カノニカル法による格子QCD
有限密度相転移現象への挑戦

Yusuke Taniguchi（University of Tsukuba)
for
Zn Collaboration

R.Fukuda (Tokyo) A.Nakamura (Hiroshima) S.Oka (Rikkyo) S.Sakai (Kyoto) A.Suzuki (Tsukuba)

Introduction

Grand canonical ensemble

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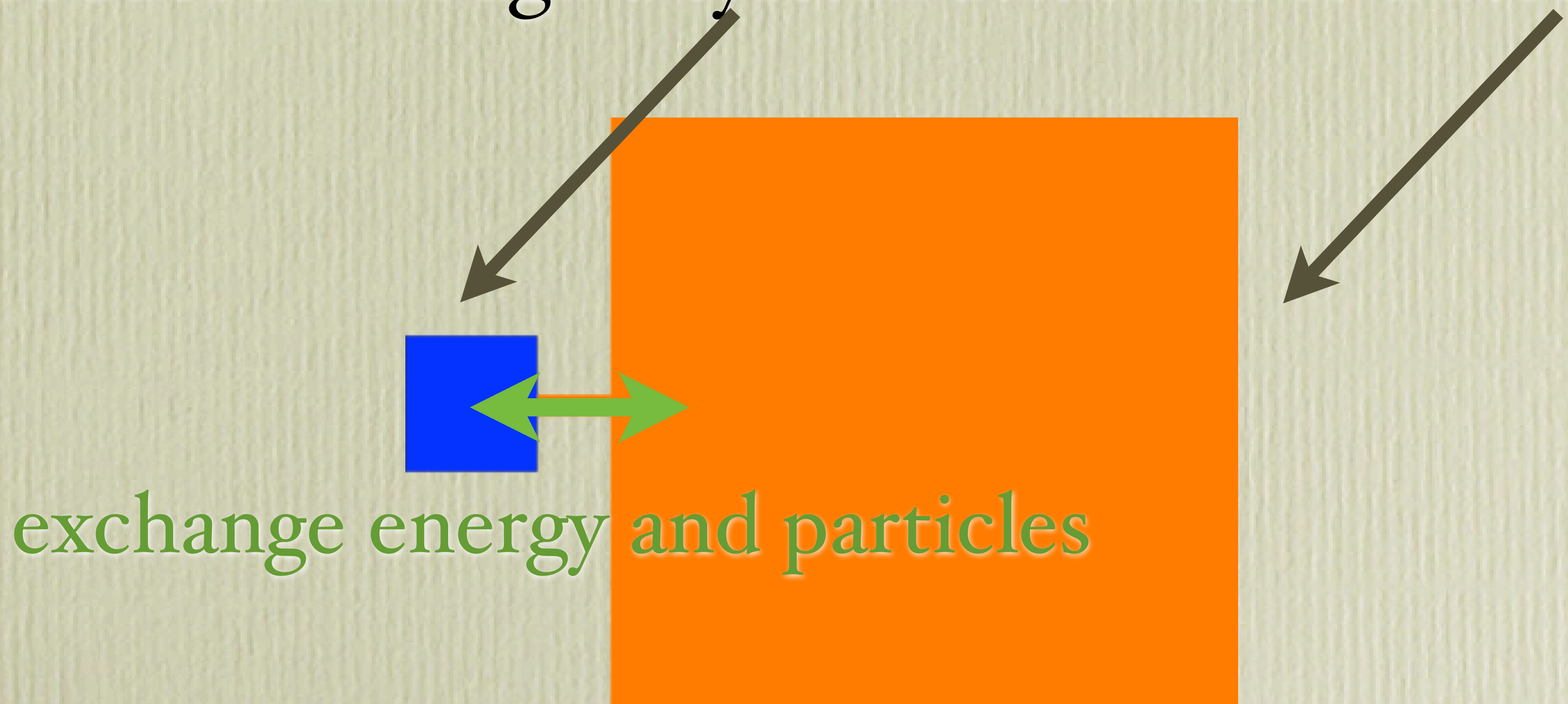
Introduced through a system attached to a reservoir



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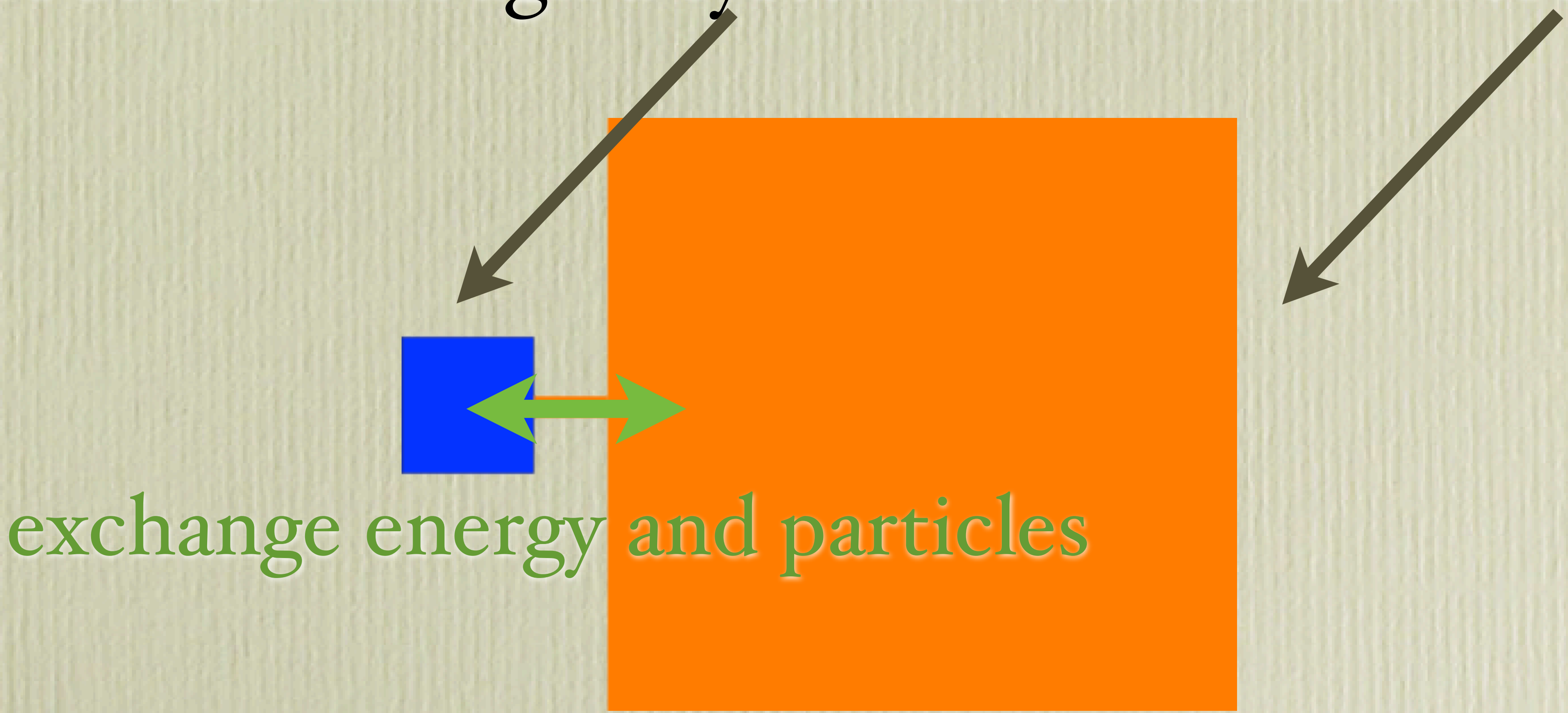

Treat total system as a micro canonical ensemble

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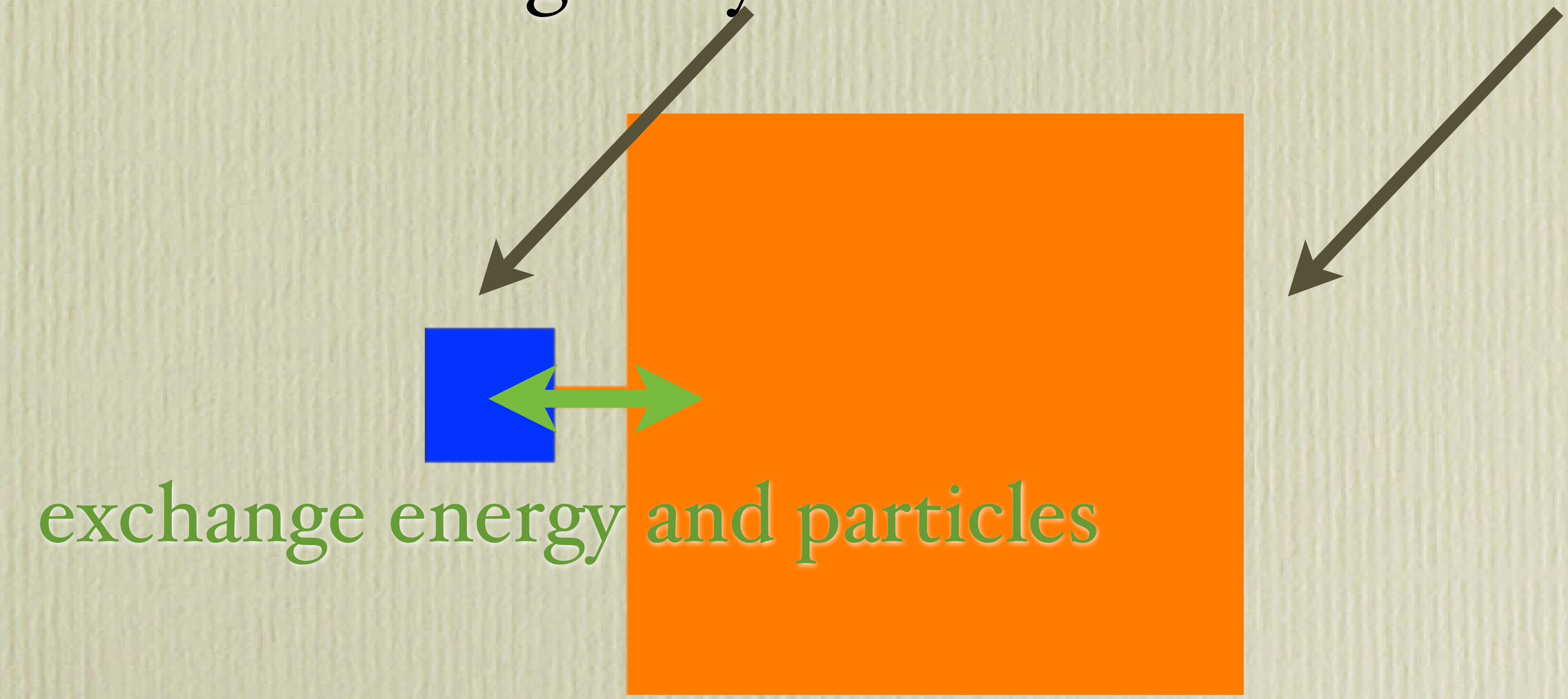

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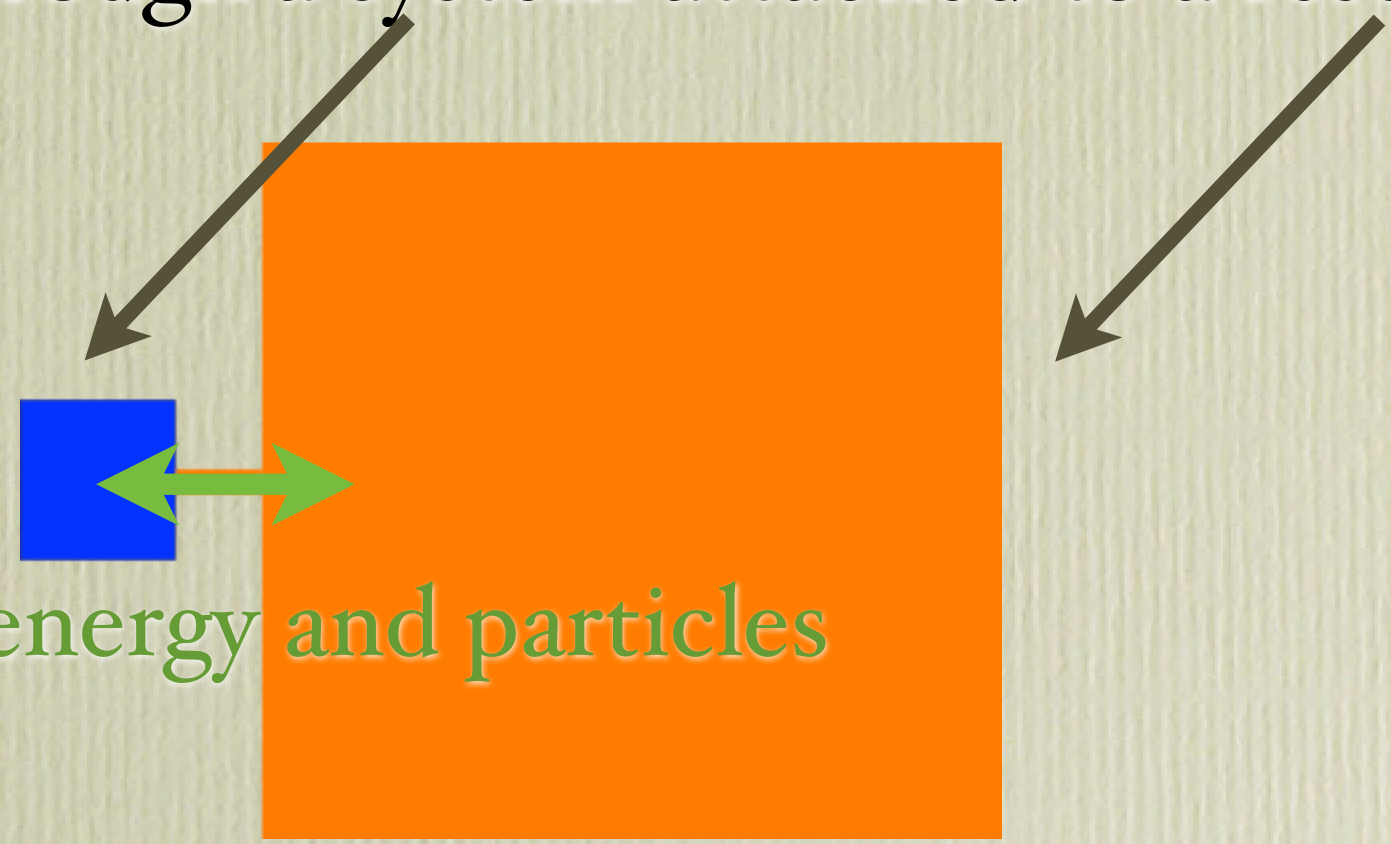

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$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

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For QCD $\left[\hat{H},\hat{N}\right]=0$

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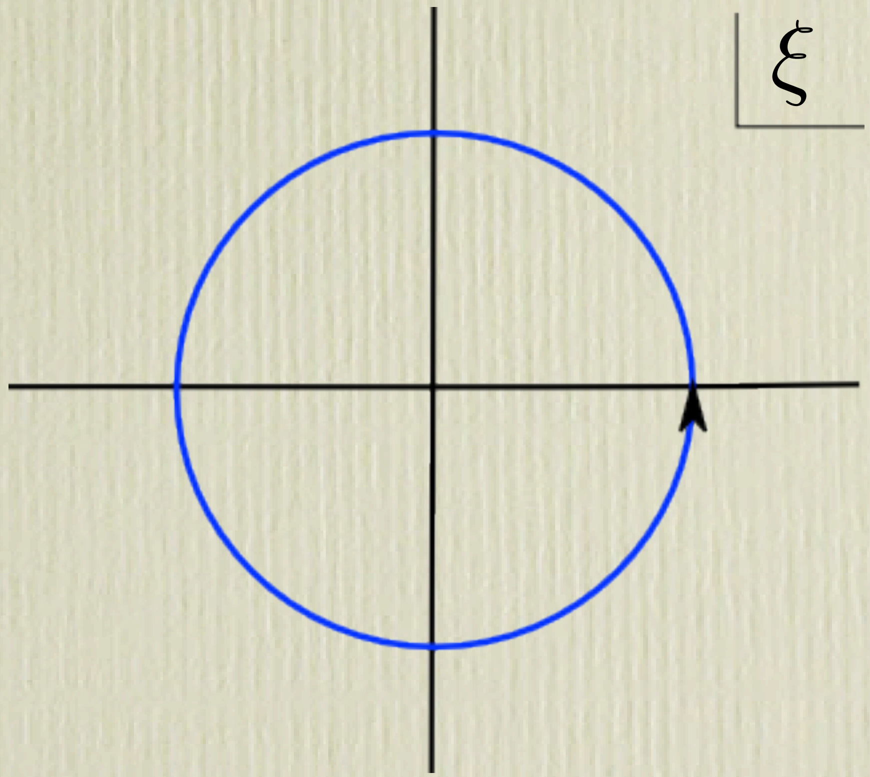

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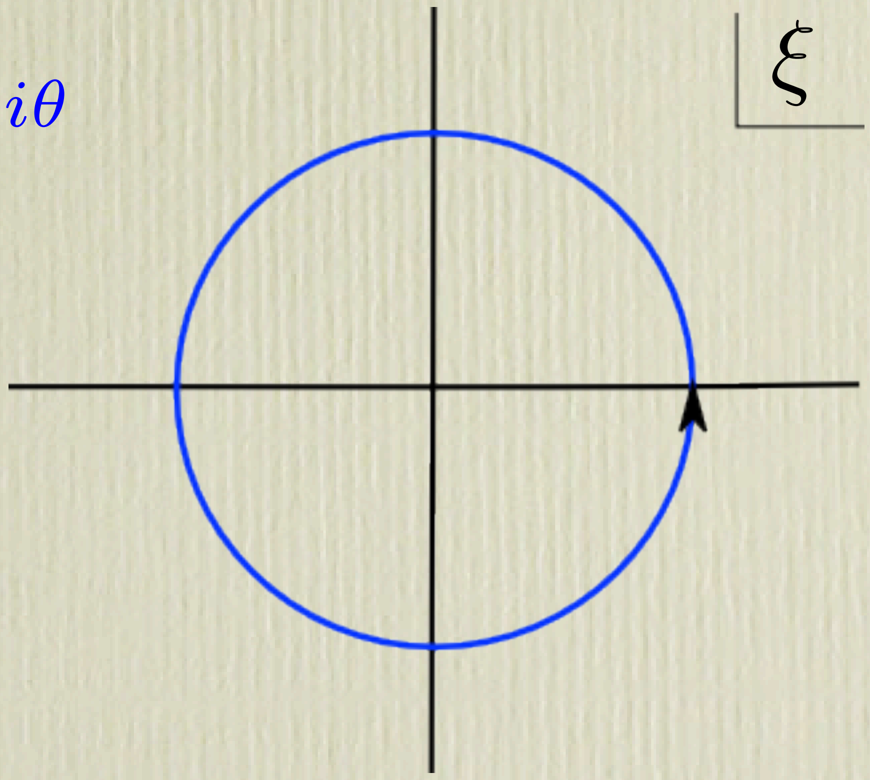

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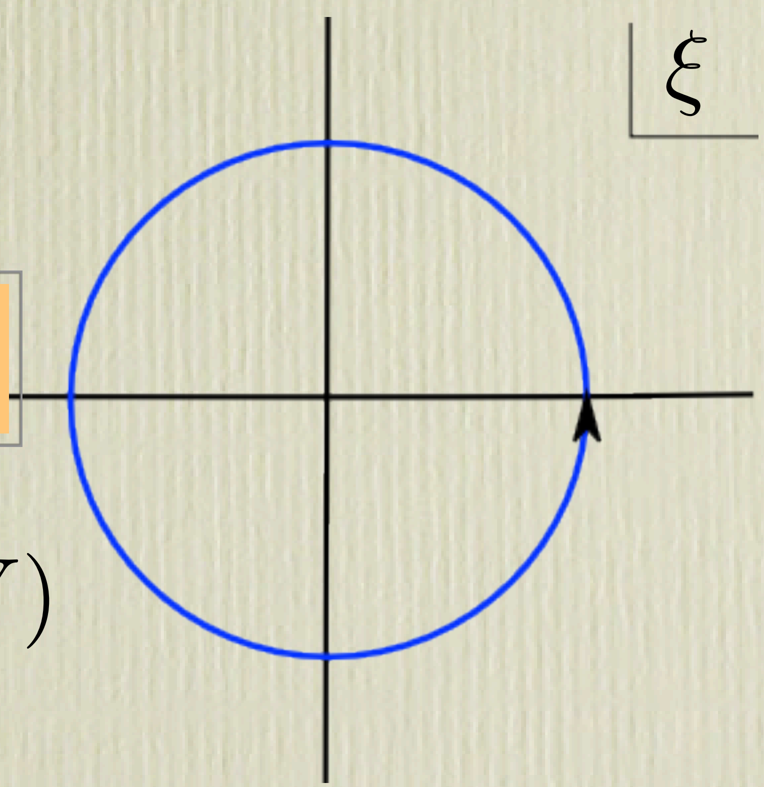

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$$Z_C\,(T,n,V) = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-in\theta} Z_G(T,e^{i\theta},V)$$

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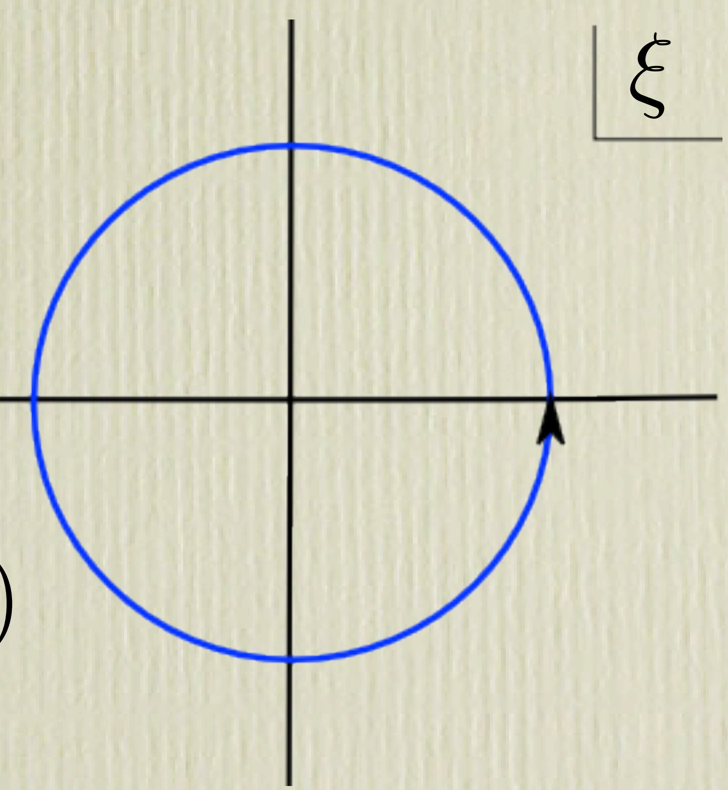

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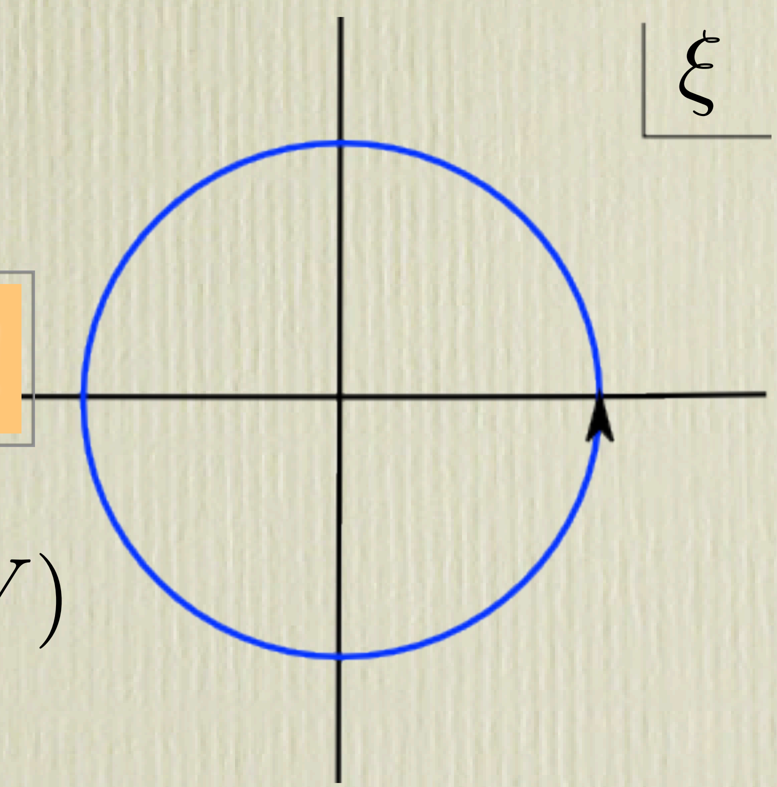

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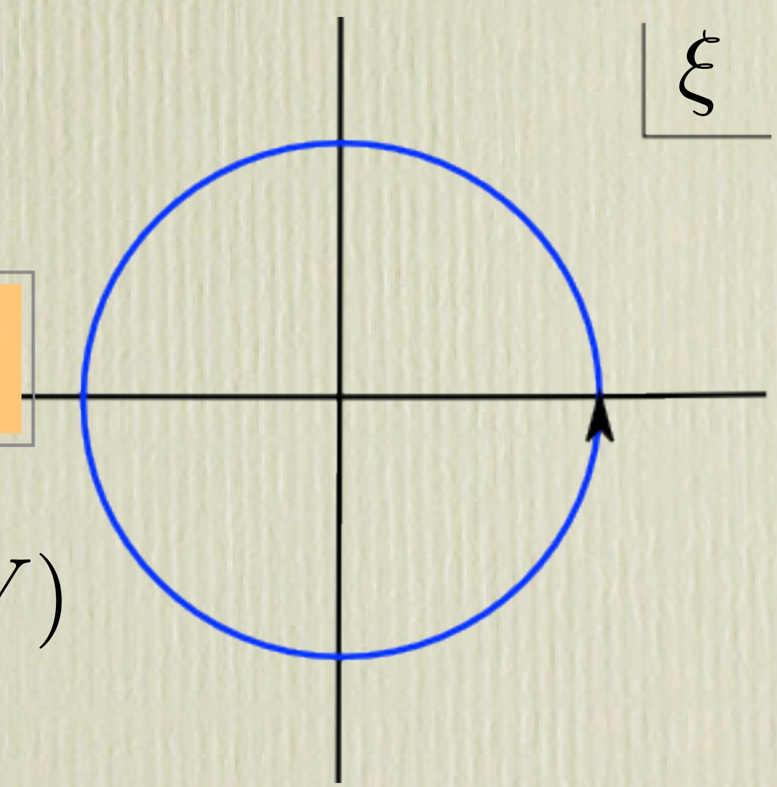

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2. Winding number expansion

3. Numerical setup

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Hopping parameter expansion

Everyone need to evaluated Det D(μ) to get $Z_G(\mu)$

Direct evaluation is still expensive...

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Direct evaluation is still expensive...

We want a cheaper method!

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Chemical potential appears in temporal hopping

$$D_W(\mu) = 1 - \kappa Q_s + \kappa e^{\mu a} Q_4^+ + \kappa e^{-\mu a} Q_4^-$$

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Meng et al. (Kentucky)

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Kentucky '08

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Fourier transformation

$$Z_C(n) = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-in\theta} \left\langle \frac{\exp\left(\sum_{k=-\infty}^{\infty} W_k e^{ik\theta}\right)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0$$

Numerical setup

★ Iwasaki gauge action

★ Clover fermion Nf=2

- APE stout smeared gauge link $c_{\mathrm{SW}} = 1.1$

★ Box sizes $8^3 \times 4$ $12^3 \times 4$ $16^3 \times 4$

β	κ	PCAC	m_π/m_ρ
0.9	0.137	0.17(13)	0.8978(55)
1.1	0.133	0.18(19)	0.9038(56)
1.3	0.133	0.088(53)	0.8656(72)
1.5	0.131	0.116(39)	0.8482(57)
1.7	0.129	0.168(21)	1.0429(46)
1.9	0.125	0.1076(68)	0.7310(95)
2.1	0.122	0.1259(11)	0.833(12)

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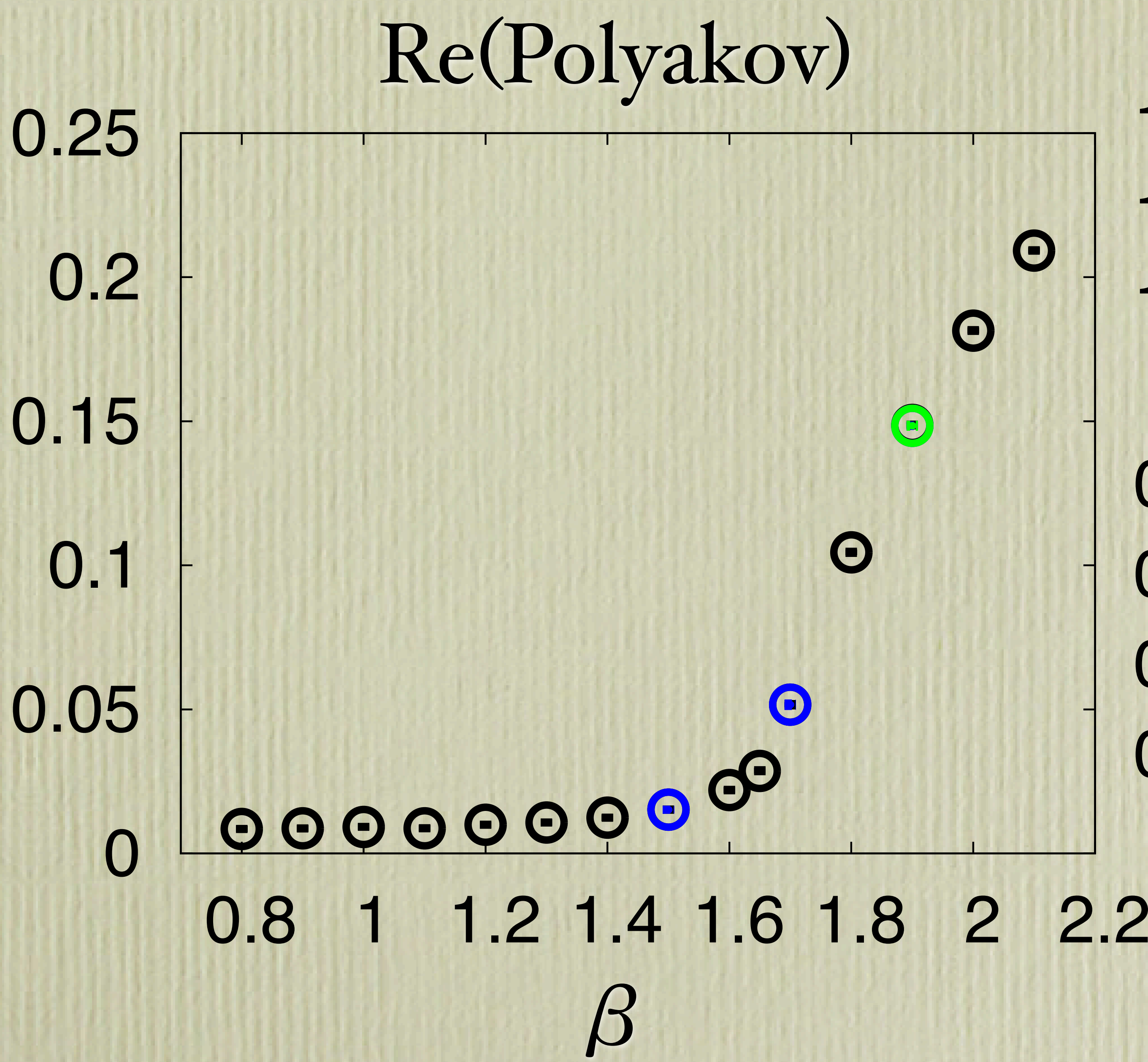

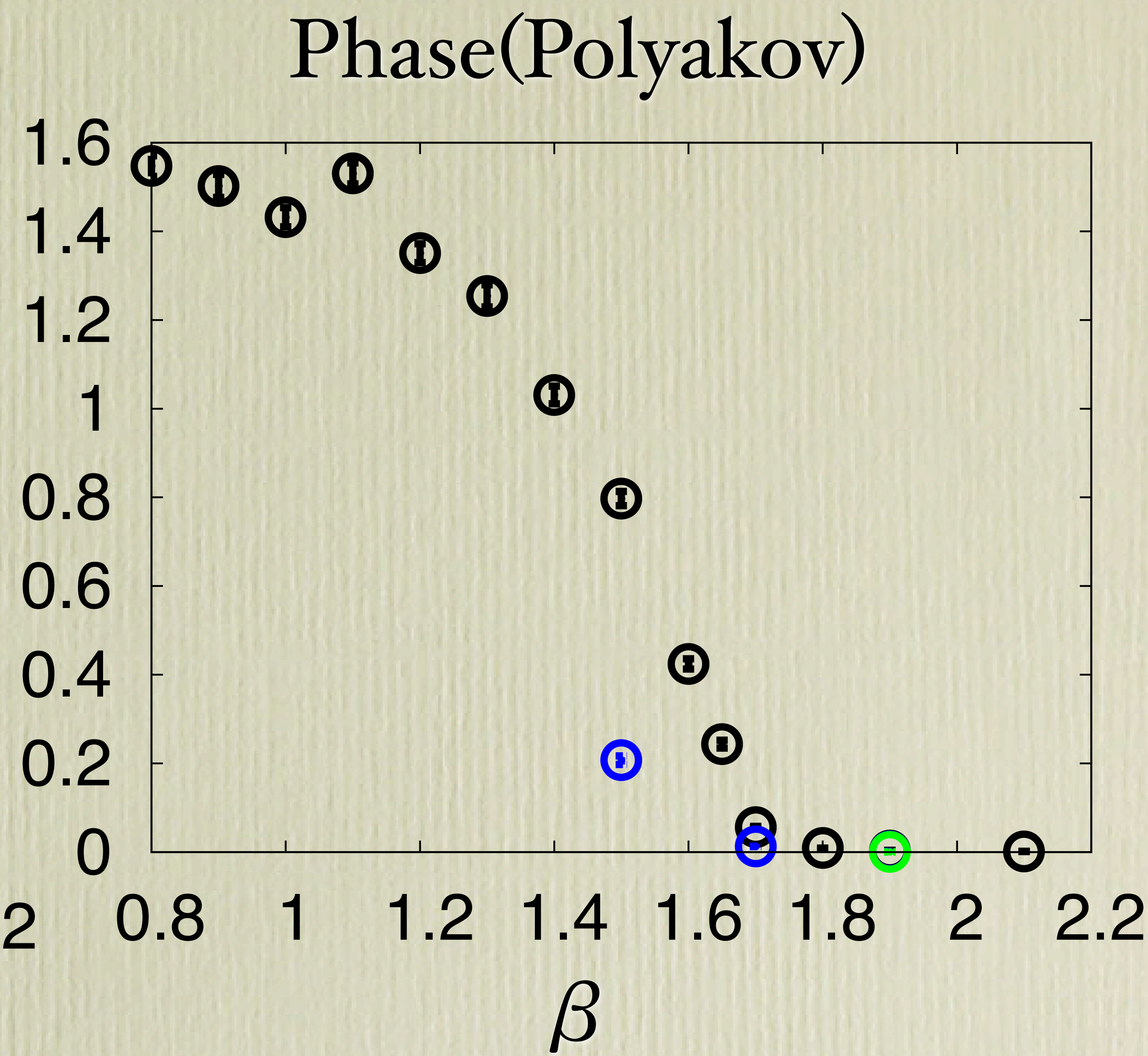

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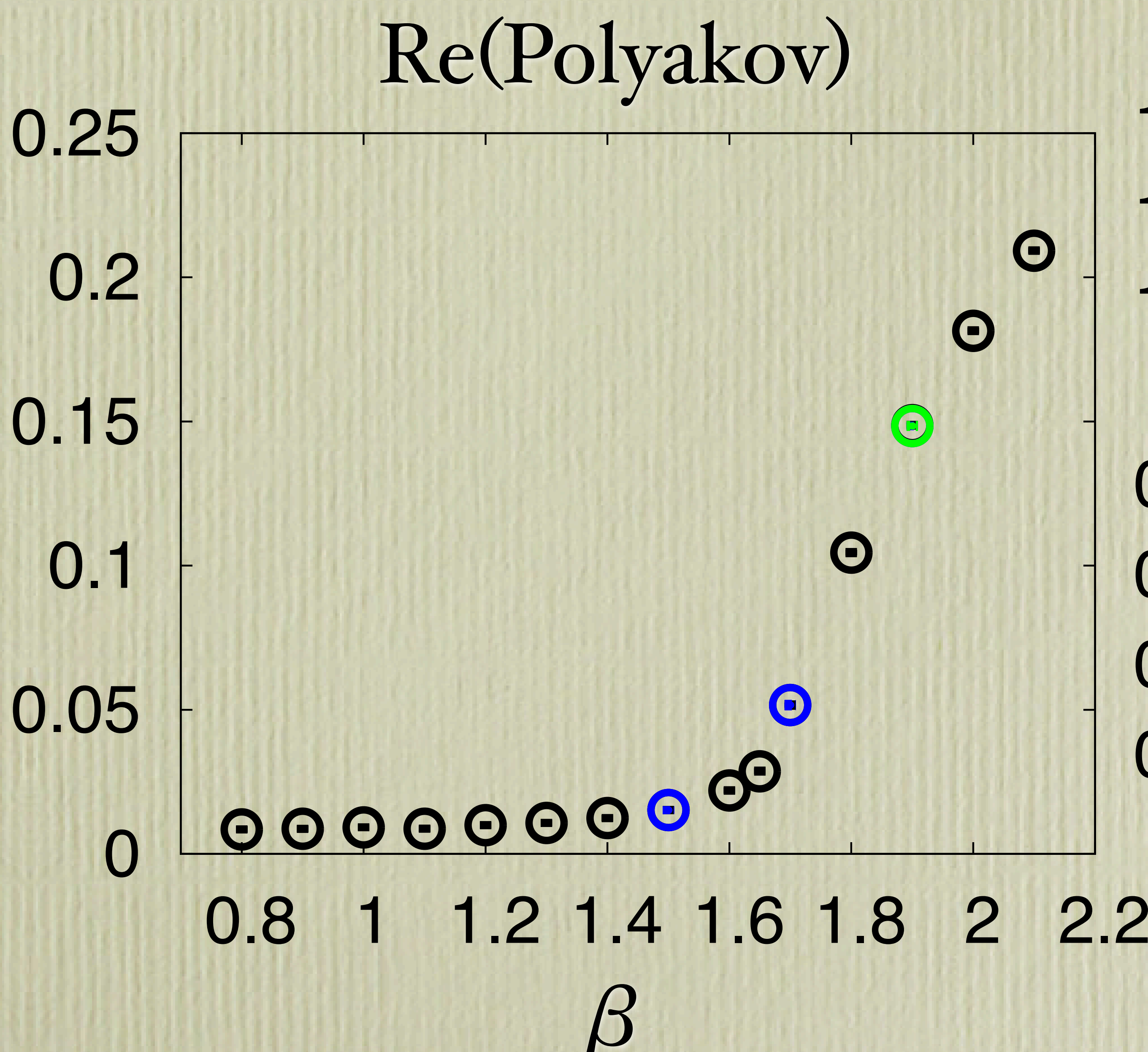

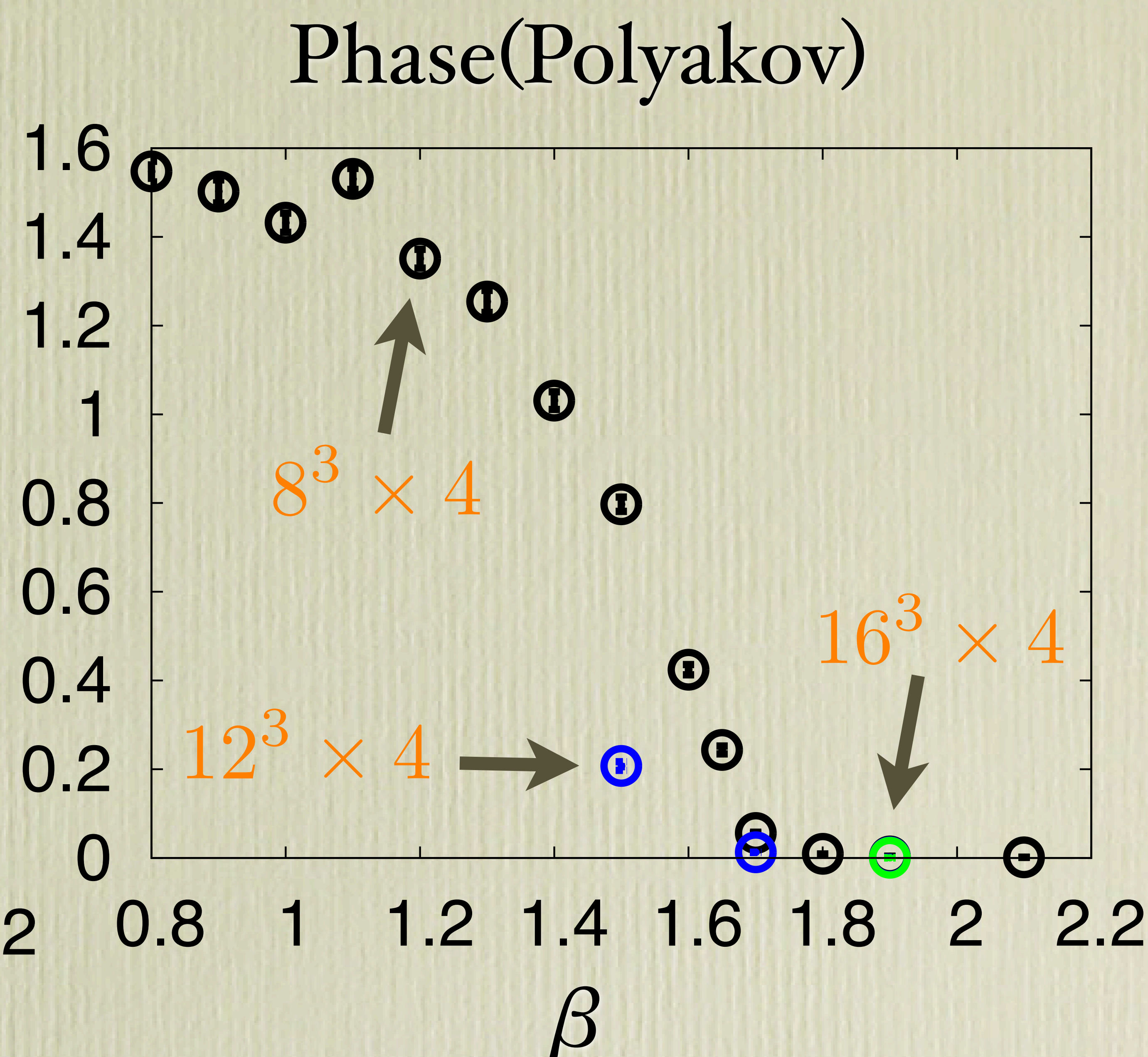

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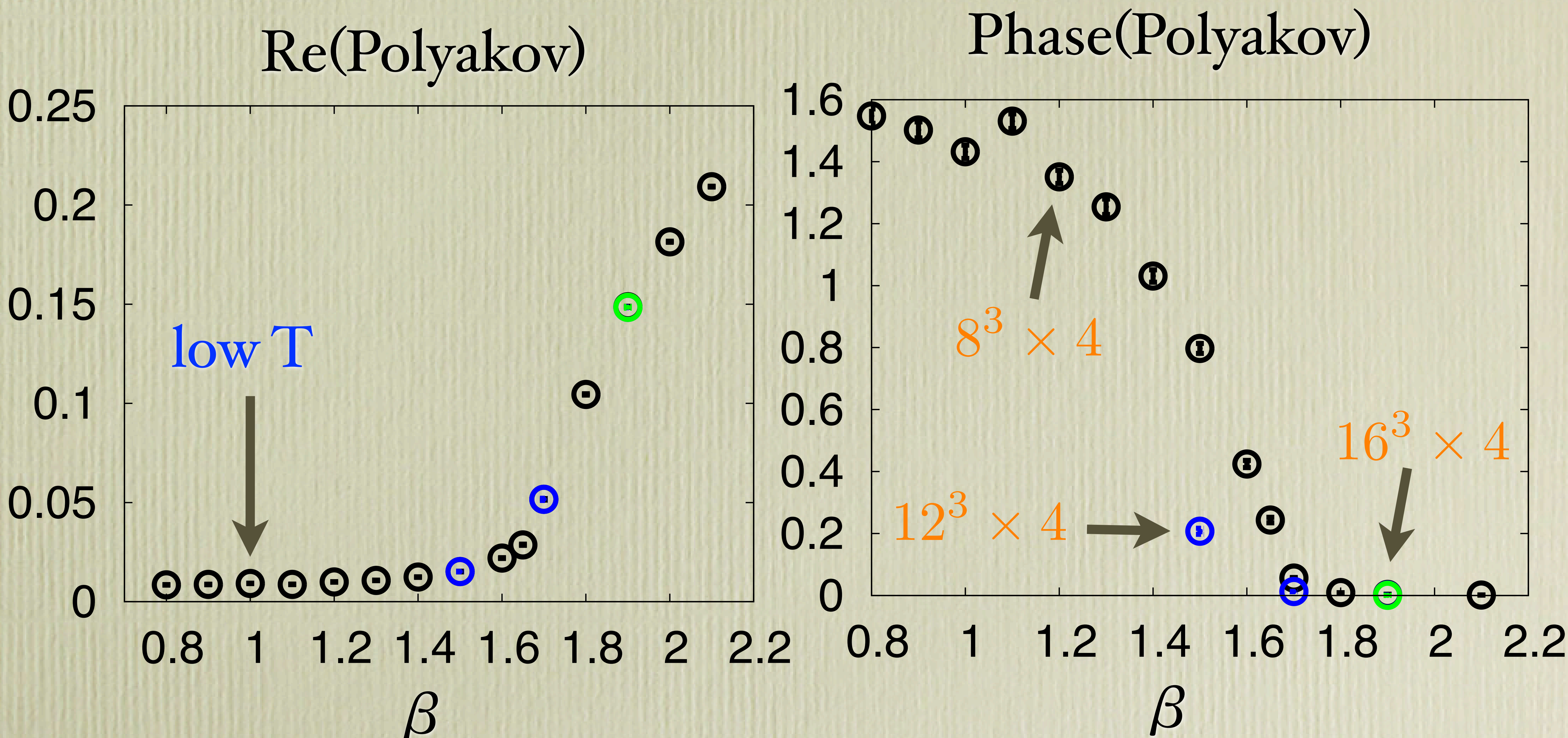

