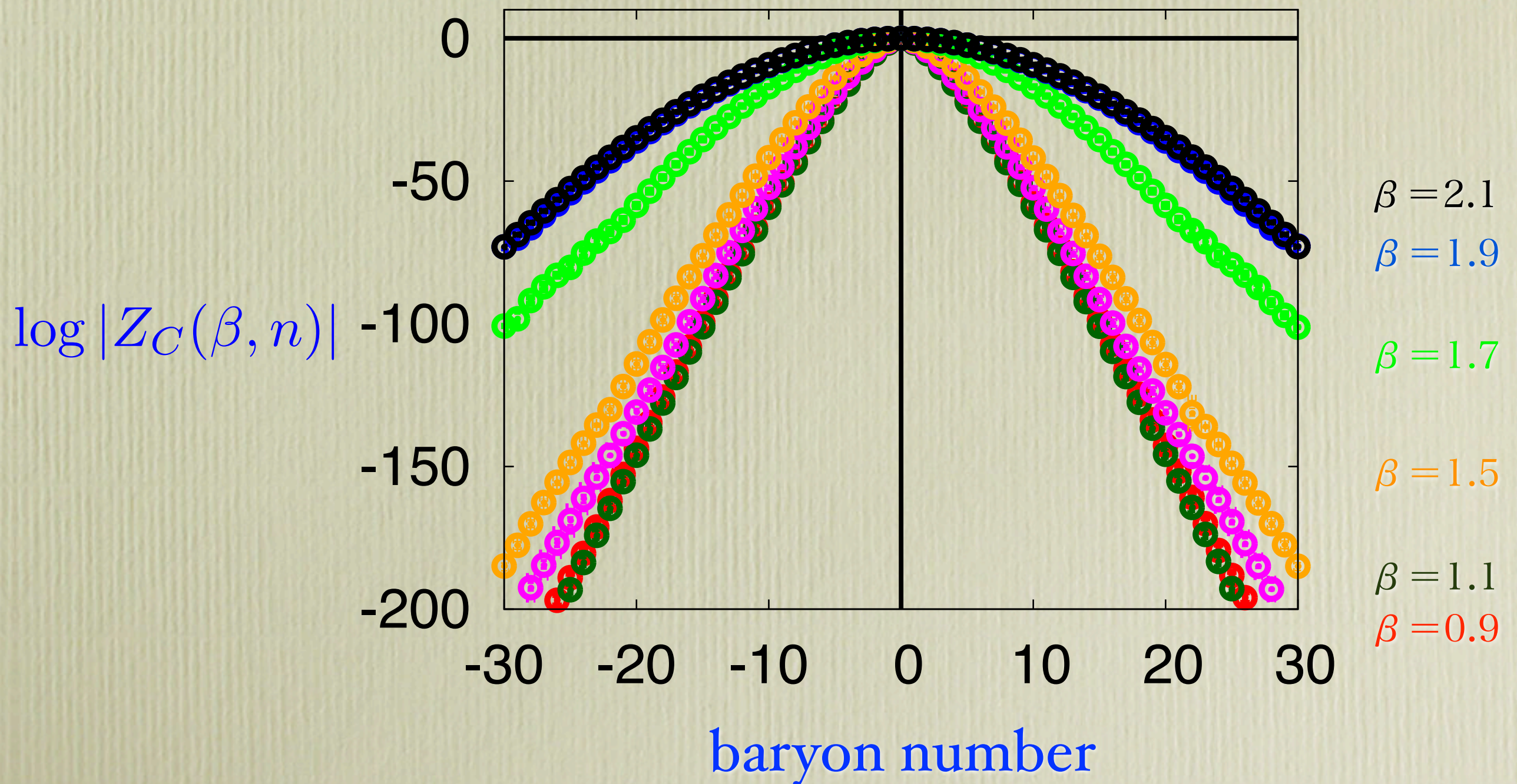


baryon number

Numerical results $|Z_C(n)|$

Canonical partition function

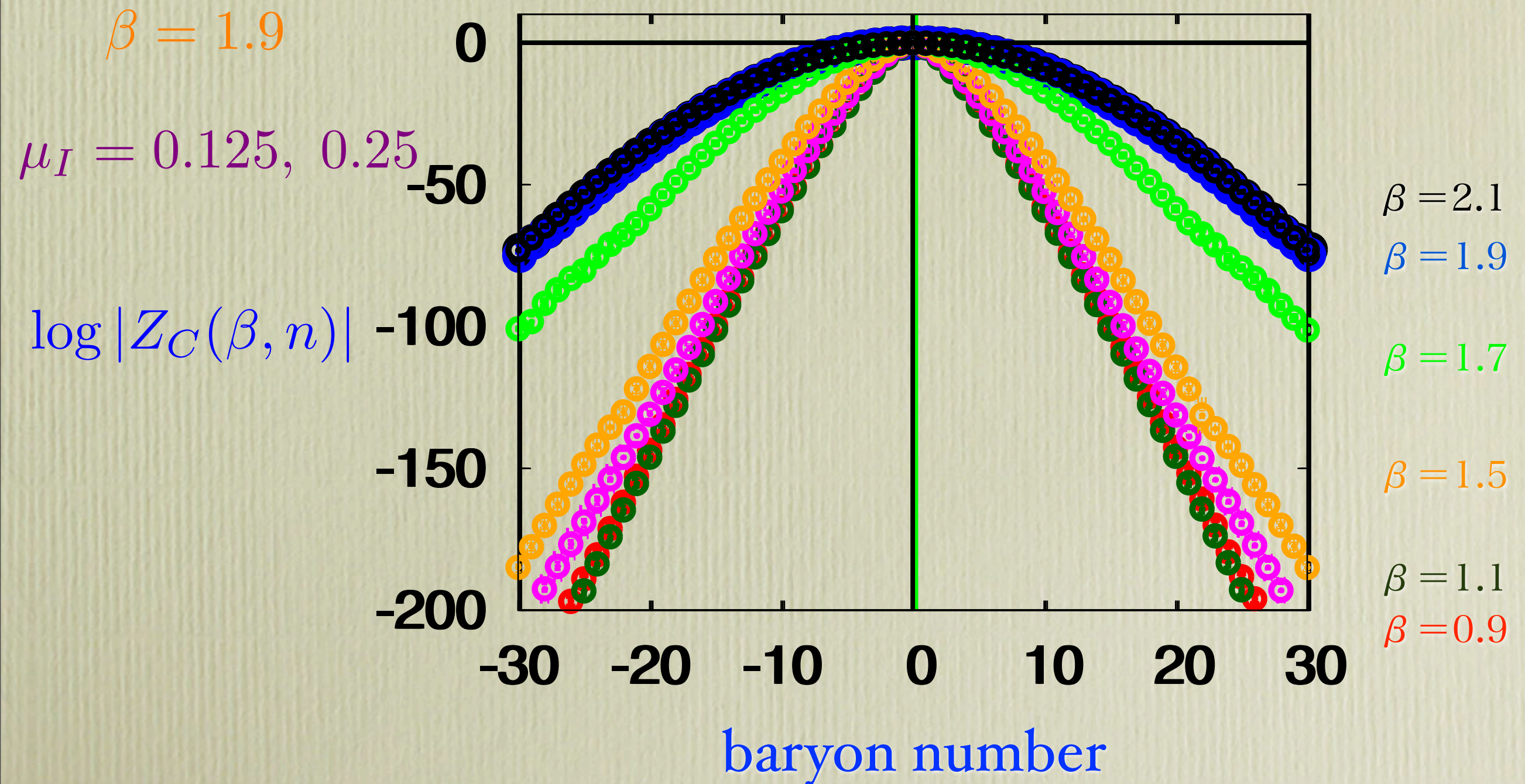
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Numerical results $|Z_C(n)|$

Canonical partition function

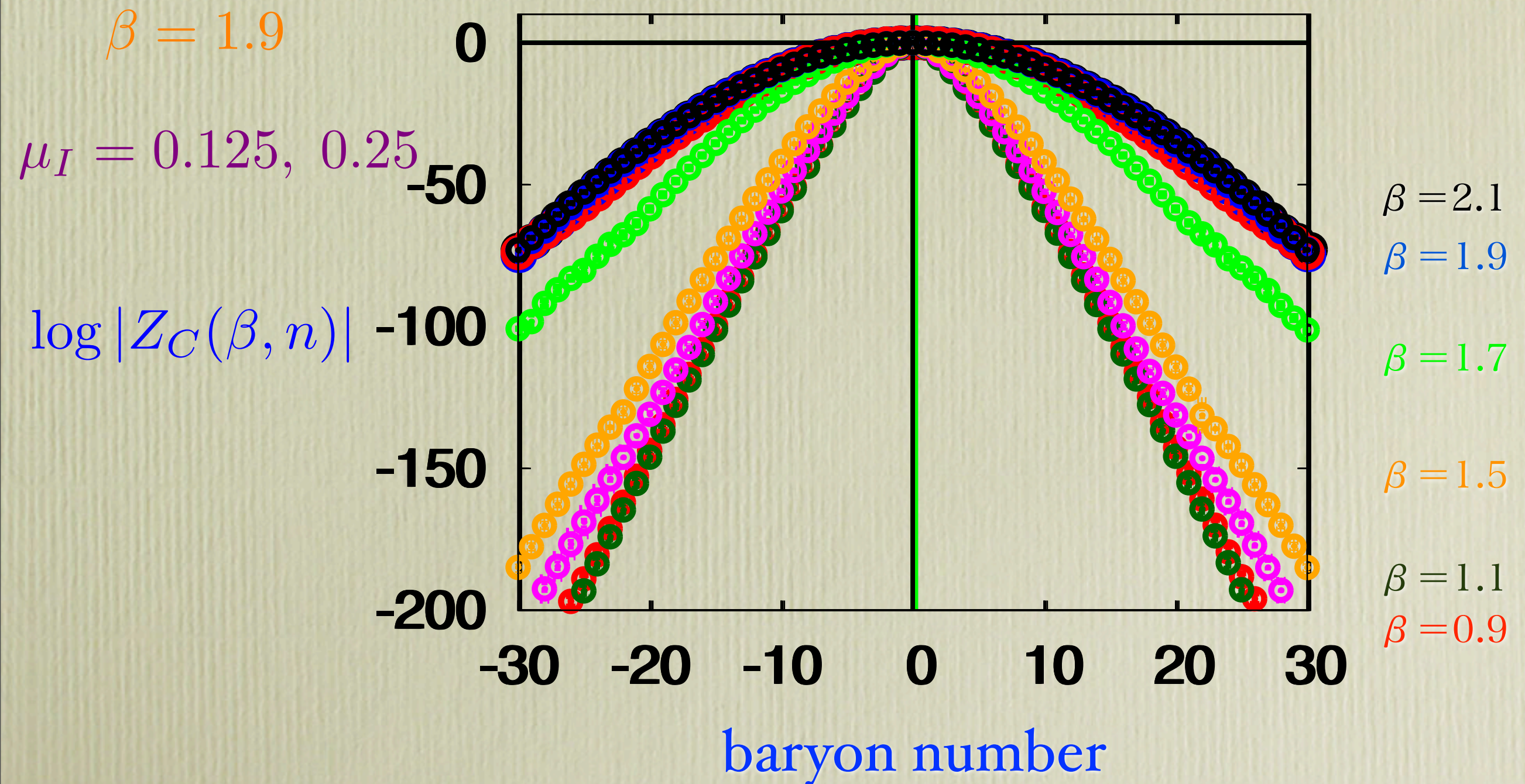
$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$



Numerical results $|Z_C(n)|$

Canonical partition function

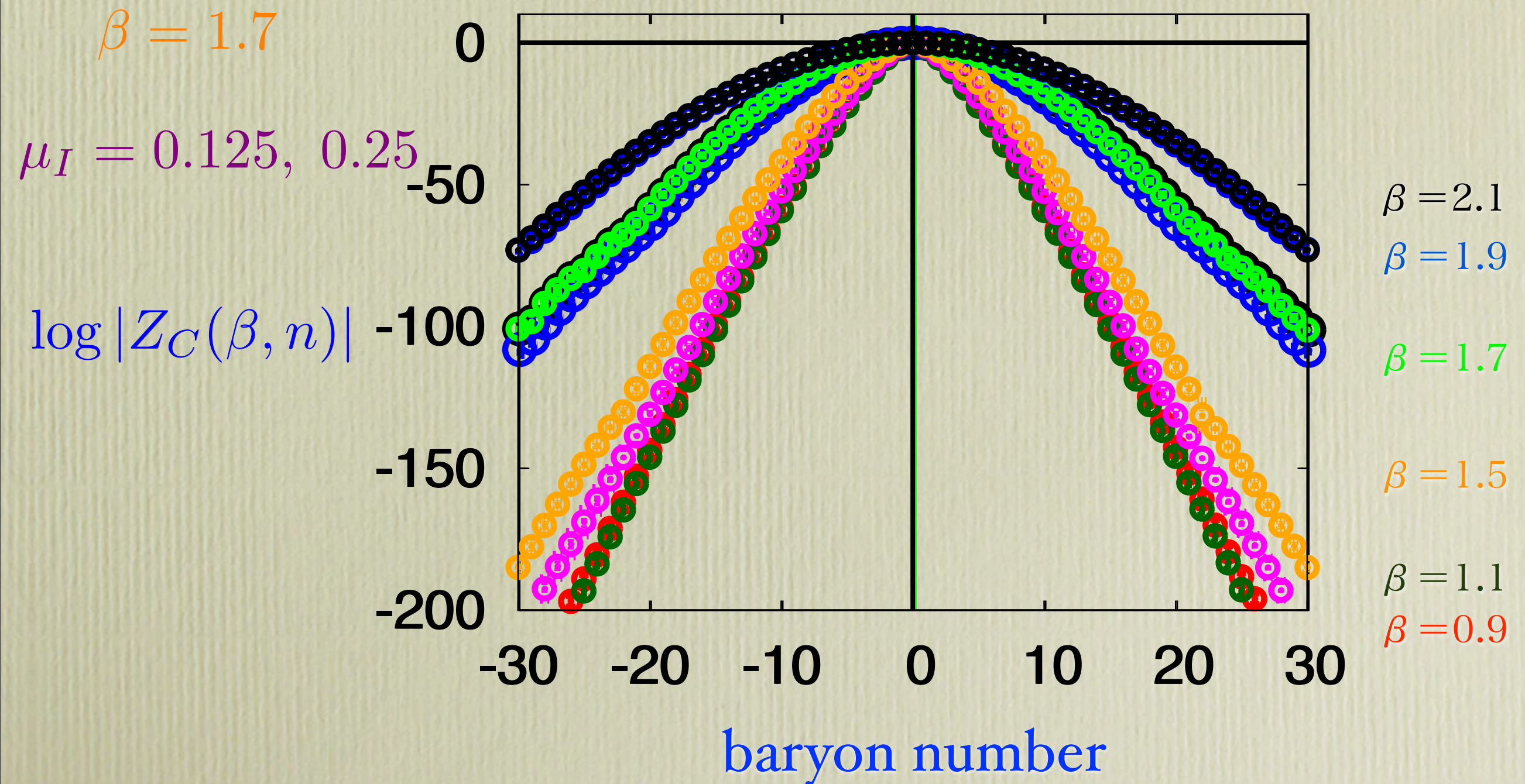
$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$



Numerical results $|Z_C(n)|$

Canonical partition function

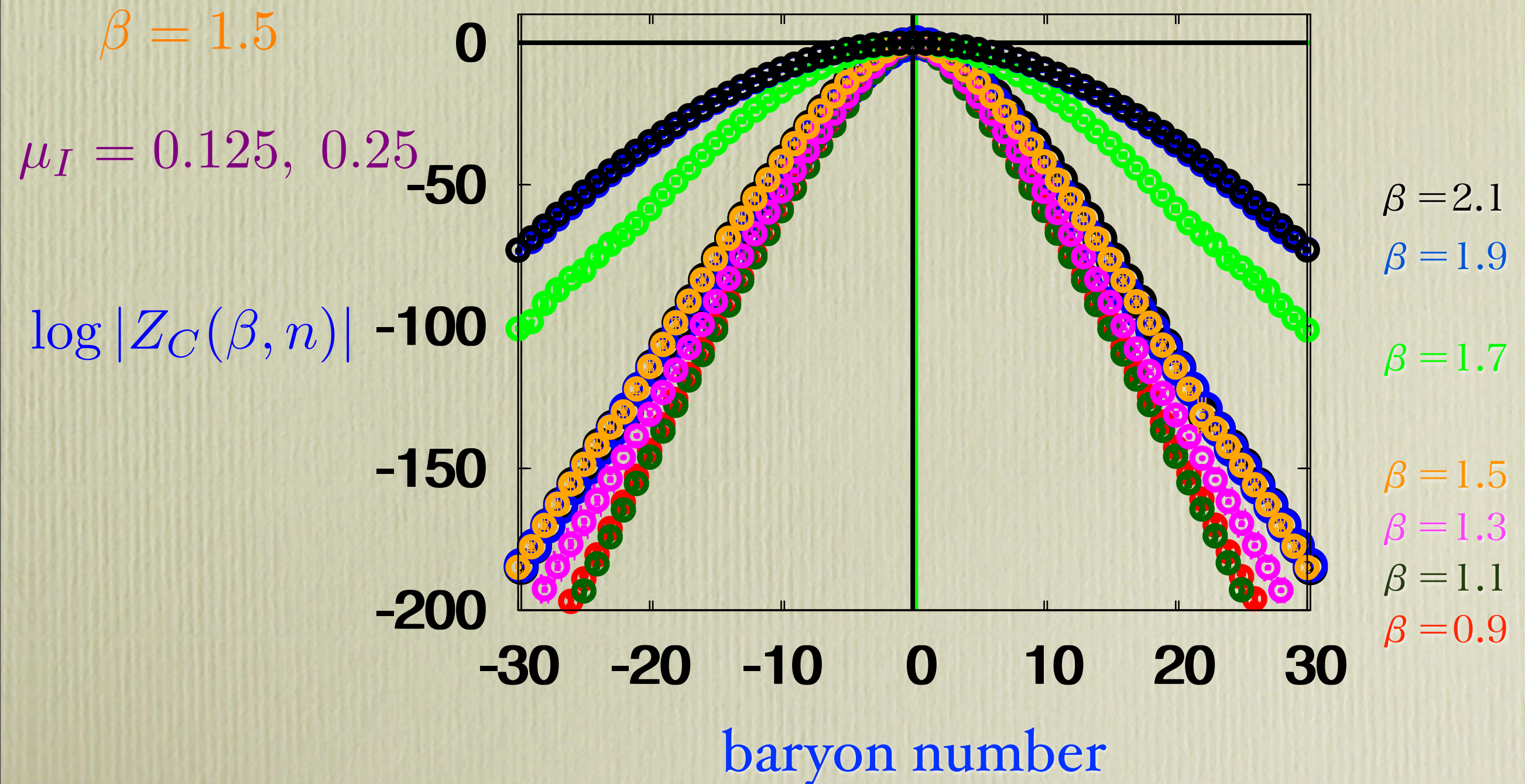
$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$



Numerical results $|Z_C(n)|$

Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$



Numerical results $\text{Phase}(Z_C(n))$

Canonical partition function

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Numerical results $\text{Phase}(Z_C(n))$

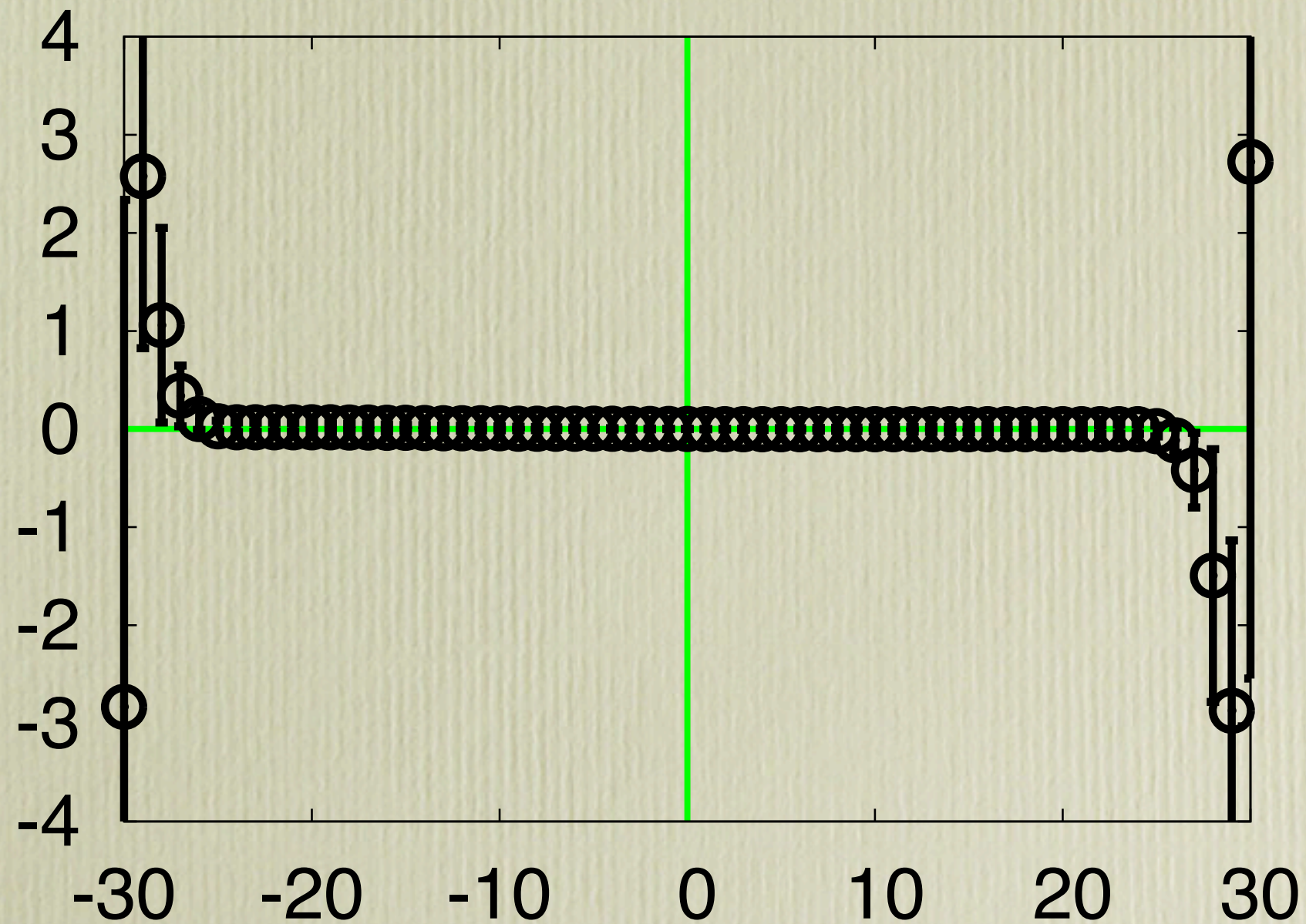
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.9$$

$$\mu = 0$$

$$\theta(\beta, n)$$



baryon number

Numerical results $\text{Phase}(Z_C(n))$

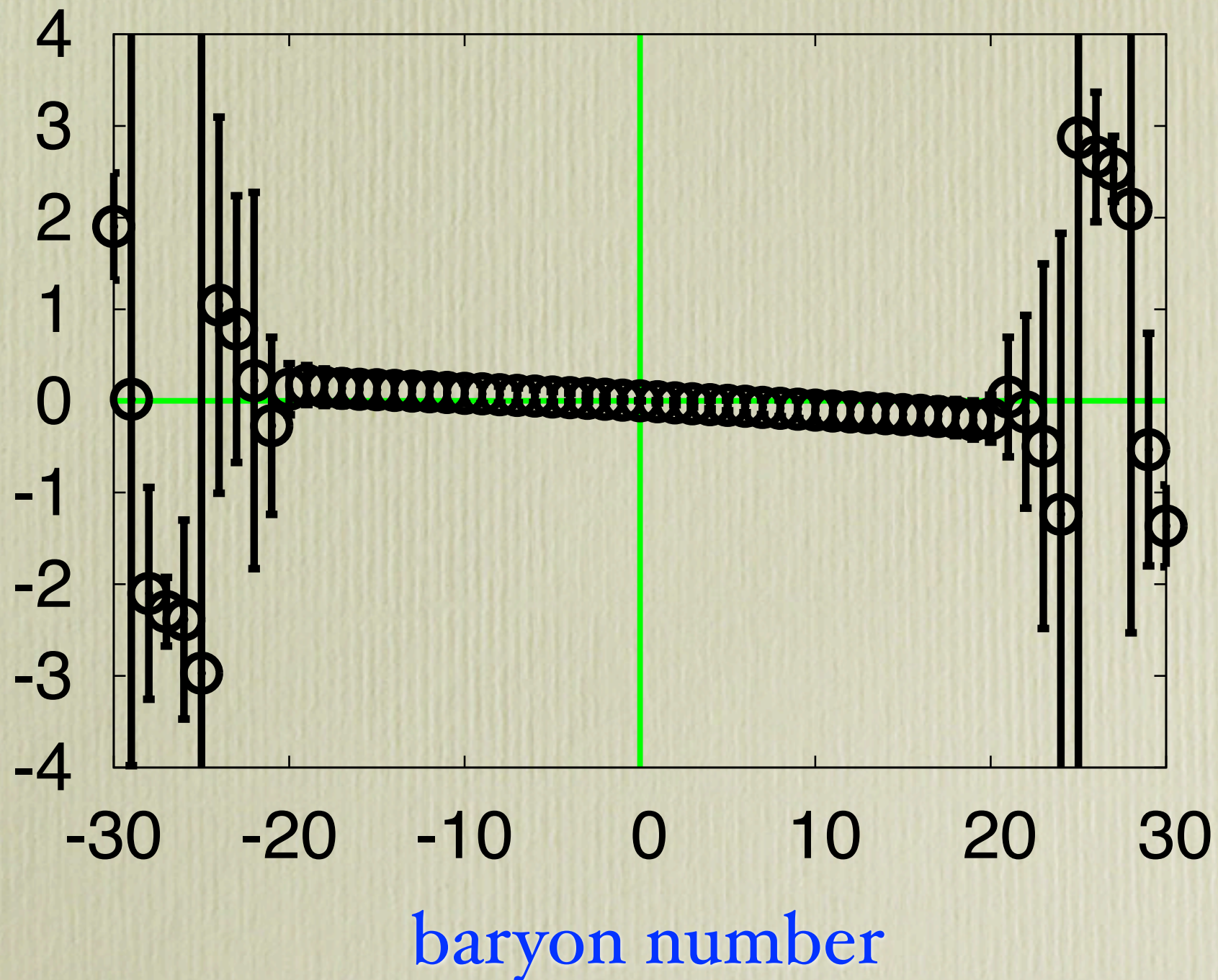
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Numerical results $\text{Phase}(Z_C(n))$

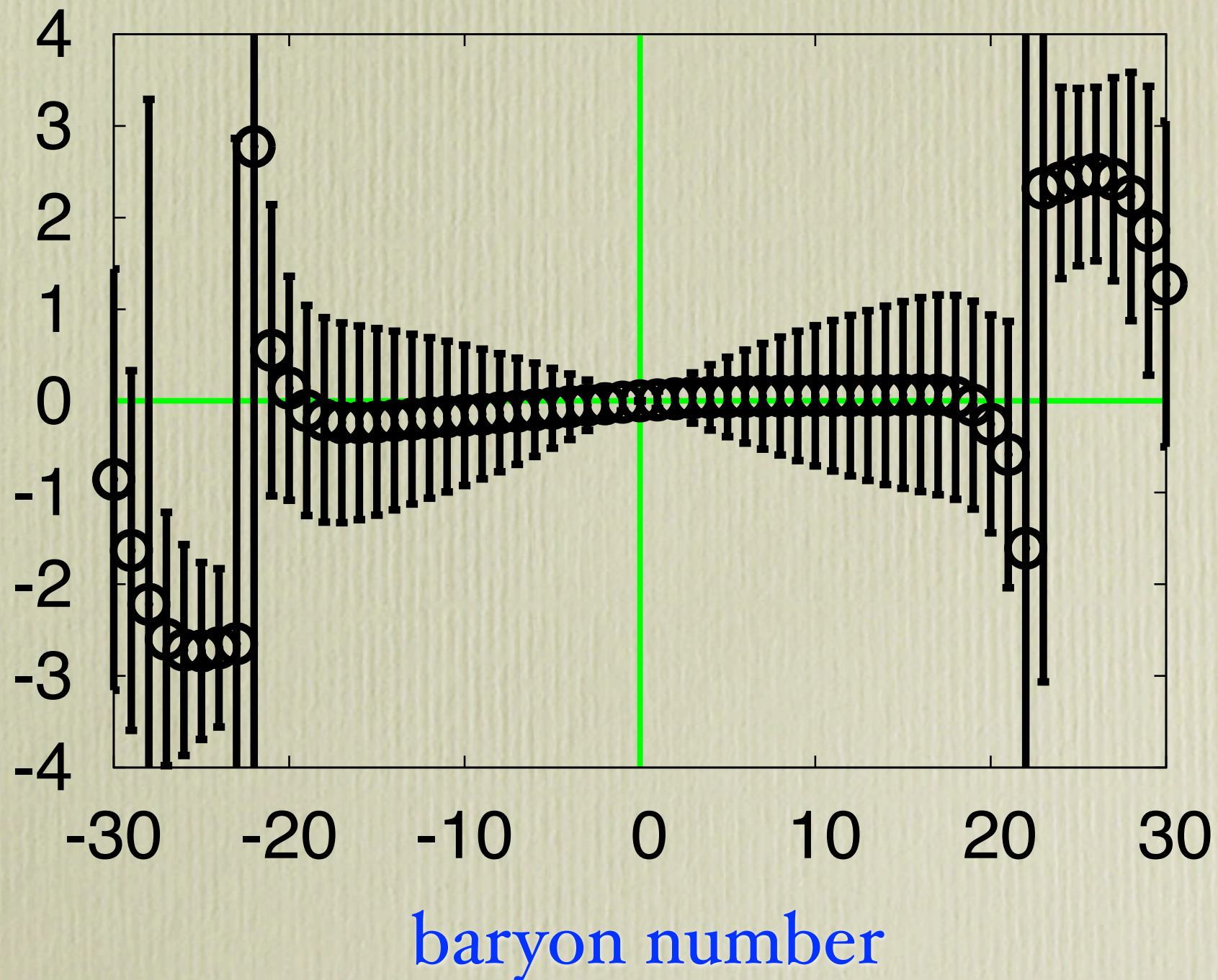
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.5$$

$$\mu = 0$$

$$\theta(\beta, n)$$



Plan of the talk

- ✓ 1. Introduction
- ✓ 2. Winding number expansion
- ✓ 3. Numerical setup
- ✓ 4. Numerical results
- 5. Hadronic observables
- 6. Conclusion

