メゾスコピック系における電流ゆらぎの普遍性

慶應大学　物理学科　齊藤圭司

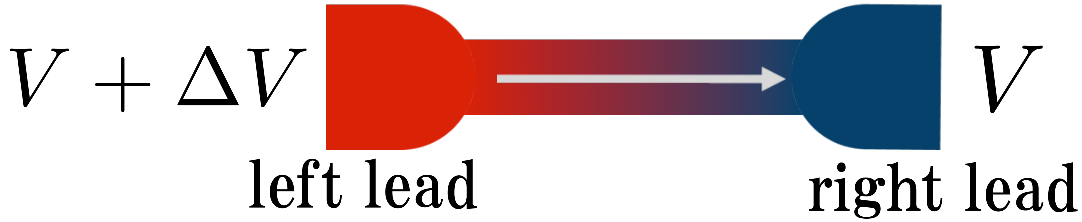
I Brief survey on recent progress in statistical physics

Equilibrium, near equilibrium, far-from-equilibrium

Jarzynski eq

Fluctuation theorem

Equilibrium, near equilibrium, far from equilibrium



ΔV

large — far from equilibrium

- Jarzynski equality
- Fluctuation theorem (FT)
- Macroscopic FT etc.

small — near equilibrium

- Linear response theory
- Onsager-Machlup Theory
- Onsager relation
- Fluctuation dissipation theory

Brownian motion

0 — equilibrium

- Statistical mechanics
- Themodynamics Second law

Jarzynski equality

$$\langle e^{-\beta \Delta W} \rangle = e^{-\beta \Delta F}$$

Jarzynski PRL (1996)

$$\langle e^{-\beta \Delta W} \rangle = \int dx(0)dp(0) e^{-\beta[H_\tau(x(\tau),p(\tau)) - H_0(x(0),p(0))]} \frac{e^{-\beta H_0(x(0),p(0))}}{Z_0}$$

$$= \int dx(0)dp(0) \frac{e^{-\beta H_\tau(x(\tau),p(\tau))}}{Z_0} = \int dx(\tau)dp(\tau) \frac{e^{-\beta H_\tau(x(\tau),p(\tau))}}{Z_0} = e^{-\beta \Delta F}$$

Liouville theorem

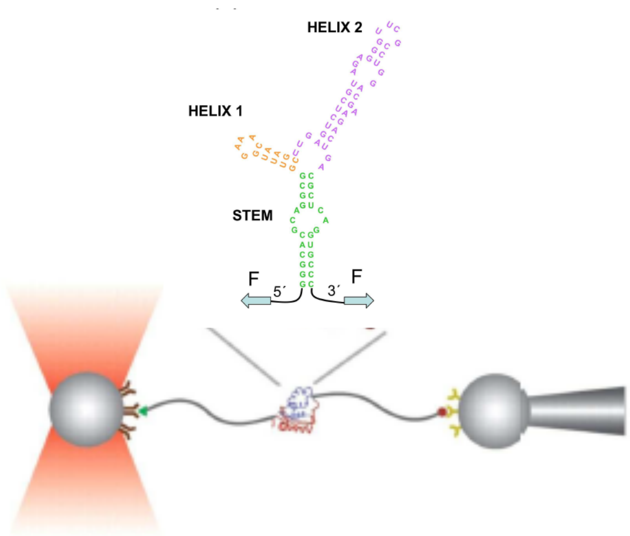
this implies

- ◆ relation between work and equilibrium free energy
- ◆ detailed description of the second law $\Delta W - \Delta F \geq 0$
- ◆ (this can provide experimental protocol to get free energy)

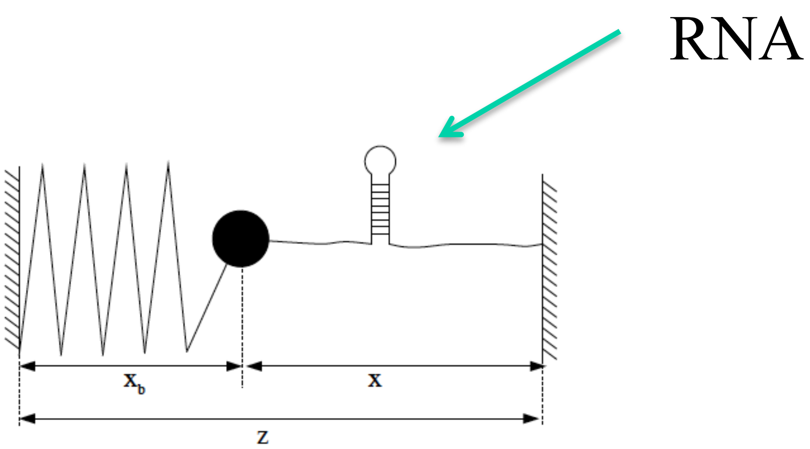
Experimental demonstration of Jarzynski equality

◇ RNA stretching experiment

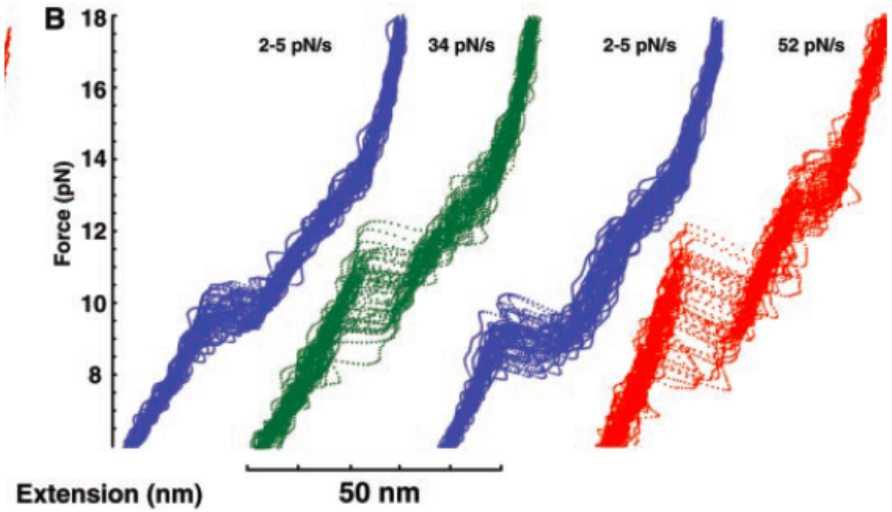
J. Liphardt et al., Science (2002)



◇ Roughly this is equivalent to

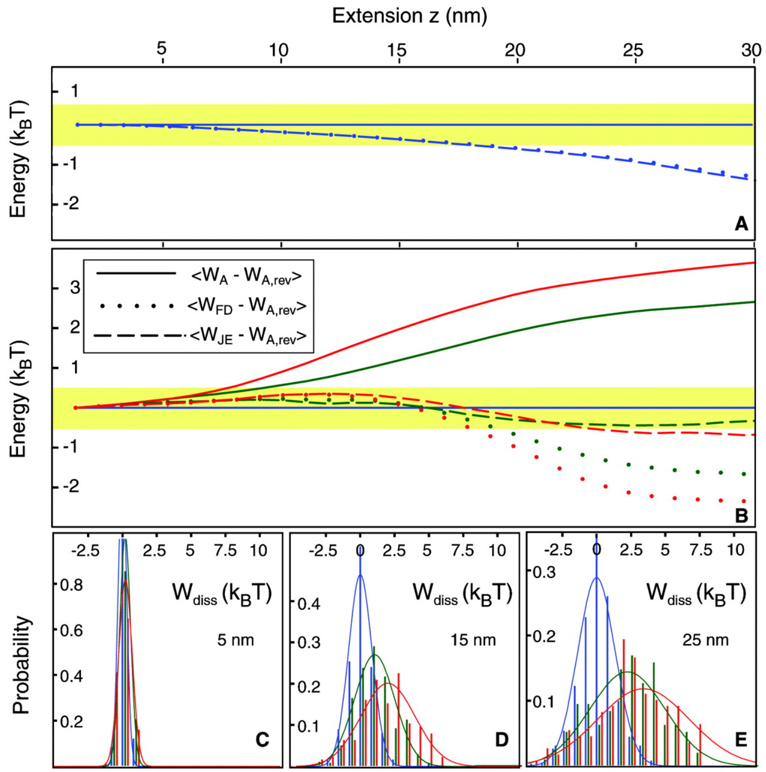


◇ measuring force as a function of stretch



◇ extracting equilibrium free energy from Jarzynski equality

$$\Delta F = -\beta^{-1} \ln \langle e^{-\beta \Delta W} \rangle$$



Reversible

Irreversible green, 34pN/s

Red, 52 pN/s

Fluctuation theorem (FT)