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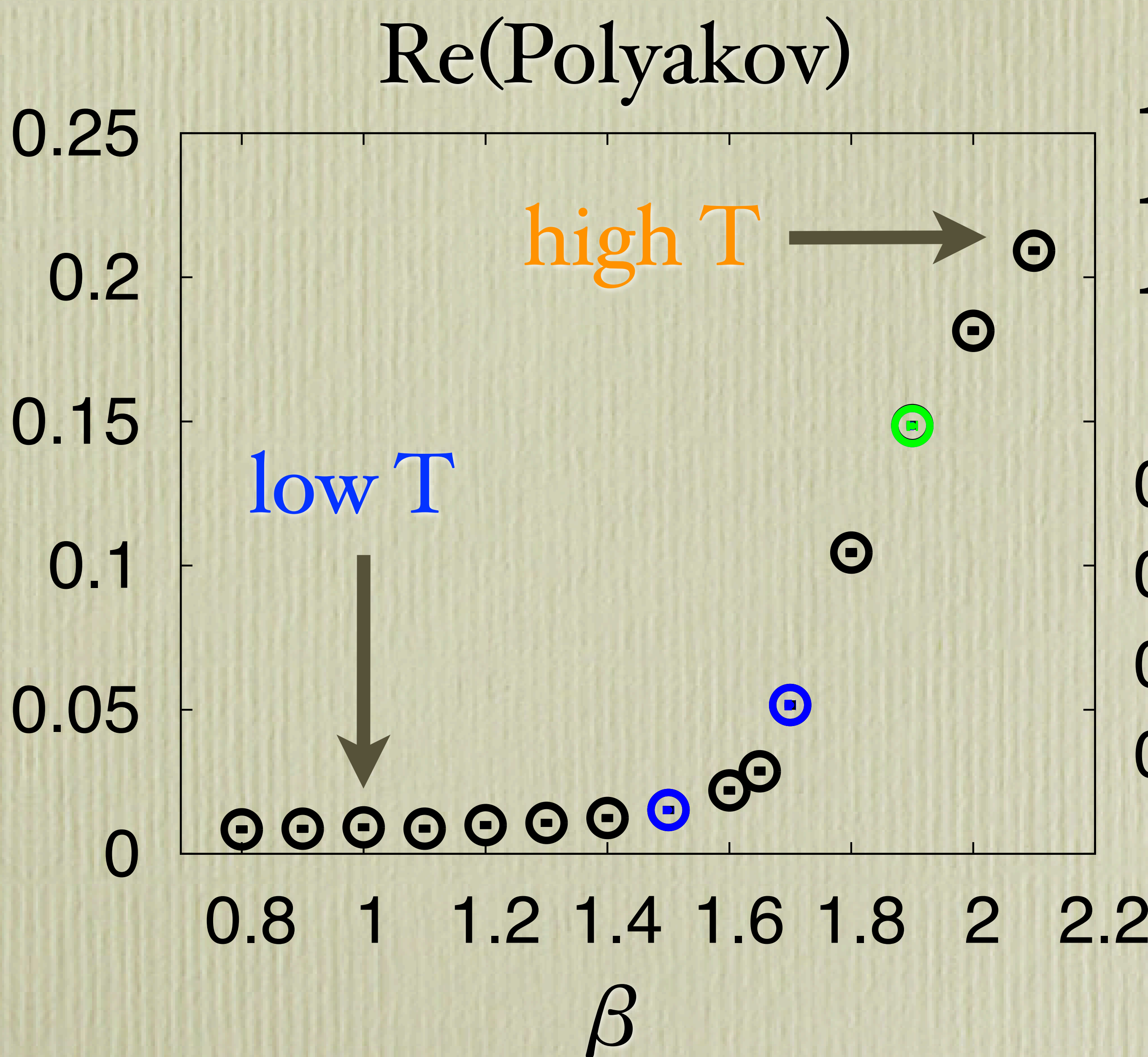

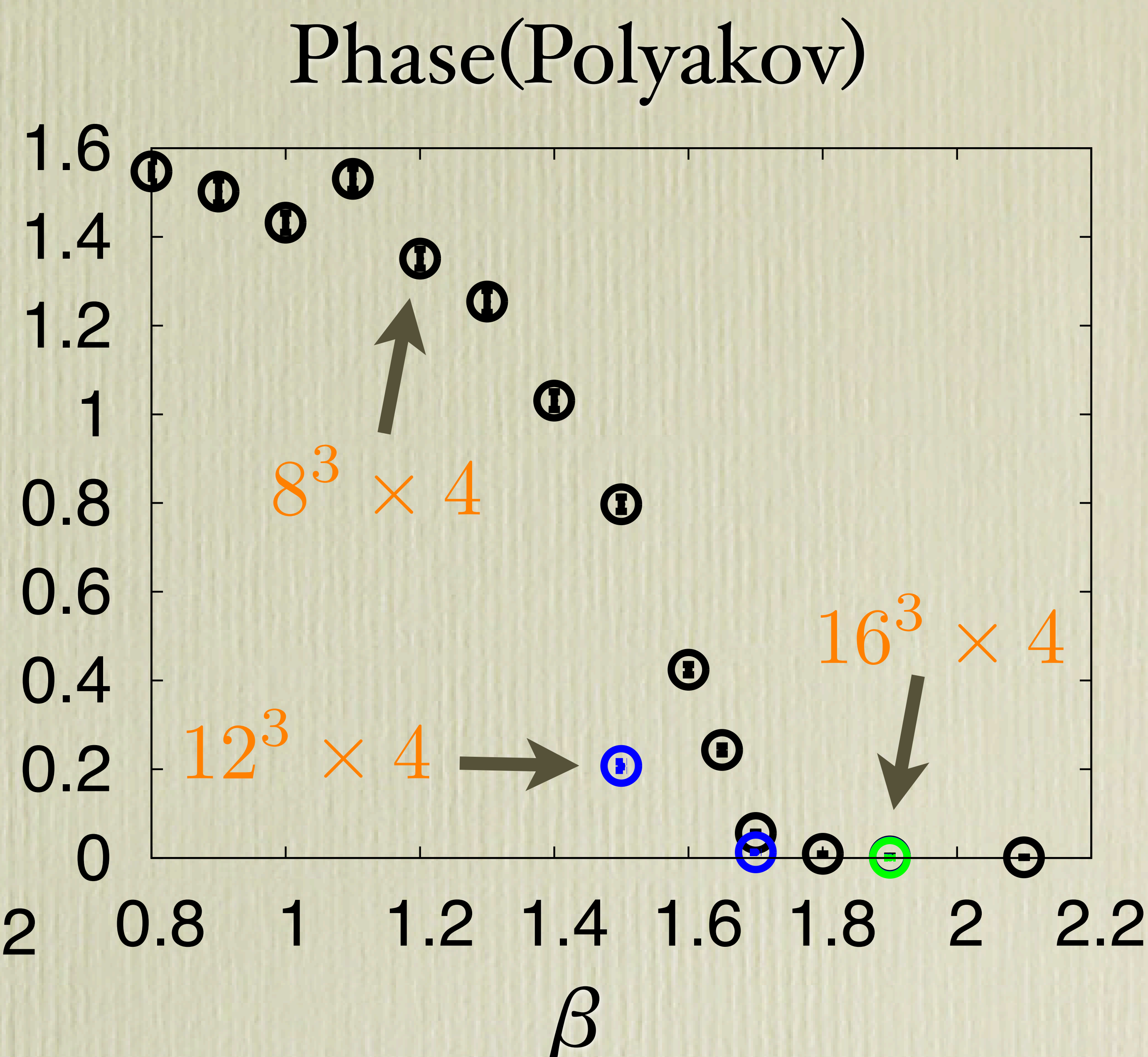

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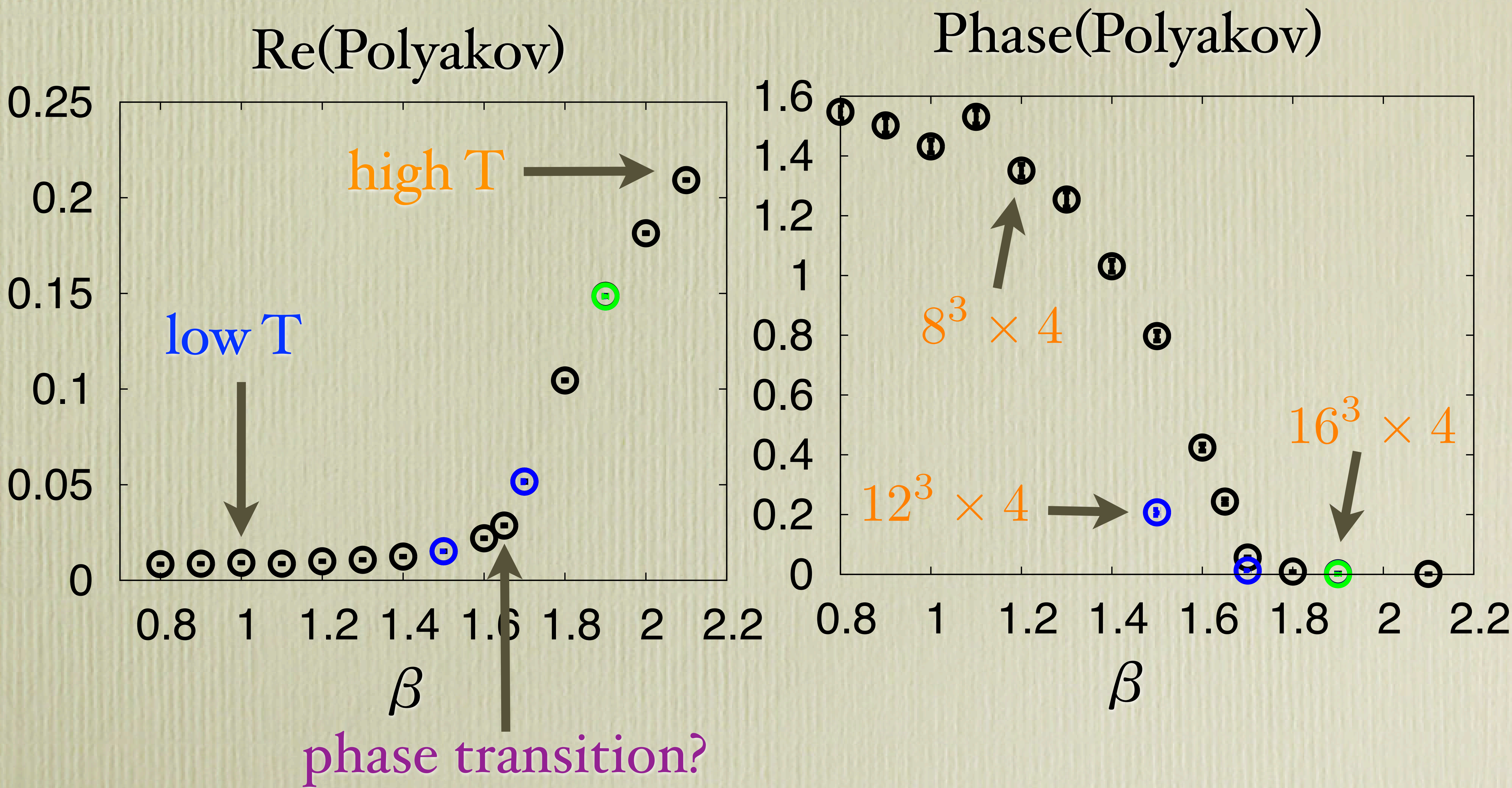

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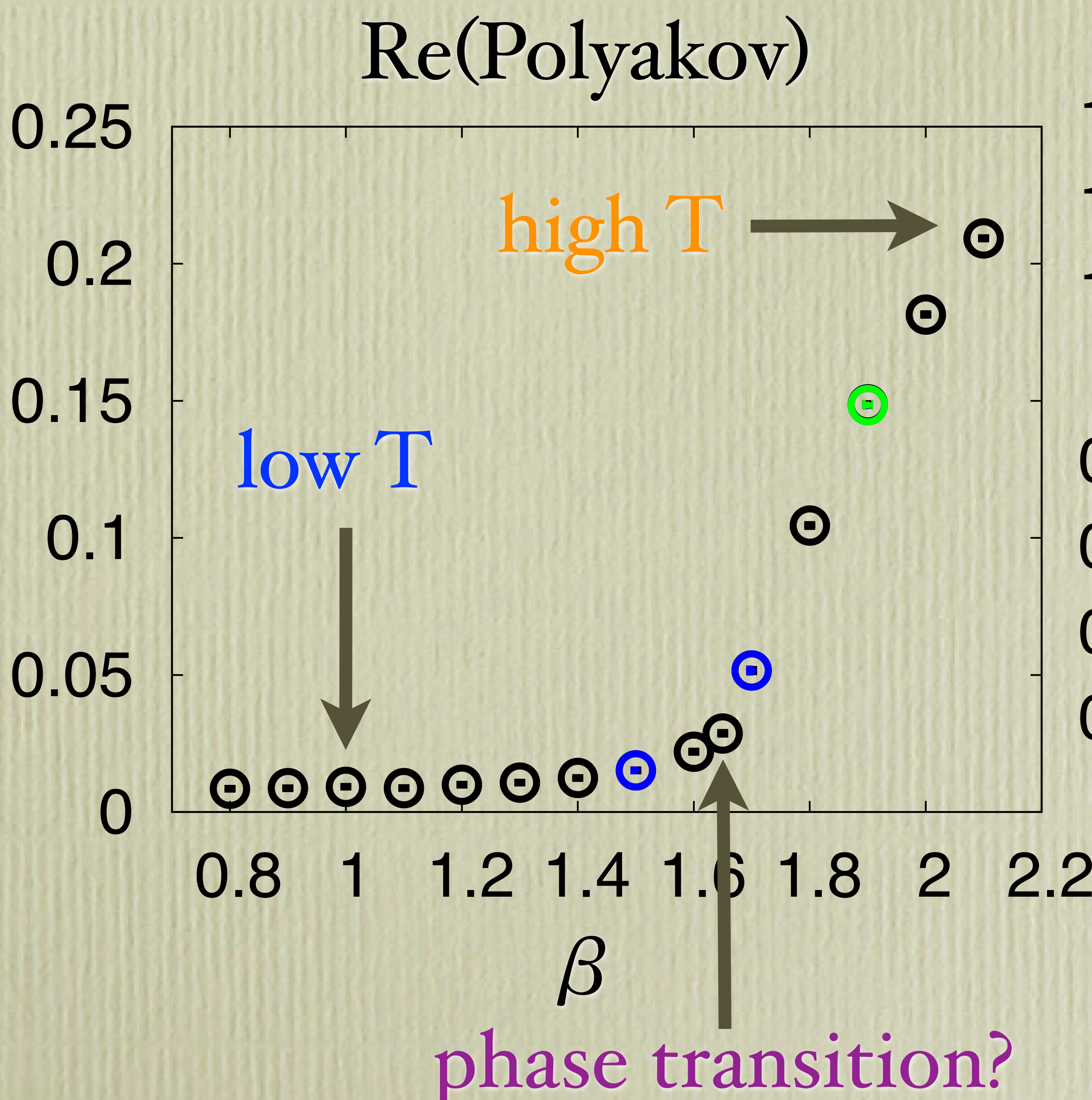

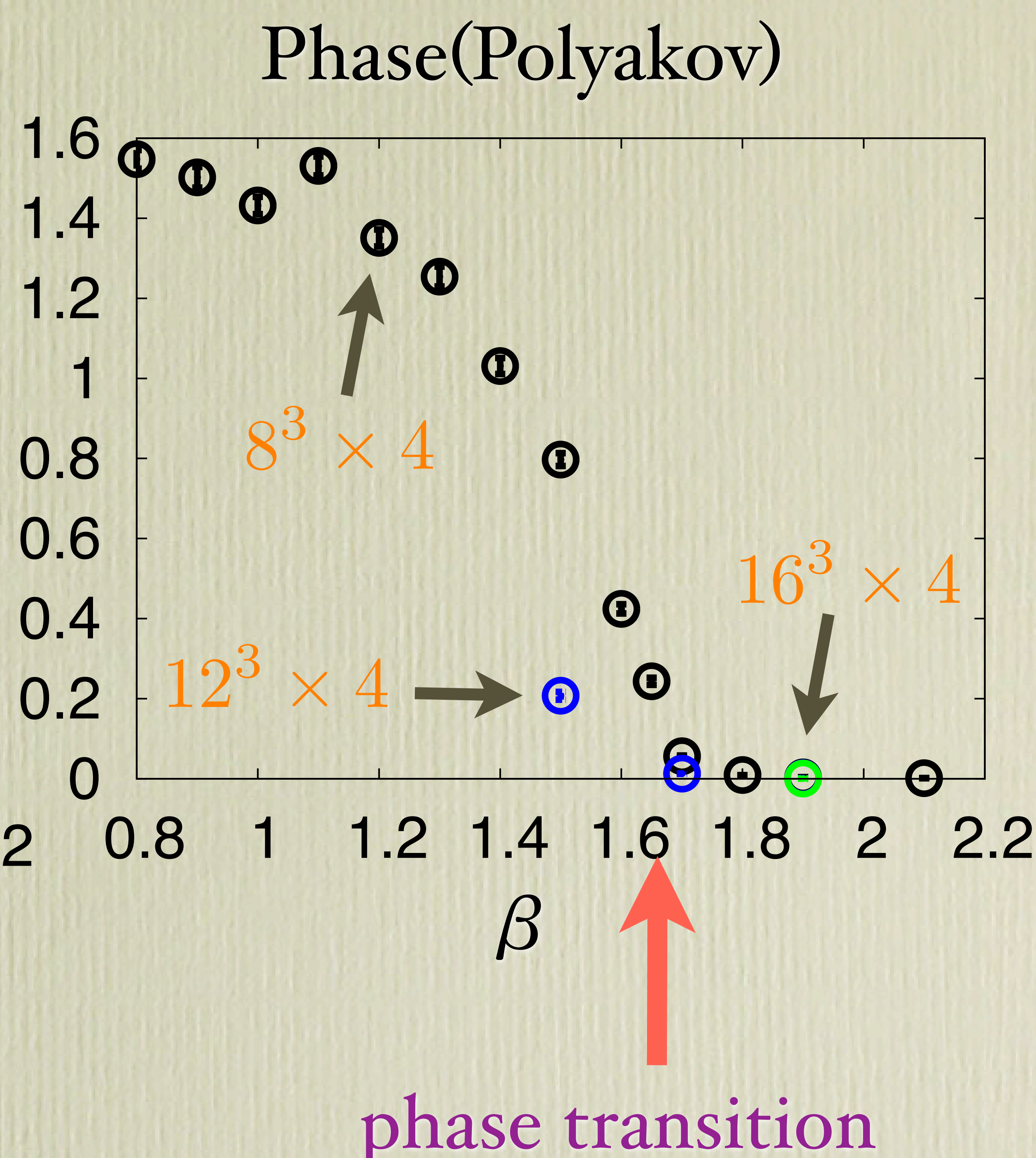

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P. Gibbs

A. Hasenfratz and D. Toussaint

for staggered

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Fugacity expansion by reduction formula

P. Gibbs

A. Hasenfratz and D. Toussaint

for staggered

K. Nagata and A. Nakamura

A. Alexandru and U. Wenger

for Wilson

Numerical results Zc(n)

Test of the hopping parameter expansion

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$$\mathrm{Det} D_W(\mu)$$

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Exact!

Computational cost is heavy...

Numerical results Zc(n)

Canonical partition function measured on a single conf.

$8^3 \times 4 \quad \beta = 1.9 \quad \kappa = 0.1250 \quad am_{\rm PCAC} = 0.1076(68) \quad \mu = 0$

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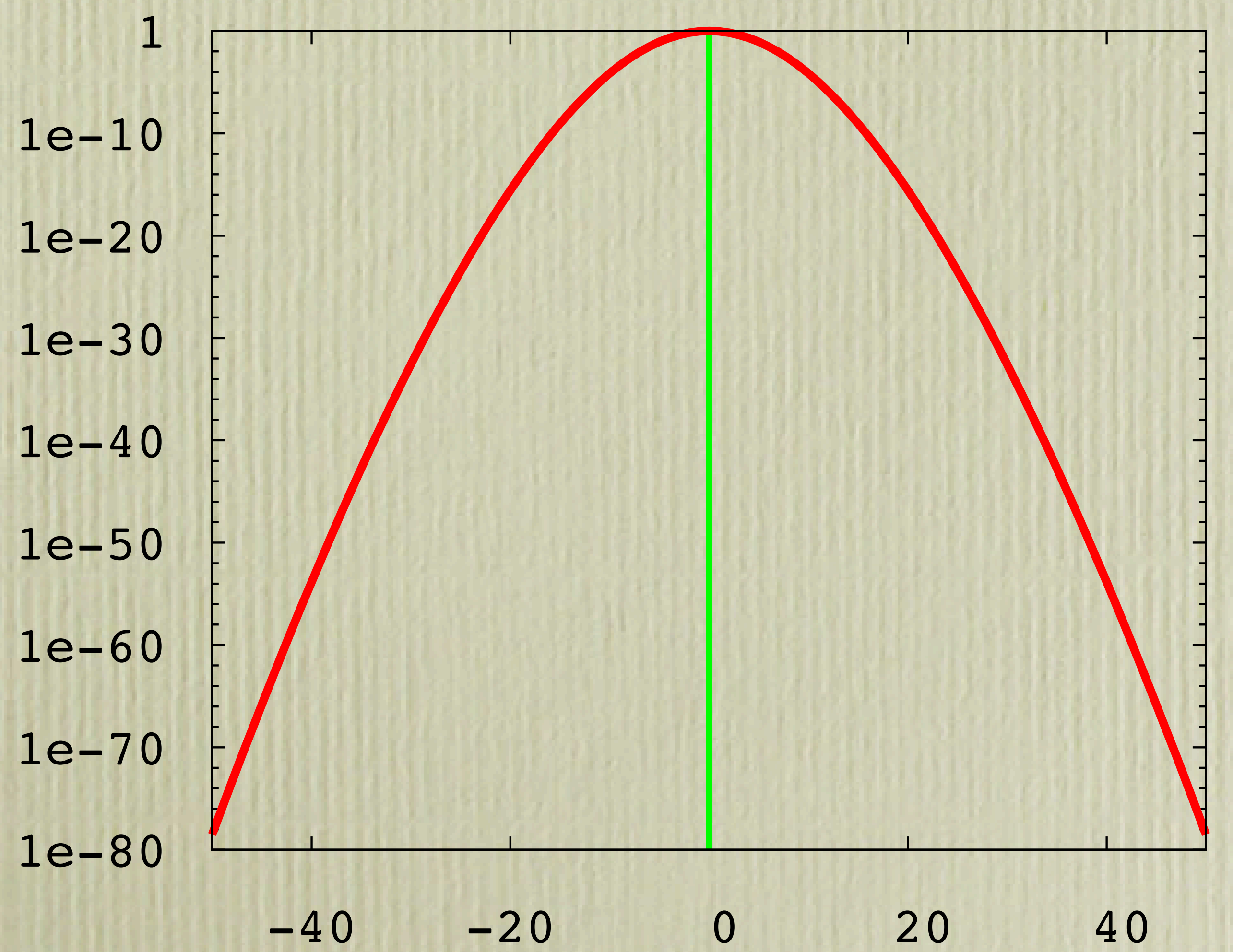

baryon number

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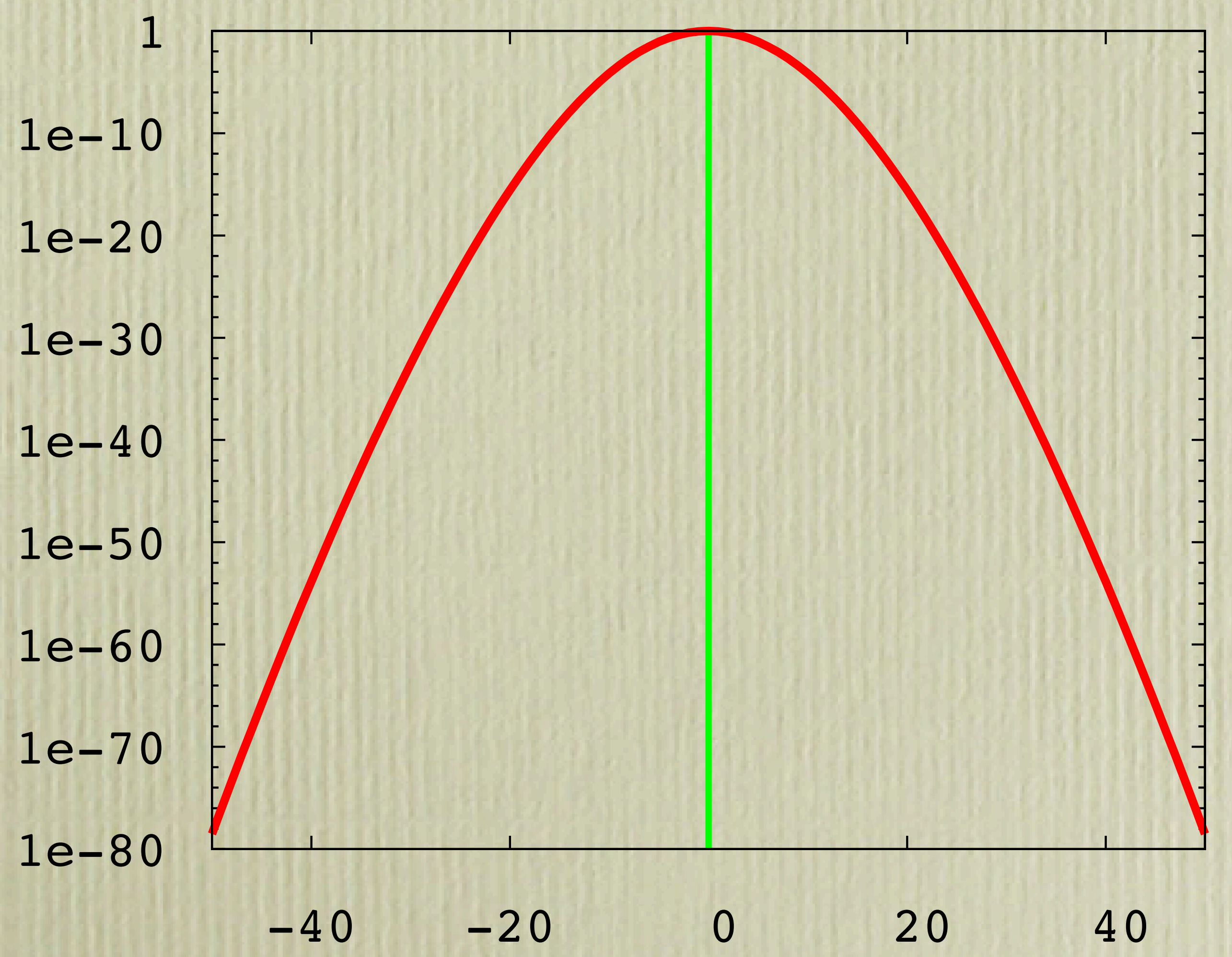
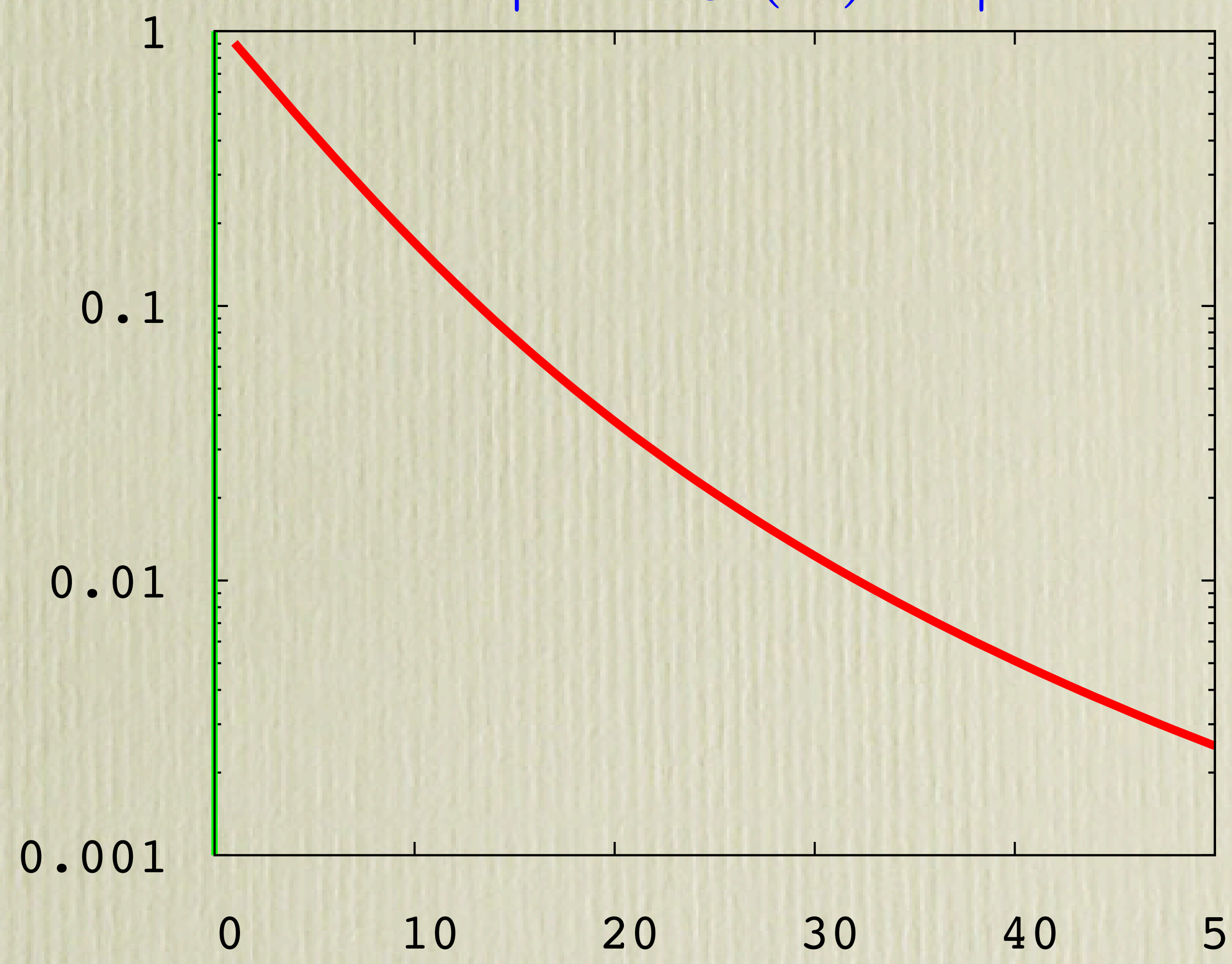


baryon number

$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$



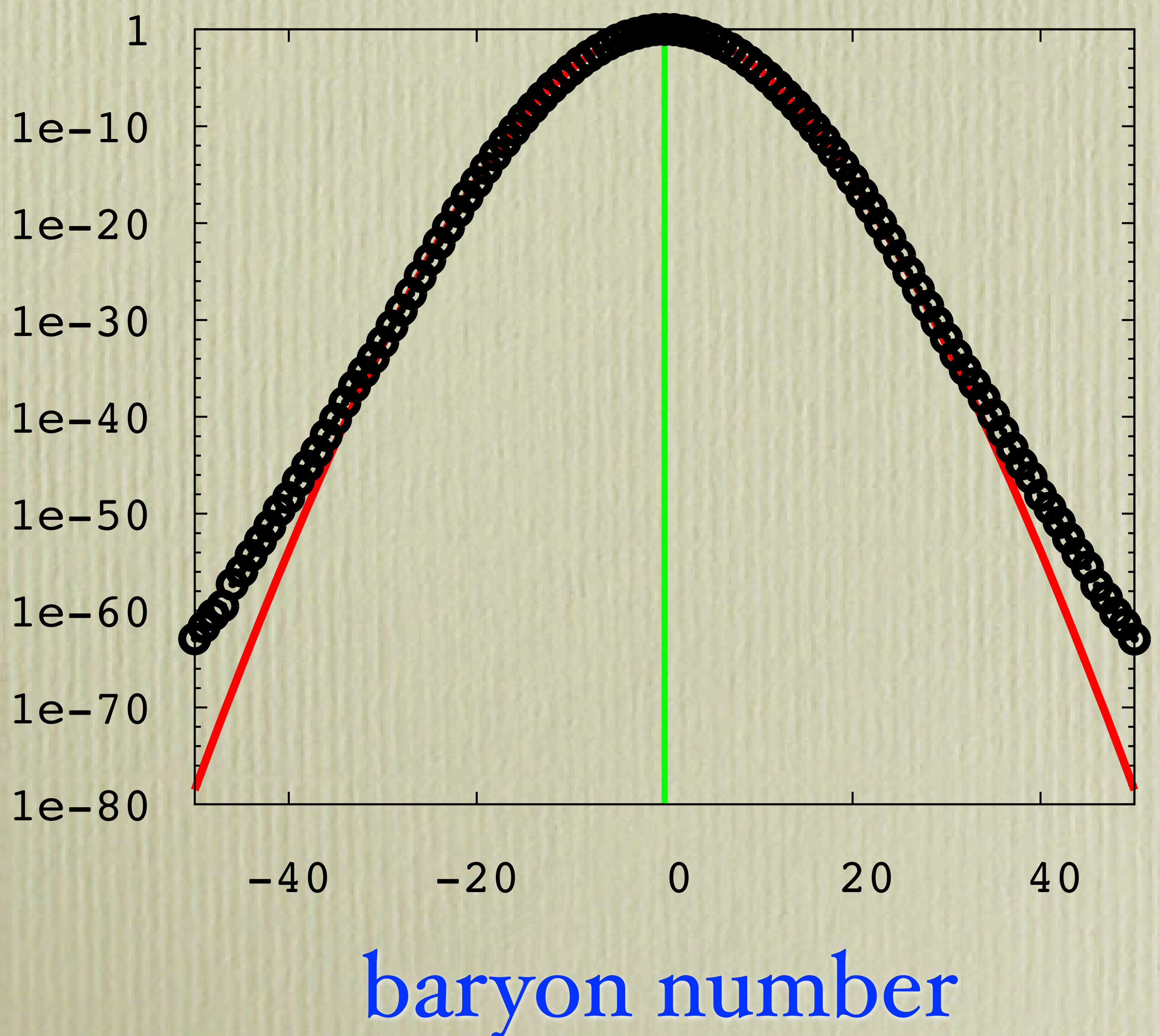
baryon number

Numerical results Zc(n)

Multi precision for Fourier transformation

winding number=120 (number of hop =480)

$\log|Z_C(n)|$



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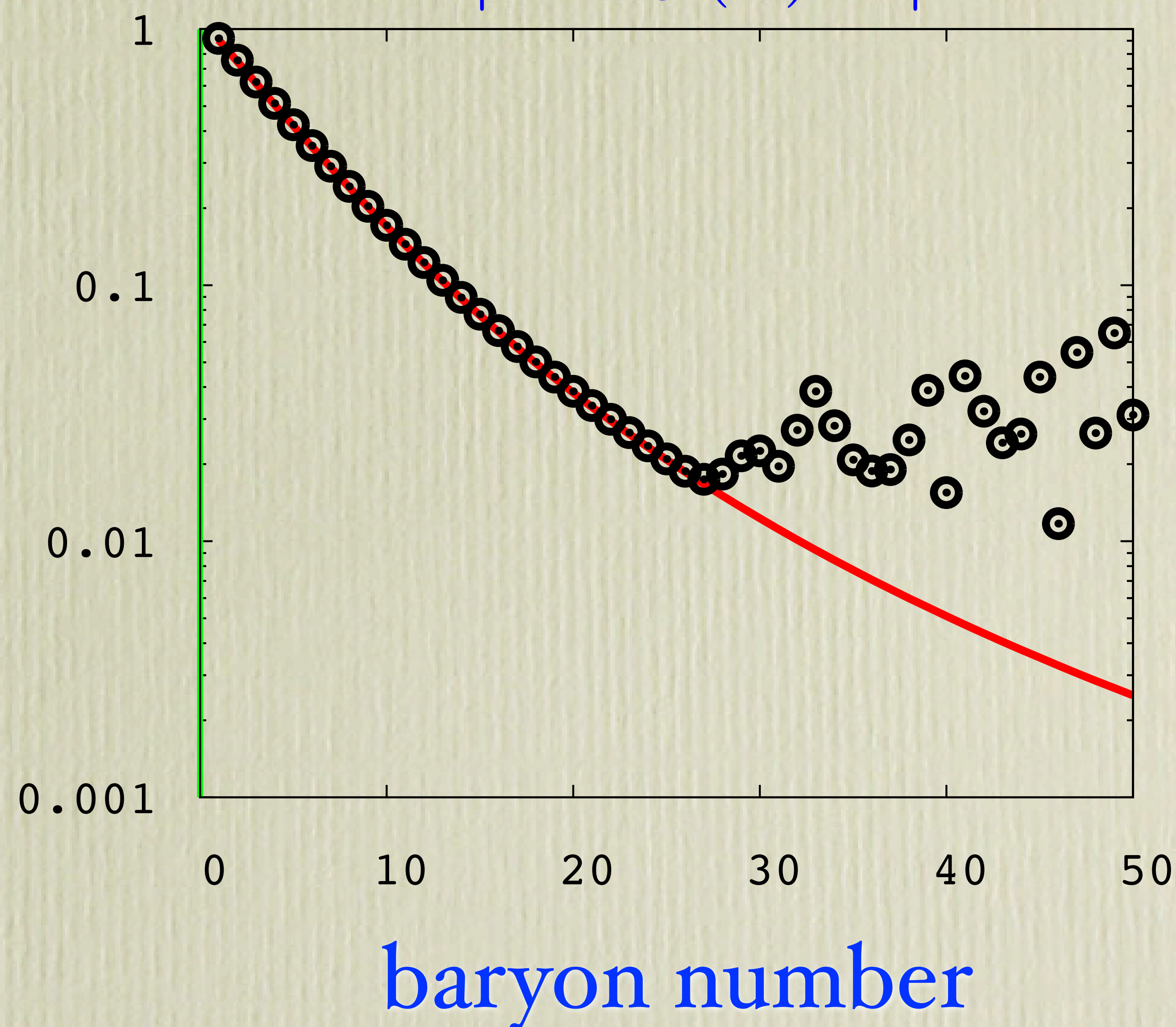
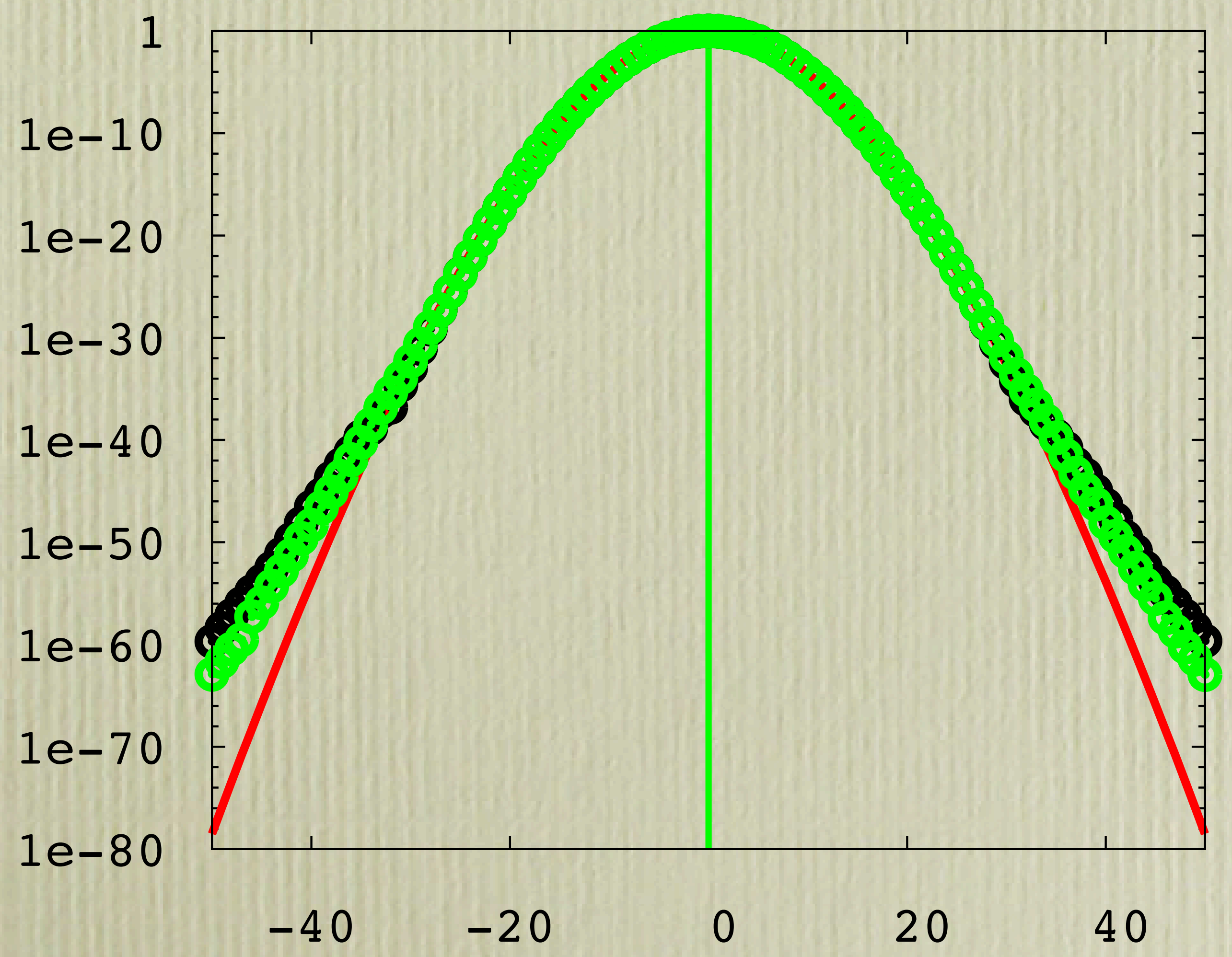


Numerical results Zc(n)

Multi precision for Fourier transformation

winding number=120 vs 240

$\log|Z_C(n)|$



baryon number

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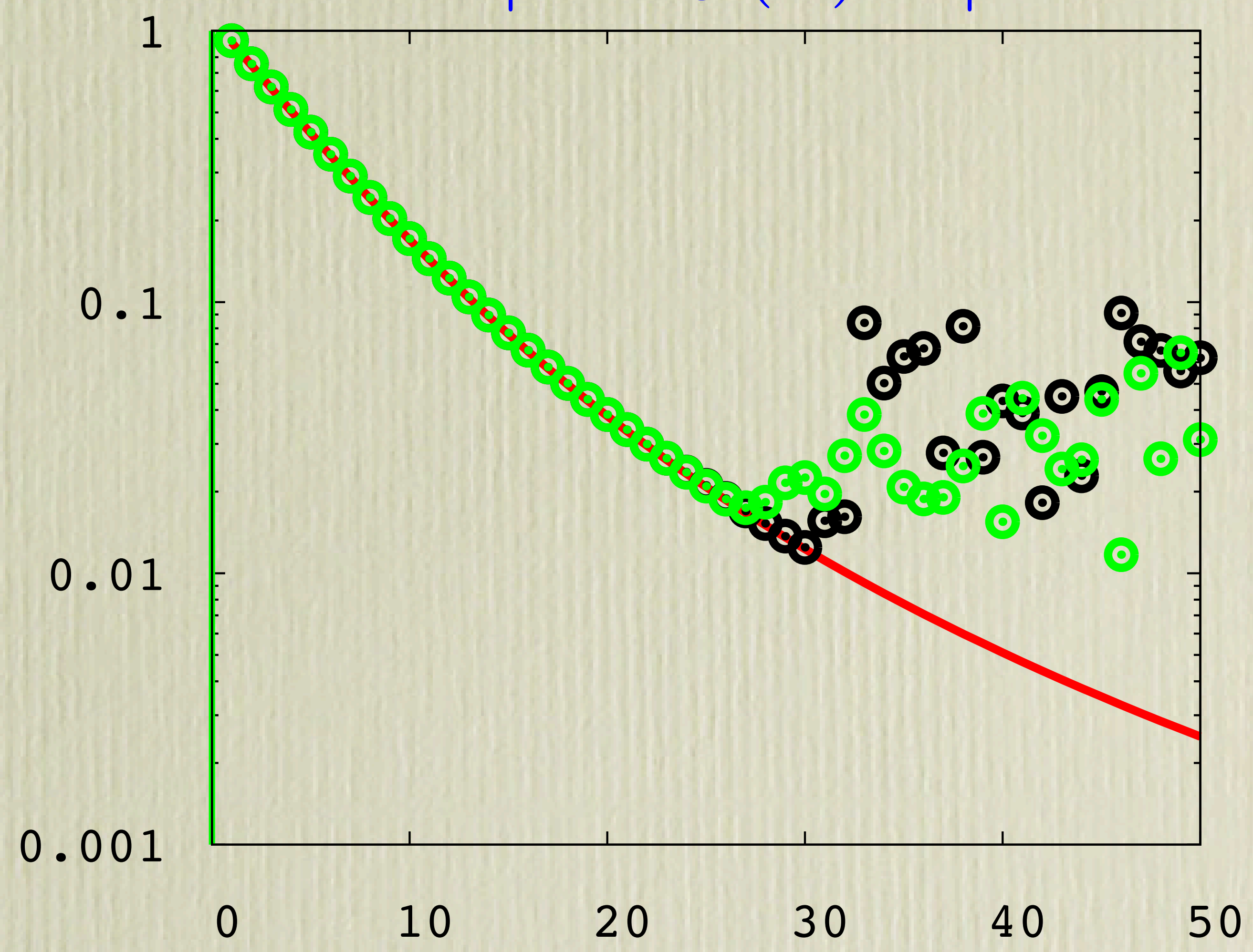

baryon number

Numerical results Zc(n)