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{\text{Tr} \left[\hat{O} \exp \left(-\beta \left(\hat{H} - \mu \hat{N} \right) \right) \right]}{\text{Tr} \left[\exp \left(-\beta \left(\hat{H} - \mu \hat{N} \right) \right) \right]}$$

Hadronic observables

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$$\text{Numerator} = \sum_{n=-\infty}^{\infty} \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle \xi^n$$

Hadronic observables

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Hadronic observables

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path integral formalism

Hadronic observables

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path integral formalism

re-weighting technique

Hadronic observables

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path integral formalism  re-weighting technique

Hadronic observables

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$$\langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{\text{Tr} \left[\hat{O} \exp \left(-\beta \left(\hat{H} - \mu \hat{N} \right) \right) \right]}{\text{Tr} \left[\exp \left(-\beta \left(\hat{H} - \mu \hat{N} \right) \right) \right]}$$

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path integral formalism  re-weighting technique

$$\left\langle O(D_W(\mu)) \frac{\text{Det} D_W(\mu)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

$$\text{Numerator} = \sum_{n=-\infty}^{\infty} \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle \xi^n \equiv \sum_{n=-\infty}^{\infty} O_n \xi^n$$

path integral formalism



re-weighting technique

$$\left\langle O(D_W(\mu)) \frac{\text{Det} D_W(\mu)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle \underline{O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)}} \right\rangle_0 Z_G(\mu_0)$$

Hadronic observables

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To see explicit functional form in ξ

Hadronic observables

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To see explicit functional form in ξ Use HPE

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

To see explicit functional form in ξ

Use HPE

$$\bar{\psi}\psi = -\text{tr} \left(\frac{1}{D_W} \right) = -\text{tr} \left(\frac{1}{1 - \kappa Q} \right) = \sum_{m=0}^{\infty} \kappa^m \text{tr} Q^m$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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resummation

Hadronic observables

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$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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resummation

$$\sum_{n=-\infty}^{\infty} S_n(U) \xi^n$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

To see explicit functional form in ξ

Use HPE

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resummation

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$\text{Det} D_W$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

To see explicit functional form in ξ

Use HPE

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resummation

$$\sum_{n=-\infty}^{\infty} S_n(U) \xi^n$$

$\text{Det} D_W$

resummation

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\text{Det} D_W(\xi)}{\text{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

To see explicit functional form in ξ

Use HPE

$$\bar{\psi}\psi = -\text{tr} \left(\frac{1}{D_W} \right) = -\text{tr} \left(\frac{1}{1 - \kappa Q} \right) = \sum_{m=0}^{\infty} \kappa^m \text{tr} Q^m$$

resummation

$$\sum_{n=-\infty}^{\infty} S_n(U) \xi^n$$

$\text{Det} D_W$

resummation

$$\exp \left(\sum_{n=-\infty}^{\infty} W_n(U) \xi^n \right)$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

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EV of canonical ensemble

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

EV of canonical ensemble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

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EV of canonical ensemble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

Fly into REAL μ !

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

EV of canonical ensemble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

Fly into REAL μ !

$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

EV of canonical ensemble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

Fly into REAL μ !

$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

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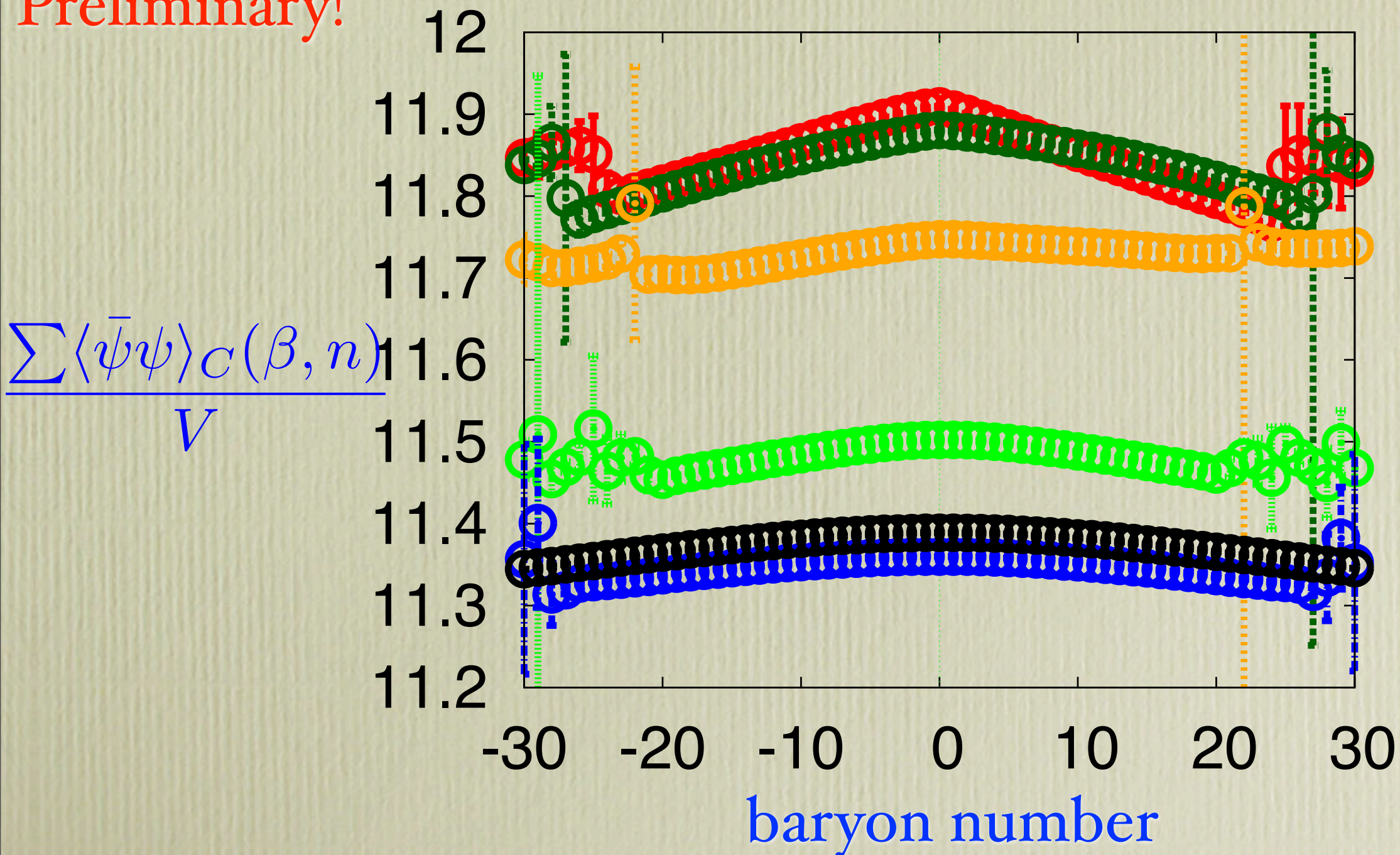
$$\longrightarrow \langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{O(\mu)}{Z(\mu)}$$

Hadronic observables

Chiral condensate in canonical ensemble

No renormalization! No subtraction! Sorry..

Preliminary!

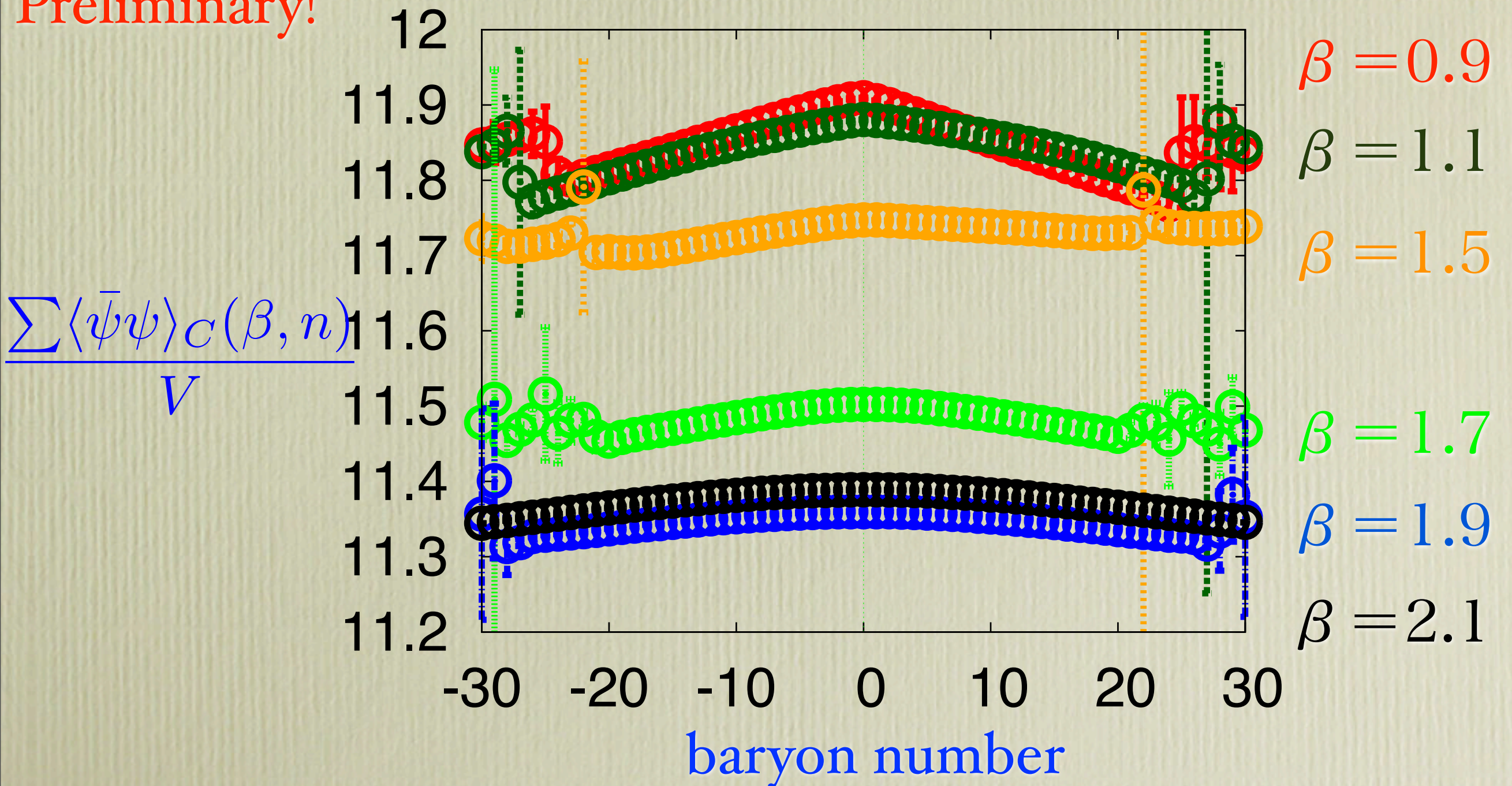


Hadronic observables

Chiral condensate in canonical ensemble

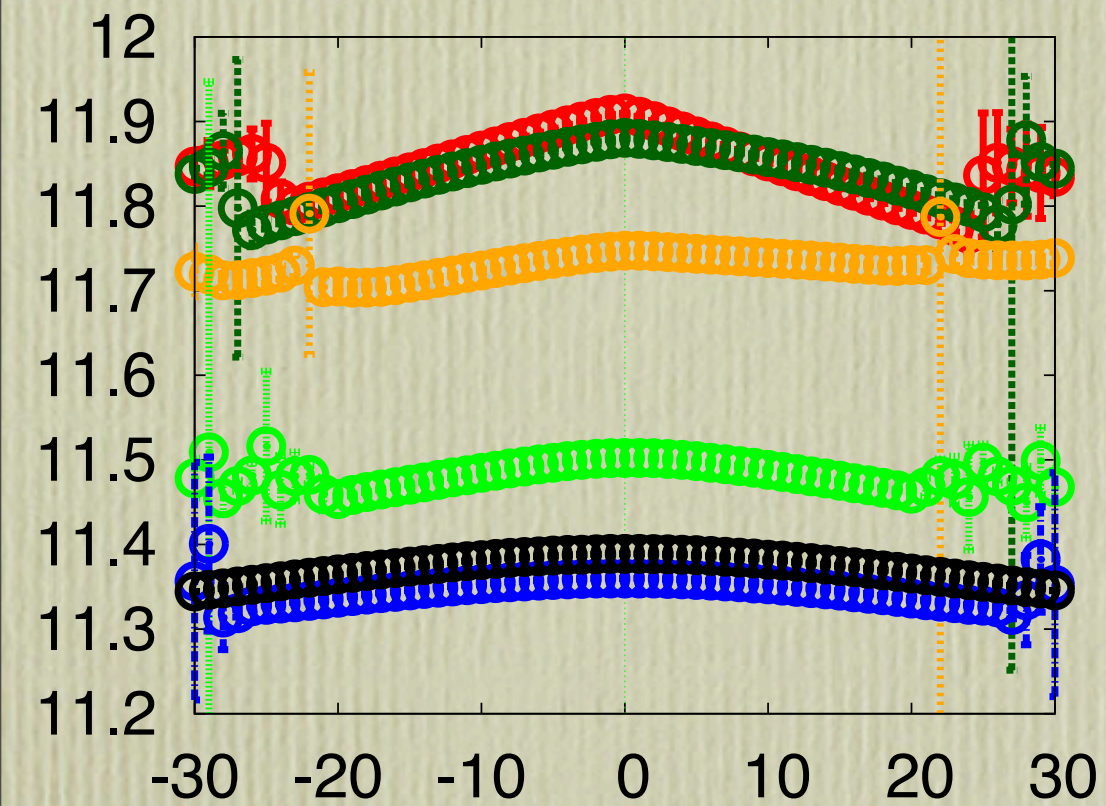
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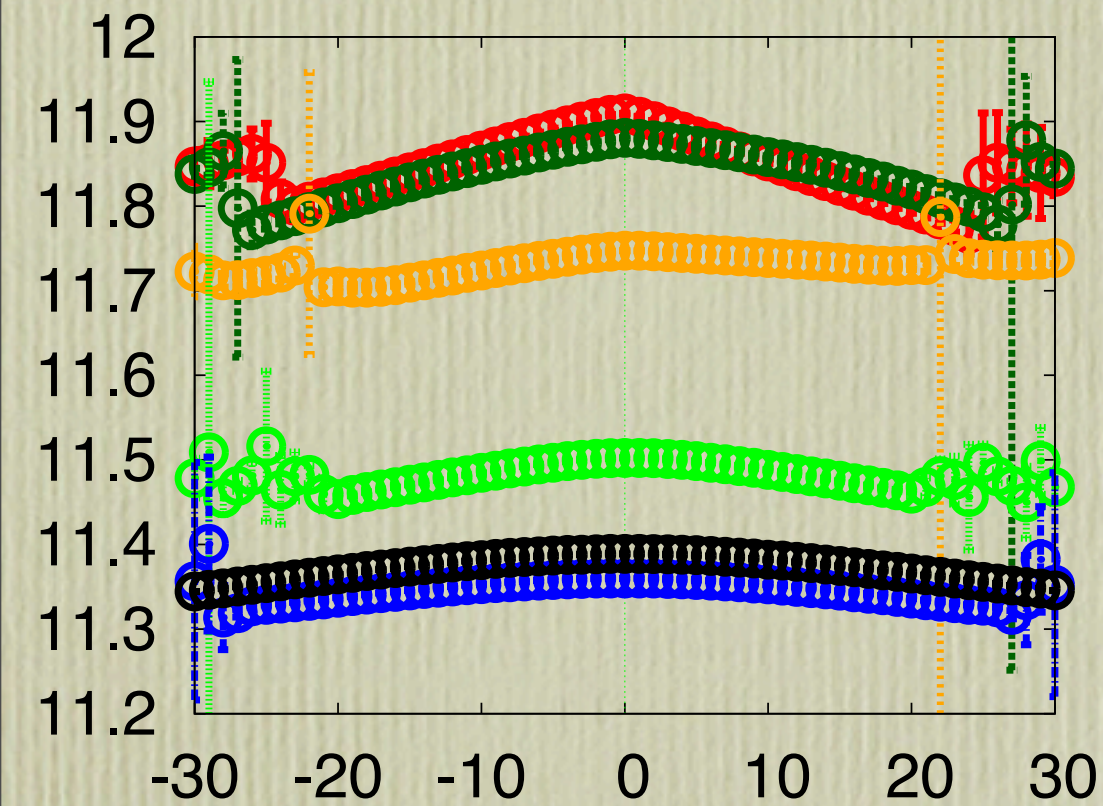
Hadronic observables

Chiral condensate in canonical ensemble



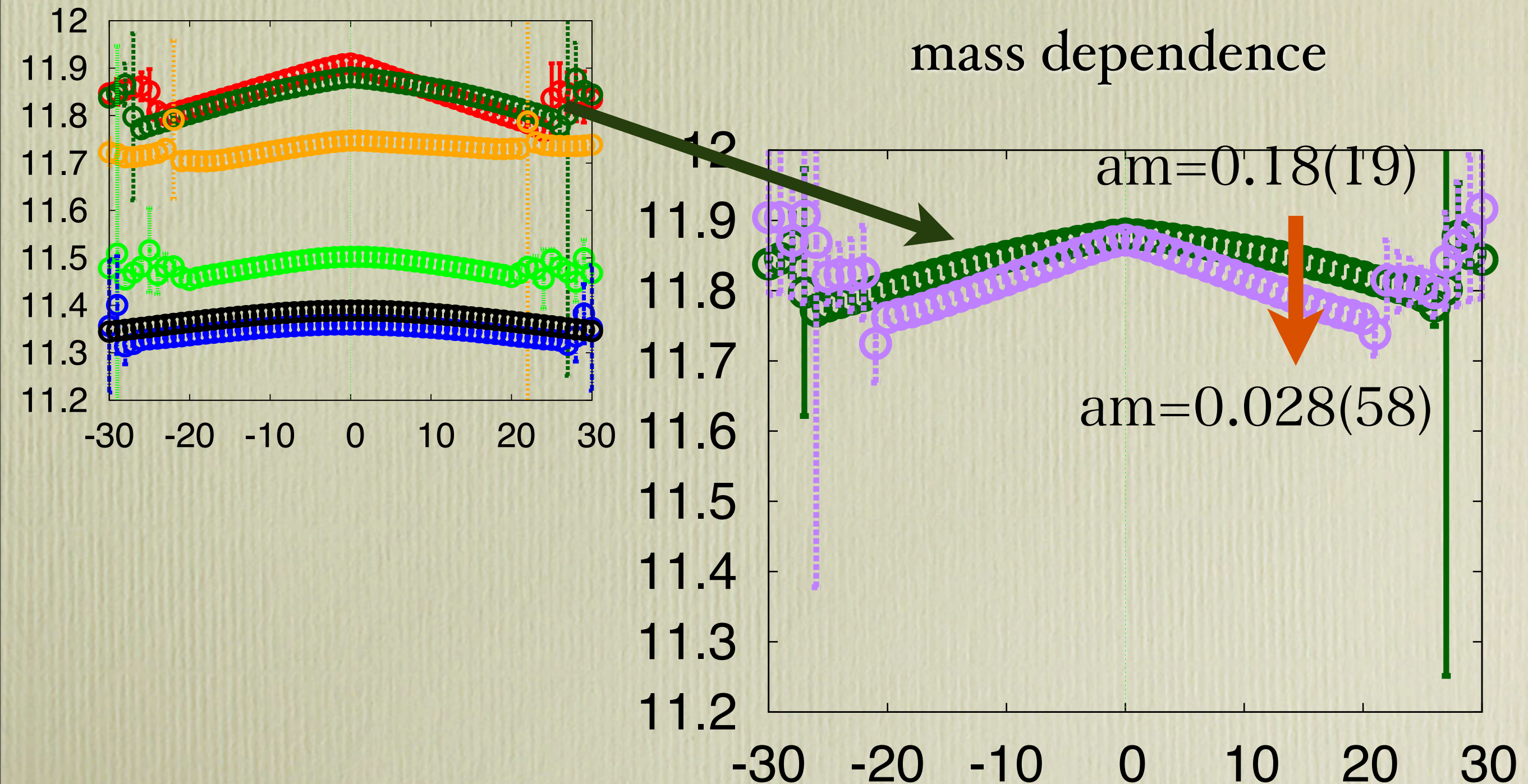
Hadronic observables

Chiral condensate in canonical ensemble



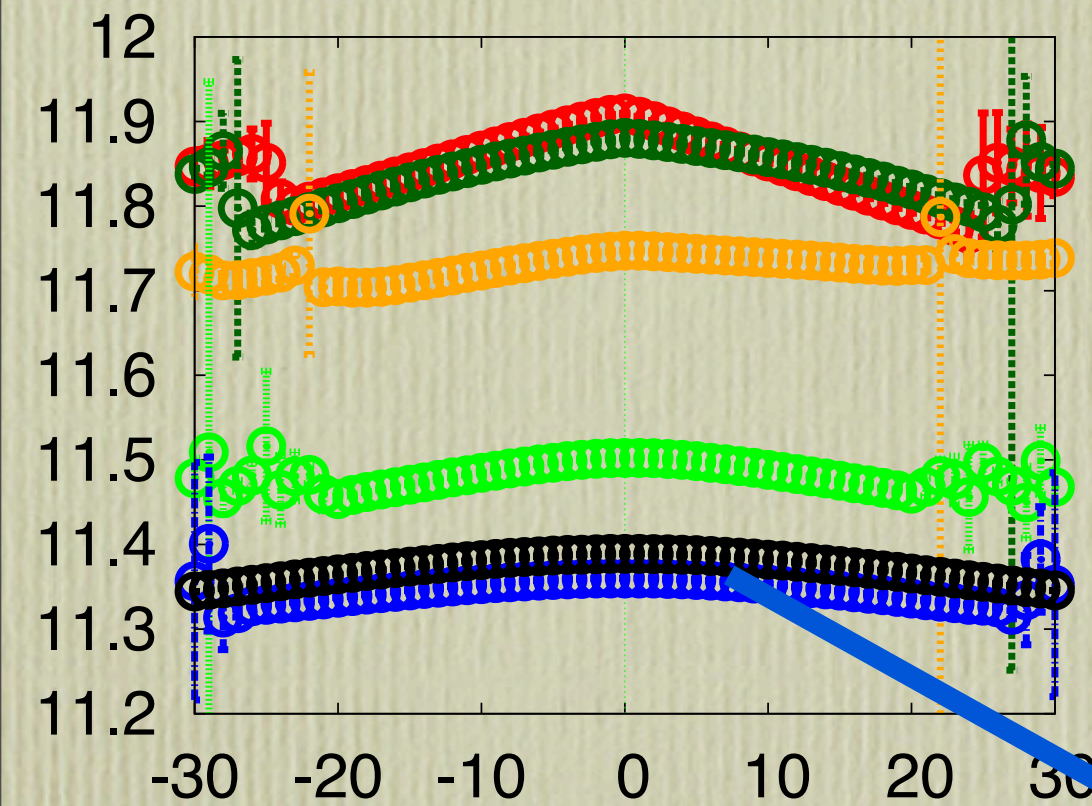
Hadronic observables

Chiral condensate in canonical ensemble

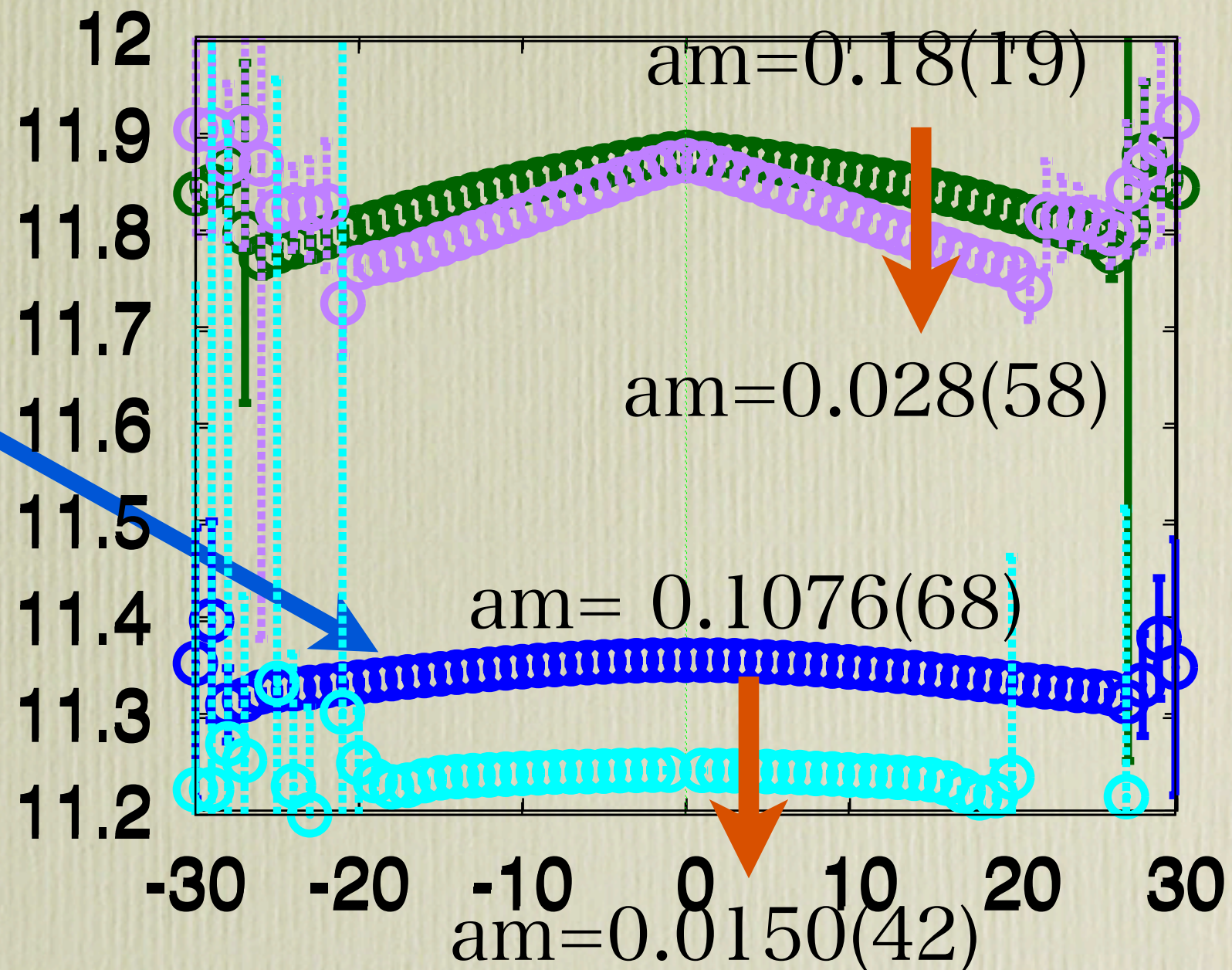


Hadronic observables

Chiral condensate in canonical ensemble



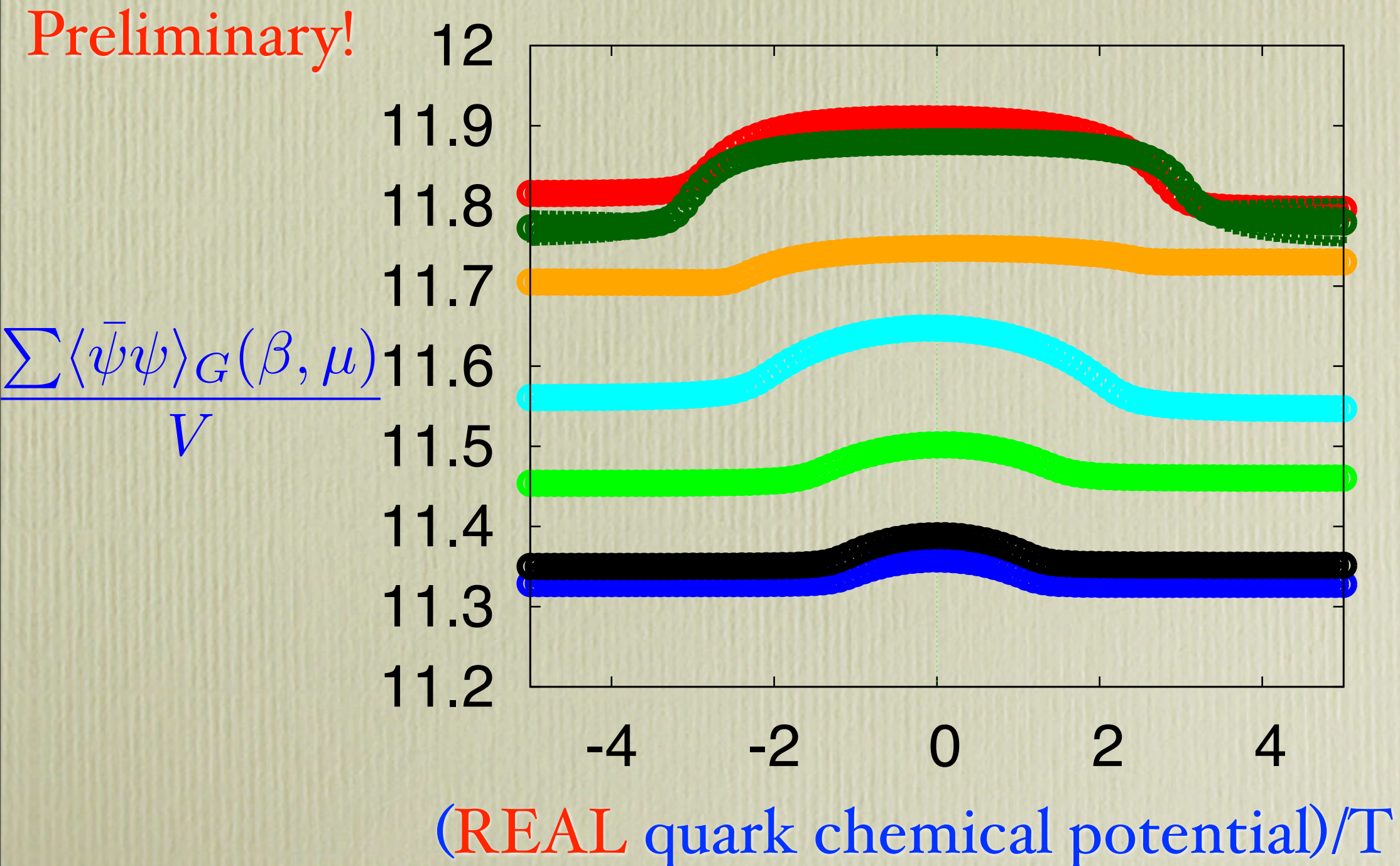
mass dependence



Hadronic observables

Chiral condensate in grand canonical ensemble

No renormalization! No subtraction! Sorry..