Evans et al. (1993)
Gallavotti and Cohen (1996)
Lebowitz and Spohn (1999)

$$P(\Delta S) = P(-\Delta S)e^{\Delta S}$$

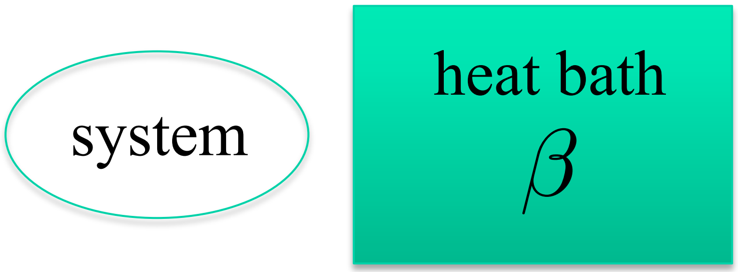
$P(\Delta S)$: distribution of entropy produced during finite time

this implies

- relation between positive and negative entropy production
- detailed description of the second law $\langle \Delta S \rangle \geq 0$
- it reproduces Onsager (-Casimir) relation $L_{ij} = L_{ji}$
- it reproduces linear response theory, fluctuation dissipation theory

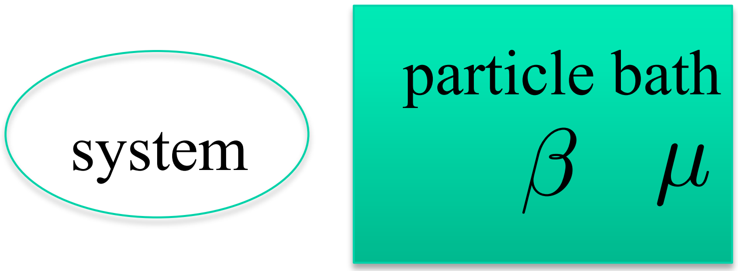
Entropy difference ?

◇ system + bath without current flow



$$\Delta S = -\ln \rho_{\mathrm{final}} + \ln \rho_{\mathrm{initial}} - \beta \Delta Q_{\mathrm{heat}}$$

◇ system + bath with current flow

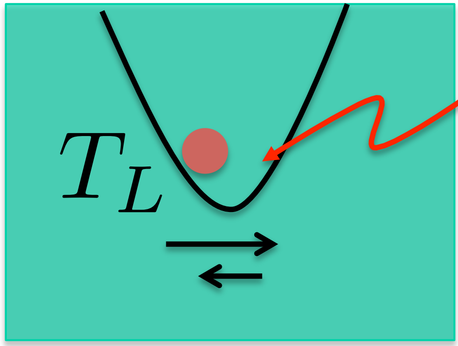


$$\Delta S = -\ln \rho_{\mathrm{final}} + \ln \rho_{\mathrm{initial}} - \beta(\Delta Q_{\mathrm{energy}} - \mu Q_{\mathrm{particle}})$$

Example of experiment on FT in classical heat transport

◇ experimental setup

Gomez-Solano, etal., EPL (2010)

