Glueball instability and thermalization driven by dark radiation

Masafumi Ishihara

Tohoku U. AIMR

Collaborators: Kazuo Ghoroku Fukuoka Inst. Tech.
Akihiro Nakamura Kagoshima U.
Fumihiko Toyoda Kinki U.

The 4D theory exists on the boundary of 5D gravity theory.

- 4D Field Theory
  - Temperature
  - Glueball

- 5D bulk gravity
  - Hawking temperature of the Black Hole
  - fluctuation of the 5D metric (closed string)
Introduction

We consider the glueball spectrum of the 4D field theory on Friedmann-Robertson-Walker ($\text{FRW}_4$) metric with negative cosmological constant ($-\lambda$).

\[ ds^2_{\text{FRW}_4} = -dt^2 + a_0^2(t)\gamma_{ij}(x)dx^i dx^j \]

\[ a_0(t): \text{scale factor} \]

\[ \gamma_{ij}(x) = \delta_{ij} \left( 1 - \frac{1}{4} \sum_{i=1}^{3} (x^i)^2 \right)^{-2} \]

By gauge/gravity correspondence, we find the 5-dimensional bulk with 4D $\text{FRW}_4$ metric at boundary ($r \to \infty$).

We also introduce the energy density $c_0$ and find that there is a phase transition of 4D boundary theory at critical $c_0$ with fixed $\lambda$. 
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Hawking Temperature

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Summary
Construction of the 5D bulk

5D bulk metric is obtained in the following ansatz,

$$ds_5^2 = \frac{r^2}{R^2} \left( -n(r)^2 dt^2 + A(r)^2 a_0^2(t) \gamma_{ij}(x) dx^i dx^j \right) + \frac{R^2}{r^2} dr^2$$

$R$: constant (AdS$_5$ radius)

We will find $n(r)$ and $A(r)$ which satisfies $n(r) \to 1$, $A(r) \to 1$ for $r \to \infty$,

The 4D boundary ($r \to \infty$) metric becomes $FRW_4$ metric.

$$ds_4^2 = -dt^2 + a_0^2(t) \gamma_{ij}(x) dx^i dx^j$$

$a_0(t)$: scale factor

$$\gamma_{ij}(x) = \delta_{ij} \left( 1 - \frac{1}{4} \sum_{i=1}^{3} (x^i)^2 \right)^{-2}$$

$$FRW_4 \xrightarrow{x,y,z} r$$

5D bulk
Friedman equation

\( a_0(t), A(r) \) and \( n(r) \) are determined by 5D Einstein Equation

\[
R_{MN} = -\Lambda g_{MN} \quad (M, N = 0 \cdots 5) \quad \left( \Lambda = \frac{4}{R^2} \right)
\]

and the 4D Friedman equation for boundary \( FRW_4 \)

\[
\left( \frac{\dot{a}_0(t)}{a_0(t)} \right)^2 - \frac{1}{a_0^2(t)} = -\lambda \quad -\lambda: \text{a negative cosmological constant}
\]

\[
(ds_{FRW_4}^2 = -dt^2 + a_0^2(t)\gamma_{ij}(x) dx^i dx^j)
\]
Then, we can get the $A(r)$ and $n(r)$ as follows

$$A = \left( \left( 1 + \left( \frac{r_0}{r} \right)^2 \right)^2 + c_0 \left( \frac{R}{r} \right)^4 \right)^{1/2}$$

$$n = \frac{\left( 1 + \left( \frac{r_0}{r} \right)^2 \right)^2 - c_0 \left( \frac{R}{r} \right)^4}{A}$$

$c_0$: energy density of dual 4D Yang-Mills theory

$r_0 \equiv \frac{R^2}{2} \sqrt{\lambda}$: cosmological constant of boundary 4d space-time.

(K.Ghoroku and A. Nakamura 2012)

for 5D bulk metric

$$ds_5^2 = \frac{r^2}{R^2} \left( -n(r)^2 dt^2 + A(r)^2 a_0^2(t) \gamma_{ij}(x) dx^i dx^j \right) + \frac{R^2}{r^2} dr^2$$

We will use $r_0$ instead of $\lambda$ as an 4D cosmological constant
Hawking temperature

5D bulk solution becomes

$$ds^2_5 = \frac{r^2}{R^2} (-n(r)^2 dt^2 + A(r)^2 a_0^2(t) \gamma_{ij}(x) dx^i dx^j) + \frac{R^2}{r^2} dr^2$$

$$A(r) = \left( \left( 1 + \left( \frac{r_0}{r} \right)^2 \right)^2 + c_0 \left( \frac{r_0}{r} \right)^4 \right)^{1/2}$$

$$n(r) = \frac{\left( 1 + \left( \frac{r_0}{r} \right)^2 \right)^2 - c_0 \left( \frac{r_0}{r} \right)^4}{A}$$

At $$r = r_H \equiv \left( \sqrt{c_0 R^2} - r_0^2 \right)^{1/2}$$, $$g_{tt} \propto n(r_H) = 0$$.

