

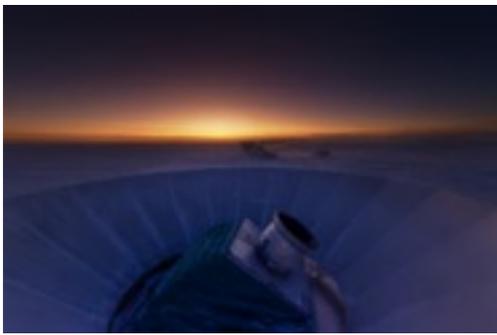


Implications of BICEP2 results for particle physics and cosmology

3rd September 2014

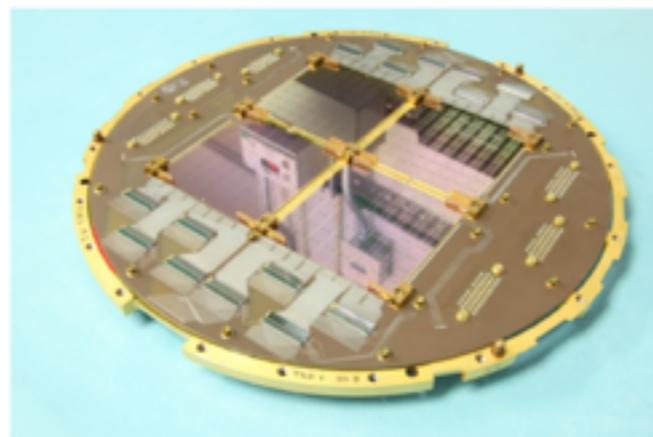
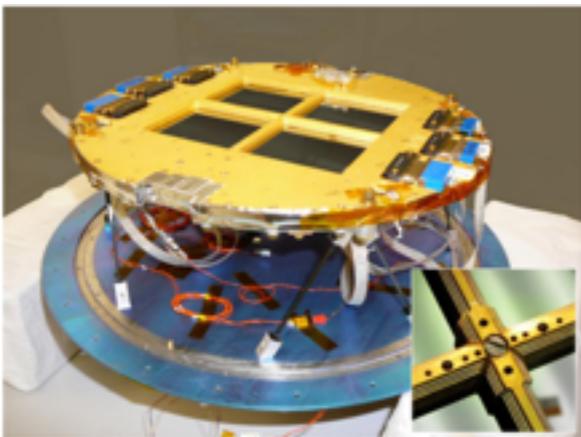
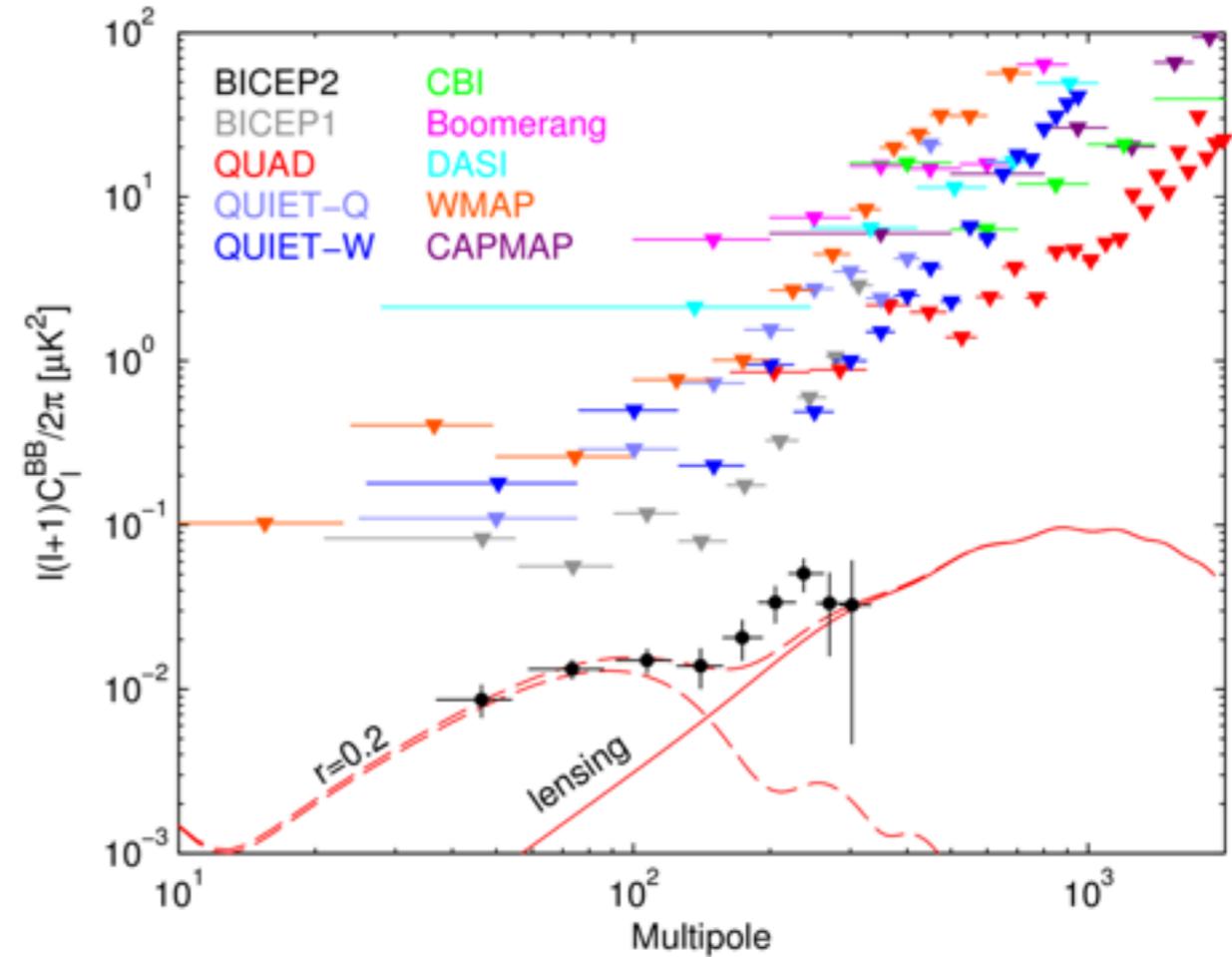
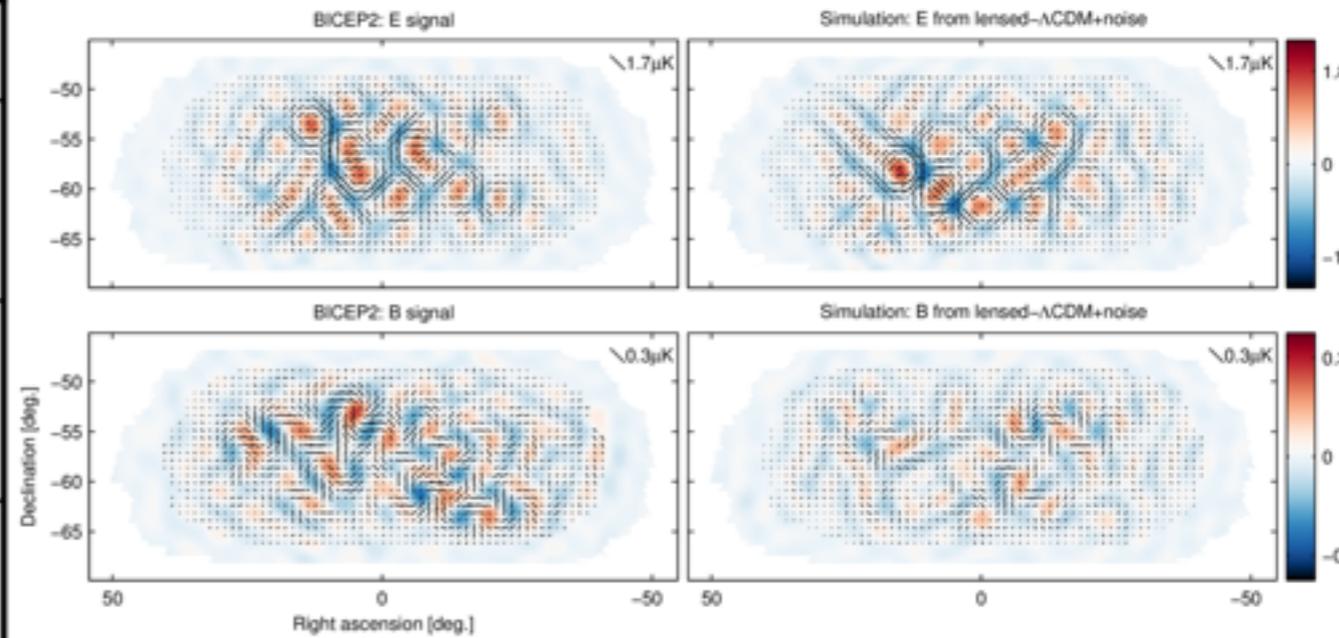
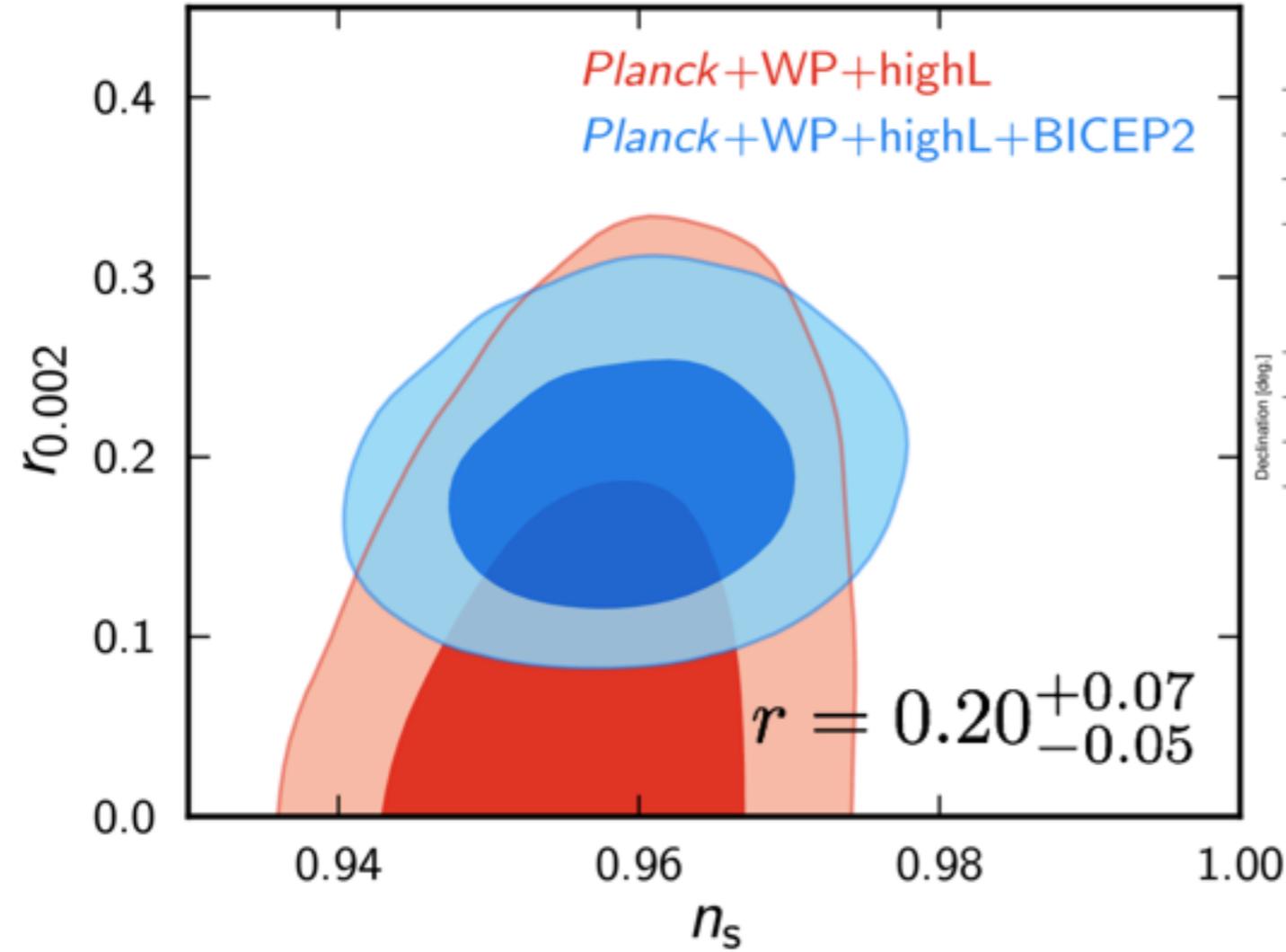
熱場の量子論@理研

Fuminobu Takahashi
(Tohoku)



BICEP2

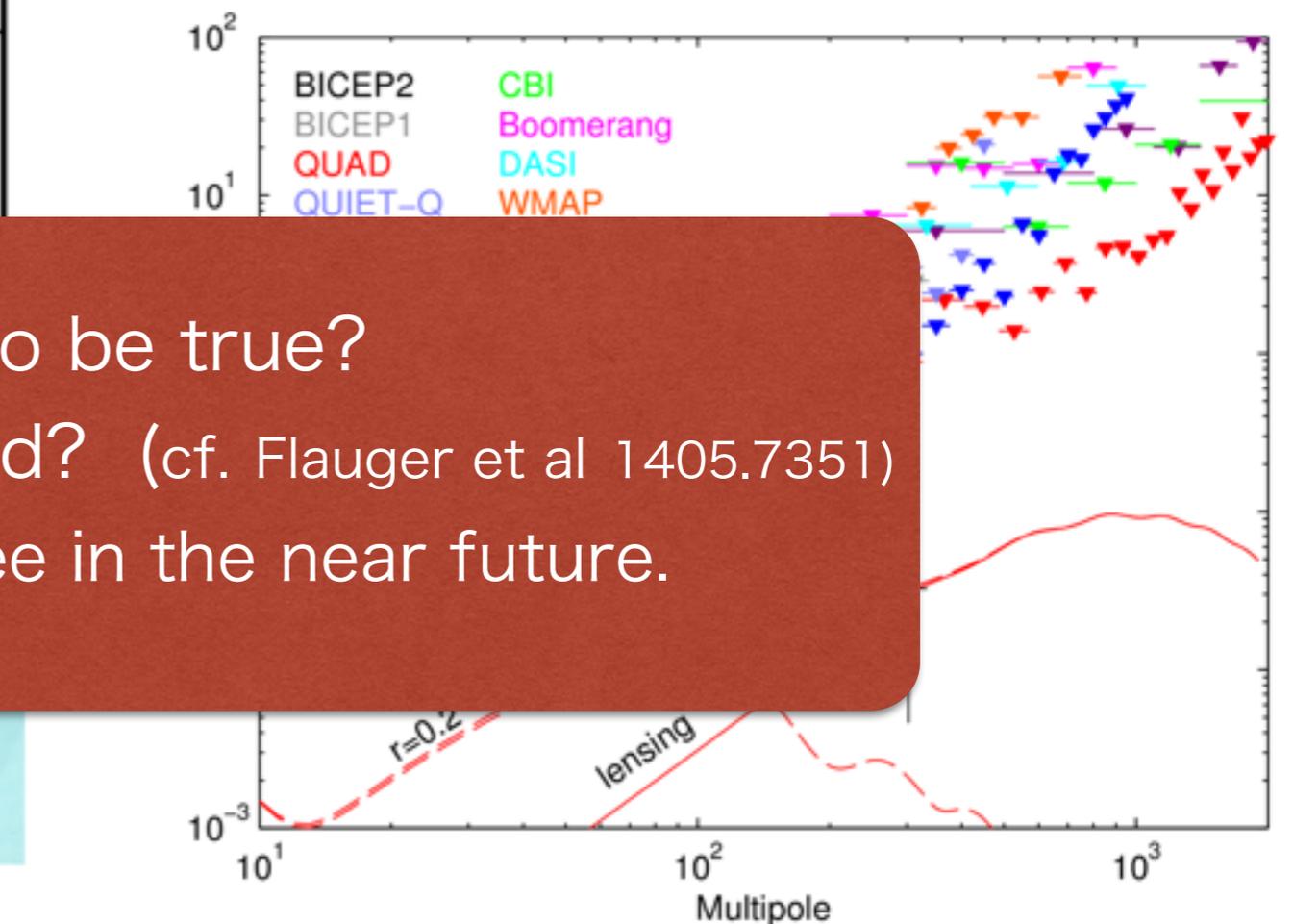
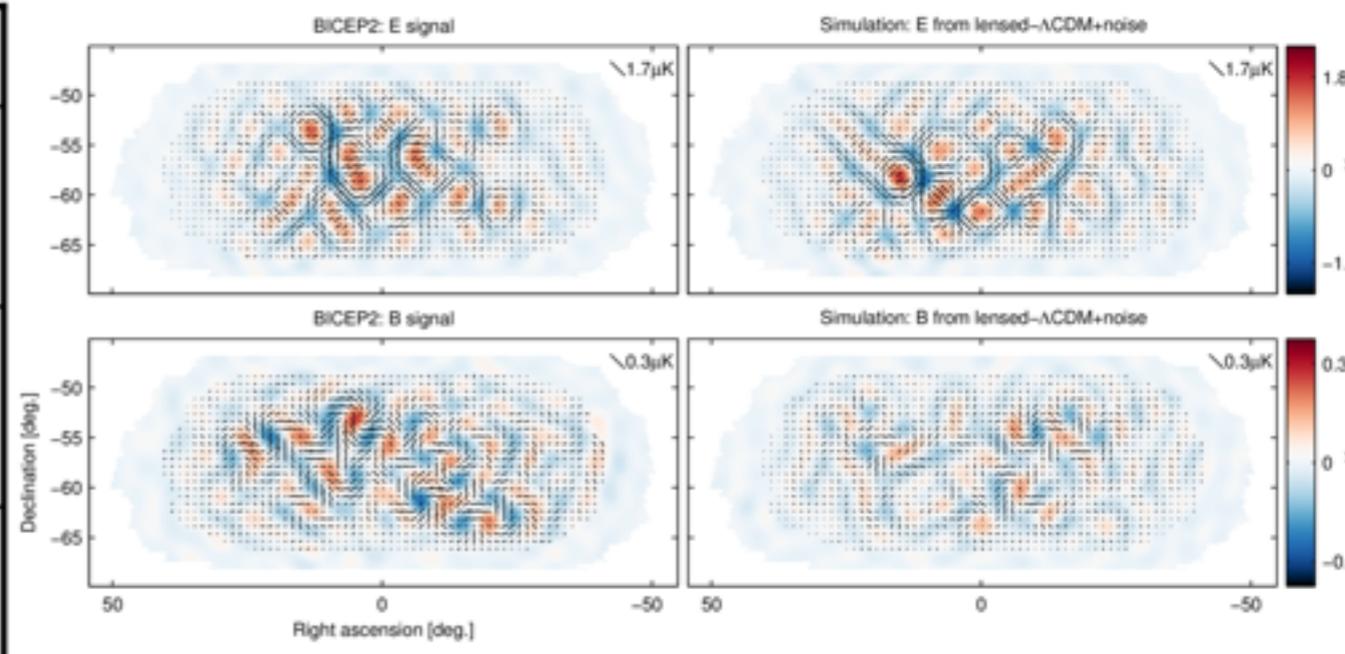
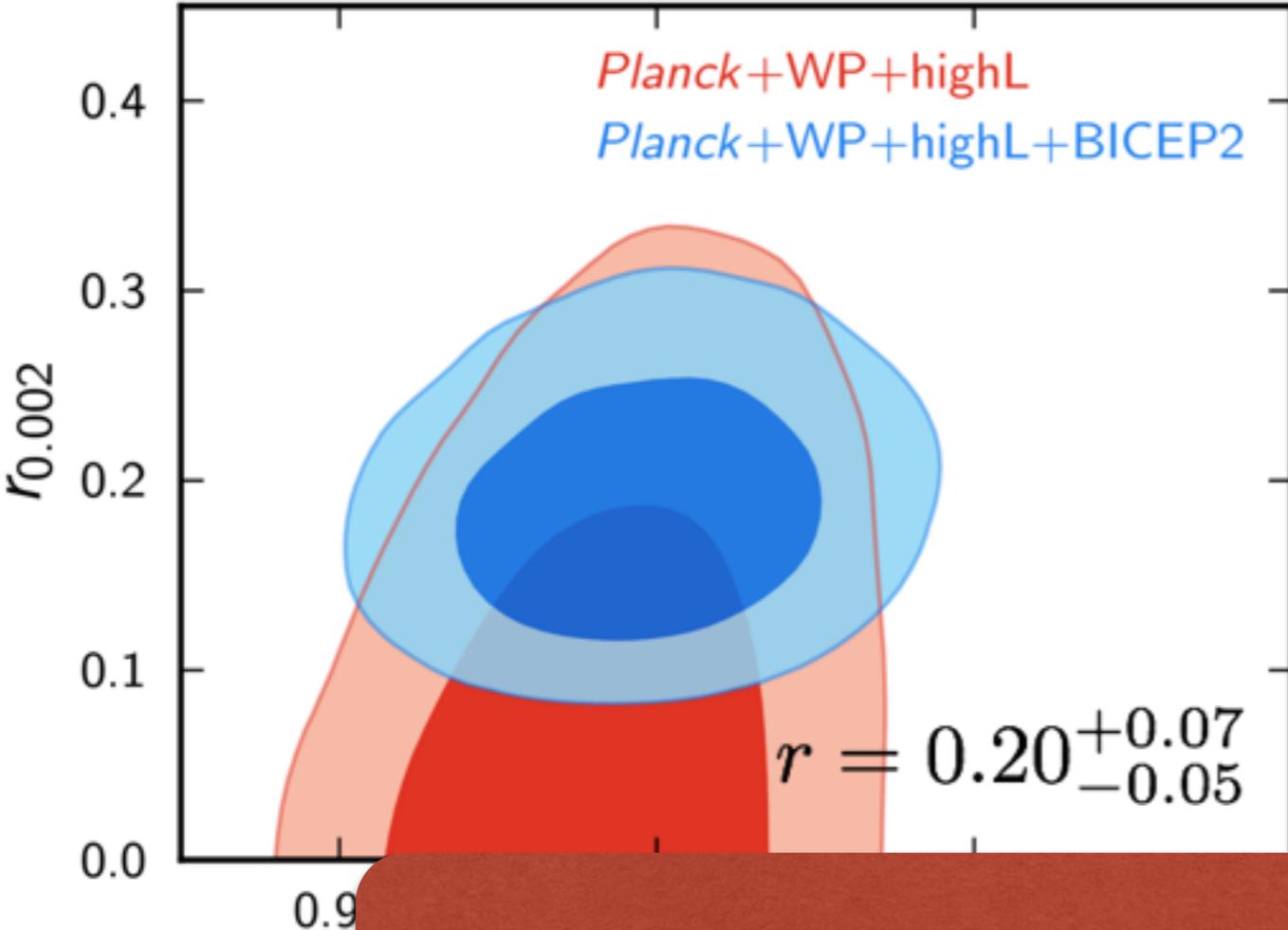
1403.3985



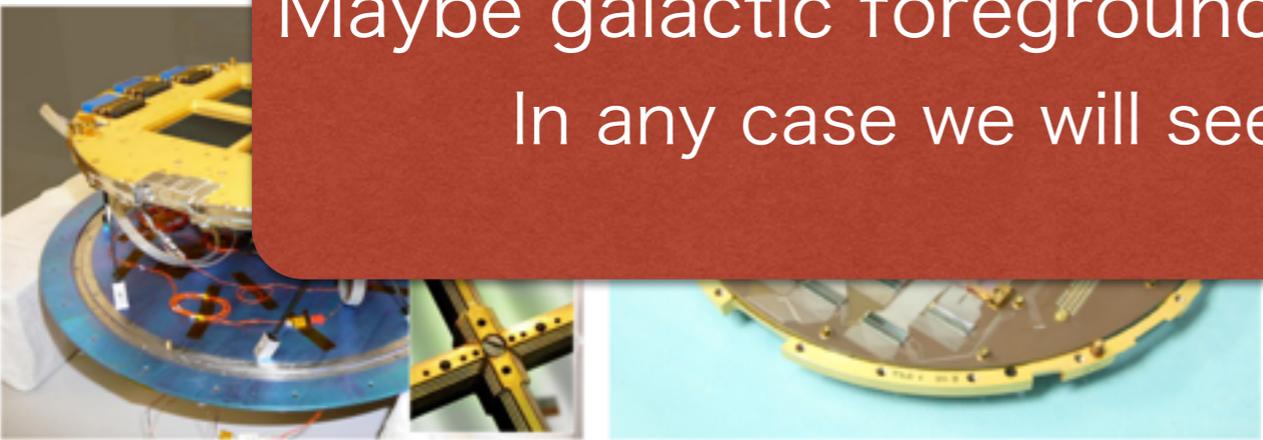


BICEP2

1403.3985



Too good to be true?
Maybe galactic foreground? (cf. Flauger et al 1405.7351)
In any case we will see in the near future.



What if $r = 0(10^{-3}-10^{-1})$?

