Glueball instability and thermalization driven by dark radiation

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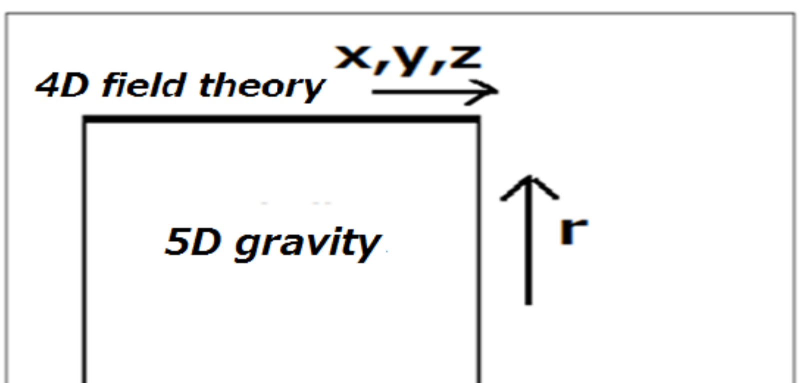
Gauge/Gravity Correspondence

4D Field Theory		**5D bulk gravity**
Temperature	⟺	Hawking temperature of the Black Hole
Glueball		fluctuation of the 5D metric (closed string)

The 4D theory exists on the boundary of 5D gravity theory



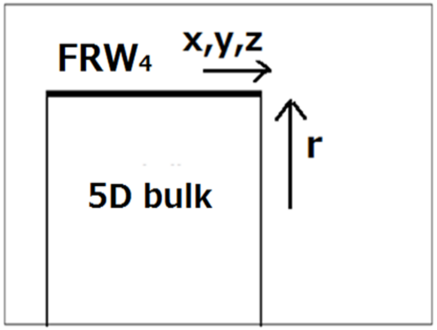
Introduction

We consider the glueball spectrum of the 4D field theory on **Friedmann-Robertson-Walker (FRW_4)** metric with negative cosmological constant ($-\lambda$).

$$ds^2_{FRW_4} = -dt^2 + a_0^2(t)\gamma_{ij}(x)dx^i dx^j$$

$a_0(t)$: scale factor

$$\gamma_{ij}(x) = \delta_{ij}\left(1 - \frac{1}{4}\Sigma_{i=1}^{3}\left(x^i\right)^2\right)^{-2}$$



By gauge/gravity correspondence, we find the 5-dimensional bulk with 4D FRW_4 metric at boundary ($r \to \infty$) .

We also introduce the energy density c_0 and find that there is an phase transition of 4D boundary theory at critical c_0 with fixed λ.

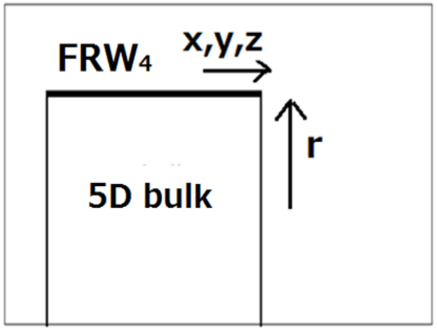
Contents

Construction of the 5D bulk

5D bulk metric is obtained in the following ansatz,

$$ds_5^2 = \frac{r^2}{R^2}\left(-n(r)^2 dt^2 + A(r)^2\, a_0^2(t)\gamma_{ij}(x)dx^i dx^j\right) + \frac{R^2}{r^2}dr^2$$

R: constant (AdS_5 radius)



We will find $n(r)$ and $A(r)$ which satisfies

$n(r) \to 1$, $A(r) \to 1$ for $r \to \infty$,

The ***4D*** boundary ($r \to \infty$) metric becomes FRW_4 metric.