When $$c_0 > \frac{r_0^4}{R^4}$$, there is an "event horizon" at $$r_H$$.

Hawking temperature $$T_H$$ is given by

$$T_H = \frac{r_H \left( 1 + \frac{r_0^2 + \sqrt{c_0 R^2}}{r_H^2} \right)}{\pi R^2 A(r_H)}$$
Hawking Temperature

Hawking temperature $T_H$ in 5D bulk BH

4D theory is in a thermal system
(deconfinement phase)

$T_H = \frac{r_H \left(1 + \frac{r_0^2 + \sqrt{c_0 R^2}}{r_H^2} \right)}{\pi R^2 A(r_H)}$

$r_H \equiv \left(\sqrt{c_0 R^2} - r_0^2\right)^{1/2}$

As $c_0$ (energy density) becomes small, $T_H$ decreases.

As $r_0$ (cosmological constant) becomes large, $T_H$ decreases.
Hawking Temperature

When $c_0 > \frac{r_0^4}{R^4}$, 5D BH bulk has a Hawking temperature

$\leftrightarrow$ dual 4D theory is in the deconfinement phase.

At $c_0 = \frac{r_0^4}{R^4}$, $T_H = 0$ and there is a phase transition between
confinement phase and deconfinement phase.

When $0 \leq c_0 < \frac{r_0^4}{R^4}$, there is no “event horizon”.

$\leftrightarrow$ Dual 4D theory is in the “confinement phase”.

Stable Glueball spectrum by the 5D bulk metric fluctuation
Glueball spectrum can be obtained by the fluctuation $h_{ij}(t, x^i, r)$ of the 5D bulk metric ($g_{MN}$). (R.C. Brower, S.D. Mathur and C.I. Tan. 2003)

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N h_{ij}) = 0$$

By decomposing $h_{ij}(x^\mu, r) = p_{ij} \chi(x^\mu) \phi(r)$

The equation of 4D part $\chi(x^\mu)$ is given by

$$\frac{1}{g_4} \partial_{\mu} \sqrt{g_4} g^{\mu\nu} \partial_{\nu} \chi(x^\mu) = m^2 \chi(x^\mu)$$

$m$: Glueball mass
Glueball mass spectrum

Glueball mass $m$ is also appeared in the equation for $\phi(r)$

$$\partial_r^2 \phi + g_2(r) \partial_r \phi + \left( \frac{R}{r} \right)^4 \frac{m^2}{n(r)^2} \phi(r) = 0$$

$$\bar{g}_2(r) = \partial_r \left( \log \left[ \left( \frac{r}{R} \right)^5 n(r) A(r)^3 \right] \right)$$
Glueball mass spectrum ($c_0 = 0$ case)

First we consider the $c_0 = 0$ case (confinement phase).

By defining $x \equiv \frac{r}{r_0}$, equation for $\phi(r)$ is given by

$$\partial_x^2 \phi + g_2(x) \partial_x \phi + \frac{R^4 m^2}{r_0^2 x^4 A^2(x)} \phi = 0$$

Where $g_2(x) = \frac{1}{x} \left( 5 - \frac{8}{x^2 A(x)} \right)$, $A(x) = 1 + \frac{1}{x^2}$

$\phi$ becomes normalizable by choosing $m$ as

$$m^2 = -\lambda (N + 1)(N + 4) \quad \lambda = \frac{4r_0^2}{R^4} \quad N = 0, 1, 2 \ldots$$

The lowest glueball mass ($N=0$) is finite as

$$m_g = 2\sqrt{\lambda}$$

This was also obtained by C. Fronsdal, 1979
Glueball mass spectrum \((c_0 > 0)\)

By factorizing \(\phi\) as

\[
\phi = e^{-\frac{1}{2} \int dr \bar{g}_2(r) f(r)}
\]

The equation for \(f(r)\) becomes the Schrödinger equation

\[
-\partial_r^2 f + V(r) f = 0
\]

with the potential \(V(r)\)

\[
V = \frac{1}{4} \bar{g}_2^2 + \frac{1}{2} \partial_r \bar{g}_2 - \frac{m^2}{n^2} \left(\frac{R}{r}\right)^4
\]
Glueball mass spectrum

WKB approximation gives

\[ \int \sqrt{-V} dr = \left( N + \frac{1}{2} \right) \pi \quad N = 0, 1, 2 \ldots \]

\[ V = \frac{1}{4} \bar{g}^2 + \frac{1}{2} \partial_r \bar{g}^2 - \frac{m^2}{n^2} \left( \frac{R}{r} \right)^4 \]

The lowest glueball mass \( m_g \) is given when \( N=0 \) in the above formula. The relation between \( m_g \) and \( c_0 \) is calculated numerically.