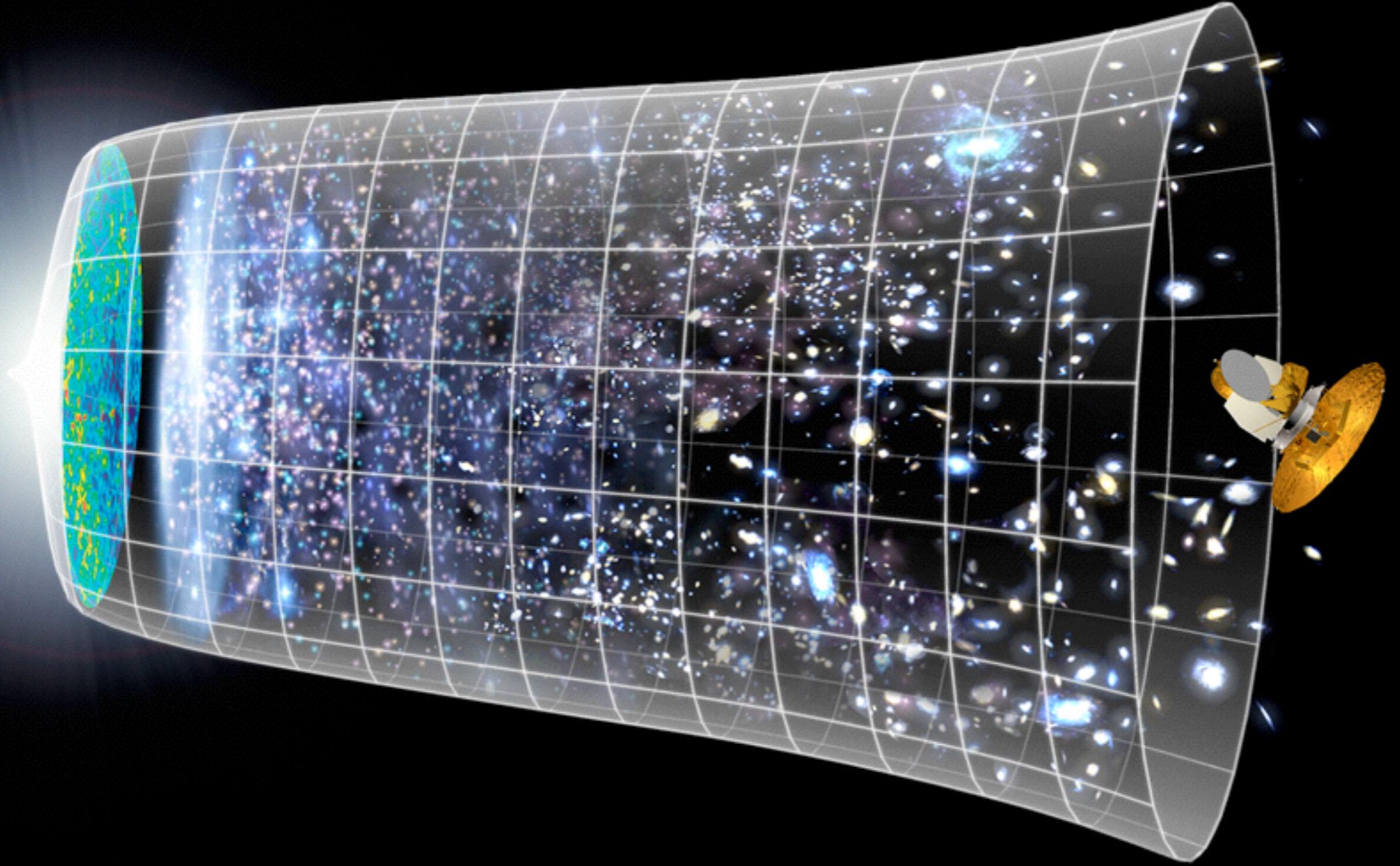
What can we say about inflation?



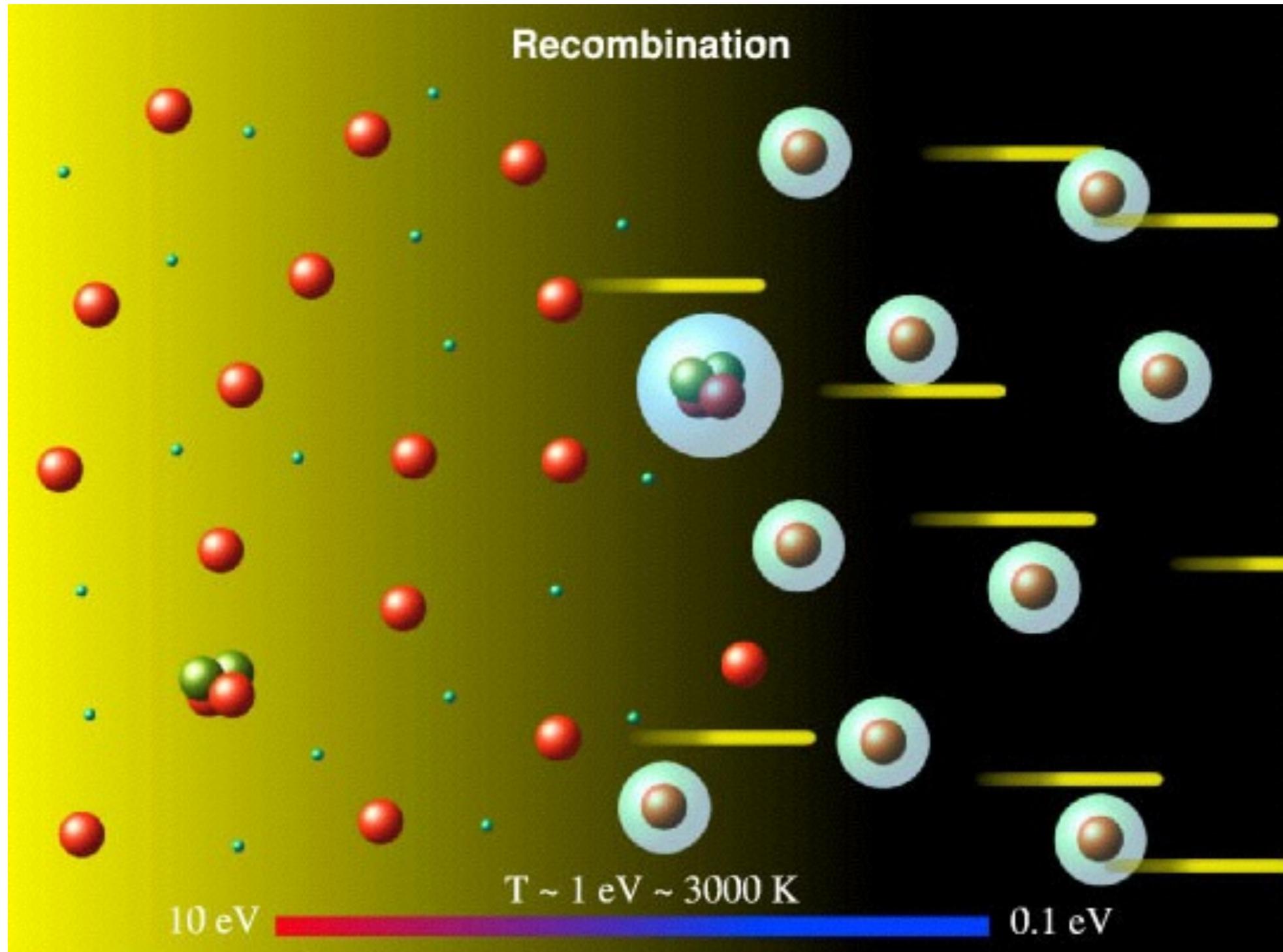
Talk plan

1. What did BICEP2 find?
2. Inflation models
 1. Chaotic inflation
 2. Natural and Multi-Natural Inflation
3. QCD axion and isocurvature perturbations (時間があれば)
4. Conclusions

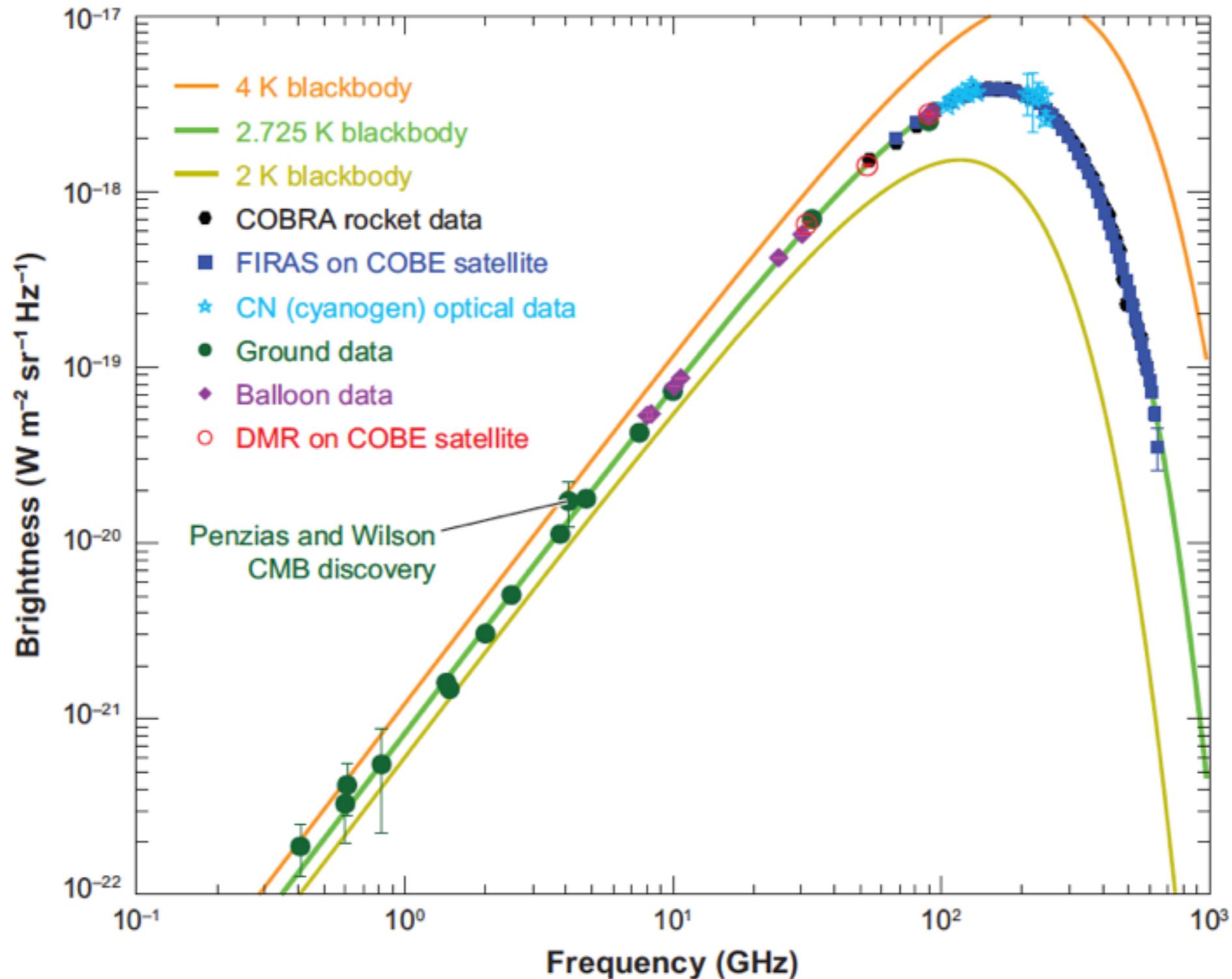
1. What did BICEP2 find?



Cosmic Microwave Background



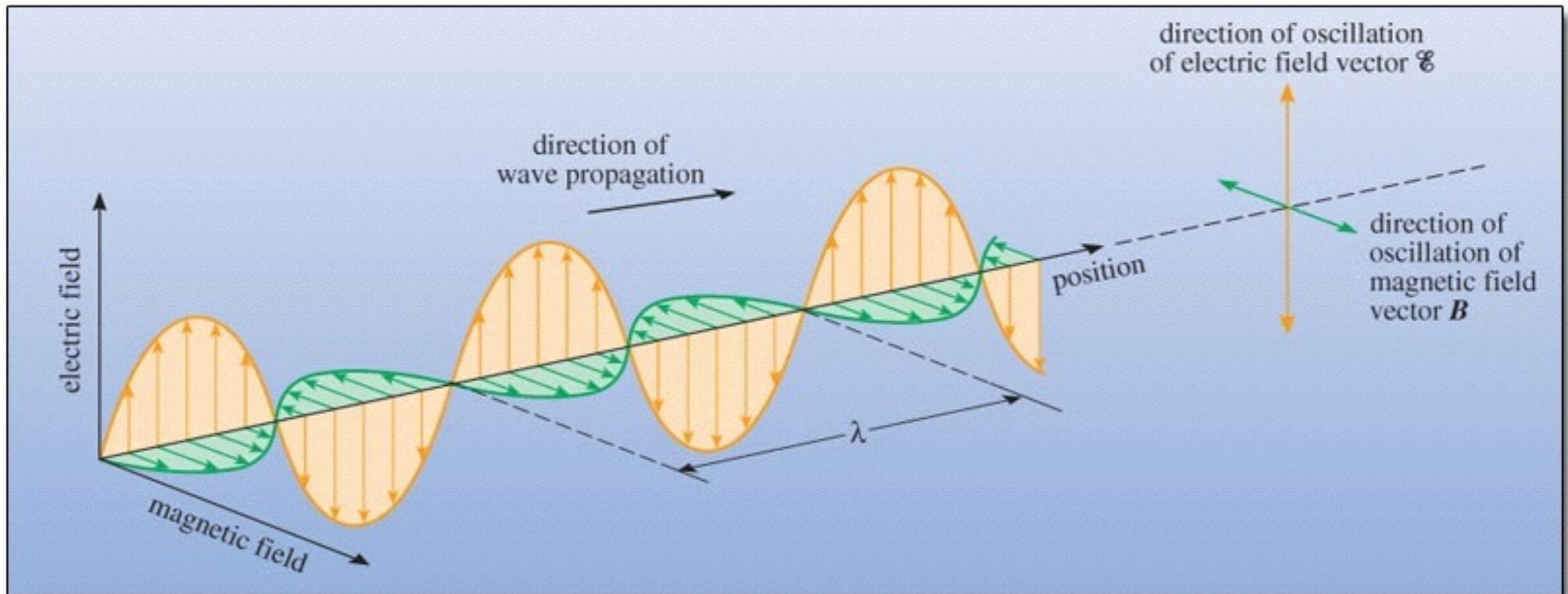
Planck distribution of CMB



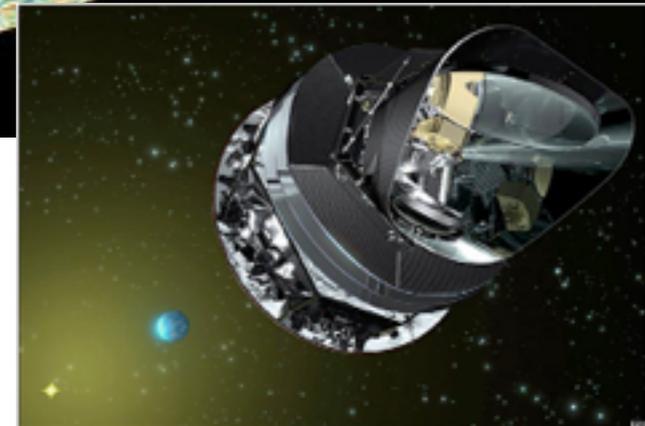
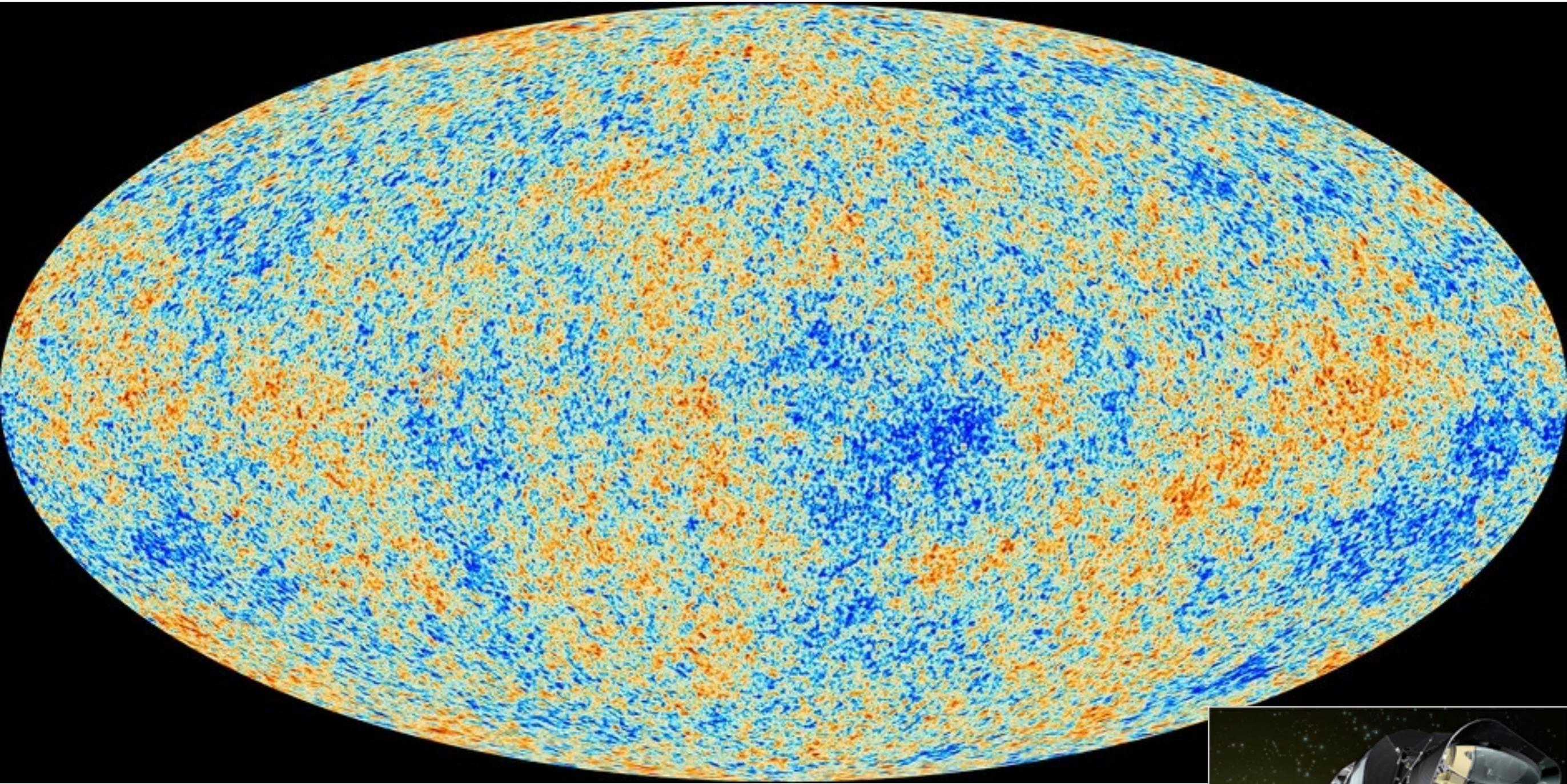
Our Universe was in equilibrium.

CMB photons are characterized by

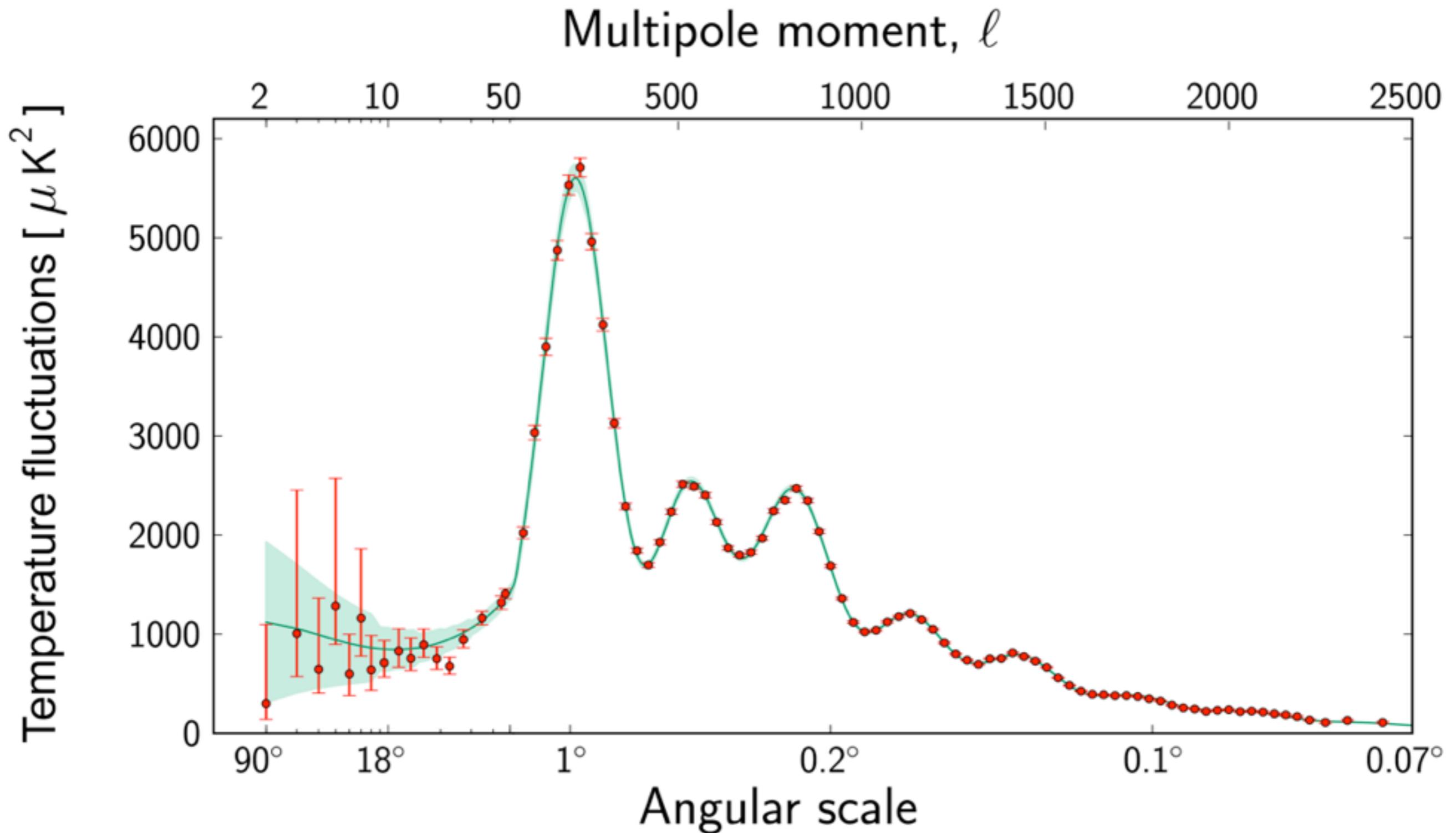
1. Energy (or temperature)
2. Polarization