$$ds_4^2 = -dt^2 + a_0^2(t)\gamma_{ij}(x)dx^i dx^j$$

$a_0(t)$: scale factor

$$\gamma_{ij}(x) = \delta_{ij}\left(1 - \frac{1}{4}\Sigma_{i=1}^3 \left(x^i\right)^2\right)^{-2}$$

Friedman equation

$a_0(t)$, $A(r)$ and $n(r)$ are determined by **5D Einstein Equation**

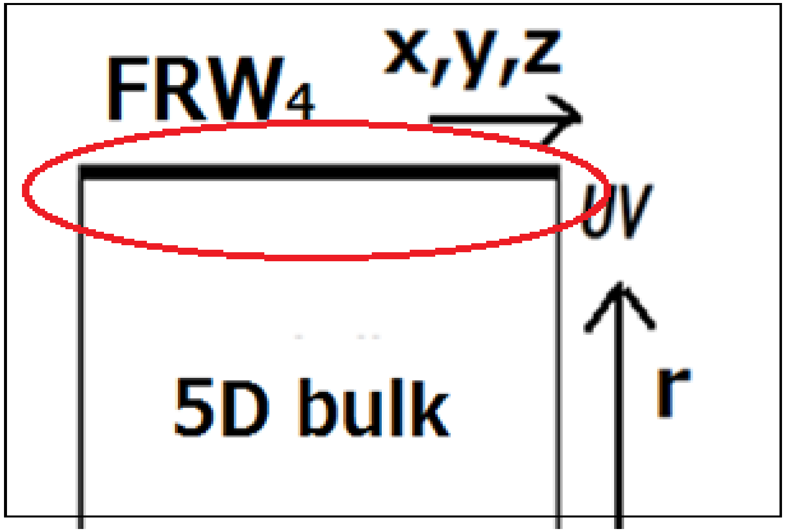
$$R_{MN} = -\Lambda g_{MN} \quad (M, N = 0 \cdots 5) \qquad \left(\Lambda = \frac{4}{R^2}\right)$$

and **the 4D Friedman equation** for boundary FRW_4

$$\left(\frac{\dot{a}_0(t)}{a_0(t)}\right)^2 - \frac{1}{a_0^2(t)} = -\lambda$$

$-\lambda$: a negative cosmological constant

$$(ds^2_{FRW_4} = -dt^2 + a_0^2(t)\gamma_{ij}(x)dx^i dx^j)$$



5D bulk metric

Then, we can get the $A(r)$ and $n(r)$ as follows

$$A = \left(\left(1 + \left(\frac{r_0}{r}\right)^2 \right)^2 + c_0 \left(\frac{R}{r}\right)^4 \right)^{1/2} \qquad n = \frac{\left(\left(1 + \left(\frac{r_0}{r}\right)^2 \right)^2 - c_0 \left(\frac{R}{r}\right)^4 \right)}{A}$$

c_0: energy density of dual 4D Yang-Mills theory

$r_0 \equiv \frac{R^2}{2}\sqrt{\lambda}$: cosmological constant of boundary 4d space-time.

(K.Ghoroku and A. Nakamura 2012)

for 5D bulk metric

$$ds_5^2 = \frac{r^2}{R^2}\left(-n(r)^2 dt^2 + A(r)^2 a_0^2(t)\gamma_{ij}(x)dx^i dx^j\right) + \frac{R^2}{r^2}dr^2$$

We will use r_0 instead of λ as an 4D cosmological constant

Hawking temperature

5D bulk solution becomes

$$ds_5^2 = \frac{r^2}{R^2}\left(-n(r)^2 dt^2 + A(r)^2 a_0^2(t)\gamma_{ij}(x)dx^i dx^j\right) + \frac{R^2}{r^2}dr^2$$