For critical \( c_0 = \frac{r_0^4}{R^4} \),

Lowest glueball mass \( m_g \) becomes zero.
Glueball as an rotating closed string

Glueball with large quantum number: a rotating string in the bulk

Spin $J_s$: angular momentum of the closed string

Energy $E_s$: energy of the closed string
Spin and Energy

Lagrangian of a closed string

\[
L = -\frac{1}{2\pi\alpha'} \int dr \frac{r^2}{R^2} A^2 \sqrt{\left(\frac{n^2}{A^2} - \omega^2 p^2 \sin^2 \theta \, a_0^2(t)\right) \left(\theta'^2 p^2 a_0(t) + \frac{1}{A^2} \left(\frac{R}{r}\right)^4\right)}
\]

\(p\): radial coordinate of \(FRW_4\)

the ansatz of the rotating closed string: \(\theta = \theta(r)\) and \(\phi = \omega t\)

Then, we can obtain the spin and Energy as follows.

**Spin**

\[
J_s = \frac{\partial L}{\partial \omega} = \frac{1}{2\pi\alpha'} \int dr \frac{a_0^2 r^2}{R^2} A^2 \omega p^2 \sin^2 \theta \sqrt{\frac{\theta'^2 p^2 a_0(t) + \frac{1}{A^2} \left(\frac{R}{r}\right)^4}{\sqrt{\frac{n^2}{A^2} - \omega^2 p^2 \sin^2 \theta a_0^2(t)}}}
\]

**Energy**

\[
E_s = \omega \frac{\partial L}{\partial \omega} - L = \frac{1}{2\pi\alpha'} \int dr \frac{r^2}{R^2} n^2 \sqrt{\frac{\theta'^2 p^2 a_0(t) + \frac{1}{A^2} \left(\frac{R}{r}\right)^4}{\sqrt{\frac{n^2}{A^2} - \omega^2 p^2 \sin^2 \theta a_0^2(t)}}}
\]
Regge Behavior

By solving the equation of motion numerically, we can get a Regge behavior

\[ J_s = \alpha_{glueball} E_s^2. \]

We calculate the relation between String tension \( k = \frac{1}{8\alpha_{glueball}} \) and \( c_0 \).

For the critical \( c_0 = \frac{r_0^4}{R^4} \), string tension becomes zero.
Summary

- We consider 5D gravity with 4D FRW boundary theory which has negative cosmological constant $\lambda = -\frac{4r_0^2}{R^4}$ and energy density $c_0$

- When $0 \leq c_0 < \frac{r_0^4}{R^4}$, 4D field theory is in the “confinement phase”.
  
  discrete glueball mass spectrum by 5D bulk metric fluctuations
  Regge behavior by closed string

- When $\frac{r_0^4}{R^4} \leq c_0$, an “event horizon” appears and 4D field theory is in the deconfinement phase.
  
  Lowest Glueball mass becomes zero
Future work

Chiral phase transition by introducing D7-brane

Entanglement Entropy by calculations of minimal surface

Introducing the chemical potential and baryon number density
Energy momentum tensor and $c_0$

The five dimensional metric is rewritten as

$$ds_5^2 = \frac{1}{\rho} \left( -n(r)^2 dt^2 + A(r)^2 a_0^2(t) \gamma_{ij}(x) dx^i dx^j \right) + \frac{d\rho^2}{4\rho^2}$$

$$\equiv \frac{1}{\rho} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + \frac{d\rho^2}{4\rho^2} \quad \text{where} \quad \rho \equiv \frac{r_0^2}{r^2} \quad \text{(and} \quad R = 1)$$

By expanding 4D metric by powers of $\rho$ as

$$\tilde{g}_{\mu\nu} = g(0)_{\mu\nu} + g(2)_{\mu\nu} \rho + g(4)_{\mu\nu} \rho^2 + \cdots$$

The energy momentum tensor of the 4D boundary ($r \to \infty$) theory are given by following formula.

$$\langle T_{\mu\nu} \rangle = \frac{4R^3}{16\pi G_N} \left( g(4)_{\mu\nu} - \frac{1}{8} g(0)_{\mu\nu} \left( (\text{Tr} g(2))^2 - \text{Tr} g^2(2) \right) - \frac{1}{2} (g^2(2))_{\mu\nu} \right.$$

$$+ \frac{1}{4} g(2)_{\mu\nu} \text{Tr} g(2) \left) \right.$$
**Energy momentum tensor and $c_0$**

Stress tensor at 4D boundary

$$\langle T_{\mu\nu} \rangle = \frac{4R^3}{16\pi G_N^5} \bar{c}_0 R^4 (3, g_{(0)ij}) + \frac{4R^3}{16\pi G_N^5} \left( \frac{3\lambda^2}{16} (-1, g_{(0)ij}) \right)$$

$g_{(0)ij} \equiv a_0(t)^2 \gamma_{ij}(x)$: boundary FRW 3D spatial metric

**First term**

the “thermal” stress tensor comes from conformal Yang-Mills fields.

$\bar{c}_0$: the energy density of the dual 4D gauge theory

**Second term**: loop corrections of SYM fields in a curves space-time ($\lambda \neq 0$).

Weyl anomaly from this term: $\langle T^\mu_\mu \rangle = -\frac{3\lambda^2}{8\pi^2} N^2$ which matches with the result of YM theory in 4d curved space-time.