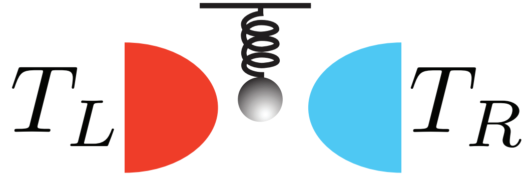


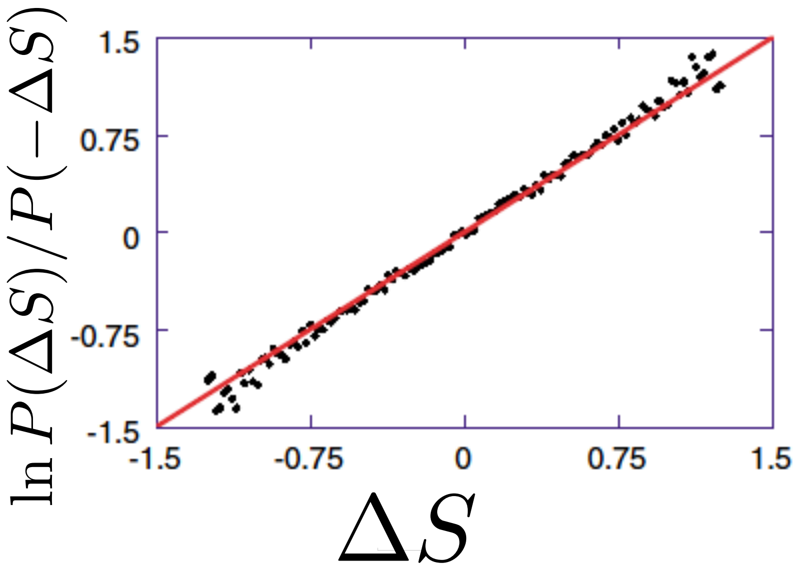
$f(t)$

External field satisfying

$\langle f(t)f(0)\rangle = 2\gamma k_B T_R \delta(t)$

$=$





$$\Delta S = \left(\frac{1}{T_L} - \frac{1}{T_R}\right) Q_{\text{heat}}$$

Steady state entropy production for electric conduction

◇ entropy produced：Jule heating $\Delta S = \beta \Delta V Q$

$$Q = \int_0^{\tau} dt I(t)$$

$I(t)$ electric current

◇ this expression and FT indicate that distribution of Q has universal relation

$$P(-Q) = e^{-\beta \Delta V Q} P(Q)$$

II Fluctuation in mesoscopic electron conduction

current noise in mesoscopic transport

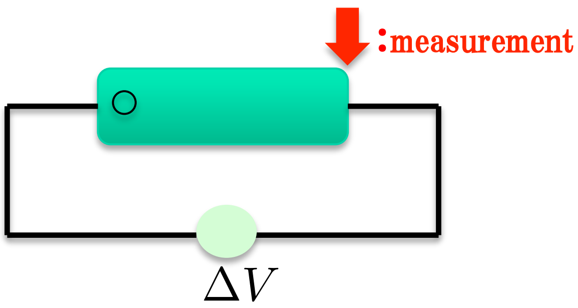
counting statistics

universal relation between transport coefficients

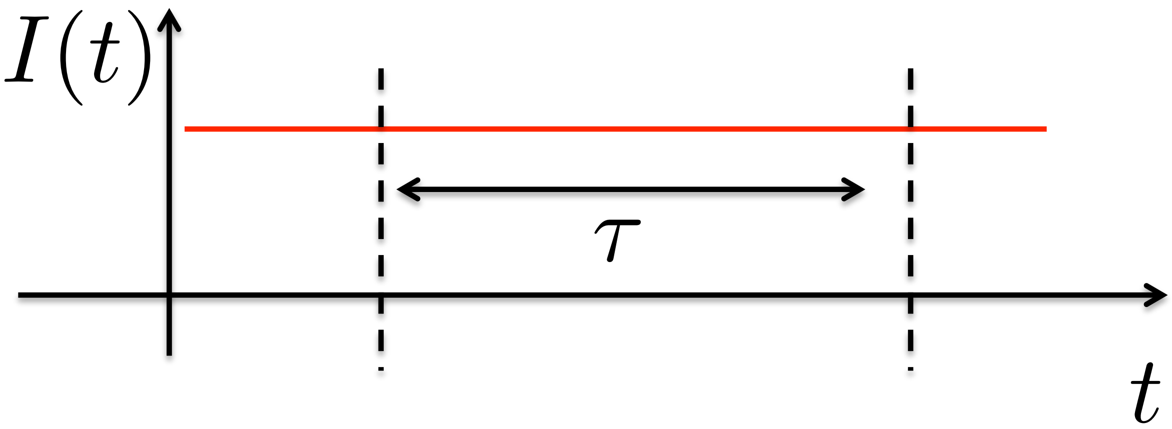
experiments

Current noise in mesoscopic transport

if the conductor is macroscopic、

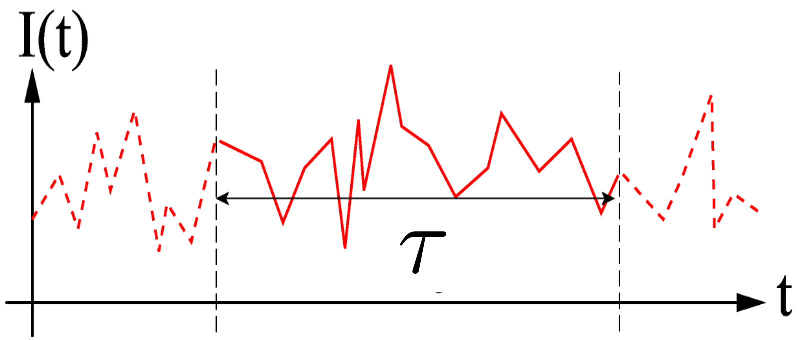


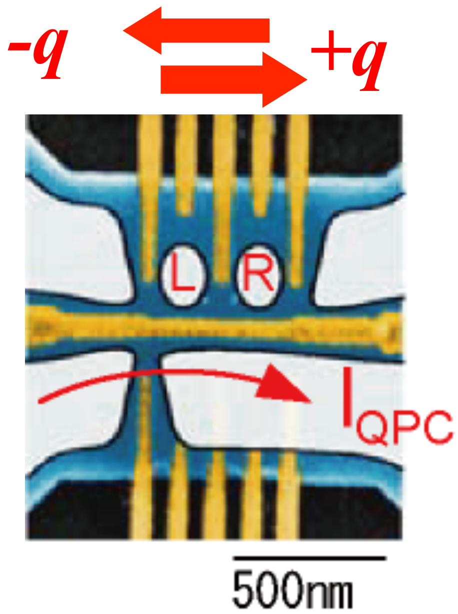
fluctuation is very small compared to average current



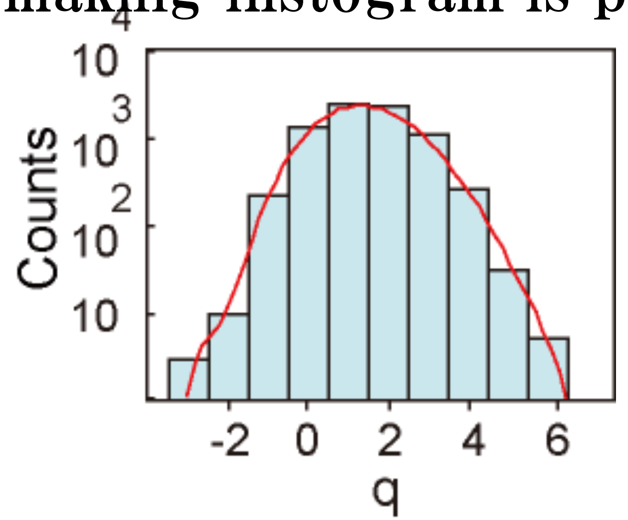
Small scale (e.g., mesoscopic scale) of conductor gives

◇large fluctuation in current



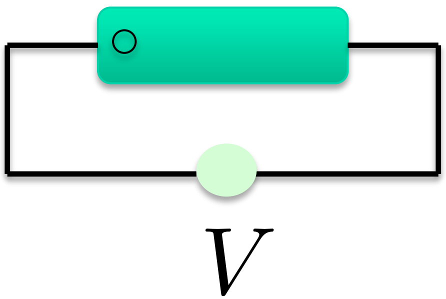


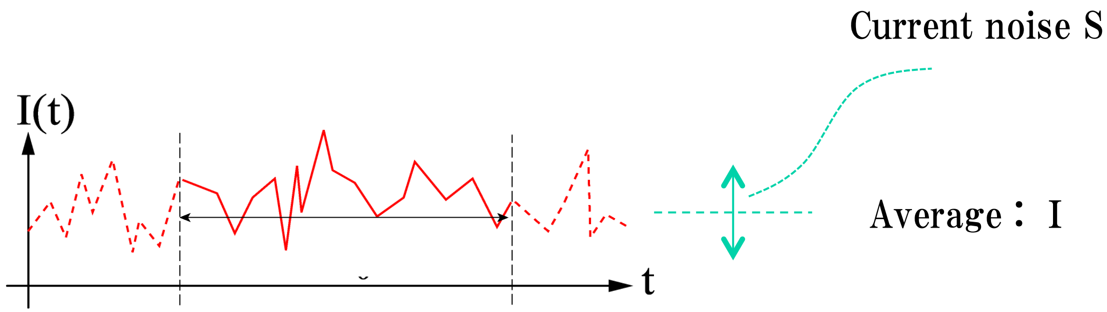
◇making histogram is possible



Elecric transport has very old history on current fluctuation

i) Zero temperature (Schottky (1918))





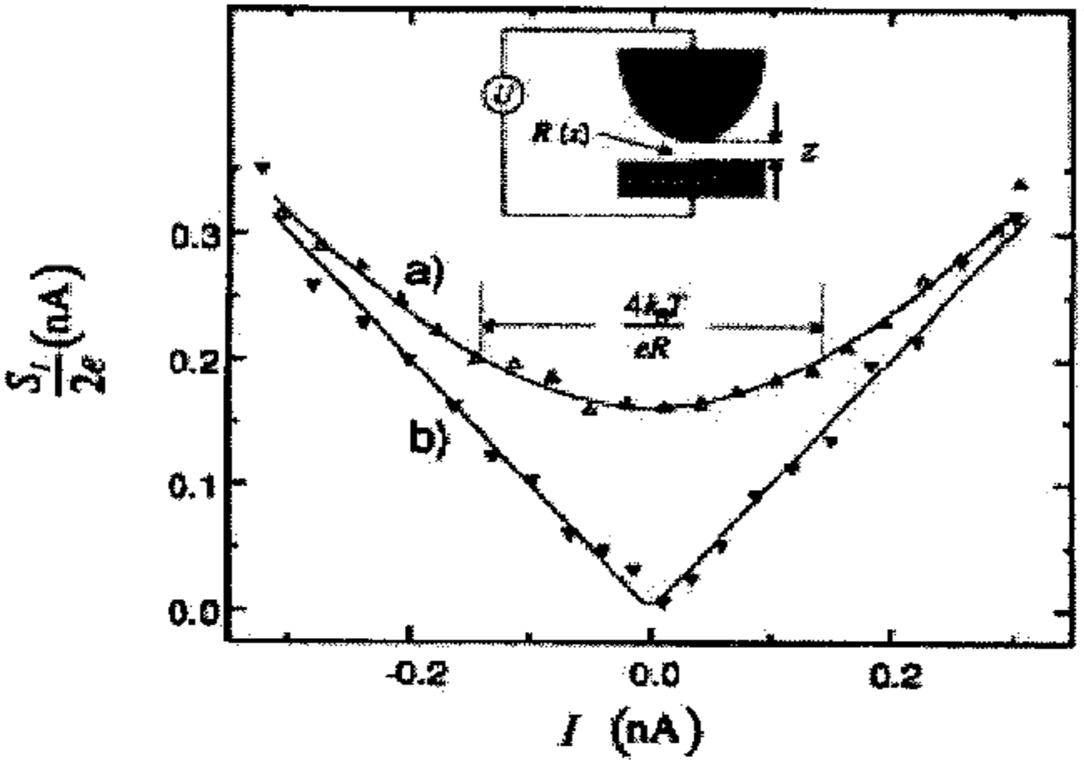
◇ Noise tells us important information: elementary charge

$$\left.\begin{aligned} \text{average}\quad I &= \frac{e^2|V|}{2\pi\hbar}T \\ \text{noise}\quad S &= \frac{e^3|V|}{\pi\hbar}T(1-T) \end{aligned}\right\} \xrightarrow{T \ll 1} F = \frac{S}{2I} = \mathrm{e}$$

"e" is obtained

‘The noise is the signal (Landauer)’