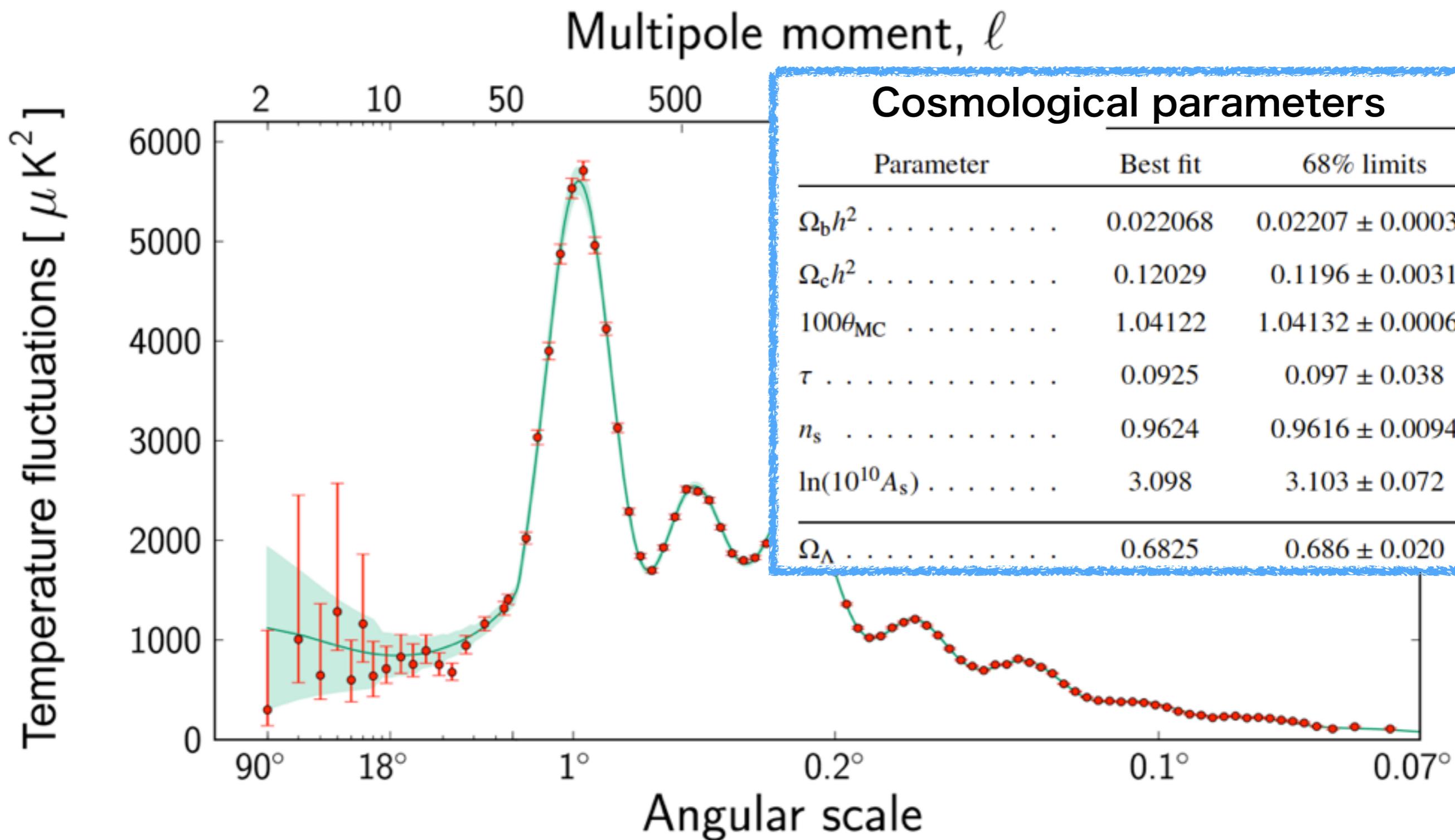
CMB temperature sky map



CMB anisotropy angular power spectrum

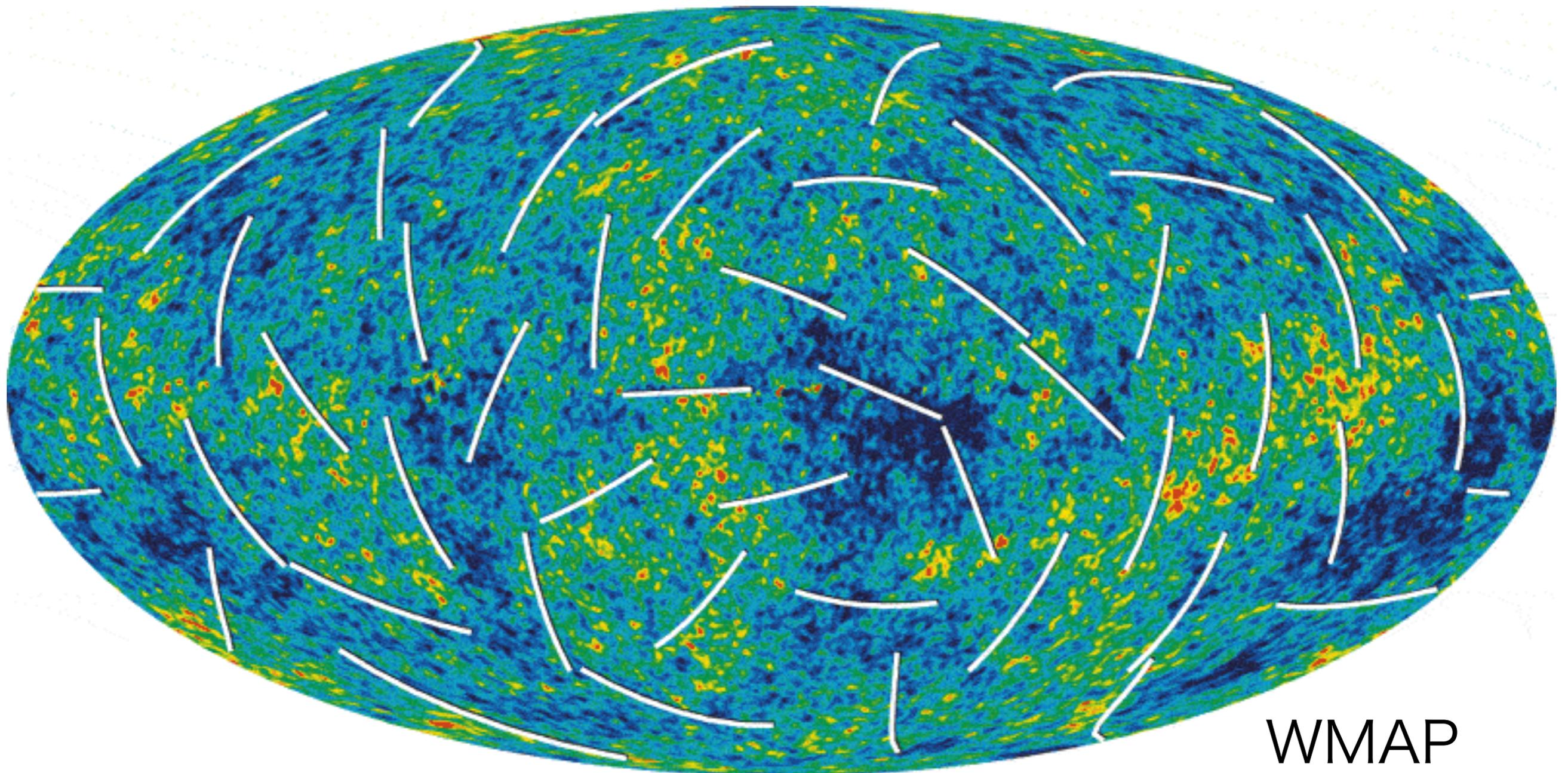


CMB anisotropy angular power spectrum



CMB polarization

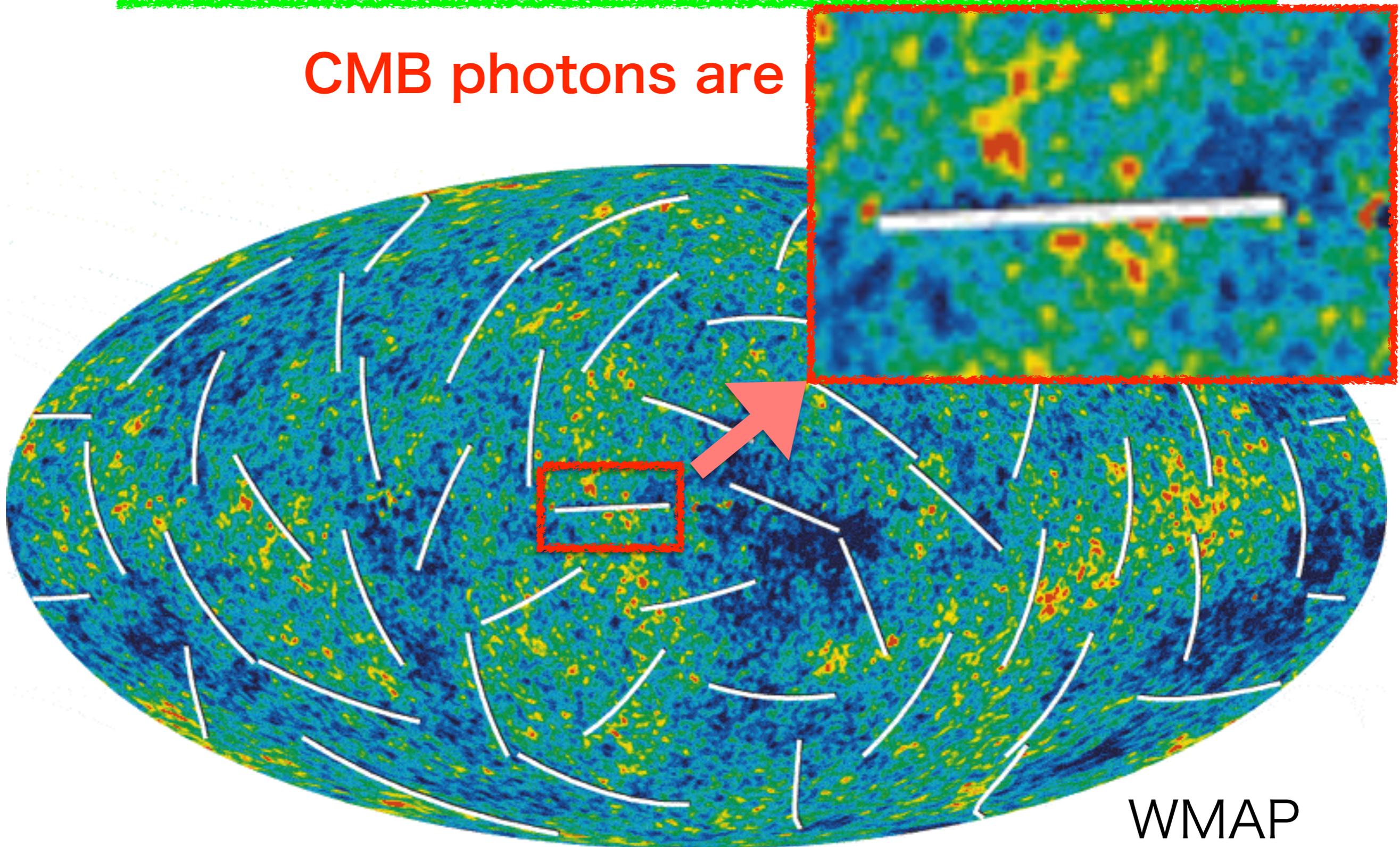
CMB photons are polarized!!



WMAP

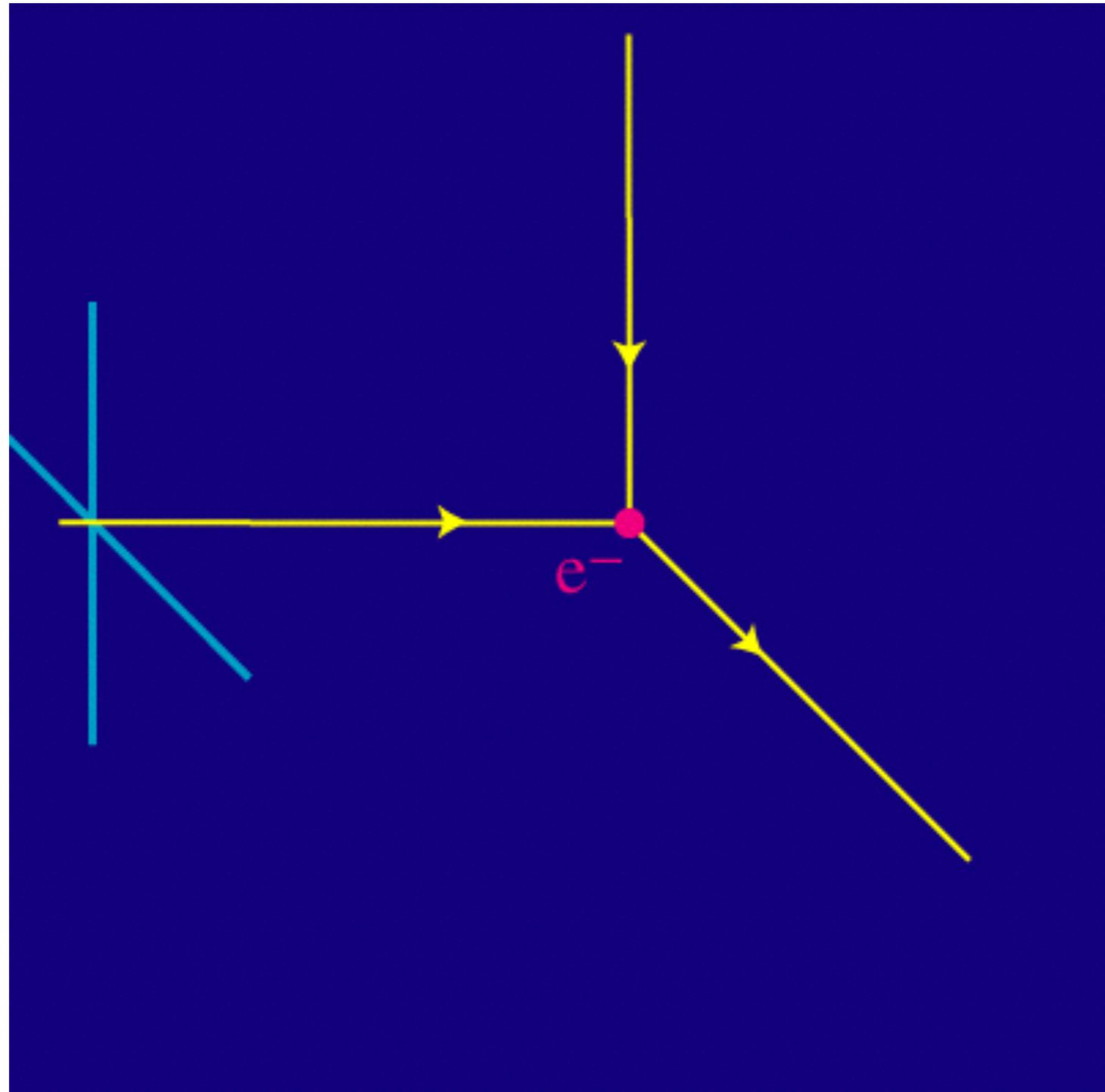
CMB polarization

CMB photons are



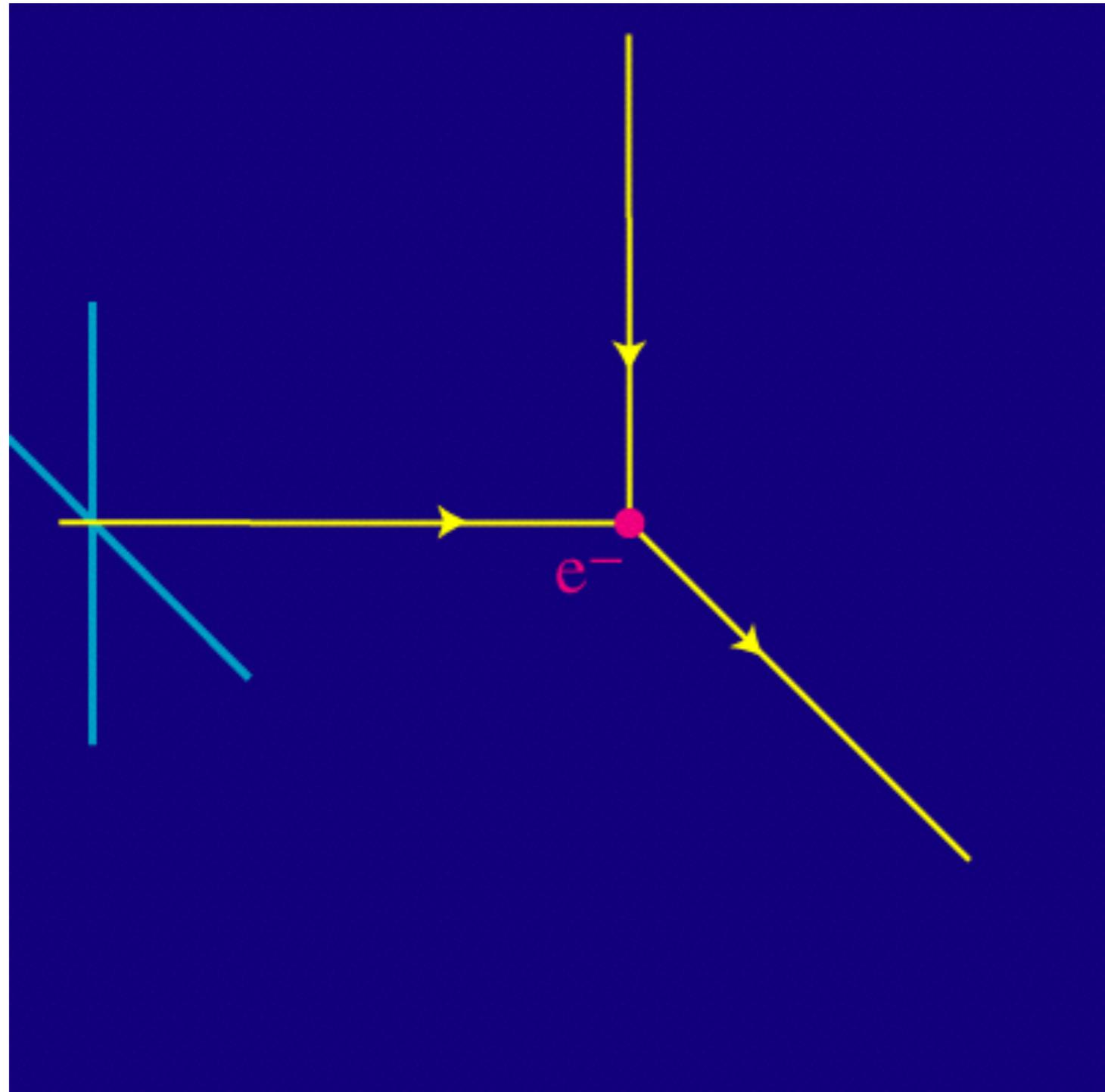
WMAP

Polarization can be easily generated
by Thomson scatterings.



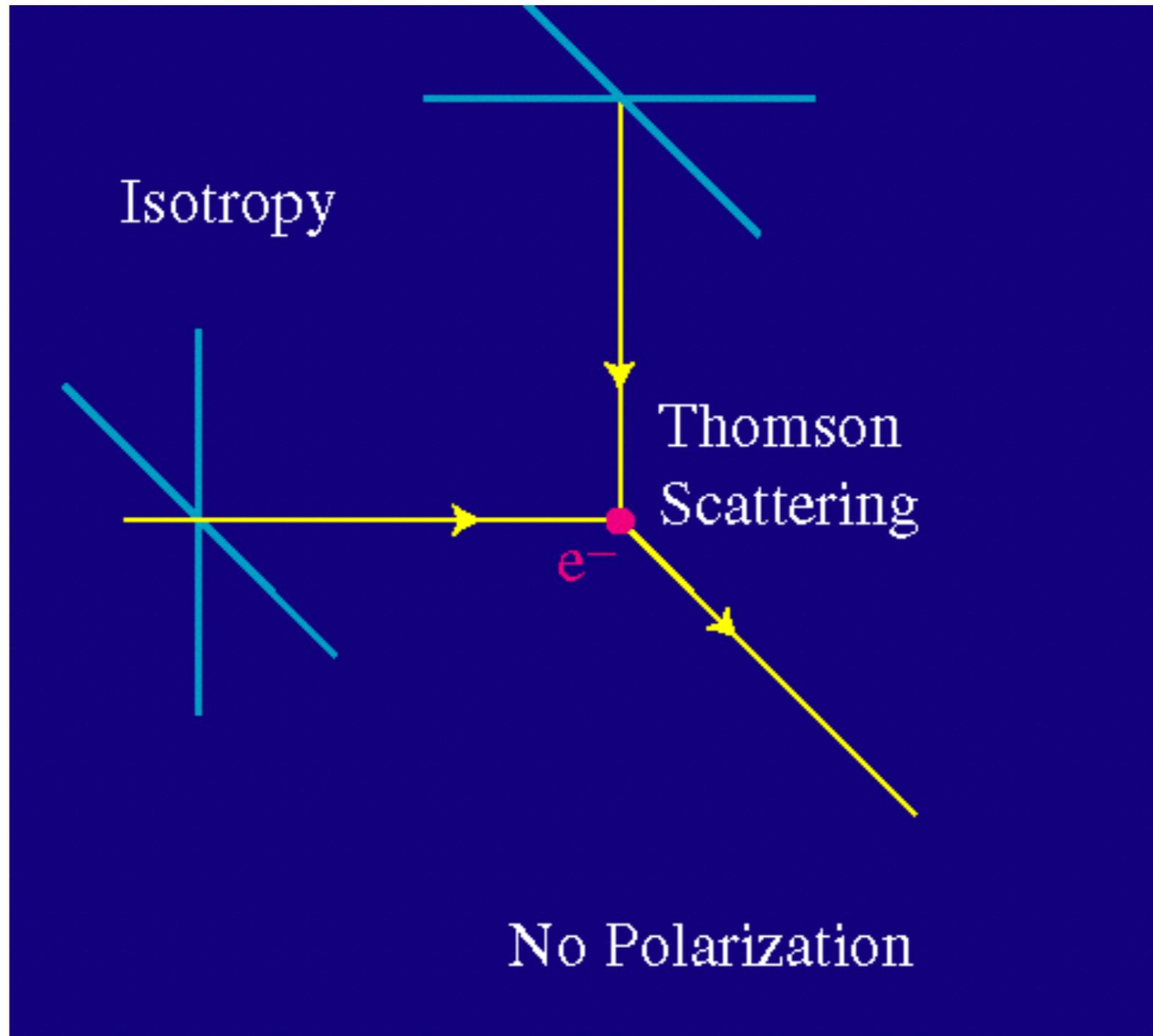
(Taken from W. Hu's webpage)

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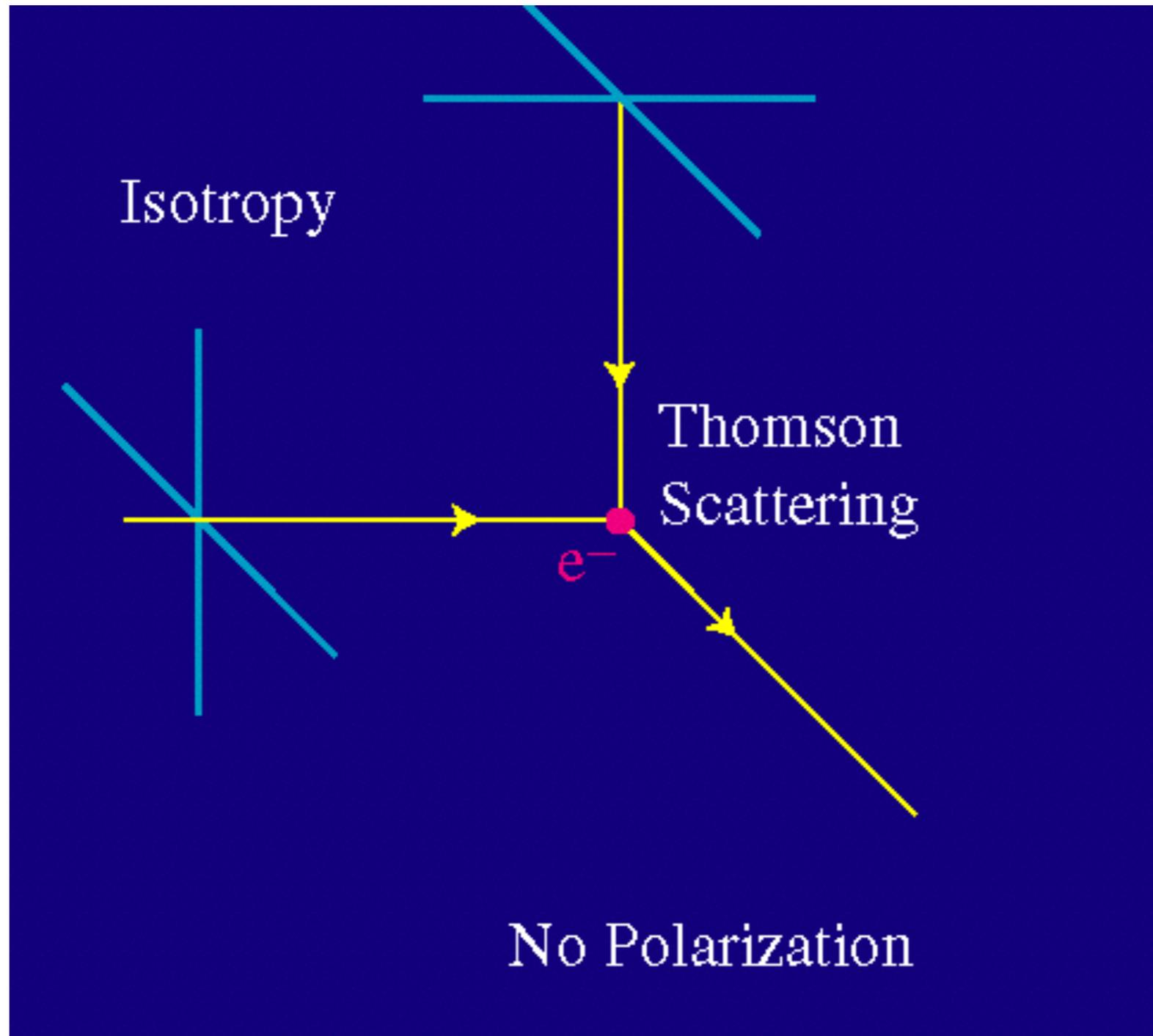
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No polarization is generated if the Universe was exactly isotropic.