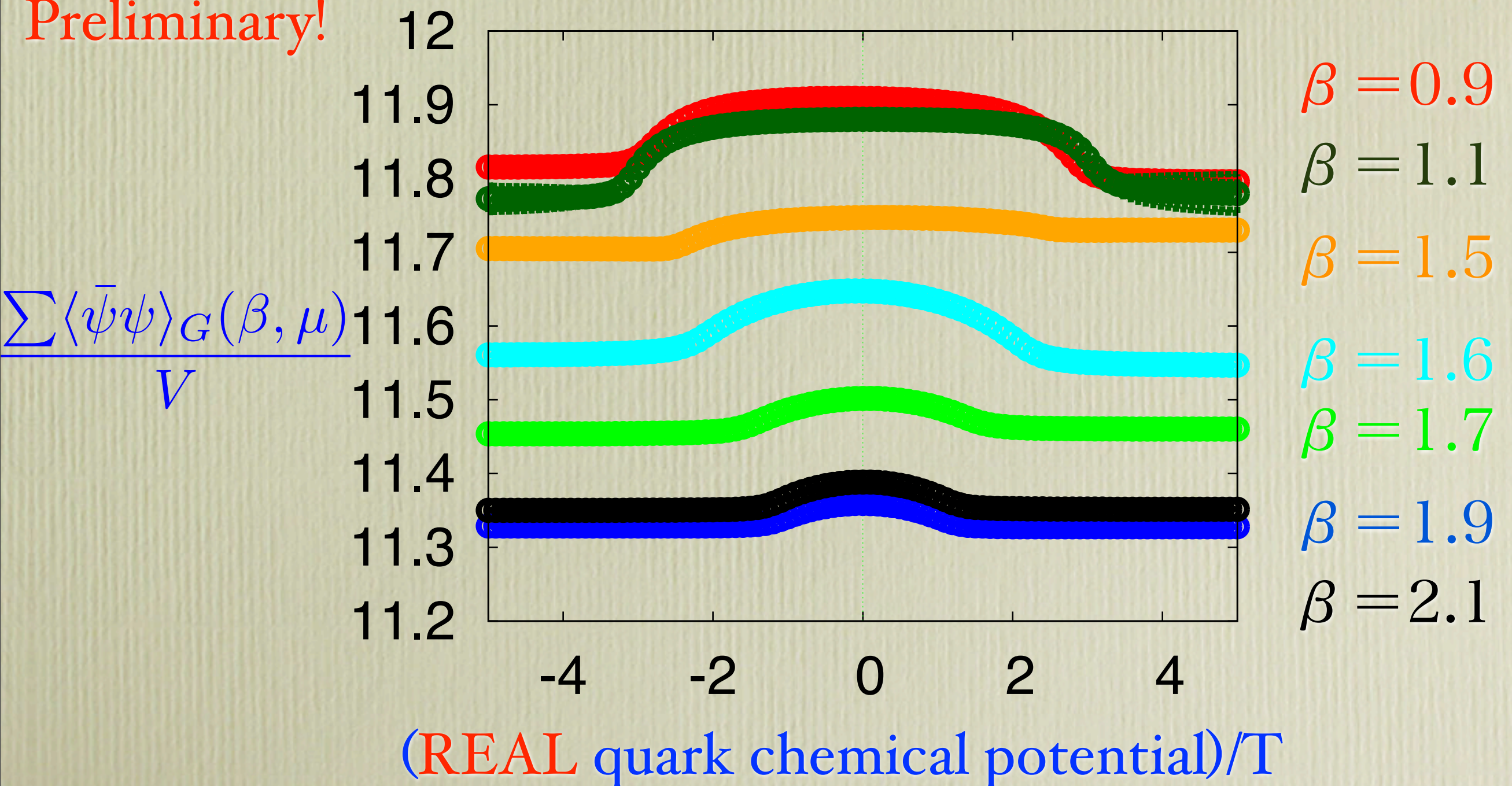


Hadronic observables

Chiral condensate in grand canonical ensemble

No renormalization! No subtraction! Sorry..

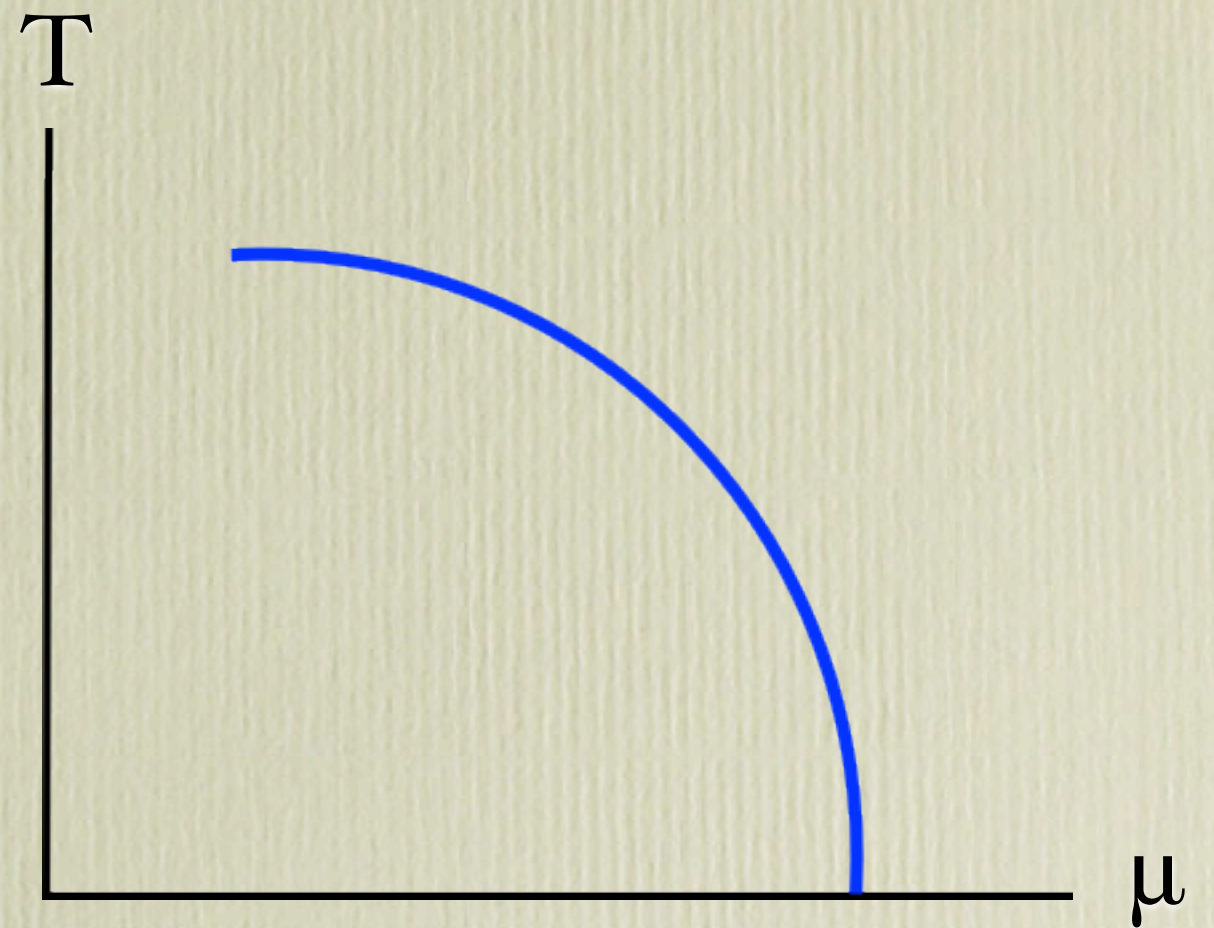
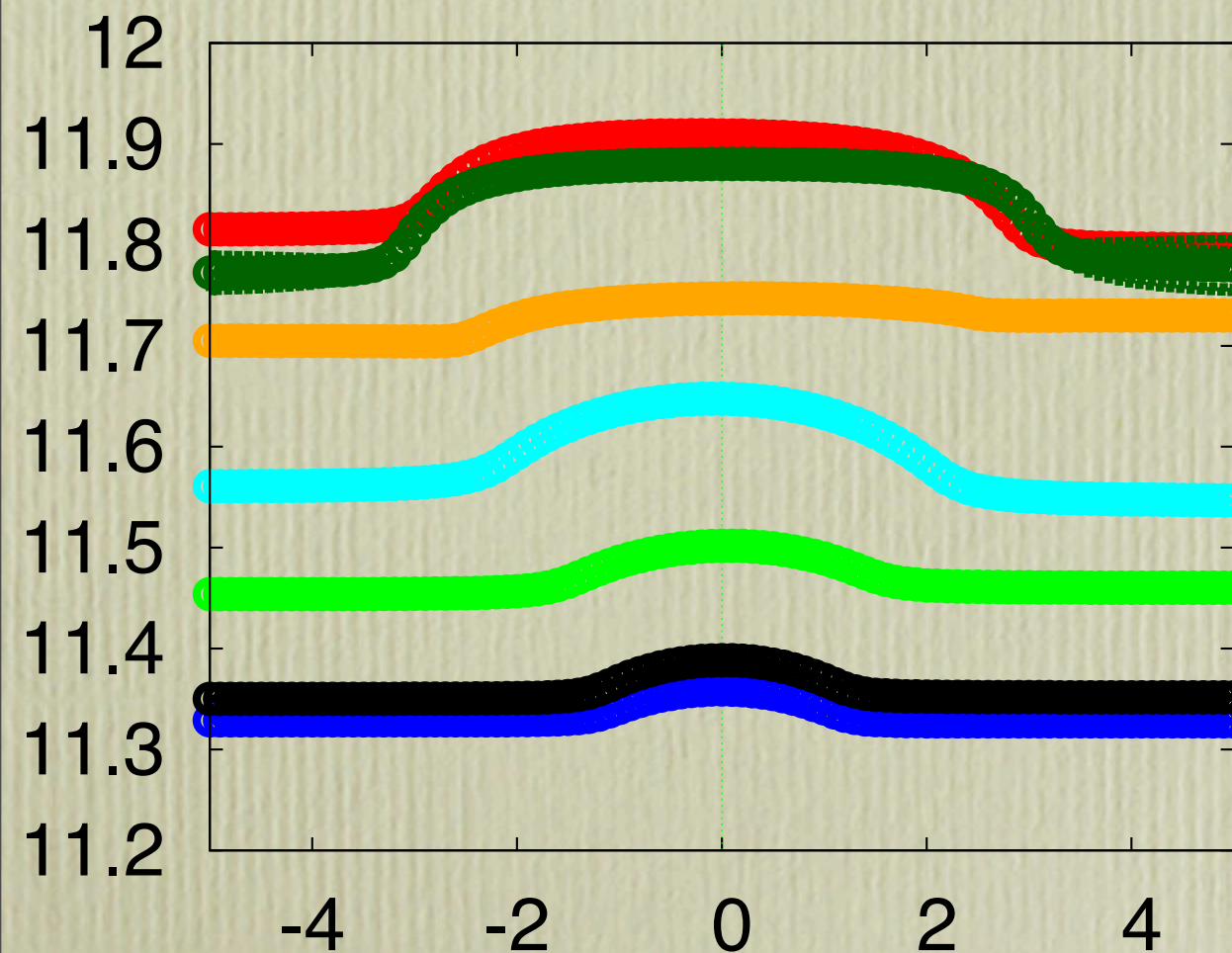
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

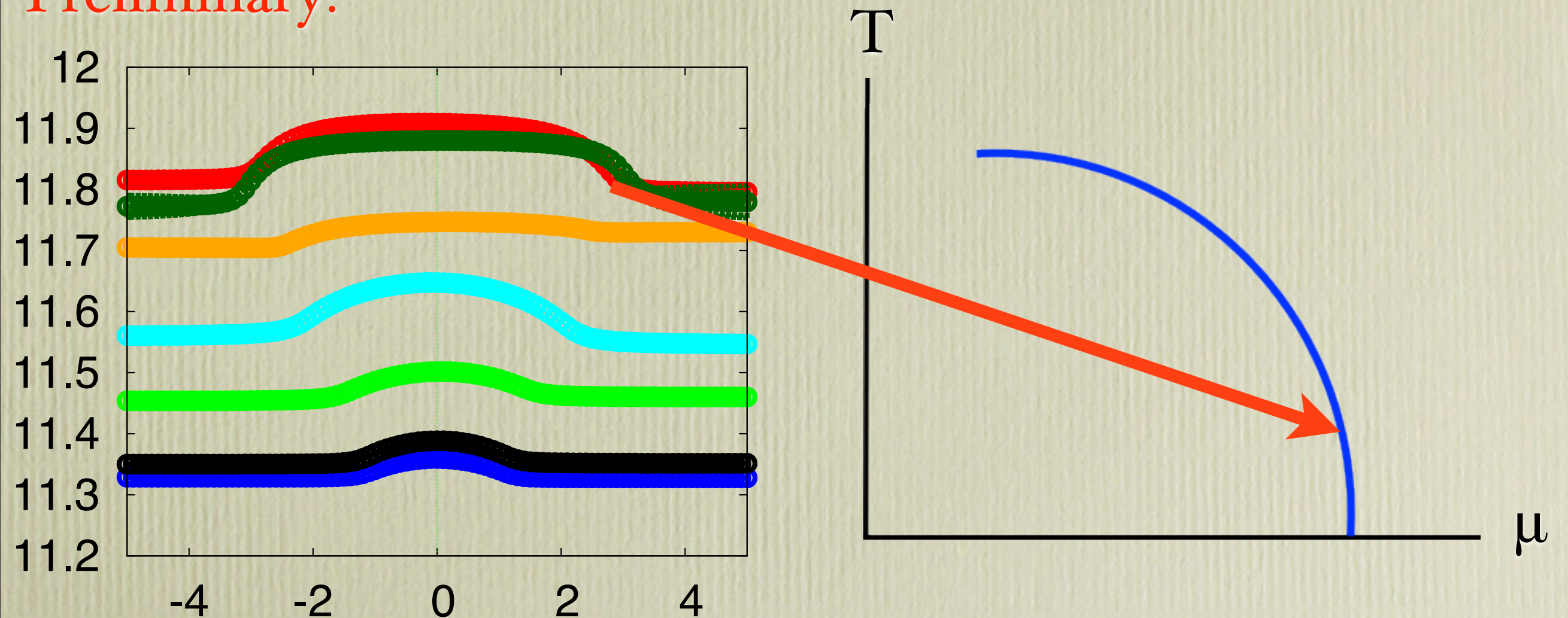
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

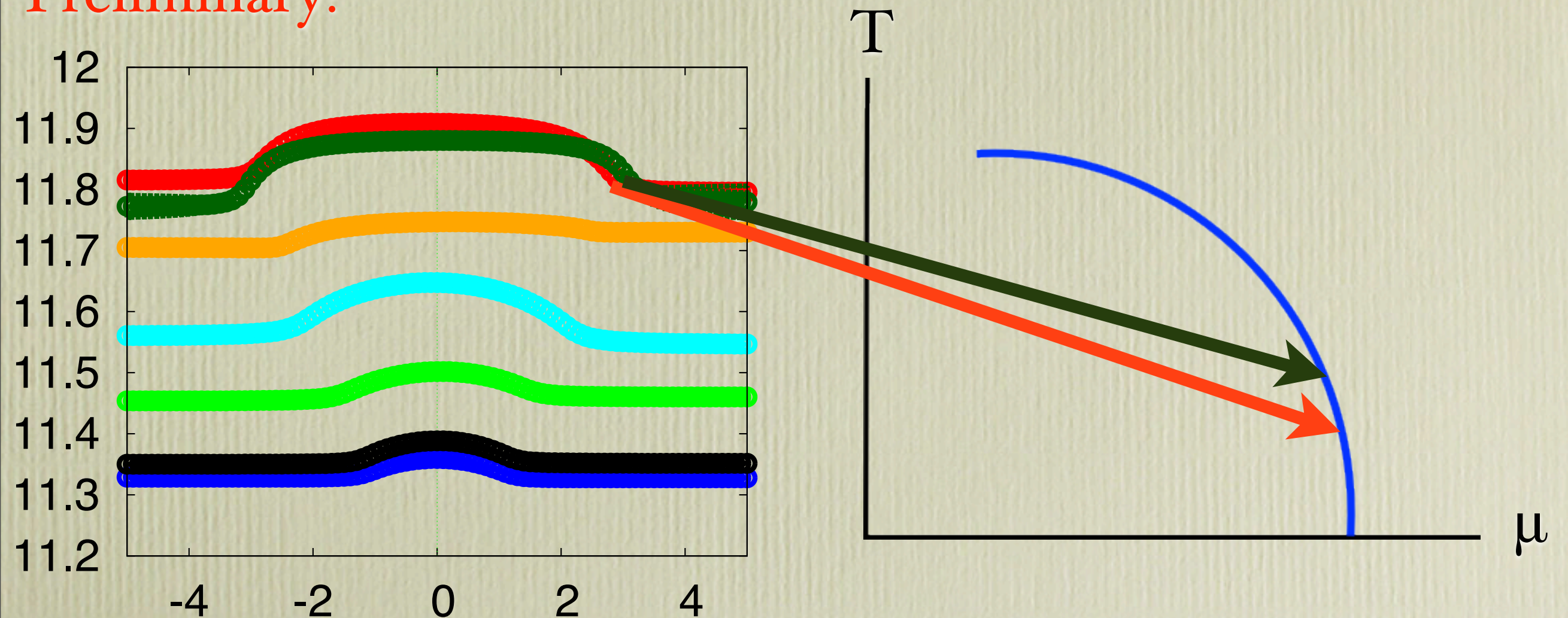
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

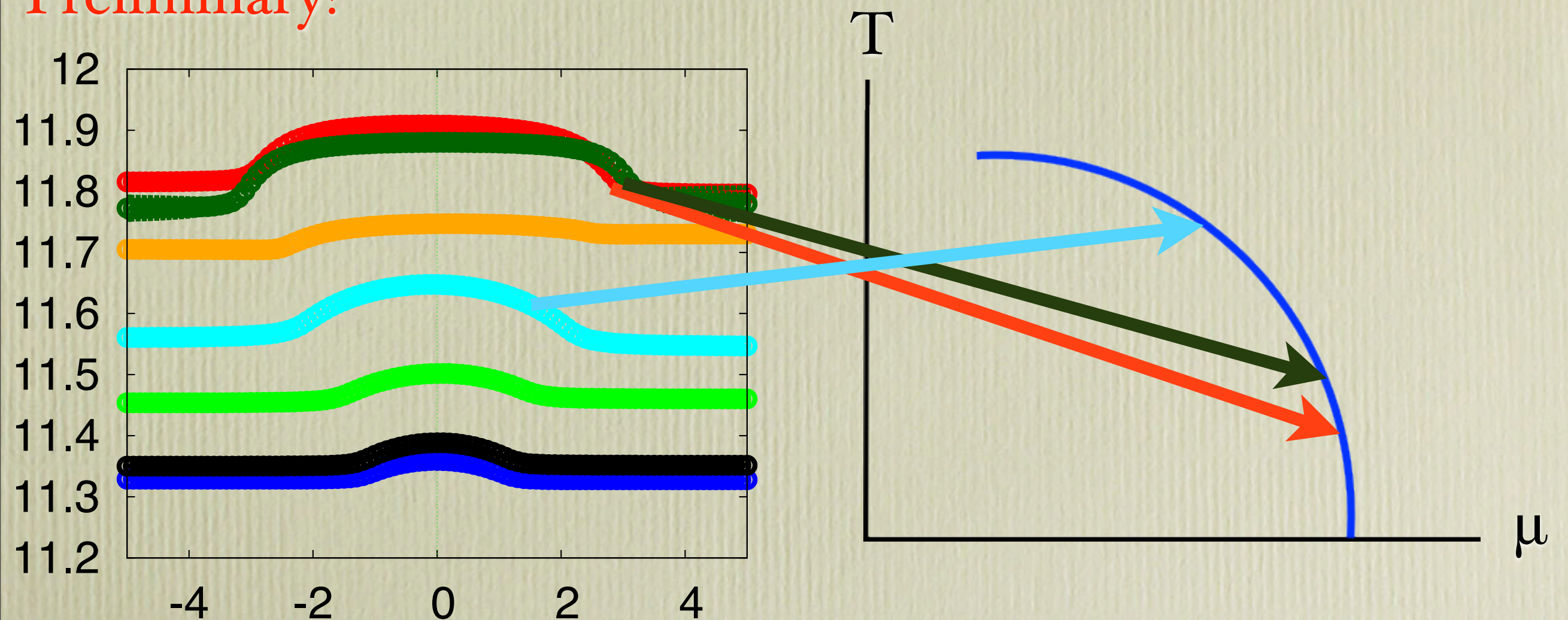
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

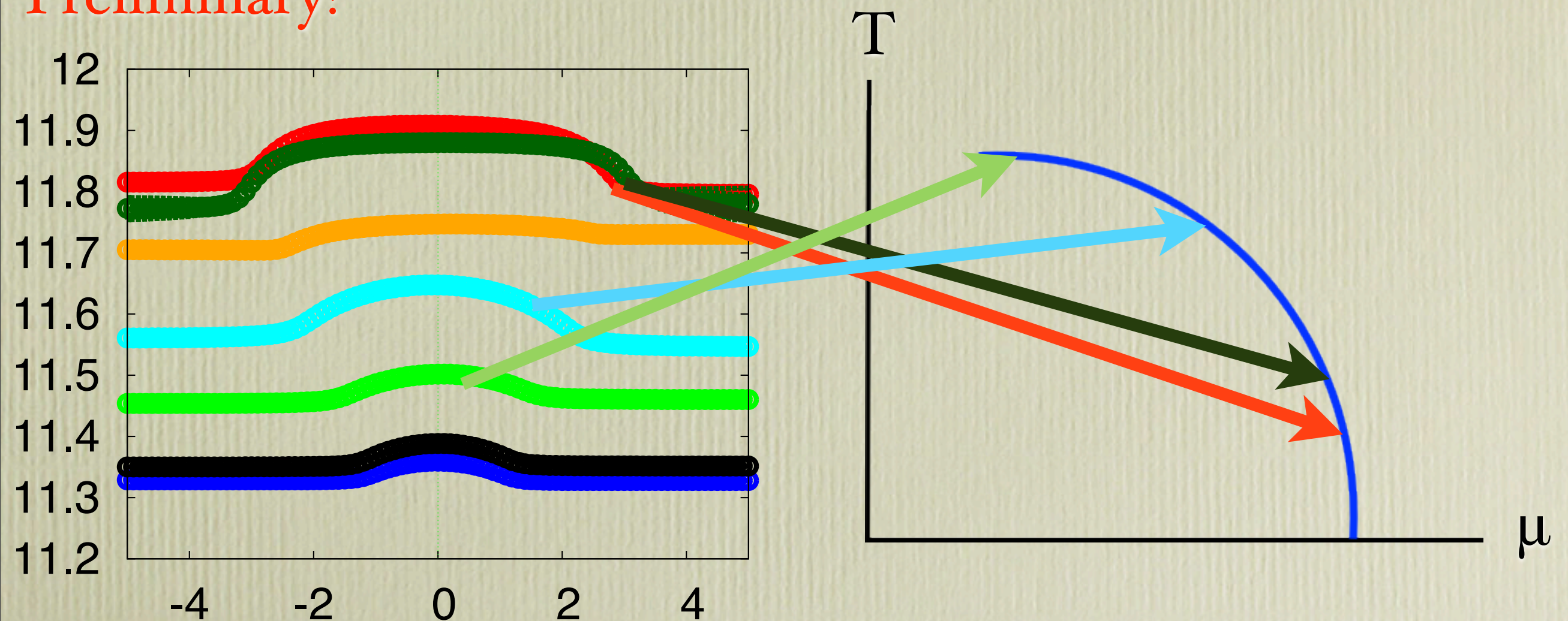
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

Preliminary!