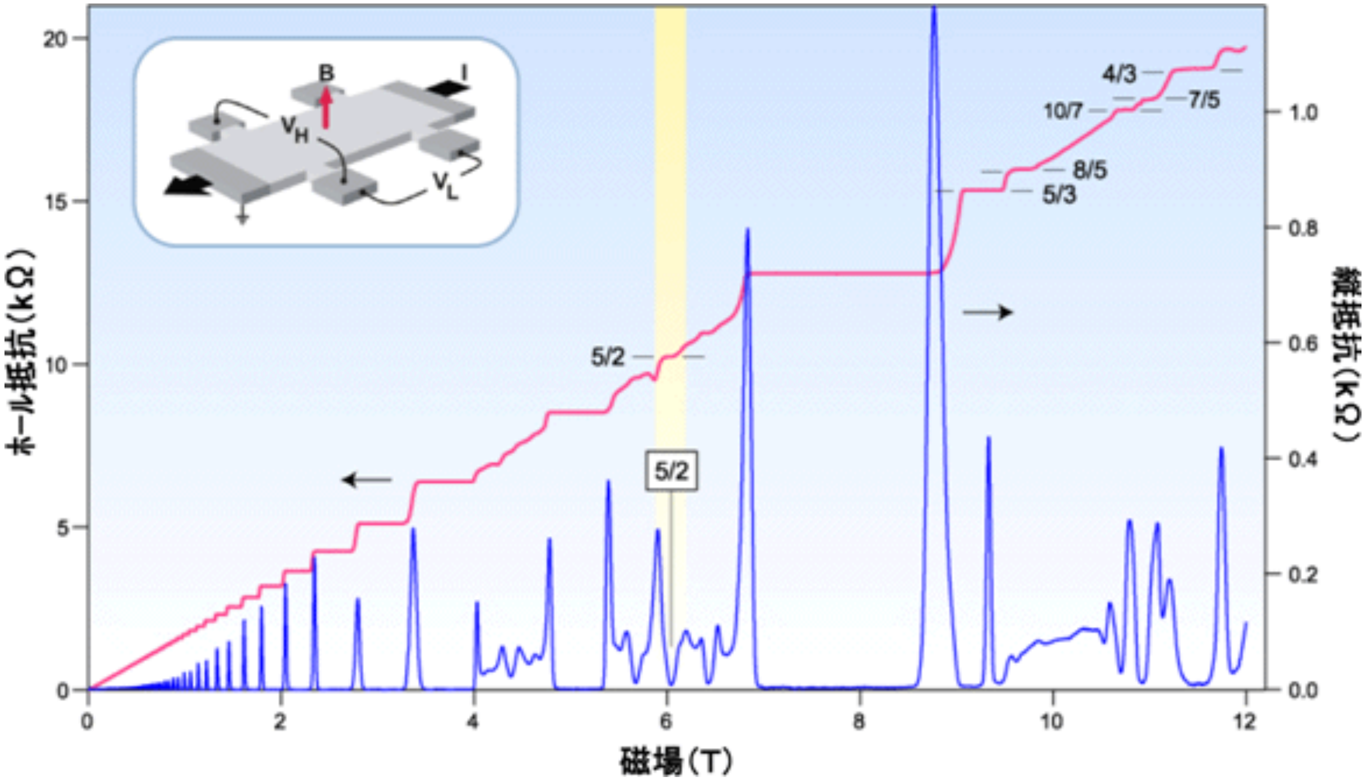
$$F = \frac{S}{2I} = \mathrm{e}$$



(Review, Blanter and Buttiker(1999))

Fractional quantum hole efect has 1/3 elementary charge and it was experimentally proved by noise measurement.
This leads to Nobel prize！

$$F = \frac{S}{2I} = \frac{\mathrm{e}}{3}$$



ii) finite temperatures

◇ expansion of average current and voltage w.r.t. voltage

$$I = G_1 \Delta V + G_2 \frac{\Delta V^2}{2!} + \cdots$$

$$S = S_0 + S_1 \Delta V + S_2 \frac{\Delta V^2}{2!} + \cdots$$

◇ relashionship

$$I = G_1 \Delta V + G_2 \frac{\Delta V^2}{2!} + \cdots$$

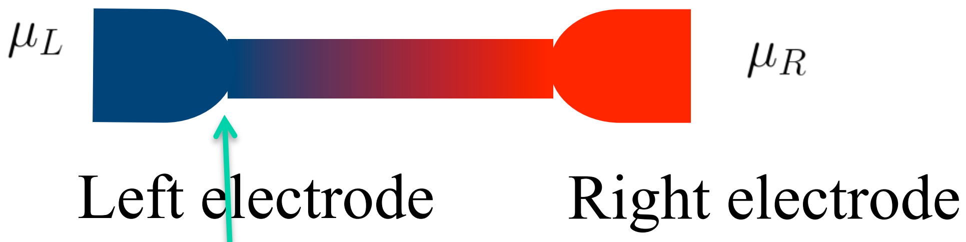
$$S = S_0 + S_1 \Delta V + S_2 \frac{\Delta V^2}{2!} + \cdots$$

Johnson-Nyquist relation (FDT)

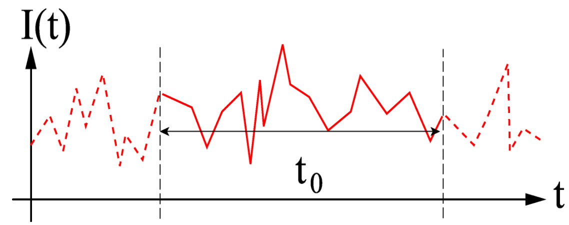
$$S_0 = 4 k_B T G_1$$

◇Only this ??

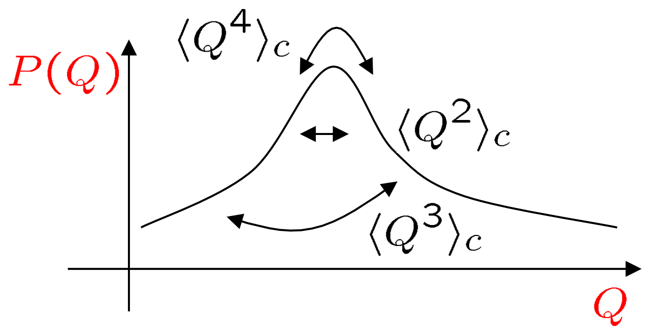
Counting statistics



$\Delta V = (\mu_R - \mu_L)/e$



◇ **measure** $Q = \int_0^\tau dt I(t)$



◇ **then,** $P(Q)$ **?**

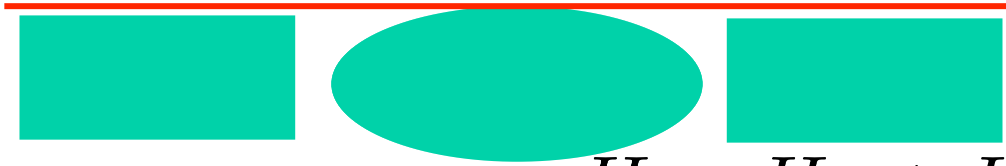
◇ equivalently, cumulant generating function ?

$$F(\chi) := \ln\Big[\int dQ P(Q) e^{i\chi Q}\Big]$$

$$\langle Q^n \rangle_c = \frac{\partial^n F(\chi)}{\partial (i\chi)^n}\Big|_{\chi=0}$$

$$P(Q) = \frac{1}{2\pi}\int d\chi e^{-i\chi Q + F}$$

(Standard) Theoretical protocol to get distribution



$$H = H_S + H_L + H_R + H_{\rm int}$$

iteration

i) switch off int. & projective measurement onto the number state

$$\hat{P}_i = |\varphi_i\rangle\langle\varphi_i| \qquad \varphi_i = \varphi_{L\,i} \otimes \varphi_{S\,i} \otimes \varphi_{Ri}$$

suppose that number of electrons in the left lead n_i

ii) switch on int. & free time evolution during τ

iii) switch off int. & projective measurement onto the number state

$$\hat{P}_j = |\varphi_j\rangle\langle\varphi_j| \qquad \varphi_j = \varphi_{L\,j} \otimes \varphi_{S\,j} \otimes \varphi_{Rj}$$

suppose that number of electrons in the left lead n_j

$(n_i - n_j)$ electrons are transmitted

$$P_{j\leftarrow i} = \delta(Q - (n_i - n_j))|\langle\varphi_j|U_\tau|\varphi_i\rangle|^2$$