Multi precision for Fourier transformation

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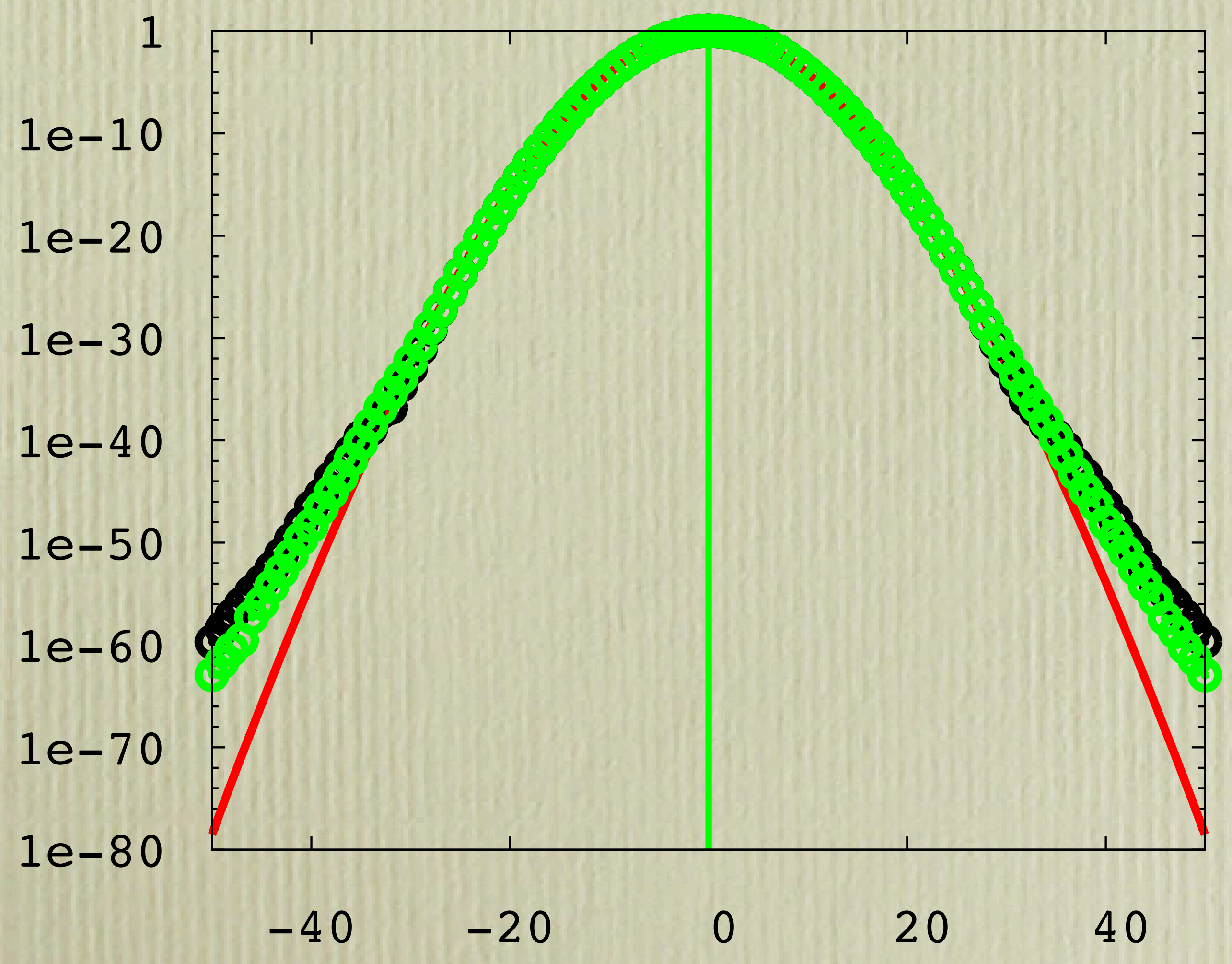

baryon number

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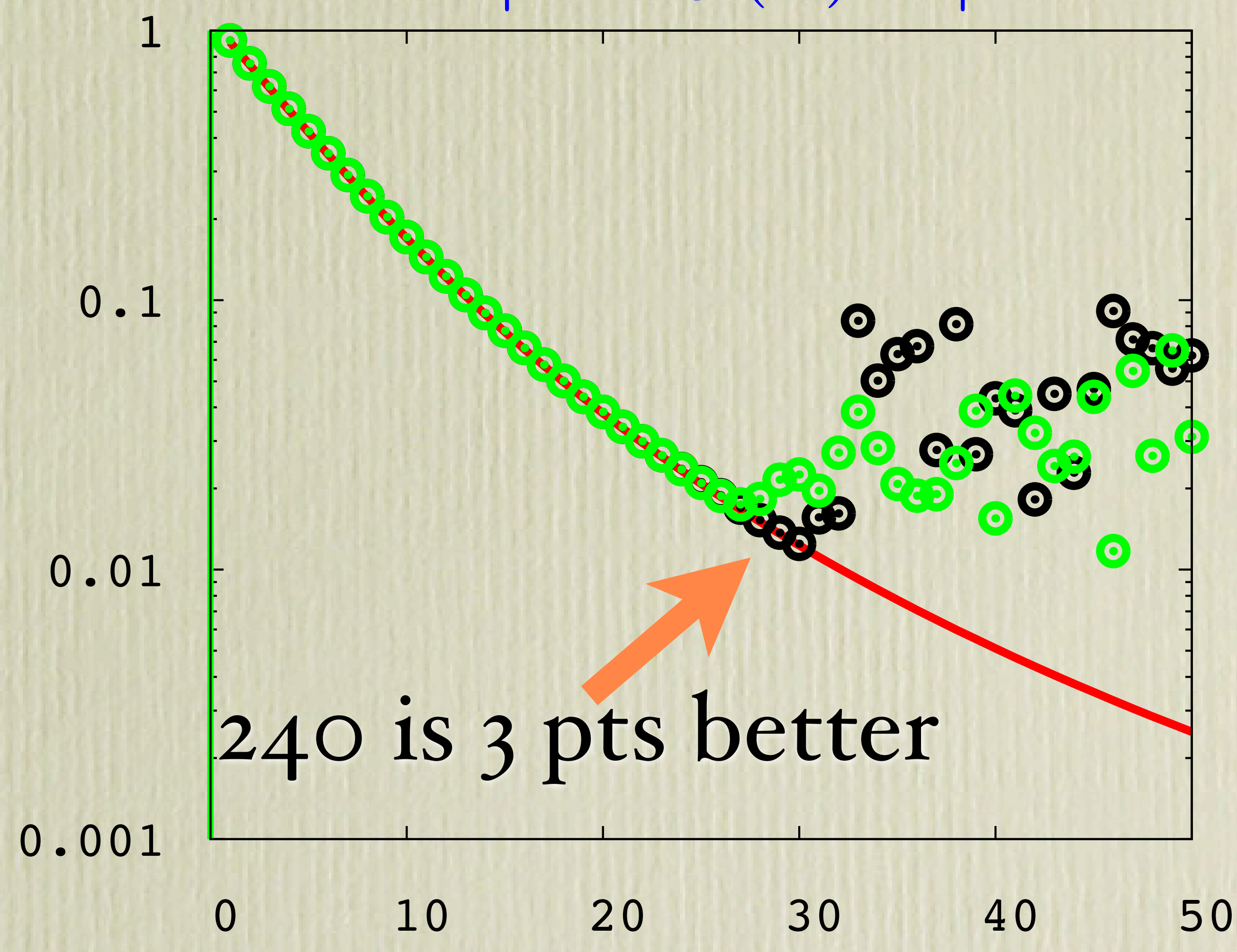


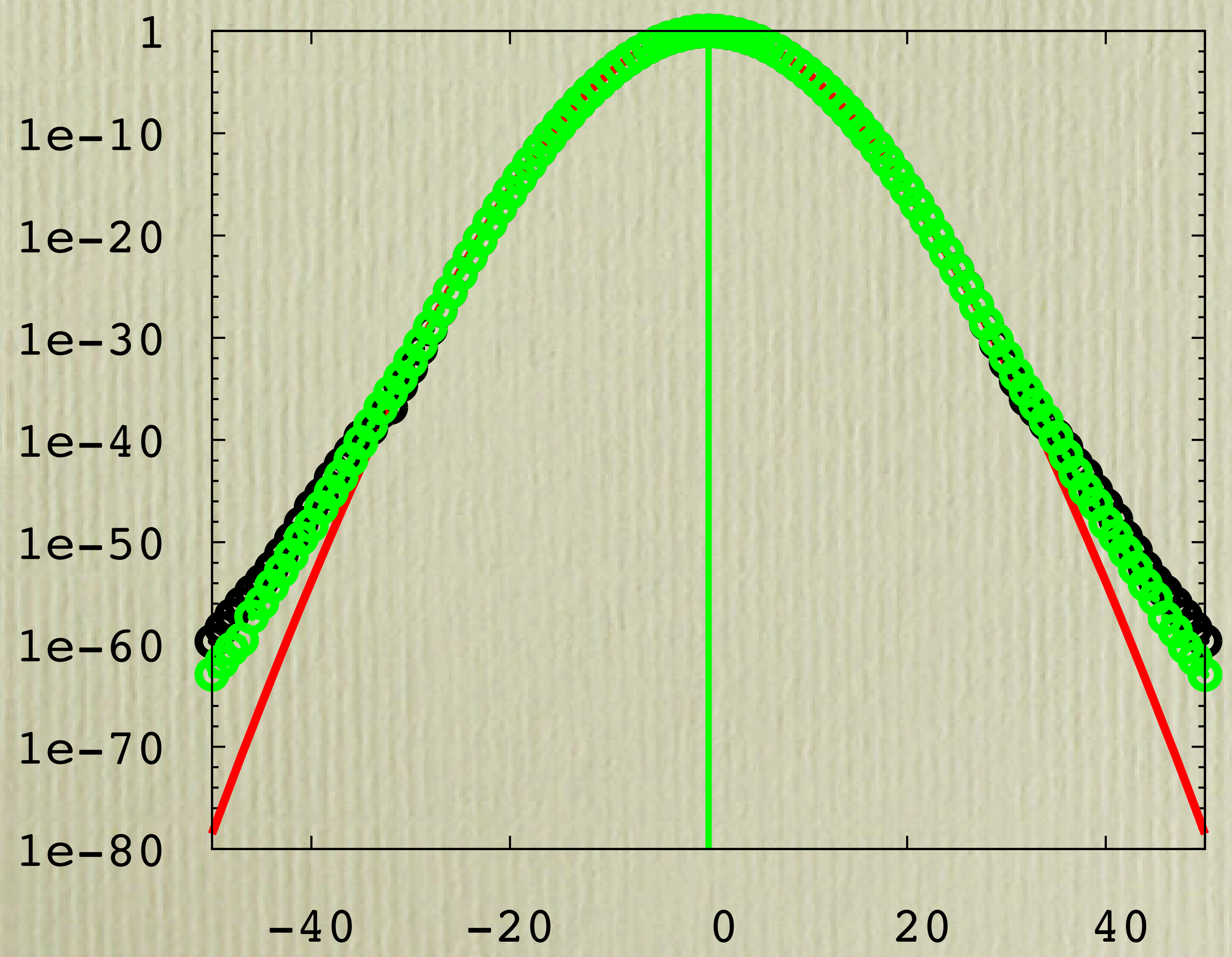
baryon number

Numerical results Zc(n)

Multi precision for Fourier transformation

We adopt winding number=120

$\log|Z_C(n)|$



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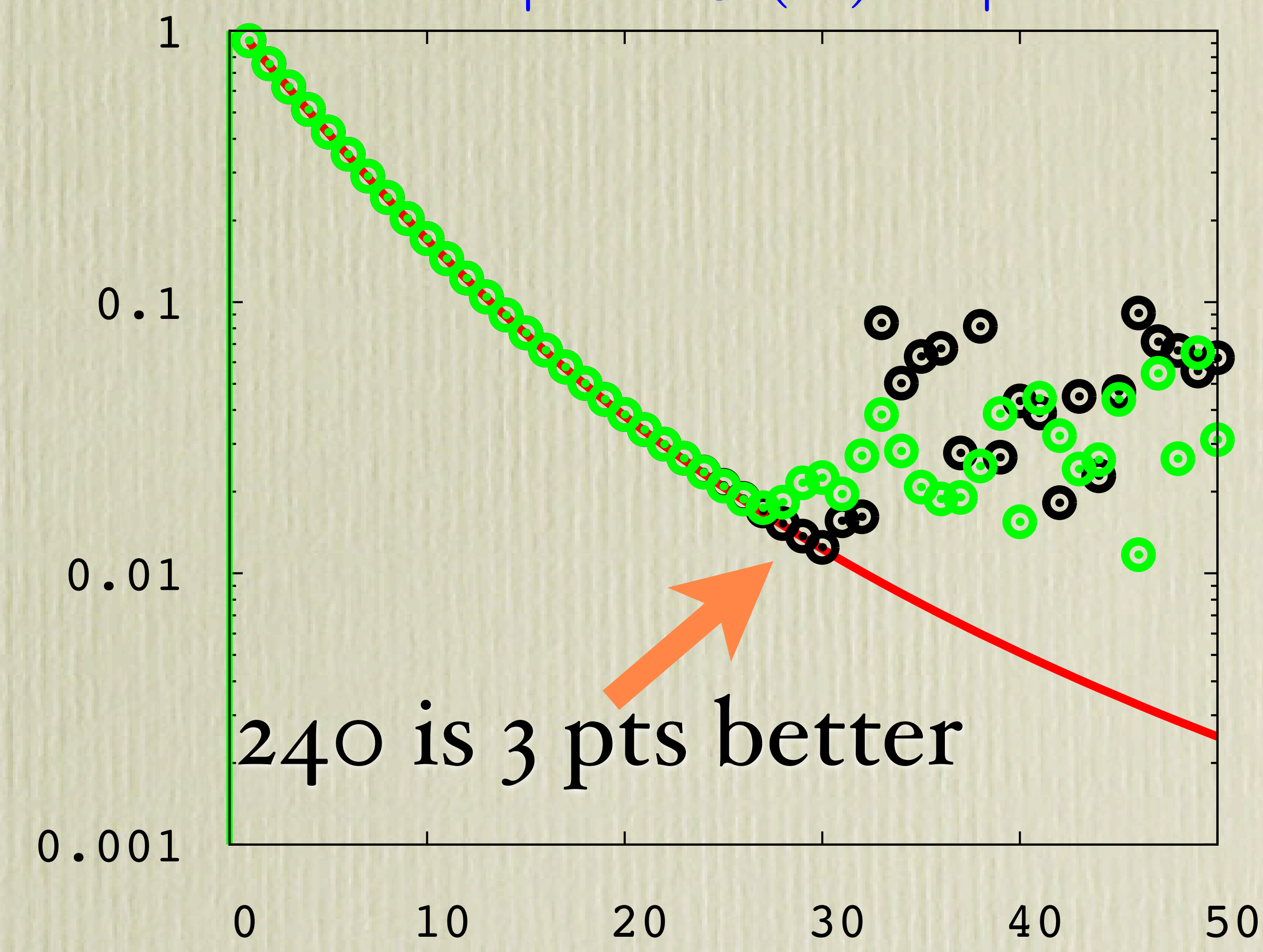

baryon number

Numerical results $|Z_C(n)|$

Canonical partition function $Z_C(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

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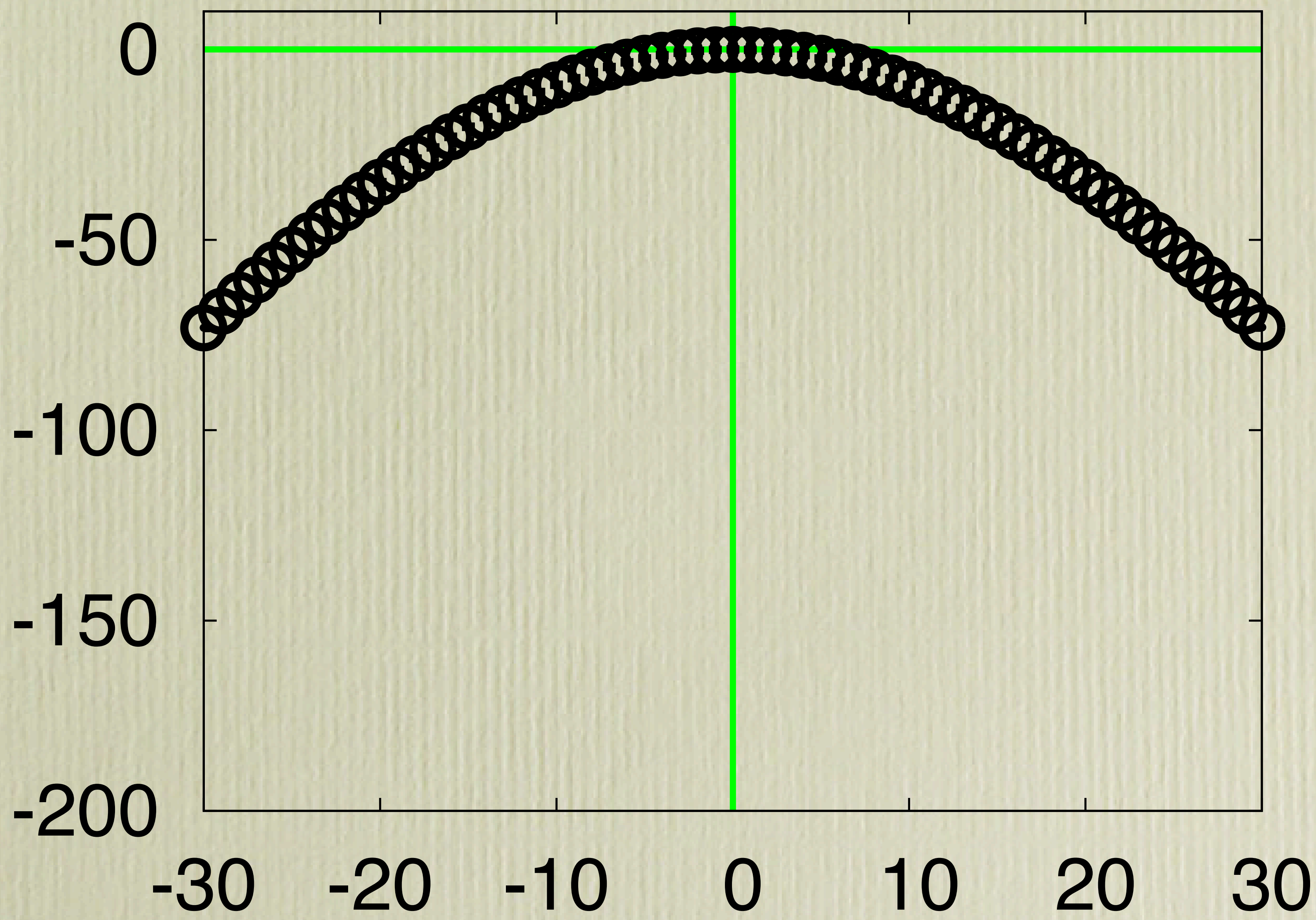
$\beta = 2.1$

$\mu = 0$

$\log|Z_C(\beta,n)|$



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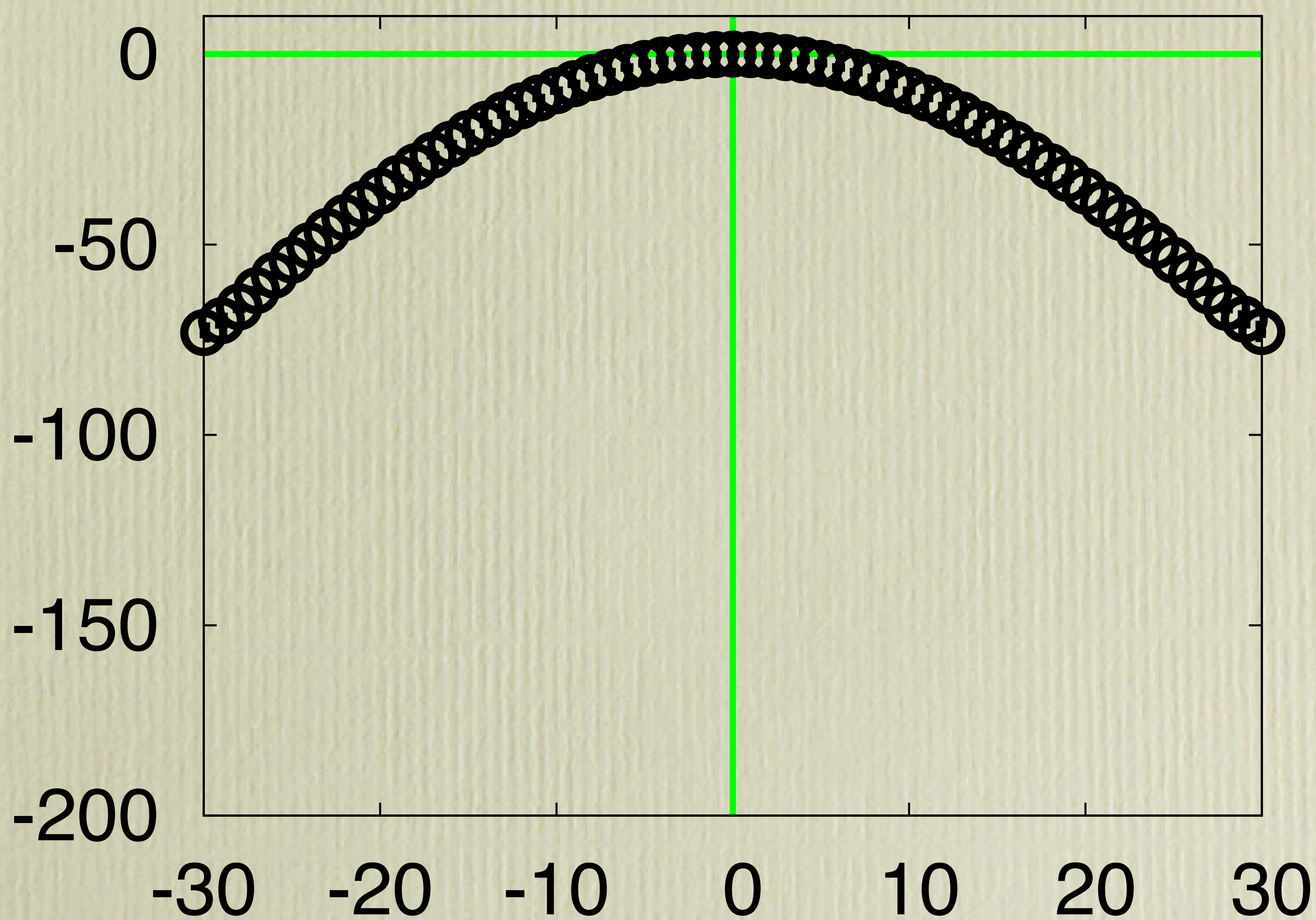


baryon number

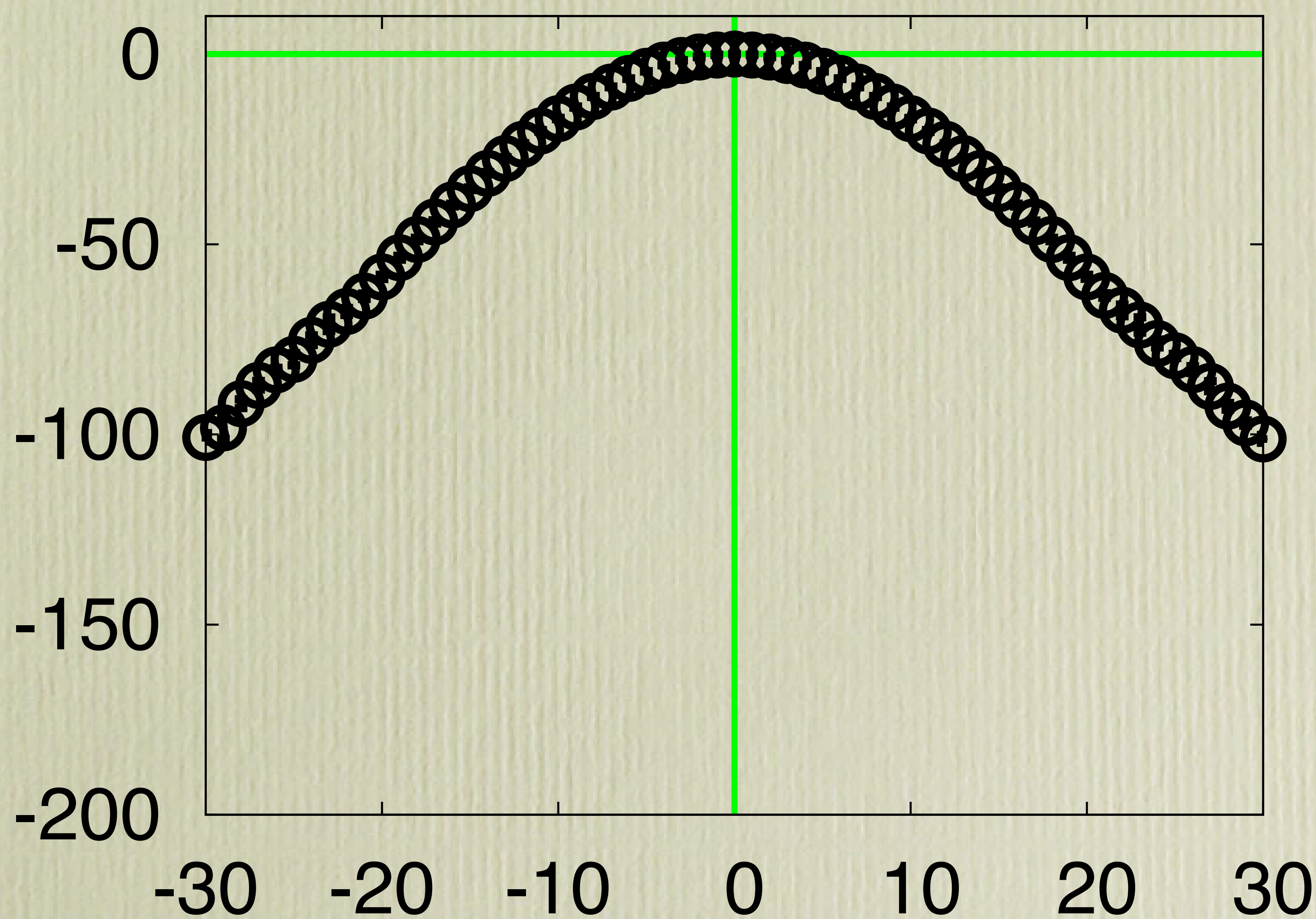
Numerical results |Zc(n)|

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.7$

$\mu = 0$

$\log|Z_C(\beta,n)|$



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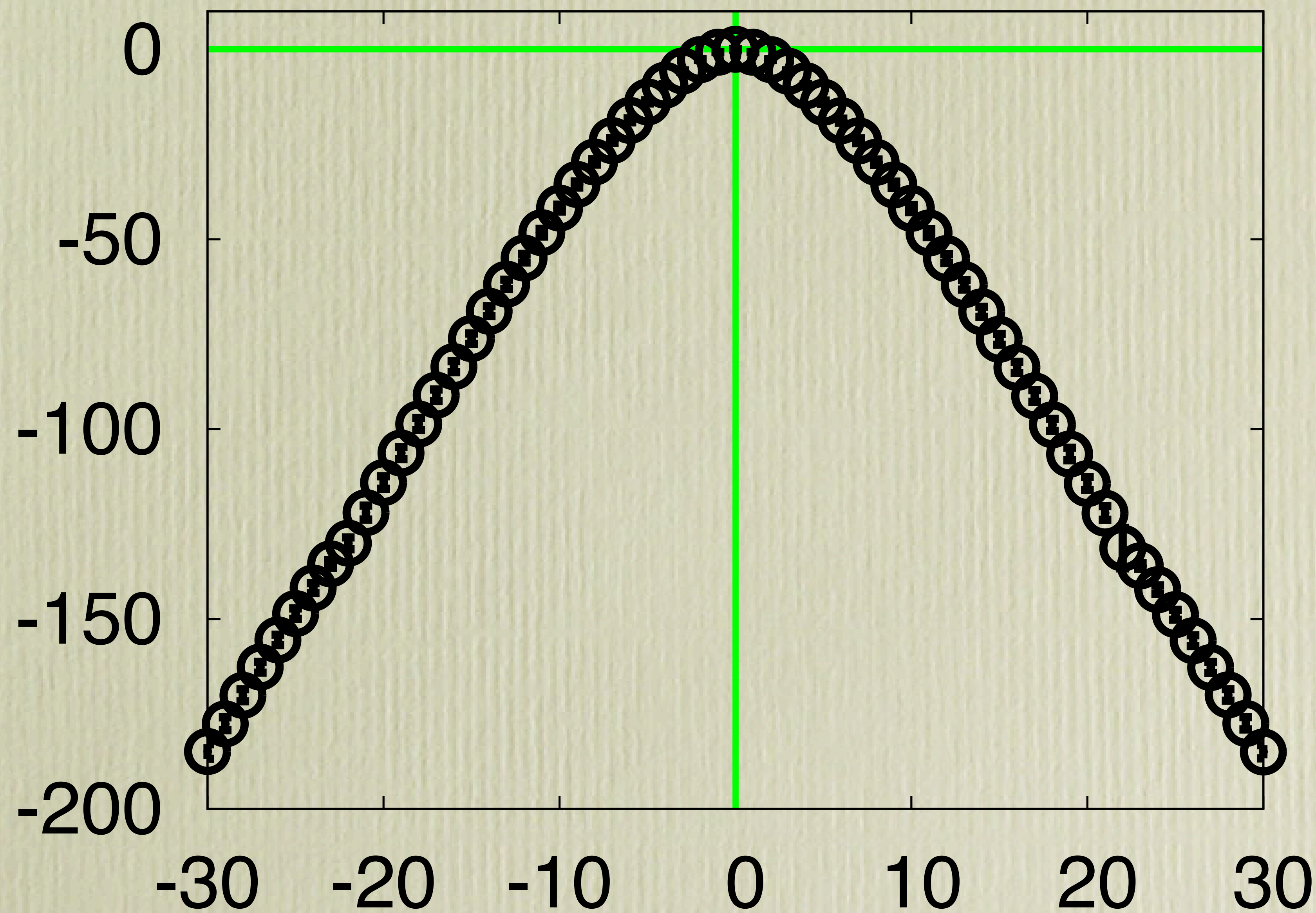
Numerical results |Zc(n)|

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.5$

$\mu = 0$

$\log|Z_C(\beta,n)|$



baryon number

Numerical results |Zc(n)|

Canonical partition function

$$Z_C\,(T,n,V)=|Z_C(\beta,n)|e^{i\theta(\beta,n)}$$

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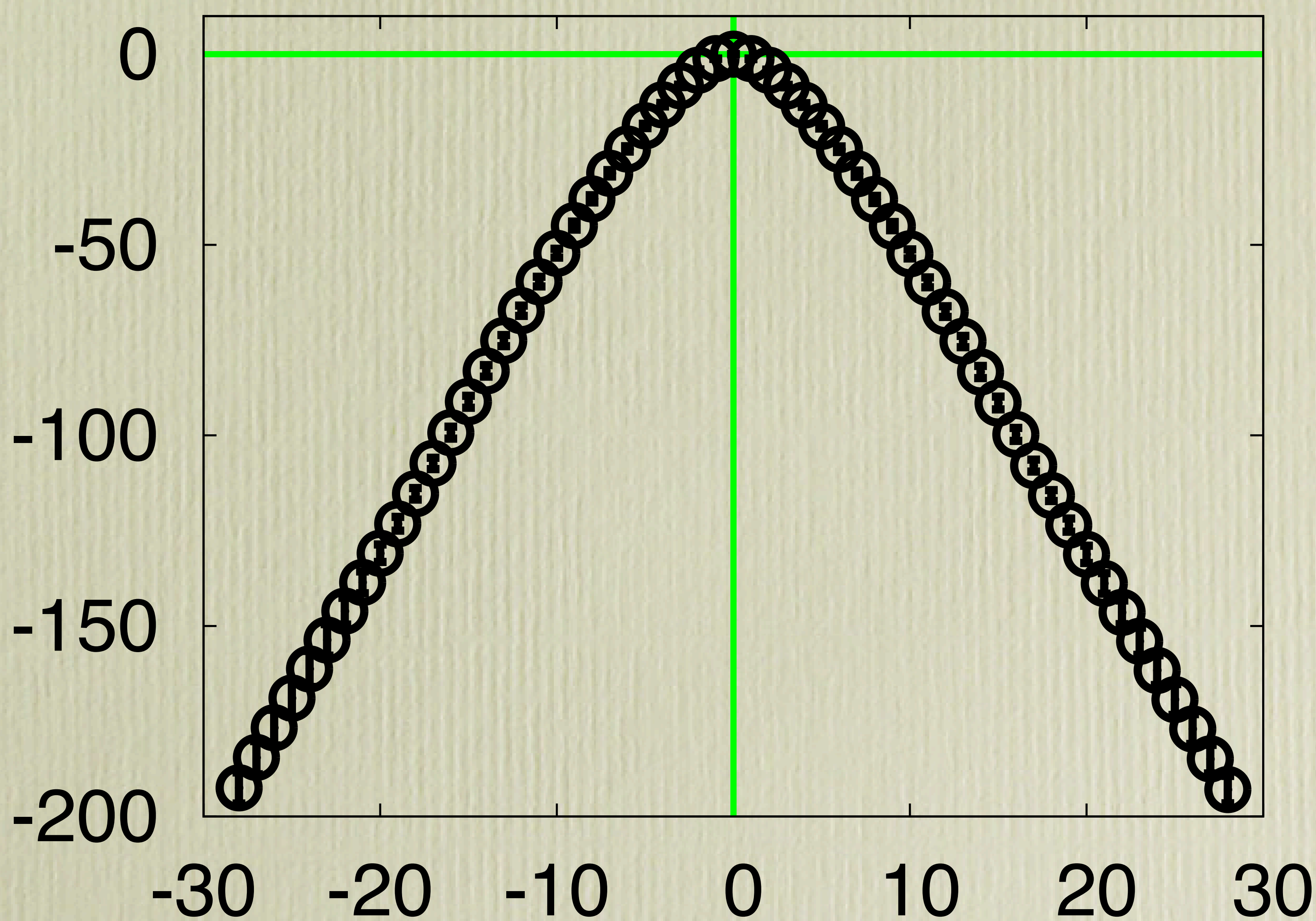

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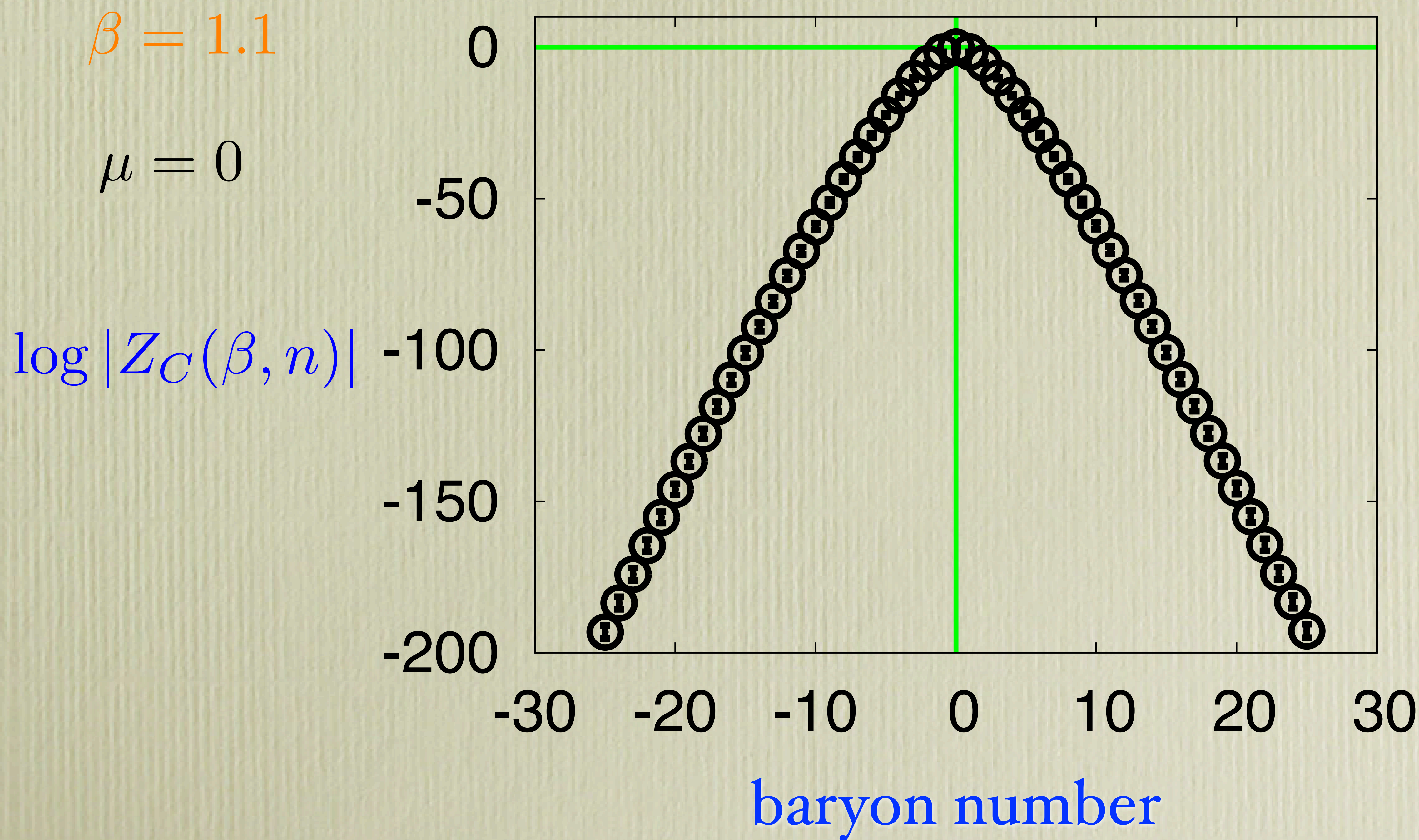

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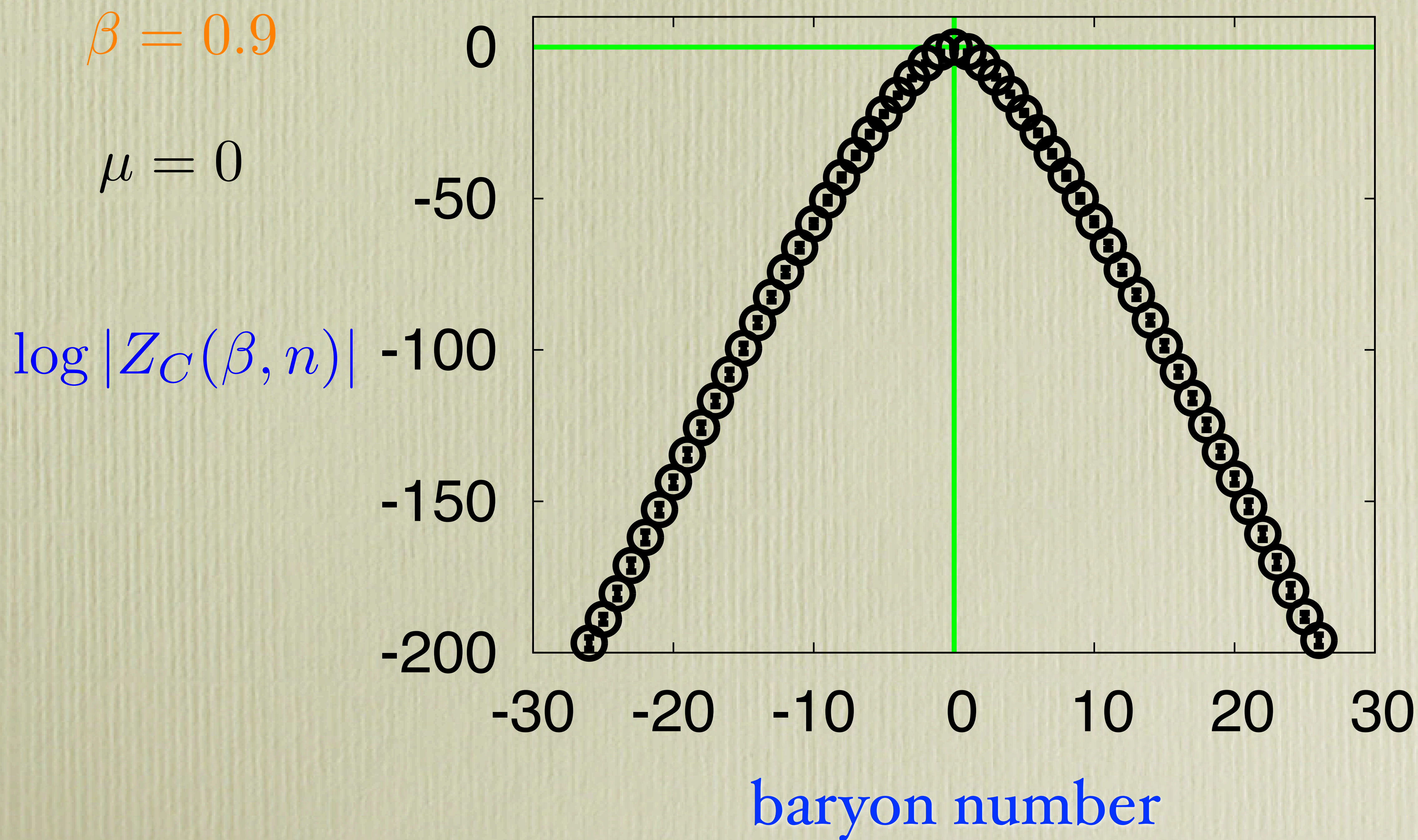

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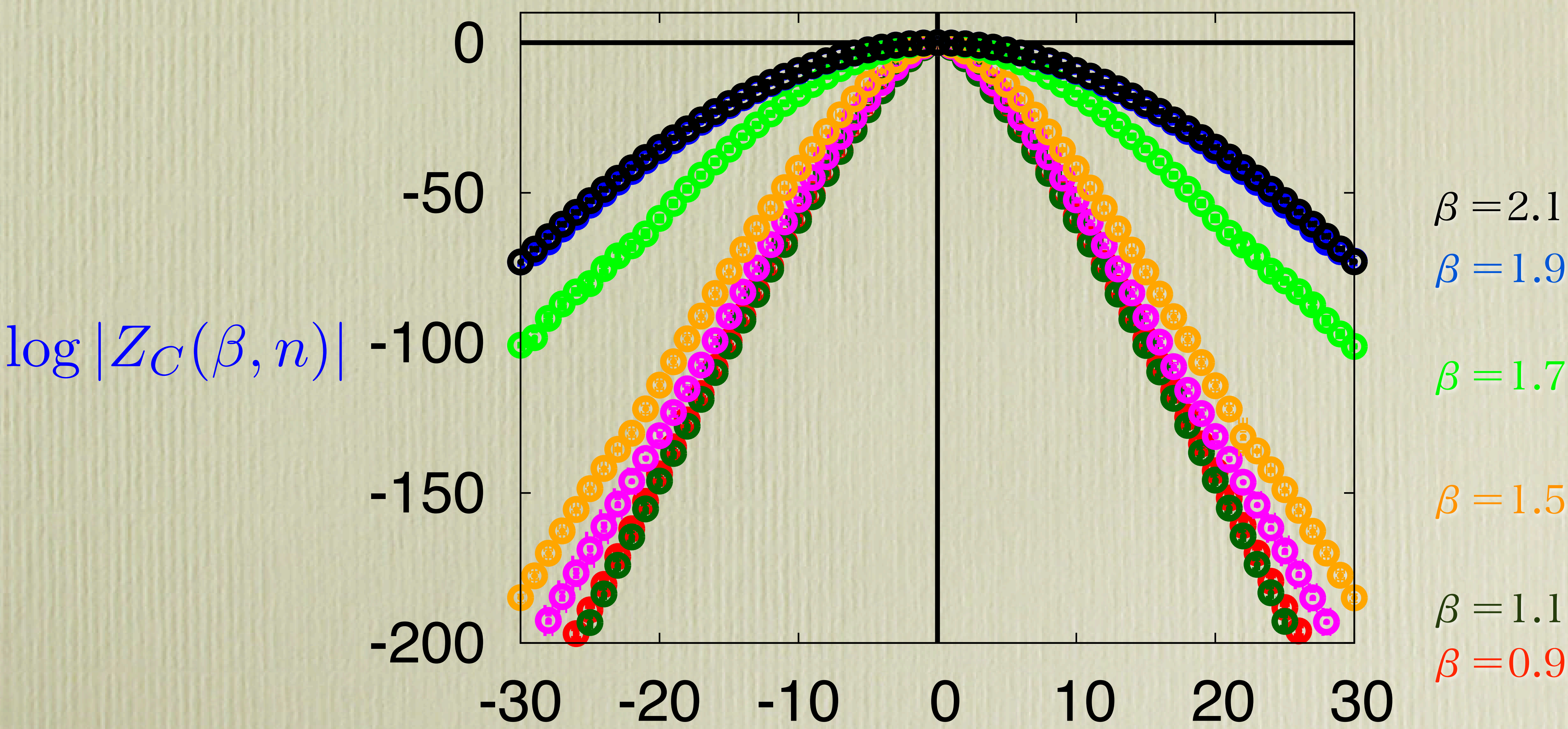