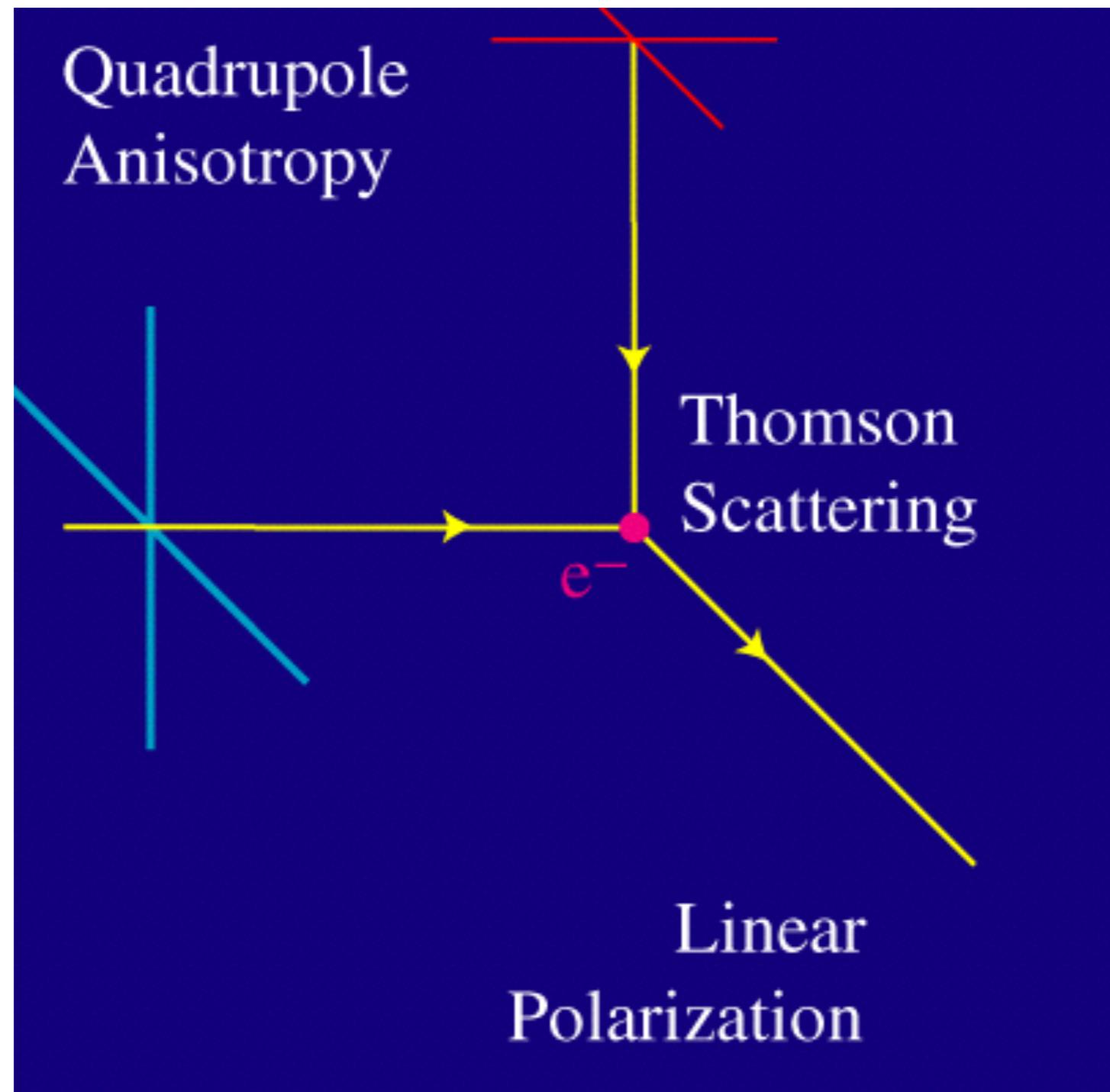
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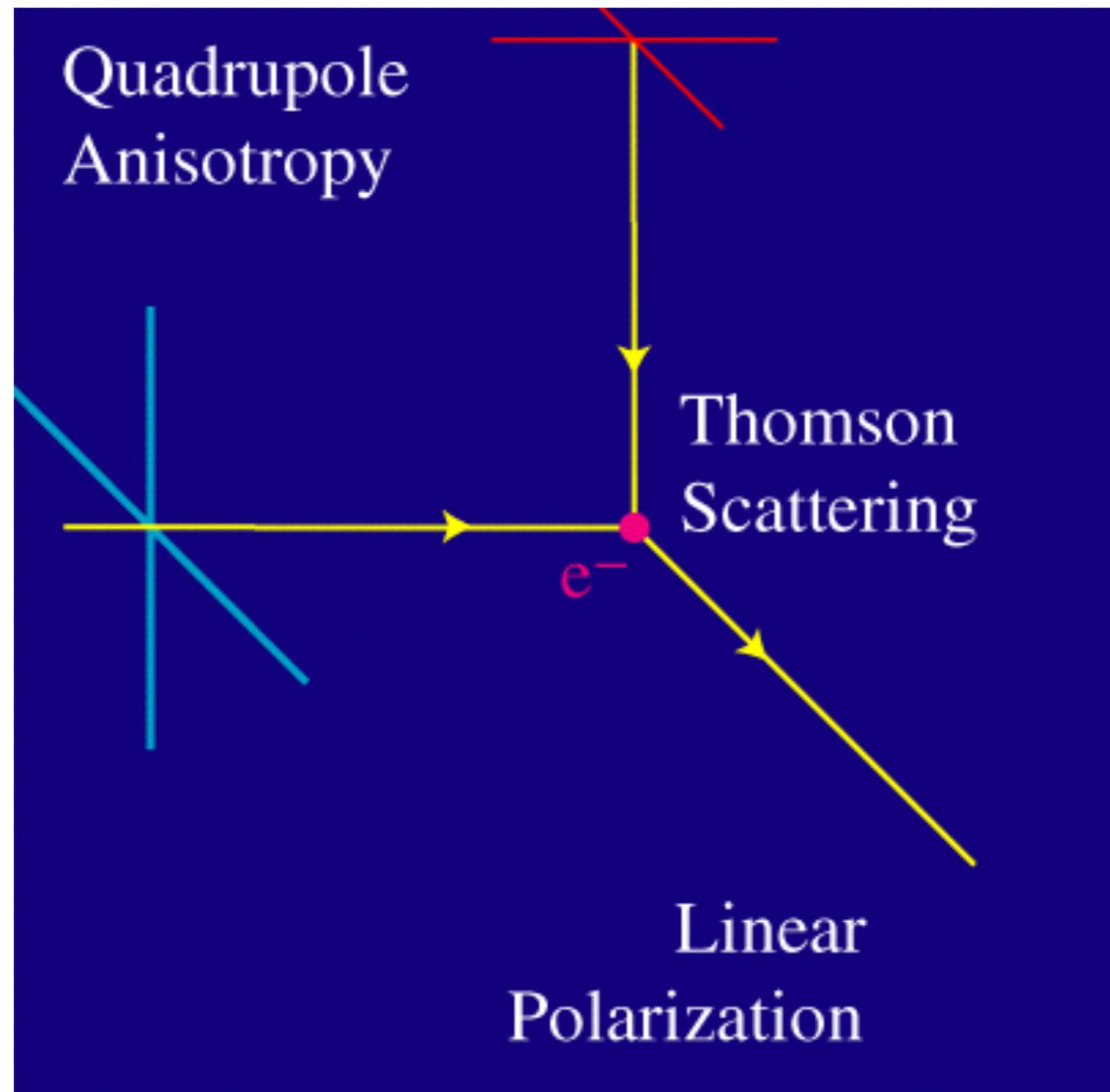
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Polarization is due to quadrupole anisotropy around electrons at recombination

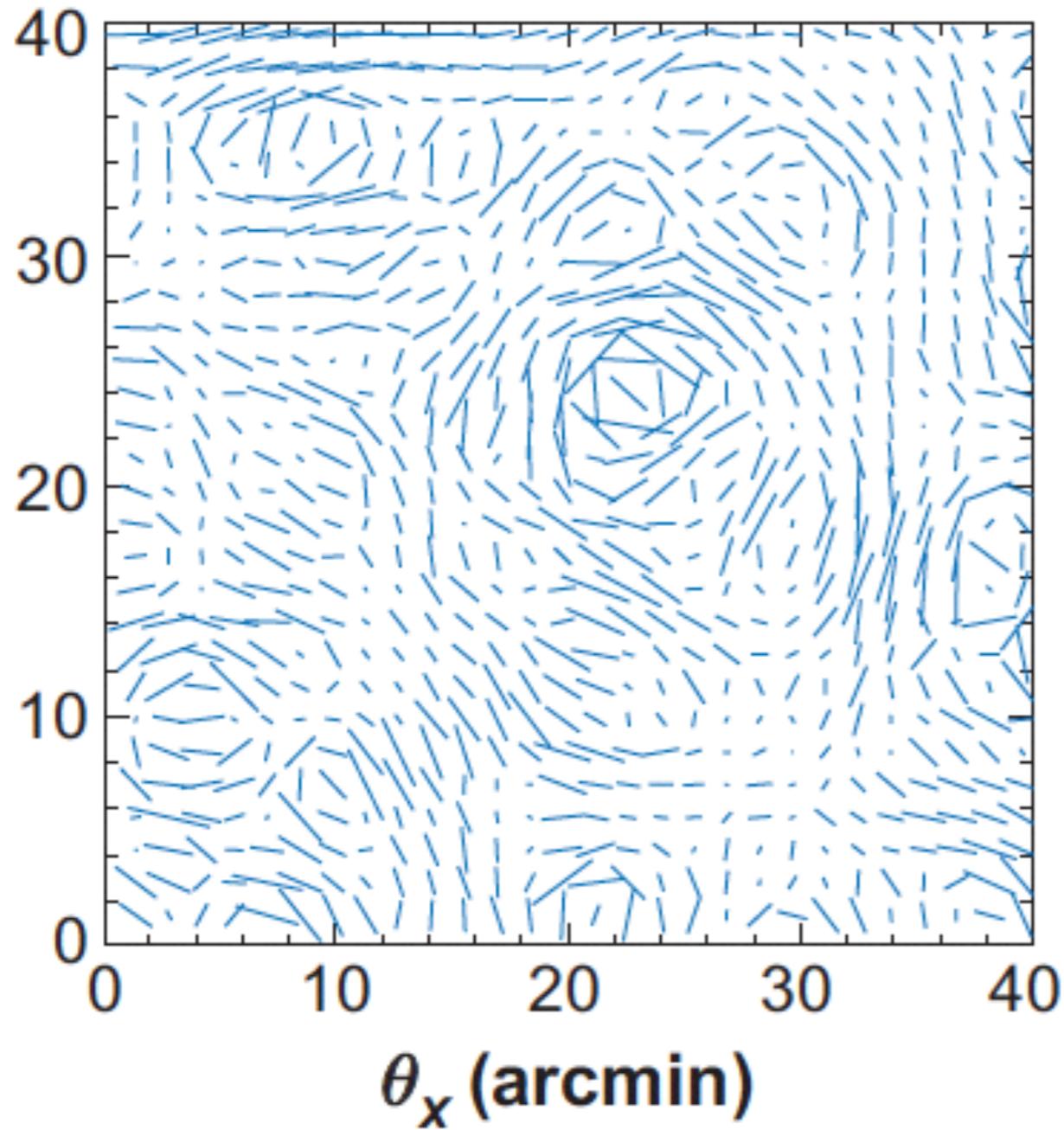
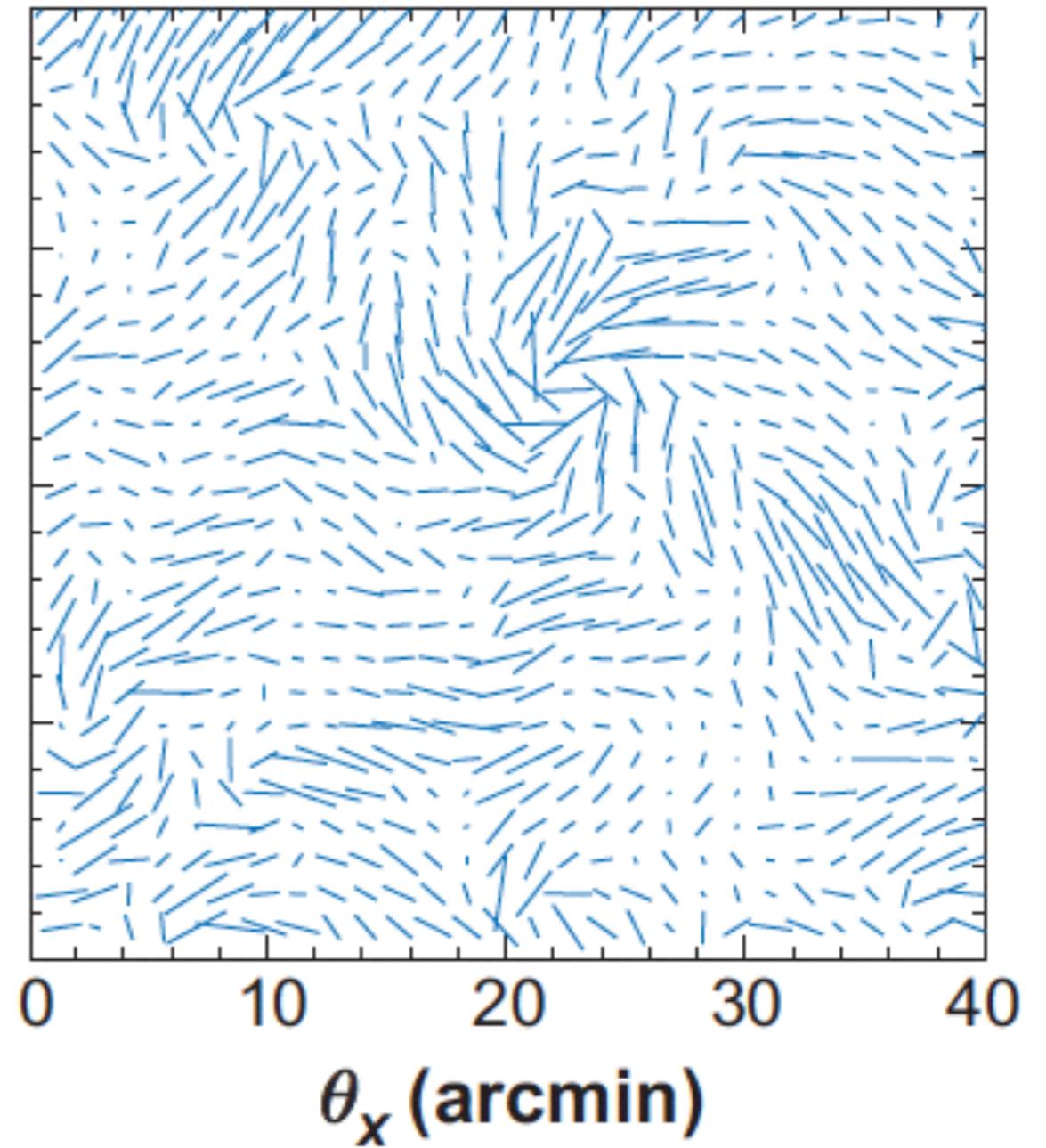


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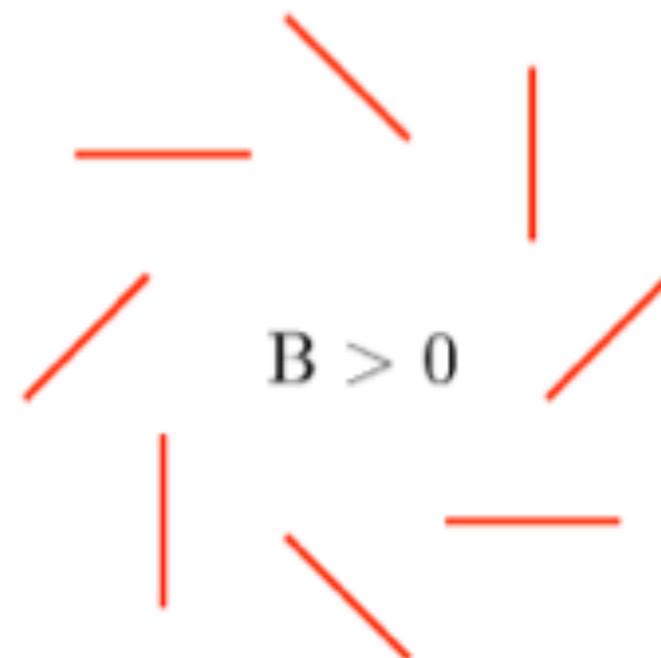
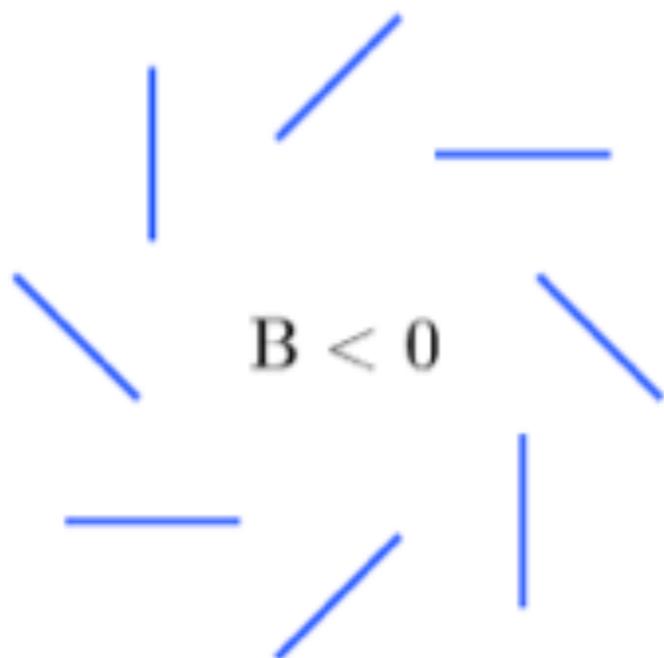
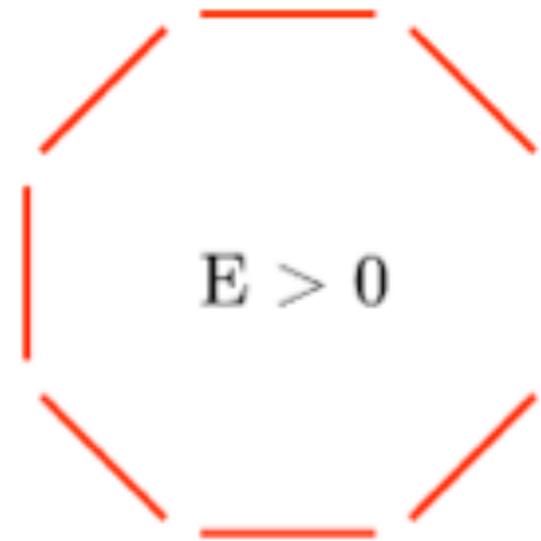
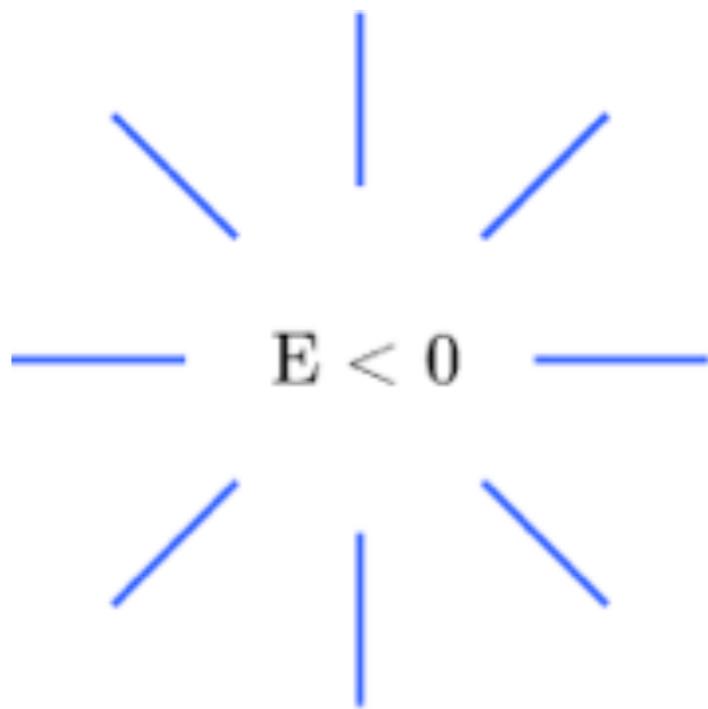
Polarization is due to quadrupole anisotropy around electrons at recombination



(Taken from W. Hu's webpage)

b**E mode****B mode**

(Taken from Samtleben et al, '07)



E-mode and B-mode are exchanged by rotating the polarization vector by 45 degrees.

Perturbations of spacetime

Flat FRW Universe:

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

Perturbations of spacetime

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$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

+ small perturbations

$$ds^2 = -(1 + 2A)dt^2 - 2aB_i dt dx^i + a^2 (\delta_{ij} + 2H_L \delta_{ij} + 2H_T ij) dx^i dx^j$$

Perturbations of spacetime

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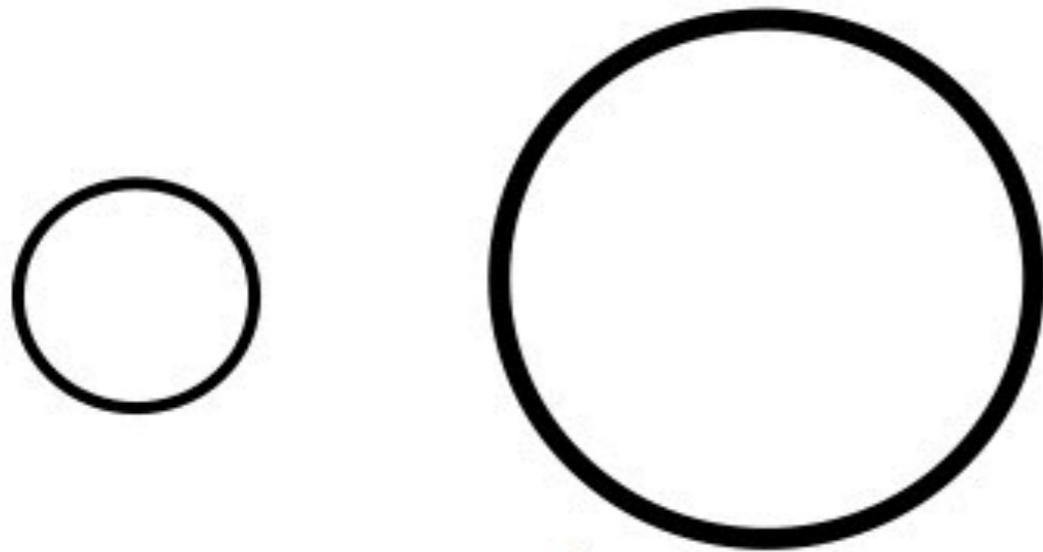
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The perturbations can be decomposed into three types.