◇ **take decoupled initial condition**

$$\rho_0 = e^{-\beta(H_L - \mu_L N_L)} \otimes e^{-\beta(H_R - \mu_R N_R)} \otimes \rho_S$$

◇ **P(Q) is expressed by the operator form**

$$P_{0,i} = \langle \varphi_i | \rho_0 | \varphi_i \rangle$$

$$P(Q) = \sum_{i,j} P_{j \leftarrow i} P_{0,i}$$

$$= \sum_{i,j} \delta(Q - (n_i - n_j)) |\langle \varphi_j | U_\tau | \varphi_i \rangle|^2 \langle \varphi_i | \rho_0 | \varphi_i \rangle$$

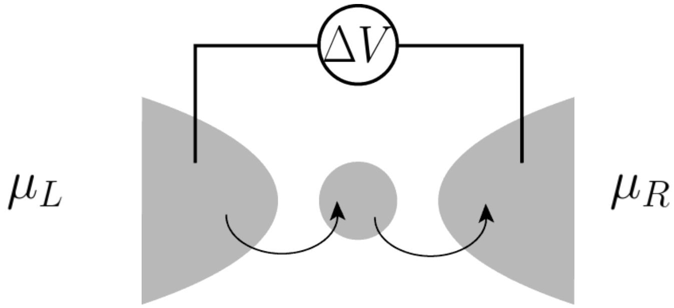
$$= \frac{1}{2\pi} \int d\chi e^{-i\chi Q} \mathrm{Tr} \left[U_\tau e^{i\chi N_L} \rho_0 U_\tau^\dagger e^{-i\chi N_L} \right]$$

◇ **cumulant generating function**

$$F(\chi) := \frac{1}{\tau} \log \mathrm{Tr} \left[U_\tau e^{i\chi N_L} \rho_0 U_\tau^\dagger e^{-i\chi N_L} \right]$$

◇ remark1 : this formulation uses two-times projective measurement

Specific case: non-interacting spinless electron



$$F(\chi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \ln\left[1 + \mathcal{T}(\omega)\left\{(e^{i\chi}-1)f_L^-(\omega)f_R^+(\omega) + (e^{-i\chi}-1)f_L^+(\omega)f_R^-(\omega)\right\}\right]$$

$\mathcal{T}(\omega)$:Transmission coefficient of electron with frequency ω

$$f_\alpha^+(\omega) = \frac{1}{e^{\beta(\omega-\mu_\alpha)}+1}$$

:Fermi distribution of Lead α=L or R

$$f_\alpha^-(\omega) = 1 - f_\alpha^+(\omega)$$

◇ this reproduces an average current and current noise

the first derivative reproduces average current expression

$$\frac{\partial F}{\partial(i\chi)}|_{\chi=0} = \langle I \rangle = \int_{-\infty}^{\infty} d\omega \mathcal{T}(\omega)[f_R(\omega) - f_L(\omega)]$$

cf. Landauer formula

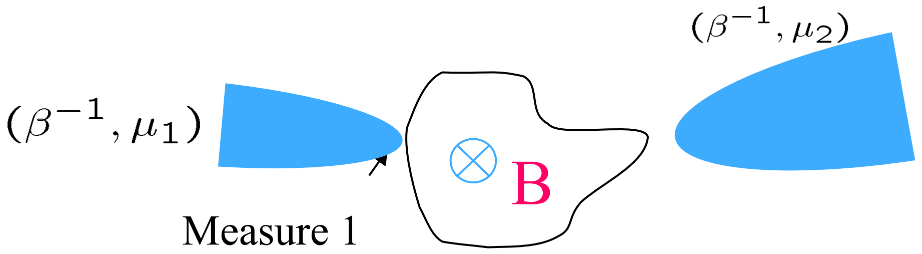
◇ symmetry and fluctuation theorem

$$F(\chi) = F(-\chi + i\mathcal{A}) \qquad \mathcal{A} = \beta(\mu_L - \mu_R)$$

$$P(Q) = \frac{1}{2\pi}\int d\chi e^{-iQ\chi + \mathcal{F}(\chi)} \Longrightarrow P(-Q) = P(Q)e^{-\mathcal{A}Q}$$

Symmetry for generating function with magnetic field

◇ Coulomb Interaction
◇ Magnetic Fields



$$F(\chi;B) = F(-\chi + i\mathcal{A}; -B)$$

$\mathcal{A} = \beta(\mu_1 - \mu_2)$: thermodynamic force (Affinity)

Quantum version of fluctuation theorem with magnetic fields

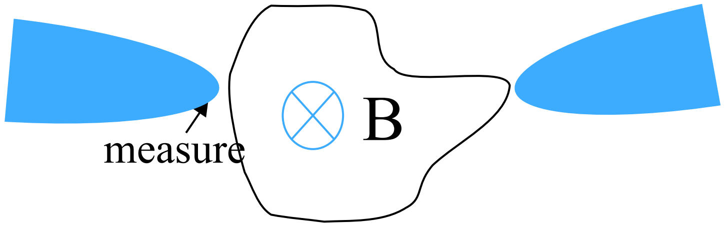
Schwinger Keldysh Calculation + Time-reversal arguments

KS, Abhishek Dhar, PRL Vol.99, 180601 (2007)
KS, Yasuhiro Utsumi, PRB vol. 78, 115429(2008)

Universal relation between transport coefficients

β^{-1}

$\mu_1 = \mu + \Delta\mu$



$\mu_2 = \mu$

In general, charge transfer is a nonlinear function of $\mathcal{A} = \beta\Delta\mu$

$$\langle Q^n(B)\rangle_c = L_0^n(B) + L_1^n(B)\mathcal{A} + \frac{L_2^n(B)}{2!}\mathcal{A}^2 + \frac{L_3^n(B)}{3!}\mathcal{A}^3 + \cdots$$

Definition of transport coefficient

$$L_k^n(B) := \left.\frac{\partial^k \langle Q^n(B)\rangle_c}{\partial \mathcal{A}^k}\right|_{\mathcal{A}=0}$$

Symmetry reproduces Linear response results

$$L_k^n(B) := \frac{\partial^k \langle Q_{c,1}^n(B)\rangle_c}{\partial \mathcal{A}^k}\Big|_{\mathcal{A}=0}$$

$$\mathcal{A} = \beta \Delta \mu$$

$$F(\chi; B) = F(-\chi + iA; -B)$$

$$\frac{\partial^n}{(i\chi)^n} F(\chi; B) = (-1)^n \frac{\partial^n}{\partial (i\chi)^n} F(\chi; -B)$$

$$\left(\frac{\partial}{\partial A}\right)^\ell F(\chi; B) = \left(\frac{\partial}{\partial A} - \frac{\partial}{\partial (i\chi)}\right)^\ell F(\chi; -B)$$

i) Johnson-Nyquist Relation

$\ell = 2 \longrightarrow L_1^1(B) = \frac{1}{2} L_0^2(B)$

ii) Onsager-Casimir relation

$n = 2 \longrightarrow L_1^1(B) = L_1^1(-B),$

Relationships beyond linear response regime

$$L_k^n(B) := \frac{\partial^k \langle Q_1^n(B)\rangle_c}{\partial \mathcal{A}^k}\Big|_{\mathcal{A}=0} \qquad \mathcal{A} = \beta\Delta\mu$$

Symmetrize the coefficients

$$L_\pm = L(B) \pm L(-B)$$

Universal relations from FT

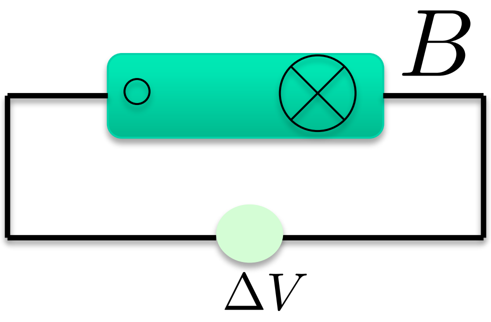
$$L_{2,+}^1 = L_{1,+}^2,$$
$$L_{0,+}^4 = 2L_{1,+}^3$$