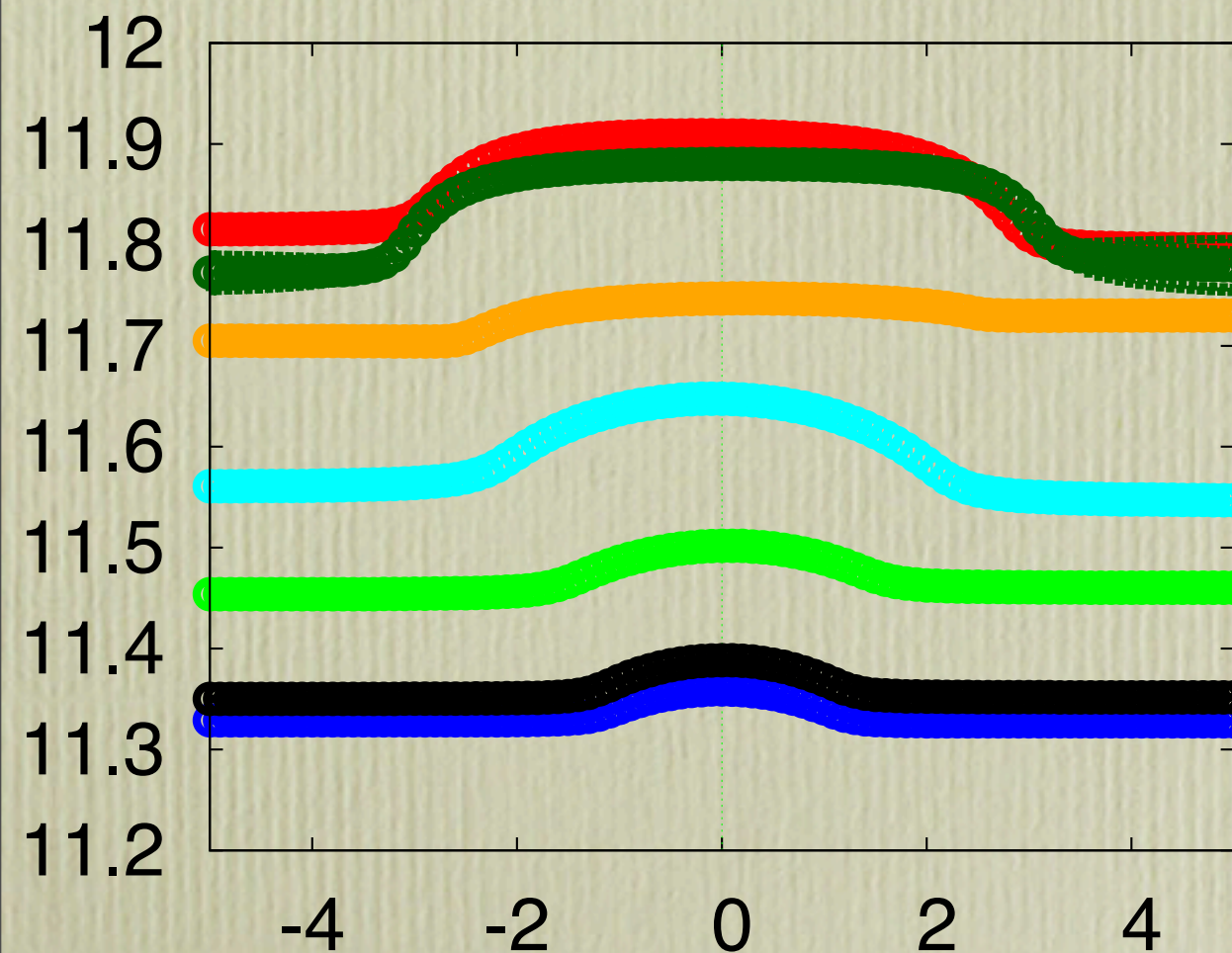


Hadronic observables

Chiral condensate in grand canonical ensemble

mass dependence

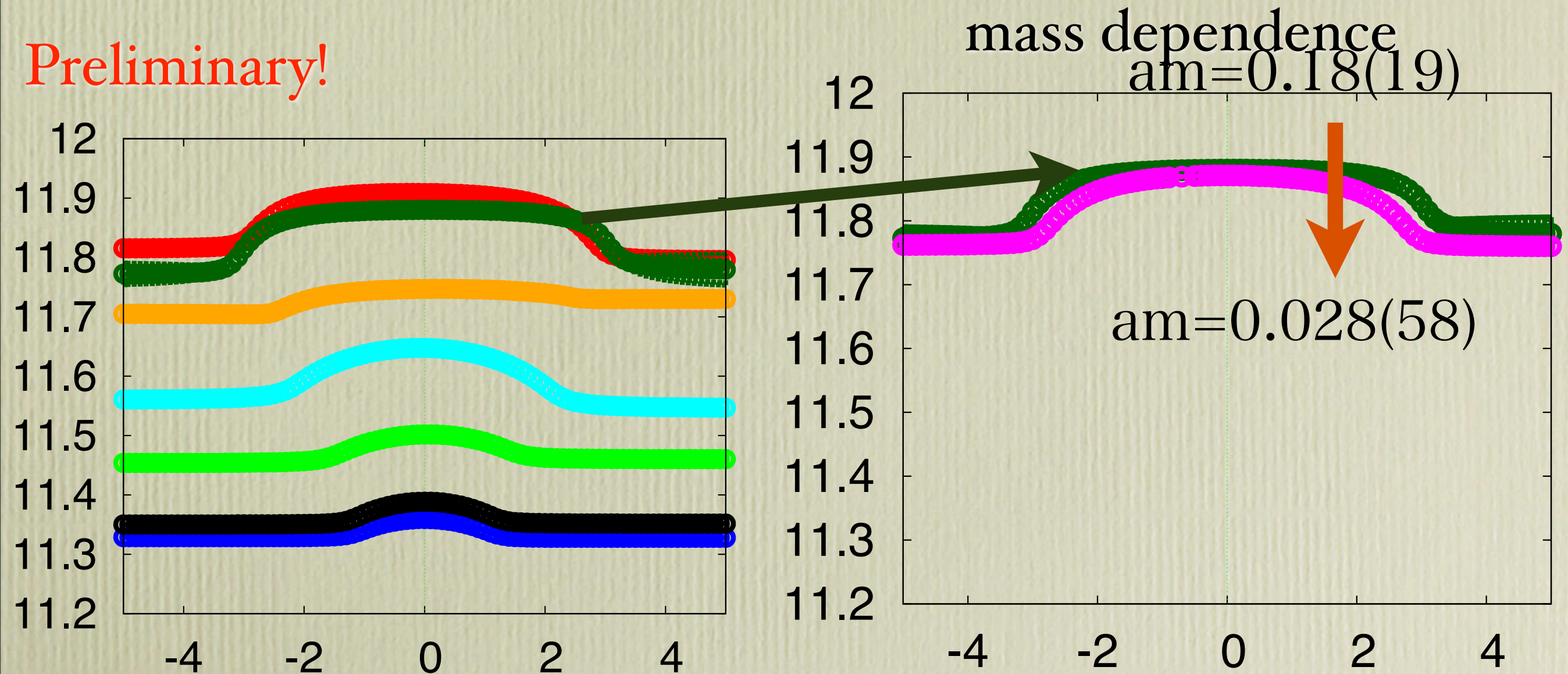
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

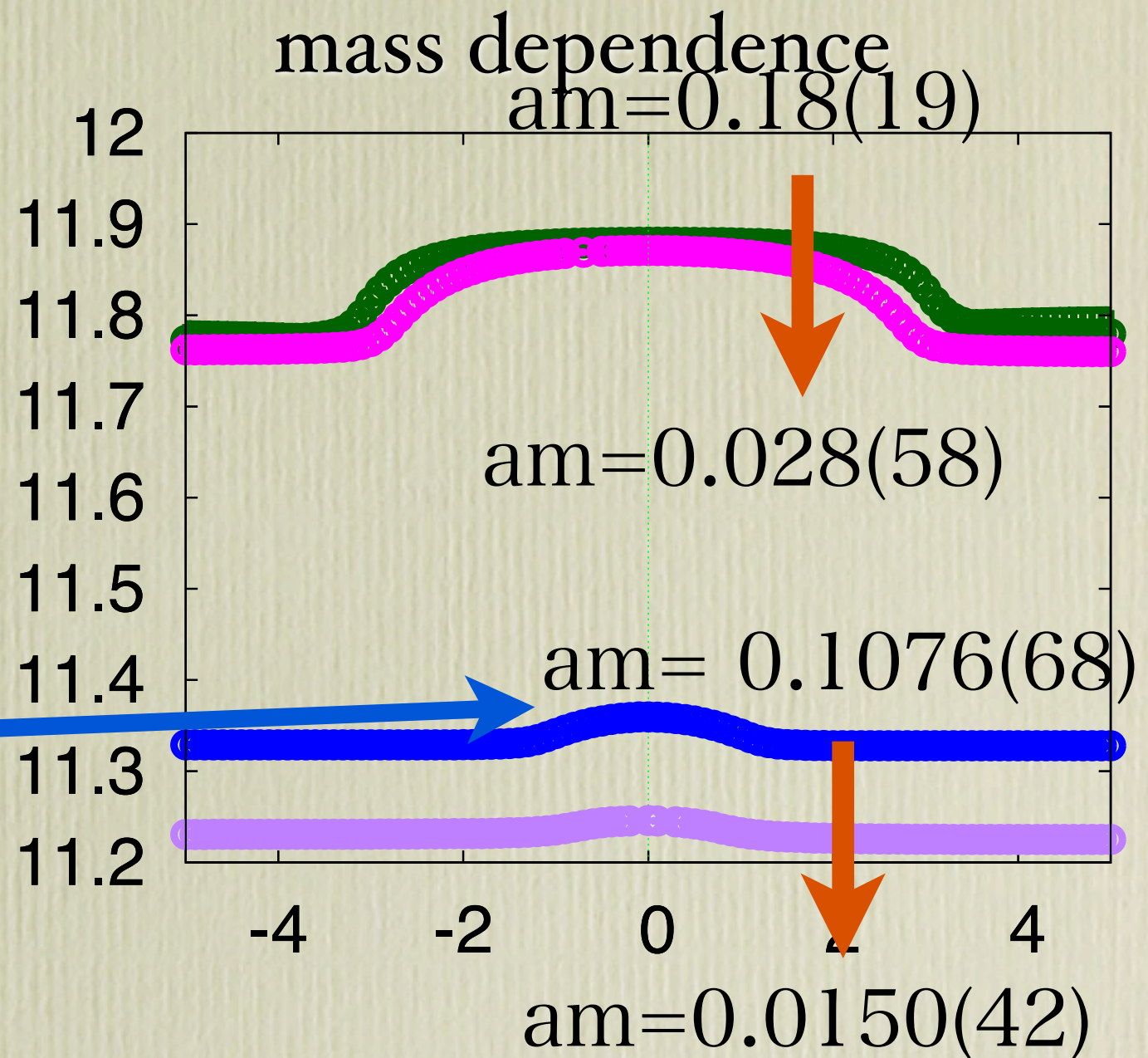
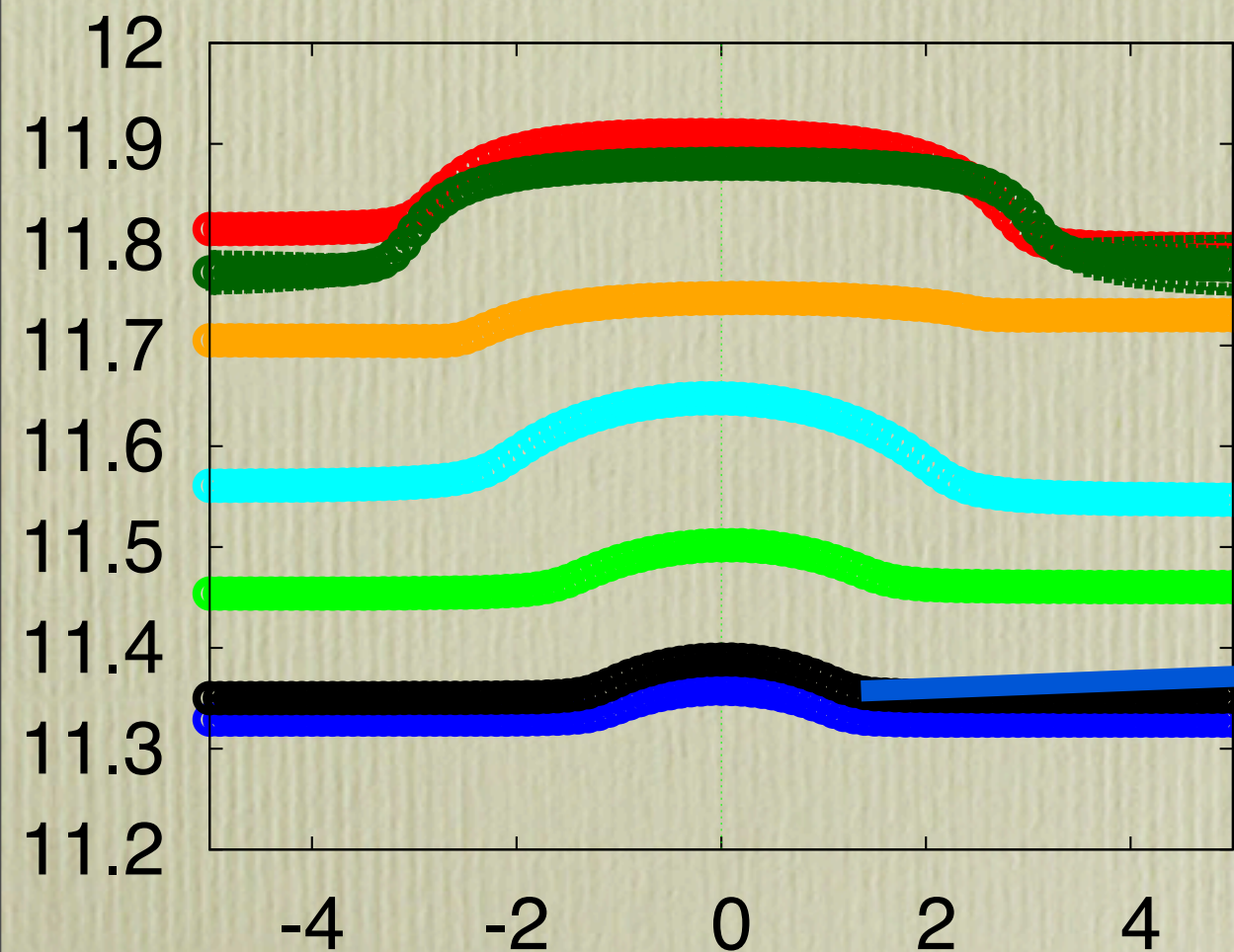
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

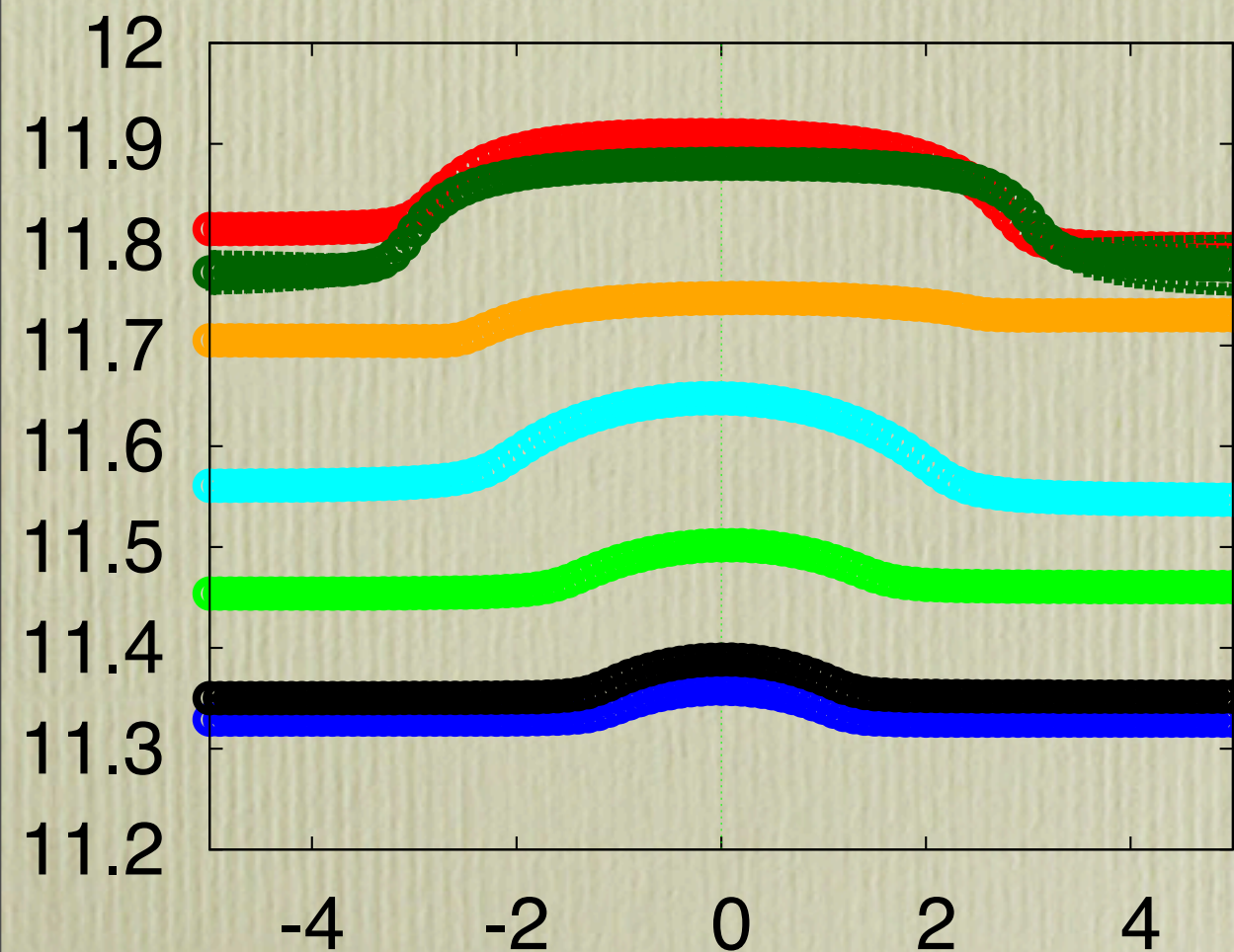
Preliminary!



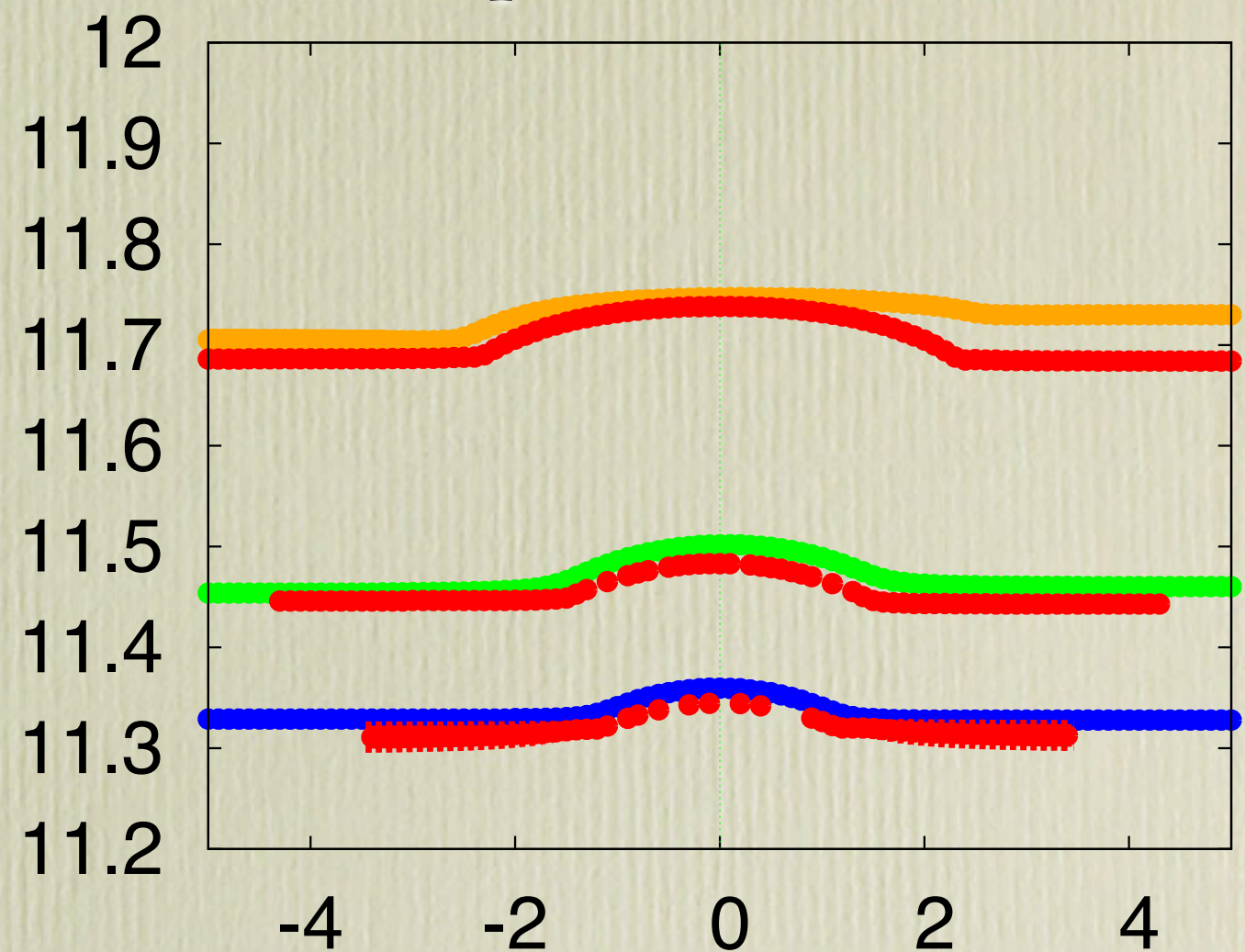
Hadronic observables

Chiral condensate in grand canonical ensemble

Preliminary!



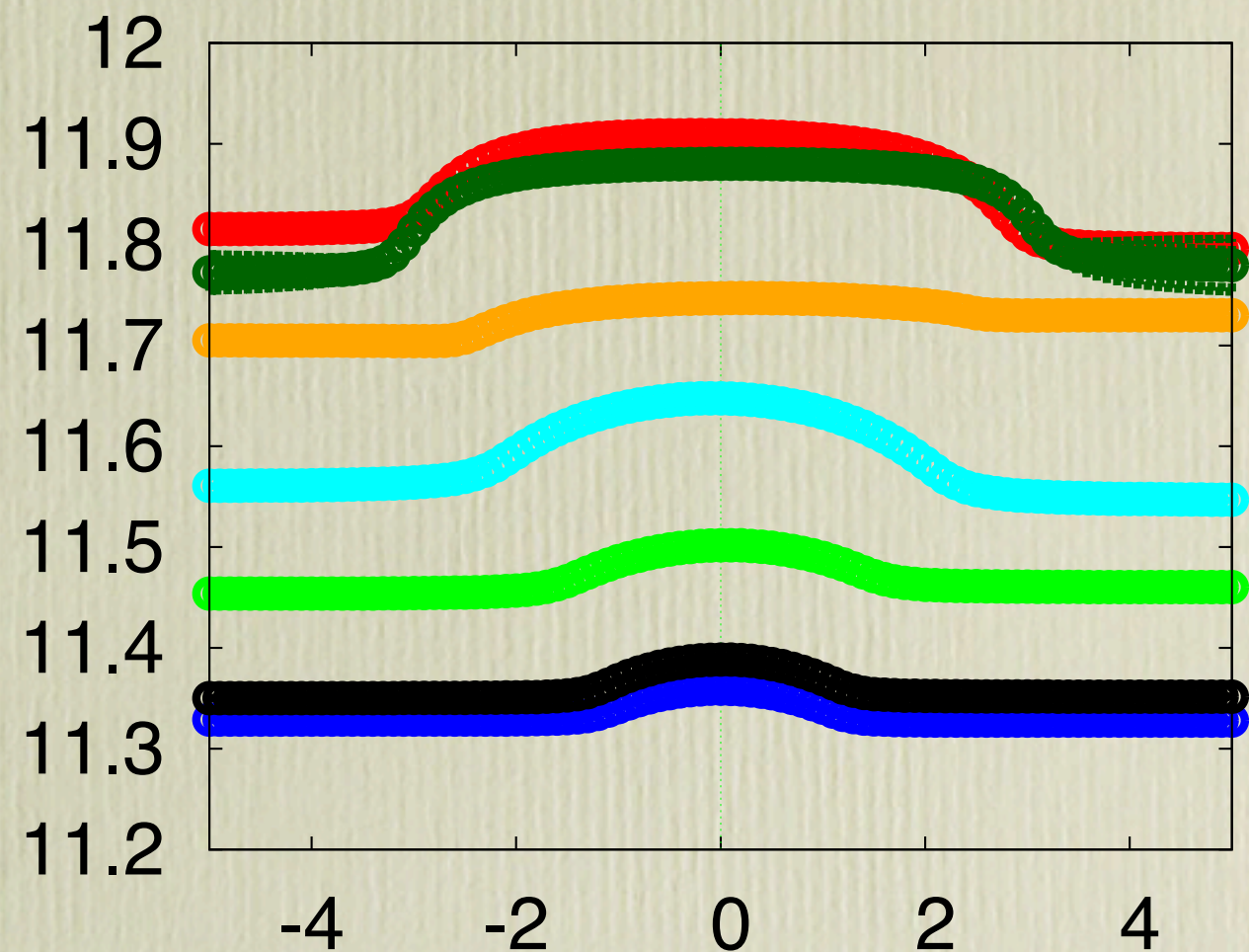
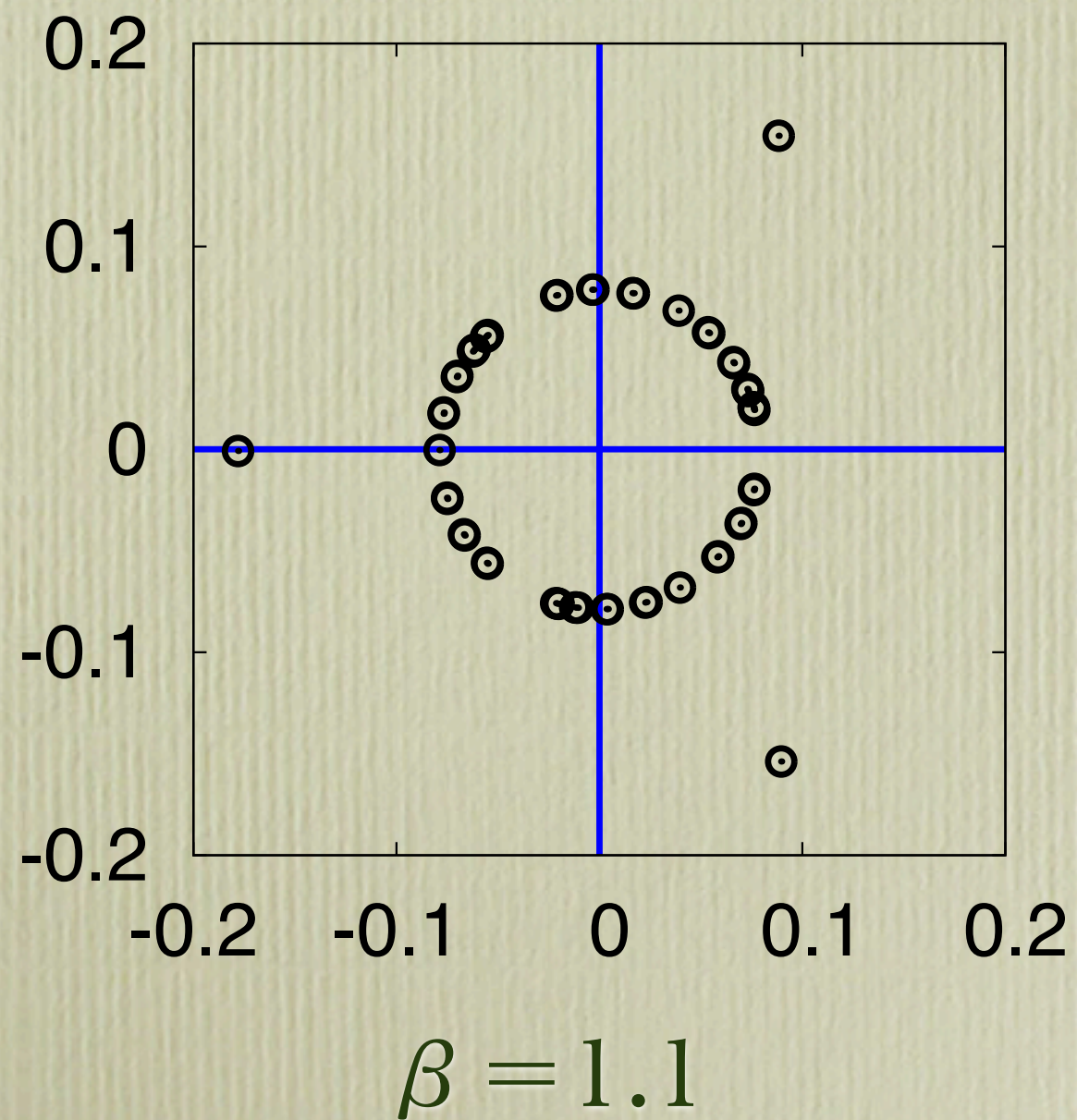
volume dependence $8 \rightarrow 1$ 2