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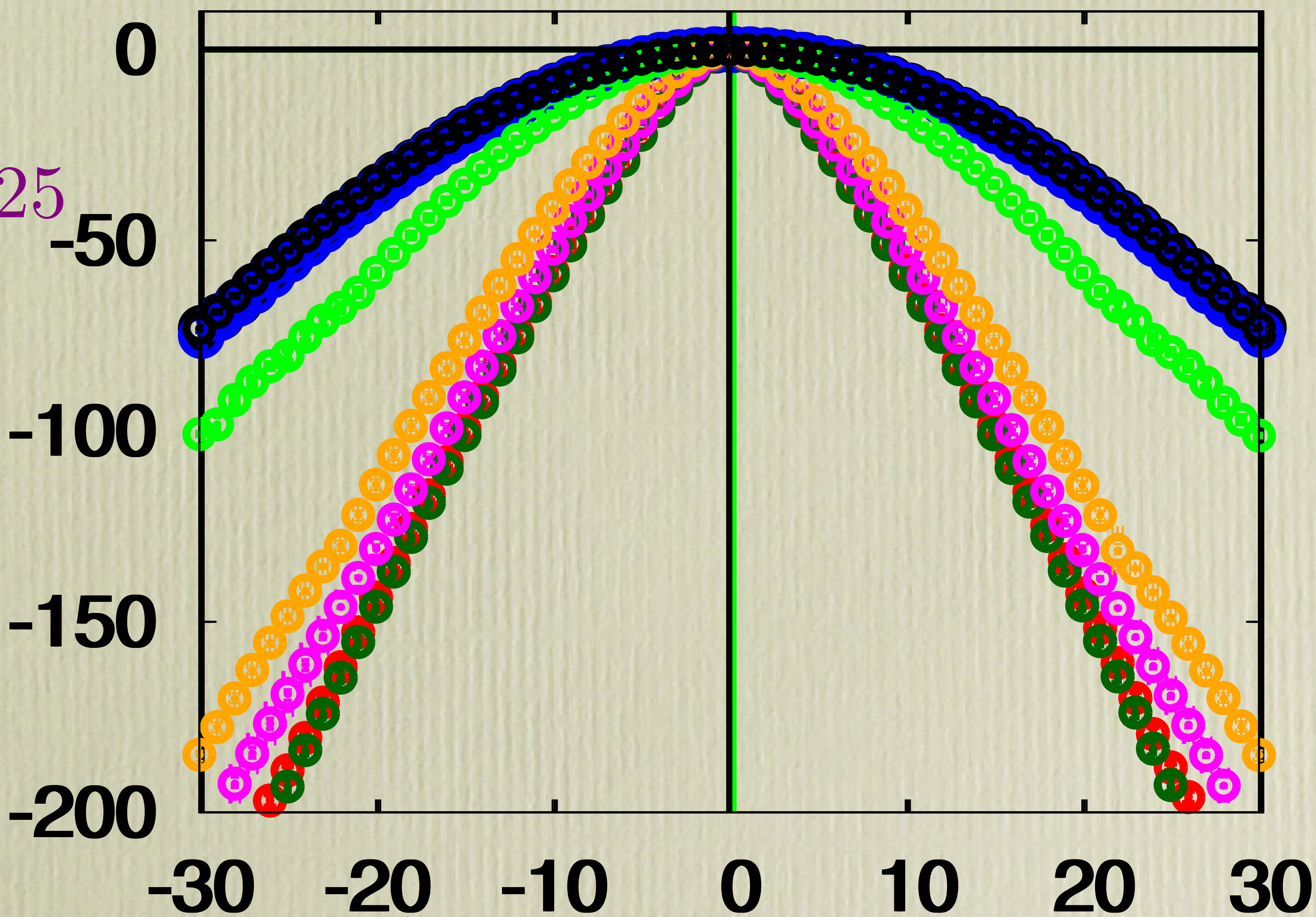
$\mu_I = 0.125,\ 0.25$

$\log|Z_C(\beta,n)|$



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baryon number

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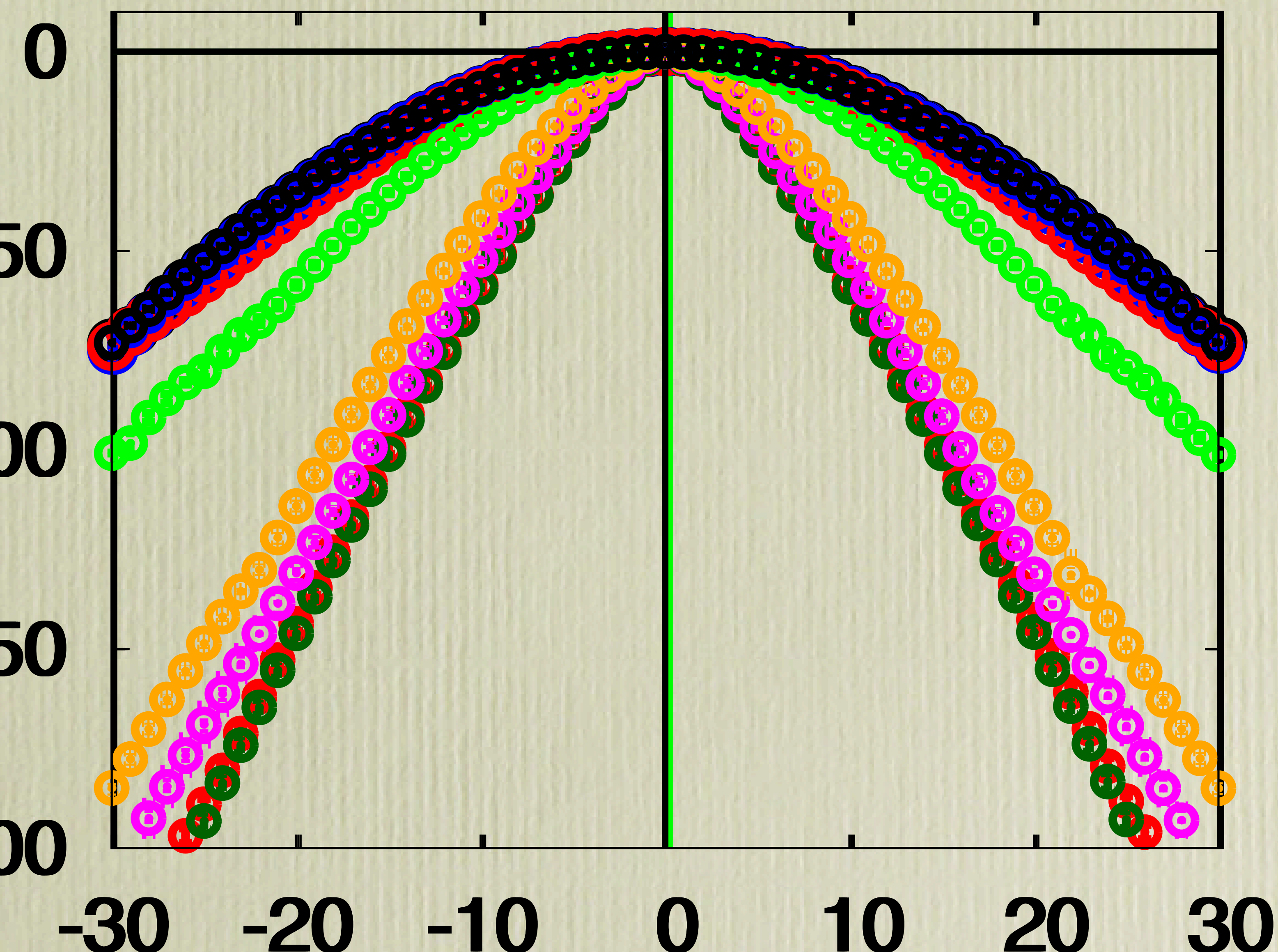

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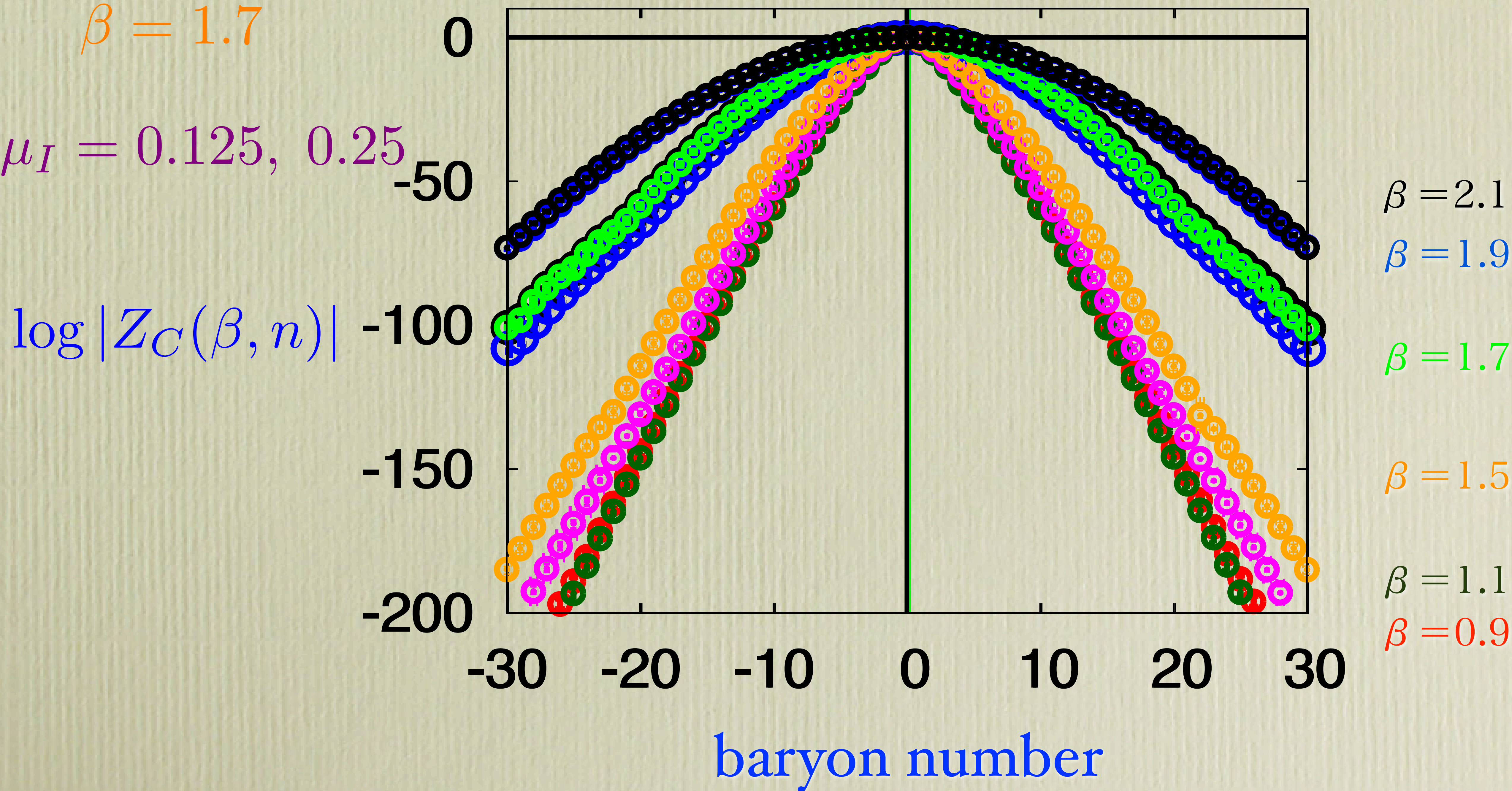

14年9月4日木曜日

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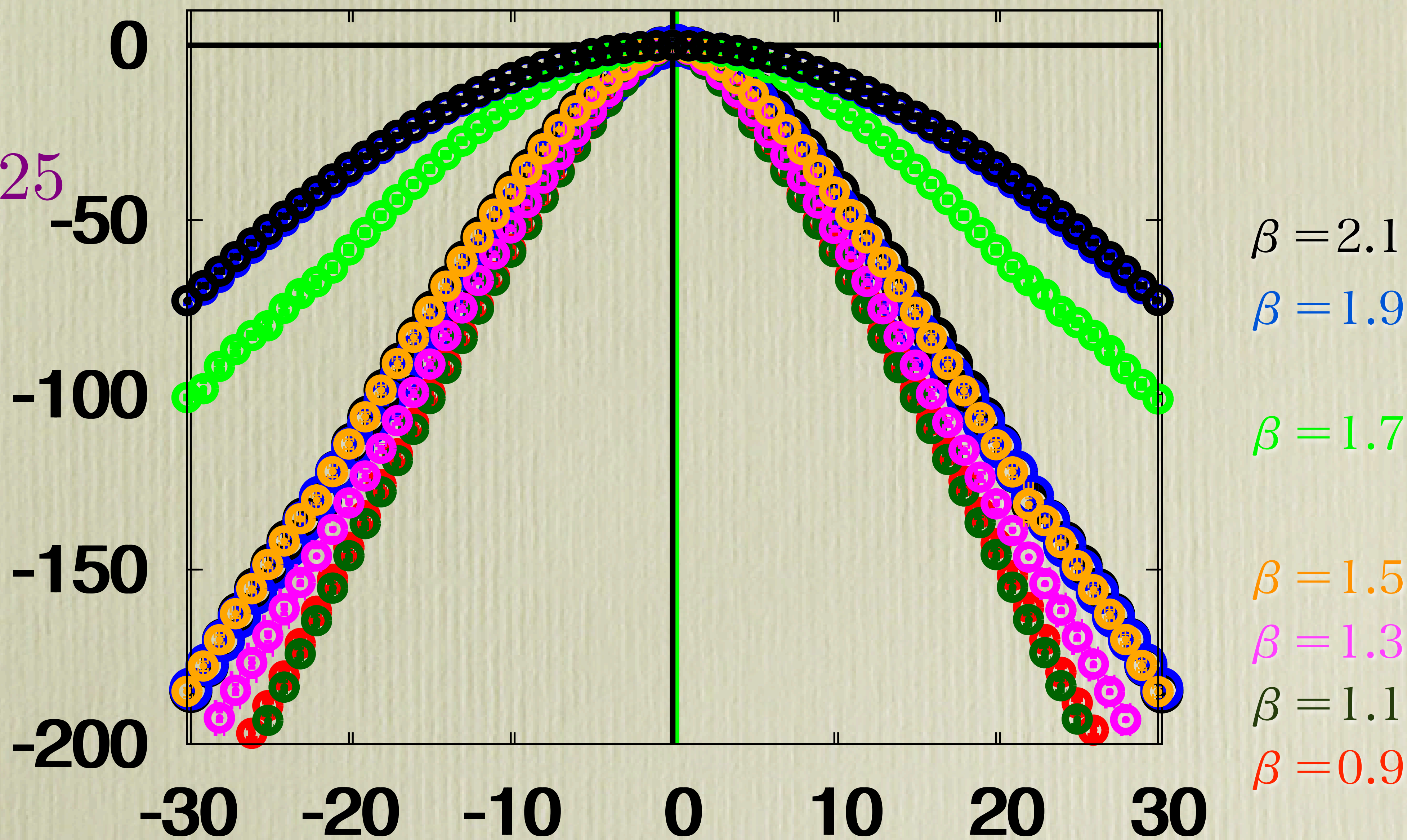

baryon number

Numerical results Phase(Zc(n))

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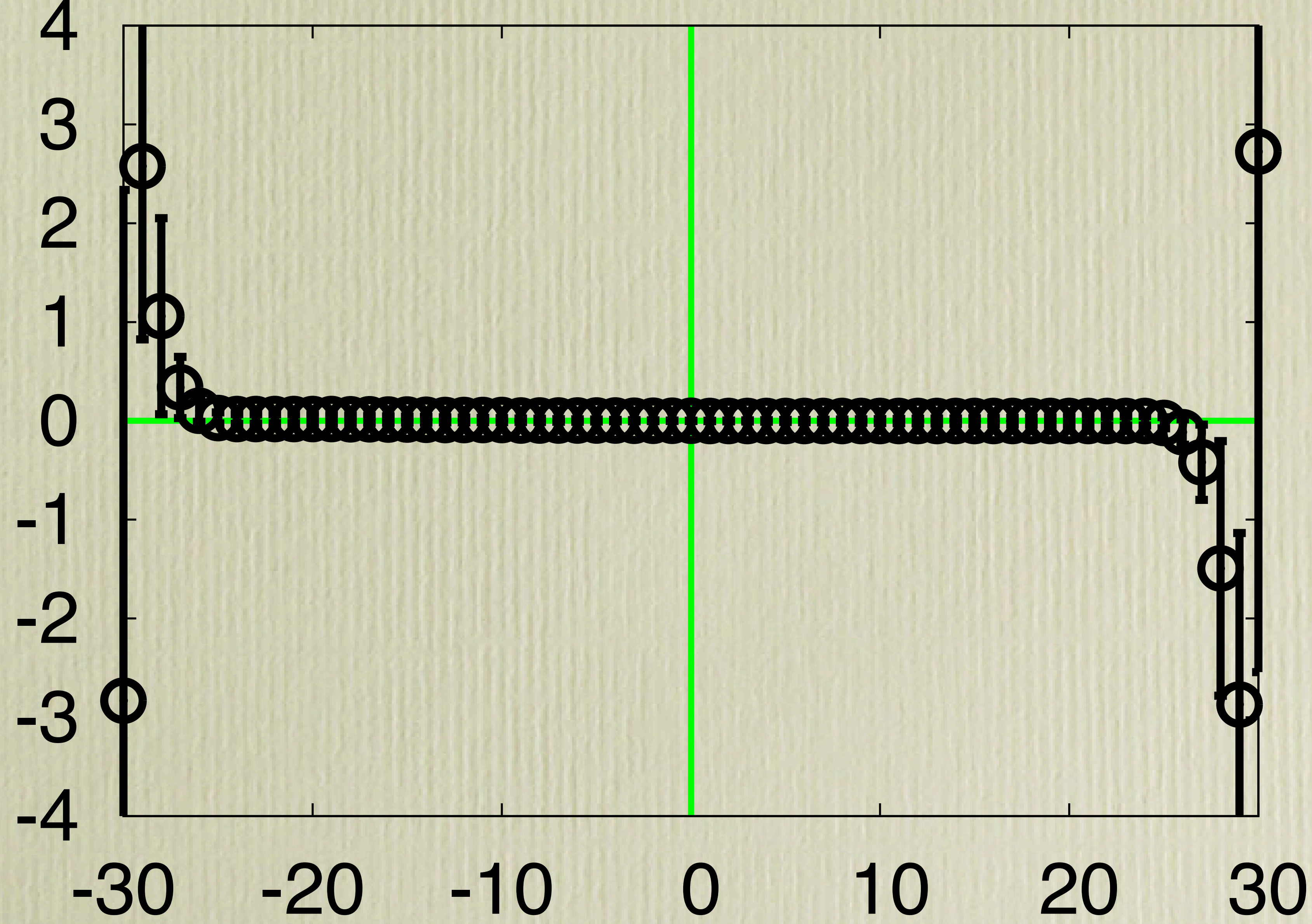

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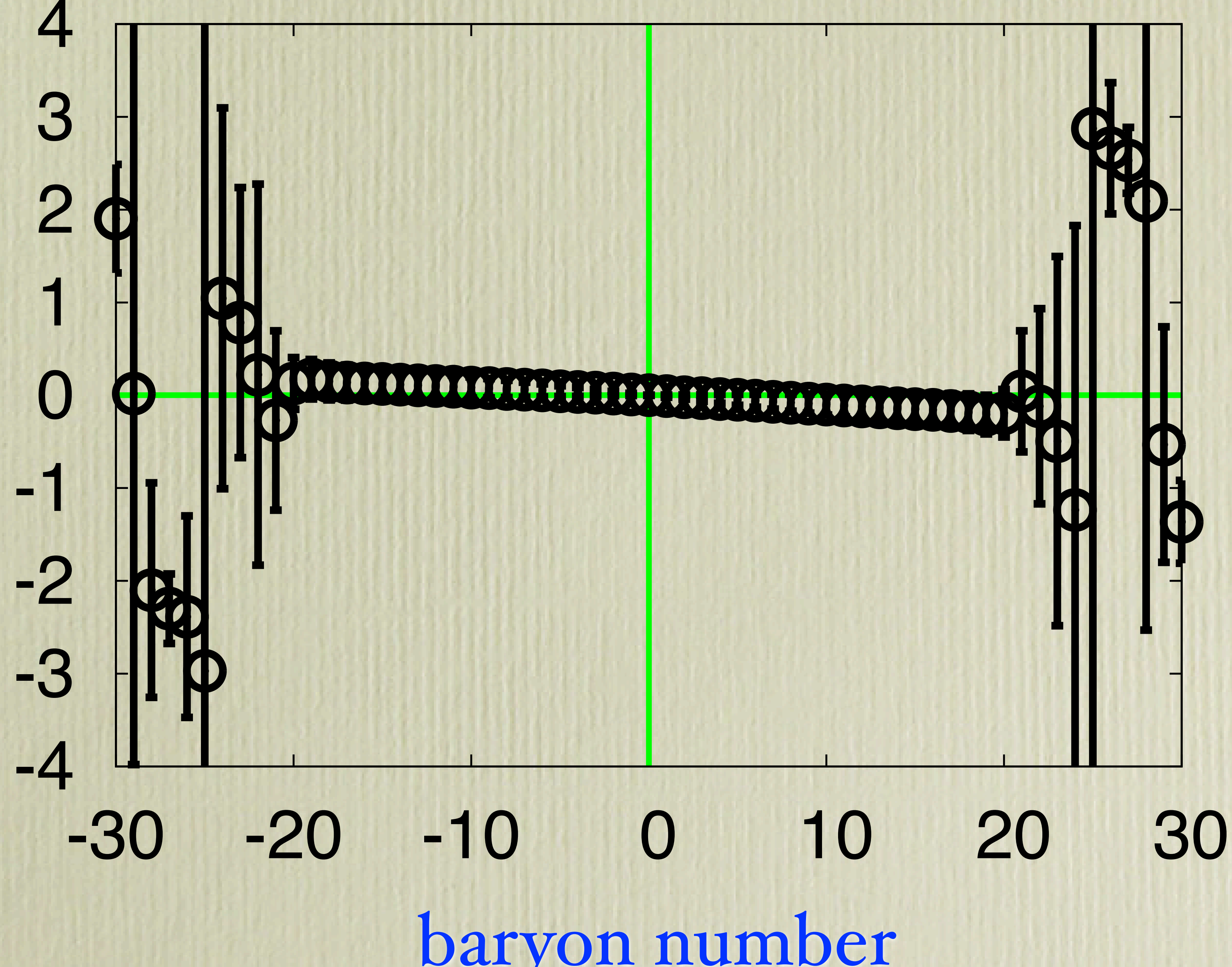

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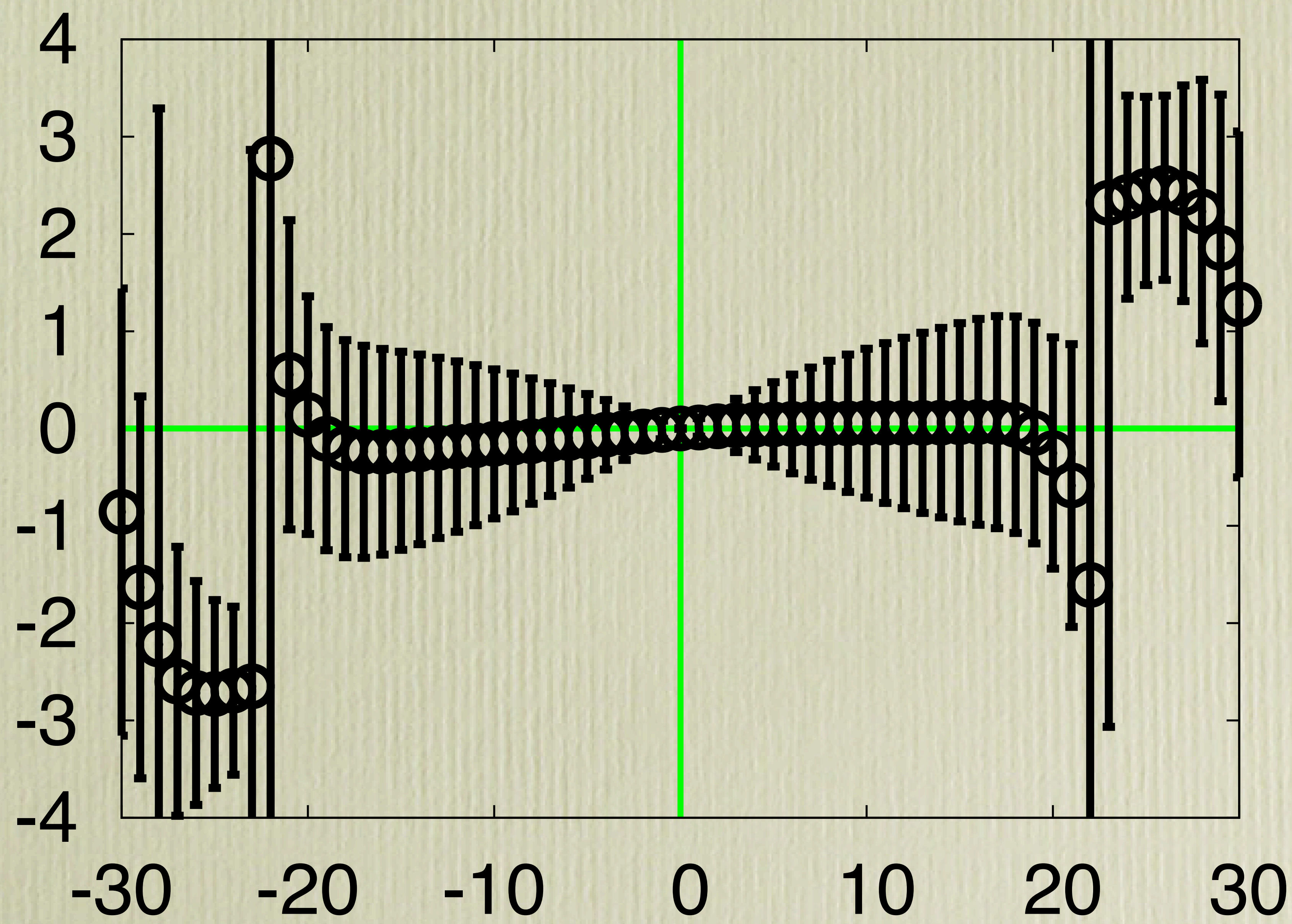

baryon number

Plan of the talk

✔ 1. Introduction

✔ 2. Winding number expansion

✔ 3. Numerical setup

✔ 4. Numerical results

5. Hadronic observables

6. Conclusion

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{\mathrm{Tr}\left[\hat{O} \exp\left(-\beta\left(\hat{H} - \mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H} - \mu\hat{N}\right)\right)\right]}$$

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path integral formalism

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path integral formalism re-weighting technique

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path integral formalism ↓ re-weighting technique

$$\left\langle O(D_W(\mu)) \frac{\mathrm{Det}\, D_W(\mu)}{\mathrm{Det}\, D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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To see explicit functional form in ξ

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To see explicit functional form in ξ   Use HPE

Hadronic observables

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To see explicit functional form in ξ

Use HPE

$$\bar{\psi}\psi = -\mathrm{tr}\left(\frac{1}{D_W}\right) = -\mathrm{tr}\left(\frac{1}{1-\kappa Q}\right) = \sum_{m=0}^{\infty} \kappa^m \mathrm{tr} Q^m$$

Hadronic observables

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resummation

Hadronic observables

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resummation →

$$\sum_{n=-\infty}^{\infty} S_n(U)\xi^n$$

Hadronic observables

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resummation

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$$\mathrm{Det}D_W$$

Hadronic observables

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resummation → $\sum_{n=-\infty}^{\infty} S_n(U)\xi^n$

$\mathrm{Det}\,D_W$ *resummation* →

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i}\xi^{-n-1}\left\langle O(D_W(\xi))\frac{\mathrm{Det}D_W(\xi)}{\mathrm{Det}D_W(\mu_0)}\right\rangle_0 Z_G(\mu_0)$$

To see explicit functional form in ξ

Use HPE

$$\bar\psi\psi = -\mathrm{tr}\left(\frac{1}{D_W}\right) = -\mathrm{tr}\left(\frac{1}{1-\kappa Q}\right) = \sum_{m=0}^{\infty}\kappa^m \mathrm{tr}Q^m$$

resummation → $\sum_{n=-\infty}^{\infty} S_n(U)\xi^n$

$\mathrm{Det}D_W$ *resummation* → $\exp\left(\sum_{n=-\infty}^{\infty} W_n(U)\xi^n\right)$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

Hadronic observables

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EV of canonical ensemble

Hadronic observables

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EV of canonical ensemble

$$\langle \hat{O} \rangle_C (\beta, n, V) = \frac{O_n}{Z_n}$$

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Fly into REAL μ !

Hadronic observables

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$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

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Fly into REAL μ !

$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

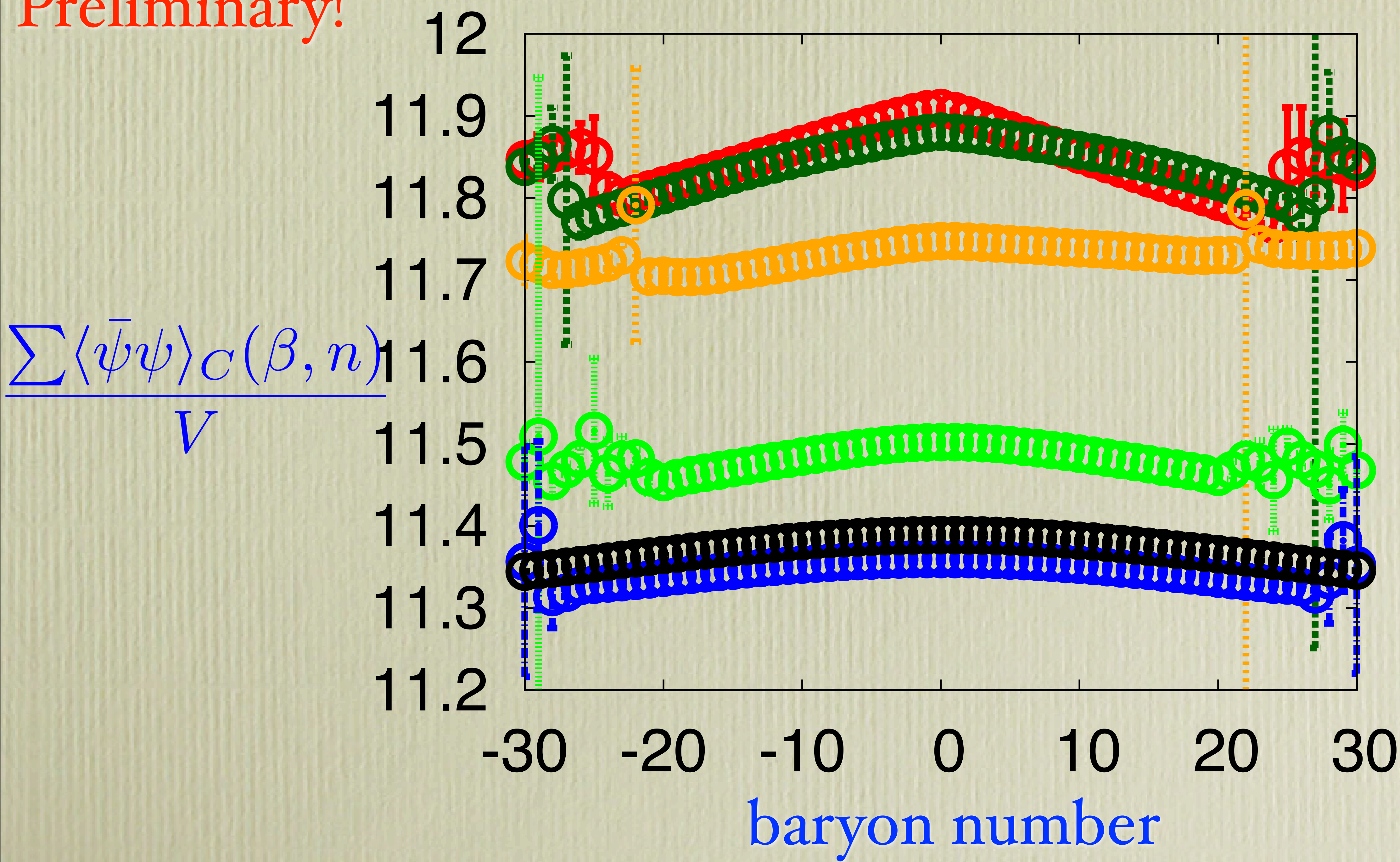
$$\longrightarrow \langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{O(\mu)}{Z(\mu)}$$

Hadronic observables

Chiral condensate in canonical ensemble

No renormalization! No subtraction! Sorry...

Preliminary!



Hadronic observables

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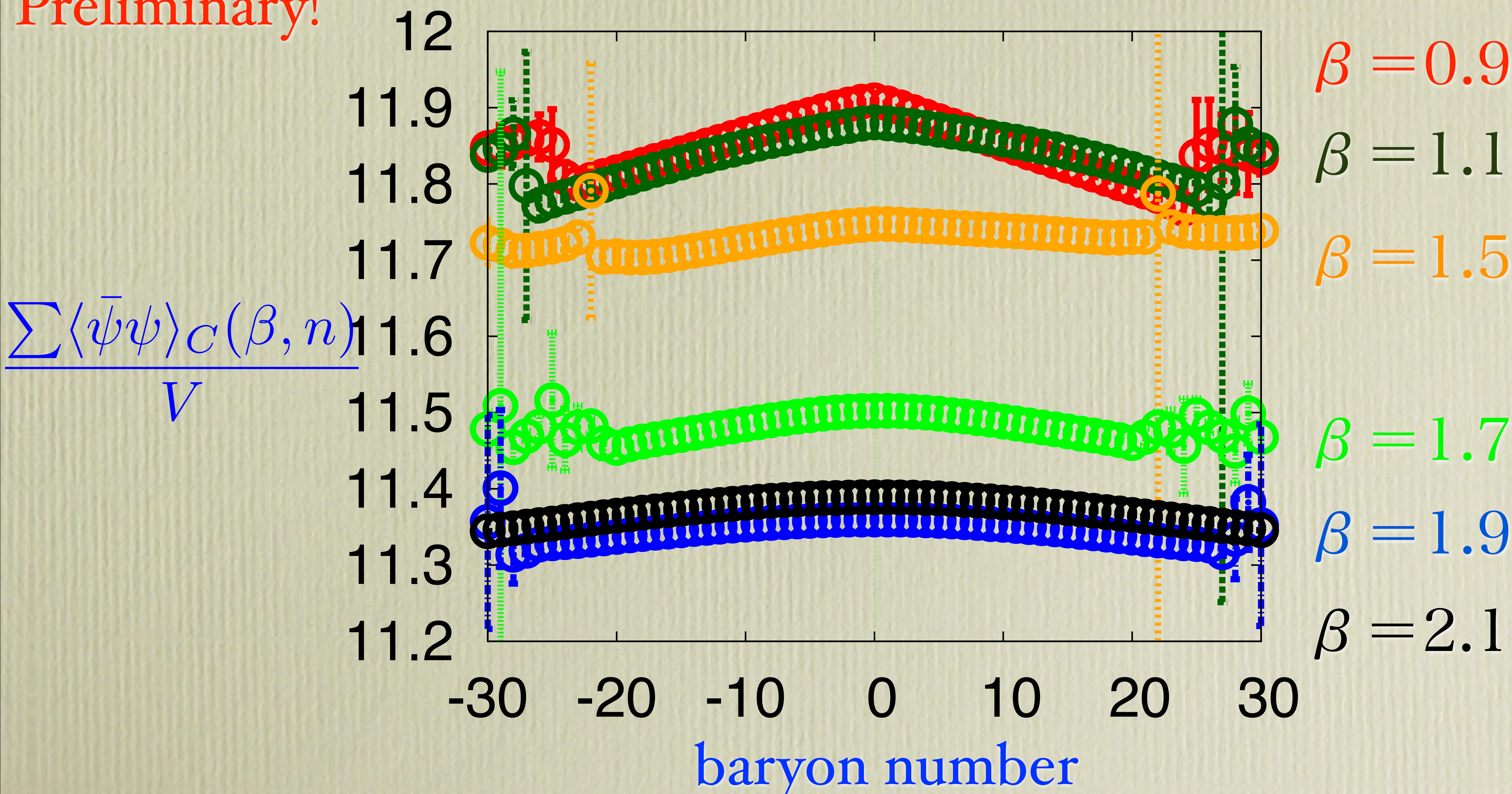
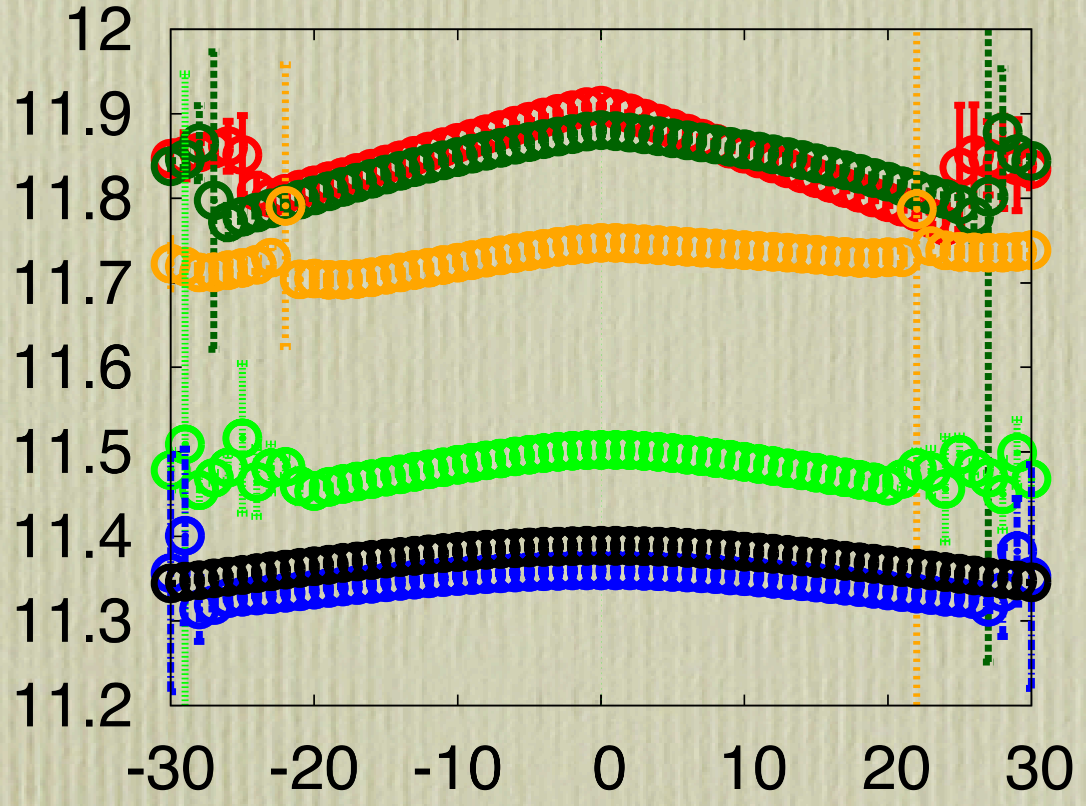