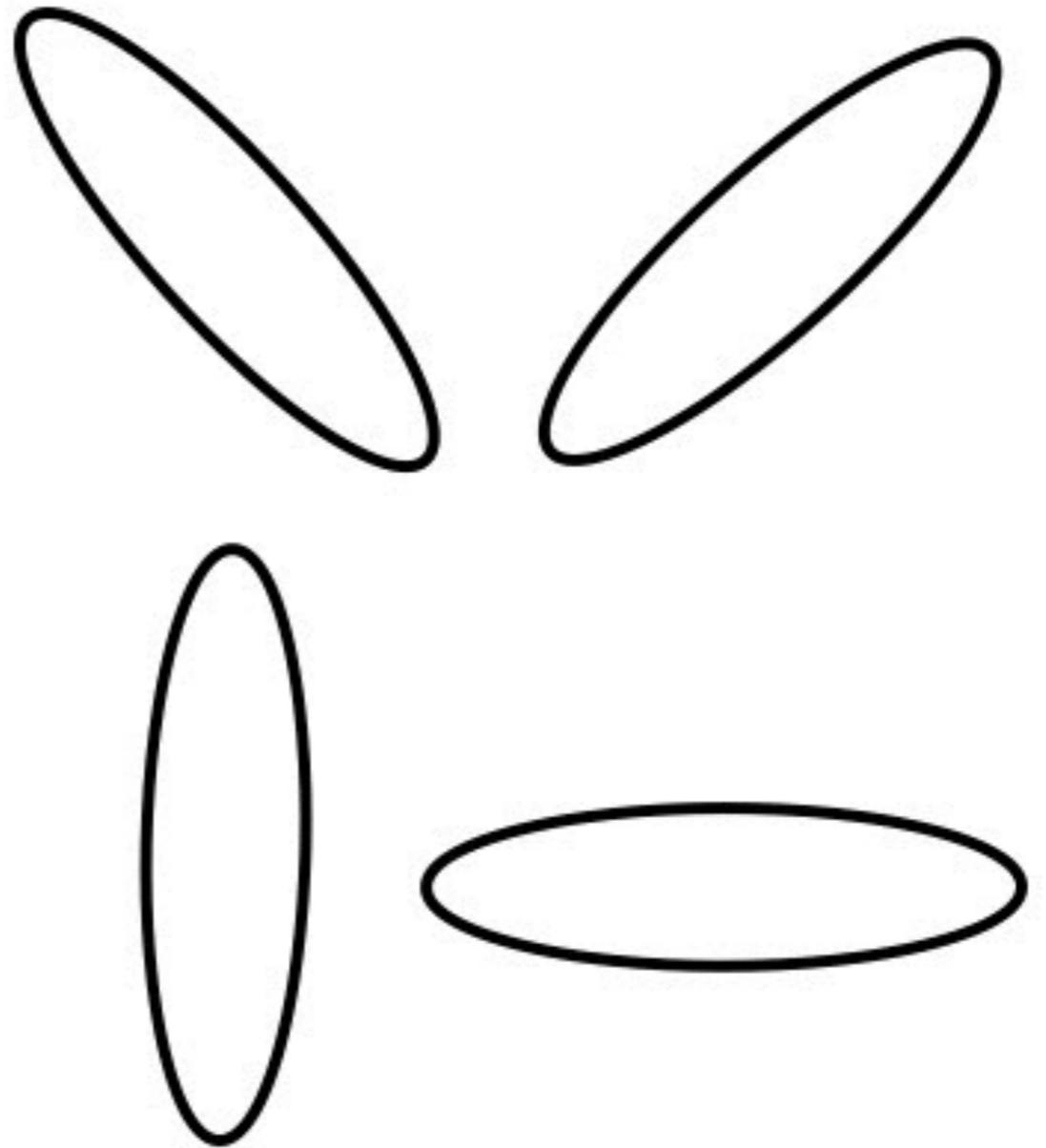
1. Scalar $ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 + 2\Psi)d\mathbf{x}^2$ **inflaton**
2. Vector
3. Tensor $ds^2 = -dt^2 + a^2 (\delta_{ij} + h_{ij}) dx^i dx^j$ **GW**

Scalar perturbations



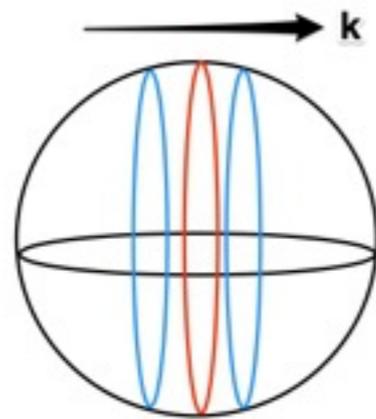
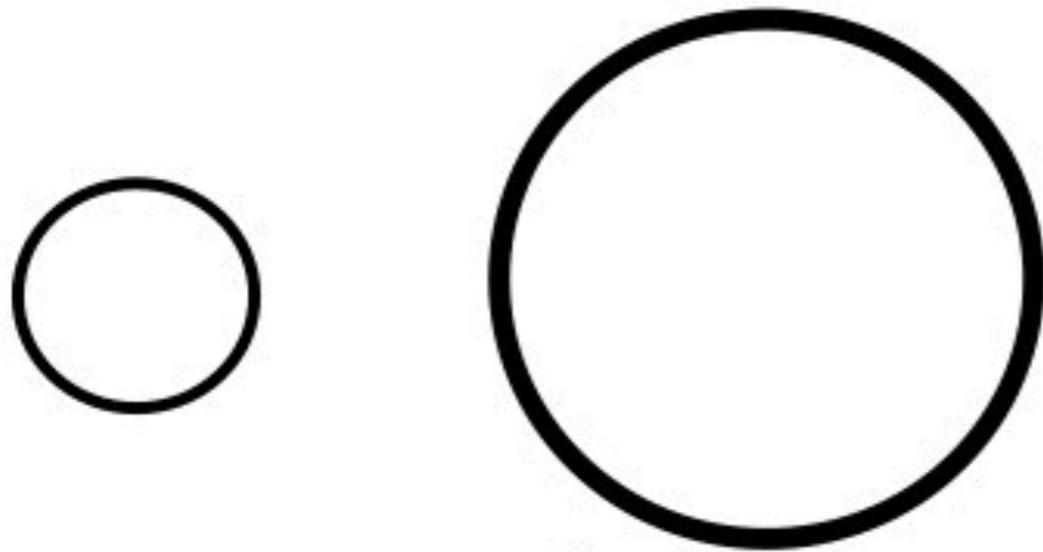
E-mode ONLY

Tensor perturbations



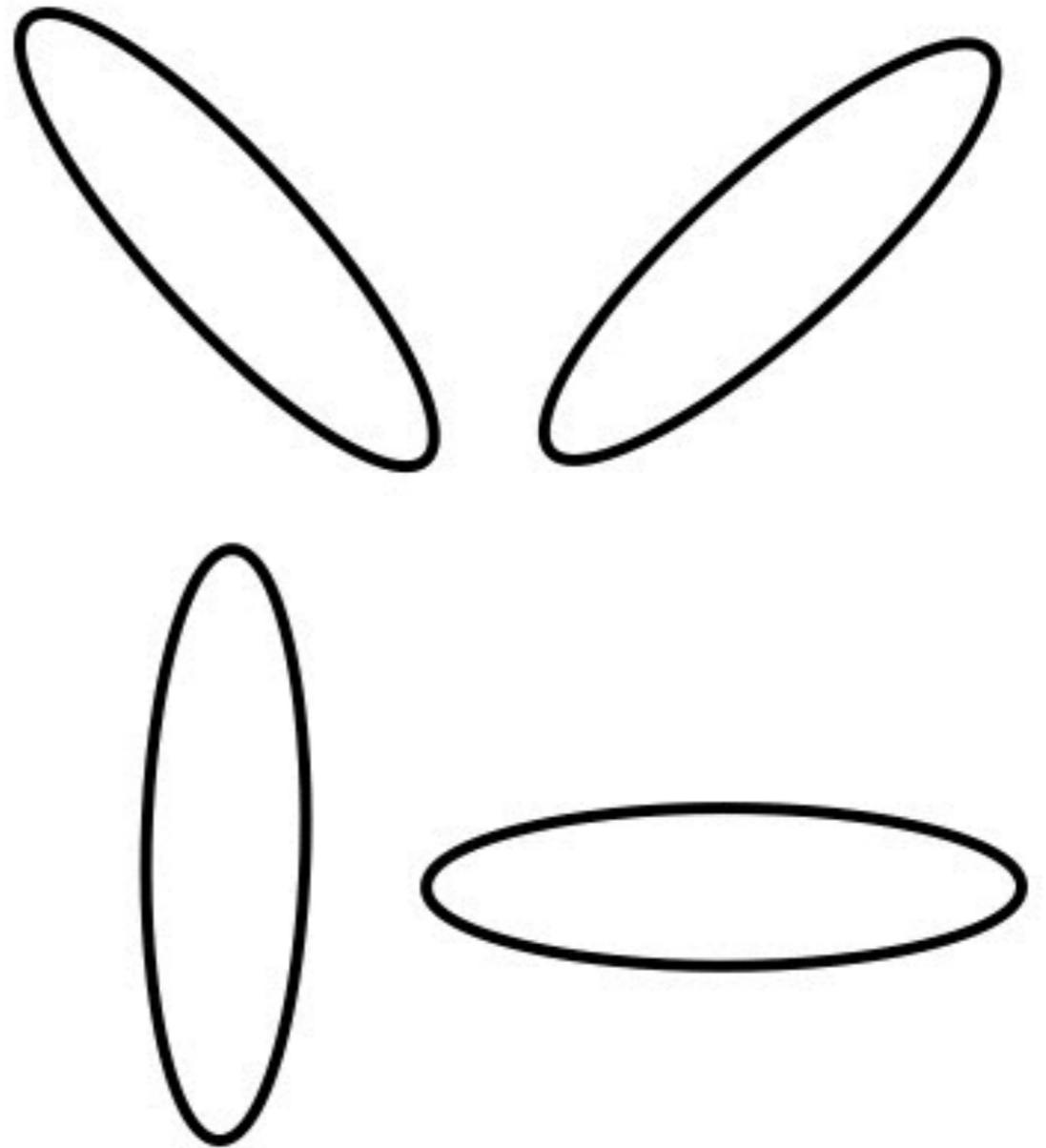
**BOTH E-mode
and B-mode**

Scalar perturbations



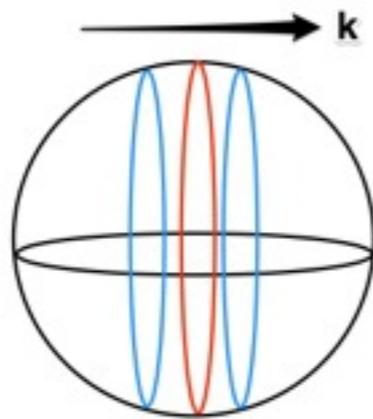
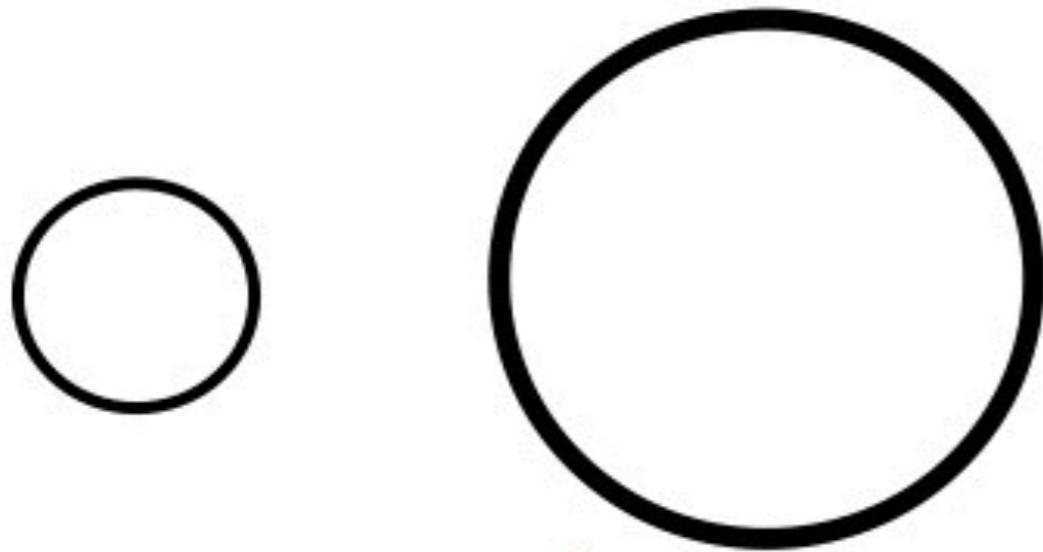
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Tensor perturbations



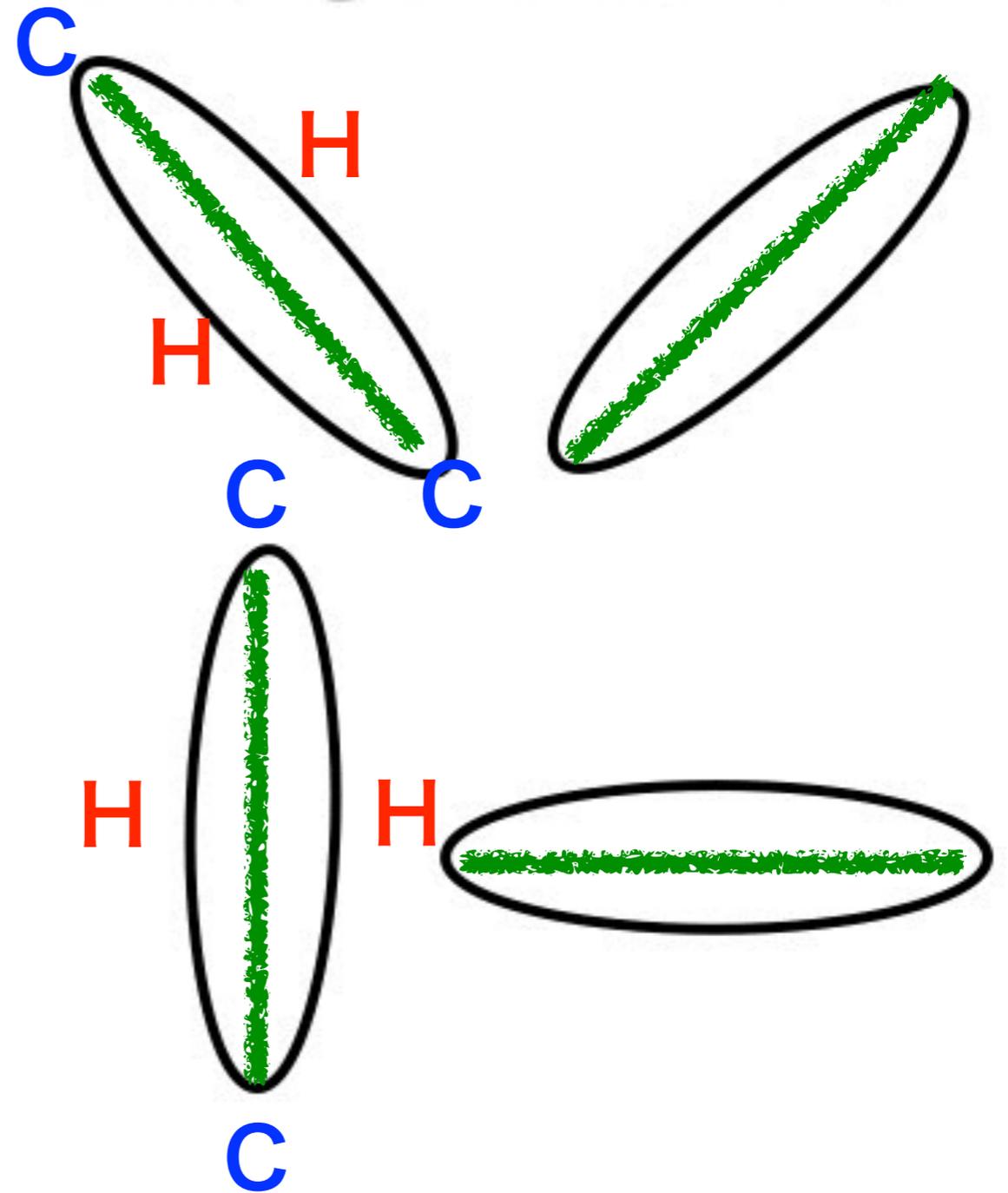
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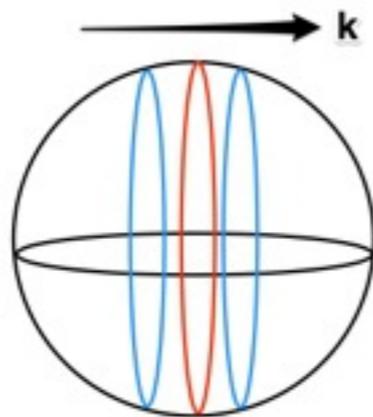
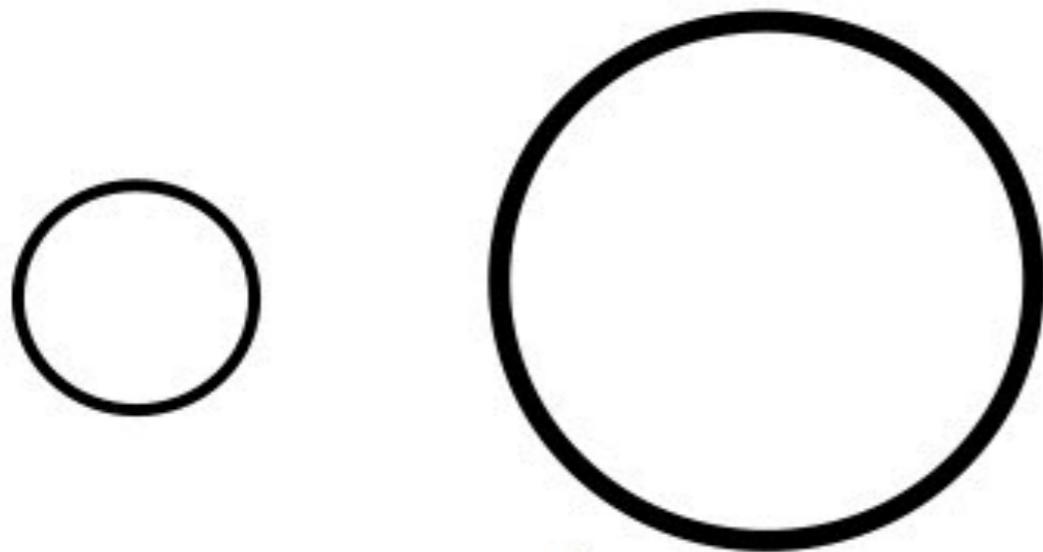
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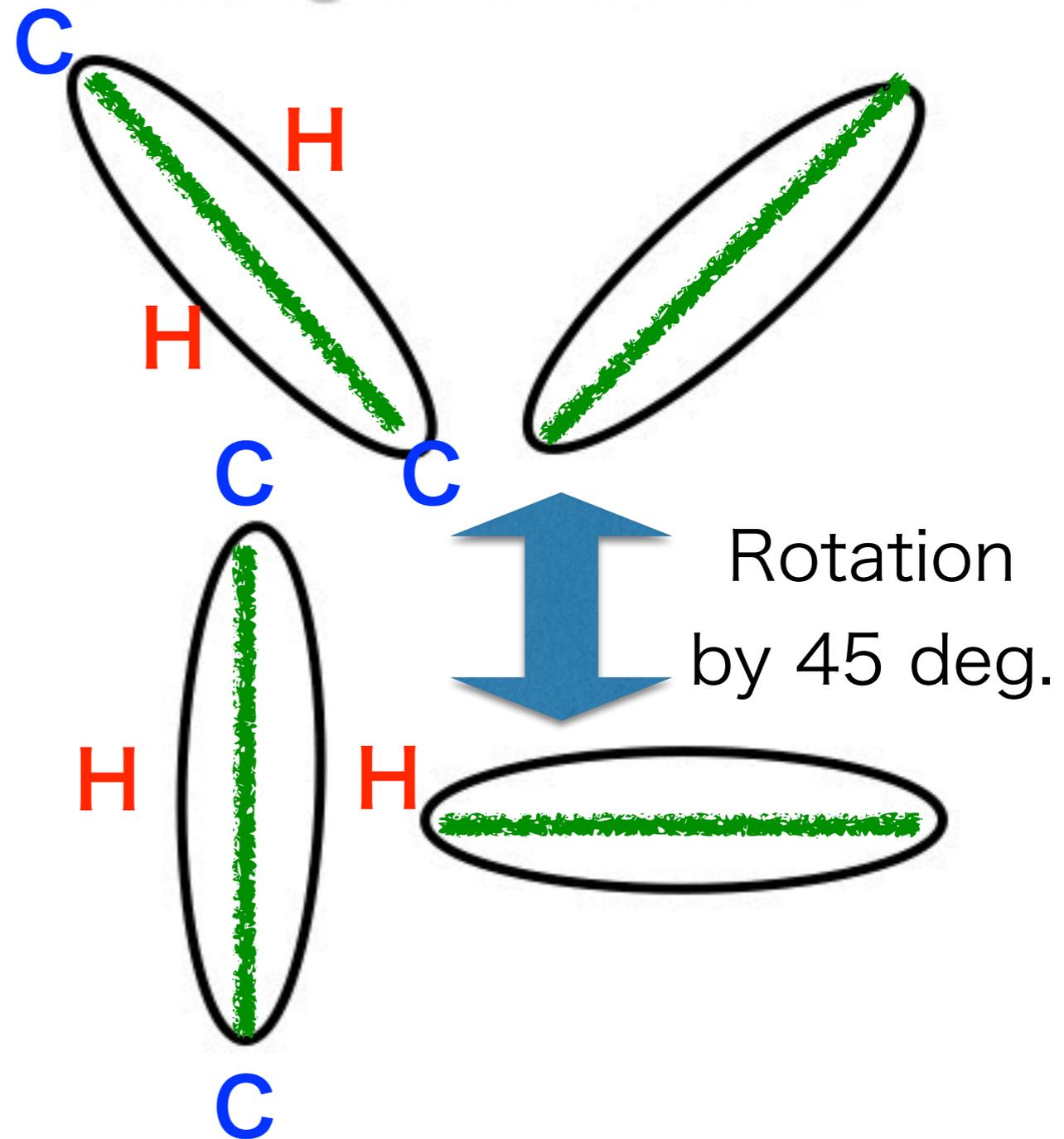
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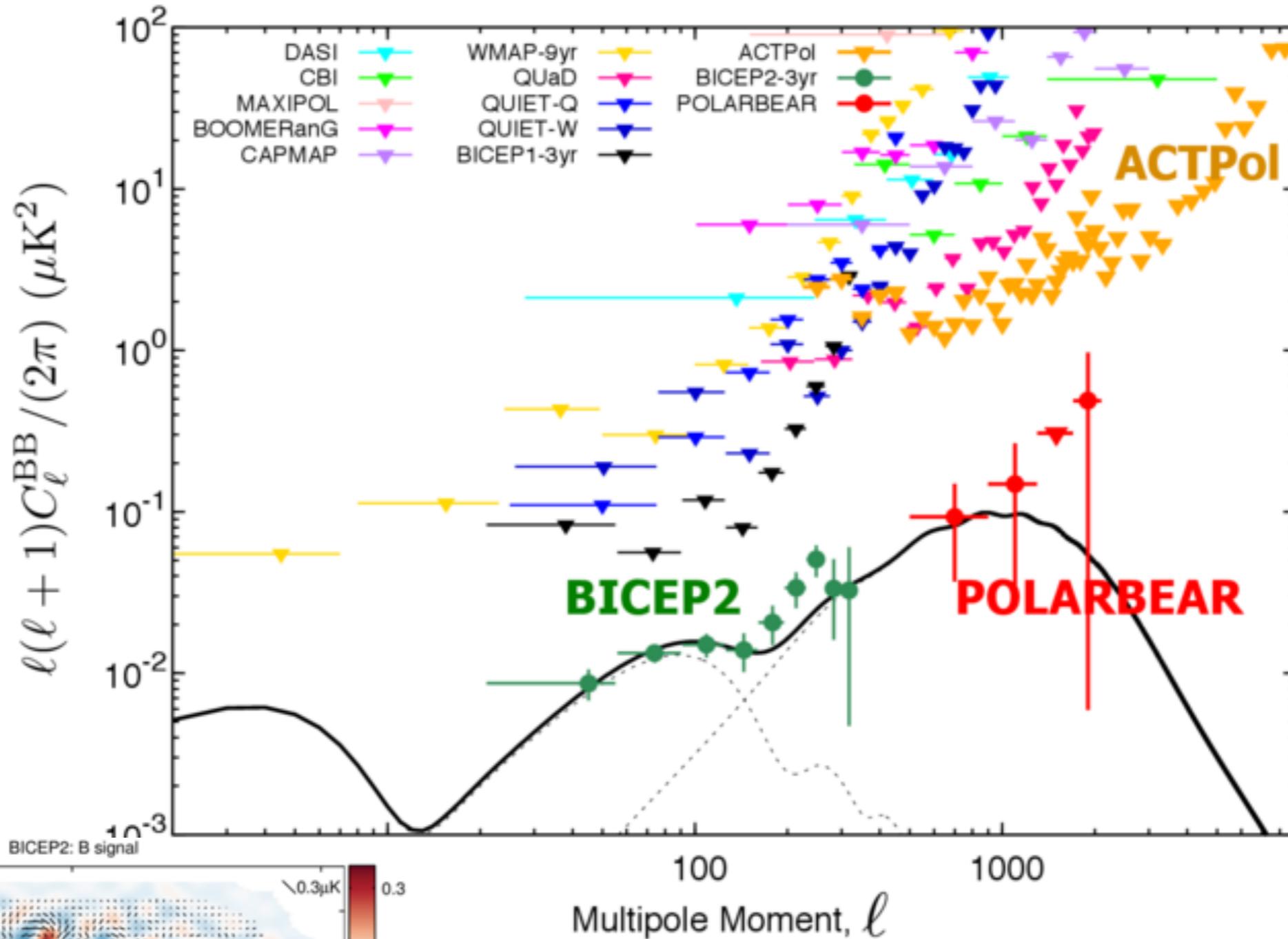
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Tensor perturbations

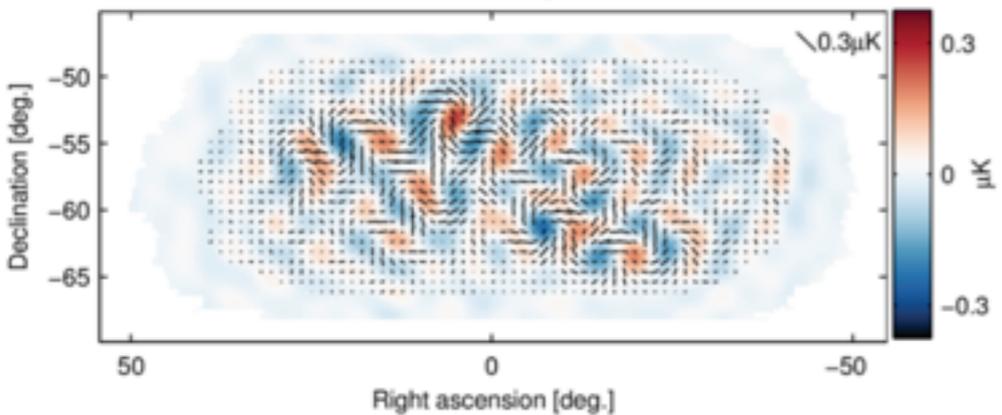


**BOTH E-mode
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BICEP2 found B-mode



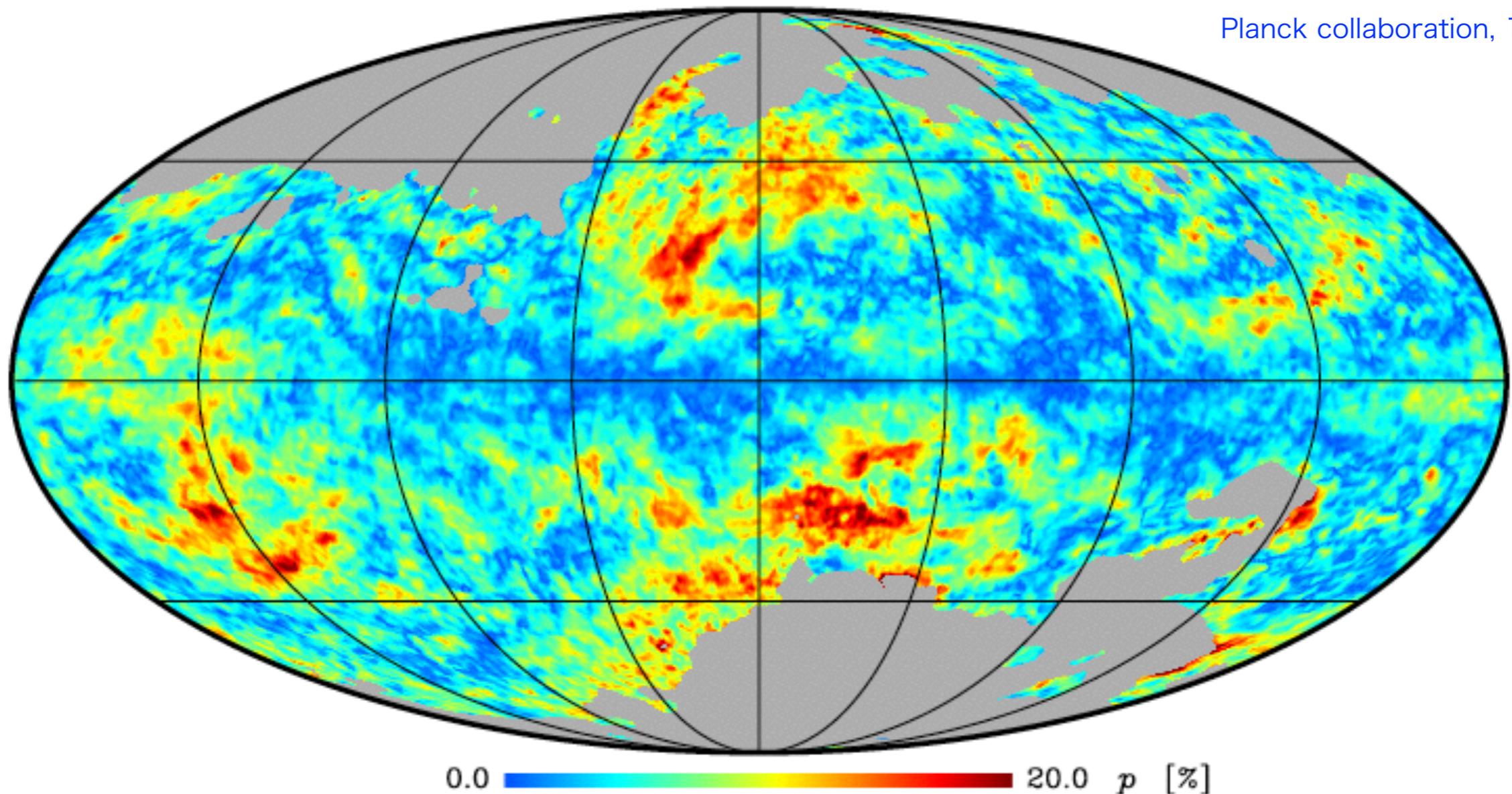
BICEP2: B signal



Taken from
Chinone's webpage

Galactic foregrounds?