Hadronic observables

Chiral condensate in grand canonical ensemble

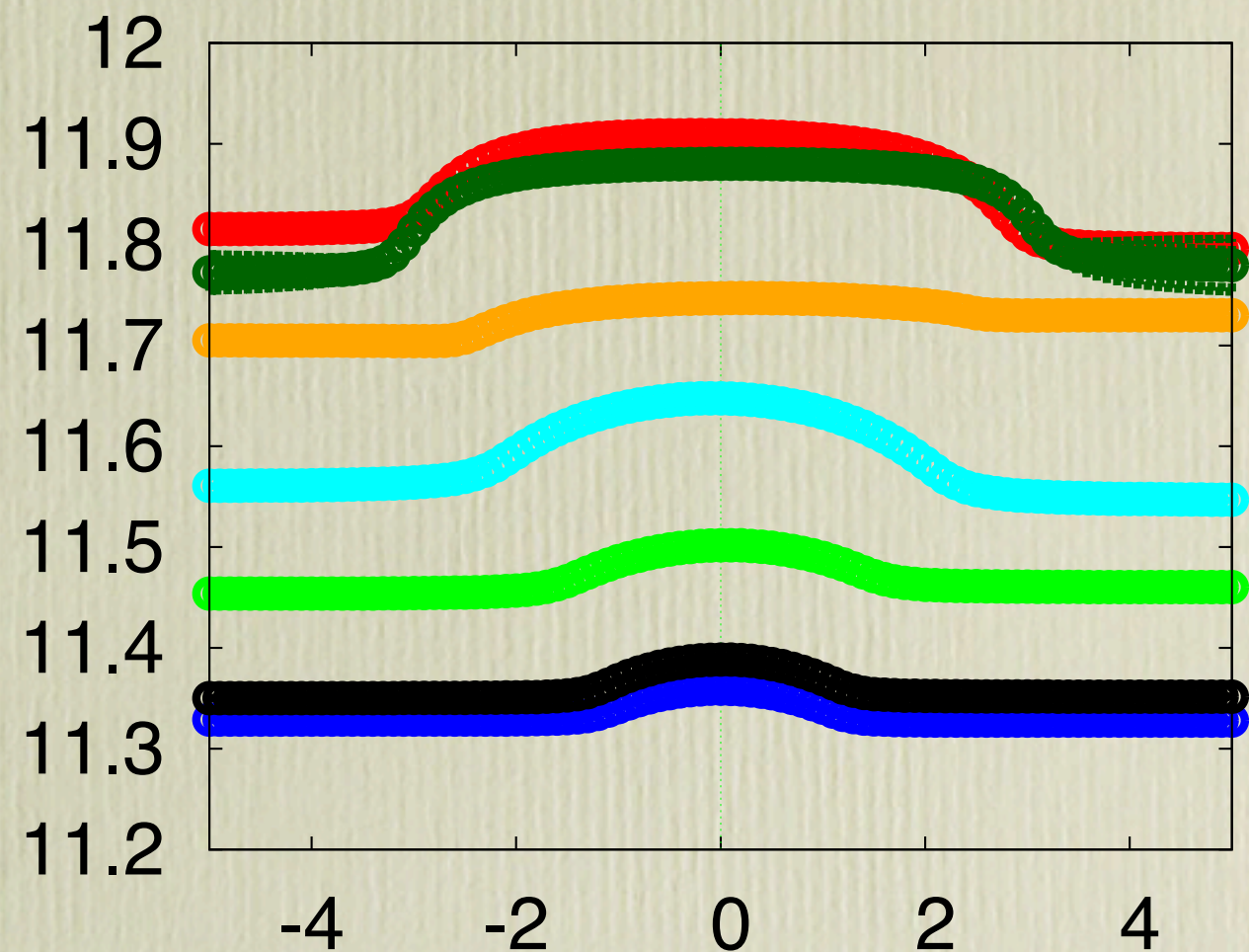
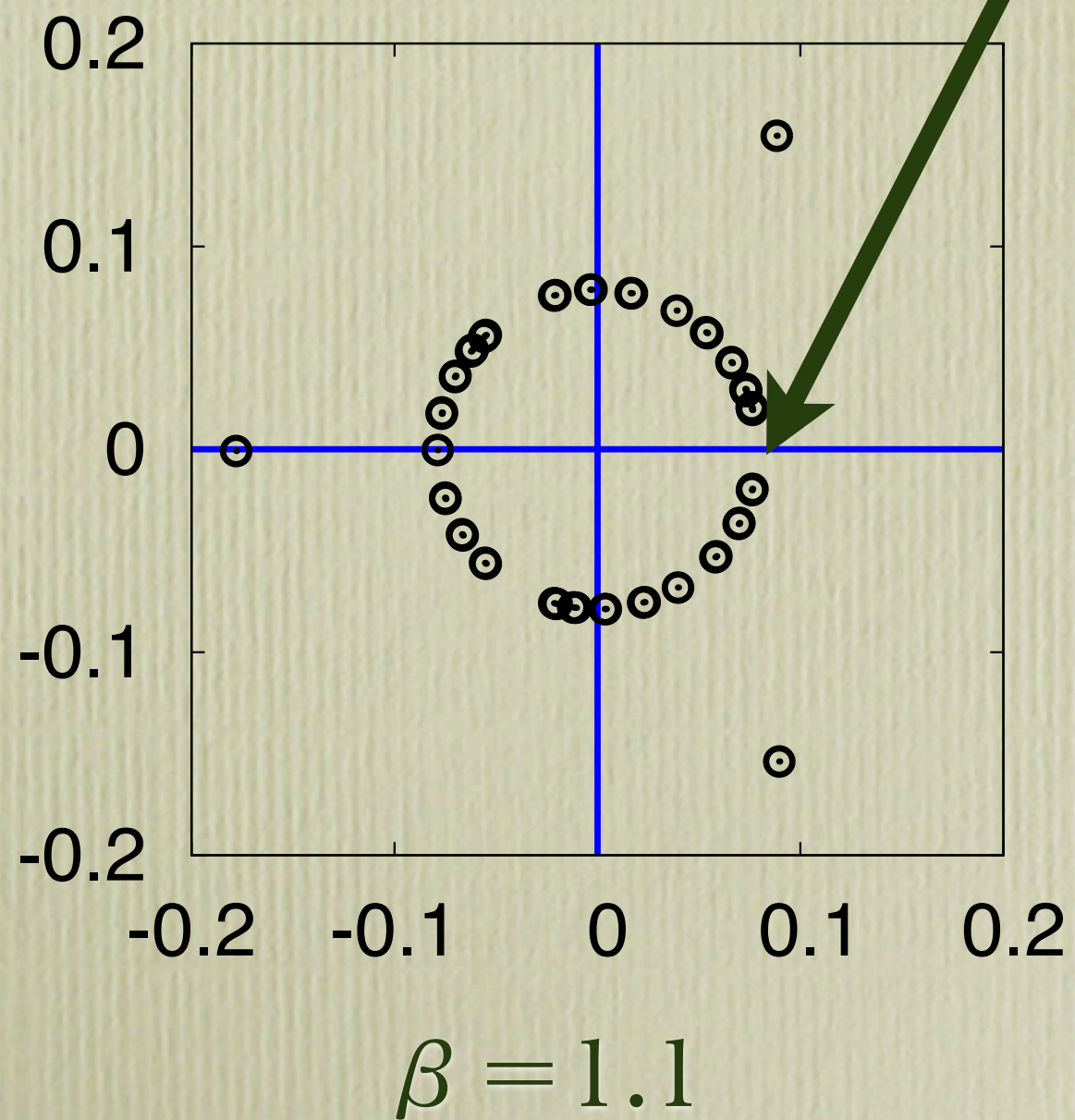
Lee-Yang zeros



Hadronic observables

Chiral condensate in grand canonical ensemble

Lee-Yang zeros $\xi \sim 0.076$

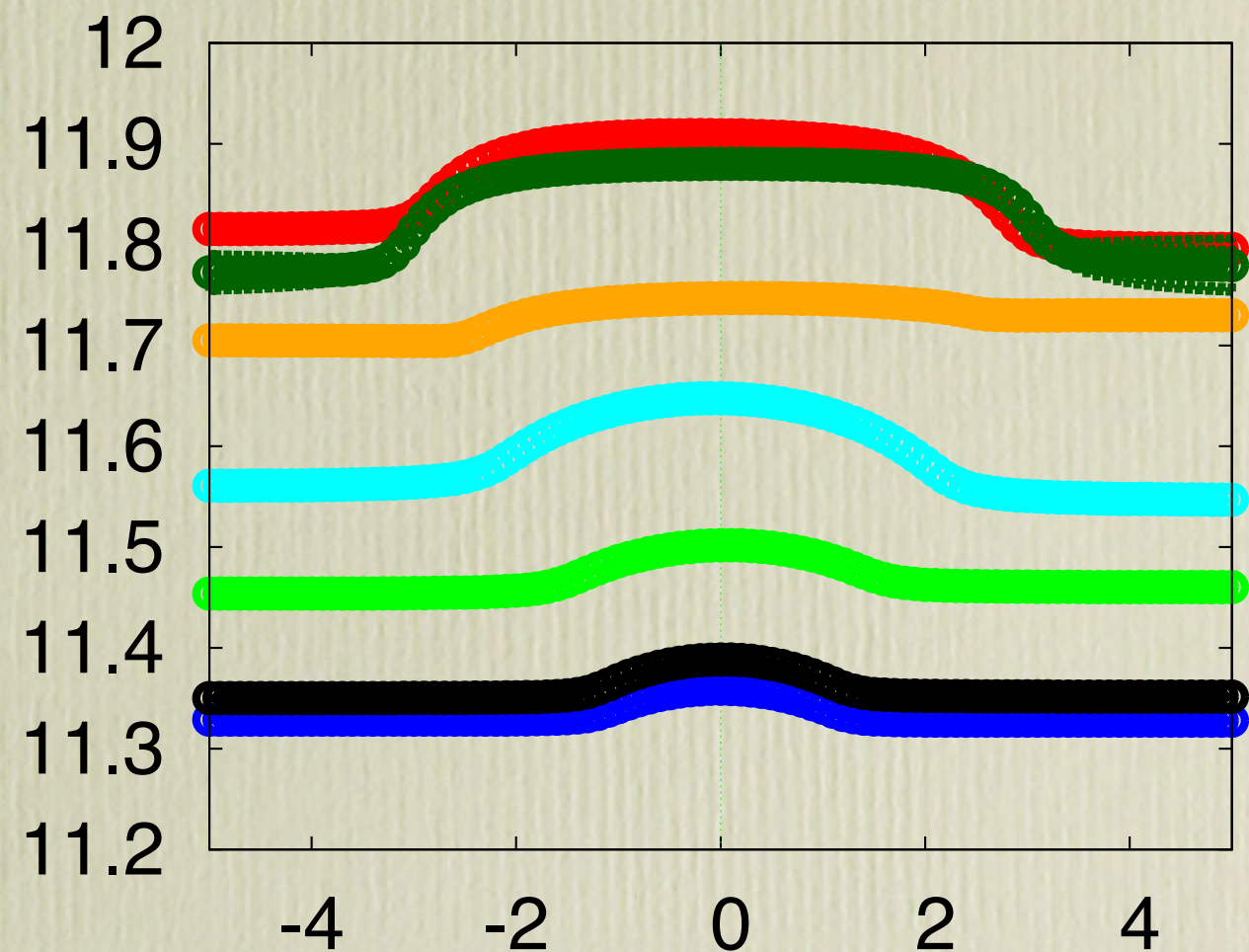
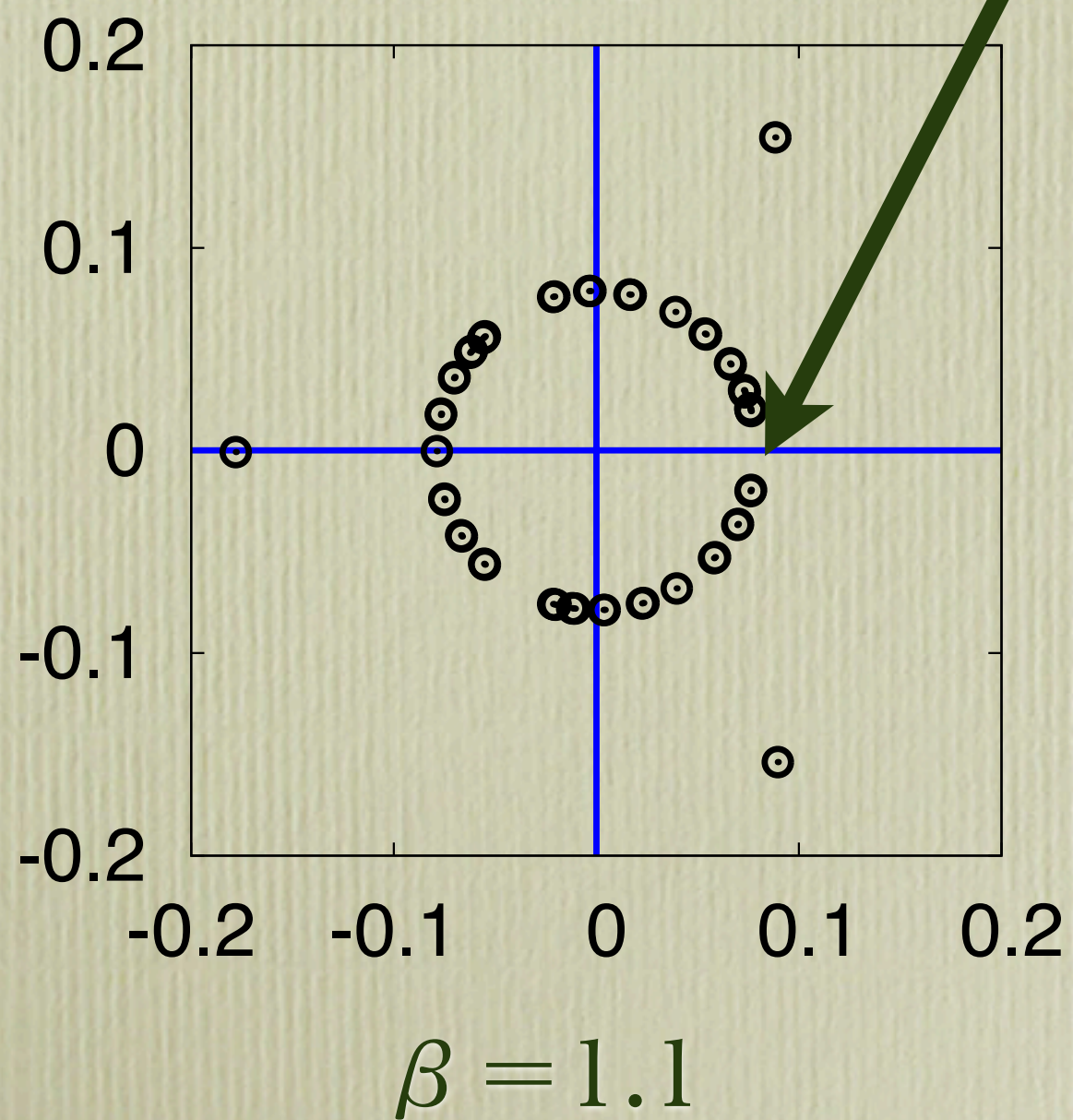


Hadronic observables

Chiral condensate in grand canonical ensemble

Lee-Yang zeros

$\xi \sim 0.076$ $\longleftrightarrow$ $\mu \sim -2.5$

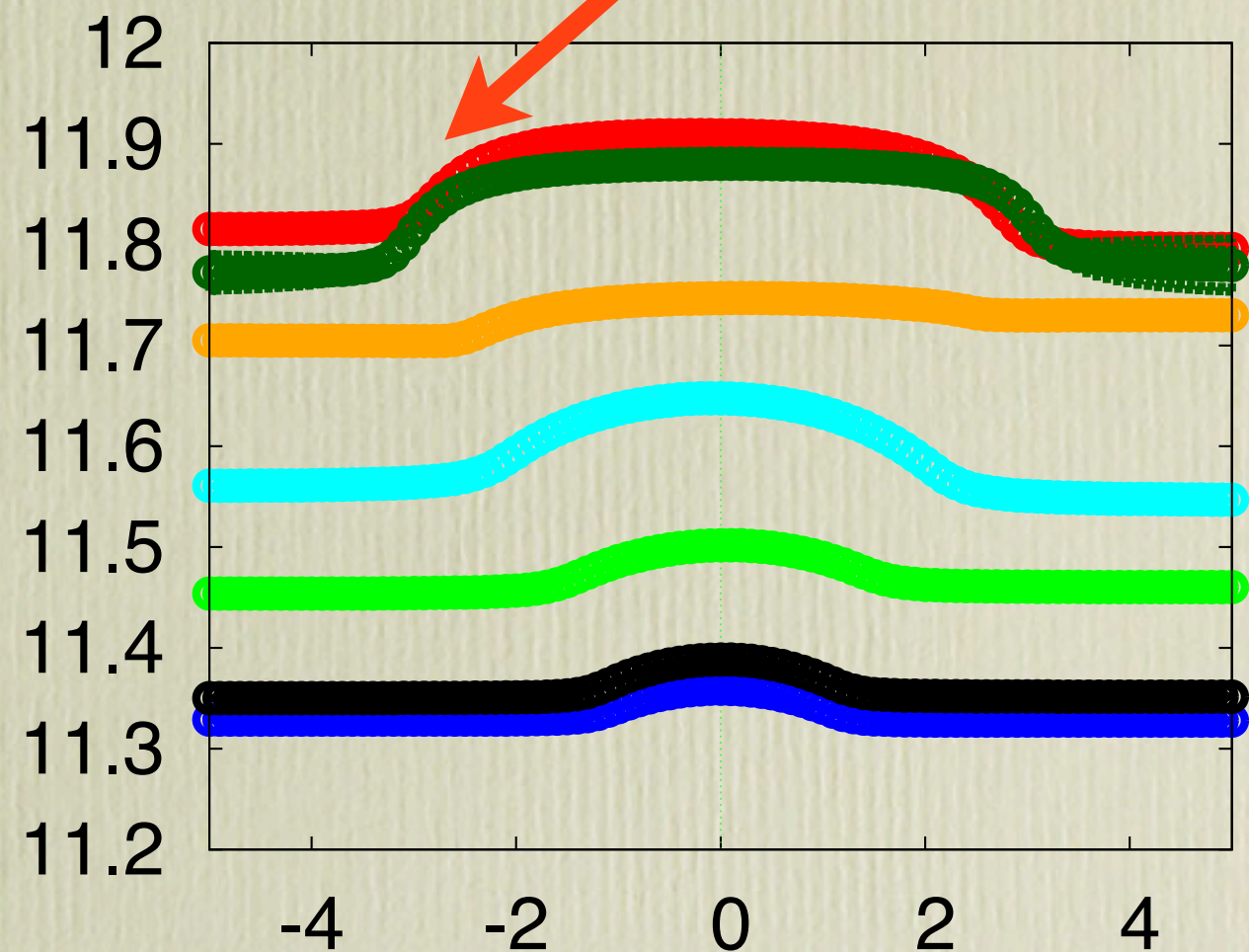
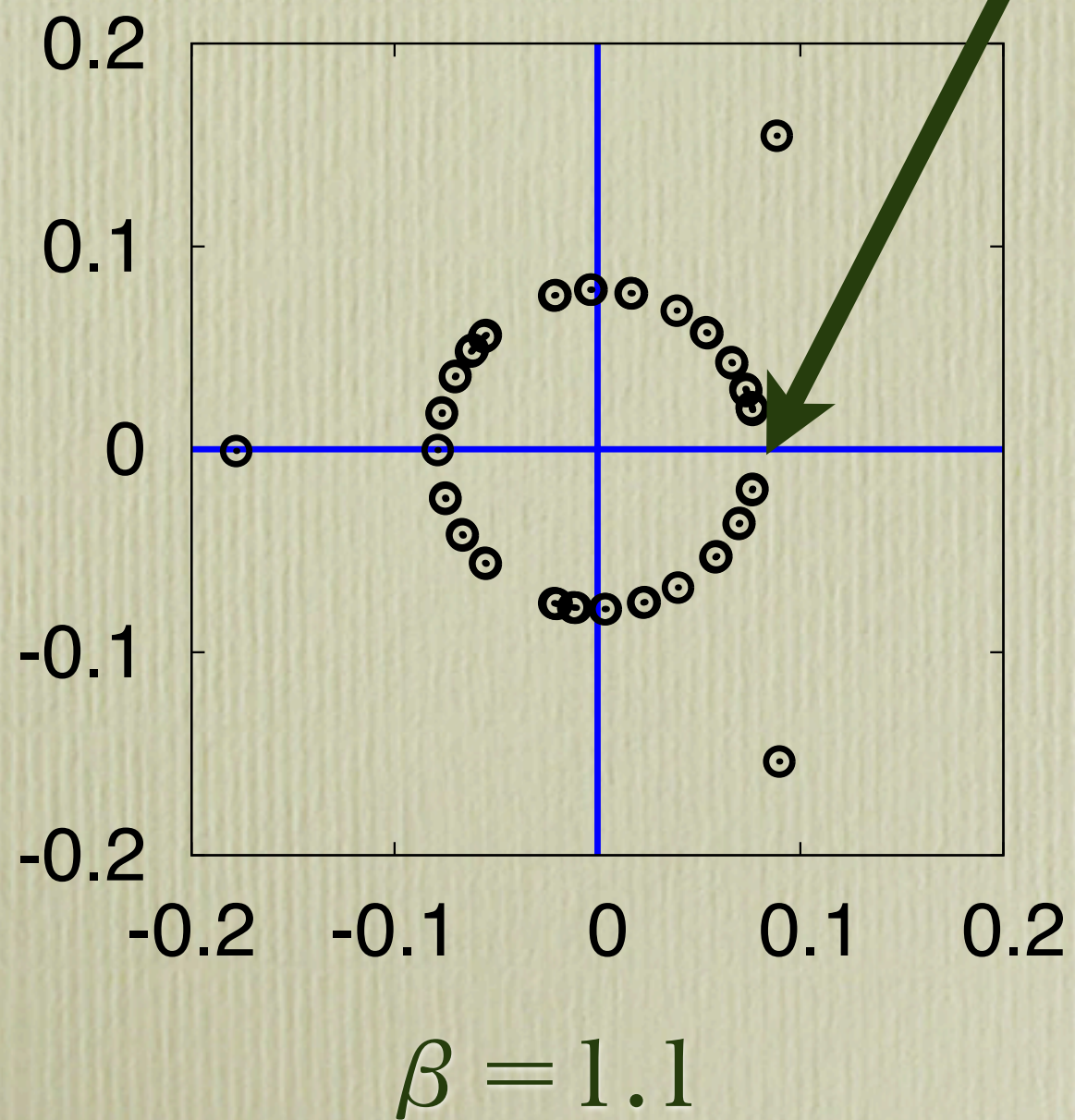


Hadronic observables

Chiral condensate in grand canonical ensemble

Lee-Yang zeros

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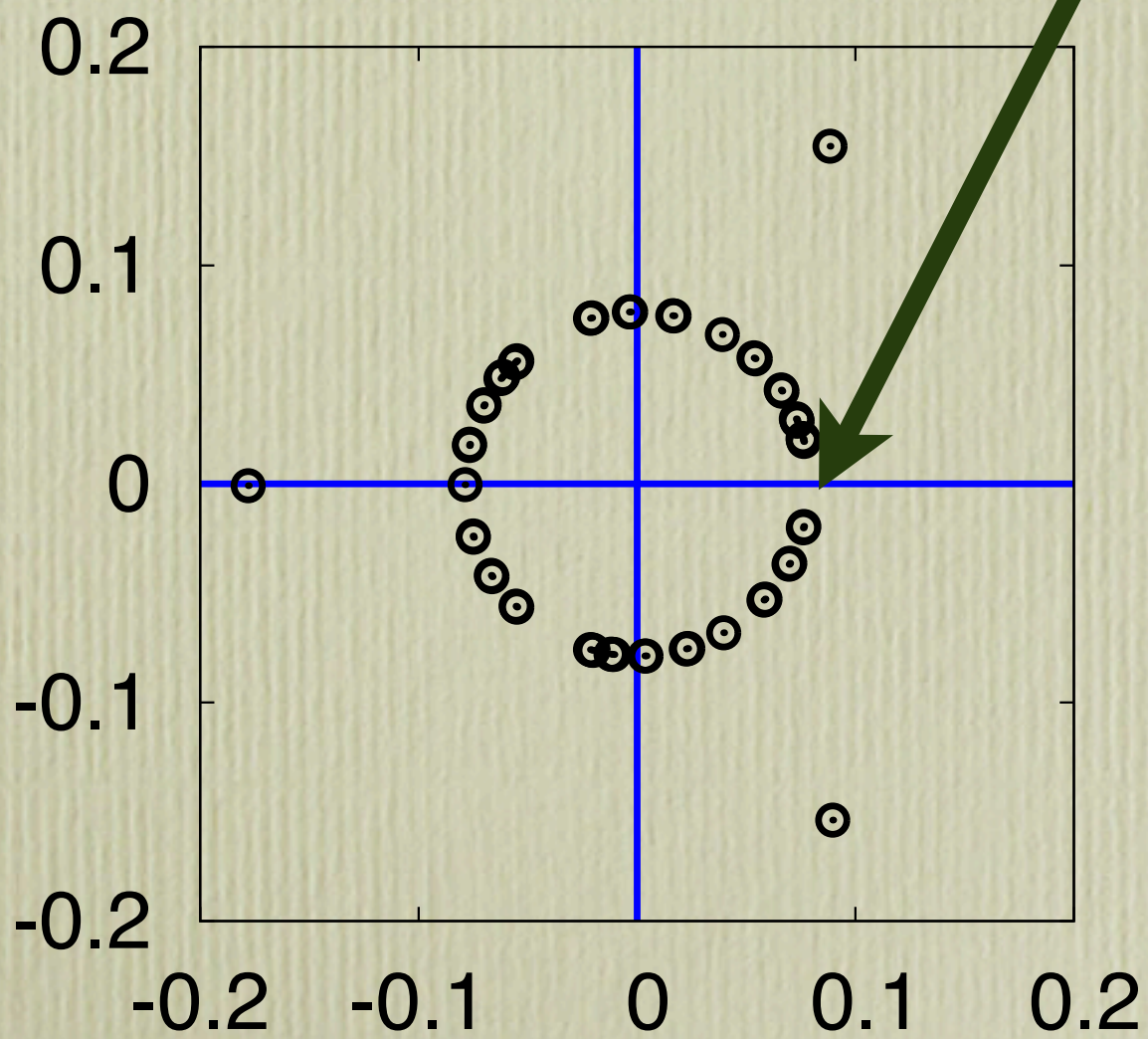


Hadronic observables

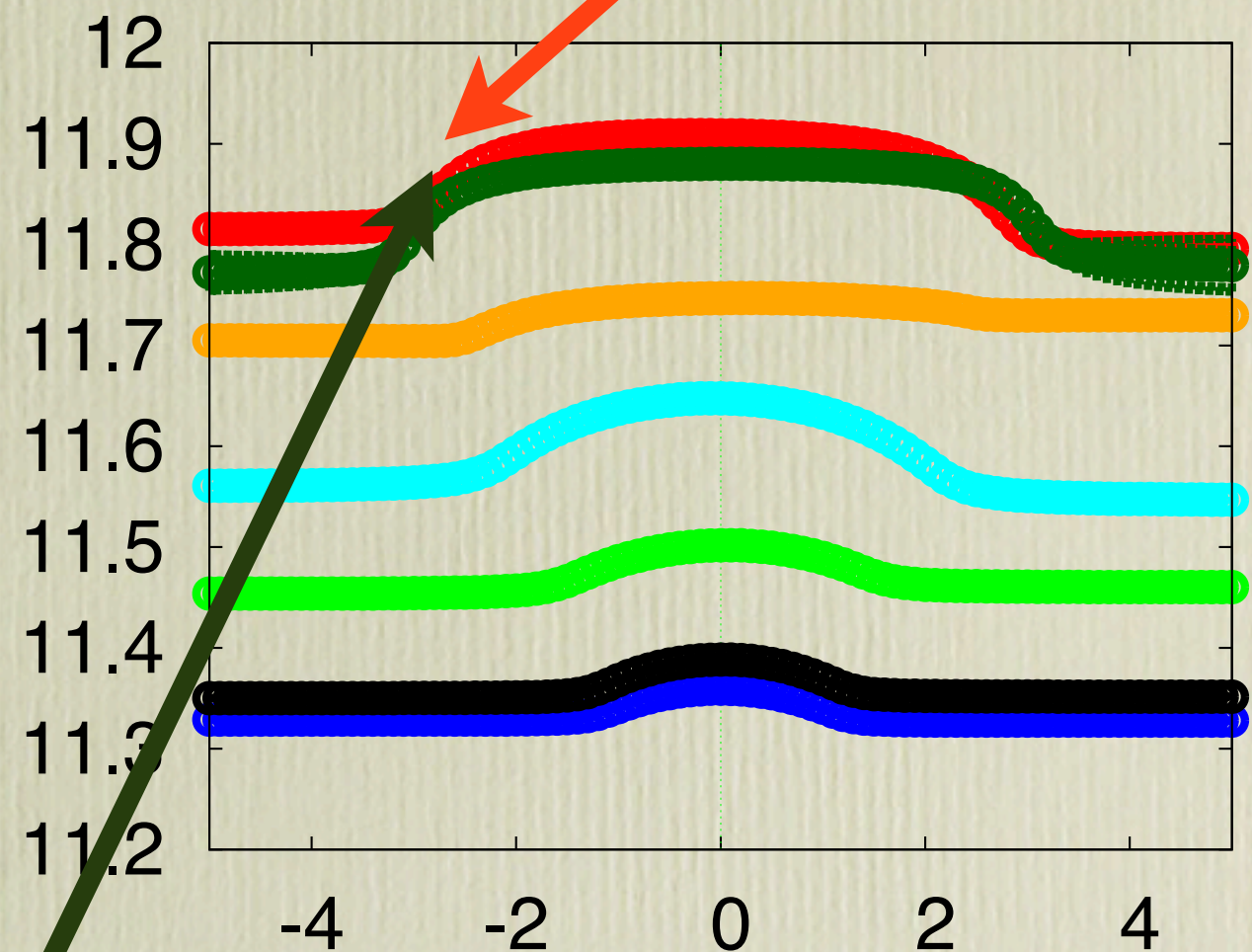
Chiral condensate in grand canonical ensemble

Lee-Yang zeros

$\xi \sim 0.076$ $\longleftrightarrow$ $\mu \sim -2.5$



$\beta = 1.1$



$\frac{m_N}{3}$? $\frac{m_\pi}{2}$? artifact?