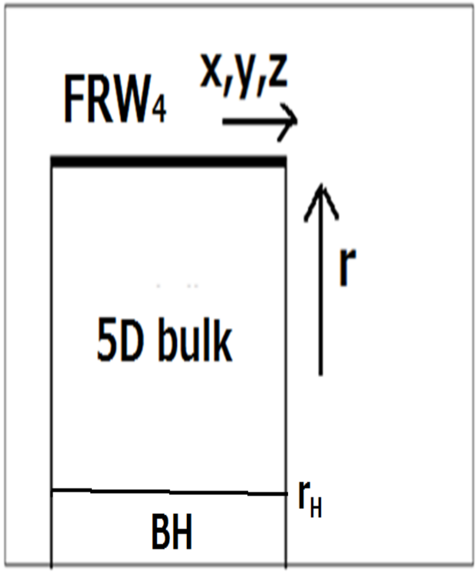
$$A(r) = \left(\left(1+\left(\frac{r_0}{r}\right)^2\right)^2 + c_0\left(\frac{R}{r}\right)^4\right)^{1/2} \qquad n(r) = \frac{\left(\left(1+\left(\frac{r_0}{r}\right)^2\right)^2 - c_0\left(\frac{R}{r}\right)^4\right)}{A}$$

At $r = r_H \equiv \left(\sqrt{c_0}R^2 - r_0^2\right)^{1/2}$, $\quad g_{tt} \propto n(r_H) = 0.$

When $c_0 > \frac{r_0^4}{R^4}$, there is an **"event horizon"** at r_H.

Hawking temperature T_H is given by

$$T_H = \frac{r_H\left(1+\frac{r_0^2+\sqrt{c_0}R^2}{r_H^2}\right)}{\pi R^2 A(r_H)}$$



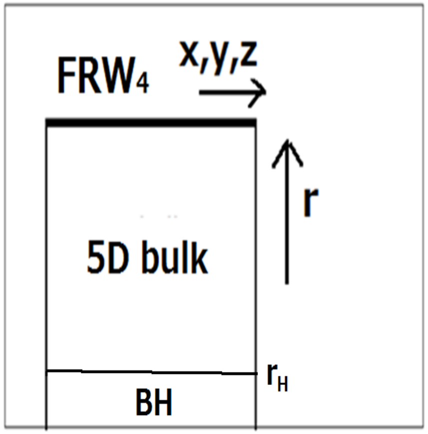
Hawking Temperature

Hawking temperature T_H **in 5D bulk BH**

$$\Updownarrow$$

4D theory is in a thermal system

(deconfinement phase)



$$T_H = \frac{r_H\left(1+\frac{r_0^2+\sqrt{c_0}R^2}{r_H^2}\right)}{\pi R^2 A(r_H)} \qquad r_H \equiv \left(\sqrt{c_0}R^2 - r_0^2\right)^{1/2}$$

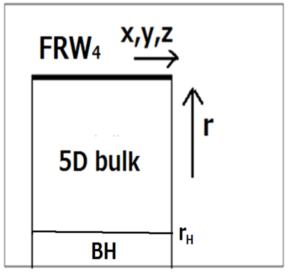
As c_0 (energy density) becomes **small**, T_H decreases.

As r_0 (cosmological constant) becomes **large**, T_H decreases.

Hawking Temperature

When $c_0 \; > \frac{r_0^4}{R^4},$ 5D BH bulk has a Hawking temperature

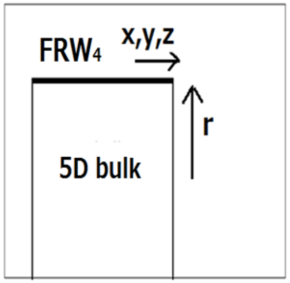
$\leftrightarrow$ dual 4D theory **is in the deconfinement phase.**



At $c_0 = r_0^4/R^4, \quad T_H = 0$ and there is a phase transition between confinement phase and deconfinement phase.

When $0 \leq c_0 \; < \frac{r_0^4}{R^4},$ there is no "event horizon" .