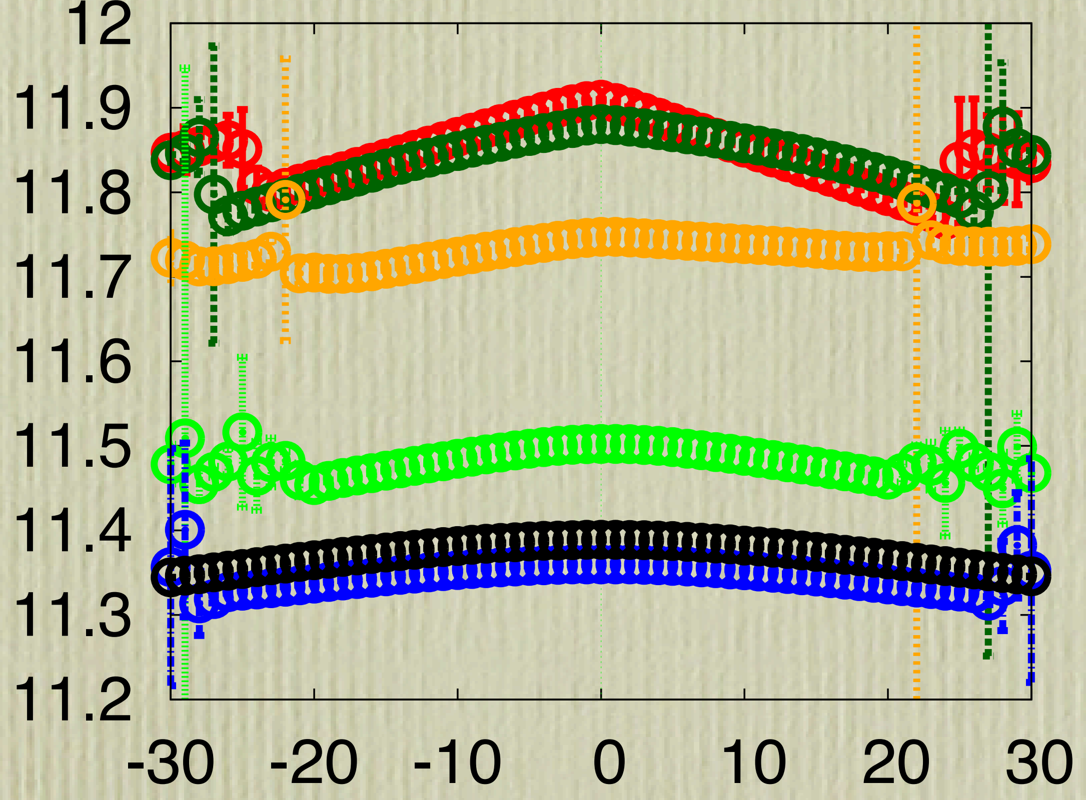
Hadronic observables

Chiral condensate in canonical ensemble



Hadronic observables

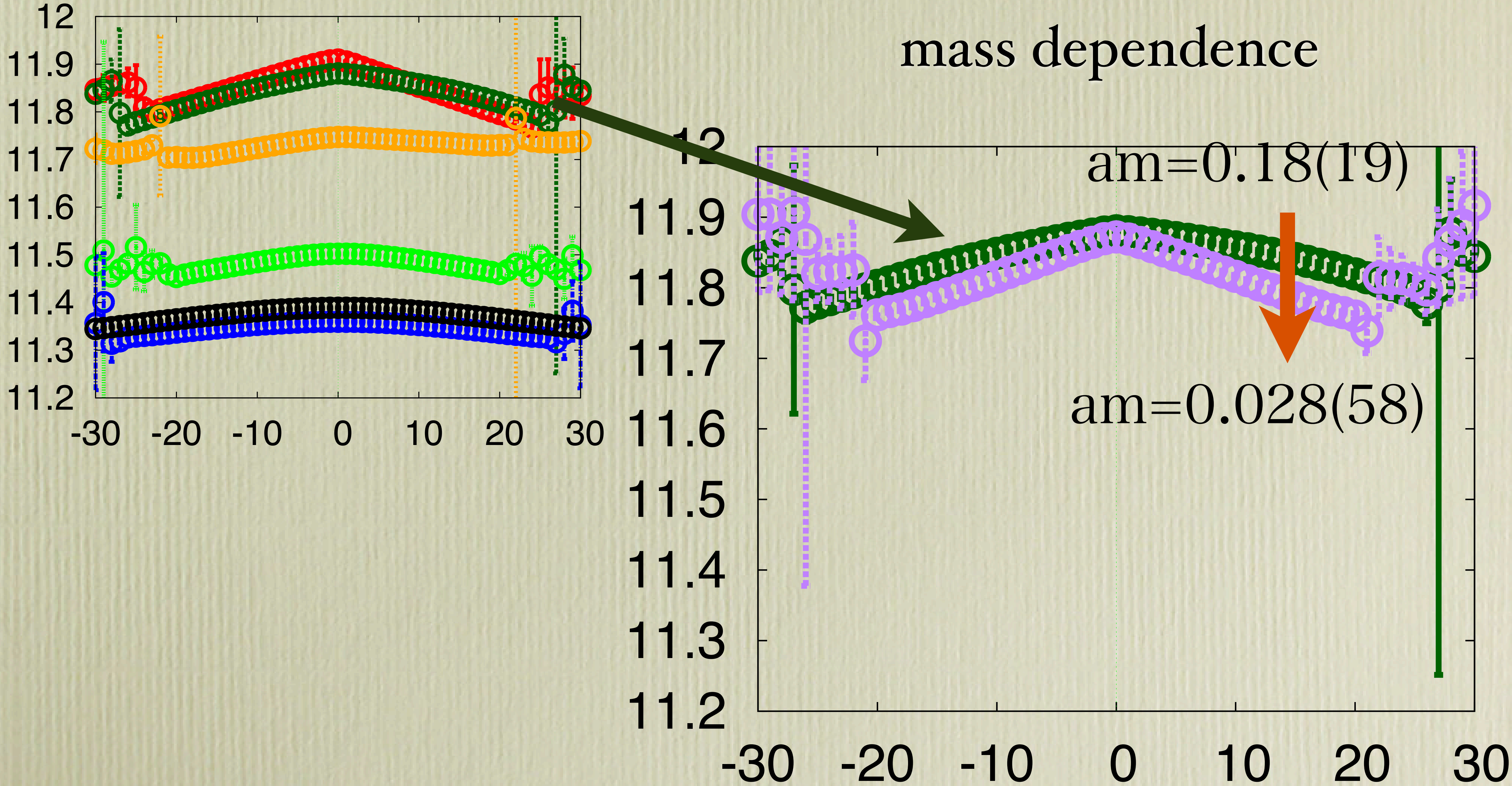
Chiral condensate in canonical ensemble



mass dependence

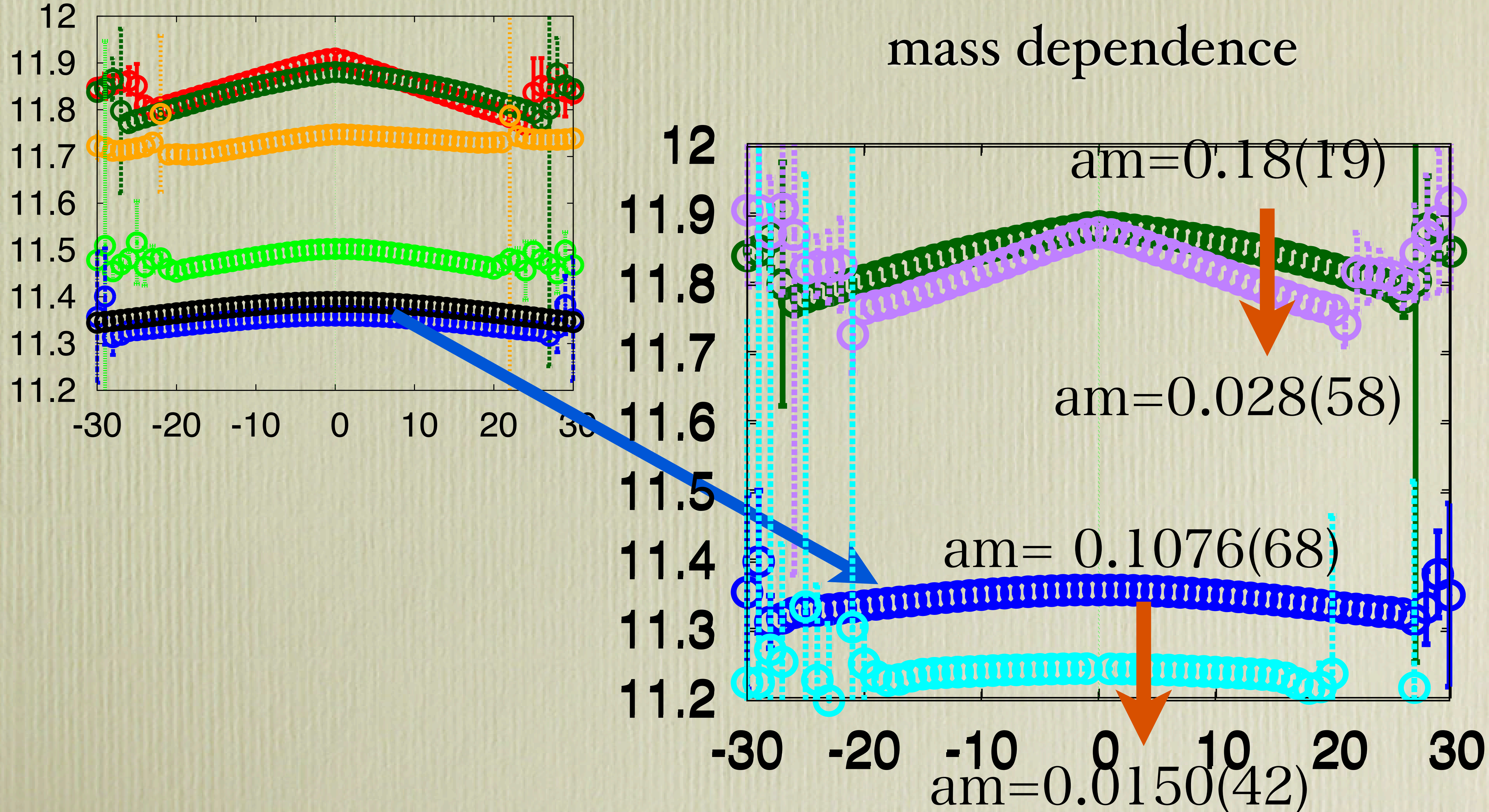
Hadronic observables

Chiral condensate in canonical ensemble



Hadronic observables

Chiral condensate in canonical ensemble



Hadronic observables

Chiral condensate in grand canonical ensemble

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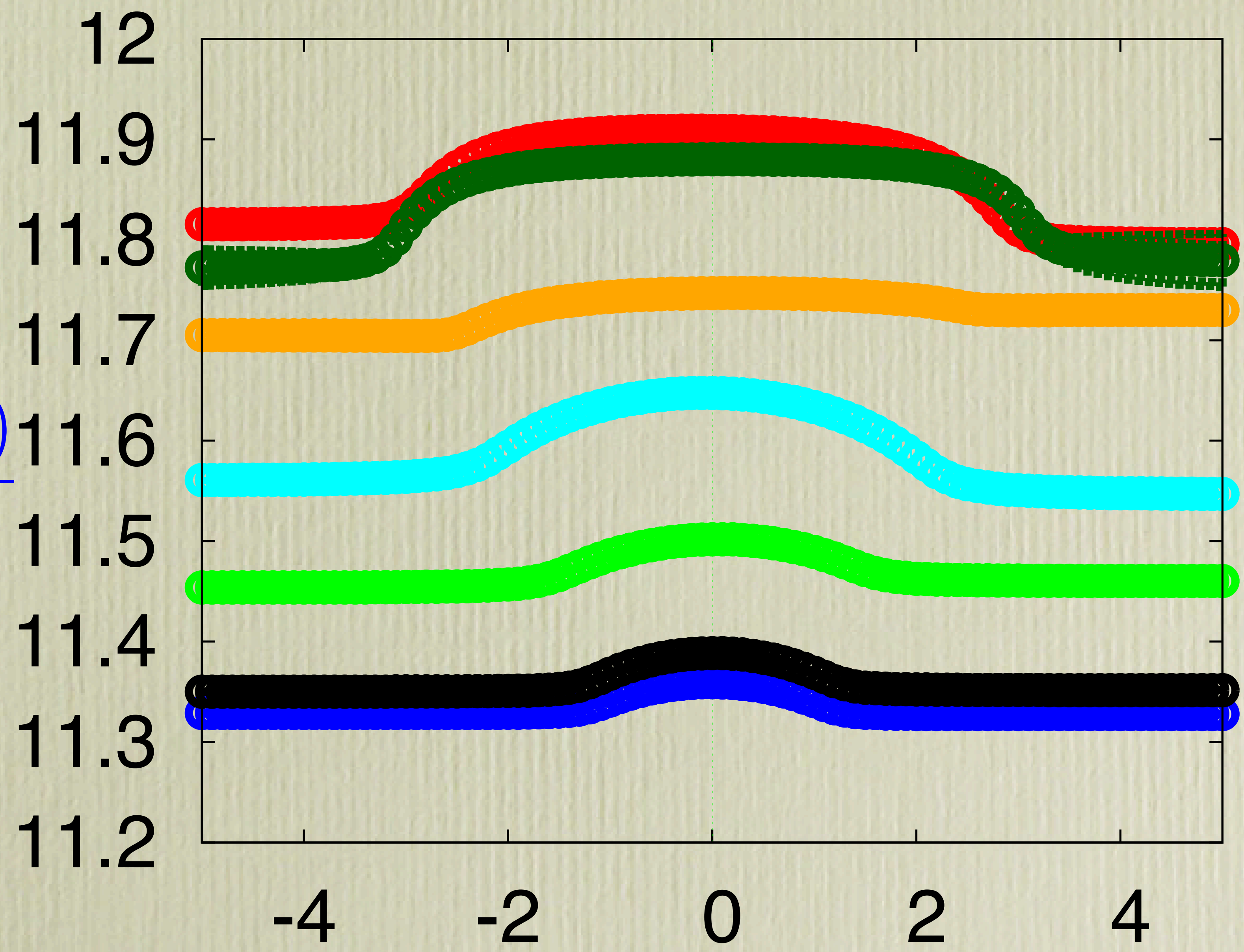
$$\frac{\sum \langle \bar{\psi}\psi \rangle_G(\beta,\mu)}{V}$$



(REAL quark chemical potential)/T

Hadronic observables

Chiral condensate in grand canonical ensemble

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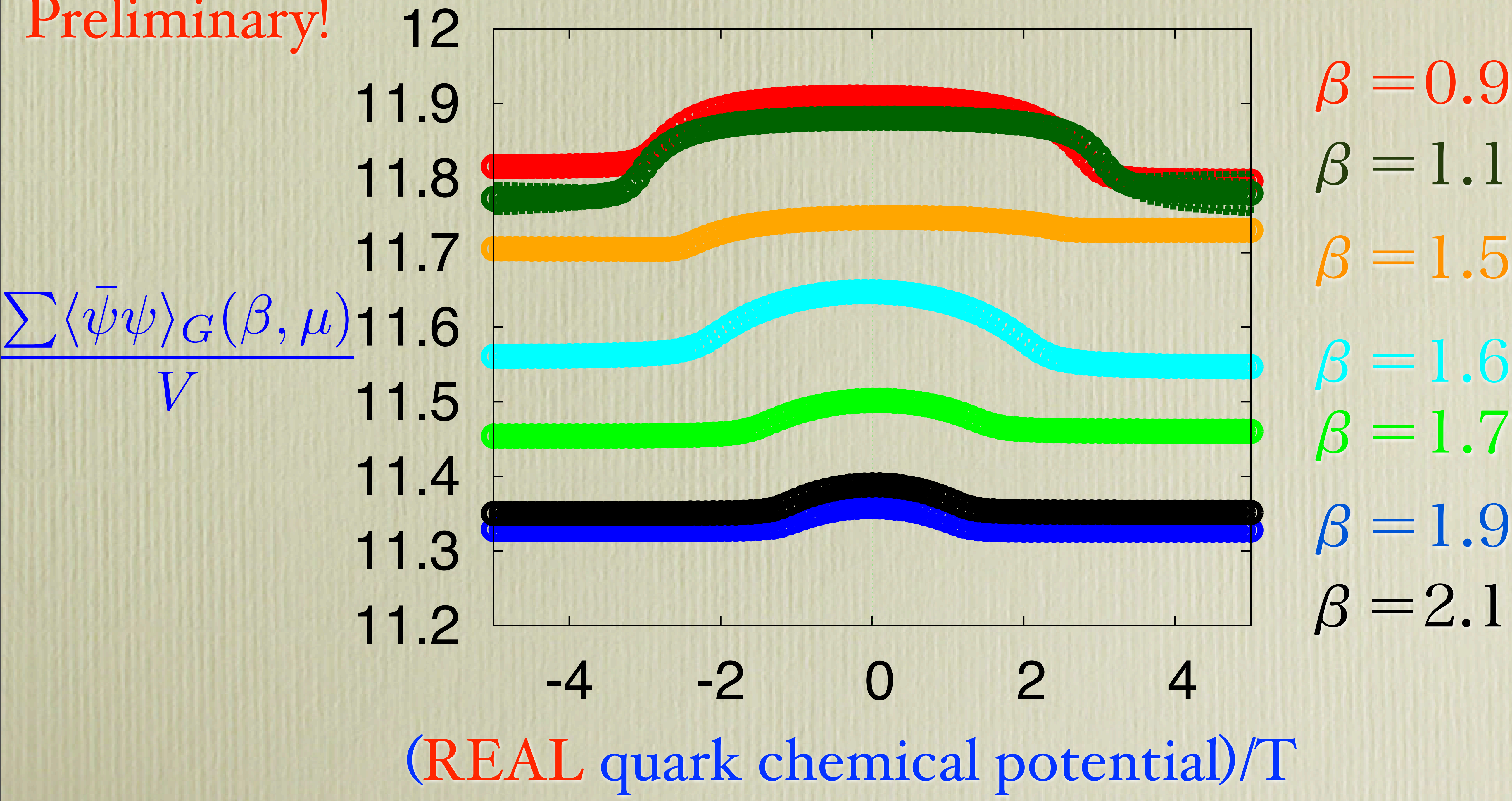
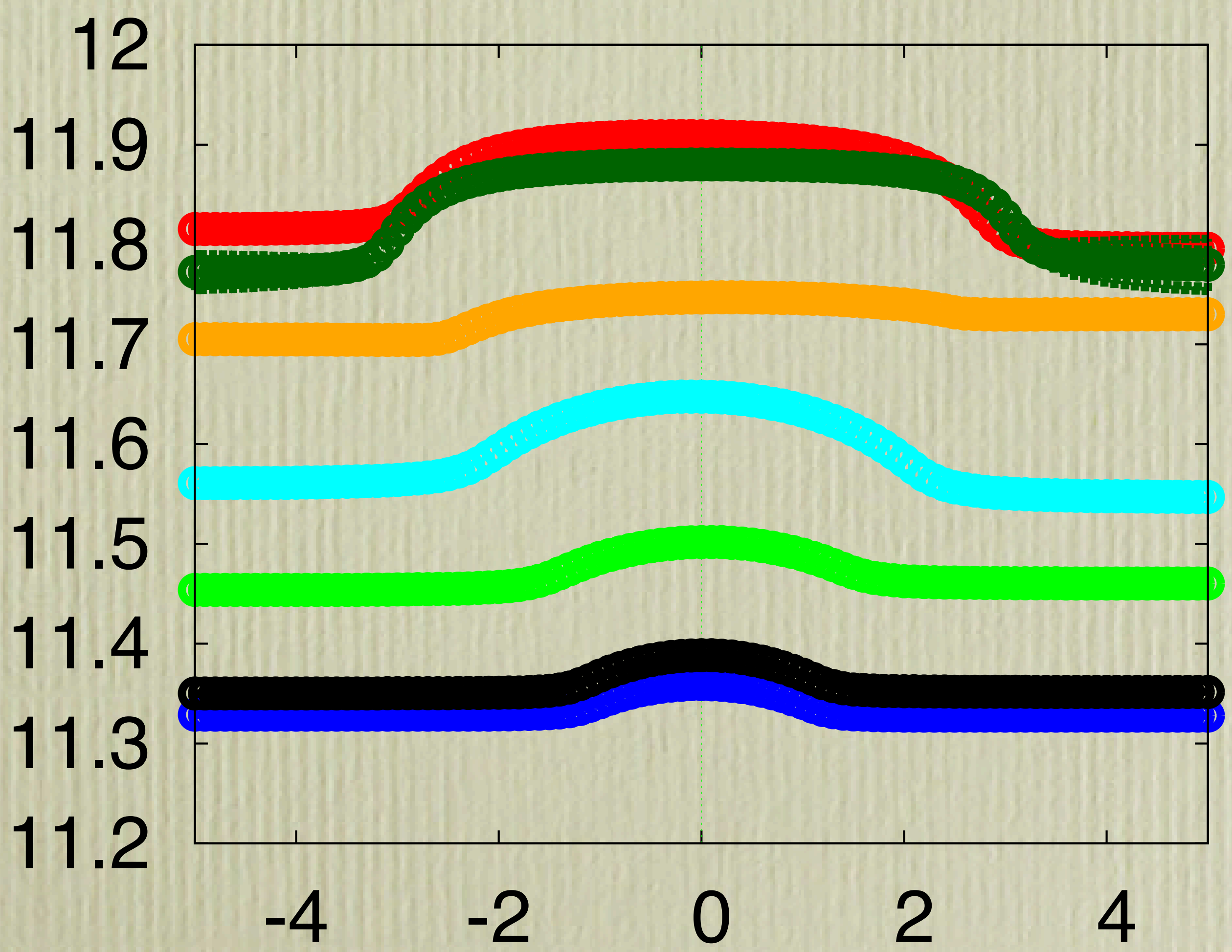
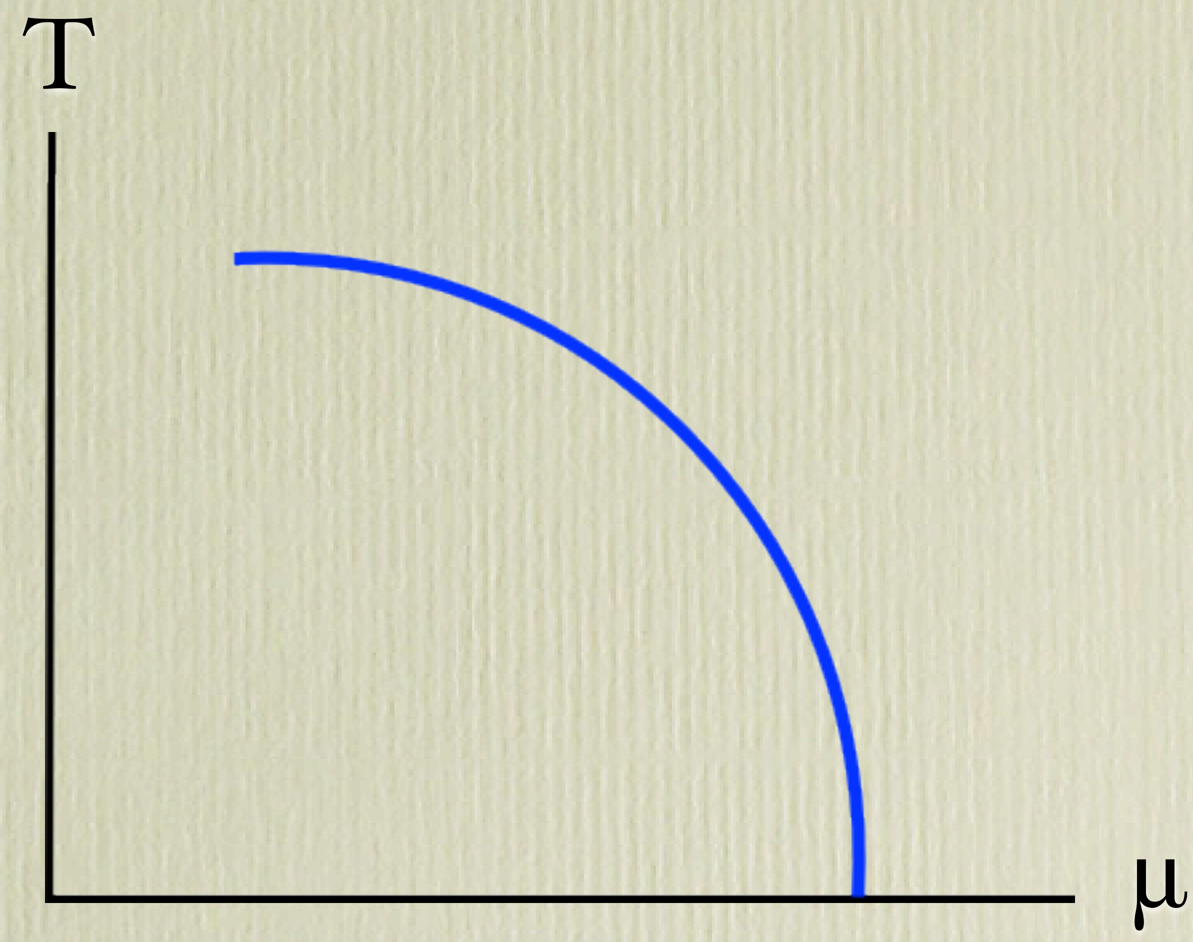


Hadronic observables

Chiral condensate in grand canonical ensemble

Preliminary!

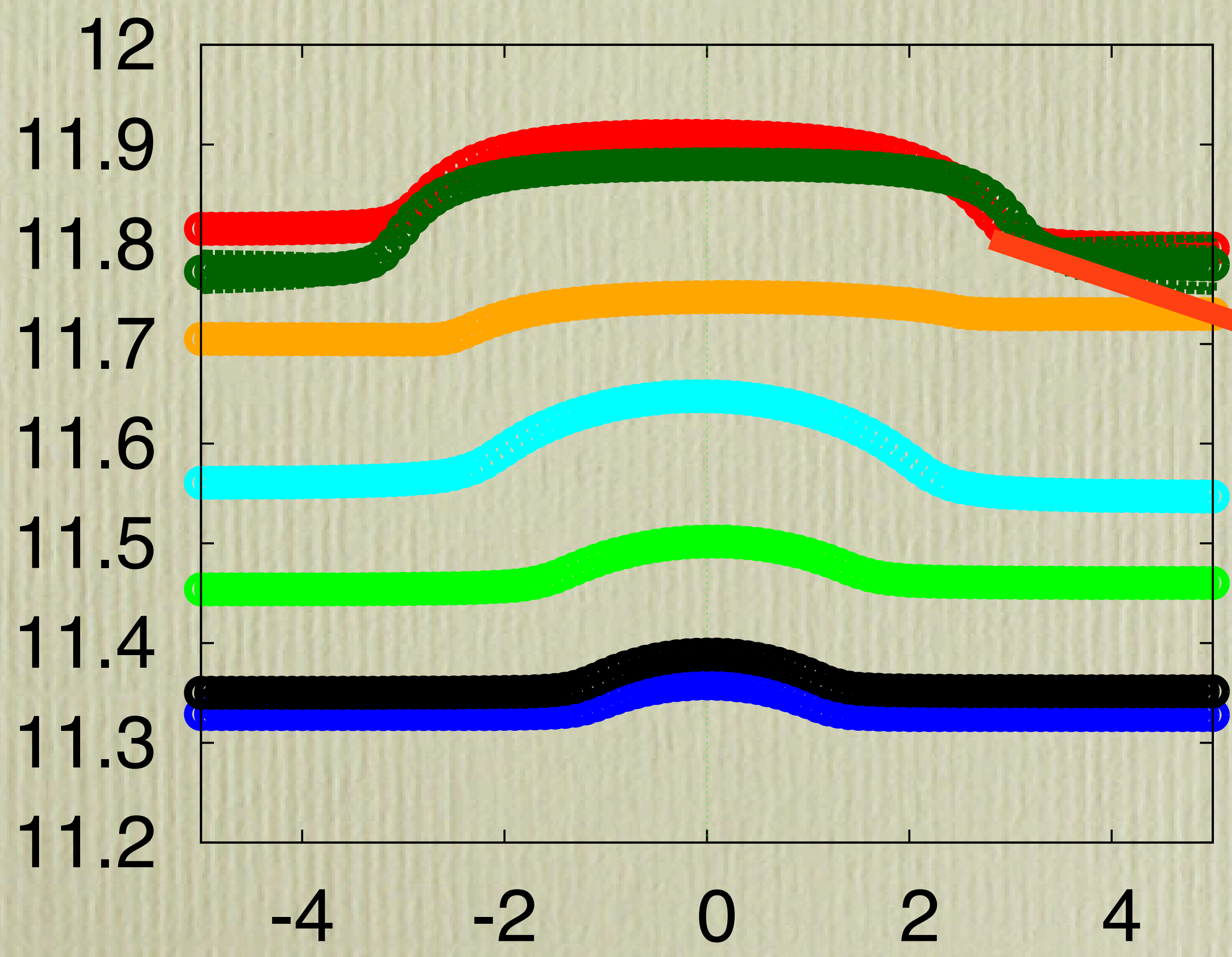


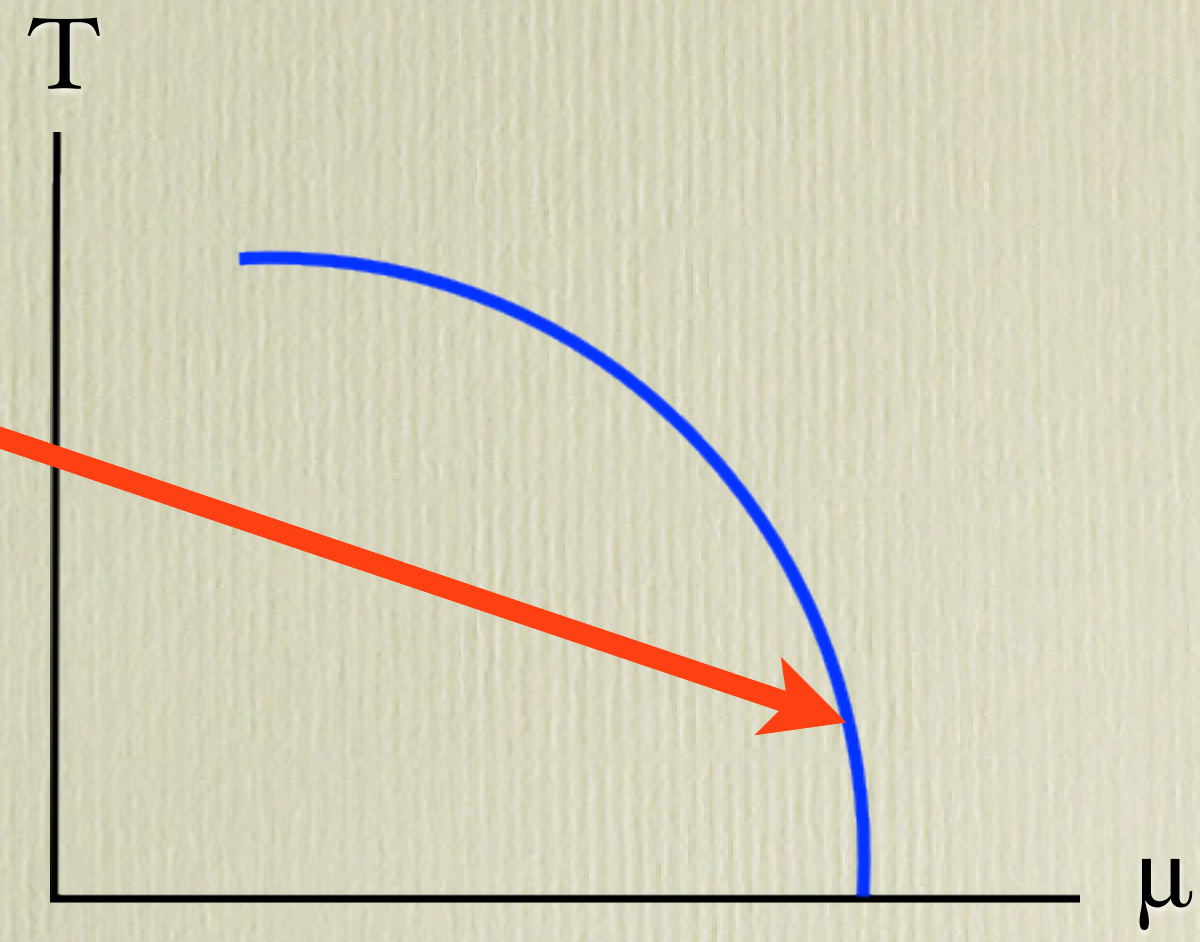


Hadronic observables

Chiral condensate in grand canonical ensemble

Preliminary!

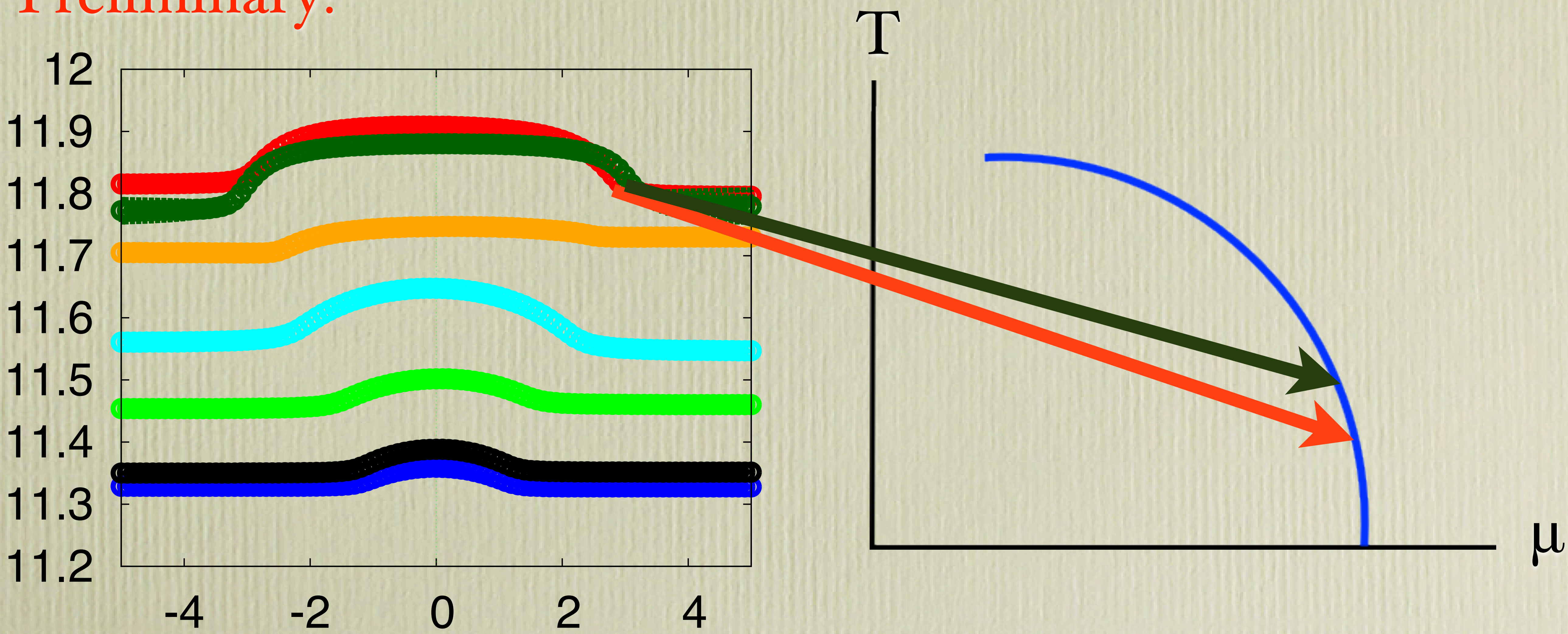




Hadronic observables

Chiral condensate in grand canonical ensemble

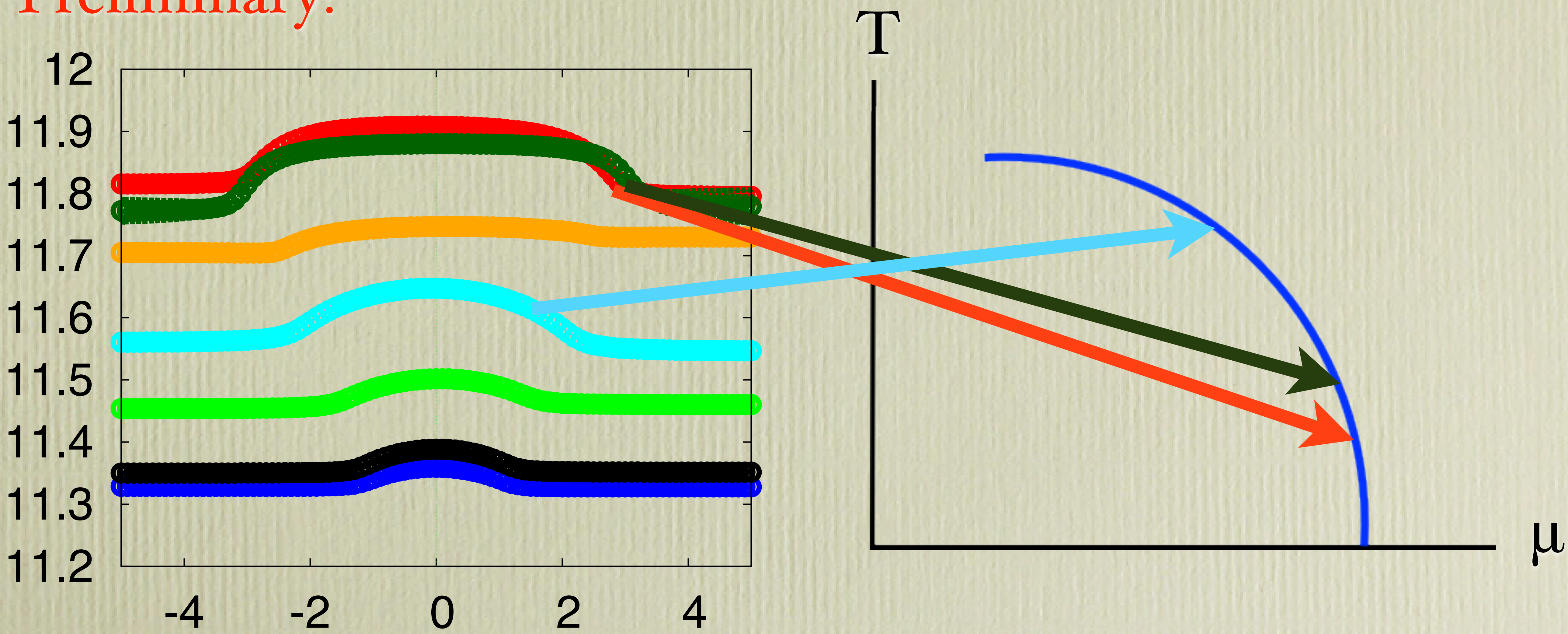
Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble

Preliminary!

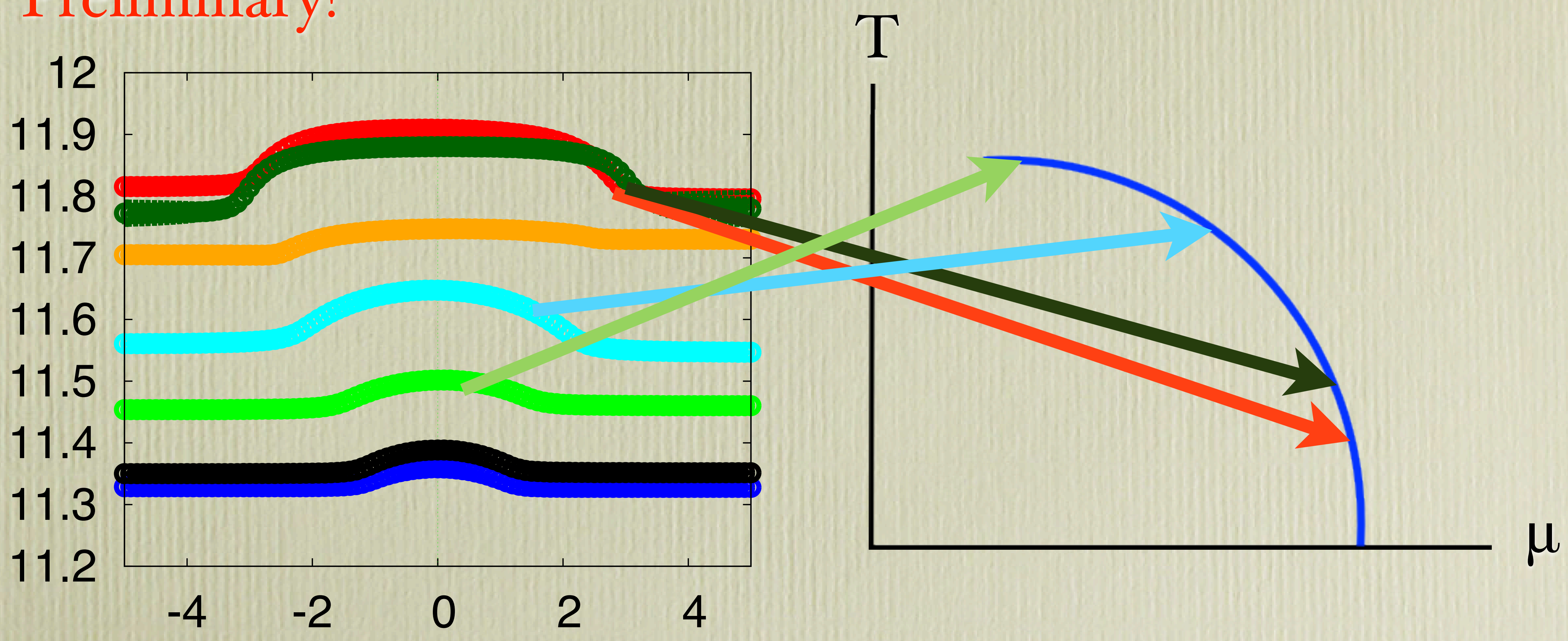


14年9月4日木曜日

Hadronic observables

Chiral condensate in grand canonical ensemble

Preliminary!

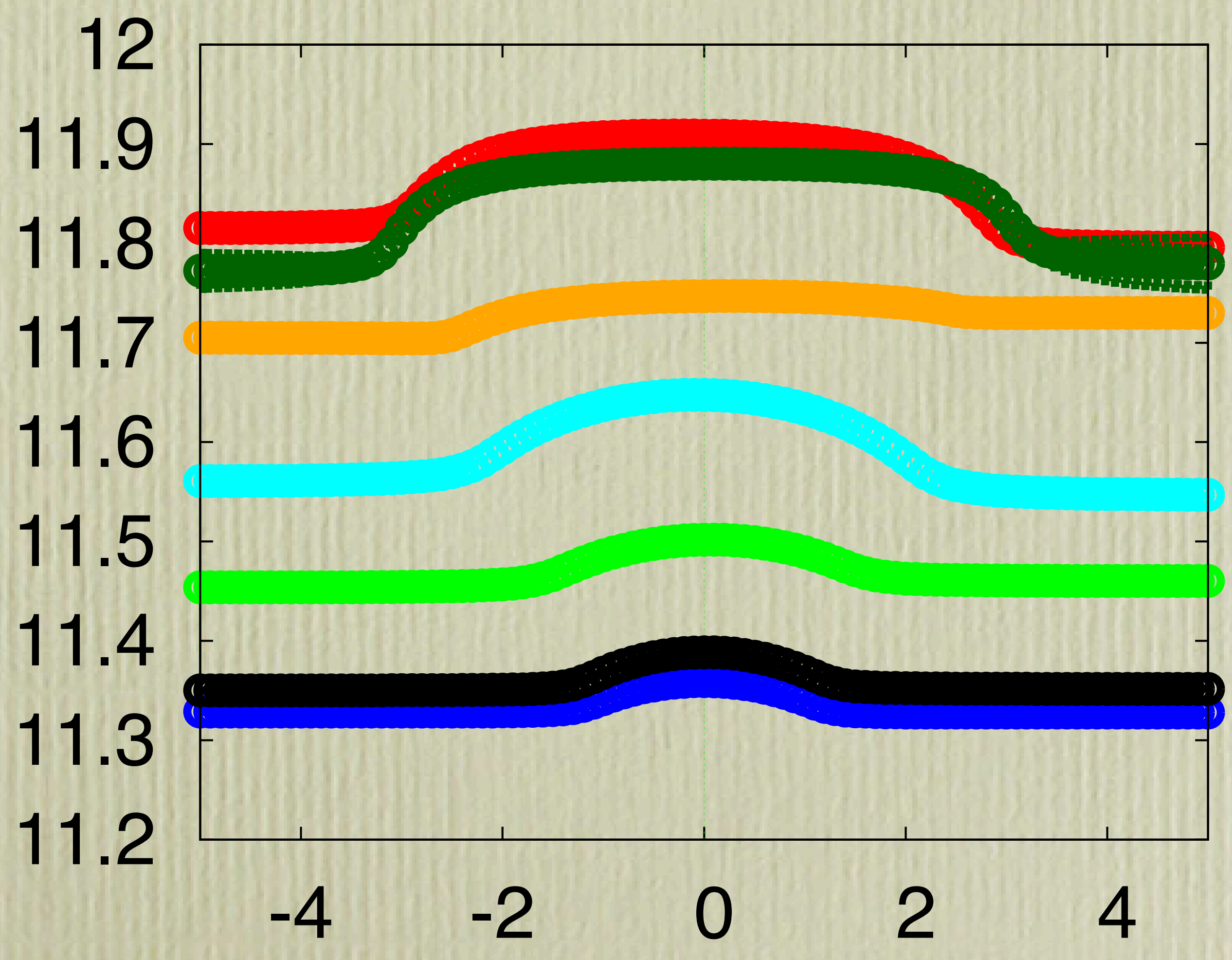


Hadronic observables

Chiral condensate in grand canonical ensemble

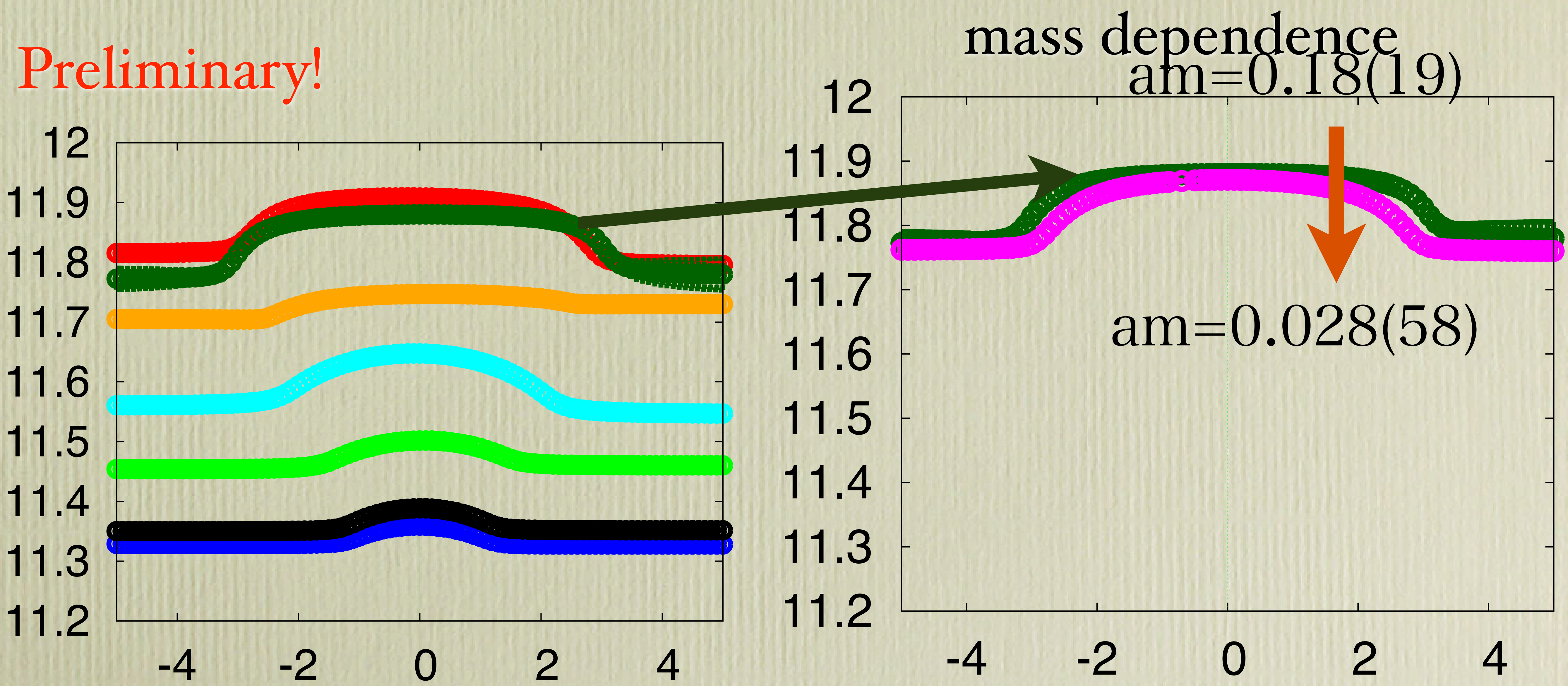
mass dependence

Preliminary!