Conclusion

Conclusion

- ✎ Canonical approach is a good choice for finite density QCD.

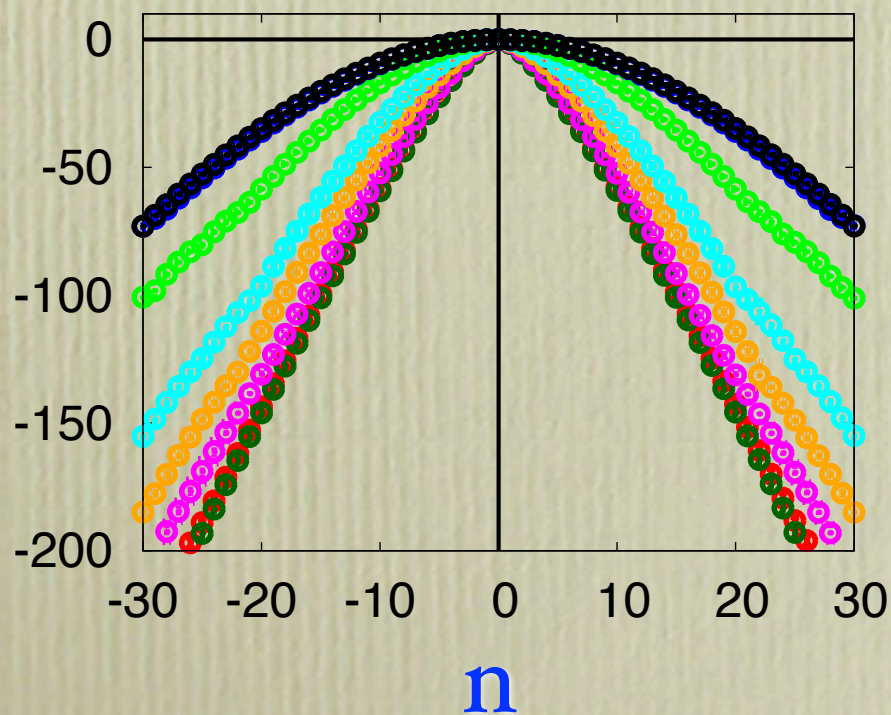
Conclusion

- ✂• Canonical approach is a good choice for finite density QCD.
- ✂• Hopping parameter expansion works more than we expected.

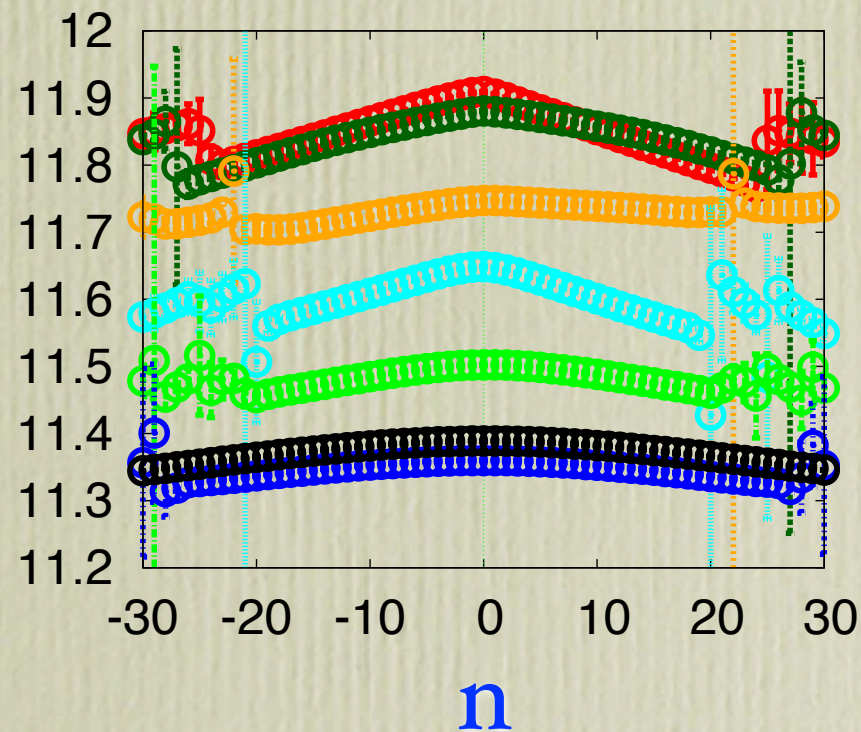
Conclusion

- Canonical approach is a good choice for finite density QCD.
- Hopping parameter expansion works more than we expected.
- We have three interesting results.

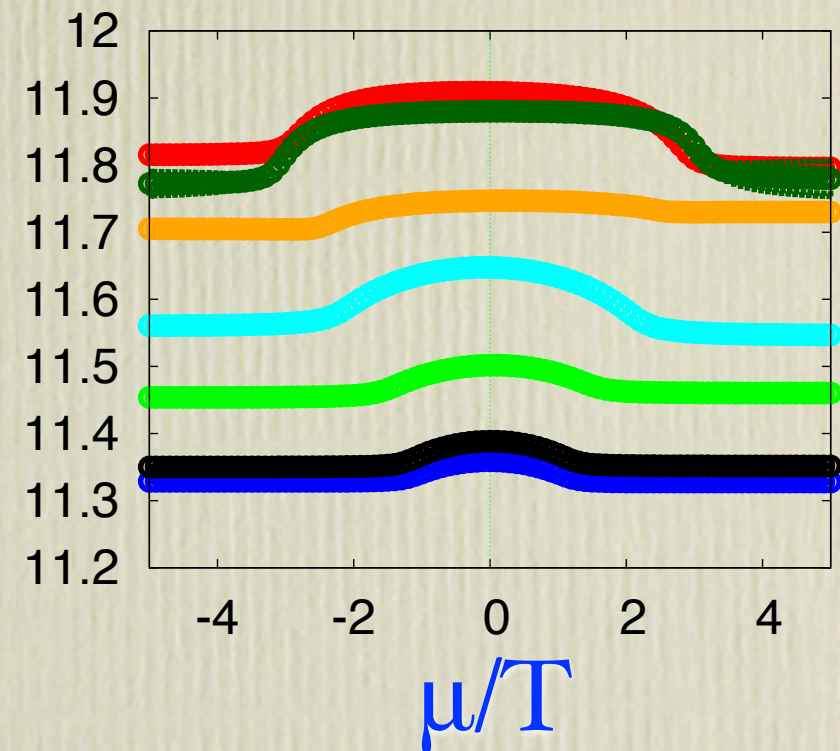
Zn



$$\frac{\sum \langle \bar{\psi}\psi \rangle_C(\beta, n)}{V}$$



$$\frac{\sum \langle \bar{\psi}\psi \rangle_G(\beta, \mu)}{V}$$



ポスターやります

酒井俊太郎
プレゼンツ

乞うご期待！

カノニカル法を用いた有限温度、密度QCDの解析

Z_N

Z_N collaboration

(岡村太郎(立教大), 酒井俊太郎(京大), 鈴木達(筑波大), 谷口裕介(筑波大), 中村純(京大), 福田龍太郎(東大))

Abstract

有限温度QCDにおいては、化学ポテンシャルによりYang行列式が複雑になることから格子QCDを用いたQCDの非摂動的性質の解析は困難になることが知られている。今回はnumerical QCDも考慮して、有限温度、密度領域におけるfavorable QCD hopping parameter展開、及びReweighting法を用いて解析した。さらに、grand canonical partition functionの2-1変換から得られるcanonical partition functionを用いてLee-Yang零点を求め、有限温度、密度領域における相構造についても解析を行った。ここで得られた結果がどのような意味を持つのか、これまでの数値計算による解析とも比較しながら報告する。

Sign problem in Lattice QCD

格子QCD-QCDの非摂動的性質を解析する有用な方法
(1)ドロン質量,有限温度相転移...

有限化学ポテンシャルへの拡張+符号問題

$$Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi) = \int D U \det(D(\mu)) \exp(-S_{\text{Gauge}})$$

$$\Rightarrow \det(D(\mu)) = \det(D(0) + \gamma_5 \mu \gamma_4) \Rightarrow \det(D(\mu)) = \det(D(0) - \mu^2)$$

⇒ 正定値性はない
($\det(D(\mu))$ を統計量としたMonte Carlo samplingはできない)

→いくつかの近似法: 数値化学ポテンシャル, テイラー展開, Reweighting, ... [1]

適用限界(Roberge-Weiss相転移, Early onset問題, 収束遅延, Overlap問題, ...)

⇒ 低温かつ高密度領域への適用は困難

Strategy: Reweighting method + Hopping parameter expansion

Reweighting method

$$Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi) = \int D U \det(D(\mu)) \exp(-S_{\text{Gauge}})$$

物理量に用いる: $\langle \mathcal{O} \rangle = \frac{\int D\psi D\bar{\psi} D U \mathcal{O} \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi)}{\int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi)}$

数値化学ポテンシャル $\beta\mu \rightarrow$ 統計量において高次元問題に陥れない

Hopping parameter expansion

$$\det(D(\mu)) = \exp(\text{Tr} \ln D(\mu))$$

grape不安定なもの(traceで残るもの)にだけloop

✓ 有限温度系(虚時間法): 時間方向は周期境界条件
⇒ 虚時間方向への格子付加での展開が可能 [2]

化学ポテンシャル依存性は $\{ \exp(i\mu) \}$ の形で現れる

$$Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi) = \int D U \det(D(\mu)) \exp(-S_{\text{Gauge}})$$

$$\Rightarrow \det(D(\mu)) = \det(D(0) + \gamma_5 \mu \gamma_4) = \det(D(0) - \mu^2)$$

⇒ 格子付加での展開より、第4成分については $\exp(i\mu)$ (整数)のみ現れる

✓ A way to the winding number expansion: 行列式の閉路公式 [3, 4]

→ 数値的安定性だが、比較的large numerical cost

⇒ Hopping parameter expansionを用いて解析(limited to small μ)

$$\text{Tr} \ln D(\mu) = \text{Tr} \ln (D(0) + \gamma_5 \mu \gamma_4) = \text{Tr} \ln D(0) + \text{Tr} \ln (1 + \gamma_5 \mu \gamma_4 D(0)^{-1})$$

$$= \text{Tr} \ln D(0) + \text{Tr} \ln (1 + \gamma_5 \mu \gamma_4 D(0)^{-1}) = \text{Tr} \ln D(0) + \text{Tr} \ln (1 + \gamma_5 \mu \gamma_4 D(0)^{-1})$$

$$\Rightarrow Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi) = \int D U \det(D(\mu)) \exp(-S_{\text{Gauge}})$$

⇒ 比較的 small numerical cost

Canonical partition function and Lee-Yang zeros

大分配関数と分配関数

$$Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi) = \int D U \det(D(\mu)) \exp(-S_{\text{Gauge}})$$

Partition trace: $Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi)$

$2N_c \times 1$ 次の多項式: 複素平面上には $2N_c \times 1$ 個の零点(Lee-Yang零点)が存在

① 粒子-反粒子の入れ替えについての対称性 ($\mu \leftrightarrow -\mu$)

⇒ $\langle \mathcal{O} \rangle(\mu) = \langle \mathcal{O} \rangle(-\mu)$ のとき、 $\mathcal{O}(\mu) = \mathcal{O}(-\mu)$ も解

② 有限温度では相転移は起こらない ⇒ 有限温度で実験上に実点はない

Lee-Yang's argument [5]

分配関数の零点が、

A) 実験を模倣する1次相転移

B) 実験に近接する2次相転移

C) それ以外cross over

⇒ Lee-Yang零点解析に基づいたQCD相転移の解析 [3, 4, 6]

Properties of the canonical partition function

実変数変換性

$$\mu \rightarrow -\mu \text{ の変換 } \Rightarrow Z(\mu) = Z(-\mu)$$

Roberge-Weiss周期性 [7]

$$Z(\mu) = Z(\mu + 2\pi i) = Z(\mu + 4\pi i) = \dots$$

⇒ $Z(\mu) = 0$ (pole 整数)

$$Z(\mu) = \int D\psi D\bar{\psi} D U \exp(-S_{\text{Gauge}} - \int d^4x \bar{\psi} (D + m) \psi) = \int D U \det(D(\mu)) \exp(-S_{\text{Gauge}})$$

Hadron observable quark condensate

Set up

- twisted gauge action
- 2-flavor Clover fermion
- APL stout smeared gauge link
- Box size, $8^3 \times 4$

クォーク凝縮 with χ フィンク

$$\chi = \frac{1}{V} \frac{\partial \ln Z(\mu)}{\partial \mu} = \frac{1}{V} \frac{\partial \ln \det(D(\mu))}{\partial \mu} = \frac{$$

If you can read this
I am on the wrong page.

Hadronic observables

Convergence radius

Hadronic observables

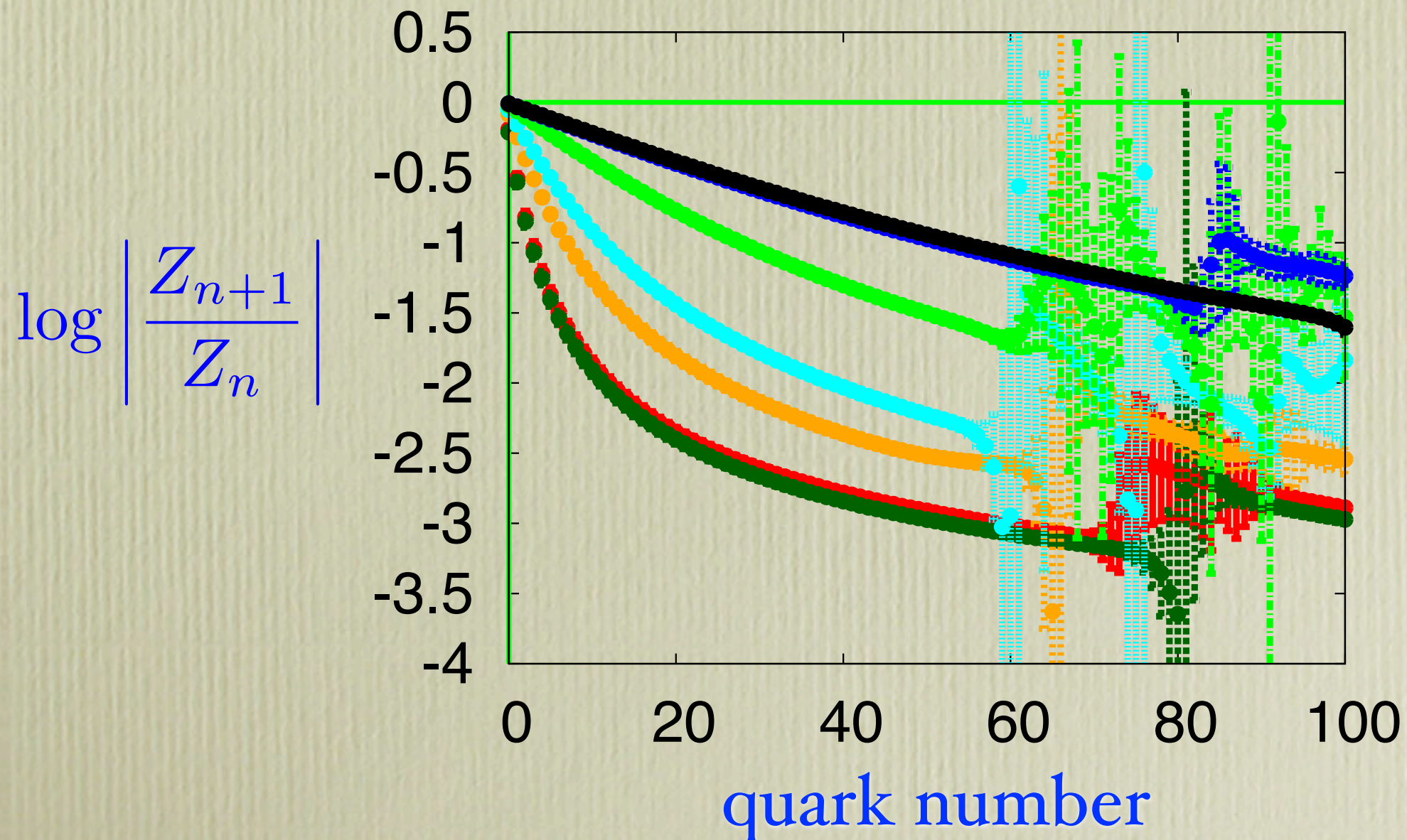
Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \dots \\ + Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \dots$$

Hadronic observables

Convergence radius

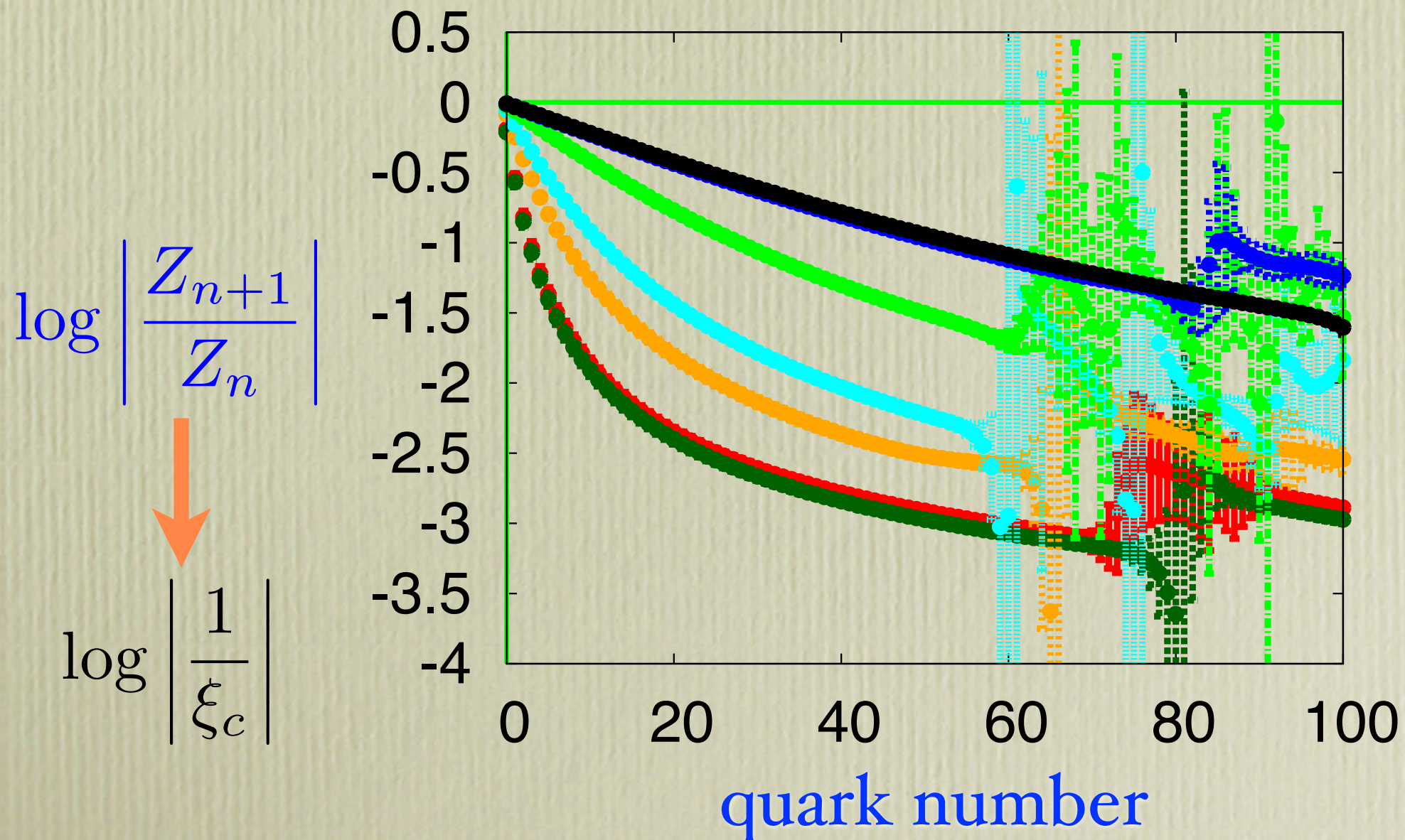
$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \dots \\ + Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \dots$$



Hadronic observables

Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \dots \\ + Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \dots$$

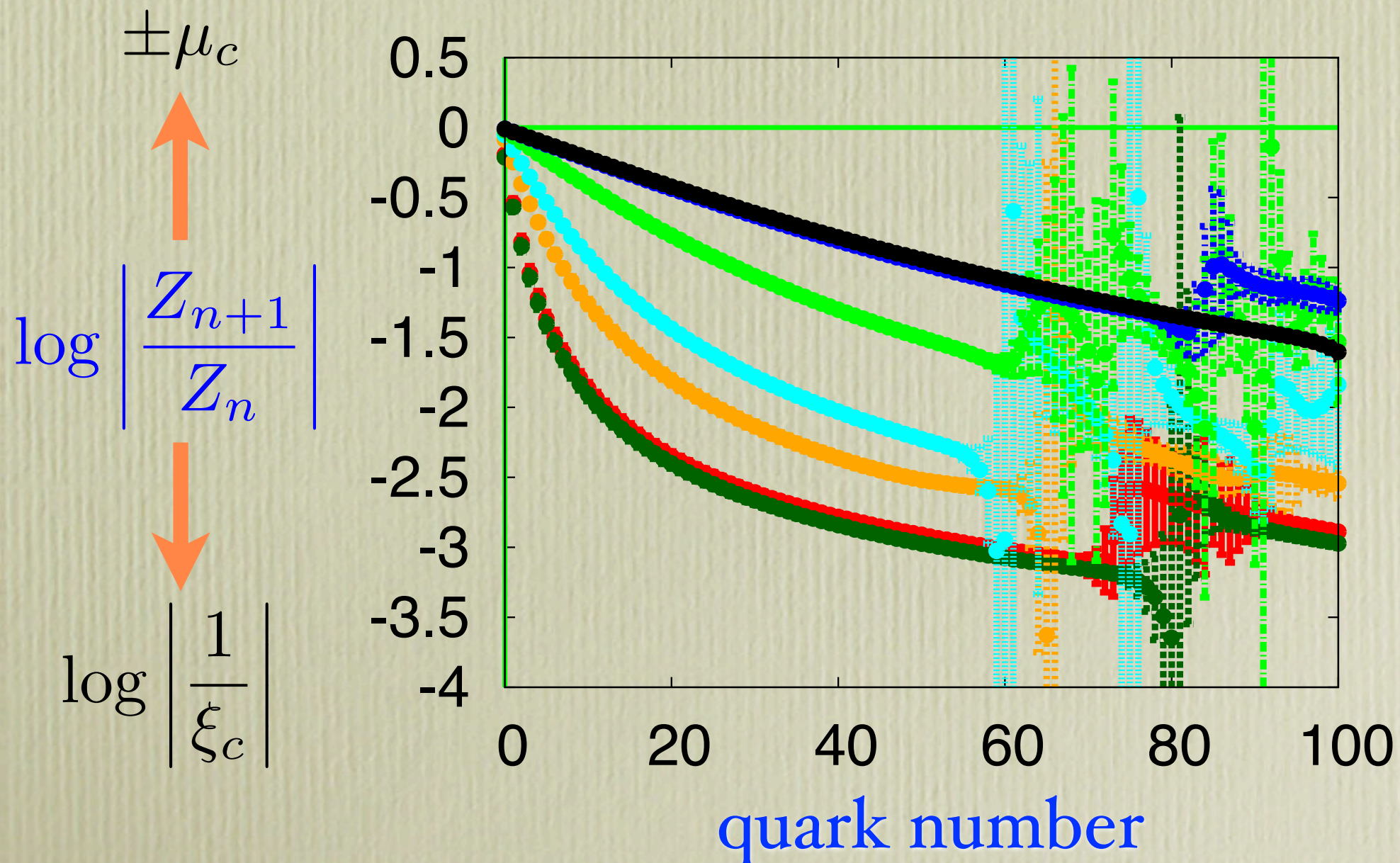


Hadronic observables

Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \dots$$

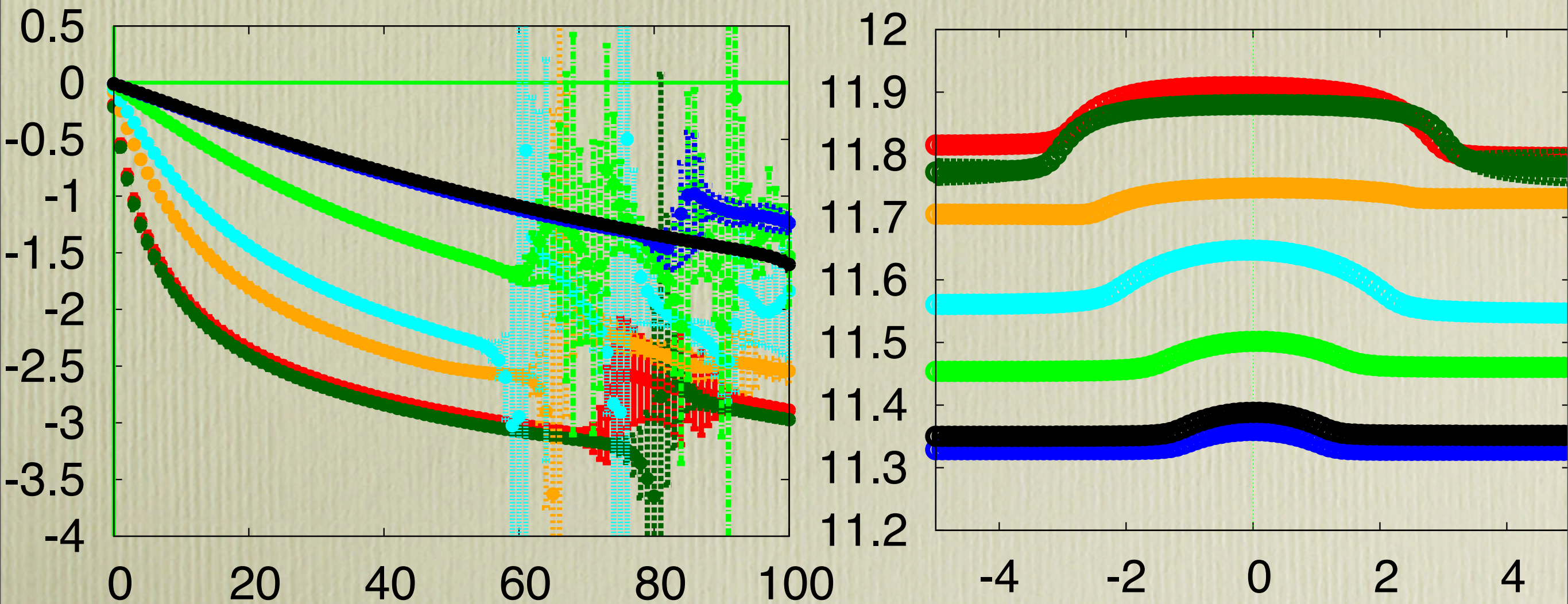
$$+ Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \dots$$



Hadronic observables

Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \dots \\ + Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \dots$$