$\leftrightarrow$ Dual 4D theory is in the **"confinement phase".**



Stable Glueball spectrum by the 5D bulk metric fluctuation

Glueball mass spectrum

Glueball spectrum can be obtained by **the fluctuation** $h_{ij}(t,\ x^i, r)$ **of the 5D bulk metric** (g_{MN}). *(R.C. Brower, S.D.Mathur and C.I. Tan. 2003)*

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}\partial_N h_{ij}\right) = 0$$

By decomposing $h_{ij}(x^\mu, r) = p_{ij}\chi(x^\mu)\phi(r)$

The equation of 4D part $\chi(x^\mu)$ is given by

$$\frac{1}{g_4}\partial_\mu\sqrt{g_4}g^{\mu\nu}\partial_\nu\chi(x^\mu) = m^2\chi(x^\mu)$$

m: Glueball mass

Glueball mass spectrum

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Glueball mass $\boldsymbol{m}$ is also appeared in the equation for $\boldsymbol{\phi(r)}$

$$\partial_r^2\phi + g_2(r)\partial_r\phi + \left(\frac{R}{r}\right)^4 \frac{m^2}{n(r)^2}\phi(r) = 0$$

$$\bar{g}_2(r) = \partial_r\left(log\left[\left(\frac{r}{R}\right)^5 n(r)A(r)^3\right]\right)$$

Glueball mass spectrum ($c_0 = 0$ case)

First we consider the $\boldsymbol{c_0 = 0}$ case (confinement phase).

By defining $\boldsymbol{x \equiv \frac{r}{r_0}}$, equation for $\boldsymbol{\phi(r)}$ is given by

$$\boldsymbol{\partial_x^2 \phi + g_2(x)\partial_x \phi + \frac{R^4 m^2}{r_0^2 x^4 A^2(x)}\phi = 0}$$

Where $\boldsymbol{g_2(x) = \frac{1}{x}\left(5 - \frac{8}{x^2 A(x)}\right)}$ $\boldsymbol{A(x) = 1 + \frac{1}{x^2}}$

$\boldsymbol{\phi}$ becomes normalizable by choosing $\boldsymbol{m}$ as

$$\boldsymbol{m^2 = -\lambda(N+1)(N+4) \quad \lambda = \frac{4r_0^2}{R^4} \quad N = 0, 1, 2 \cdots}$$

The lowest glueball mass (***N=0***) is finite as

$$\boldsymbol{m_g = 2\sqrt{\lambda}}$$

This was also obtained by ***C.Fronsdal, 1979***

Glueball mass spectrum ($c_0 > 0$)

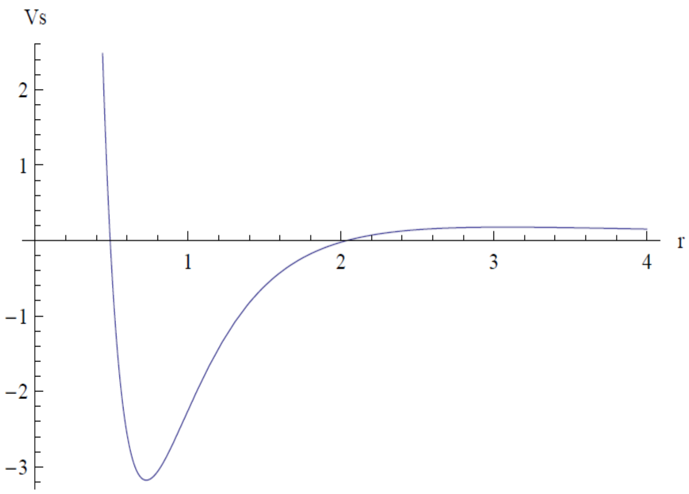
By factorizing **ϕ as** $\phi = e^{-\frac{1}{2}\int dr\bar{g}_2(r)} f(r)$