Planck collaboration, 1405.0871



@353 GHz (cf. BICEP2 @150 GHz)

The polarization fraction tends to be higher where the dust emission is smaller.

2. Inflation models

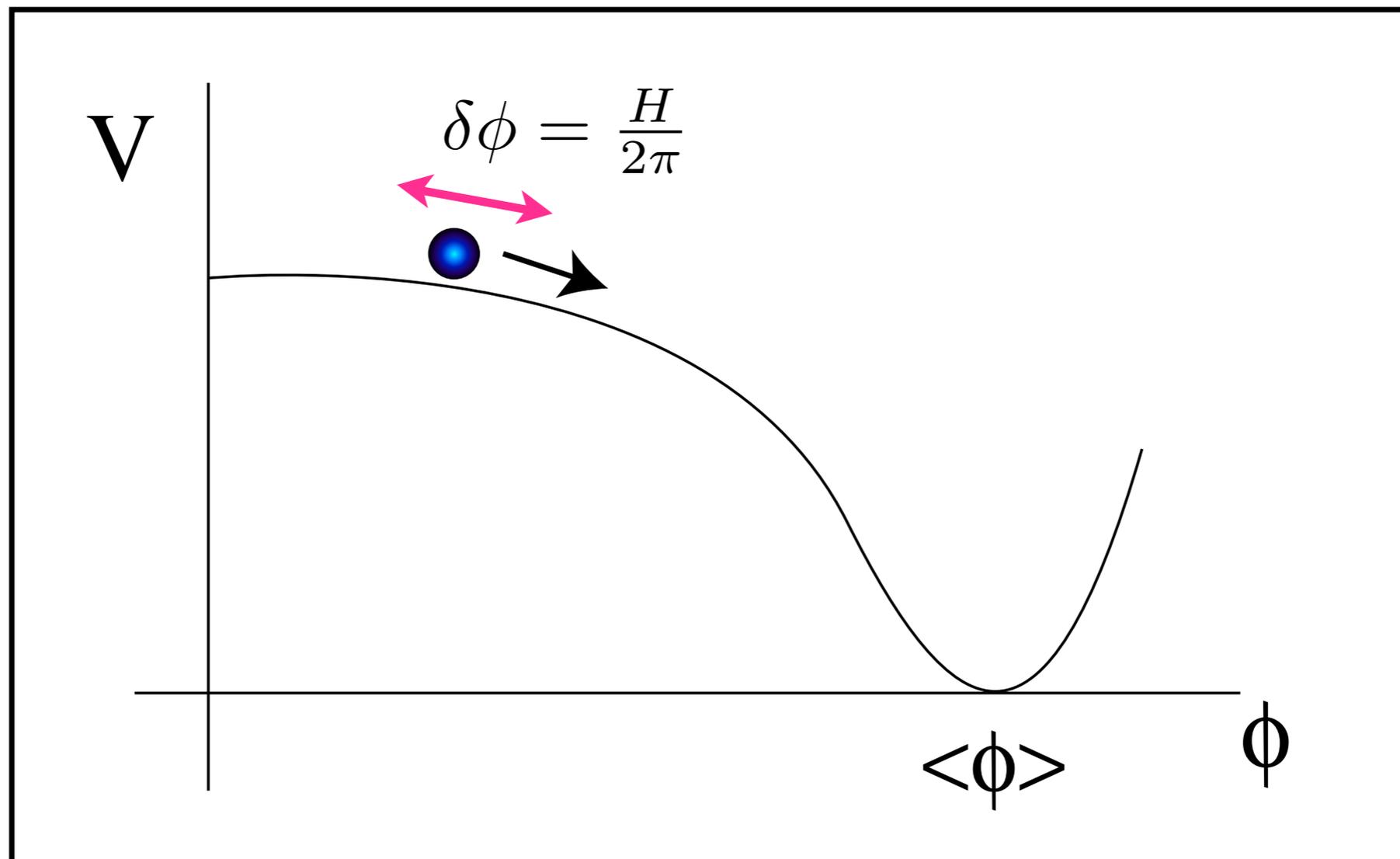
Inflation

Accelerated cosmic expansion solves various theoretical problems of the std. big bang cosmology.

Guth '81, Sato '80, Starobinsky '80, Kazanas '80, Brout, Englert, Gunzig, '79

One way to realize the inflationary expansion is the slow-roll inflation.

Linde '82, Albrecht and Steinhardt '82



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$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

+ small perturbations

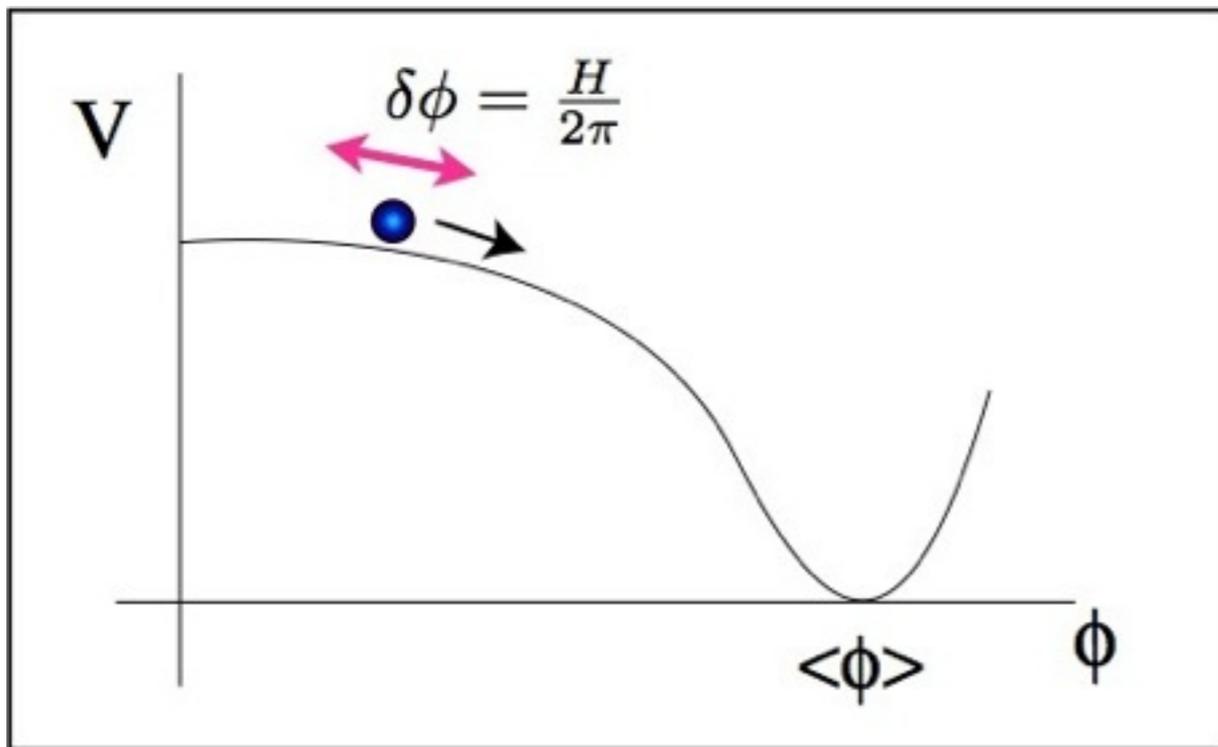
$$ds^2 = -(1 + 2A)dt^2 - 2aB_i dt dx^i + a^2 (\delta_{ij} + 2H_L \delta_{ij} + 2H_T ij) dx^i dx^j$$

The perturbations can be decomposed into three types.

1. Scalar $ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 + 2\Psi)d\mathbf{x}^2$ **Inflaton**
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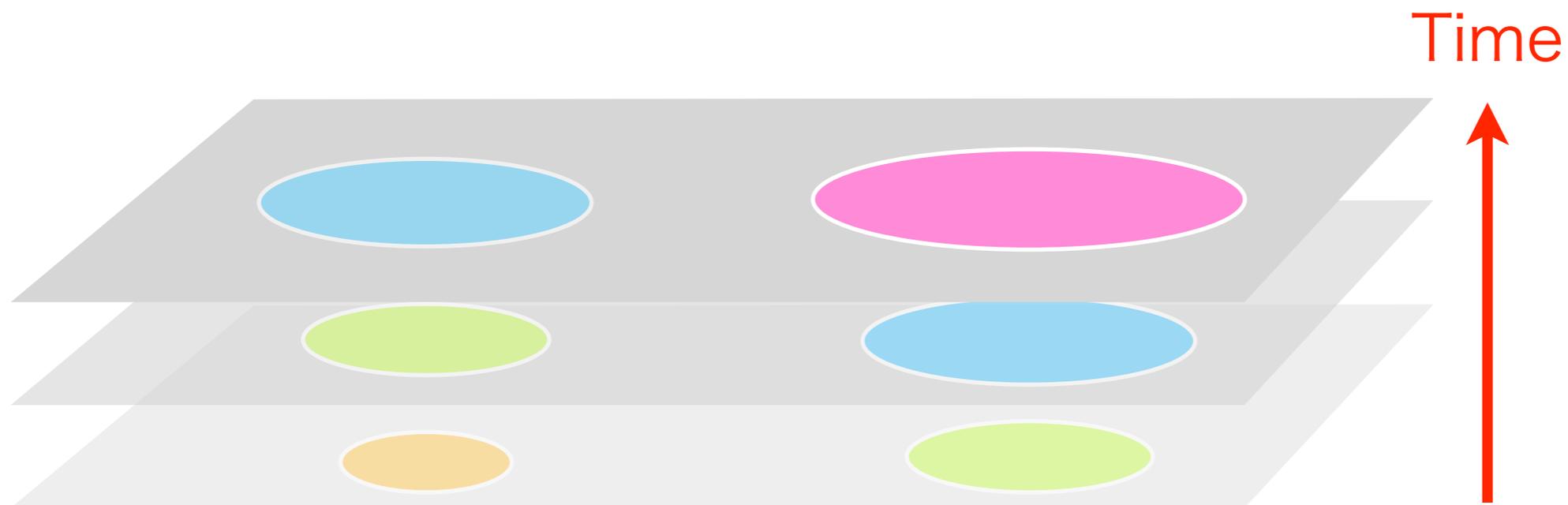
Scalar mode

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 + 2\Psi)d\mathbf{x}^2$$



It is due to **fluctuations in time** induced by the inflaton's quantum fluctuation.

$$\Phi \sim \frac{\delta\rho}{\rho} \sim H\delta t \sim H_{\text{inf}} \frac{\delta\phi}{\dot{\phi}} \sim \left| \frac{V^{3/2}}{V' M_P^3} \right|$$



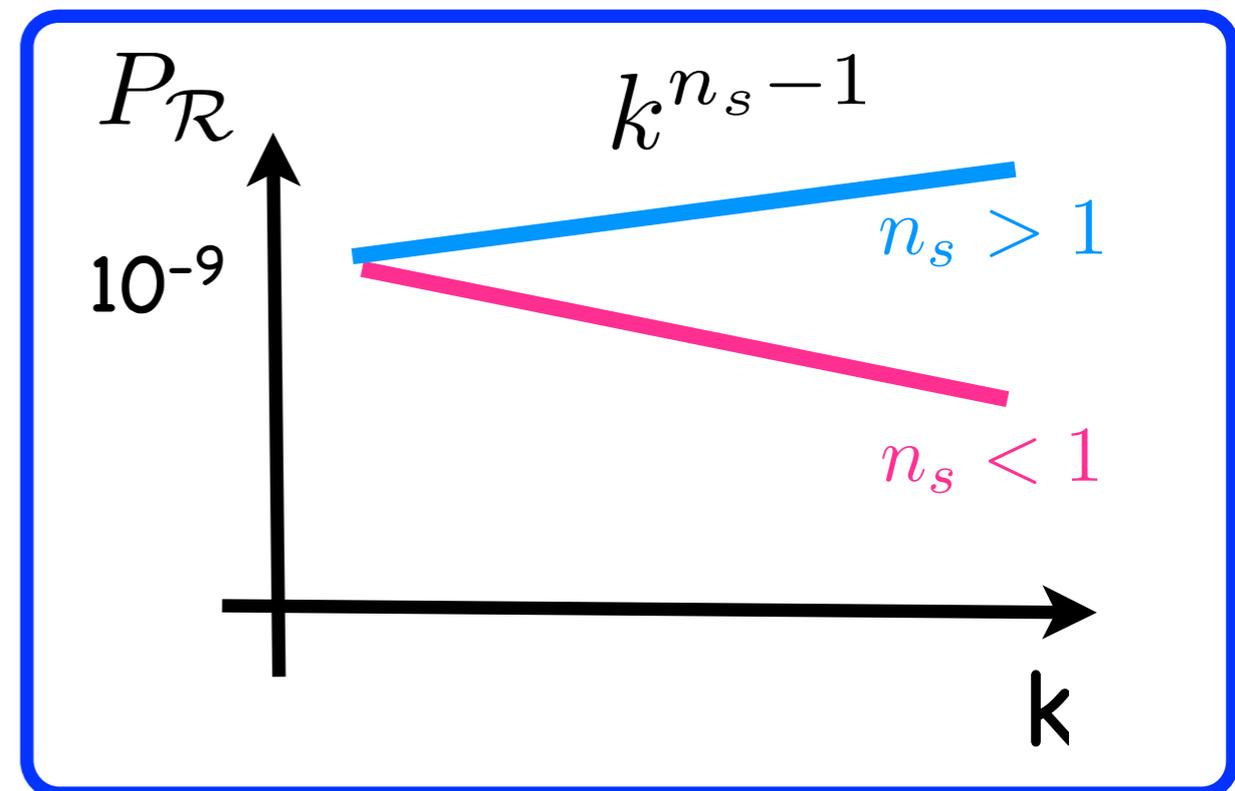
Scalar mode

Amplitude:

$$\frac{\delta\rho}{\rho} \sim \left| \frac{V^{3/2}}{V' M_P^3} \right| \sim 10^{-5} \quad \text{:COBE normalization}$$

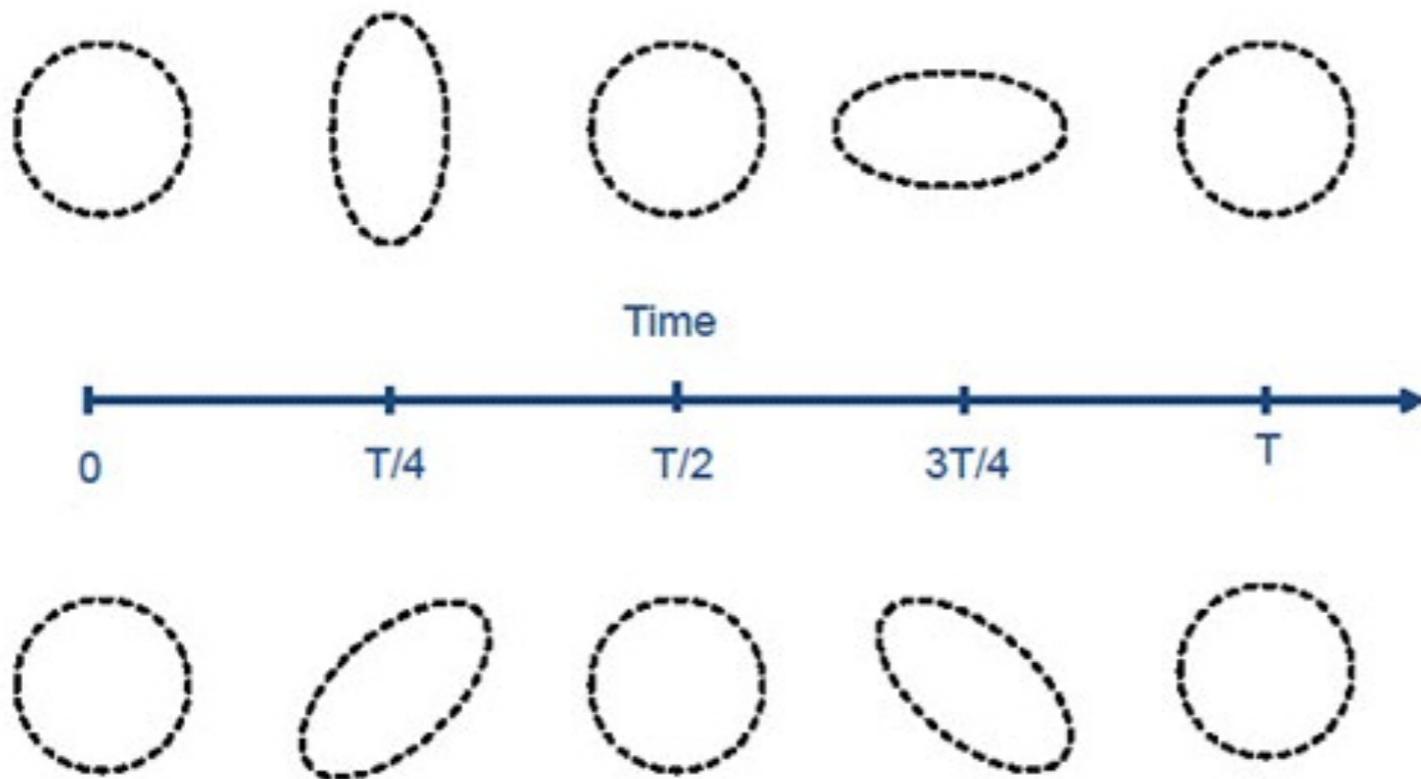
Spectral index:

$$\begin{aligned} n_s - 1 &= \frac{d \ln(\delta\rho_k/\rho)^2}{d \ln k} \\ &\simeq -\frac{V'}{V} \frac{d}{d\phi} \ln \left(\frac{V^3}{V'^2} \right) \\ &= -3 \frac{V'^2}{V^2} + 2 \frac{V''}{V} \end{aligned}$$



Tensor mode

$$ds^2 = -dt^2 + a^2 (\delta_{ij} + h_{ij}) dx^i dx^j$$



It is due to **fluctuations of graviton itself.**

$$h_{ij} \sim \frac{H_{\text{inf}}}{M_P}$$

Observation vs Theory

Scalar mode

$$P_{\mathcal{R}} = A_s \left(\frac{k}{k_0} \right)^{n_s - 1}$$

$$A_s = \frac{V^3}{2\sqrt{3}V'^2},$$

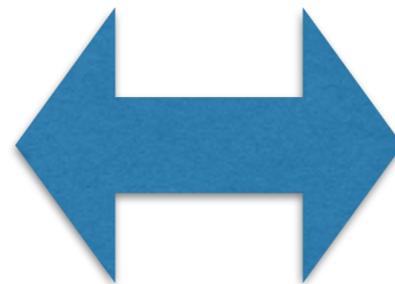
$$n_s = 1 + 2\frac{V''}{V} - 3\left(\frac{V'}{V}\right)^2,$$

Tensor mode

$$P_t = A_t \left(\frac{k}{k_0} \right)^{n_t}$$

$$r = 8\left(\frac{V'}{V}\right)^2$$

$$A_s, n_s, r \equiv \frac{A_t}{A_s}$$



$$V, V', V''$$

V : the inflaton potential

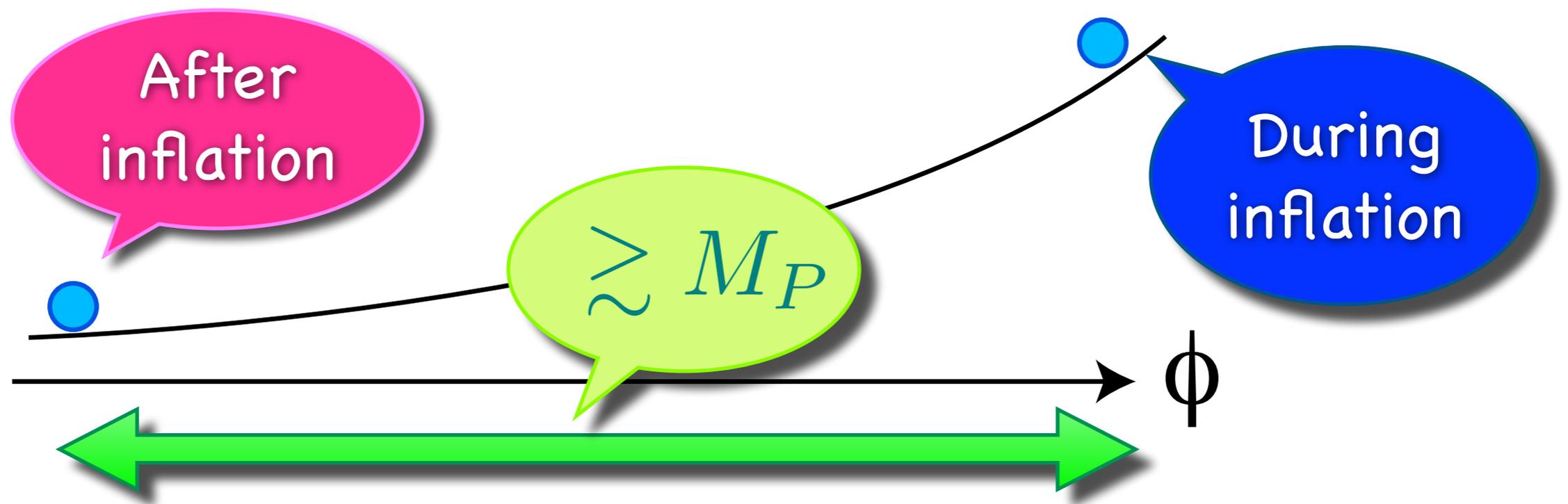
It's GUT-scale inflation!

$$V_{\text{inf}} \simeq (2.1 \times 10^{16} \text{ GeV})^4 \left(\frac{r}{0.16} \right)$$

$$H_{\text{inf}} \simeq 1.0 \times 10^{14} \text{ GeV} \left(\frac{r}{0.16} \right)^{\frac{1}{2}},$$

Large-field inflation

The inflaton excursion exceeds the Planck scale.



Lyth bound:
$$\Delta\phi \gtrsim 8M_P \left(\frac{r}{0.2}\right)^{\frac{1}{2}} \left(\frac{N}{50}\right)$$

GUT-scale, large-field inflation

GUT-scale, large-field inflation

- **Inflation model building in sugra/string**

GUT-scale, large-field inflation

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 - Shift symmetry is likely.

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GUT-scale, large-field inflation

- Inflation model building in sugra/string
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GUT-scale, large-field inflation

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 - The QCD axion less likely? PQ symmetry restoration?