Numerical results $\text{Phase}(Z_C(n))$

Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

Numerical results $\text{Phase}(Z_C(n))$

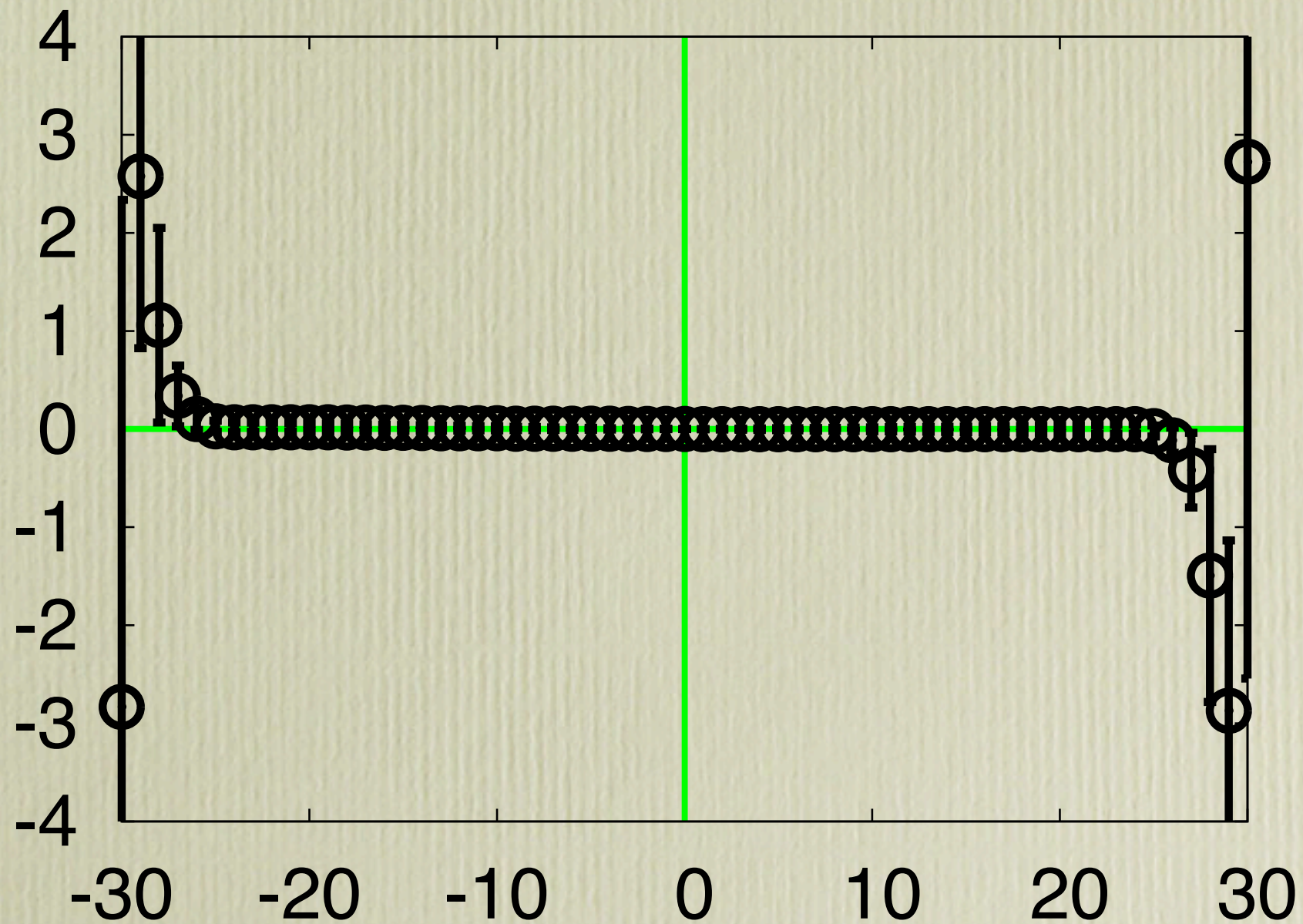
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.9$$

$$\mu = 0$$

$$\theta(\beta, n)$$



baryon number

Numerical results $\text{Phase}(Z_C(n))$

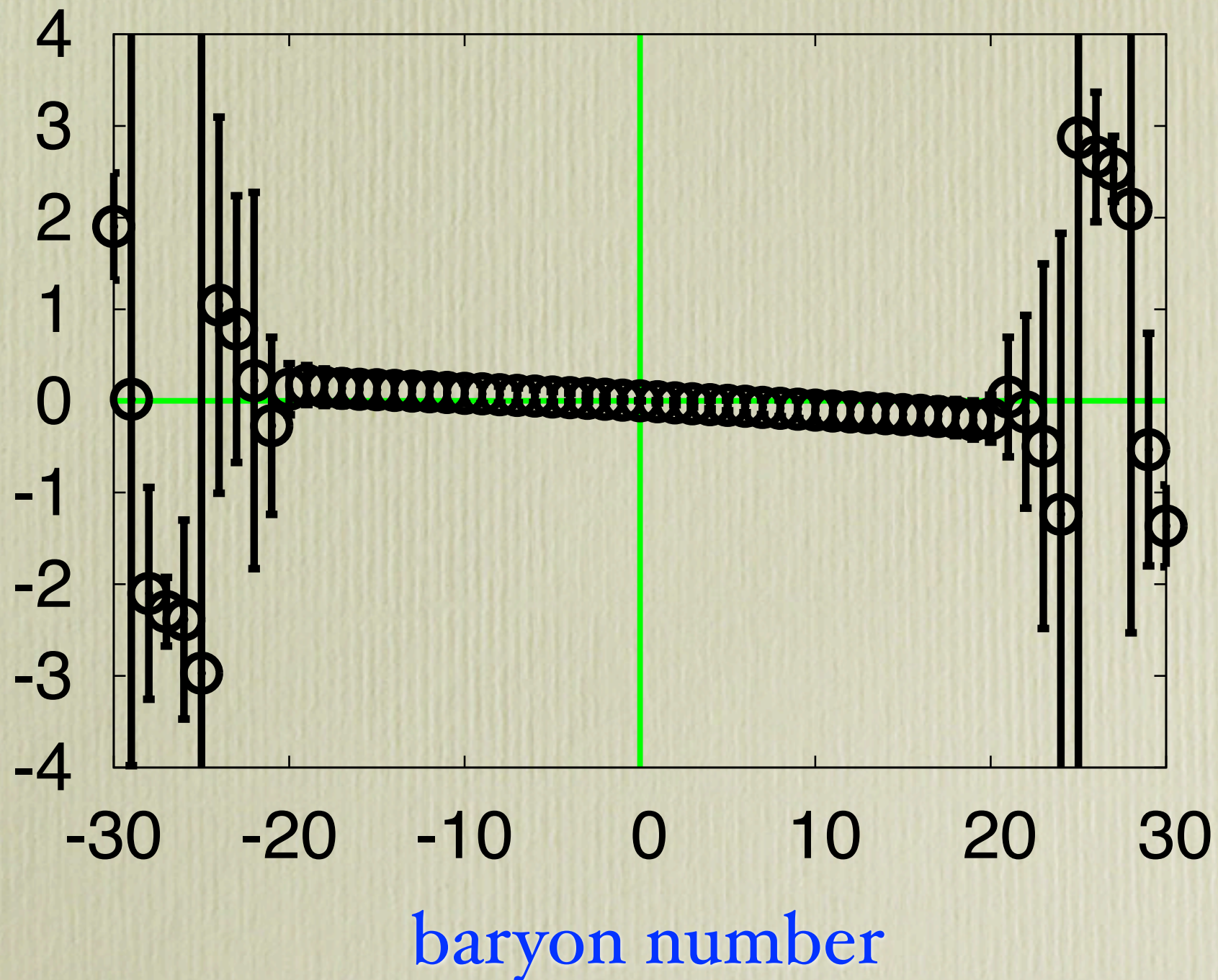
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.7$$

$$\mu = 0$$

$$\theta(\beta, n)$$



Numerical results $\text{Phase}(Z_C(n))$

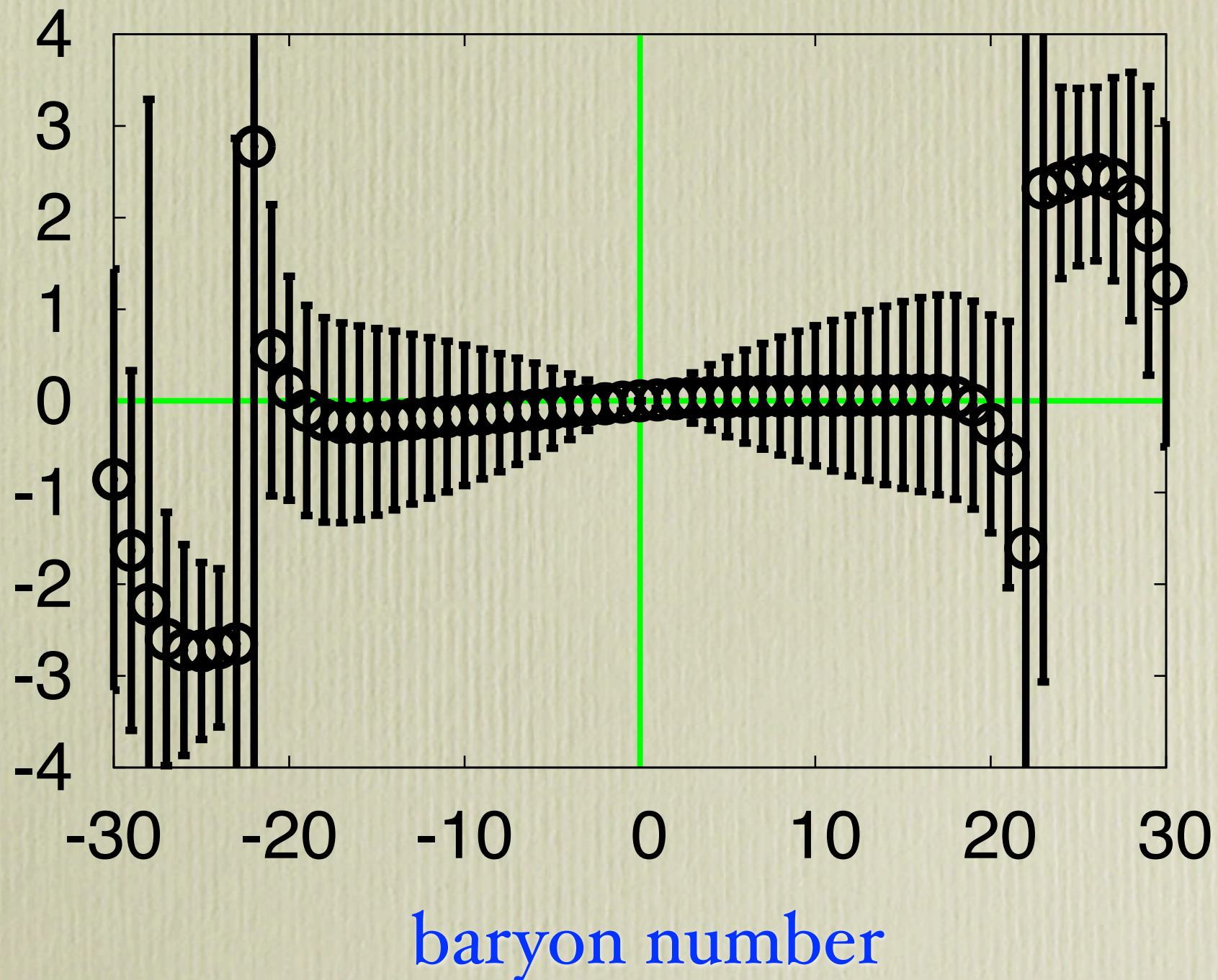
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.5$$

$$\mu = 0$$

$$\theta(\beta, n)$$



Numerical results $\text{Phase}(Z_C(n))$

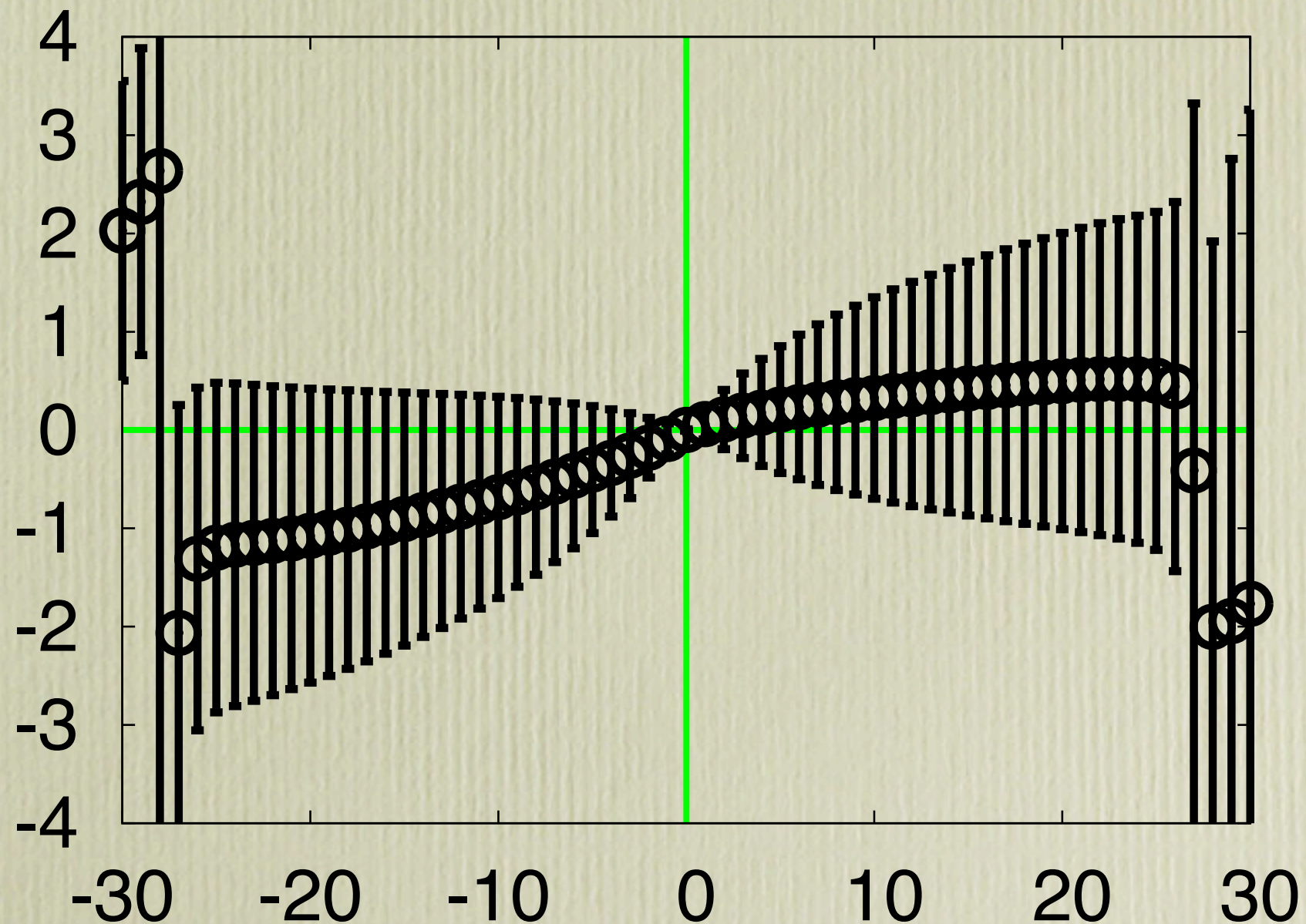
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.1$$

$$\mu = 0$$

$$\theta(\beta, n)$$



baryon number

Numerical results $\text{Phase}(Z_C(n))$

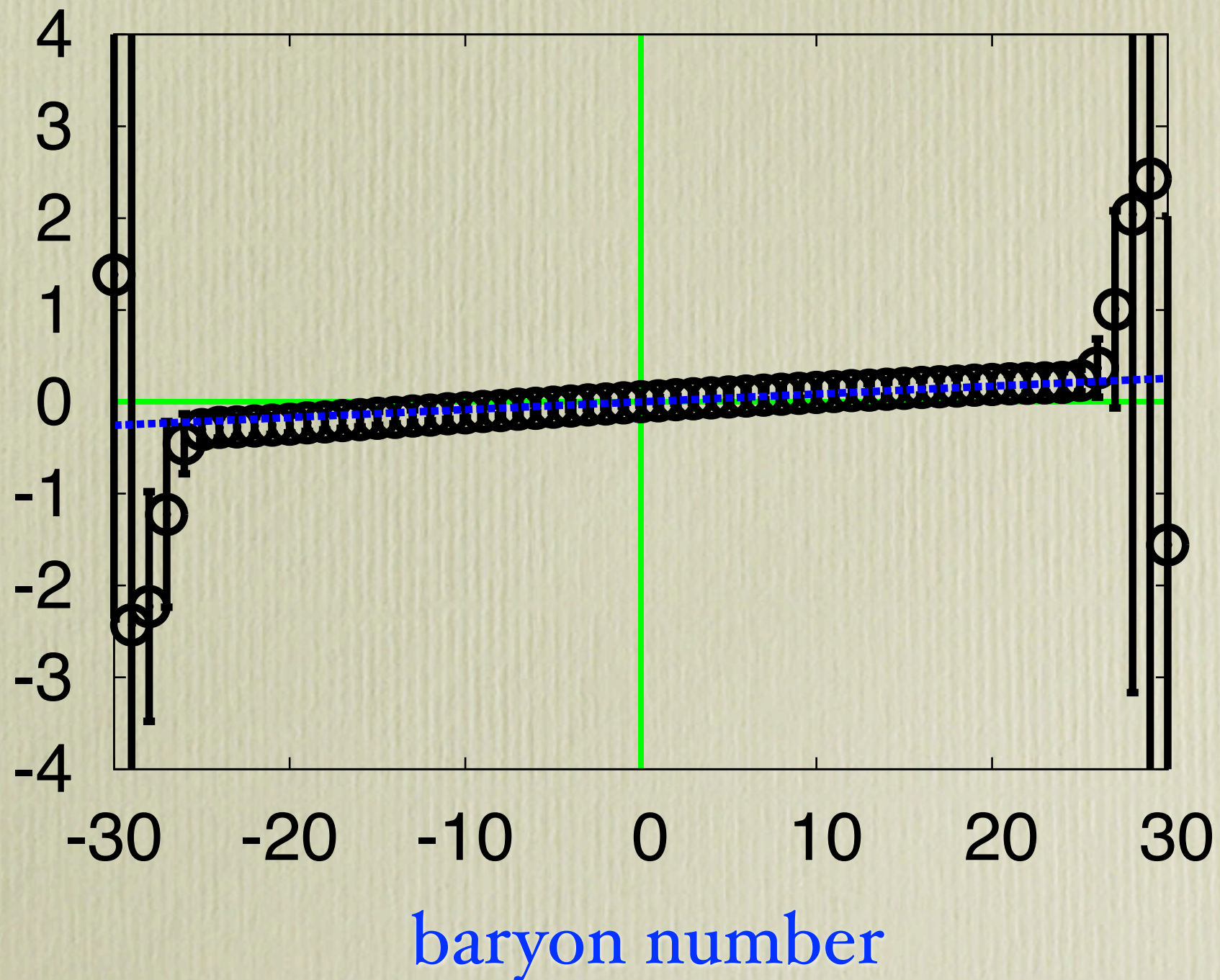
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.9$$

$$\mu = 0.125$$

$$\theta(\beta, n)$$



Numerical results $\text{Phase}(Z_C(n))$

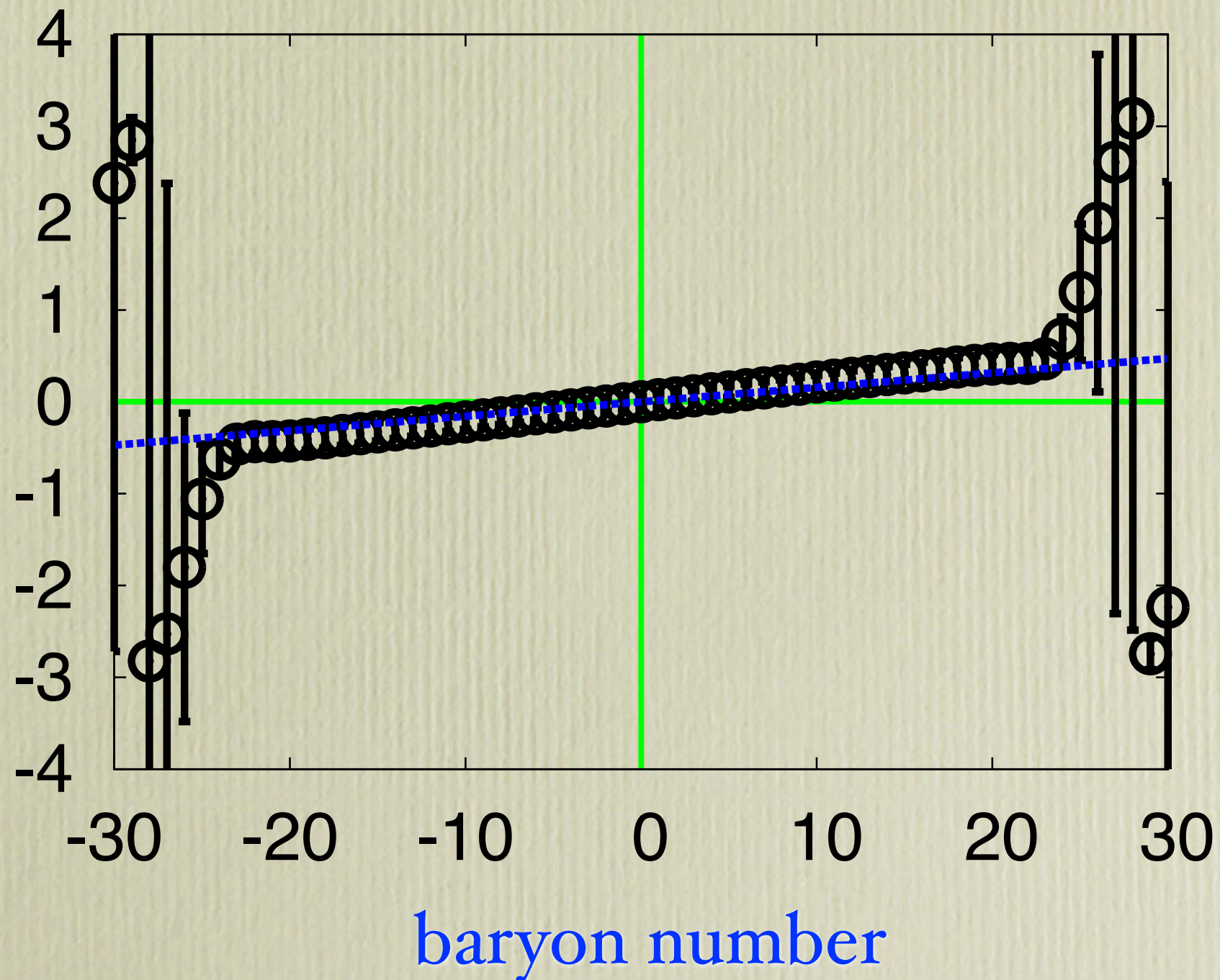
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.9$$

$$\mu = 0.25$$

$$\theta(\beta, n)$$



Numerical results $\text{Phase}(Z_C(n))$

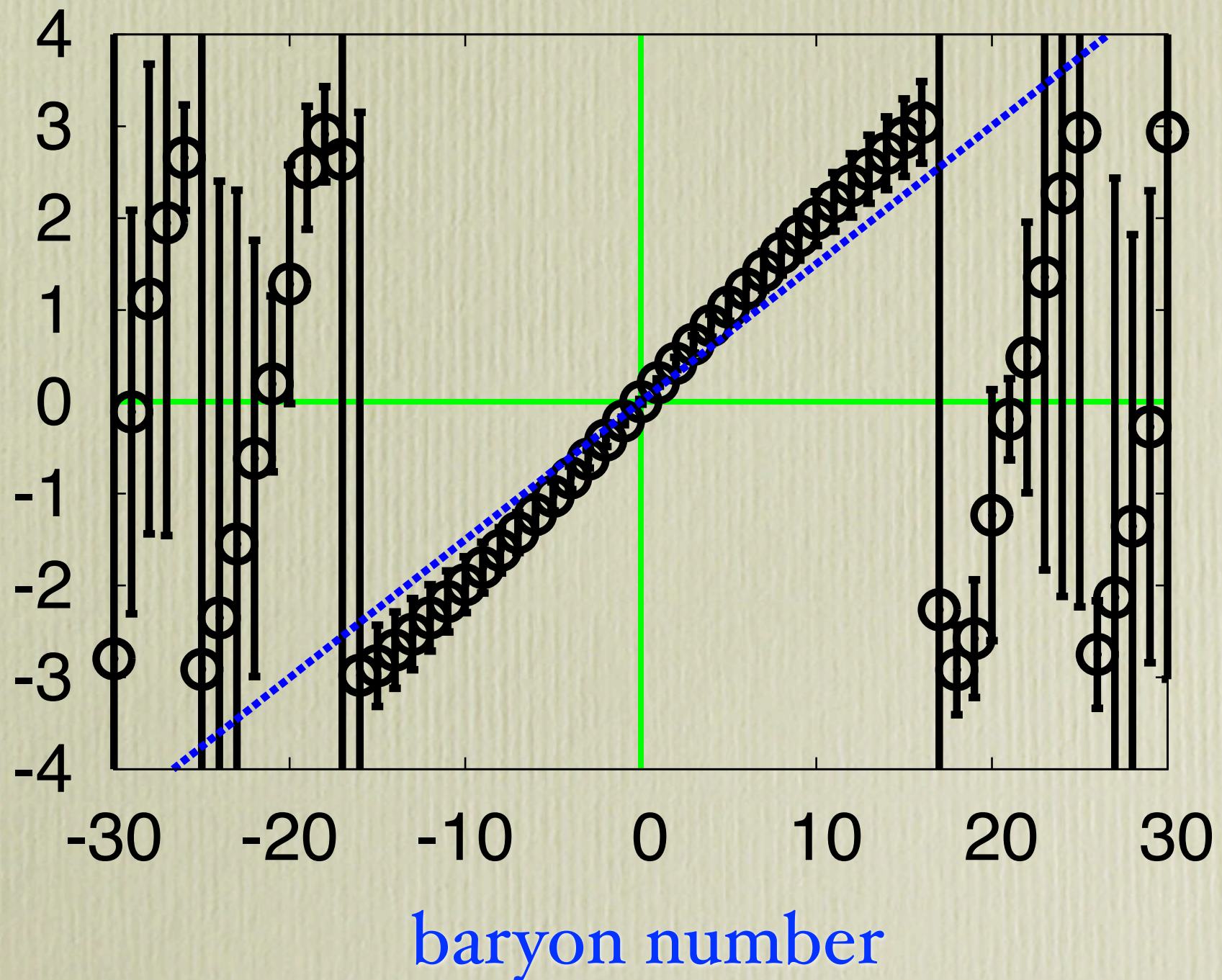
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.7$$

$$\mu = 0.125$$

$$\theta(\beta, n)$$



Numerical results $\text{Phase}(Z_C(n))$

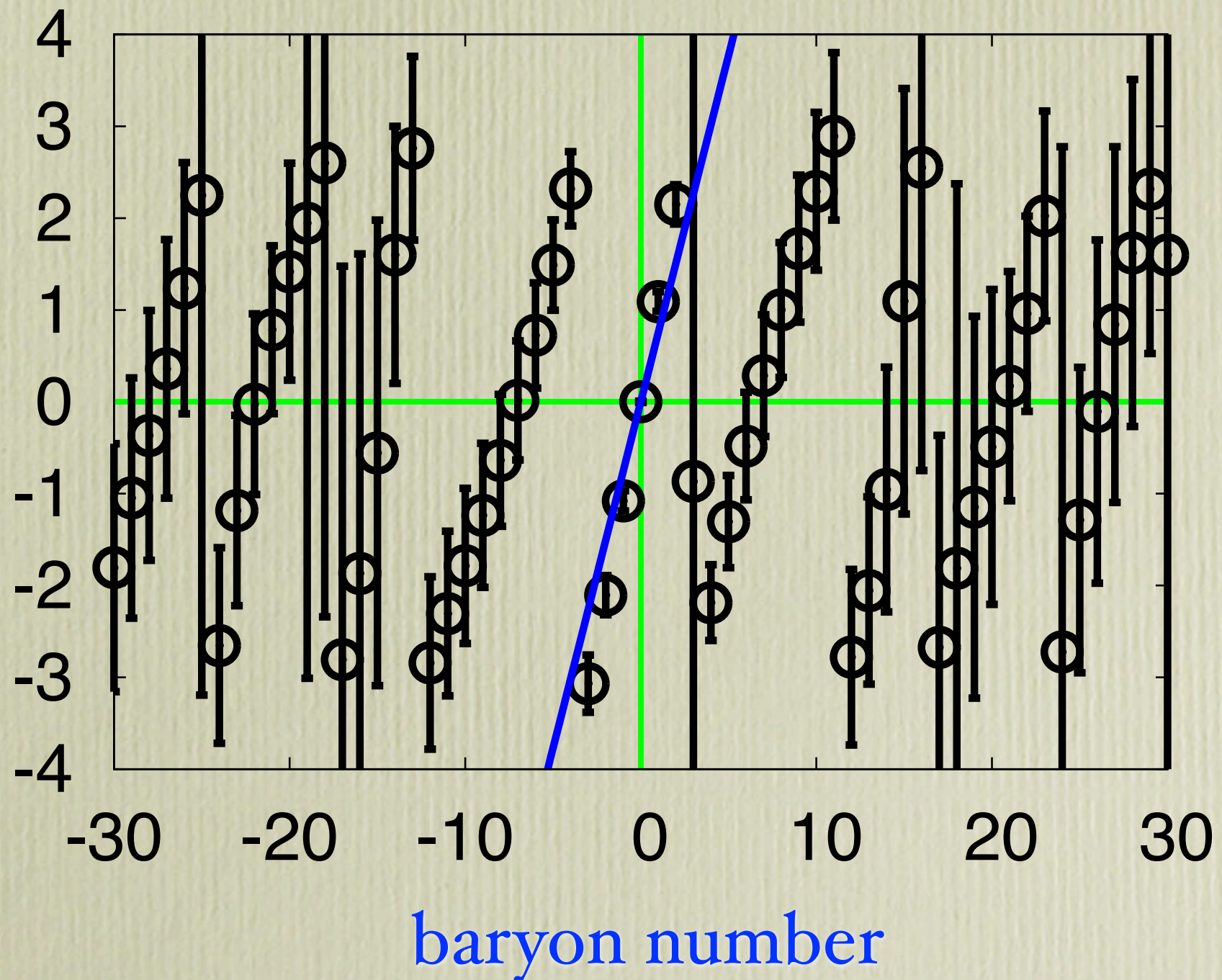
Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$$\beta = 1.5$$

$$\mu = 0.125$$

$$\theta(\beta, n)$$

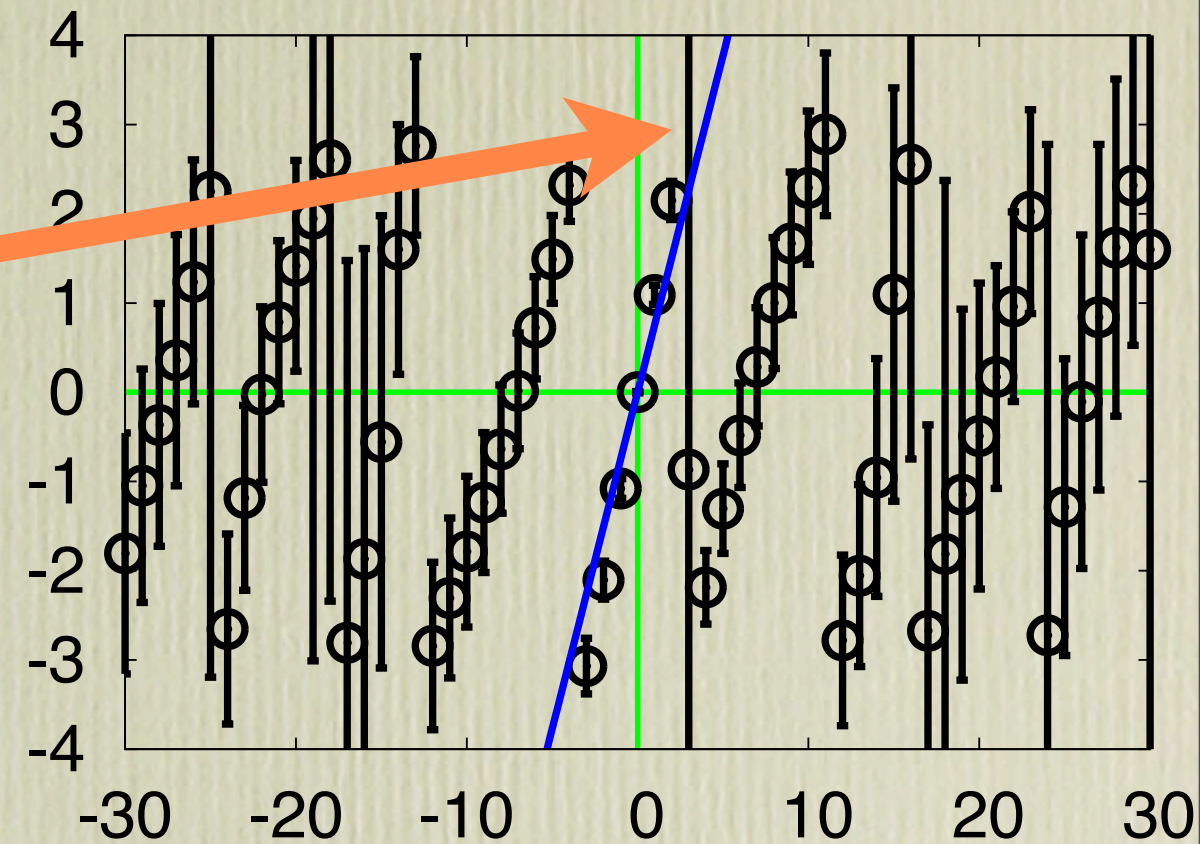


Numerical results $\text{Phase}(Z_C(n))$

Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$\text{slope} \sim 2 \times \text{phase}(W_1)$