The equation for $f(r)$ becomes the Schrodinger equation

$$-\partial_r^2 f + V(r) f = 0$$

with the potential ***V(r)***

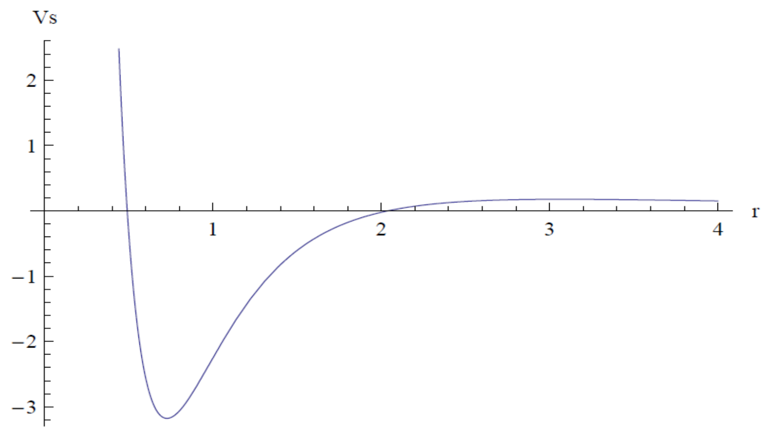
$$V = \frac{1}{4}\bar{g}_2^2 + \frac{1}{2}\partial_r \bar{g}_2 - \frac{m^2}{n^2}\left(\frac{R}{r}\right)^4$$



Glueball mass spectrum

WKB approximation gives

$$\int \sqrt{-V}\,dr = \left(N+\frac{1}{2}\right)\pi \quad N = 0,1,2\cdots$$

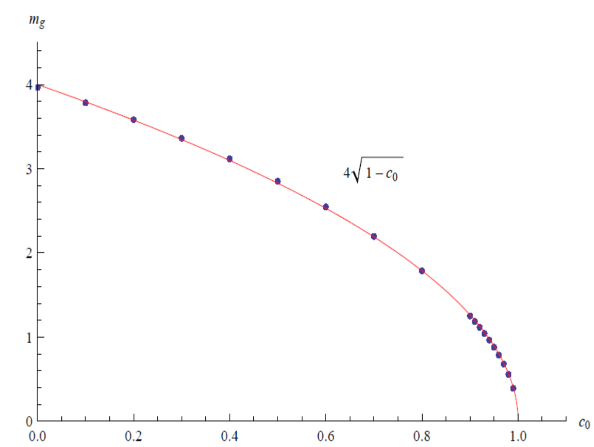


$$V = \frac{1}{4}\bar{g}_2^2 + \frac{1}{2}\partial_r \bar{g}_2 - \frac{m^2}{\bar{n}^2}\left(\frac{R}{r}\right)^4$$

The lowest glueball mass m_g is given when ***N=0*** in the above formula**.** The relation between $\boldsymbol{m_g}$ and $\boldsymbol{c_0}$ is calculated numerically.

For critical $\boldsymbol{c_0 = \frac{r_0^4}{R^4}}$,

Lowest glueball mass $\boldsymbol{m_g}$ becomes zero.



Glueball as an rotating closed string

Glueball with large quantum number: a rotating string in the bulk

Spin J_s **: angular momentum of the closed string**

Energy E_s**: energy of the closed string**

Spin and Energy

Lagrangian of a closed string

$$L = -\frac{1}{2\pi\alpha'}\int dr\,\frac{r^2}{R^2}A^2\sqrt{\left(\frac{n^2}{A^2}-\omega^2p^2\sin^2\theta\,a_0^2(t)\right)\left(\theta^2p^2a_0(t)+\frac{1}{A^2}\left(\frac{R}{r}\right)^4\right)}$$

p **:** radial coordinate of FRW_4

the ansatz of the rotating closed string : $\boldsymbol{\theta = \theta(r)}$ and $\boldsymbol{\phi = \omega t}$

Then, we can obtain the spin and Energy as follows.