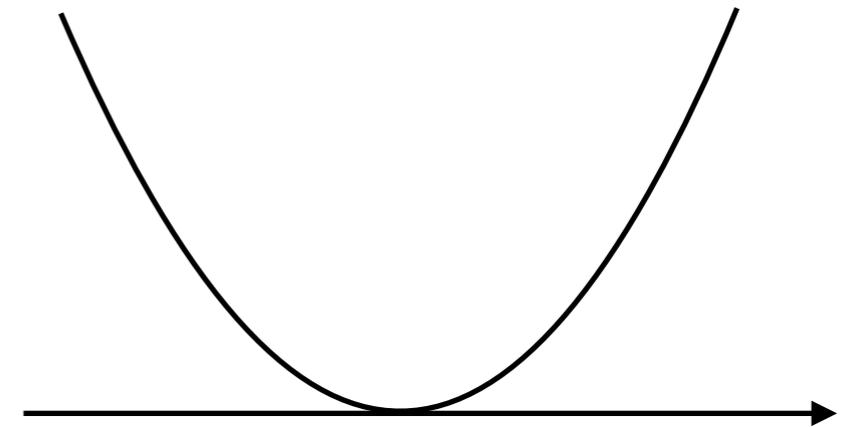
Various large-field inflation models

Quadratic chaotic inflation

Linde '83

$$V = \frac{1}{2}m^2\phi^2$$

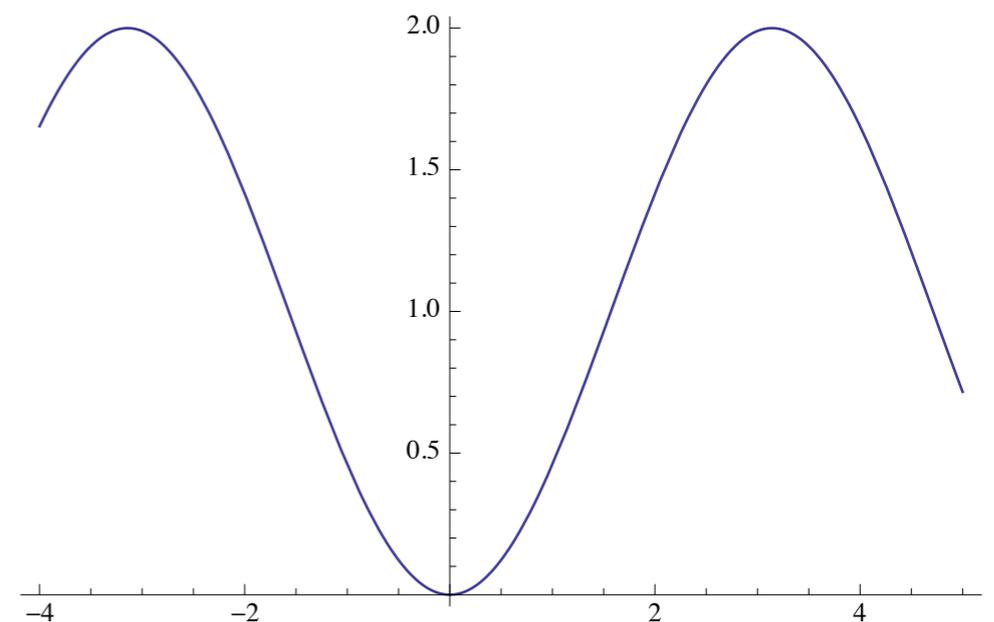
$$m \simeq 2 \times 10^{13} \text{ GeV} \quad \phi_{60} \sim 16M_P$$



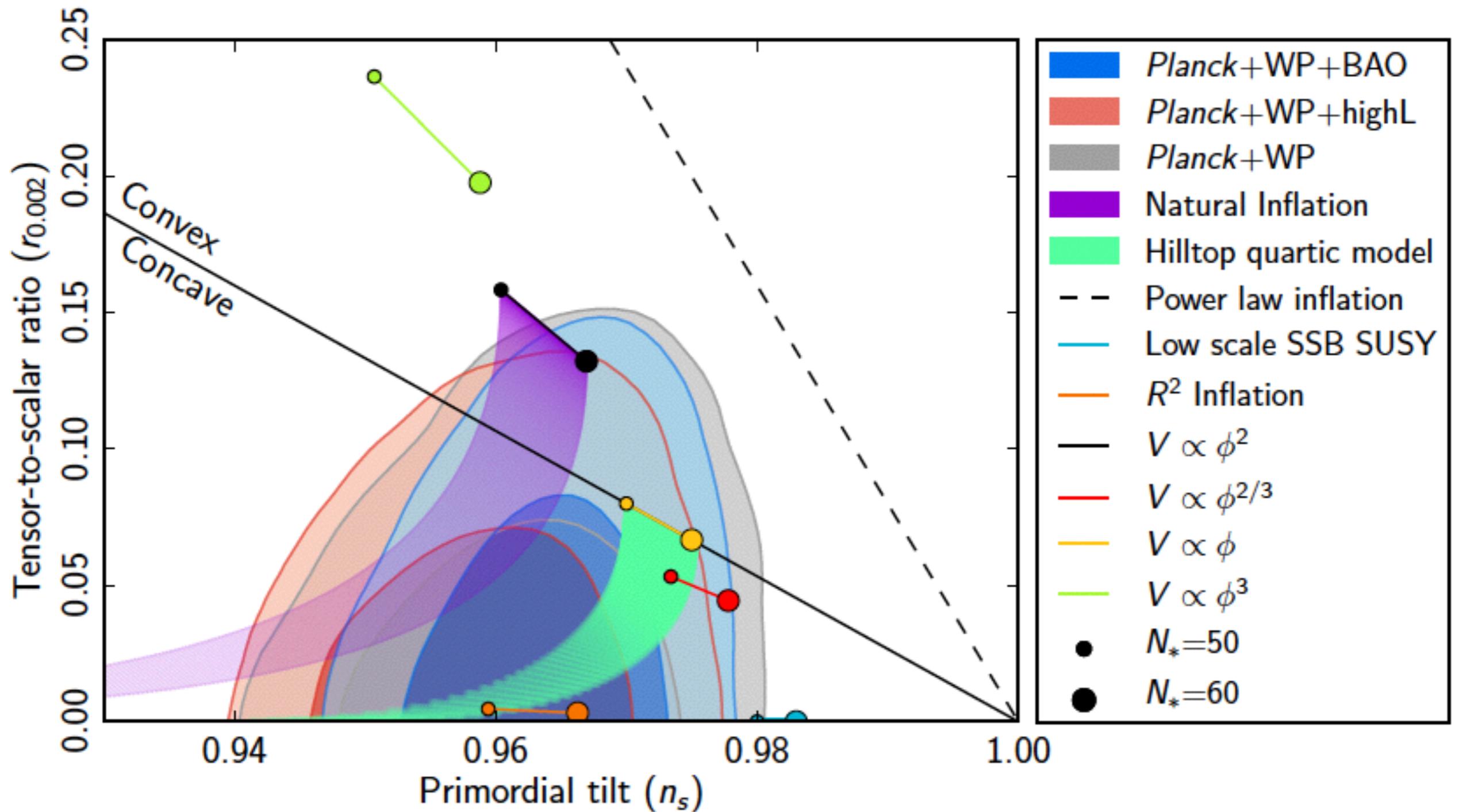
Natural inflation

Freese et al, '90

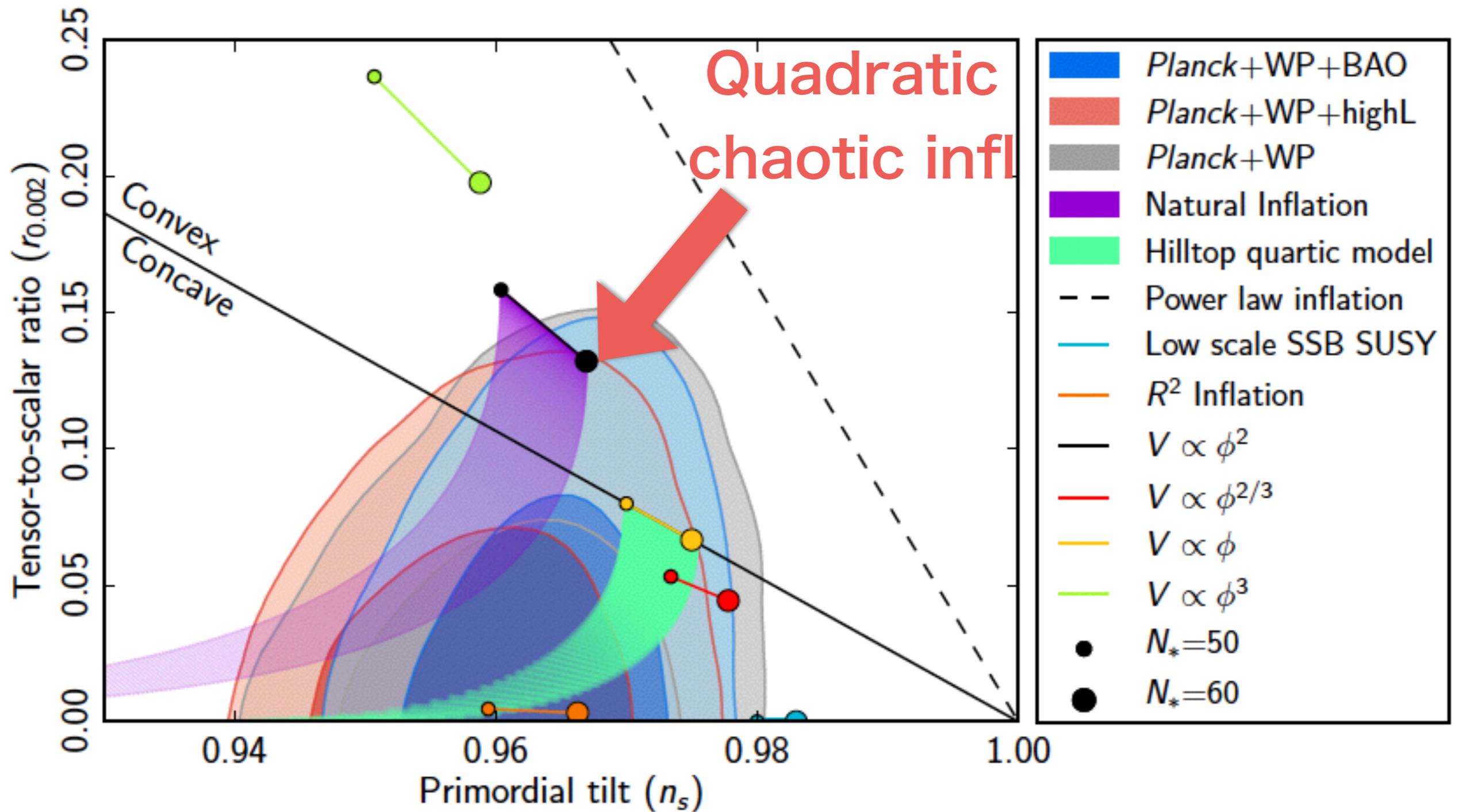
$$V = \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right)$$



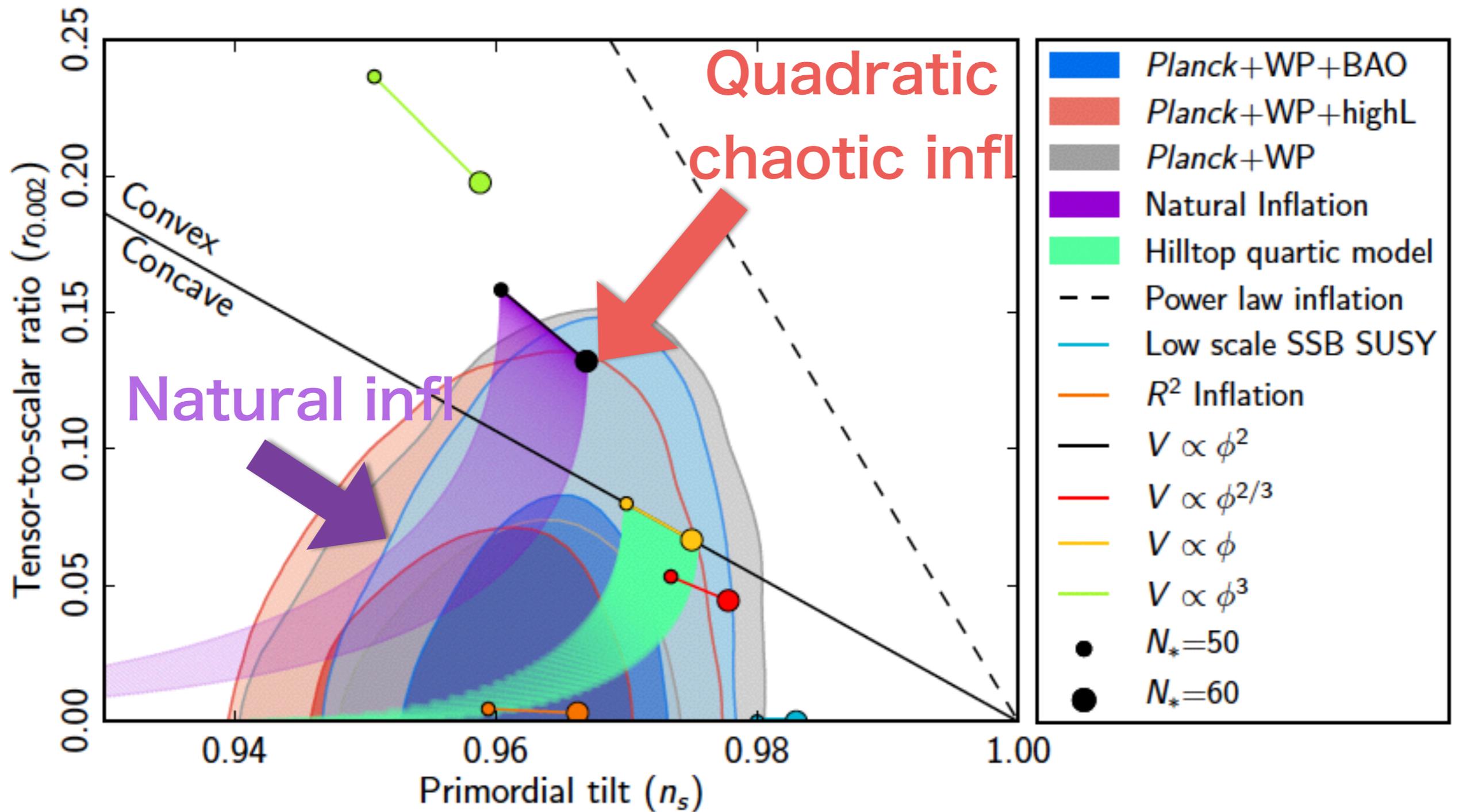
Predicted values of (n_s, r)



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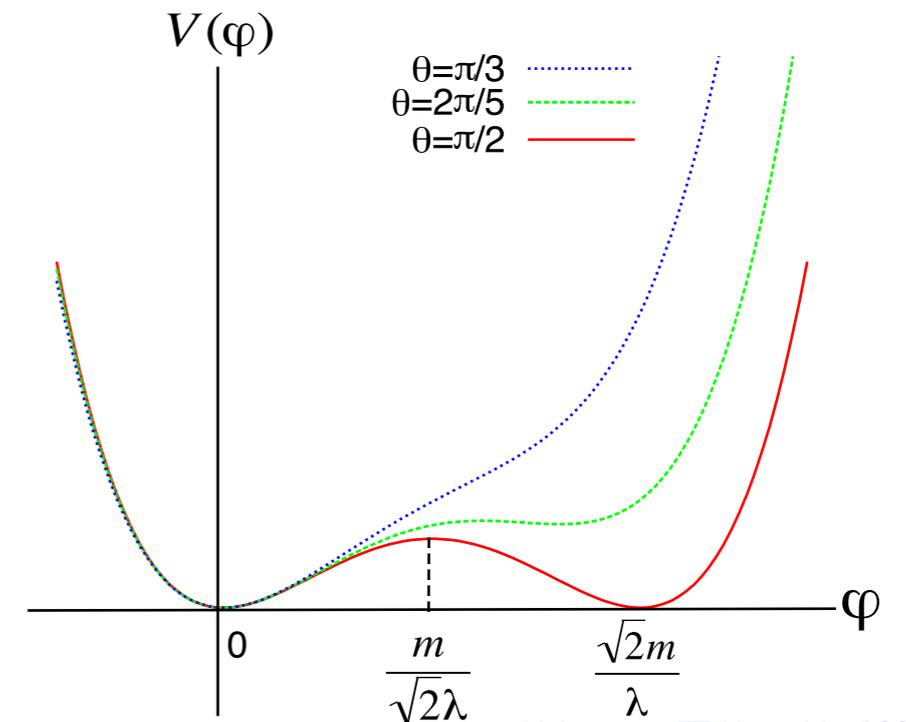


Various large-field inflation models

Polynomial chaotic inflation

Destri, de Vega, Sanchez [astro-ph/0703417]
 Nakayama, FT, Yanagida 1303.7315
 (see also Kobayashi, Seto 1403.5055
 Kallosh, Linde, Wesphal 1405.0270)

$$V = \frac{1}{2}m^2\phi^2 + \frac{\kappa}{3}\phi^3 + \frac{\lambda}{4}\phi^4 + \dots$$



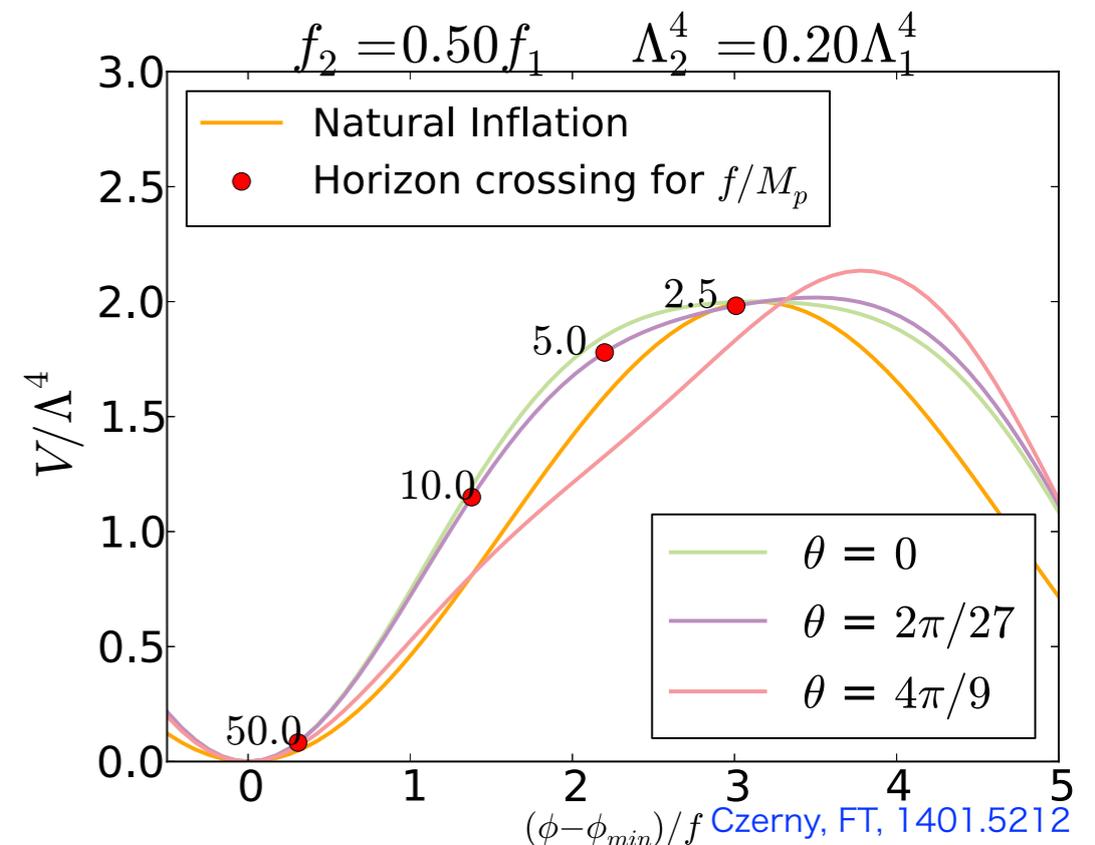
Nakayama, FT, Yanagida, 1303.7315

Multi-Natural inflation (MNI)

Czerny, FT 1401.5212
 Czerny, Higaki FT 1403.0410, 1403.5883

$$V(\phi) = C - \Lambda_1^4 \cos(\phi/f_1) - \Lambda_2^4 \cos(\phi/f_2 + \theta),$$

Sub-Planckian decay constants are allowed as hilltop inflation can be realized.



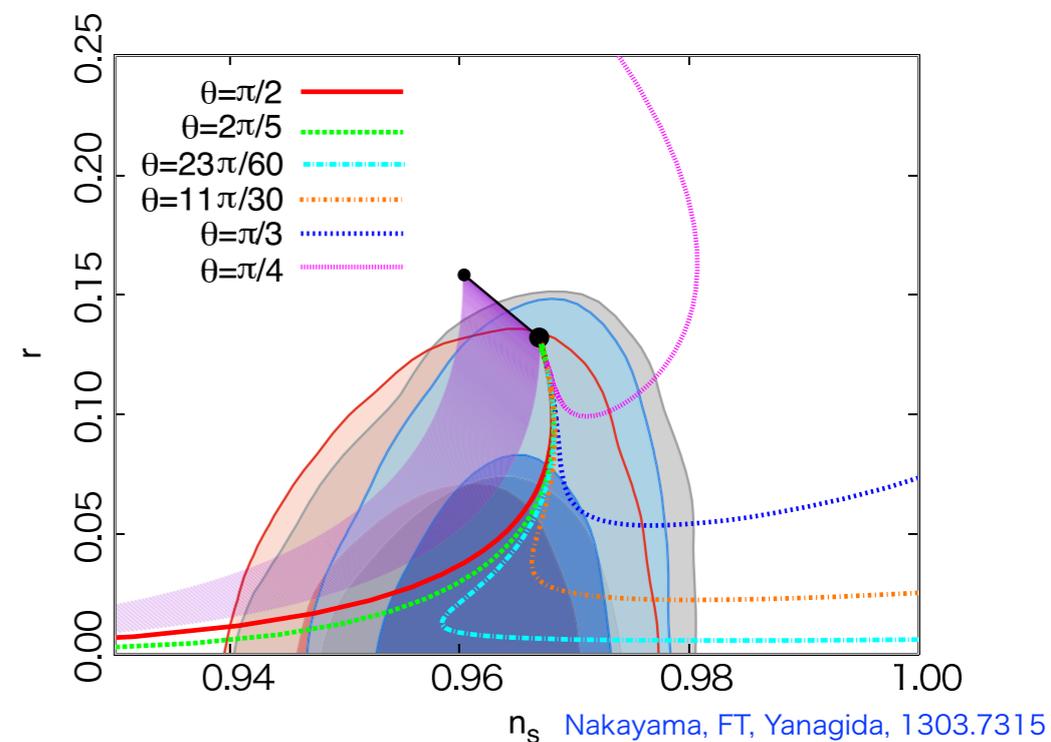
Czerny, FT, 1401.5212

Various large-field inflation models

Polynomial chaotic inflation

Destri, de Vega, Sanchez [astro-ph/0703417]
 Nakayama, FT, Yanagida 1303.7315
 (see also Kobayashi, Seto 1403.5055
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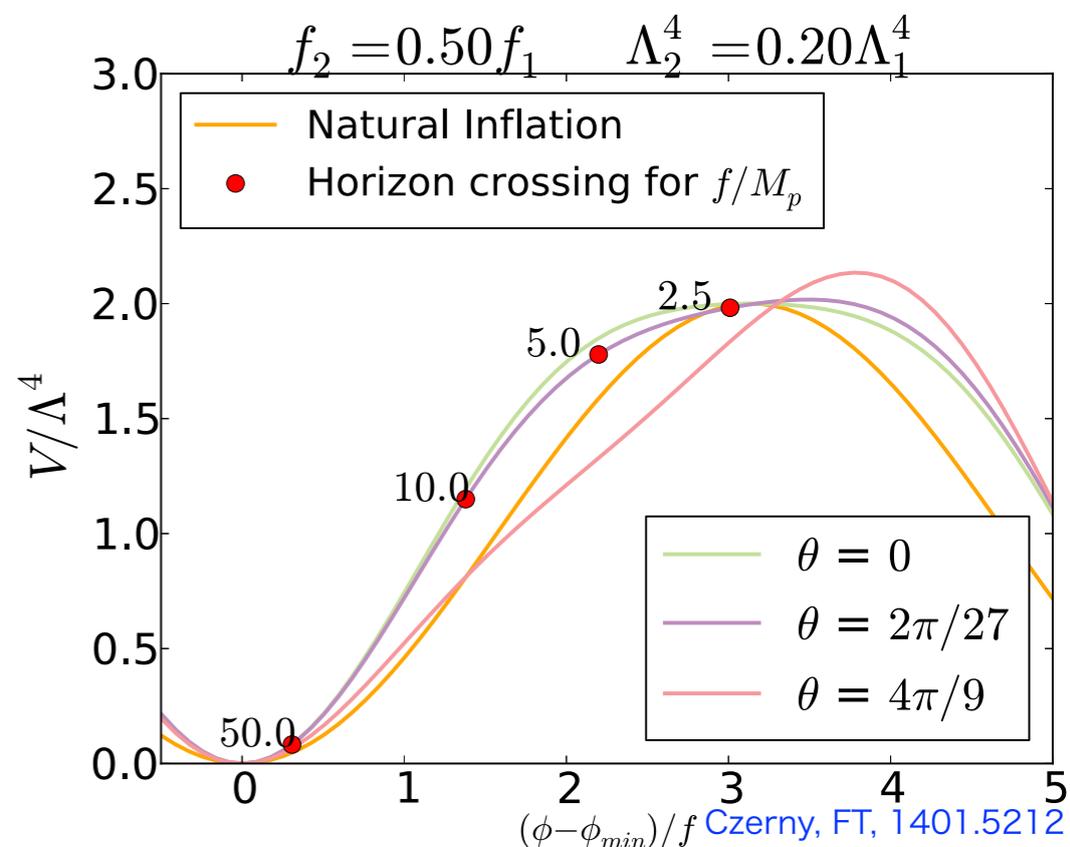