Hadronic observables

Chiral condensate in grand canonical ensemble



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Chiral condensate in grand canonical ensemble

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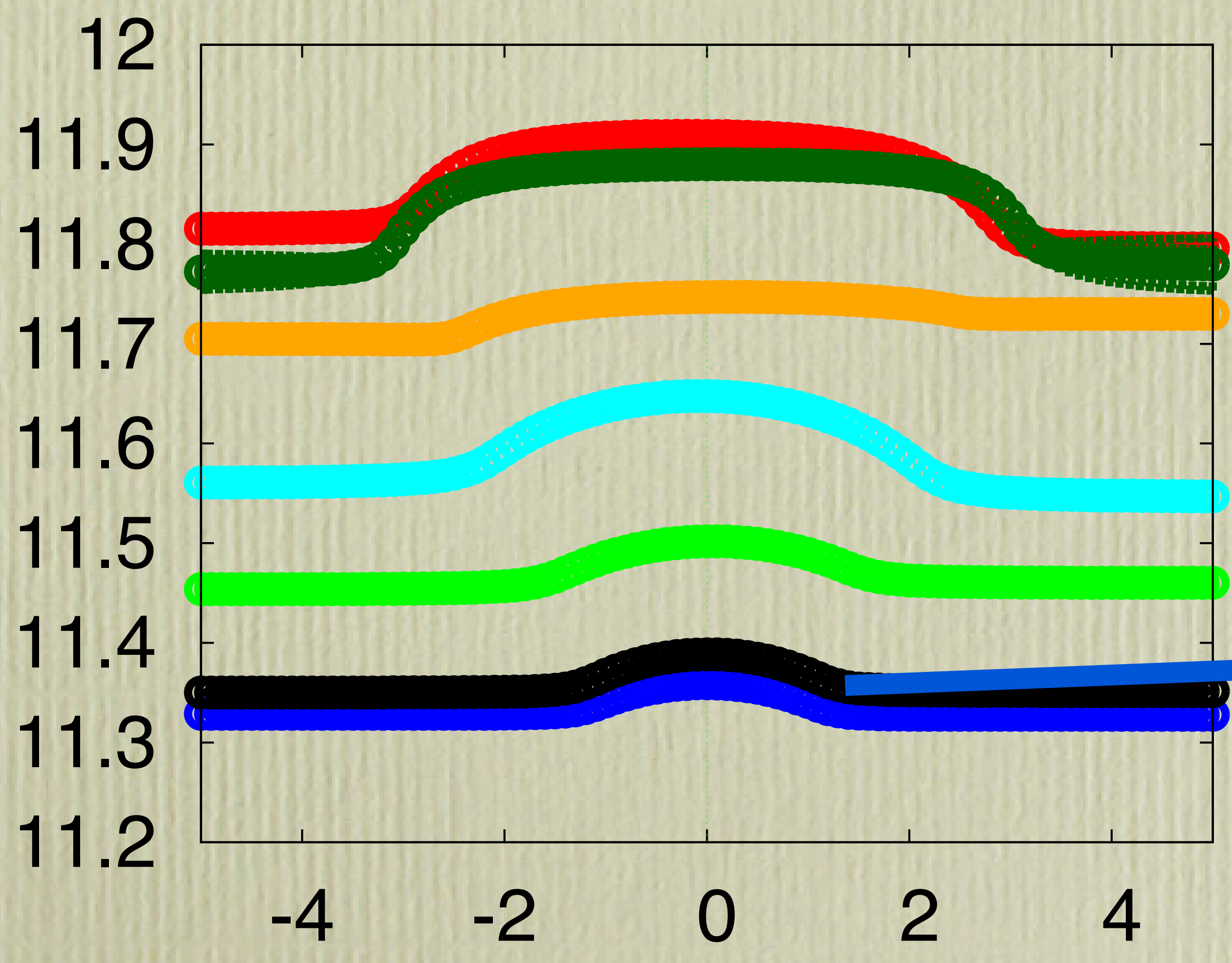

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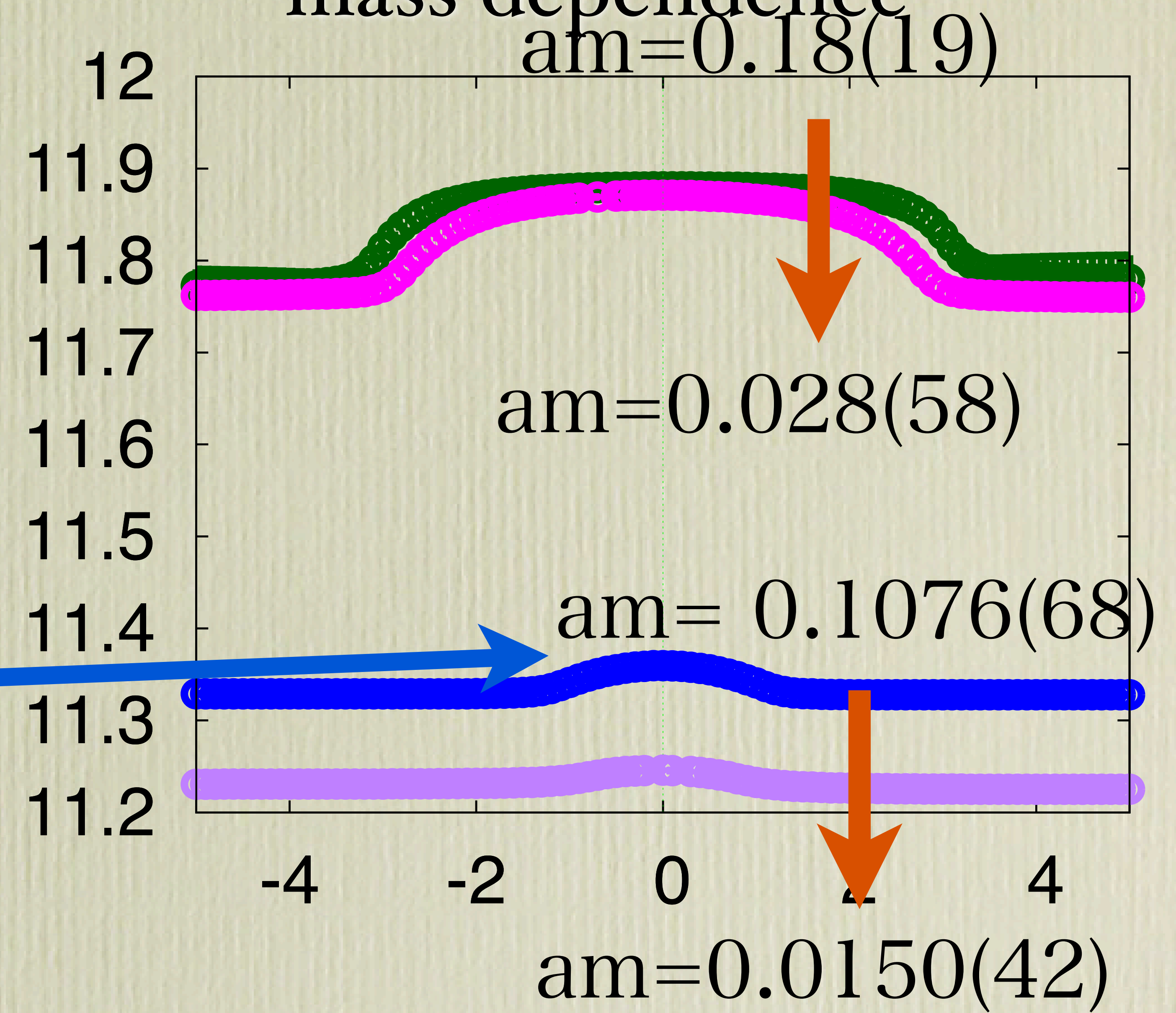

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Chiral condensate in grand canonical ensemble

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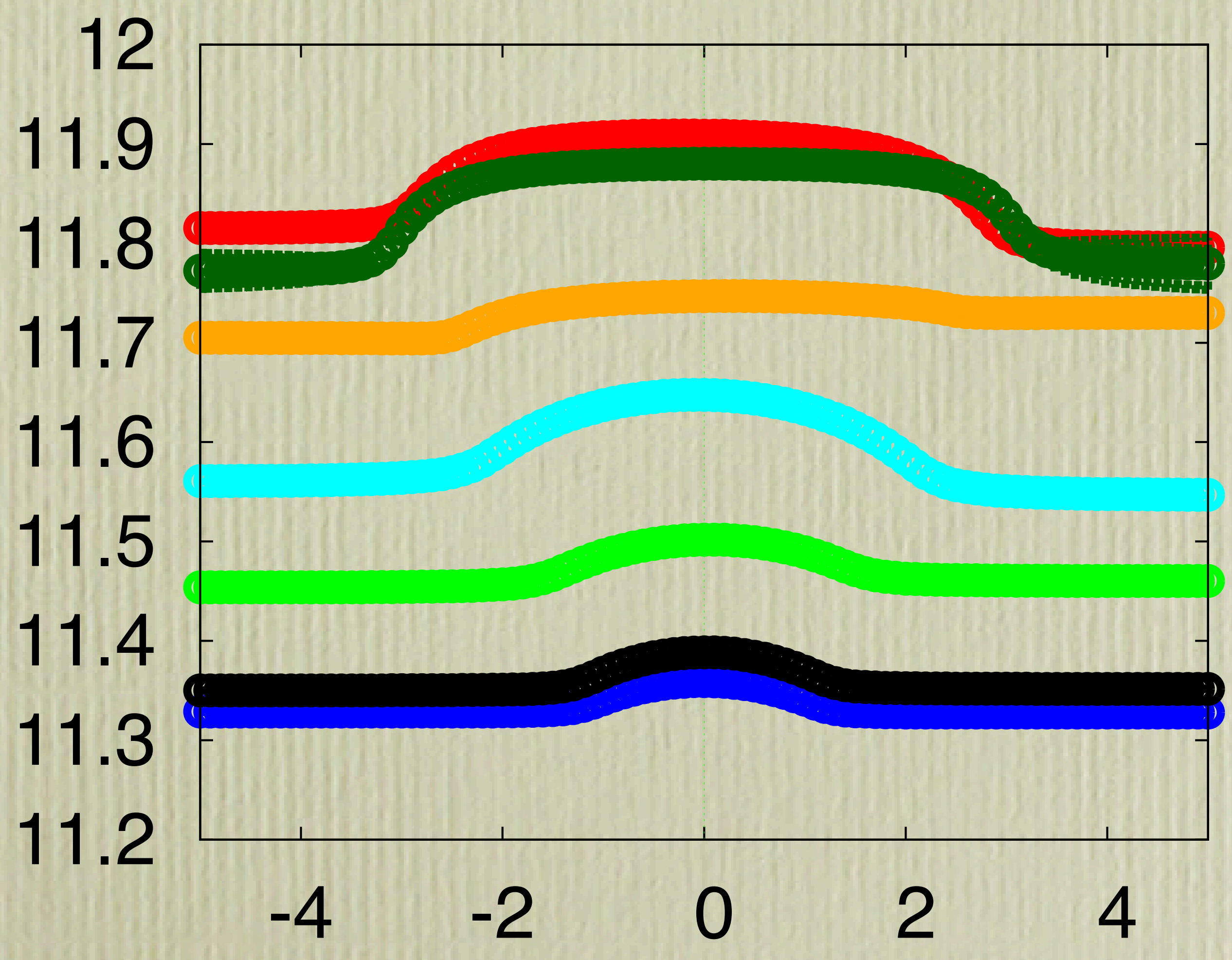
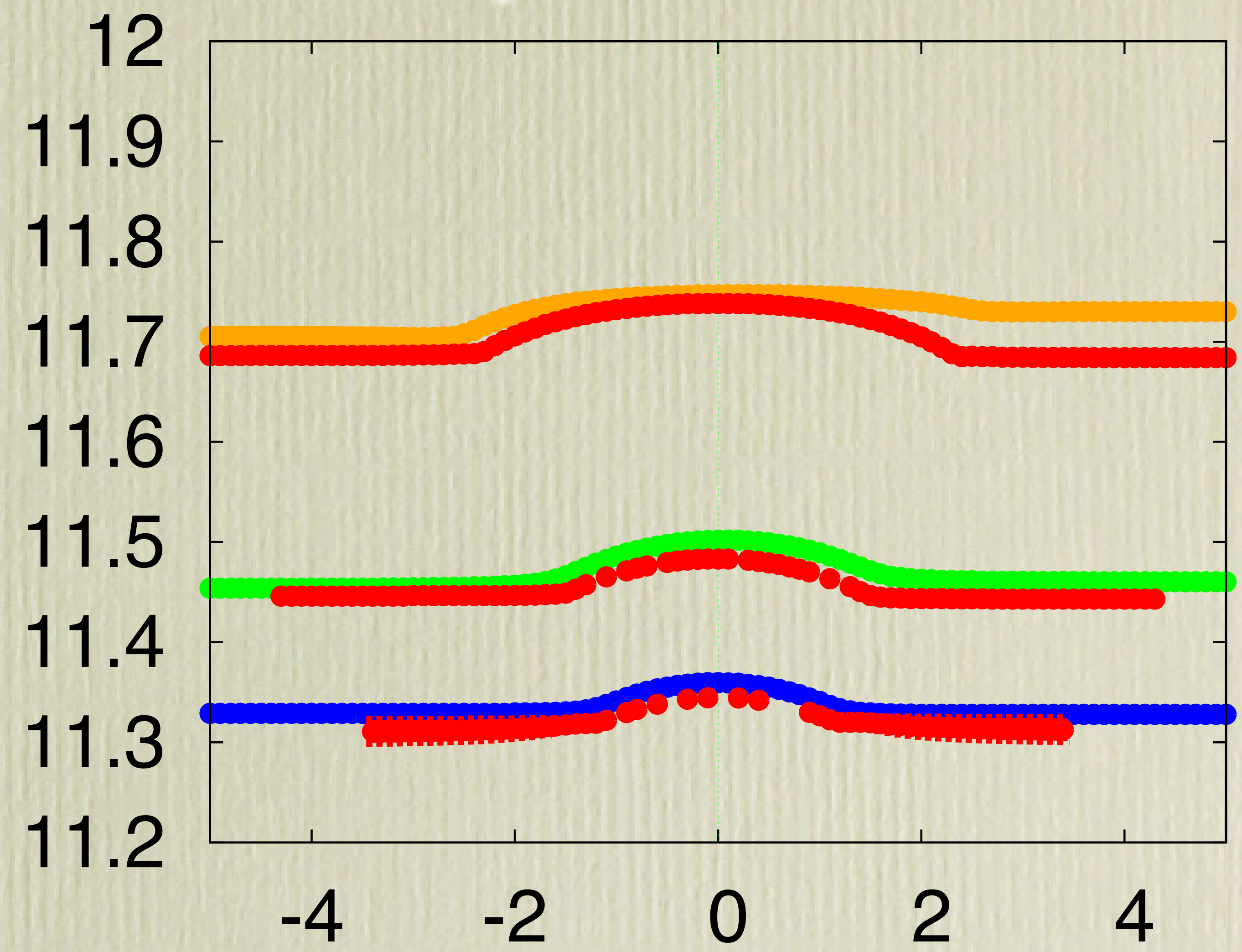


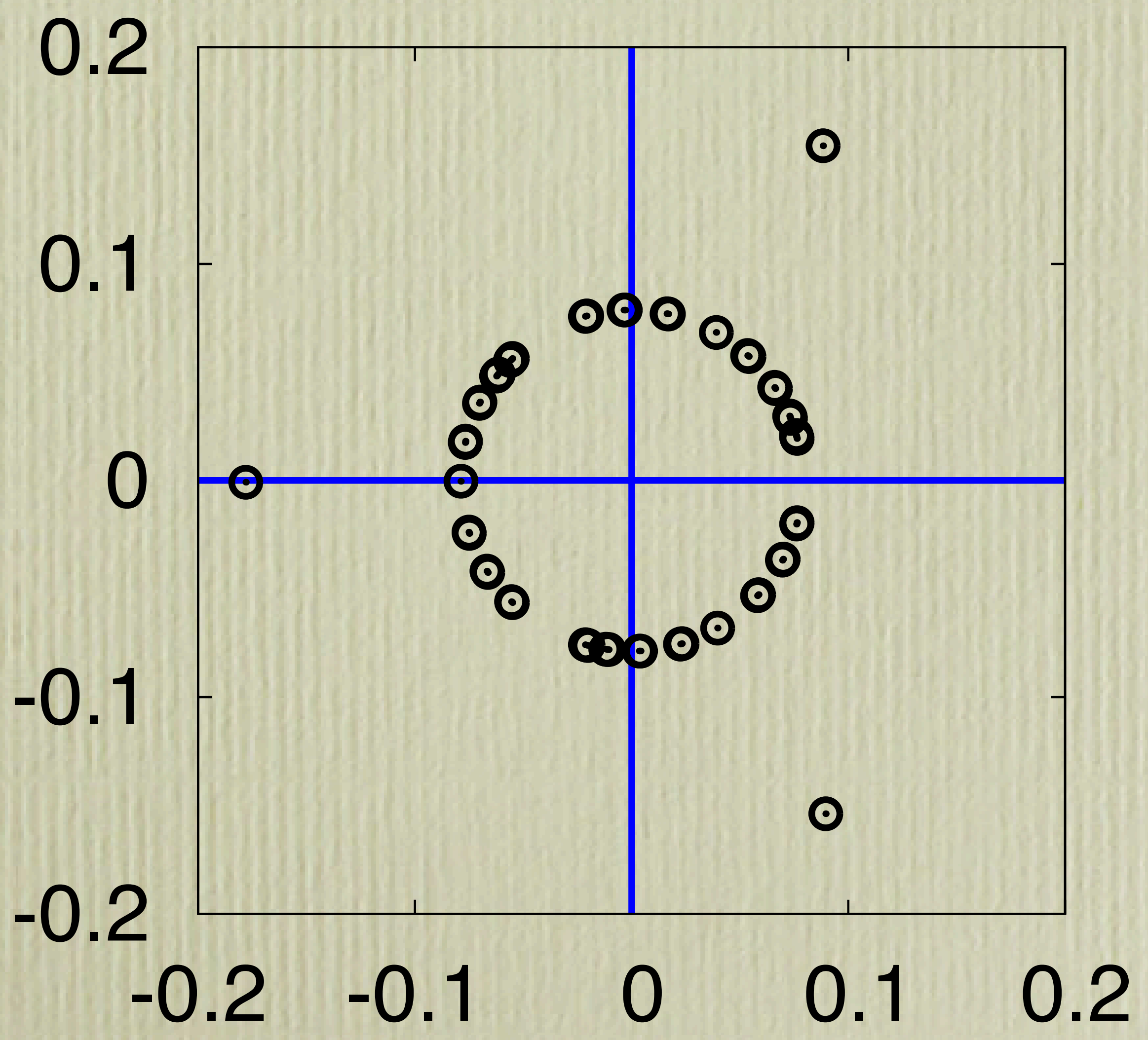
volume dependence 8 → 1 2

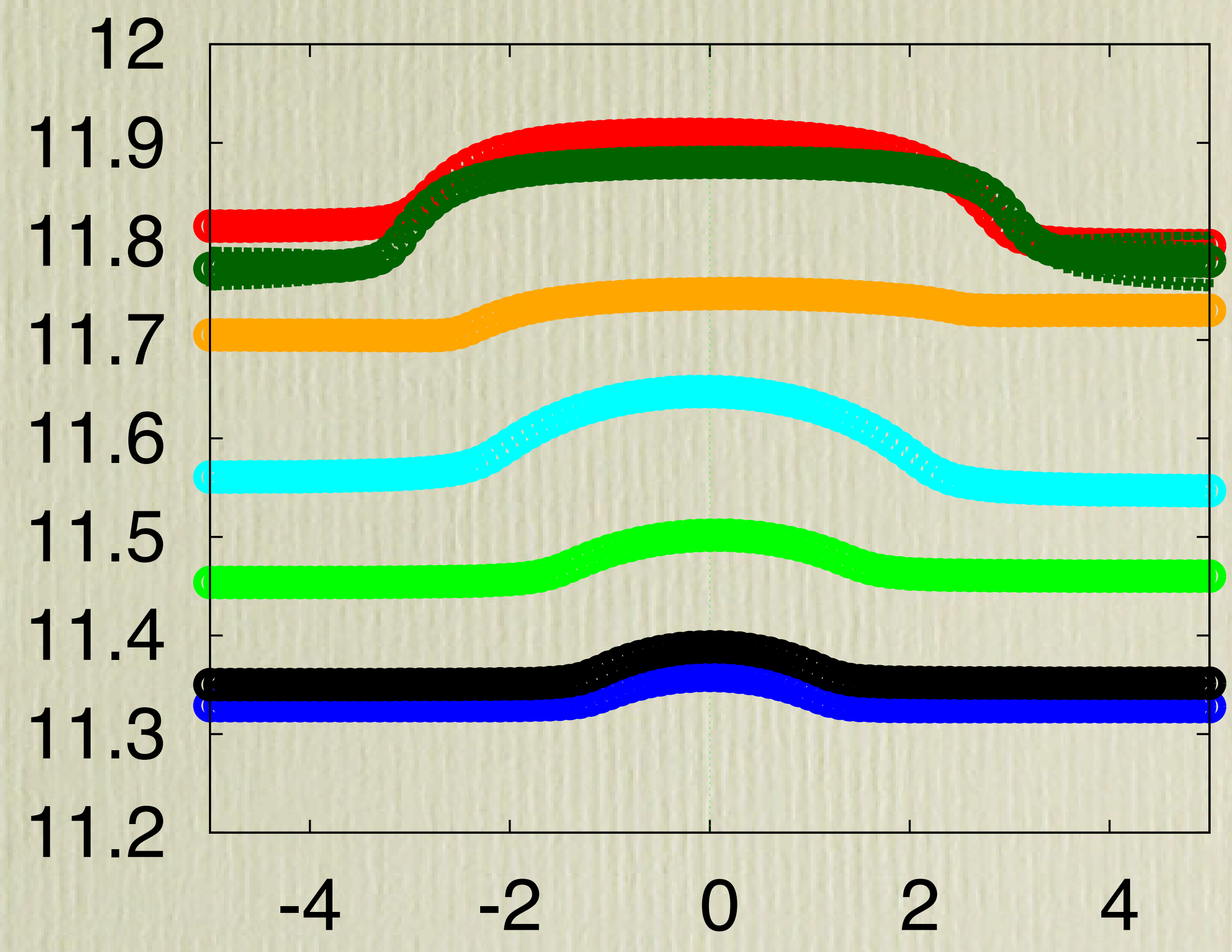


Hadronic observables

Chiral condensate in grand canonical ensemble

Lee-Yang zeros





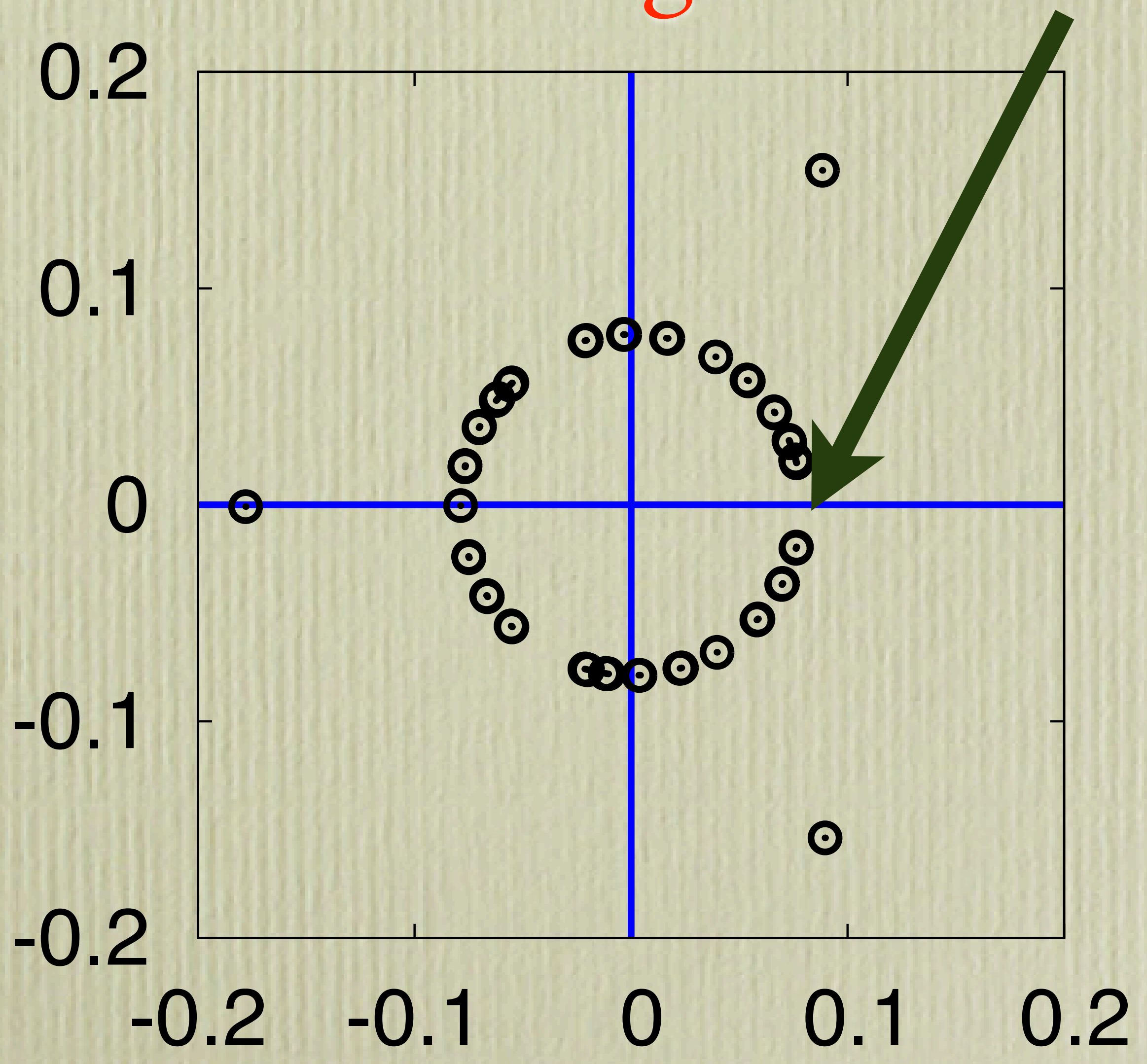
$\beta = 1.1$

Hadronic observables

Chiral condensate in grand canonical ensemble

Lee-Yang zeros

$\xi \sim 0.076$



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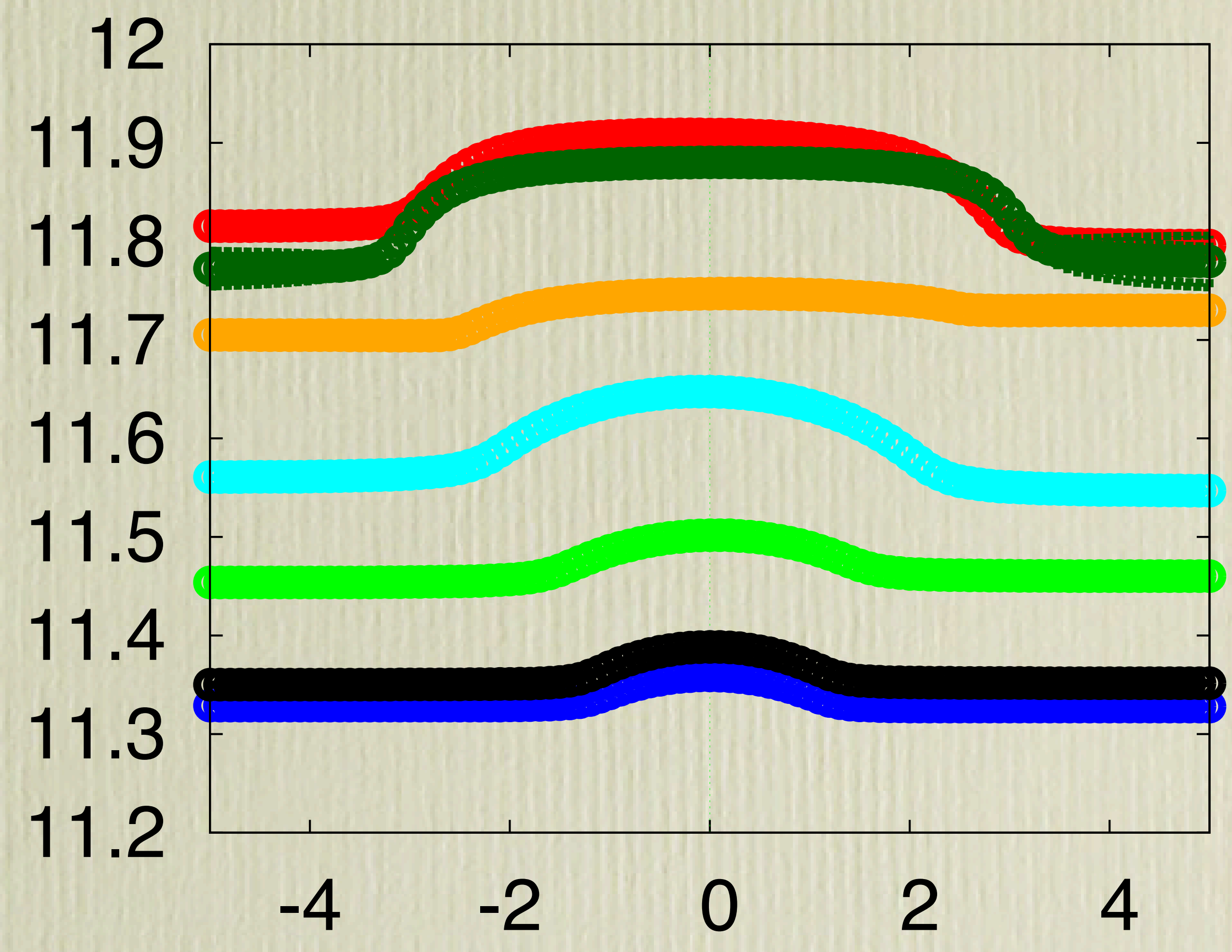
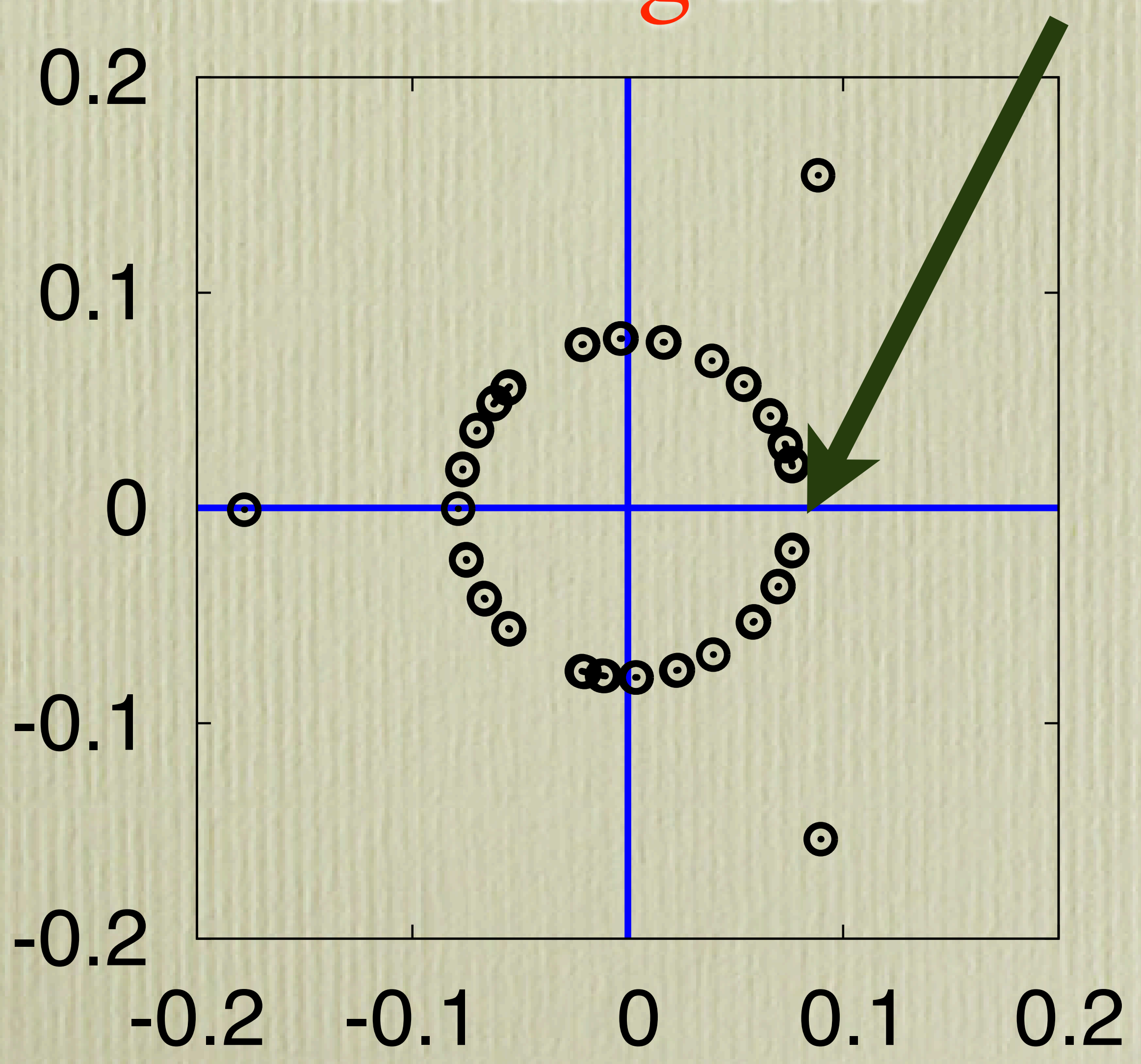


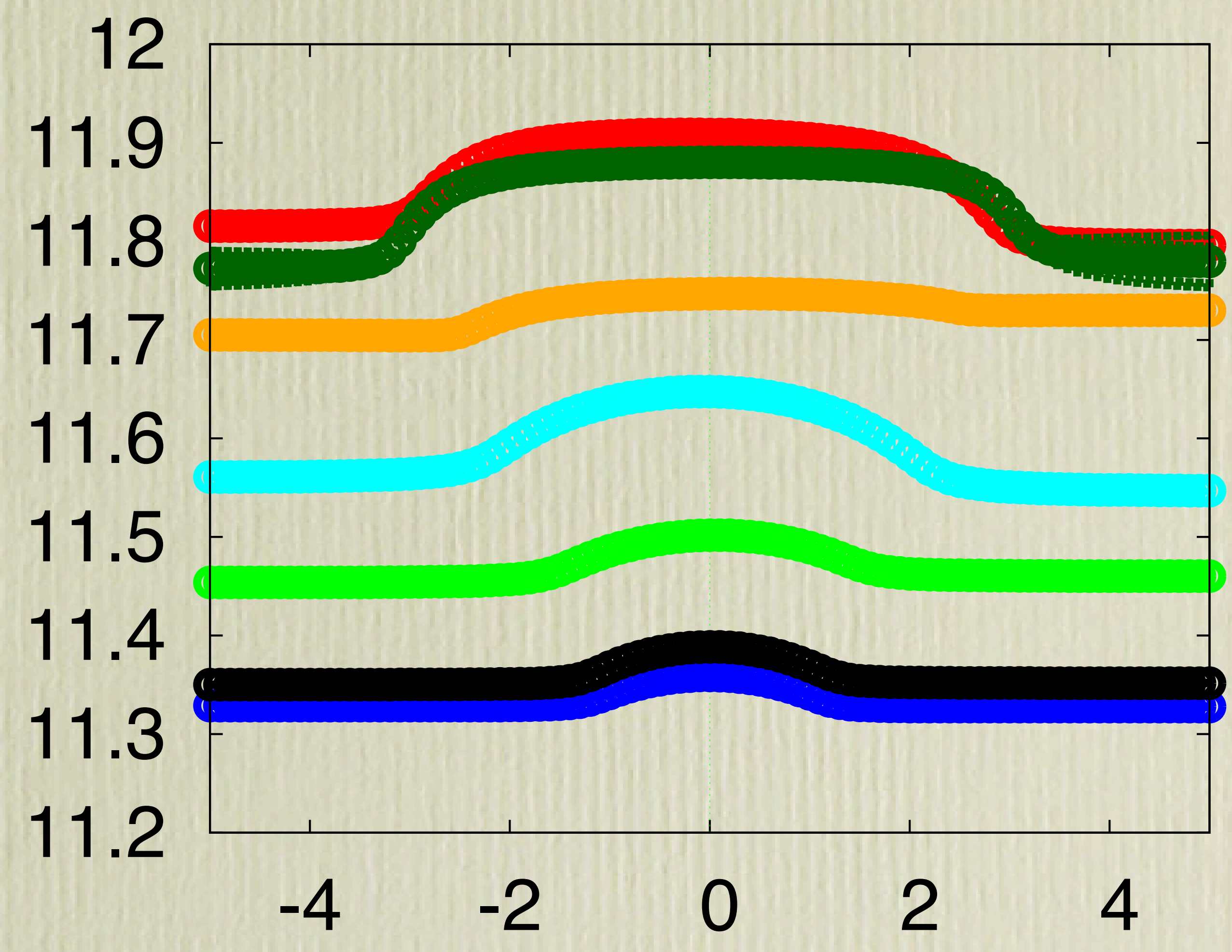
Hadronic observables

Chiral condensate in grand canonical ensemble

$\xi \sim 0.076$ ⟷ $\mu \sim -2.5$

Lee-Yang zeros





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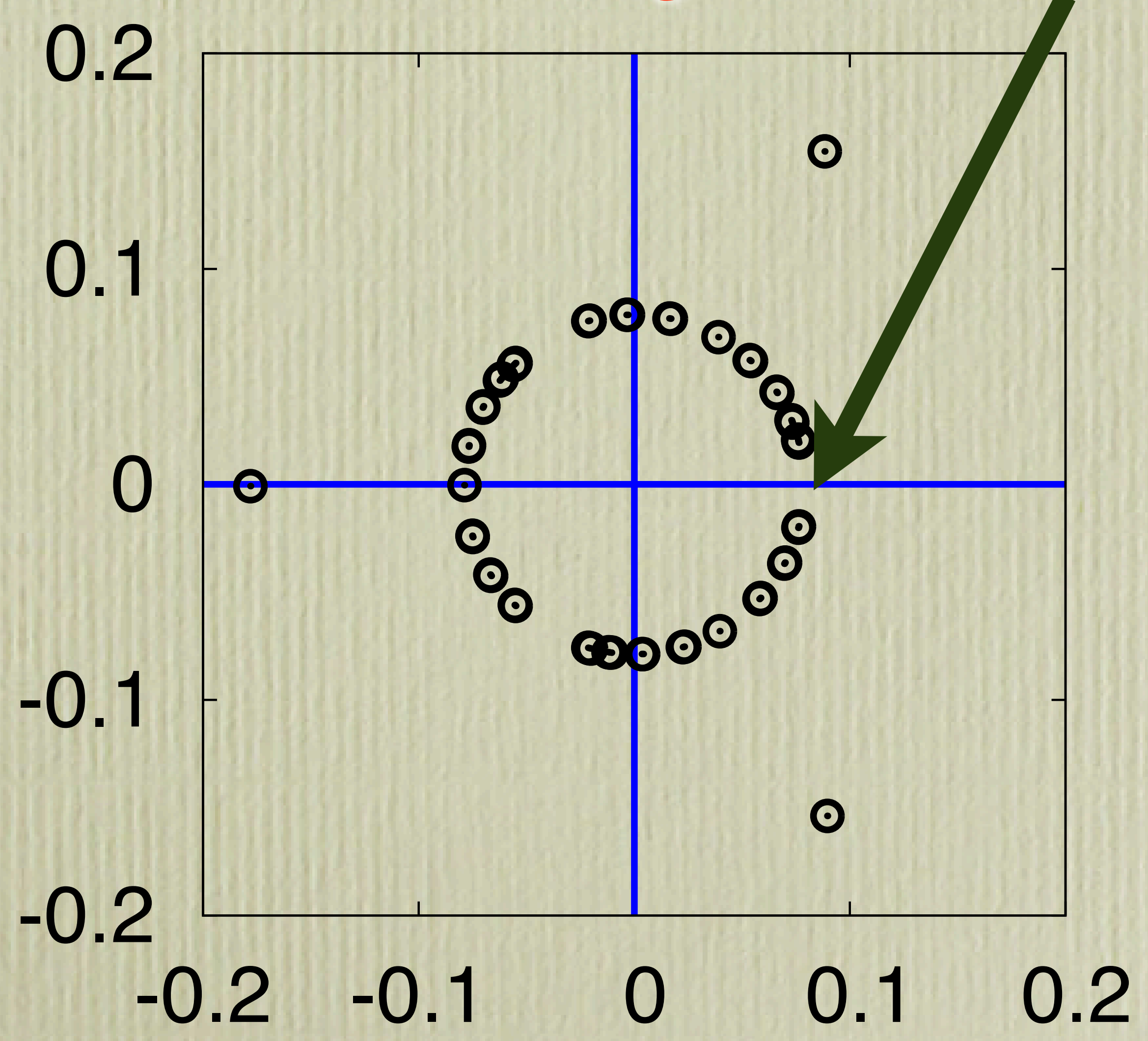
Chiral condensate in grand canonical ensemble

Lee-Yang zeros

$\xi \sim 0.076$ ↔ $\mu \sim -2.5$



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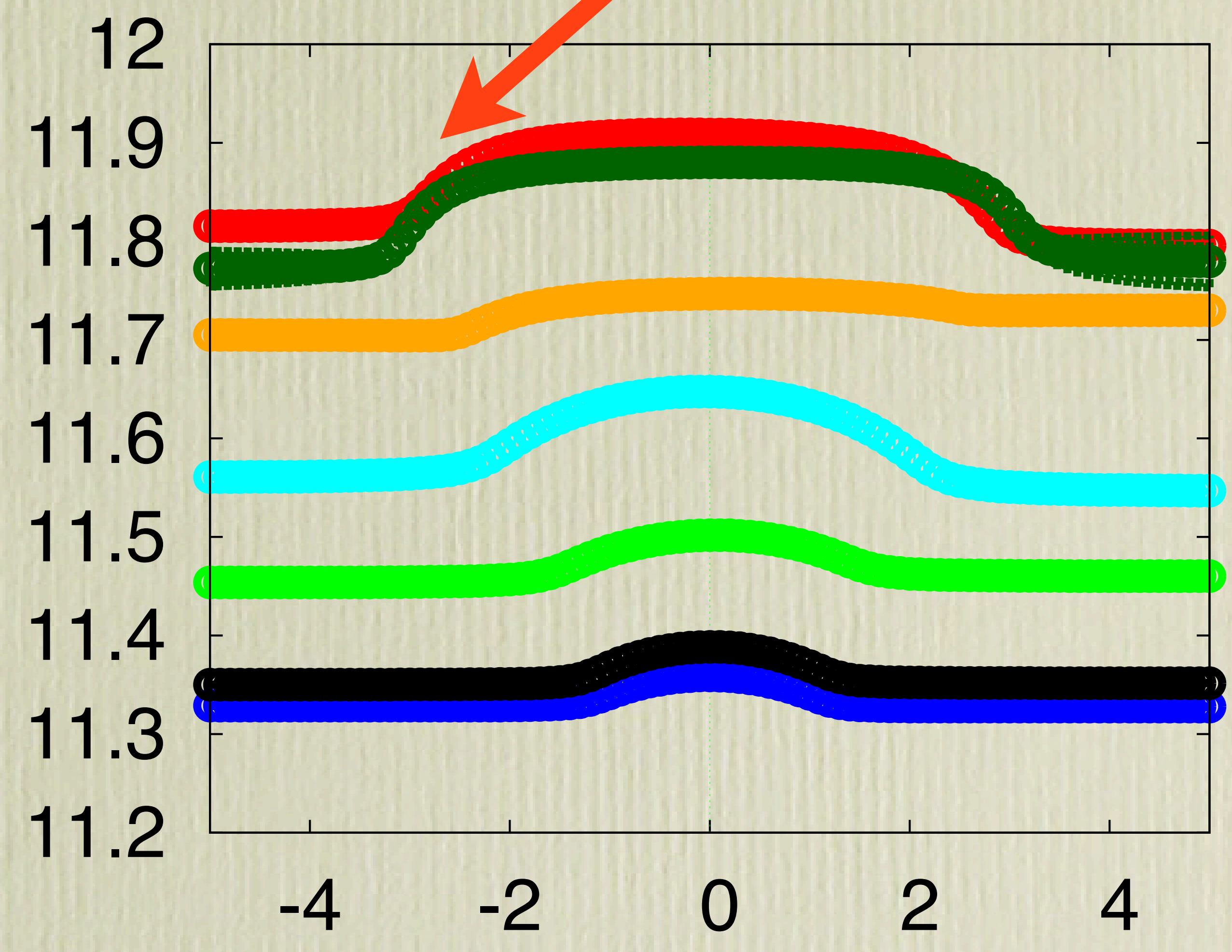
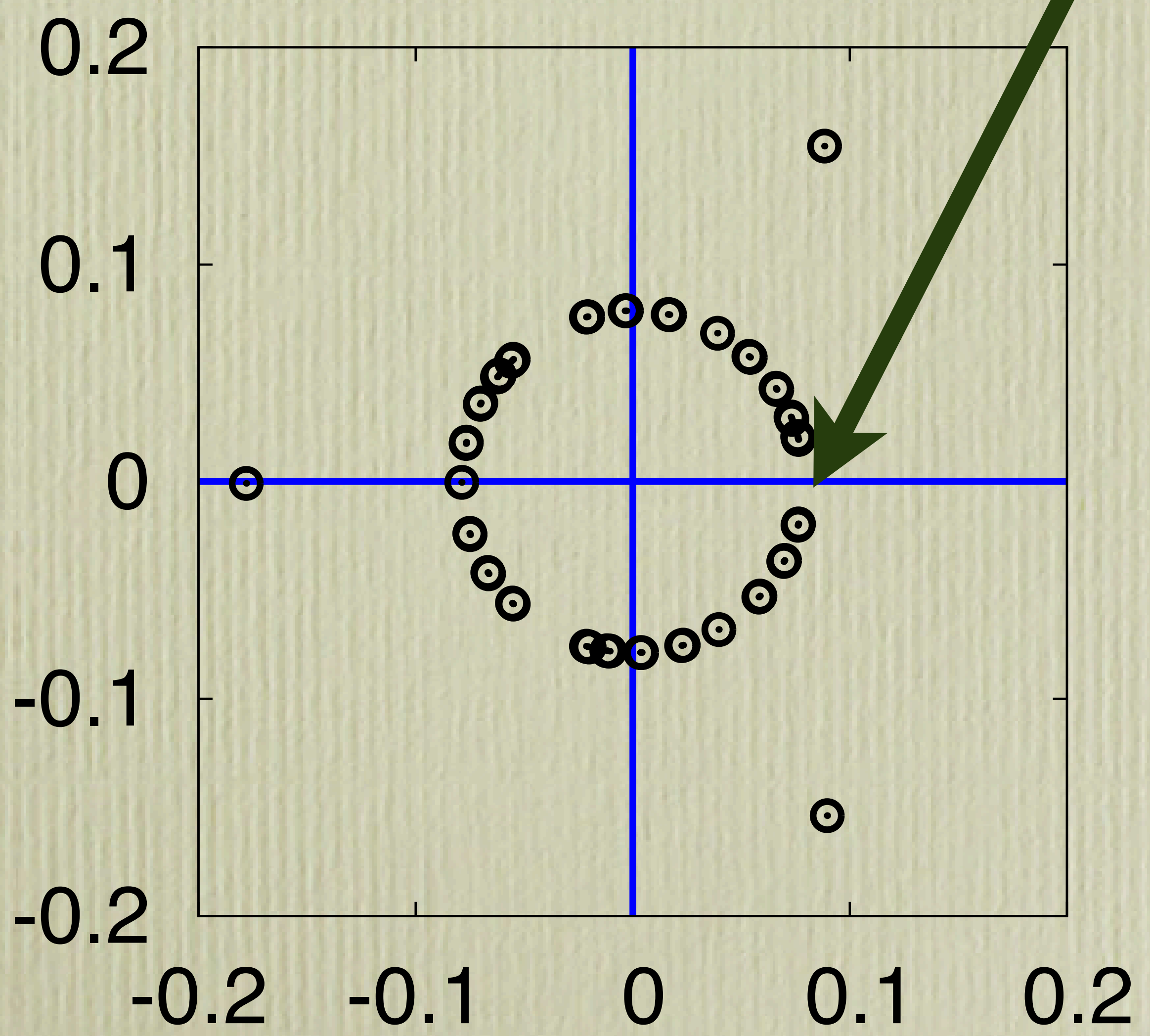


Hadronic observables

Chiral condensate in grand canonical ensemble

Lee-Yang zeros $\xi \sim 0.076 \longleftrightarrow \mu \sim -2.5$



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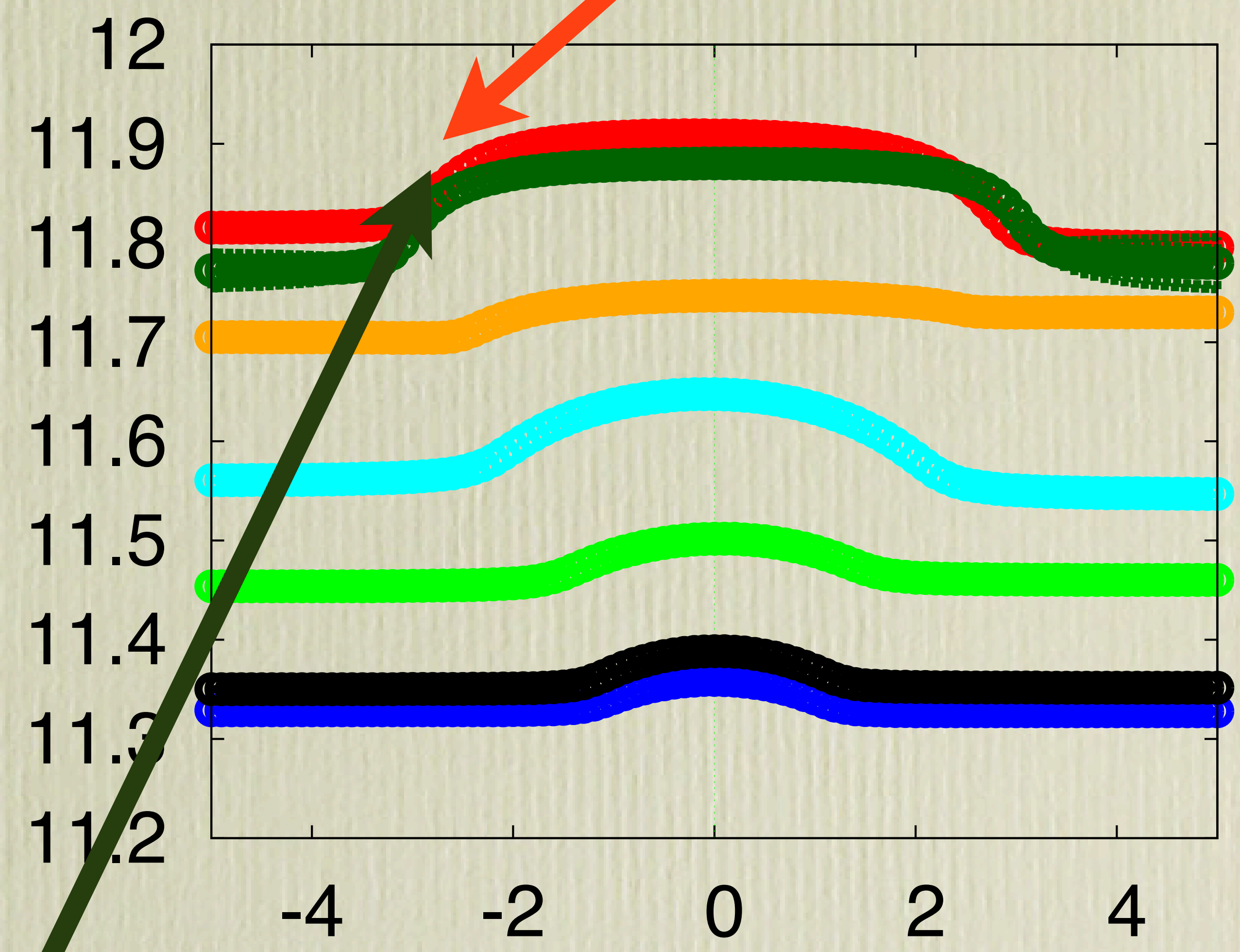

$\frac{m_N}{3}$? $\frac{m_\pi}{2}$? artifact?