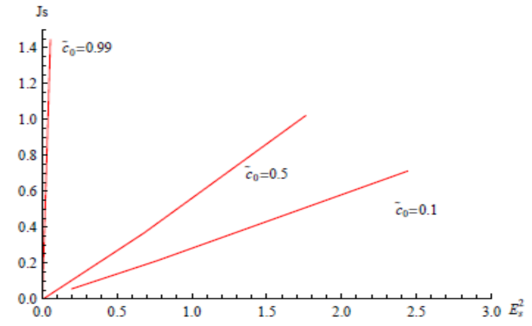
Spin $$J_s = \frac{\partial L}{\partial\omega} = \frac{1}{2\pi\alpha'}\int dr\,\frac{a_0^2r^2}{R^2}A^2\omega p^2\,sin^2\,\theta\,\sqrt{\frac{\theta^2p^2a_0(t)+\frac{1}{A^2}\left(\frac{R}{r}\right)^4}{\frac{n^2}{A^2}-\omega^2p^2\,sin^2\,\theta a_0^2(t)}}$$

Energy $$E_s = \frac{\omega\partial L}{\partial\omega} - L = \frac{1}{2\pi\alpha'}\int dr\,\frac{r^2}{R^2}n^2\sqrt{\frac{\theta'^2p^2a_0(t)+\frac{1}{A^2}\left(\frac{R}{r}\right)^4}{\frac{n^2}{A^2}-\omega^2p^2\,sin^2\,\theta a_0^2(t)}}$$

Regge Behavior

By solving the equation of motion numerically, we can get a Regge behavior

$$J_s = \alpha_{gluball} E_s^2 \, .$$

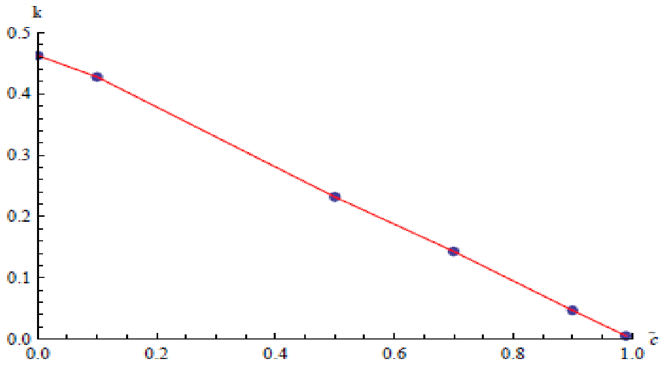


$J_s - E_s$ relation for $r_0 = 1, \ R = 1$

We calculate the relation between String tension $\left(k = \frac{1}{8\alpha_{glueball}}\right)$ and c_0 .

For the critical $c_0 = \frac{r_0^4}{R^4}$,

string tension becomes zero.



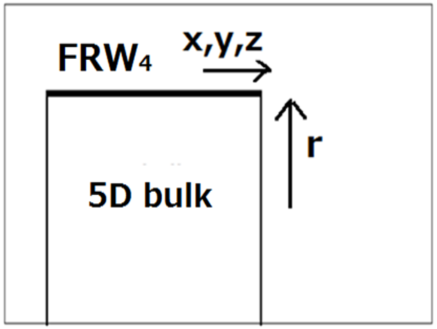
Summary

・We consider 5D gravity with 4D FRW boundary theory which has negative cosmological constant $-\lambda\left(=-\frac{4r_0^2}{R^4}\right)$ and energy density c_0

・When $0 \leq c_0 < \frac{r_0^4}{R^4}$, 4D field theory is in the "confinement phase".