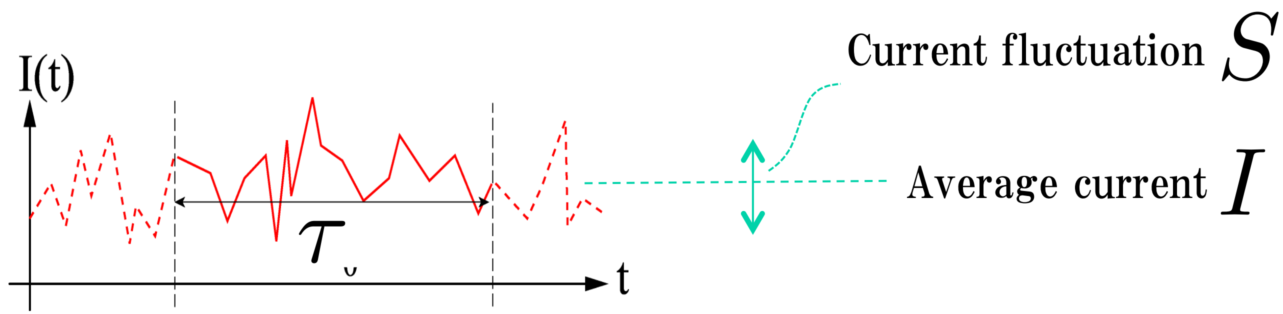
$$L_{2,-}^1 = L_{1,-}^2/3 = L_{0,-}^3/6, \quad L_{0,+}^3 = 0.$$

$$\vdots$$

In more understandable way

KS and A. Dhar, PRL (2007)
KS and Y. Utsumi PRB (2008)
KS and A. Dhar, PRL(2010)





$$I = G_1 \Delta V + G_2 \frac{\Delta V^2}{2!} + \cdots$$

$$S = S_0 + S_1 \Delta V + S_2 \frac{\Delta V^2}{2!} + \cdots$$

Symmetrized conductance

$$G^{S,A} = G(B) \pm G(-B)$$

$$S^{S,A} = S(B) \pm S(-B)$$

$$S_0 = 4 k_B T G_1$$

$$S_1^S = 2 k_B T G_2^S \qquad S_1^A = 6 k_B T G_2^A$$

Experimental approach

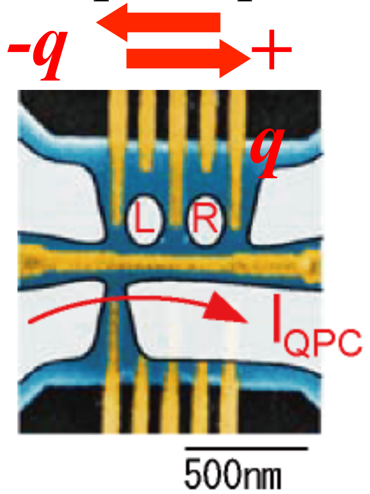
◇ classsical case

◇ quantum case

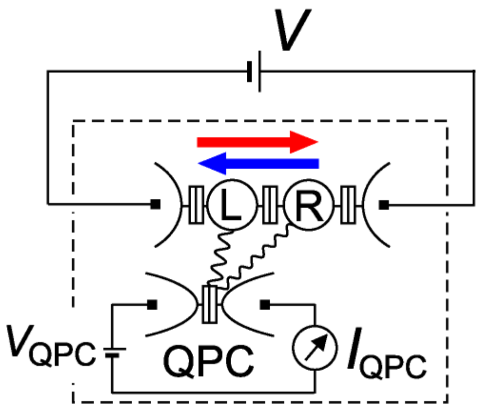
Experimental approach in classical regime

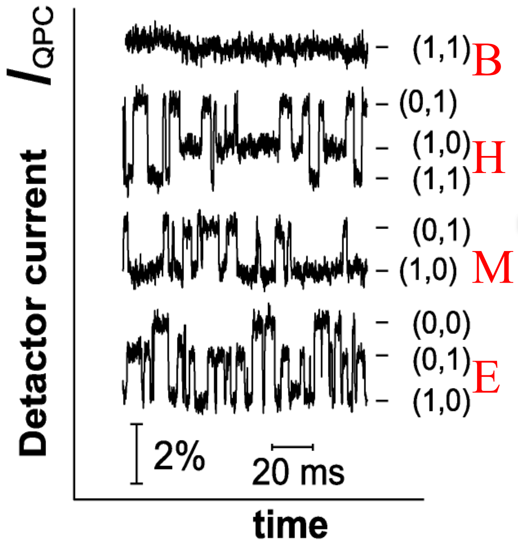
T. Fujisawa et al., Scince (2006)

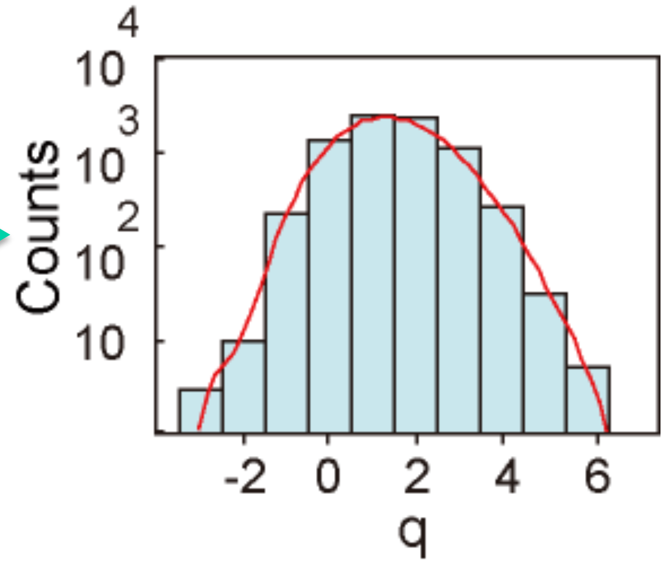
◇ coupled quantum-dots



◇ state-dependent I_QPC



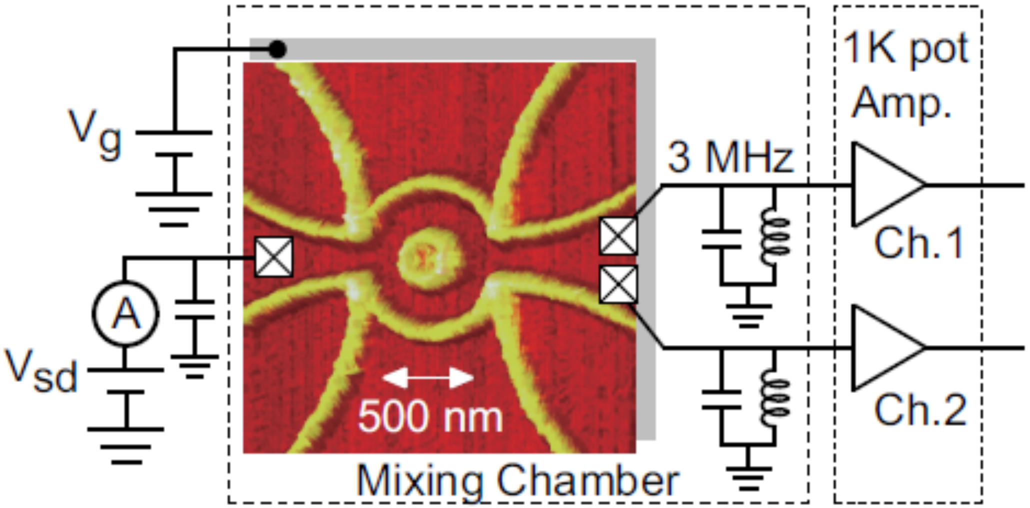




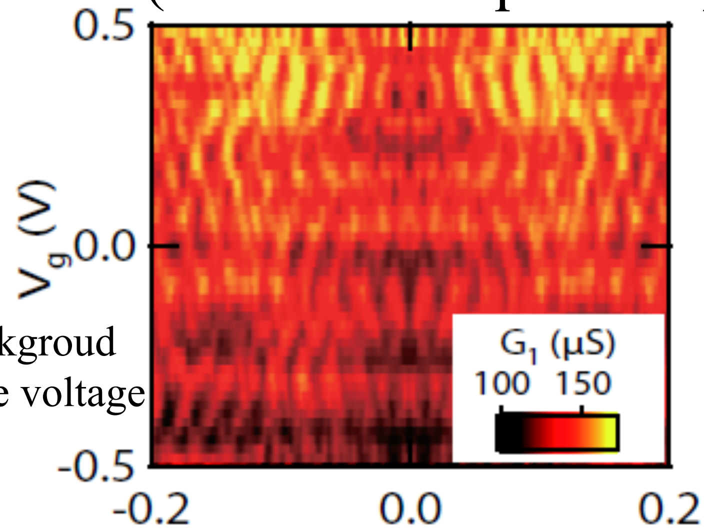
Experimental approach in quantum regime

Kobayashi group PRL (2010)