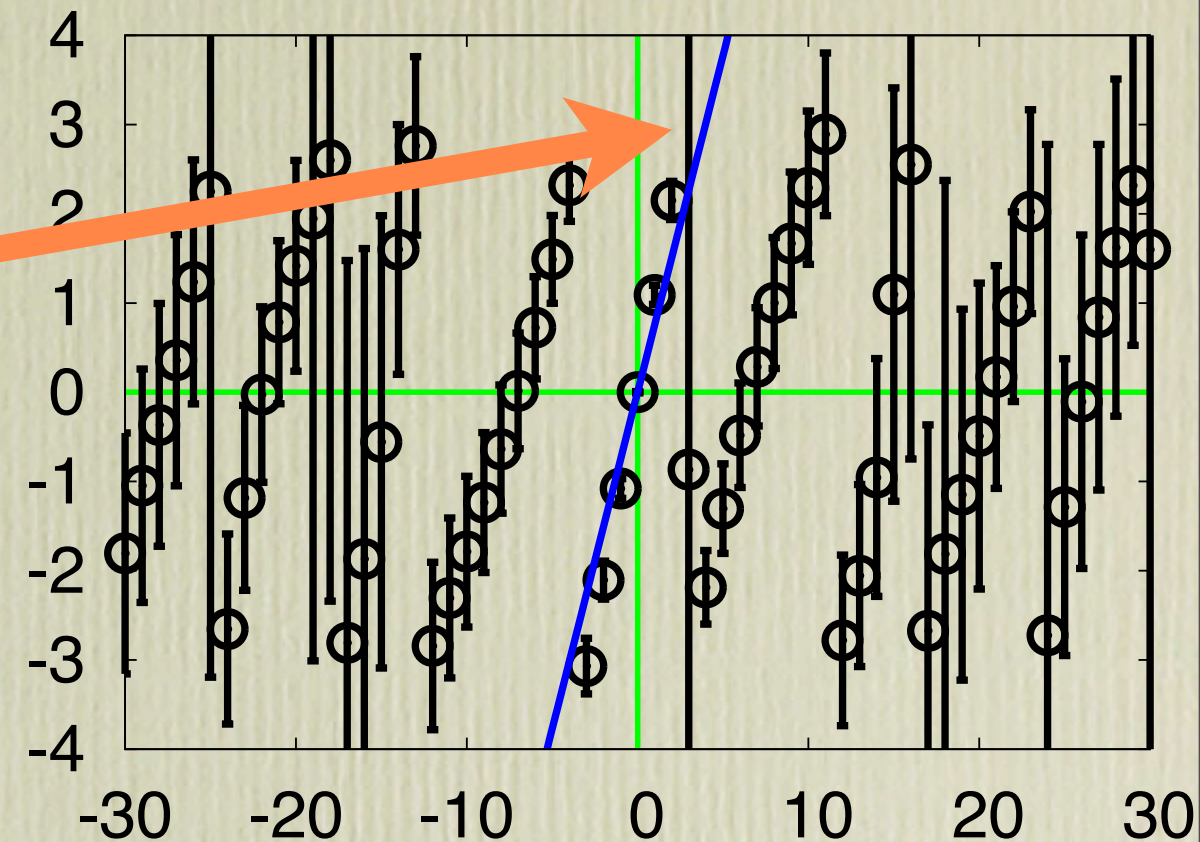
Numerical results $\text{Phase}(Z_C(n))$

Canonical partition function

$$Z_C(T, n, V) = |Z_C(\beta, n)| e^{i\theta(\beta, n)}$$

$\text{slope} \sim 2 \times \text{phase}(W_1)$

$$\text{TrLog} D_W(\mu) = \sum_{N=-\infty}^{\infty} W_N \xi^N$$



Numerical results $\text{Phase}(Z_C(n))$

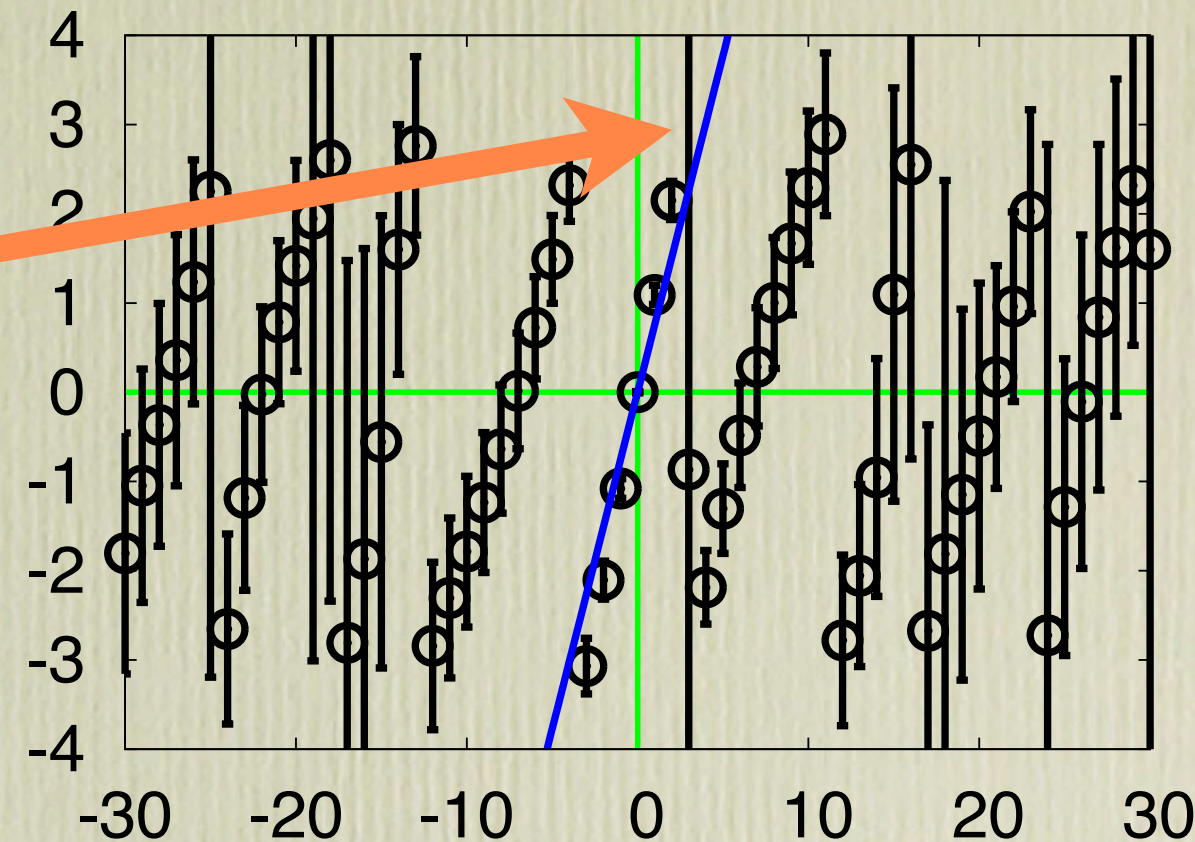
Canonical partition function

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Numerical results $\text{Phase}(Z_C(n))$

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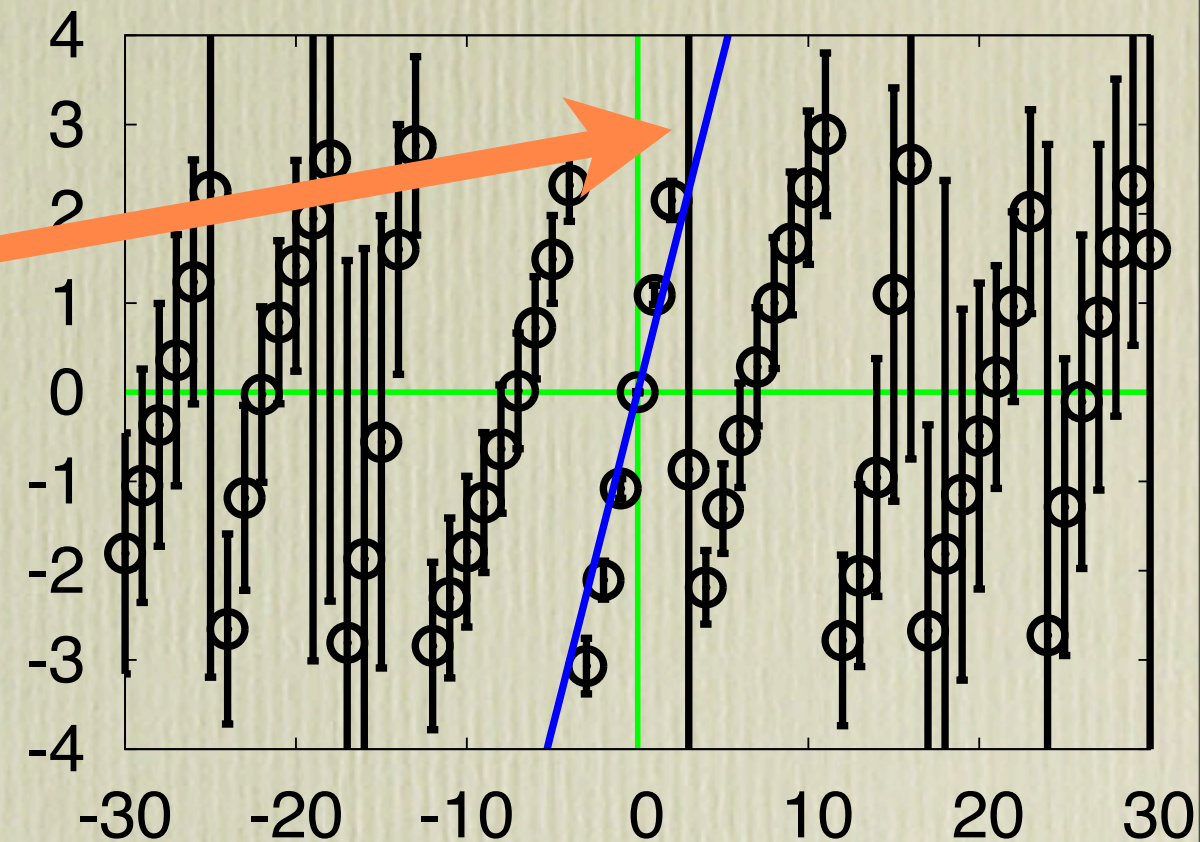
slope $\sim 2 \times \text{phase}(W_1)$

$$\text{TrLog} L_W(\mu) = \sum_{N=-\infty}^{\infty} W_N \xi^N$$

phase of $\text{Det} D_W(\mu)$

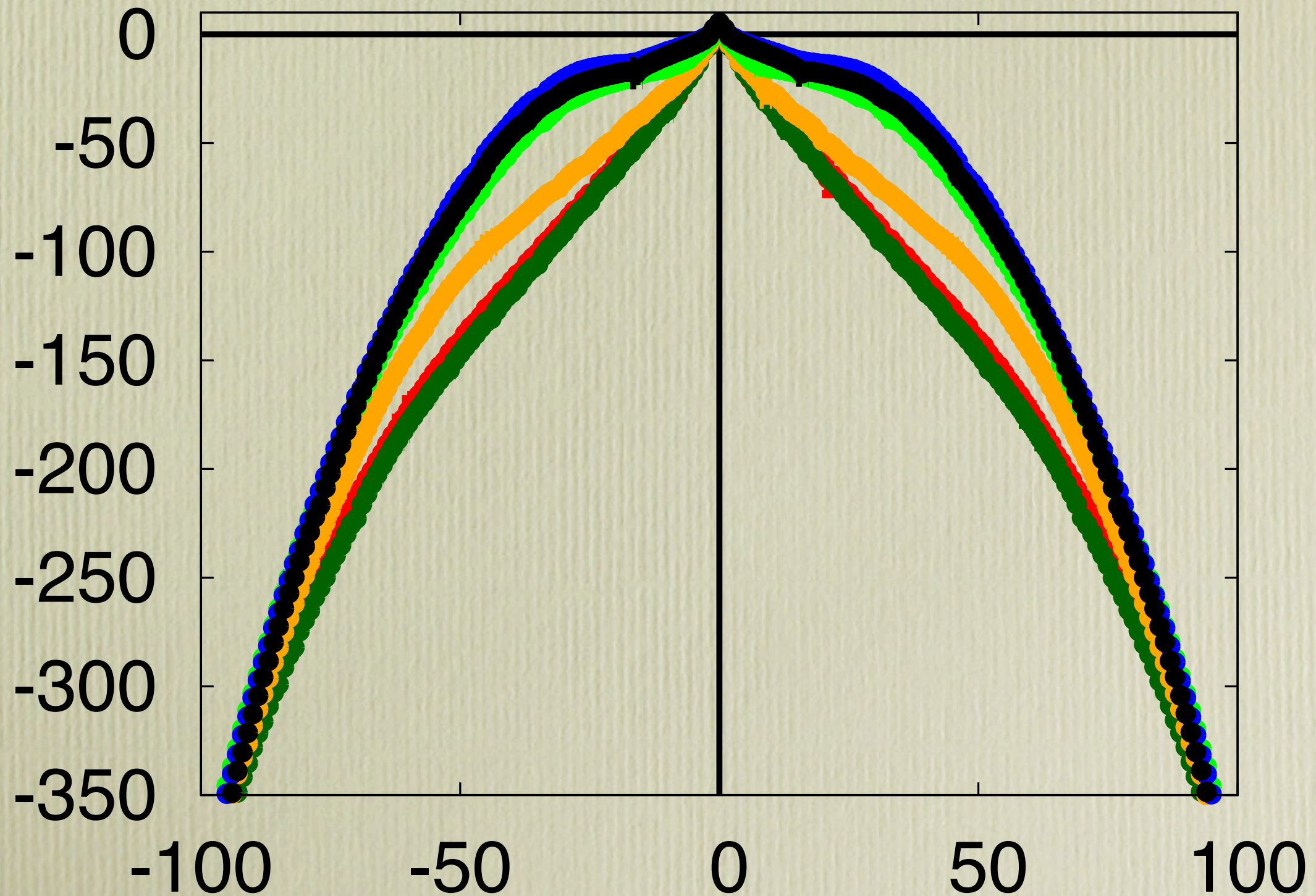
Pion condensate

Ipsen and Splittorff (2012)



Hadronic observables

W_n

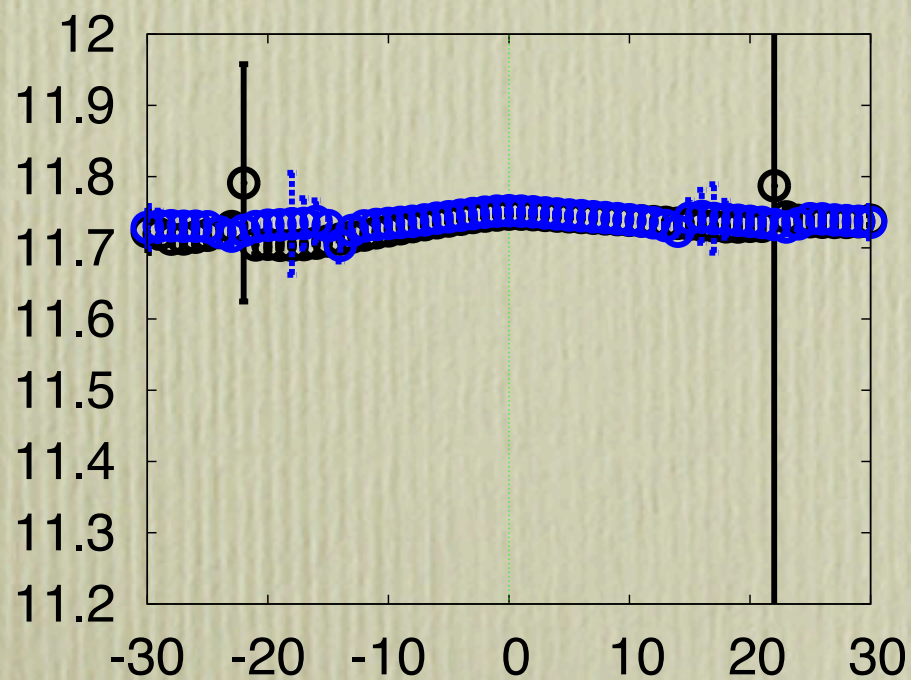


Hadronic observables

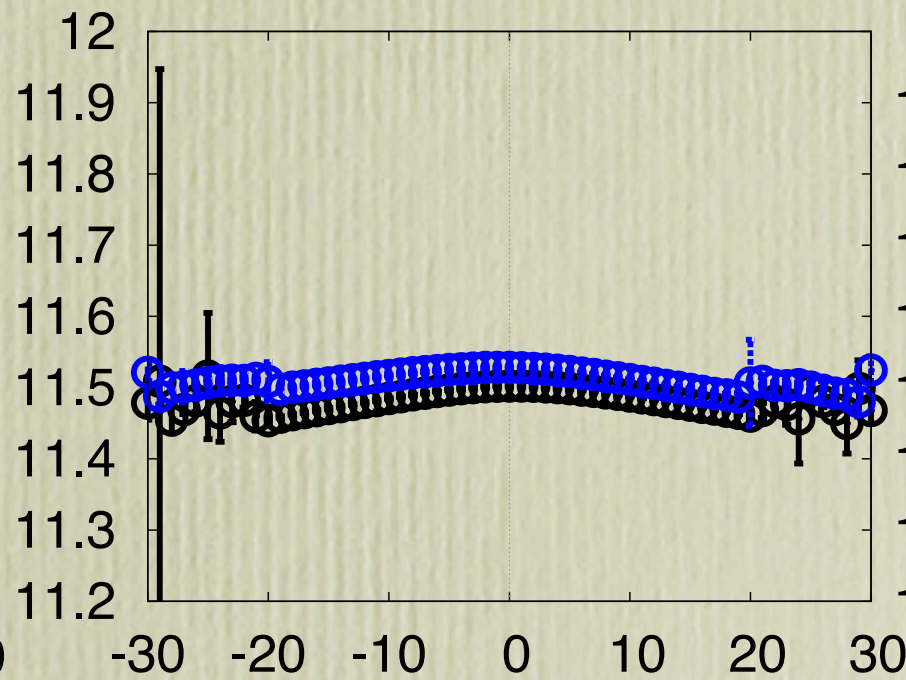
Chiral condensate in canonical ensemble

$$\frac{\sum \langle \bar{\psi} \psi \rangle_C(\beta, n)}{V}$$

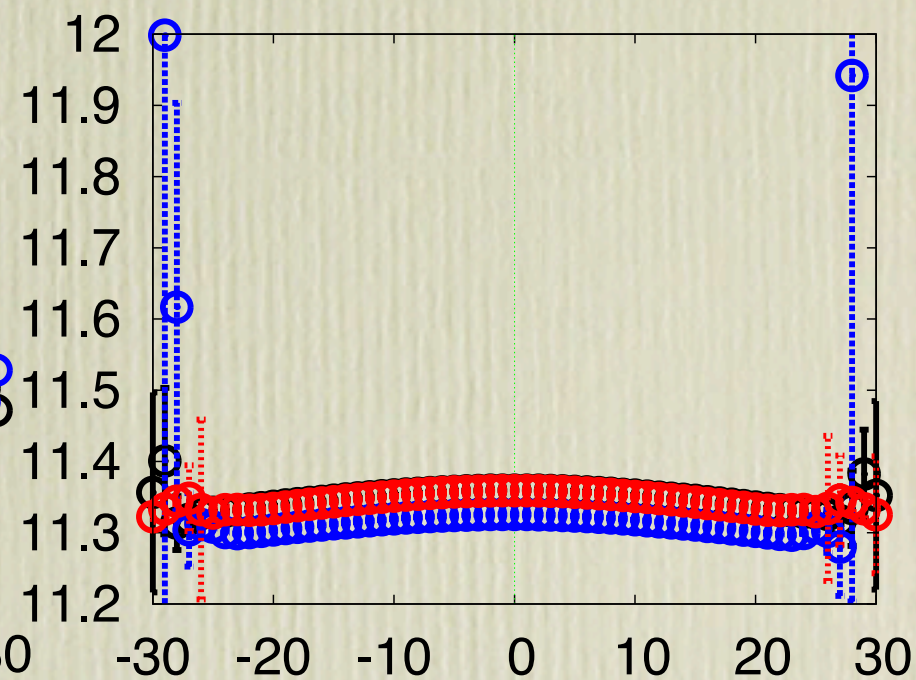
μ dependence



$\beta = 1.5$



$\beta = 1.7$



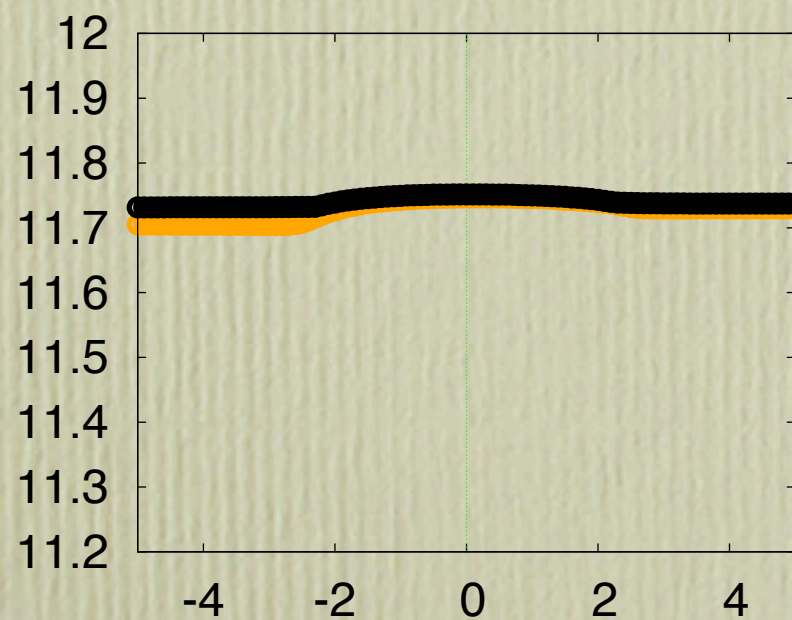
$\beta = 1.9$

Hadronic observables

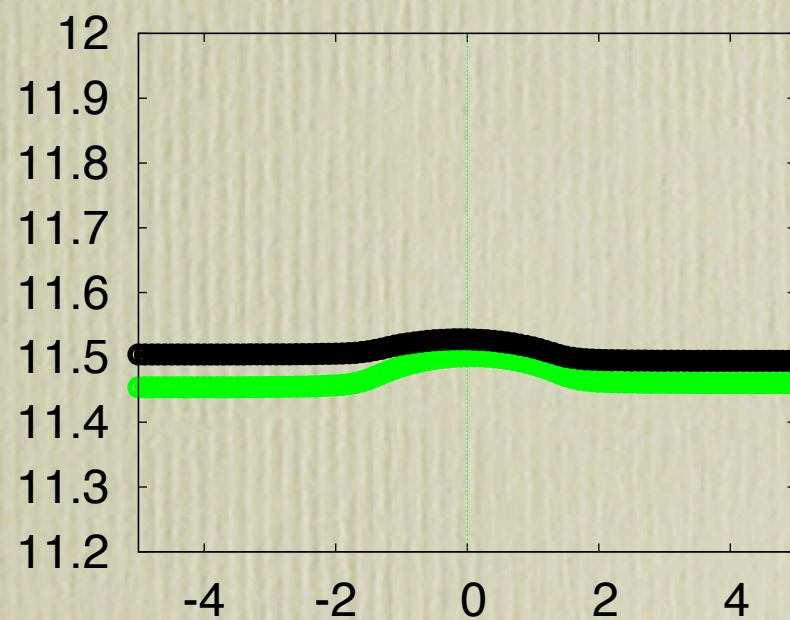
Chiral condensate in canonical ensemble

$$\frac{\sum \langle \bar{\psi} \psi \rangle_G(\beta, \mu)}{V}$$

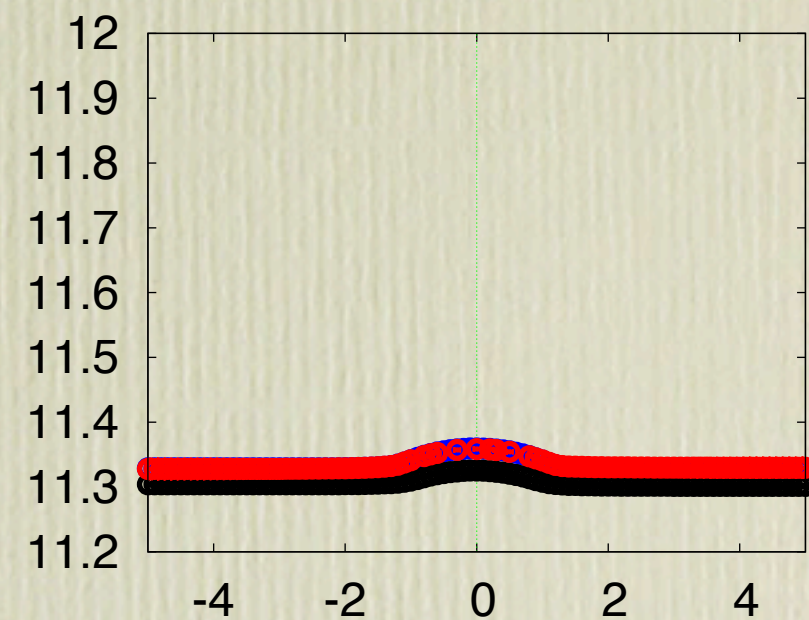
μ dependence



$\beta = 1.5$



$\beta = 1.7$



$\beta = 1.9$