Conclusion

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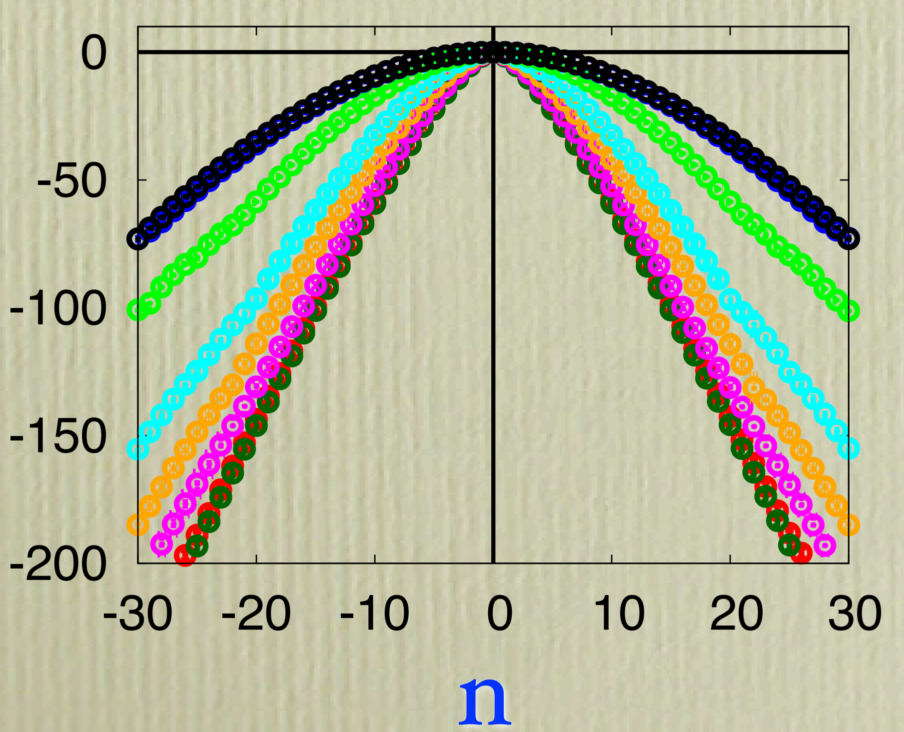
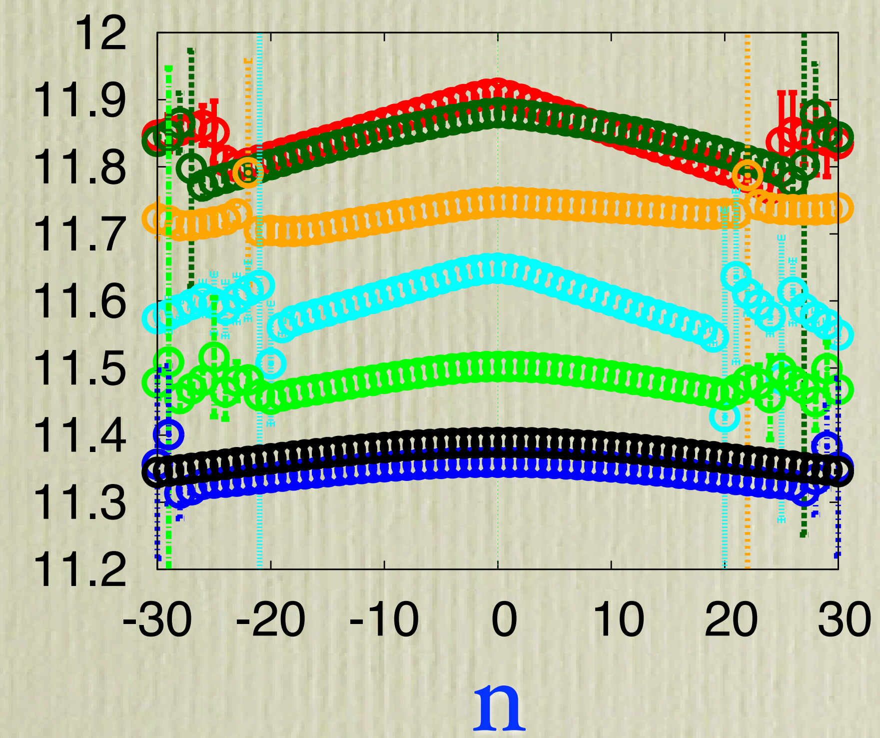
- Hopping parameter expansion works more than we expected.
- We have three interesting results.

Zn



$$\frac{\sum \langle \bar{\psi}\psi \rangle_C(\beta, n)}{V}$$



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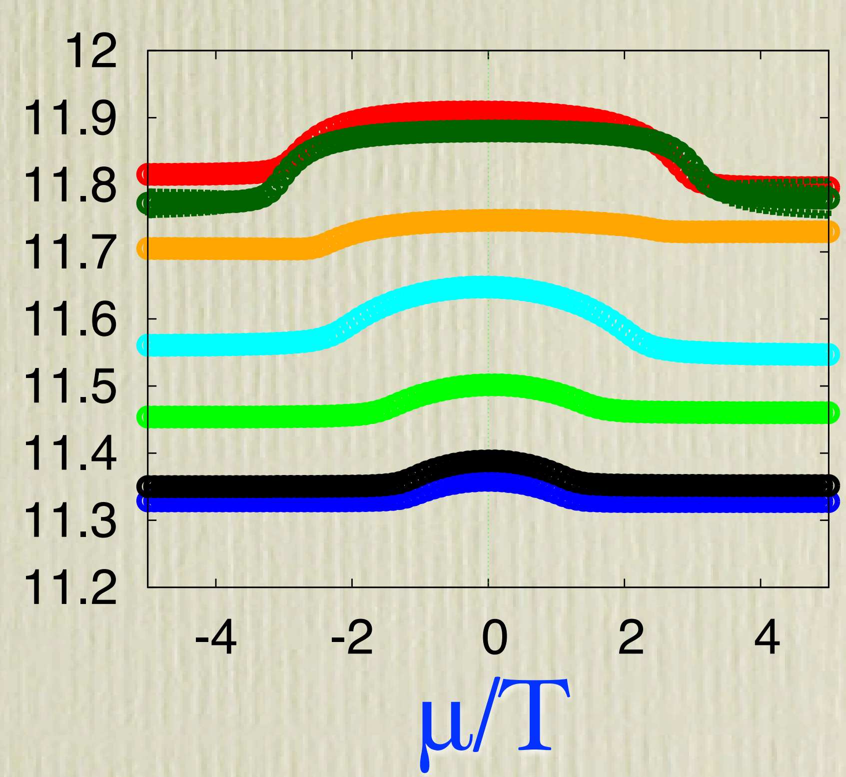
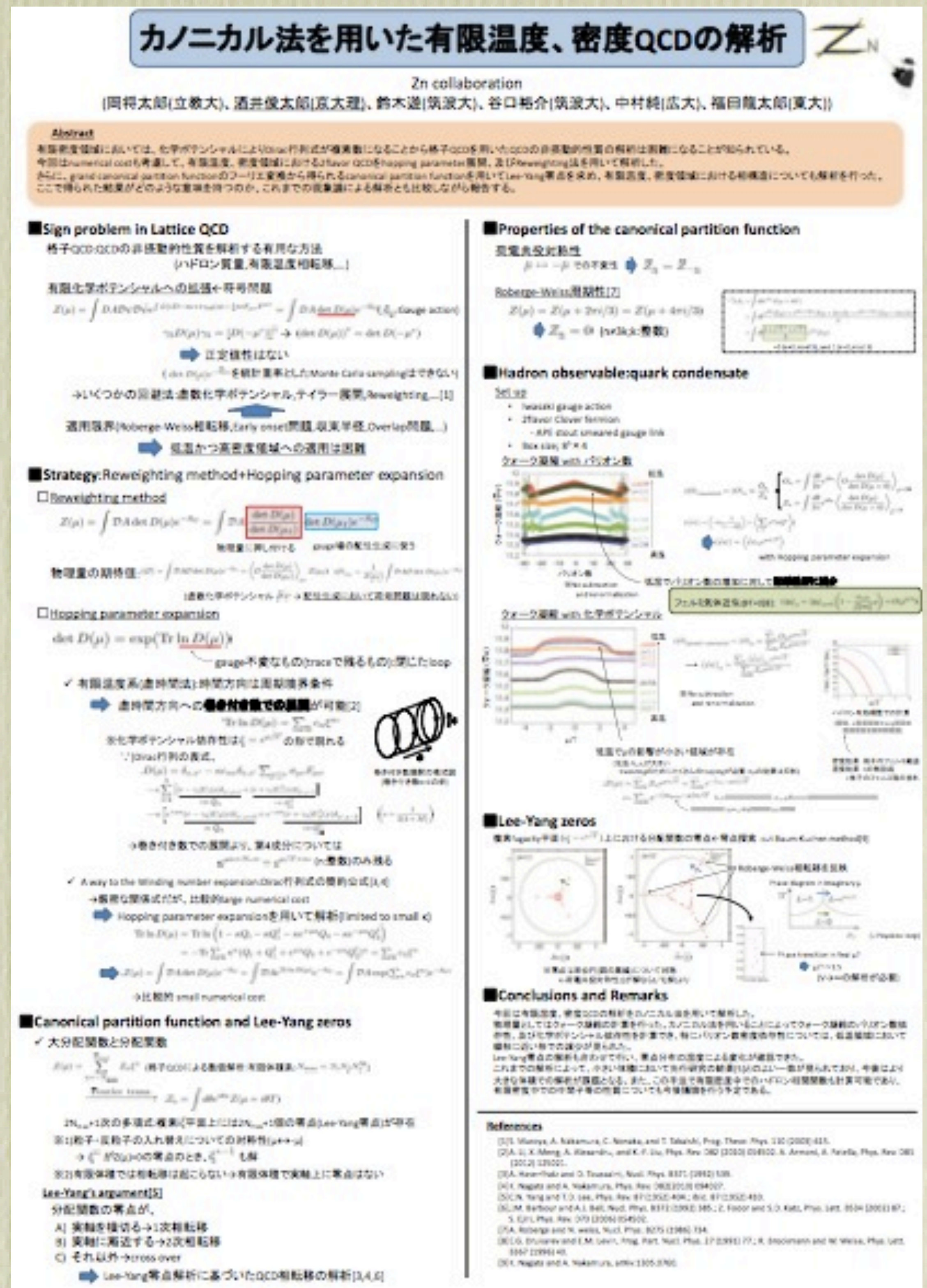


ポスターやります

酒井俊太郎

プレゼンツ

乞うご期待！



If you can read this
I am on the wrong page.

Hadronic observables

Convergence radius

Hadronic observables

Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \cdots$$
$$+ Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \cdots$$

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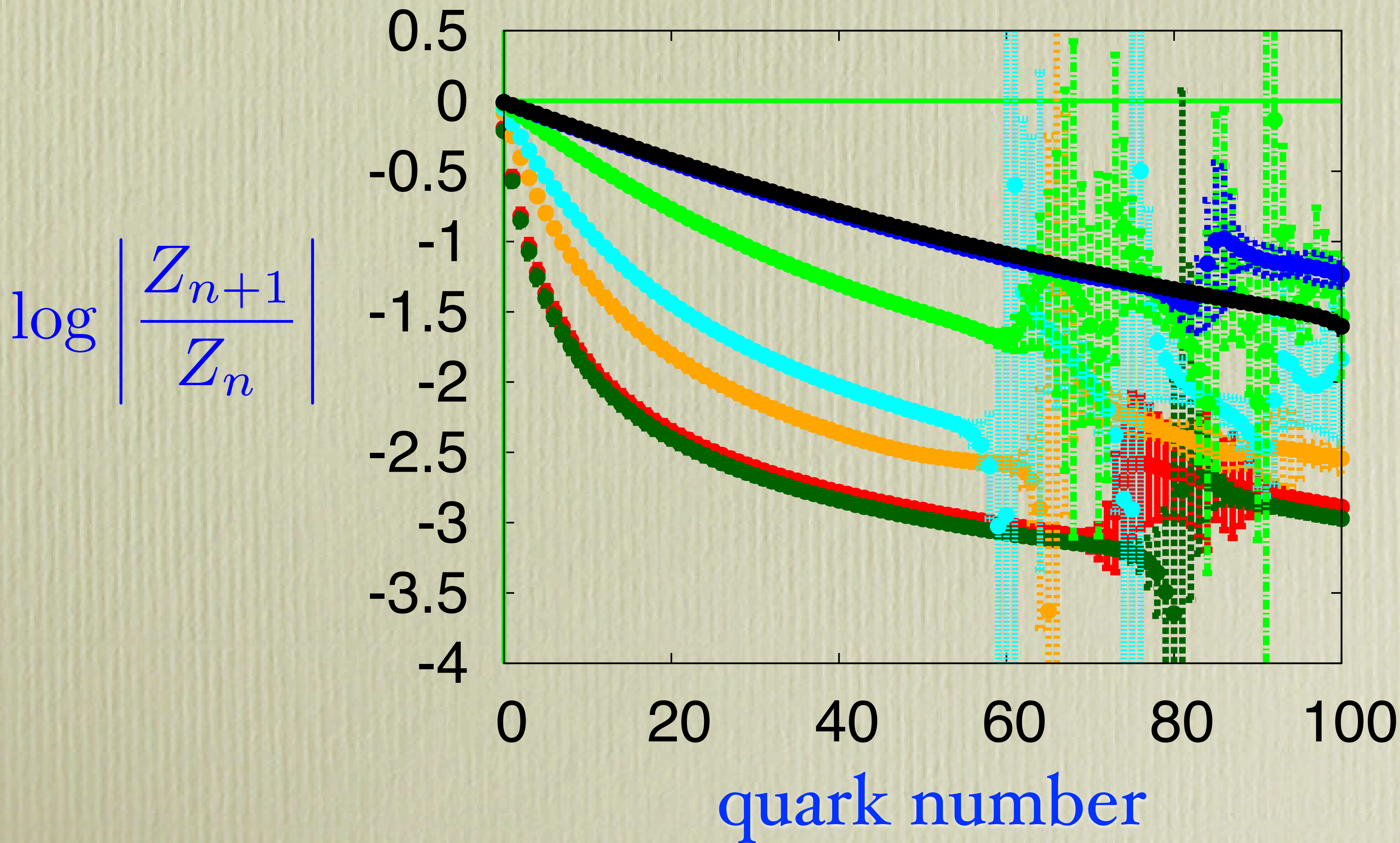

Hadronic observables

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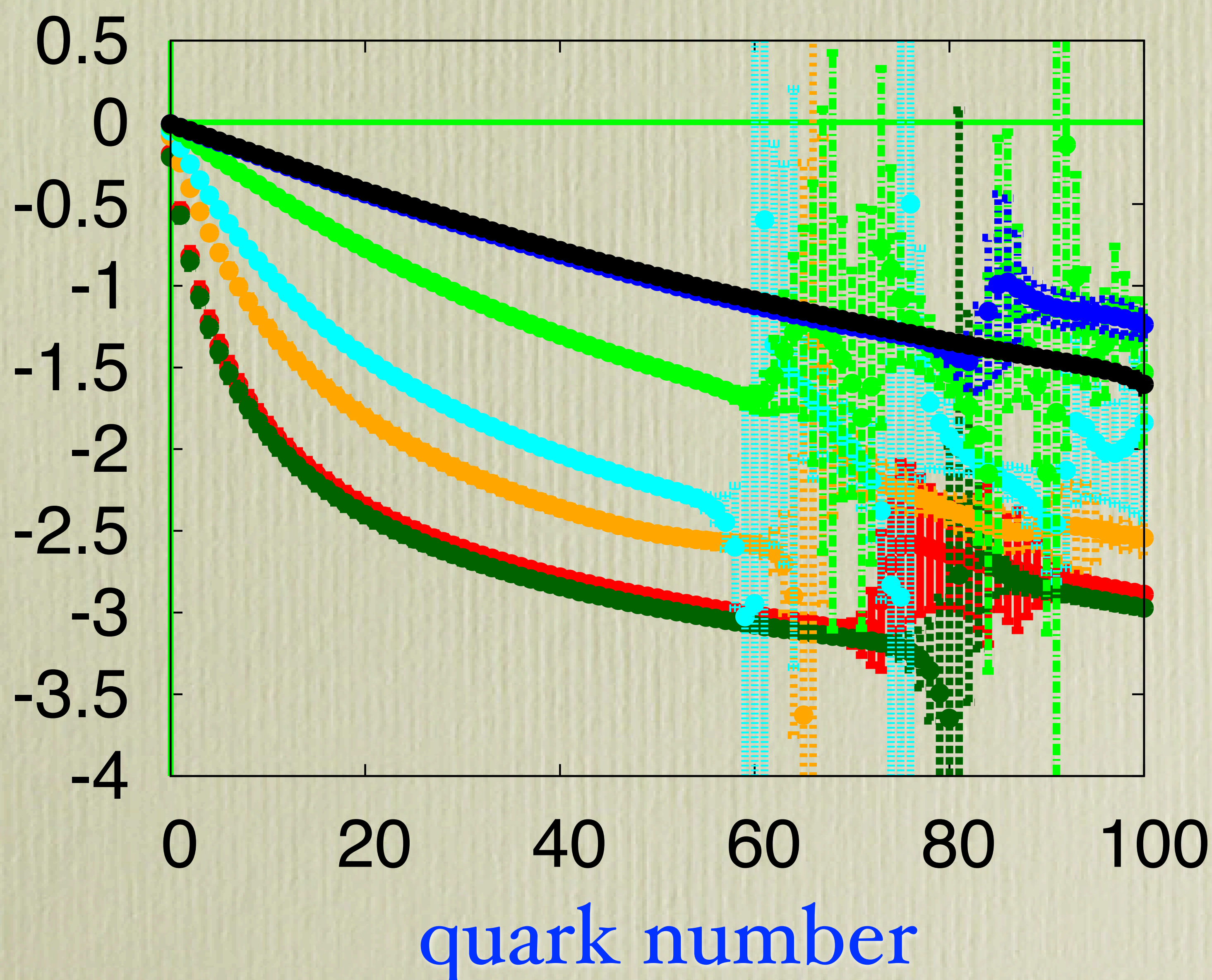
$\log \left| \frac{Z_{n+1}}{Z_n} \right| \rightarrow \log \left| \frac{1}{\xi_c} \right|$



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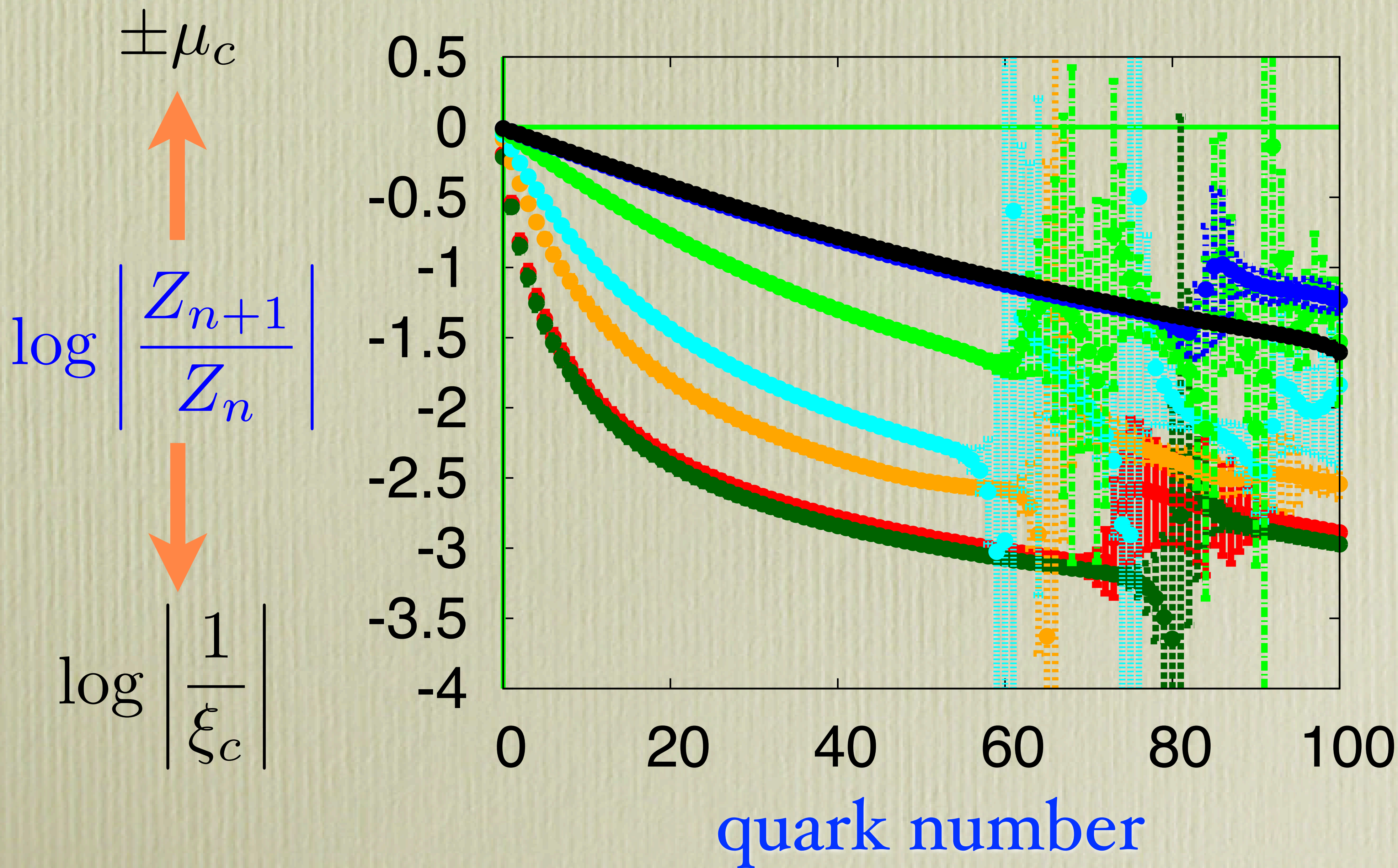

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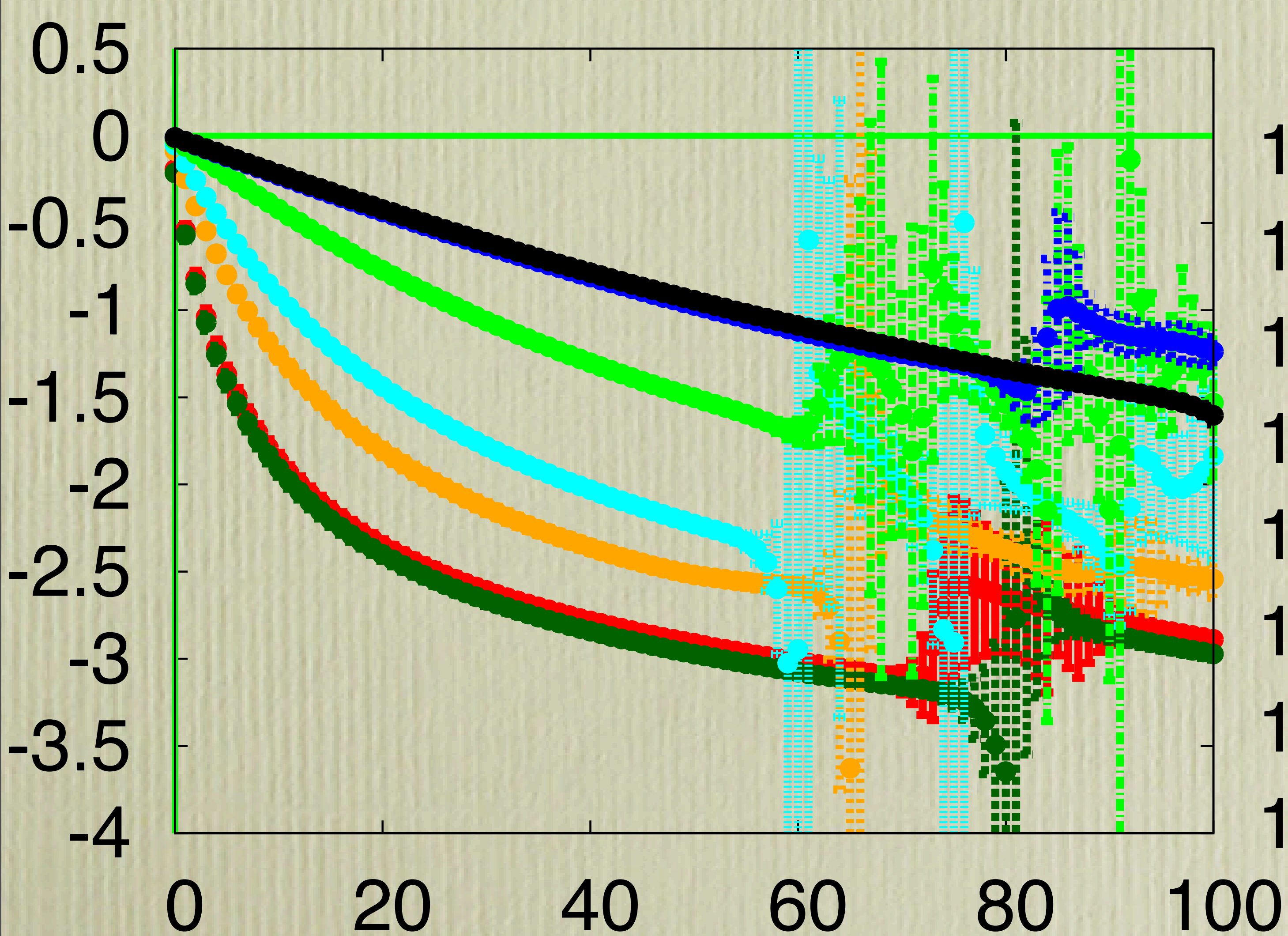

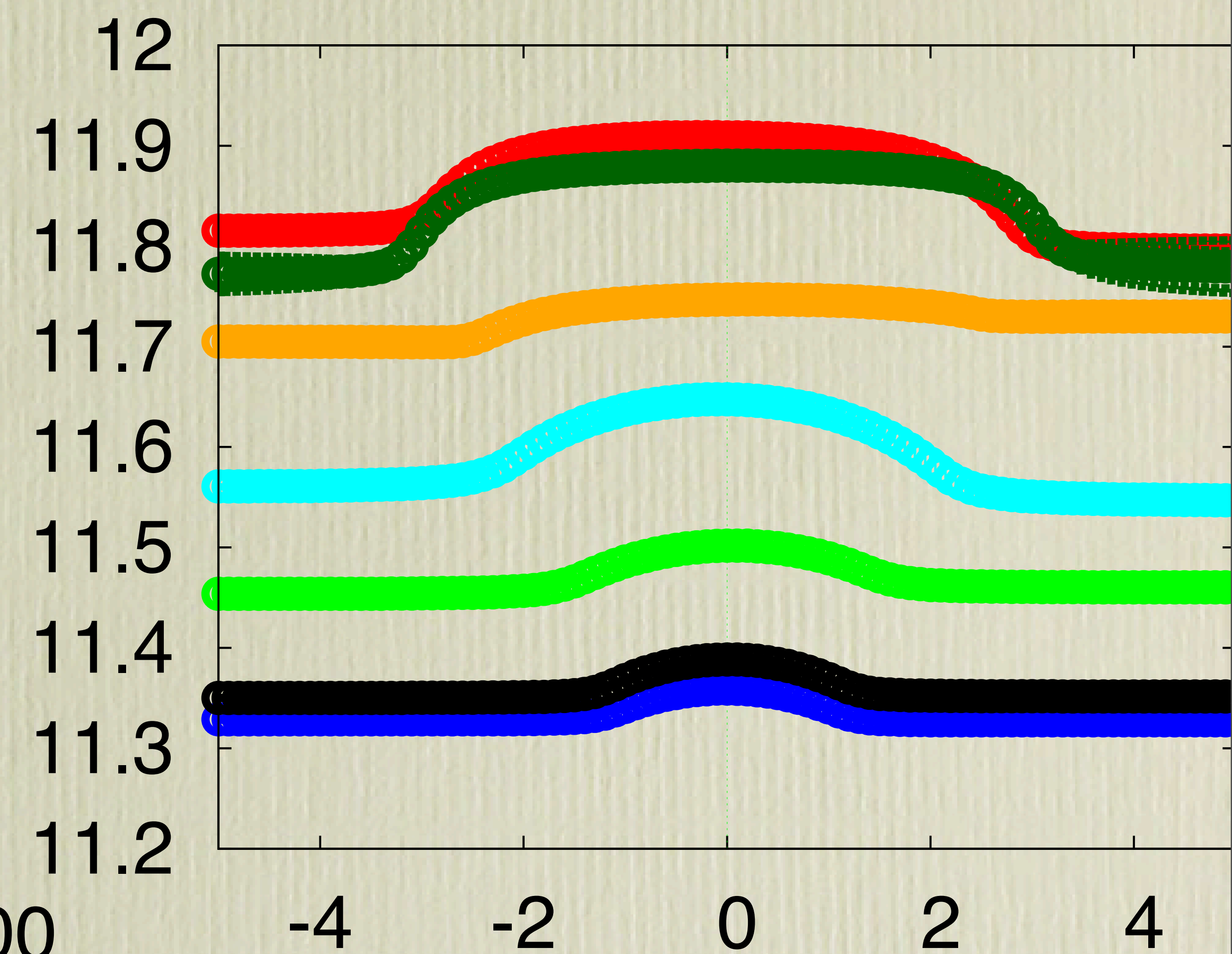

Numerical results Phase(Zc(n))

Canonical partition function $Z_C\left(T,n,V\right) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

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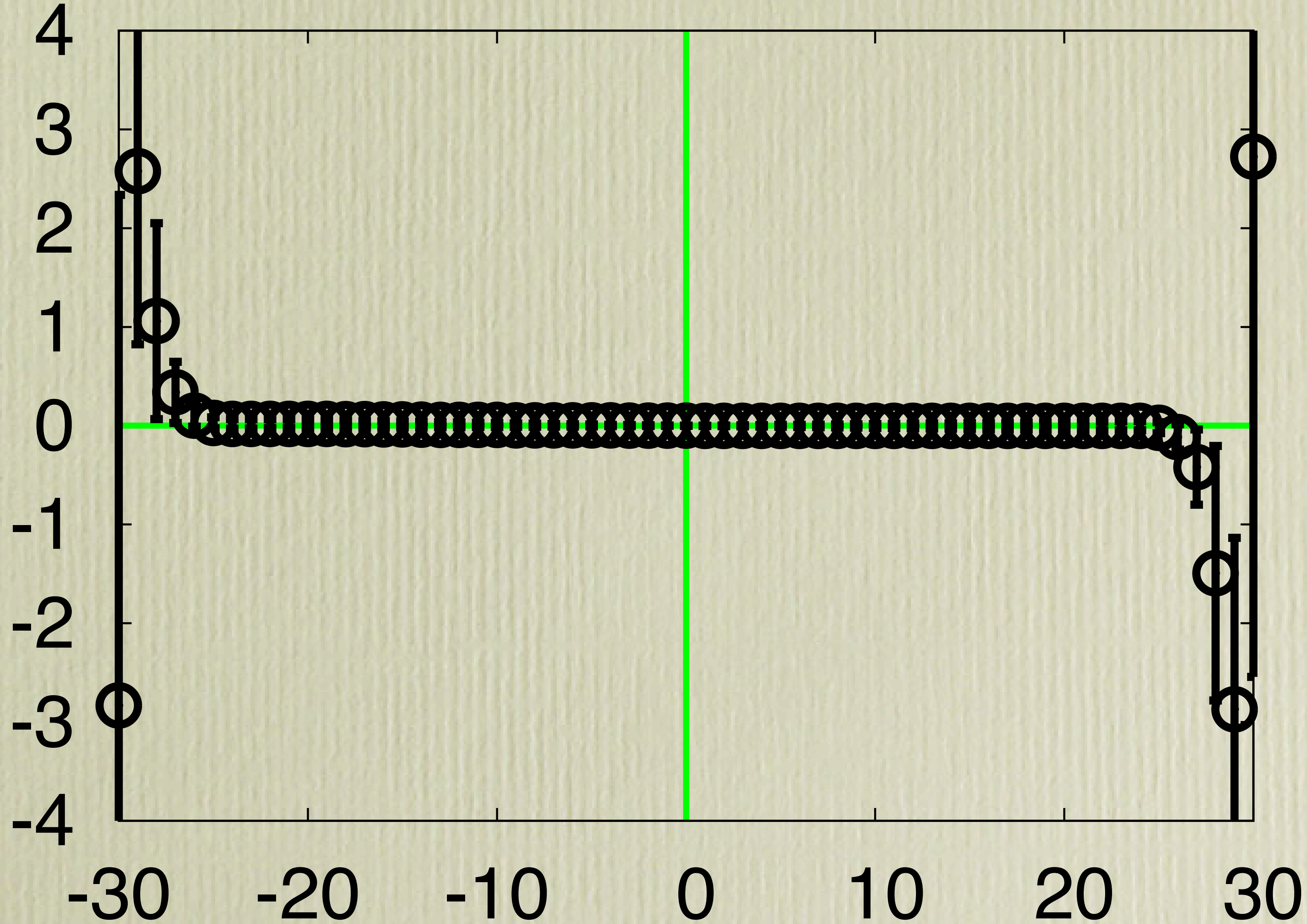
Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.9$

$\mu = 0$

$\theta(\beta,n)$



baryon number

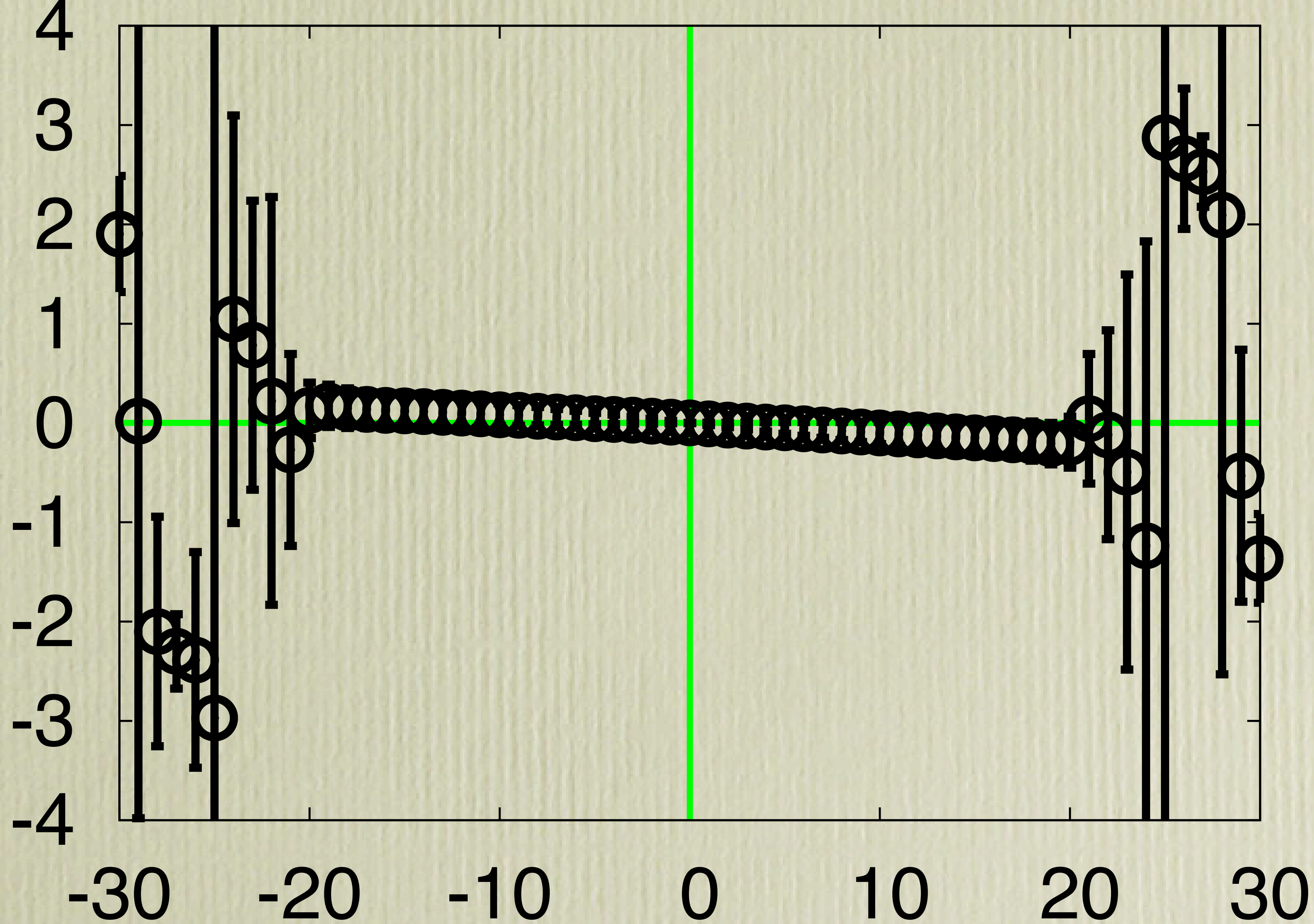
Numerical results Phase(Zc(n))

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.7$

$\mu = 0$

$\theta(\beta,n)$



baryon number

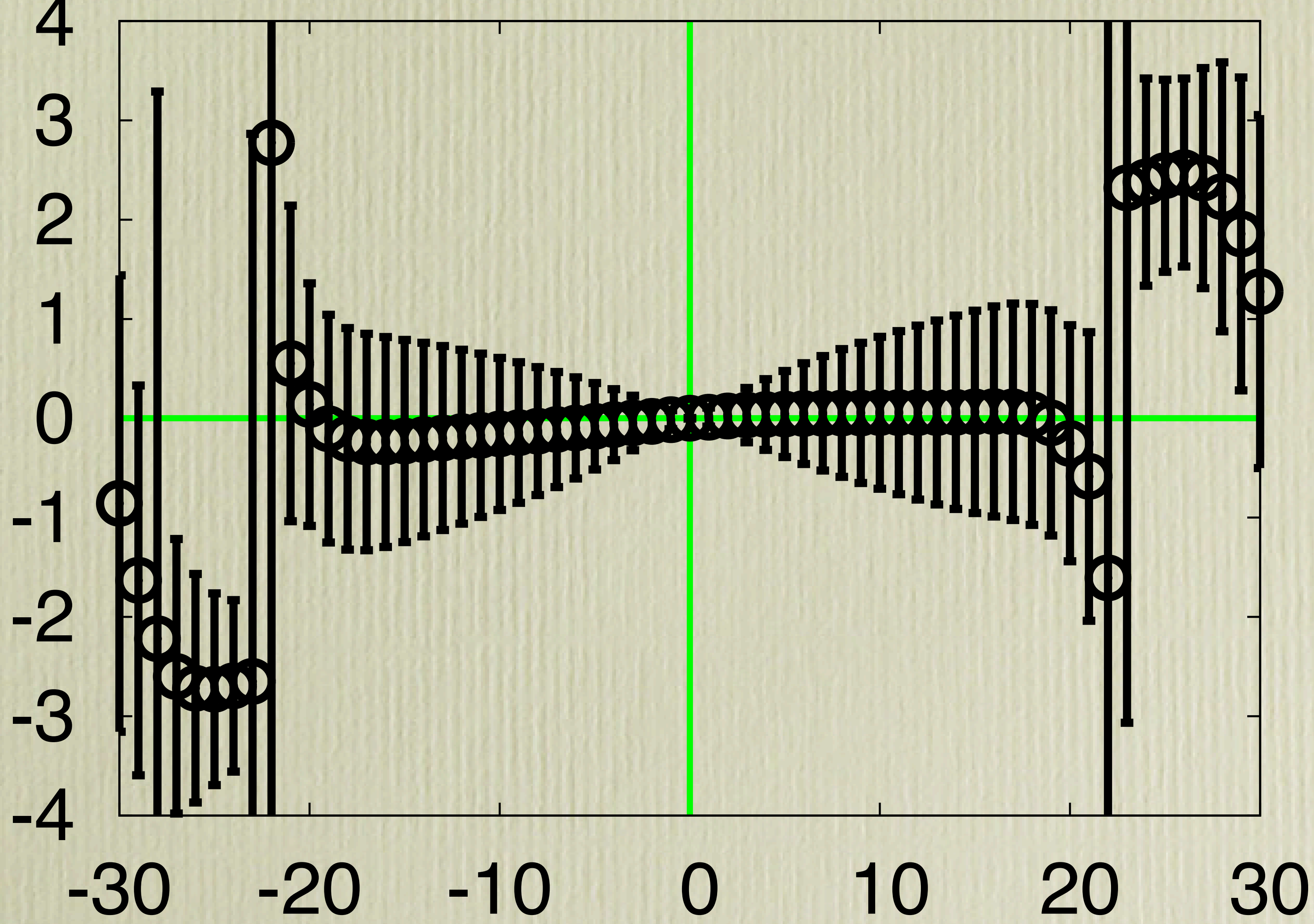
Numerical results Phase(Zc(n))