◇Aharonov-Bhom Interferometer



Interference of current (evidence of quantum)



Backgroud Gate voltage

Magnetic Field

◇ full-distribution is difficult to get even in the present-day experiments. instead, current noise is possible to measure

$$I = G_1 \Delta V + \frac{\Delta V^2}{2!} G_2 + \cdots$$

$$S = S_0 + S_1 \Delta V + \frac{\Delta V^2}{2!} S_2 + \cdots$$

$$S_1^S = 2 k_B T G_2^S$$

$$S_1^A = 6 k_B T G_2^A$$

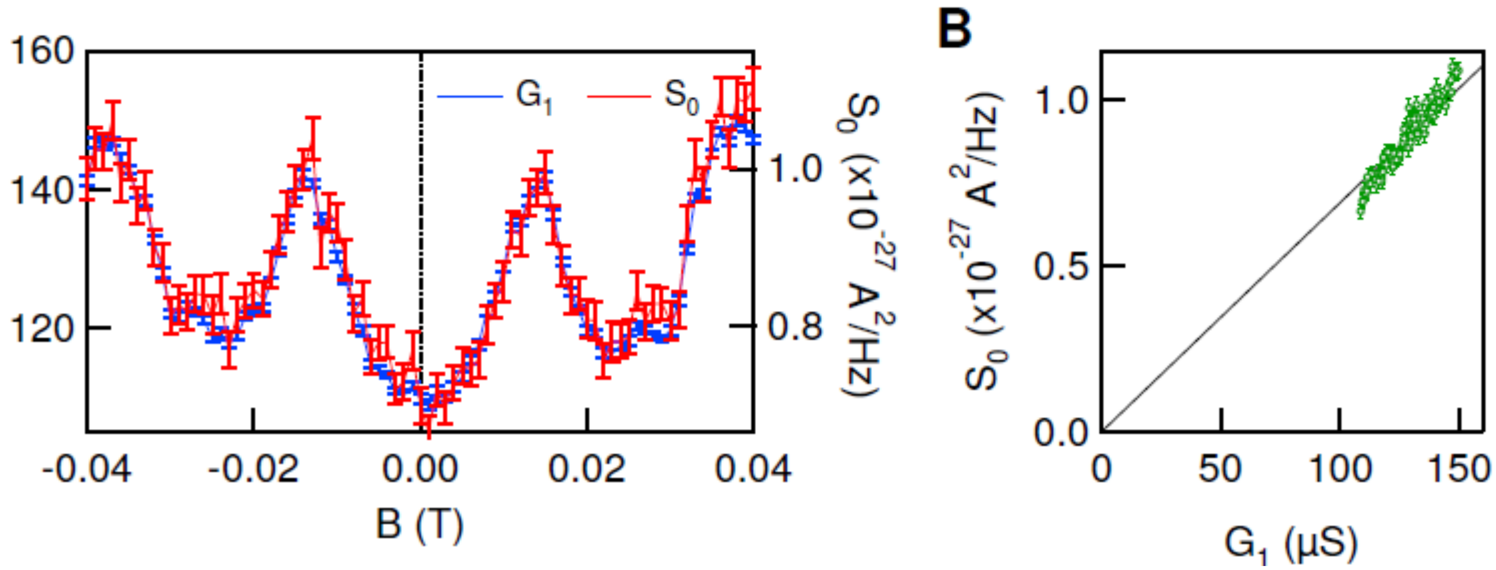
Results of Experiments

$$I = G_1 \Delta V + \frac{\Delta V^2}{2!} G_2 + \cdots$$

$$S = S_0 + S_1 \Delta V + \frac{\Delta V^2}{2!} S_2 + \cdots$$

Check of Johnson-Nyquist Relation (Kubo Formula)