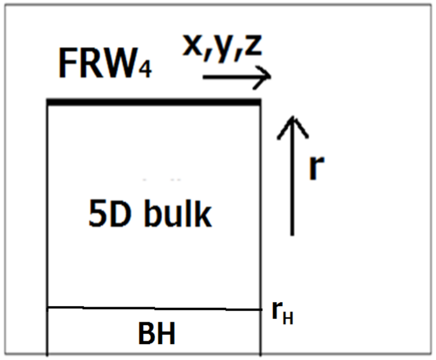
discrete glueball mass spectrum by 5D bulk metric fluctuations

Regge behavior by closed string



・When $\frac{r_0^4}{R^4} \leq c_0$, an "event horizon" appears and 4D field theory is in the deconfinement phase.

Lowest Glueball mass becomes zero



Future work

Chiral phase transition by introducing D7-brane

Entanglement Entropy by calculations of minimal surface

Introducing the chemical potential and baryon number density

Energy momentum tensor and c_0

The five dimensional metric is rewritten as

$$ds_5^2 = \frac{1}{\rho}\left(-n\,(r)^2 dt^2 + A\,(r)^2 a_0^2(t)\gamma_{ij}(x)dx^i dx^j\right) + \frac{d\rho^2}{4\rho^2}$$

$$\equiv \frac{1}{\rho}\hat{g}_{\mu\nu}dx^\mu dx^\nu + \frac{d\rho^2}{4\rho^2} \qquad \text{where } \rho \equiv \frac{r_0^2}{r^2} \quad (\text{and } \quad R = 1)$$

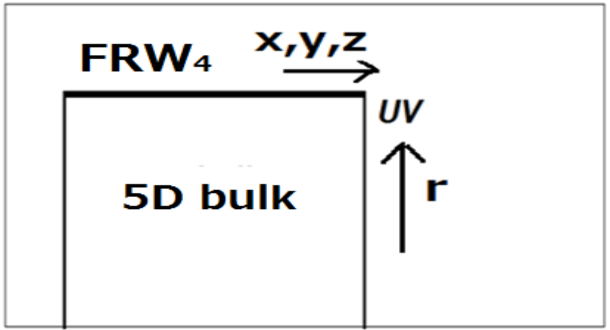
By expanding 4D metric by powers of ρ as

$$\hat{g}_{\mu\nu} = g_{(0)\mu\nu} + g_{(2)\mu\nu}\rho + g_{(4)\mu\nu}\rho^2 + \cdots$$

The energy momentum tensor of the 4D boundary ($r \to \infty$) theory are given by following formula.

$$\langle T_{\mu\nu}\rangle = \frac{4R^3}{16\pi G_N}\Big(\mathbf{g}_{(4)\mu\nu} - \frac{1}{8}\mathbf{g}_{(0)\mu\nu}\left(\left(\mathrm{Tr}\mathbf{g}_{(2)}\right)^2 - \mathrm{Tr}\mathbf{g}_{(2)}^2\right) - \frac{1}{2}\left(\mathbf{g}_{(2)}^2\right)_{\mu\nu}$$
$$+\frac{1}{4}\mathbf{g}_{(2)\mu\nu}\mathrm{Tr}\mathbf{g}_{(2)}\Big)$$

(S.de Haro, et al. 2000)



Energy momentum tensor and c_0

Stress tensor at 4D boundary

$$\langle T_{\mu\nu}\rangle = \frac{4R^3}{16\pi G_N^5}\frac{\bar{c}_0}{R^4}\left(3, g_{(0)ij}\right) + \frac{4R^3}{16\pi G_N^5}\left(\frac{3\lambda^2}{16}\left(-1, g_{(0)ij}\right)\right)$$

$g_{(0)ij} \equiv a_0(t)^2\gamma_{ij}(x)$: boundary FRW 3D spatial metric

First term

the "thermal" stress tensor comes from conformal Yang-Mills fields.

$\bar{c}_0$: the energy density of the dual 4D gauge theory

Second term: loop corrections of SYM fields in a curves space-time ($\lambda \neq 0$).

Weyl anomaly from this term: $\langle T^{\mu}_{\mu}\rangle = -\frac{3\lambda^2}{8\pi^2}N^2$ which matches with the result of YM theory in 4d curved space-time.