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.5$

$\mu = 0$

$\theta(\beta,n)$



baryon number

Numerical results Phase(Zc(n))

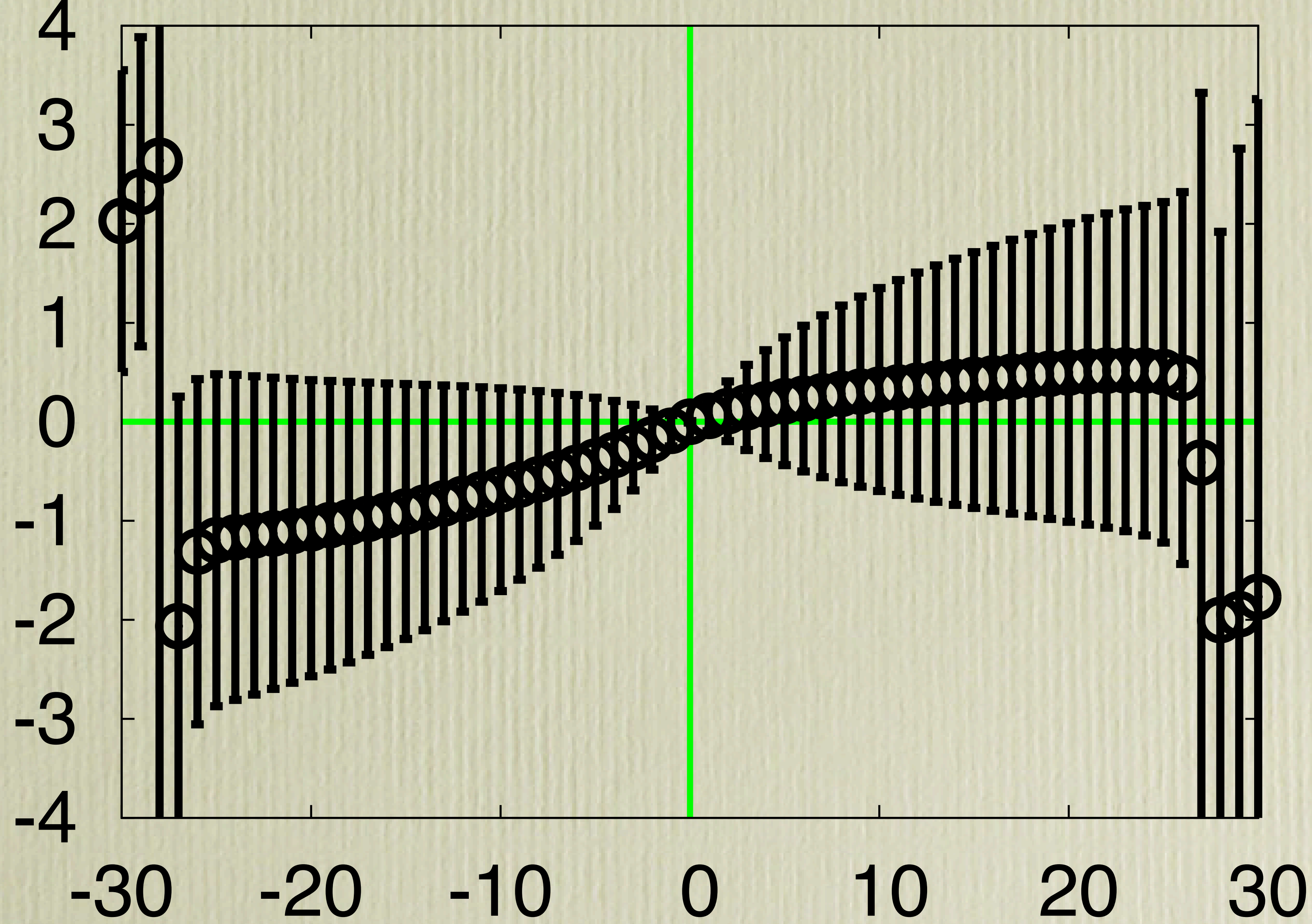
Canonical partition function

$$Z_C\,(T, n, V) = |Z_C(\beta, n)|e^{i\theta(\beta, n)}$$

$\beta = 1.1$

$\mu = 0$

$\theta(\beta, n)$



baryon number

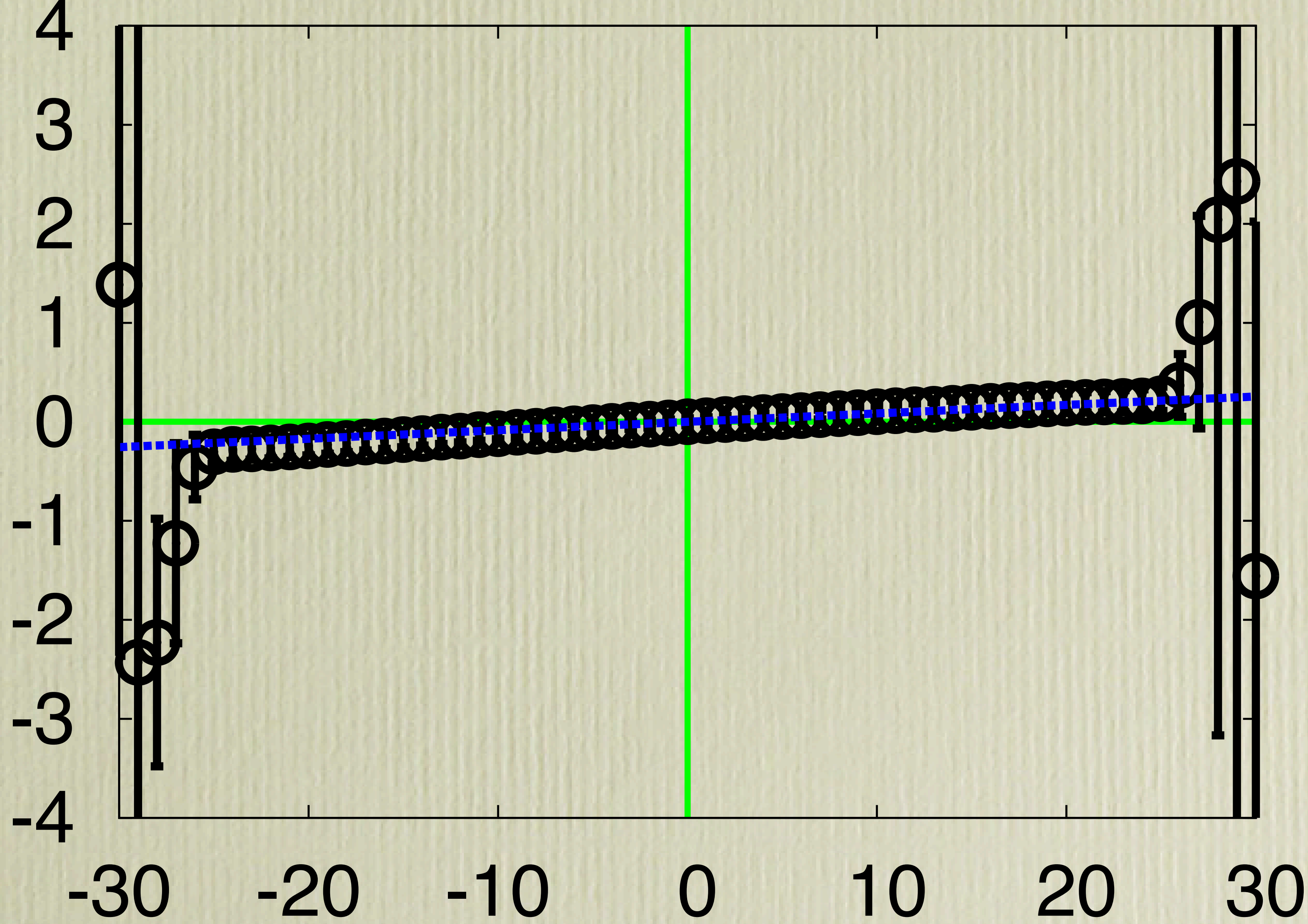
Numerical results Phase(Zc(n))

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.9$

$\mu = 0.125$

$\theta(\beta,n)$



baryon number

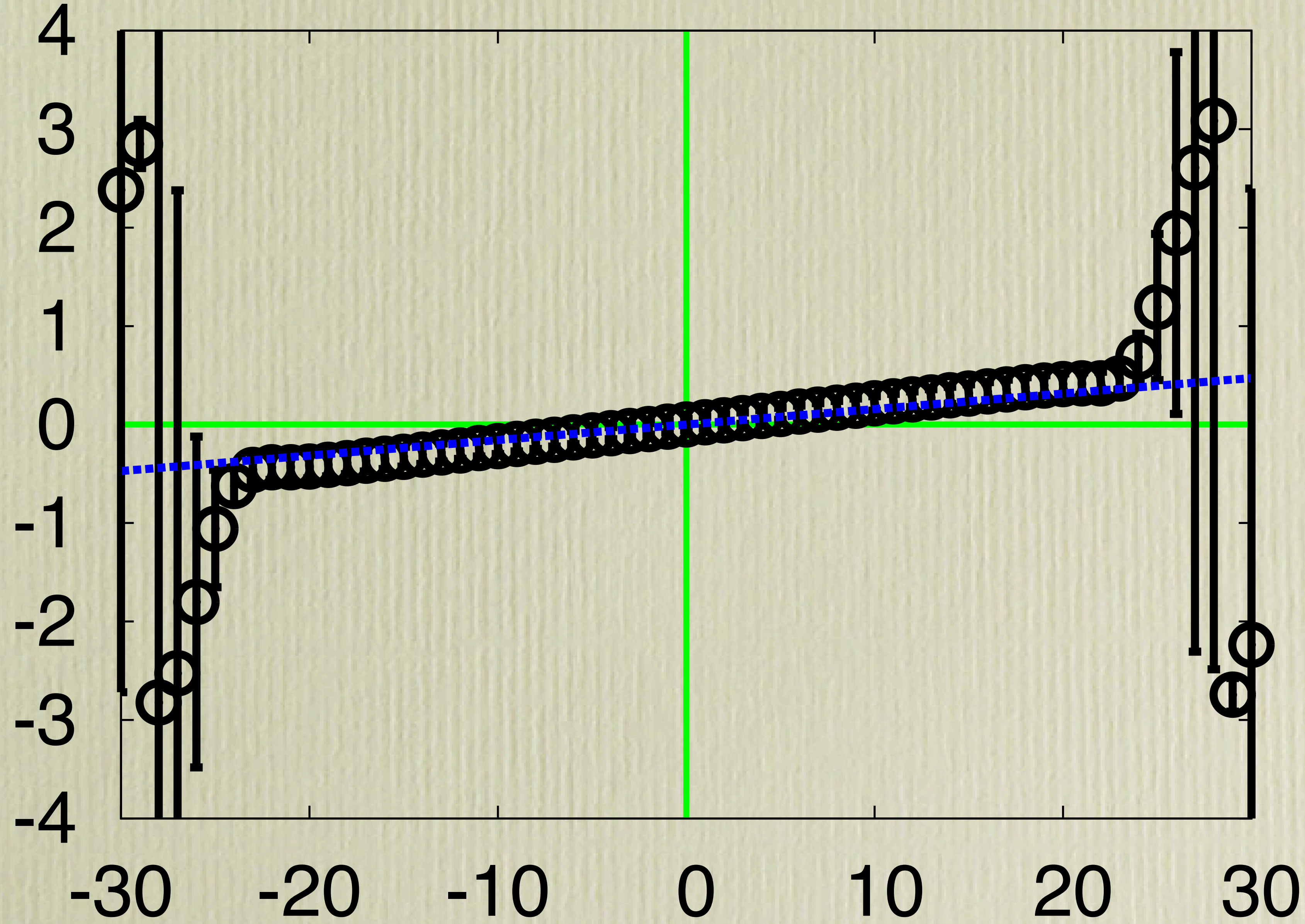
Numerical results Phase(Zc(n))

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.9$

$\mu = 0.25$

$\theta(\beta,n)$



baryon number

Numerical results Phase(Zc(n))

Canonical partition function $Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$

$\beta = 1.7$

$\mu = 0.125$

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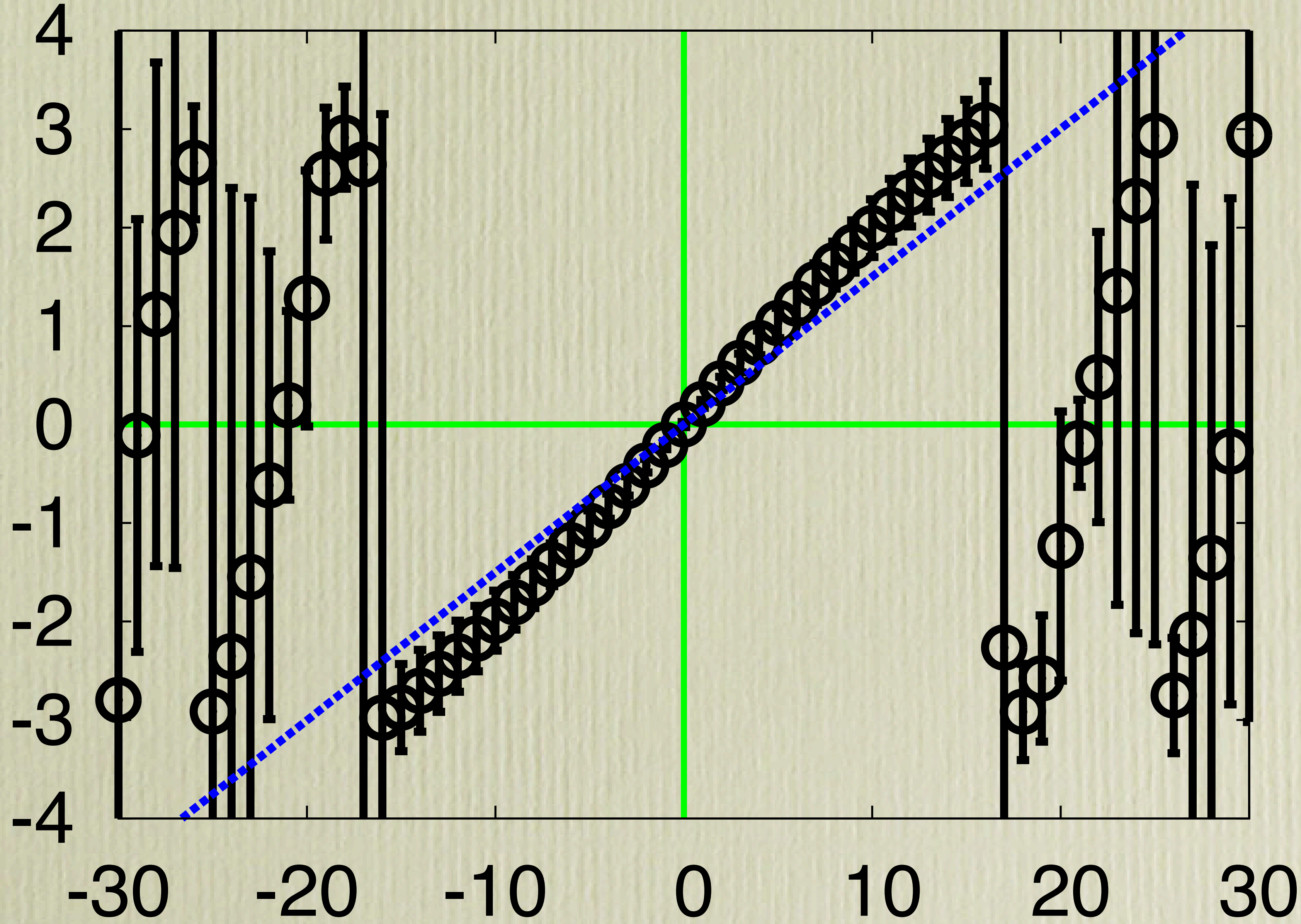


baryon number

Numerical results Phase(Zc(n))

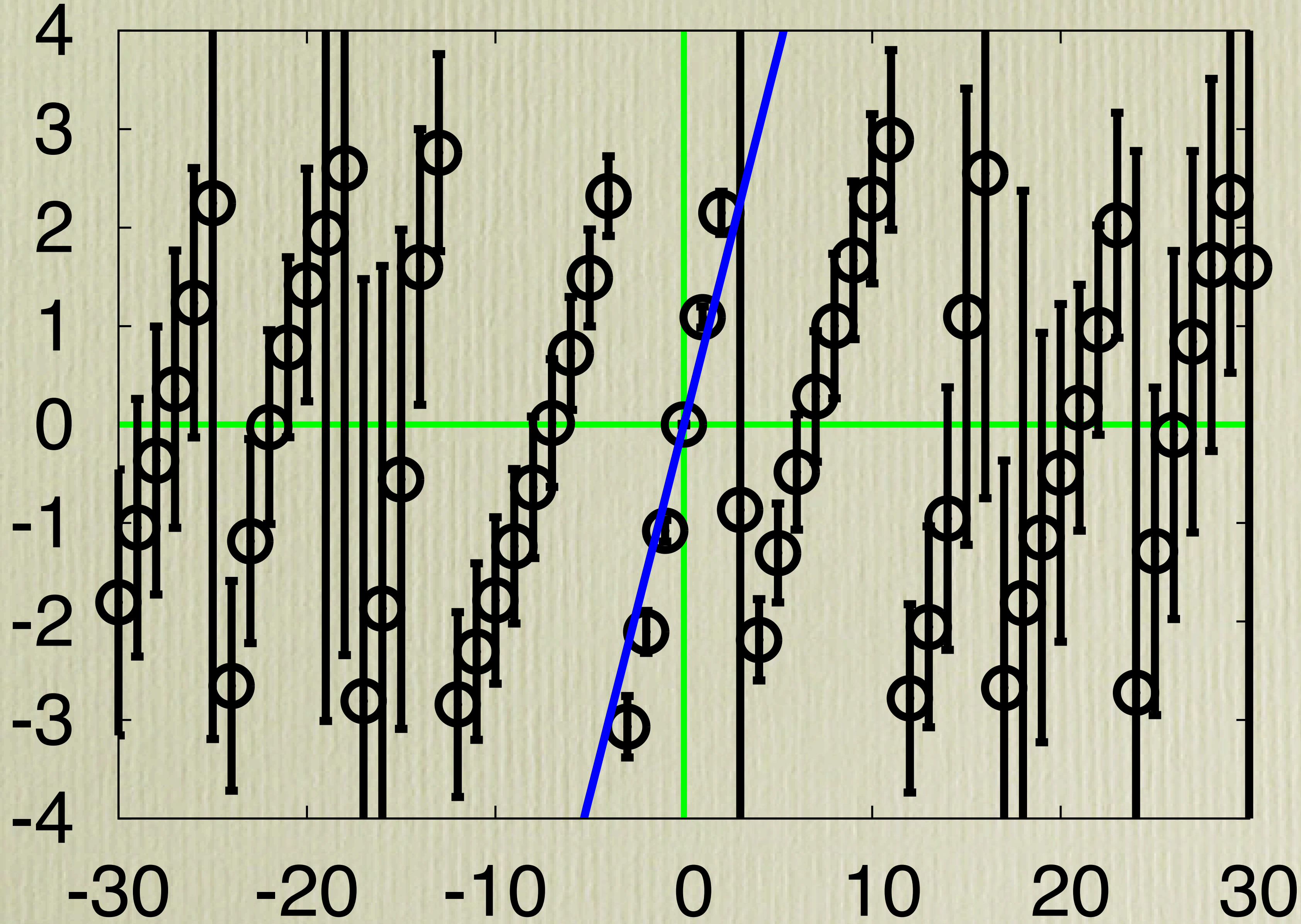
Canonical partition function

$$Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$$

$\beta = 1.5$

$\mu = 0.125$

$\theta(\beta, n)$



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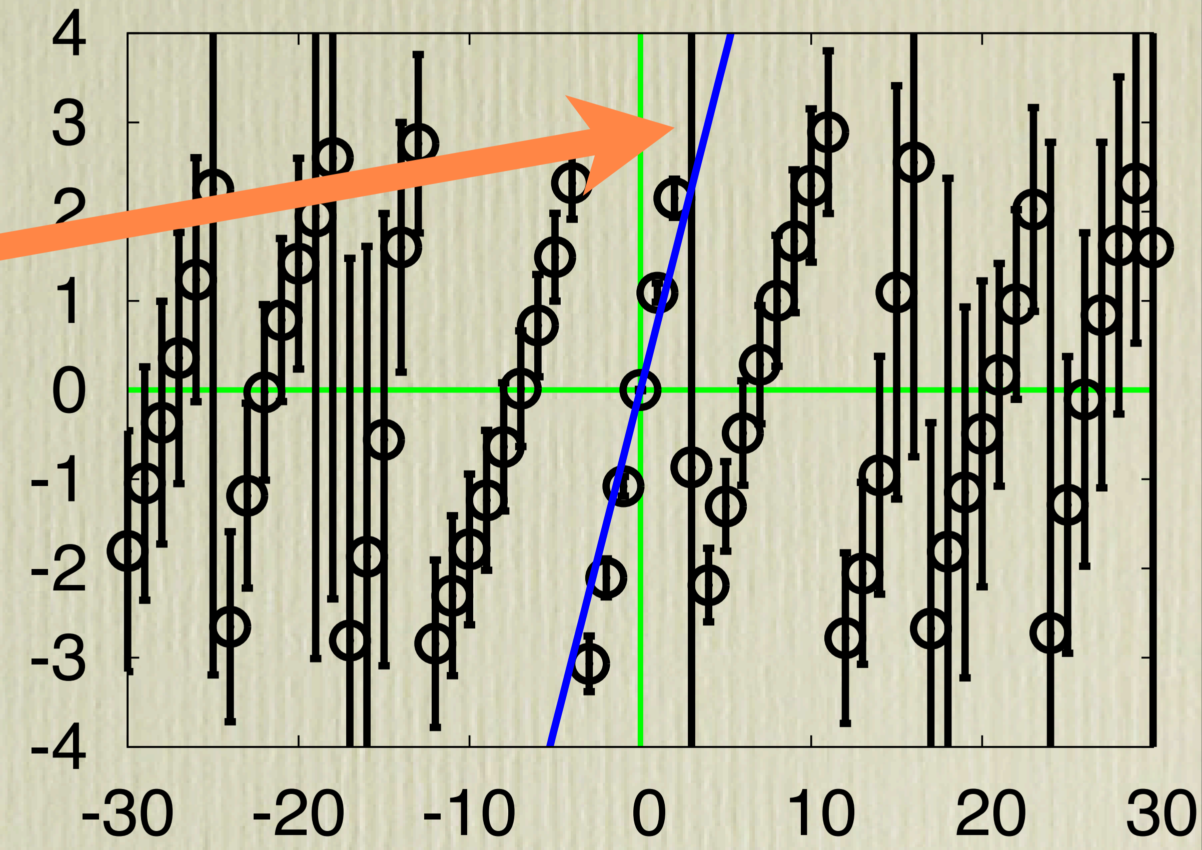
Canonical partition function $Z_C\,(T, n, V) = |Z_C(\beta, n)|e^{i\theta(\beta, n)}$

slope〜 2 ×phase(W_1)



Numerical results Phase(Zc(n))

Canonical partition function

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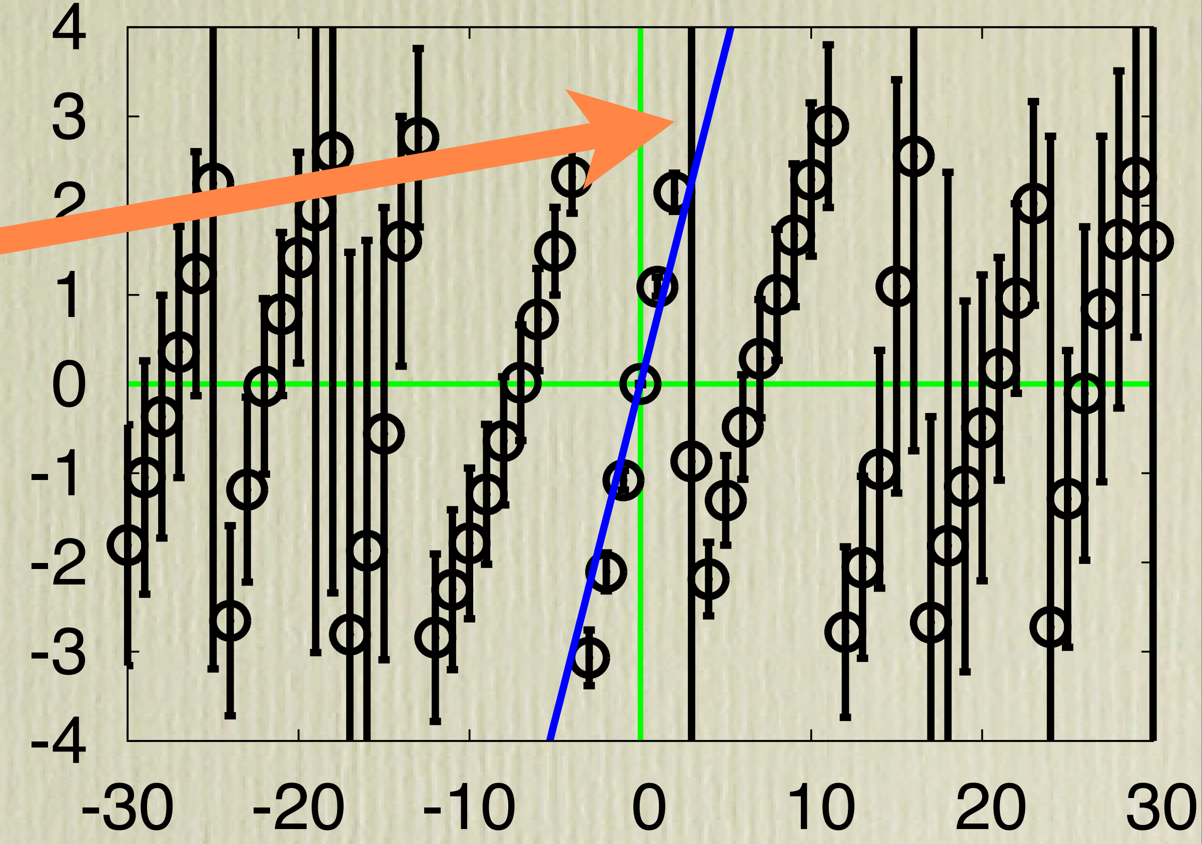

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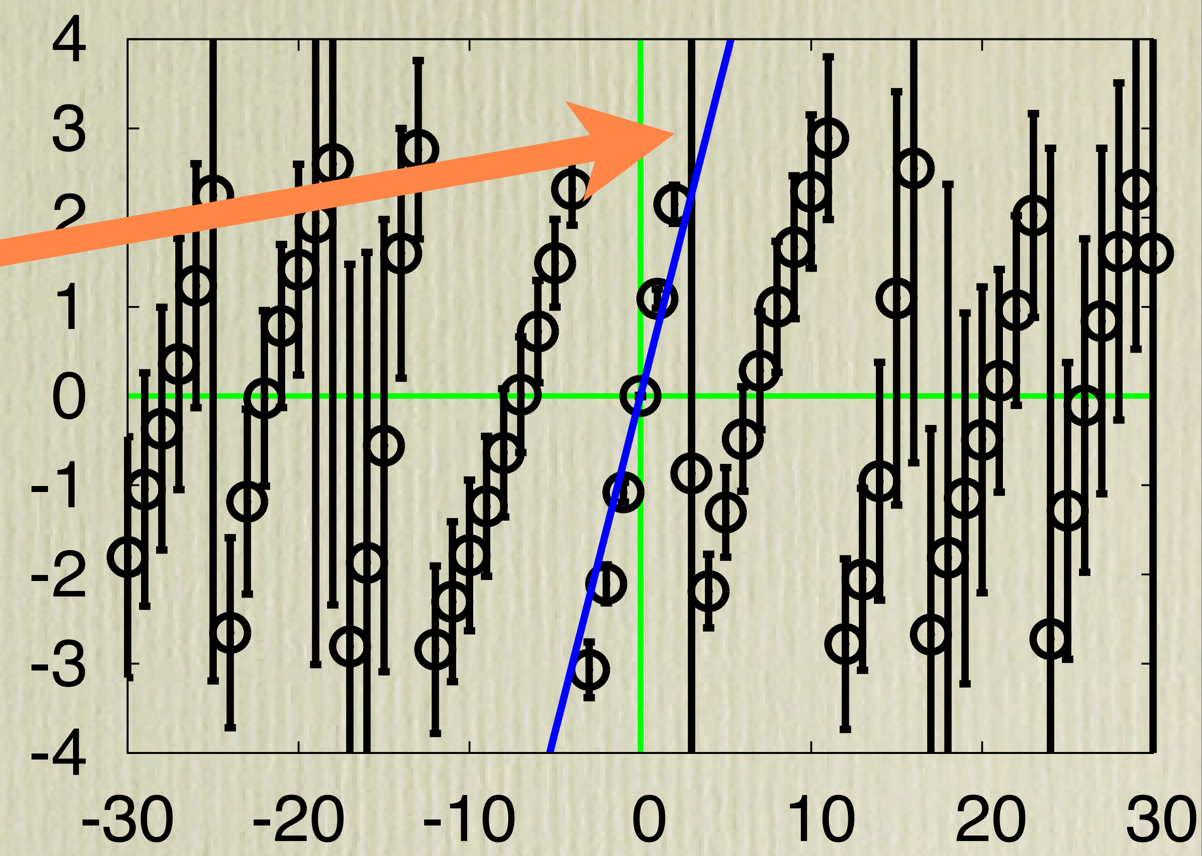

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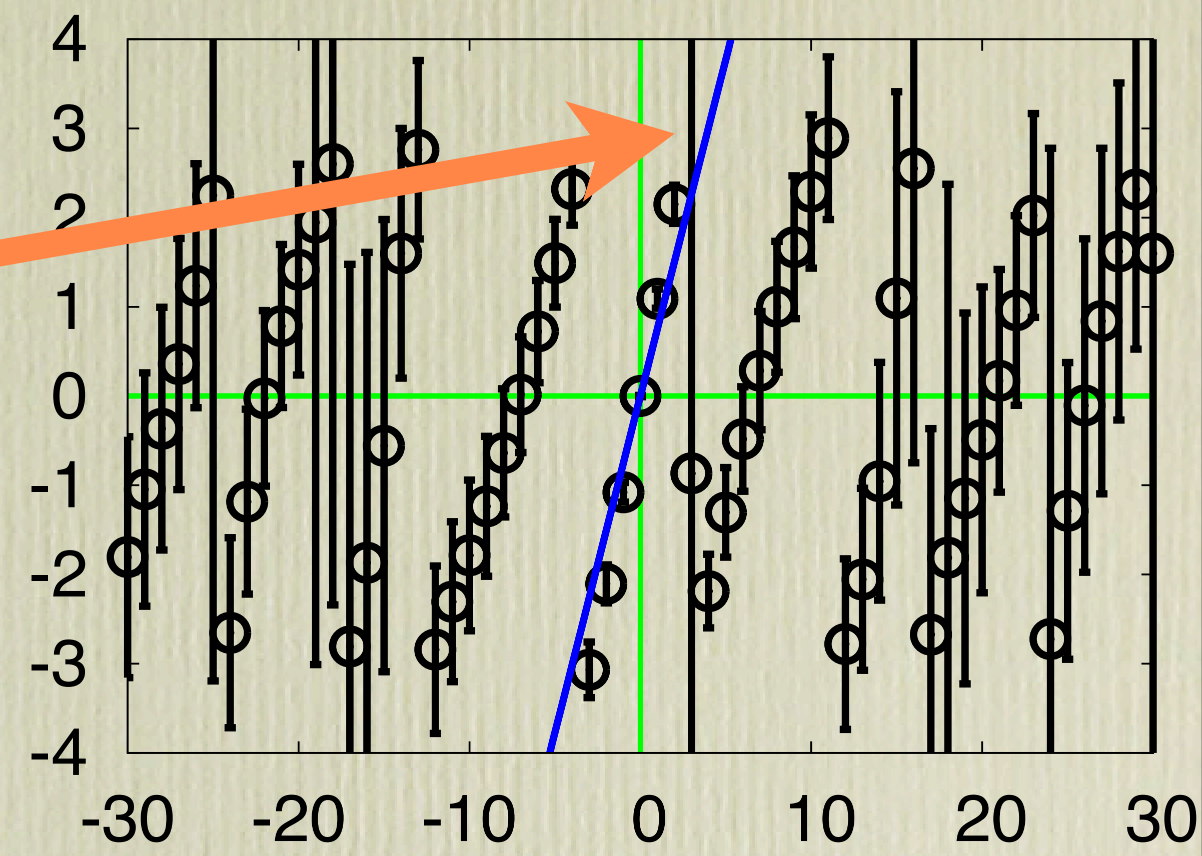

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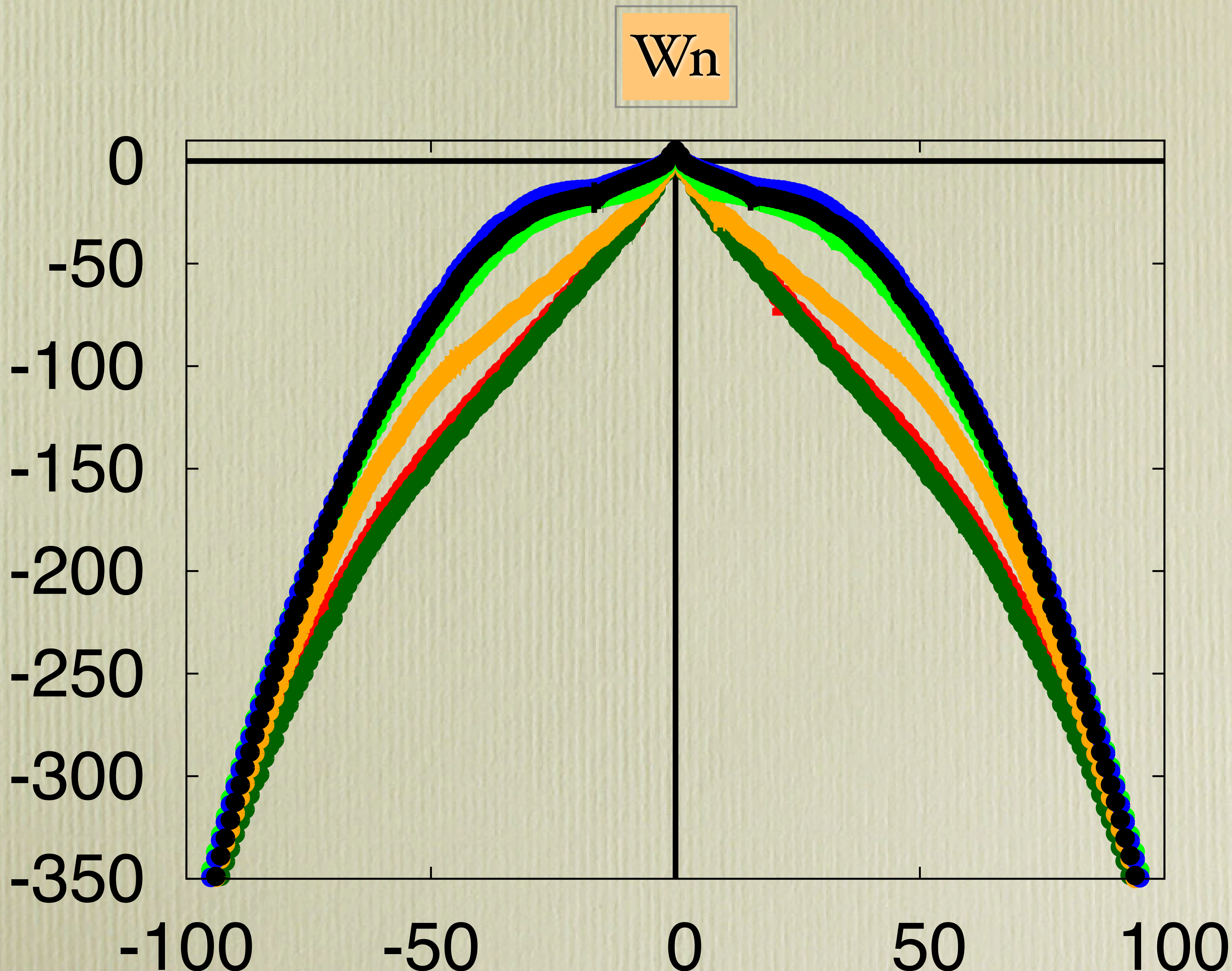
Pion condensate

Ipsen and Splittorff (2012)

Hadronic observables

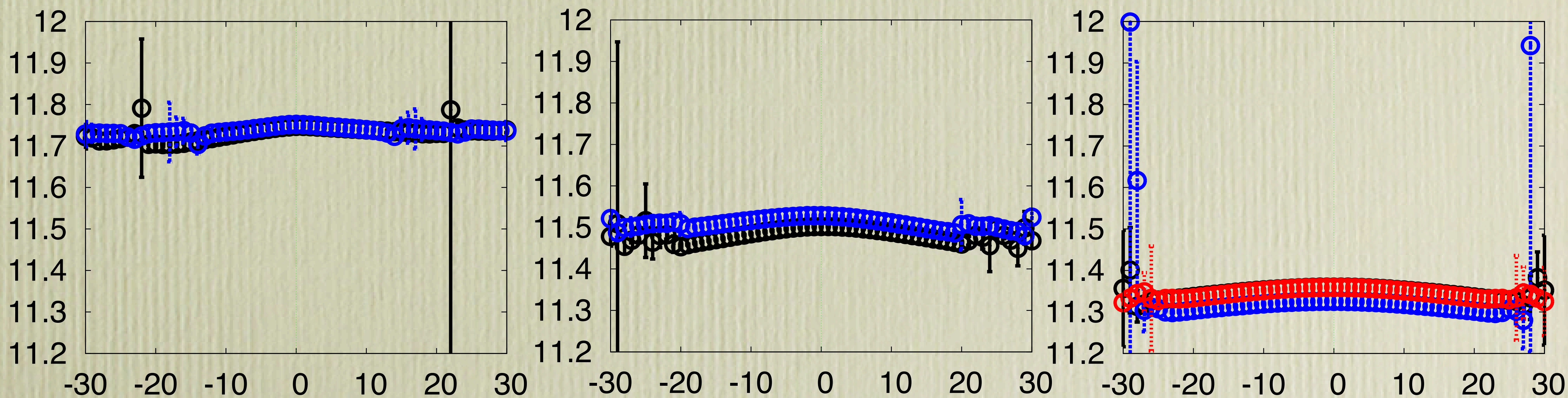


Hadronic observables

Chiral condensate in canonical ensemble

$$\frac{\sum \langle \bar{\psi}\psi \rangle_C(\beta, n)}{V}$$

μ dependence



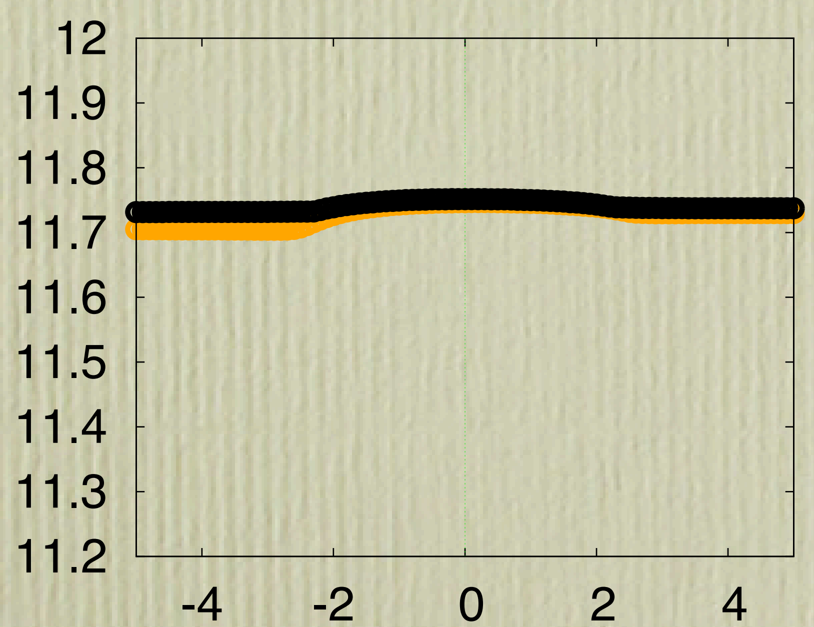
$\beta = 1.5$ $\beta = 1.7$ $\beta = 1.9$

Hadronic observables

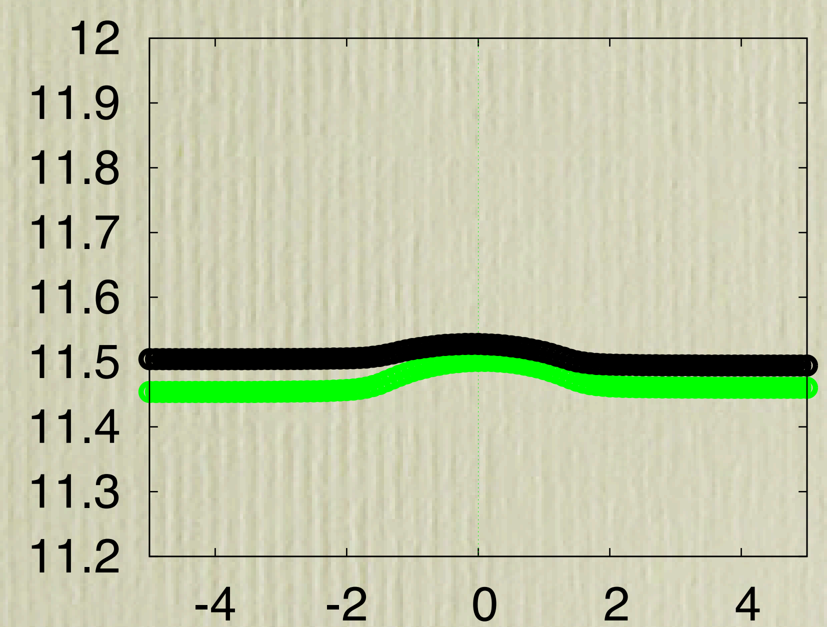
Chiral condensate in canonical ensemble

$$\frac{\sum \langle \bar{\psi}\psi \rangle_G(\beta,\mu)}{V}$$

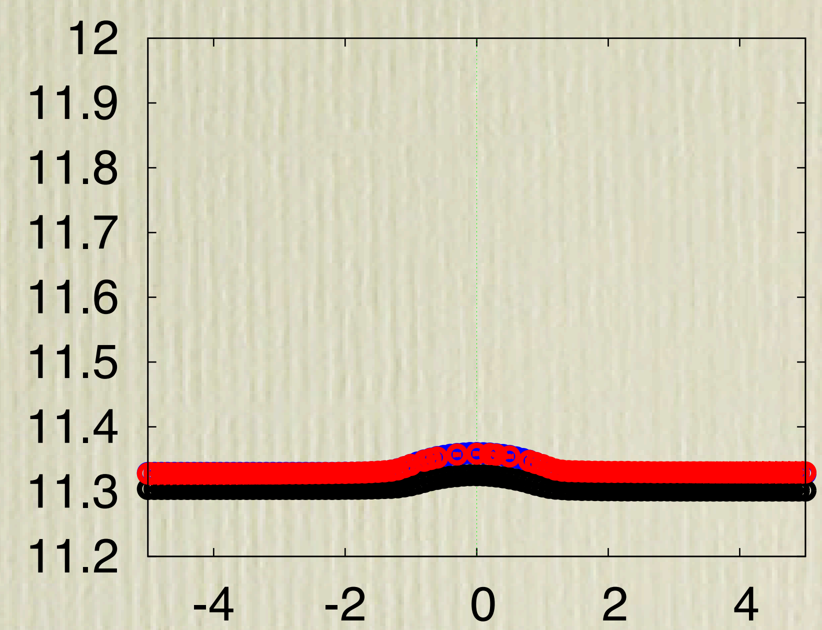
μ dependence



$\beta = 1.5$



$\beta = 1.7$



$\beta = 1.9$