$$S_0(B) = 4 k_B T G_1(B)$$

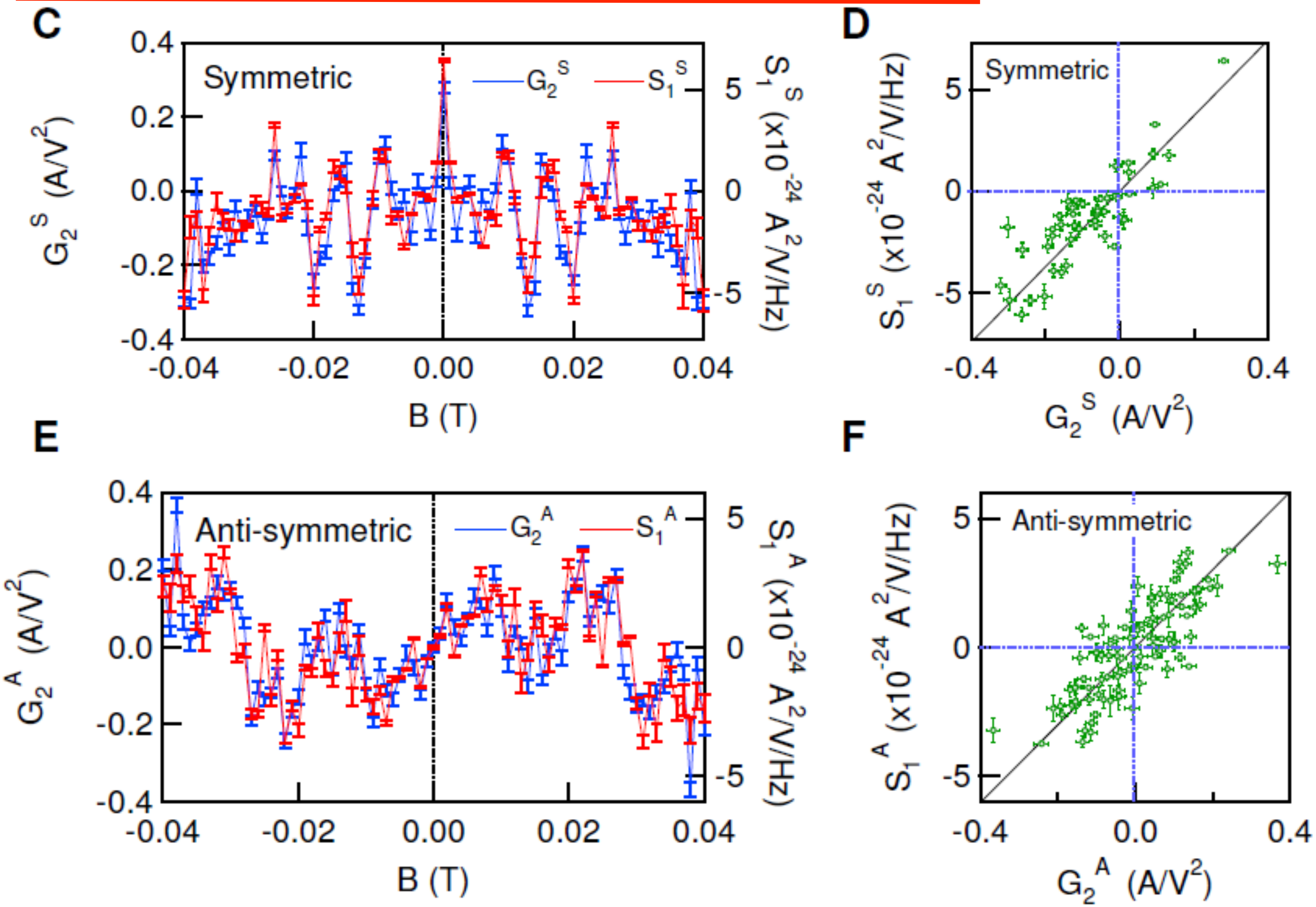


Results on New elations

$$S_1^S = 2k_B T G_2^S \quad S_1^A = 6k_B T G_2^A$$

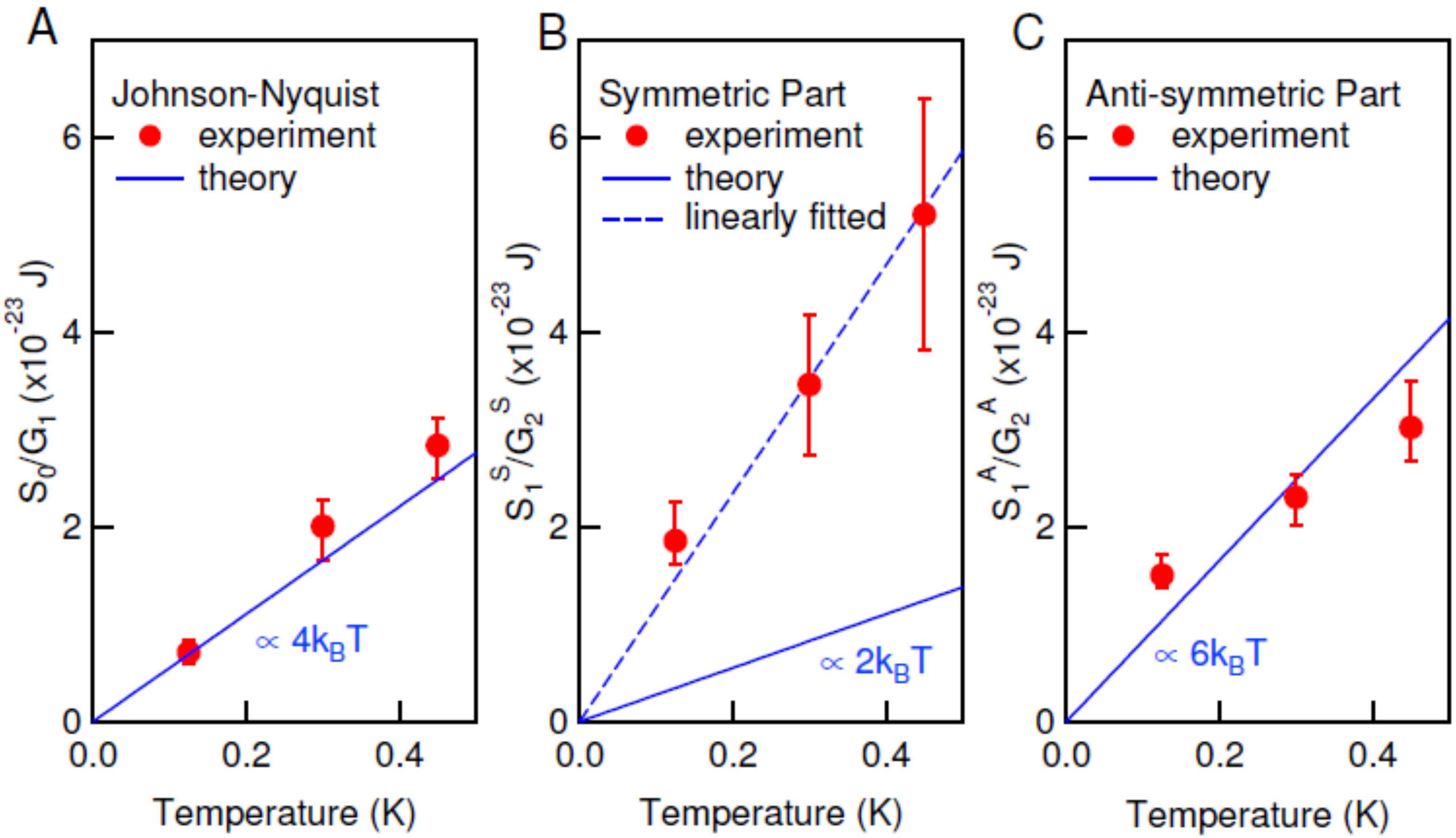
$$I = G_1 \Delta V + \frac{\Delta V^2}{2!} G_2 + \cdots$$

$$S = S_0 + S_1 \Delta V + \frac{\Delta V^2}{2!} S_2 + \cdots$$



Experiment versus theory

$$S_1^S = 2k_B T G_2^S \qquad S_1^A = 6k_B T G_2^A$$



まとめと展望

◇量子測定に基づく理論の構築と実験との比較を行った。

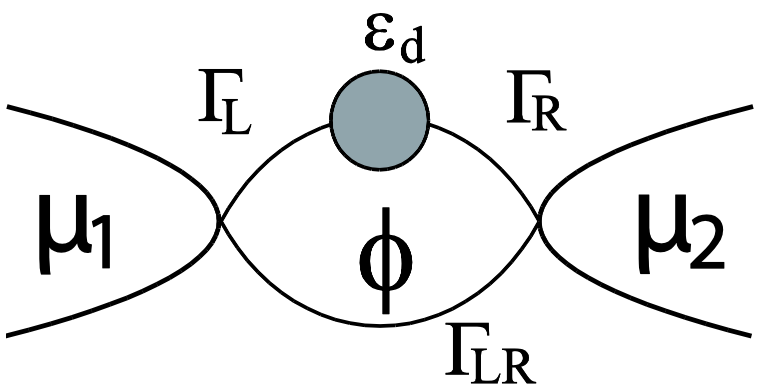
-recent studies -

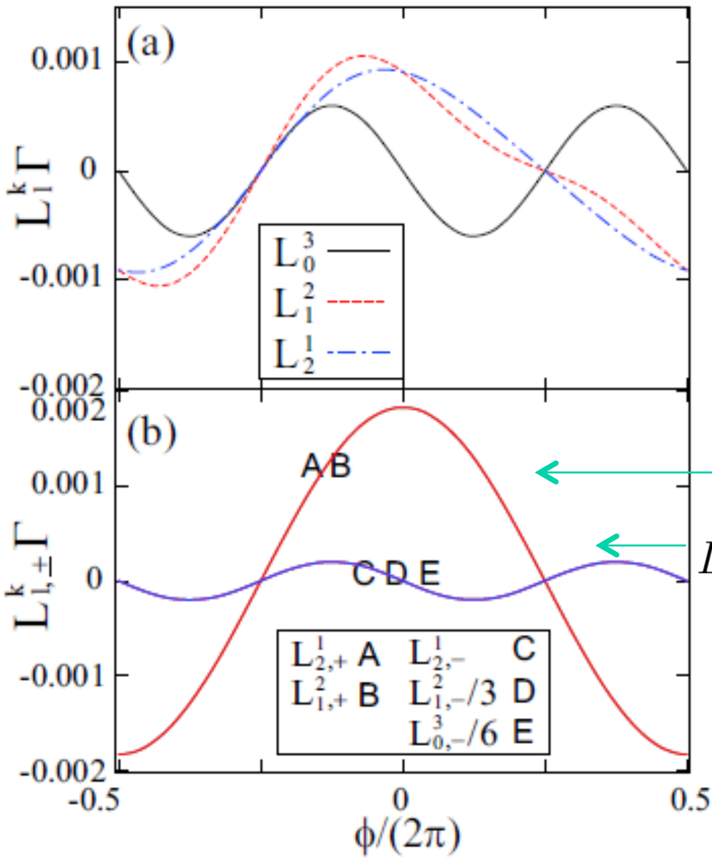
◇　量子観測の効果

◇　測定による反跳効果

◇　測定の精度限界による効果

Demonstration *- 2 terminal interacting Aharonov-Bohm interferometer*





$L^{1}_{2,+} = L^{2}_{1,+}$

$L^{1}_{2,-} = L^{2}_{1,-}/3 = L^{3}_{0,-}/6$

Microcopic -Reversibility preserving Hartree-Level calculation

FIG. 2: (a) Flux dependent third order nonlinear transport coefficients. (b) The extension of Onsager relation (4). Parameters are the same as in Fig. 1.