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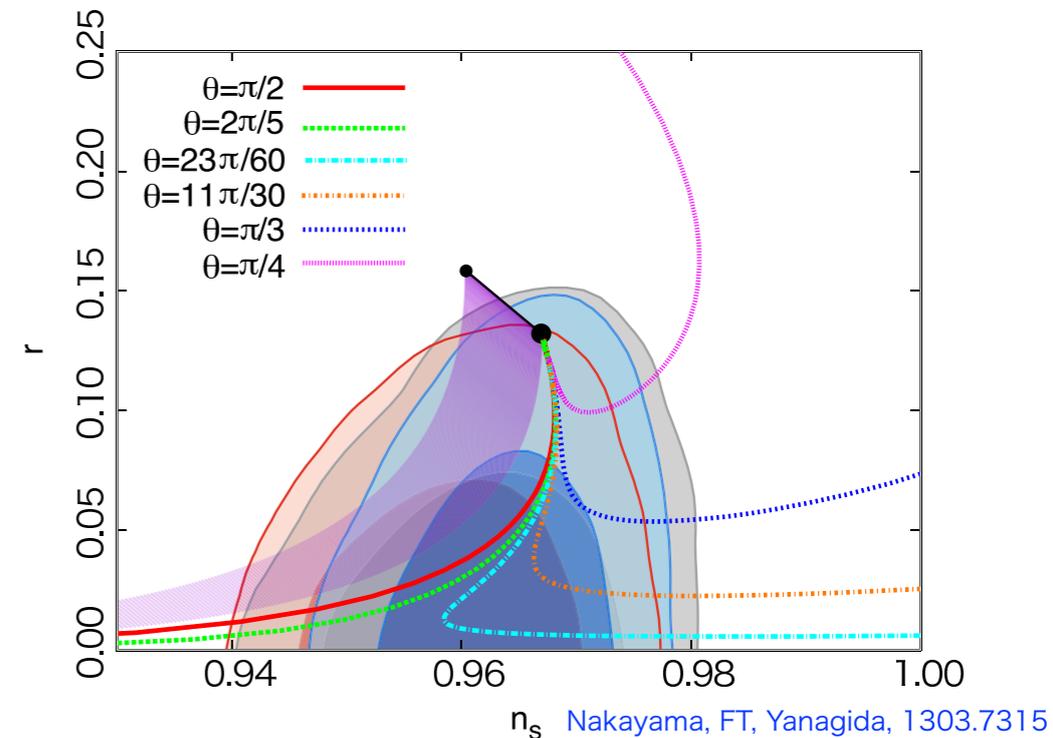


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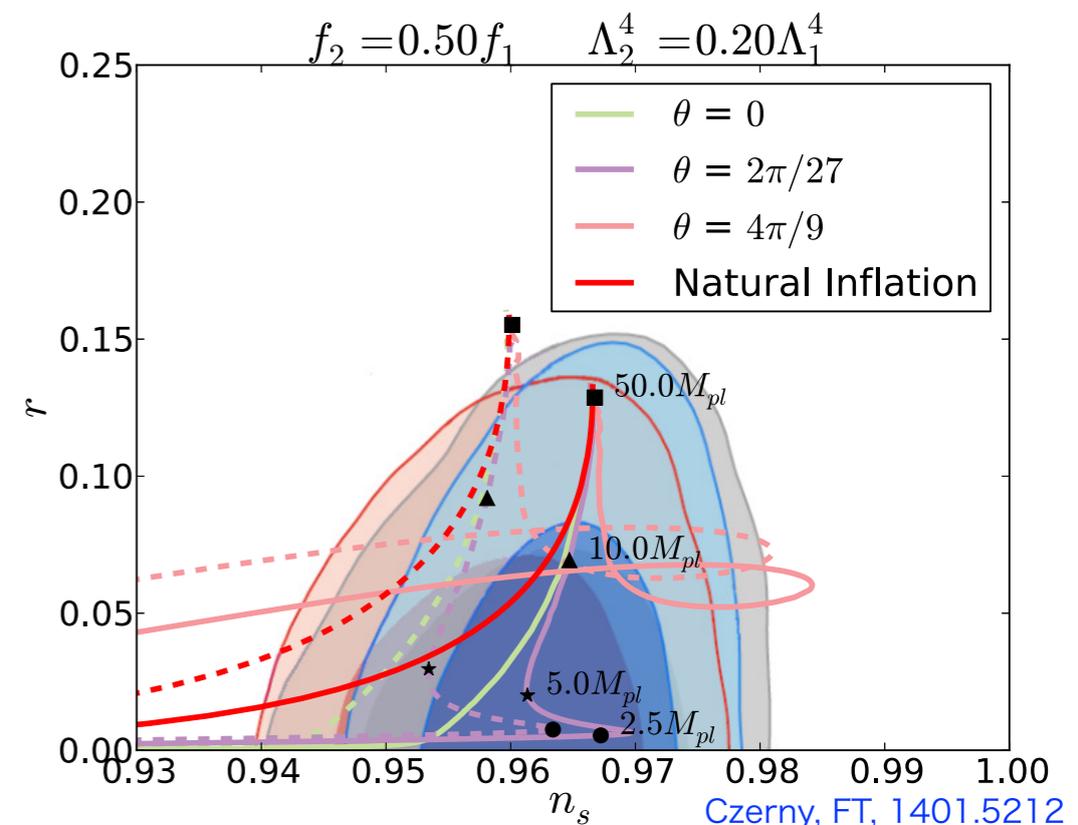


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Sub-Planckian decay constants are allowed as hilltop inflation can be realized.



Chaotic inflation in SUGRA

Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243 ,hep-ph/0011104

To have a good control over the inflaton field values greater than the Planck scale, we impose a shift symmetry;

$$\phi \rightarrow \phi + iC,$$

which is explicitly broken by the superpotential.

$$K_{\text{inf}} = c(\phi + \phi^\dagger) + \frac{1}{2}(\phi + \phi^\dagger)^2 + |X|^2 - k|X|^4 + \dots$$

$$W_{\text{inf}} = mX\phi,$$

$$V_{\text{sugra}} = e^K \left((D_i W) K^{i\bar{j}} (D_{\bar{j}} W)^* - 3|W|^2 \right).$$

$$V \simeq \frac{1}{2} m^2 \varphi^2$$

$$\varphi \equiv \sqrt{2} \text{Im}[\phi]$$

even for $\varphi \gg M_p$

Polynomial chaotic inflation in SUGRA

Nakayama, FT, Yanagida 1303.7315,1305.5099

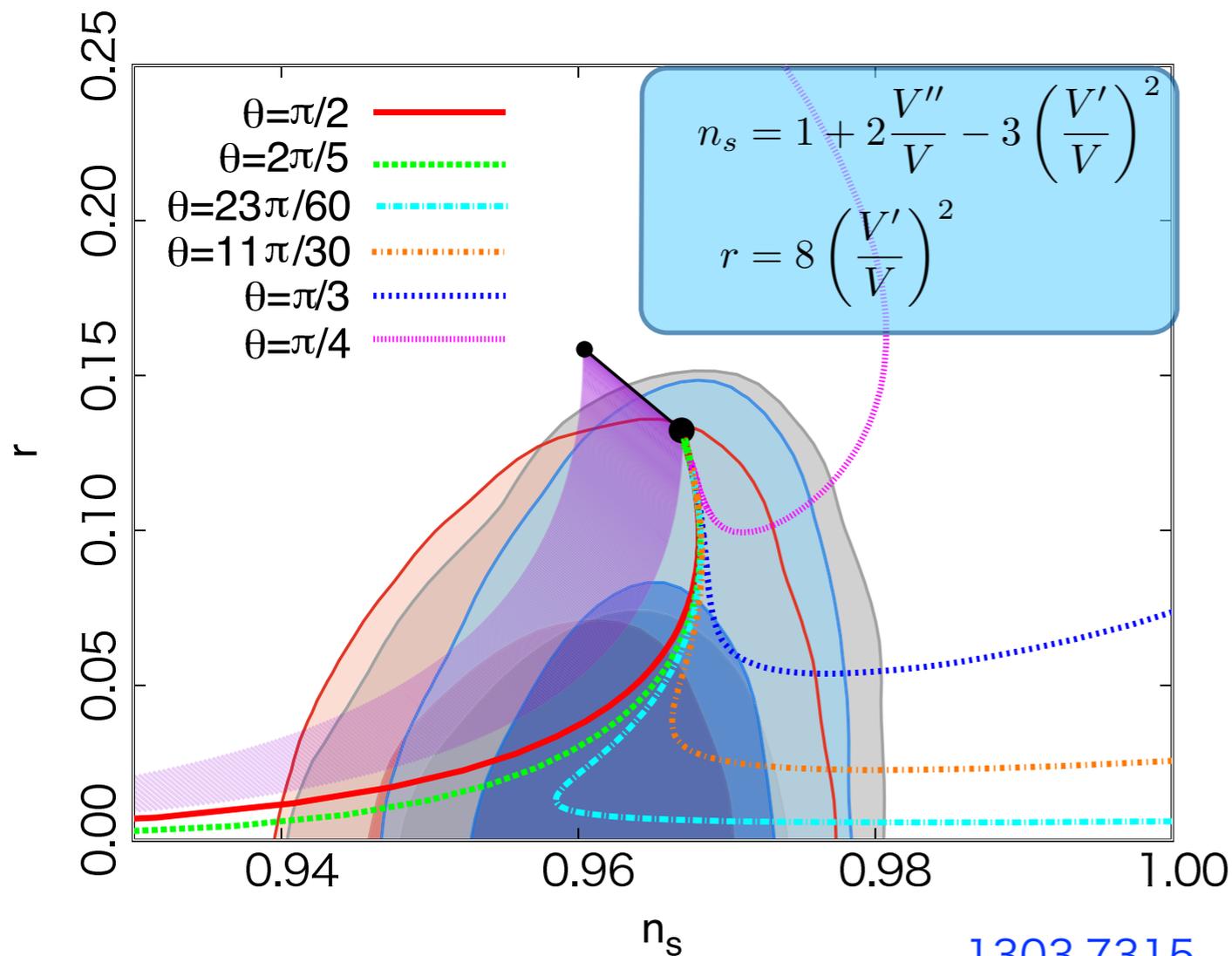
(cf. Kallosh, Linde, Westphal 1405.0270)

$$K = \frac{1}{2}(\phi + \phi^\dagger)^2 + |X|^2 + \dots,$$

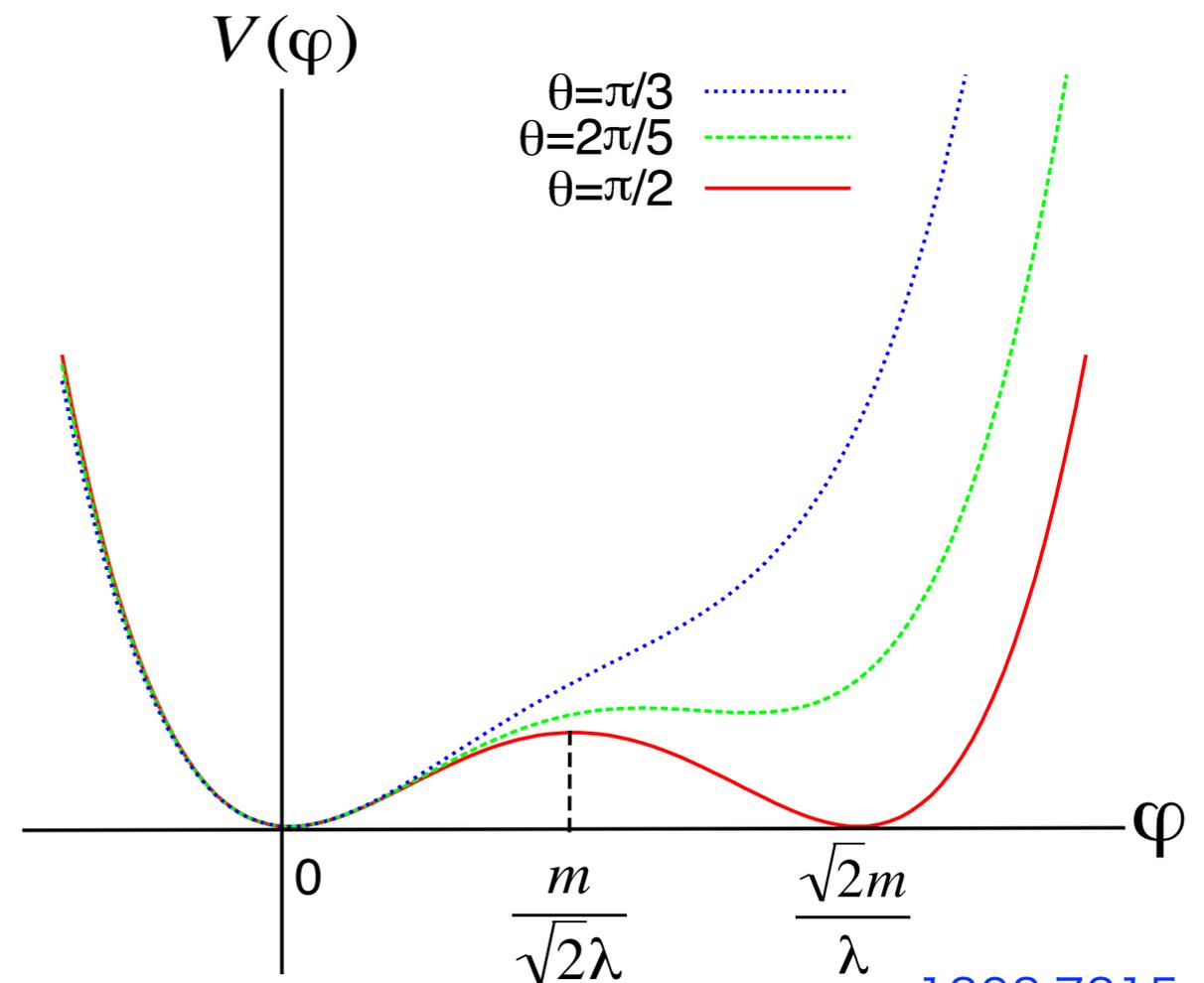
$$W = X(m\phi + k_2\phi^2 + \dots),$$

$$V \simeq \frac{1}{2}\varphi^2 \left(m^2 - \sqrt{2}m\lambda \sin\theta \varphi + \frac{\lambda^2}{2}\varphi^2 \right)$$

$$\lambda = |k_2| \quad \theta = \arg[k_2] \quad \text{Re}[\phi] \lesssim 1$$



1303.7315



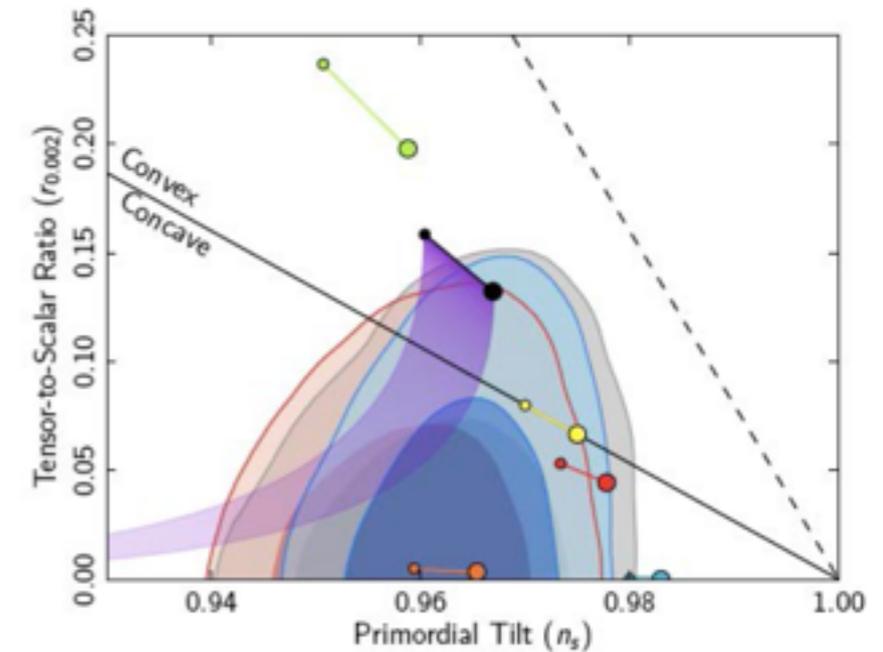
1303.7315

Natural and Multi-Natural Inflation

- Natural inflation [Freese, Frieman, Olinto '90](#)

$$V = \Lambda^4 \left(1 - \cos \left(\frac{\phi}{f} \right) \right)$$

Only large-field inflation is possible,
and f is bounded below: $f \gtrsim 5M_P$

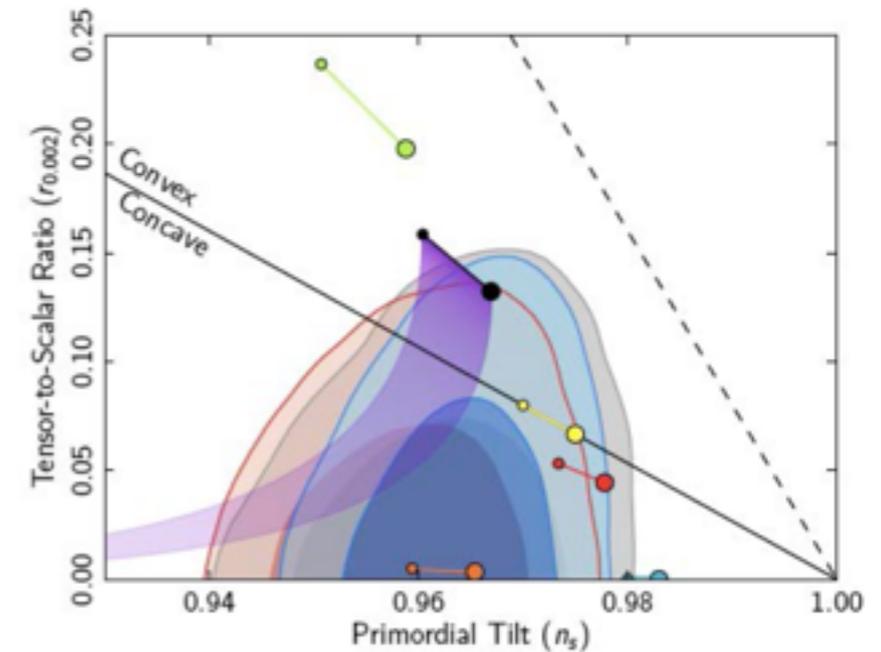


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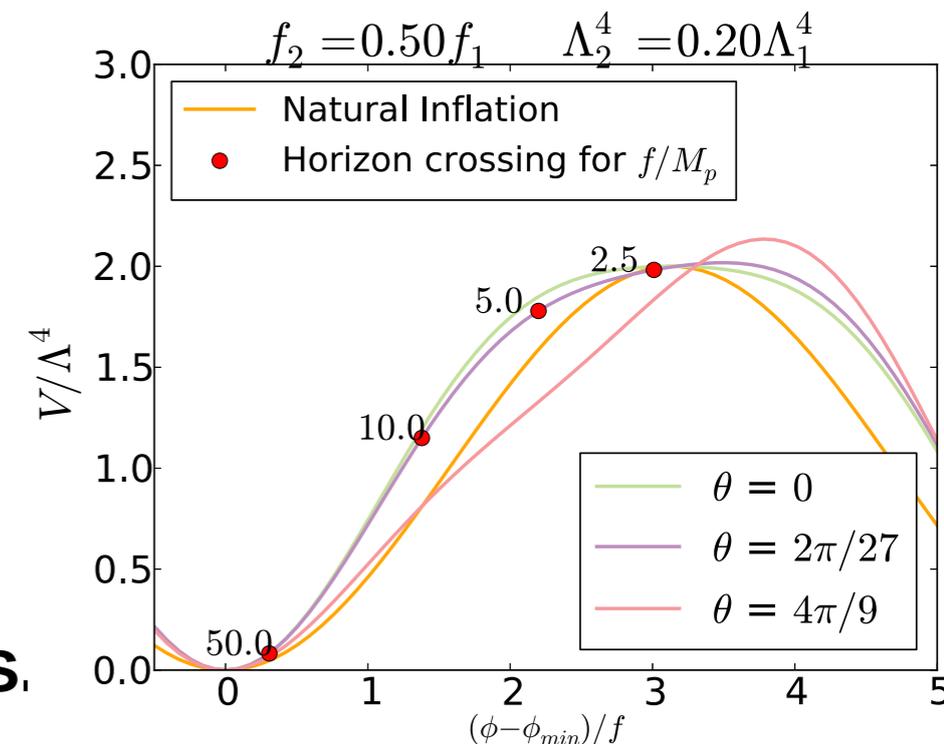
- Multi-Natural inflation

[Czerny, FT 1401.5212](#), [Czerny, Higaki, FT 1403.0410](#), [1403.5883](#)

$$V = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\frac{\phi}{f_i} + \theta_i \right) + \text{const.}$$

For $N_{\text{source}} = 2$, various values of (n_s, r) are possible as in the polynomial chaotic inf.

No lower bound on the decay constants.

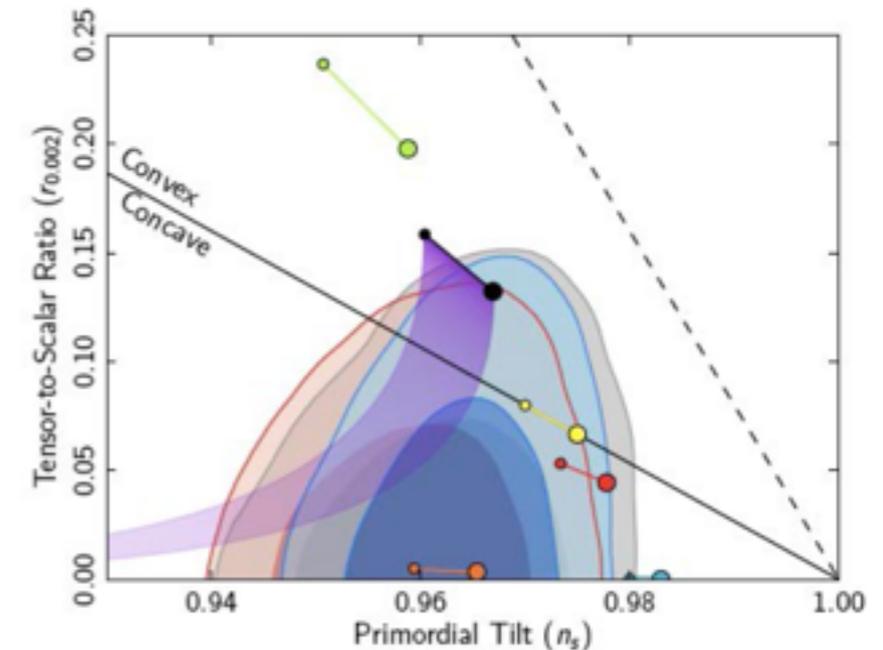


Natural and Multi-Natural Inflation

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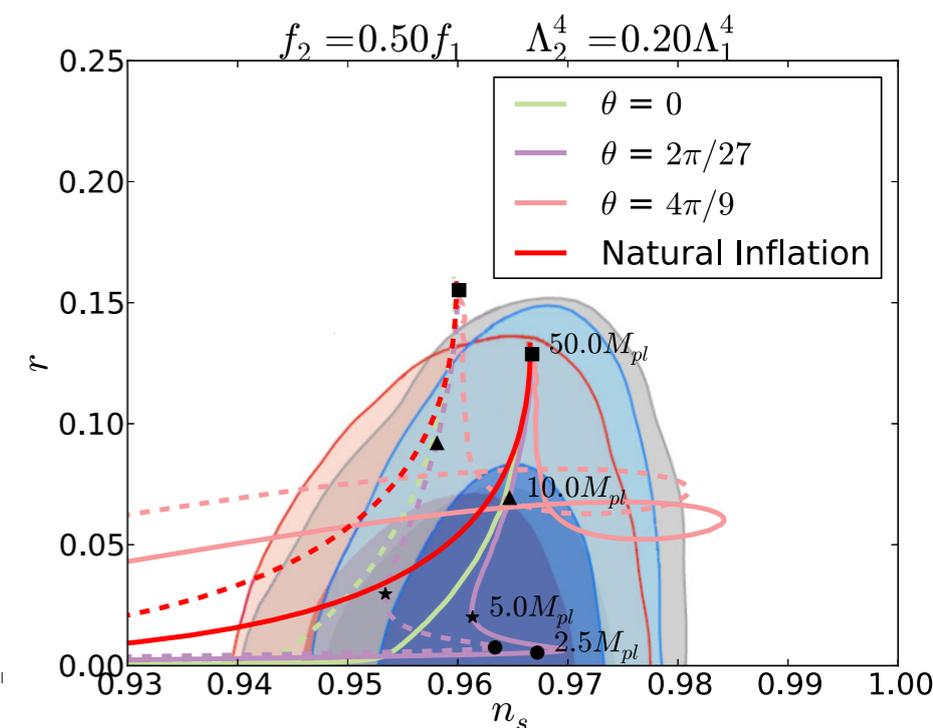
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Axion hilltop inflation

(Small-field Multi-Natural inflation)

Czerny, FT 1401.5212

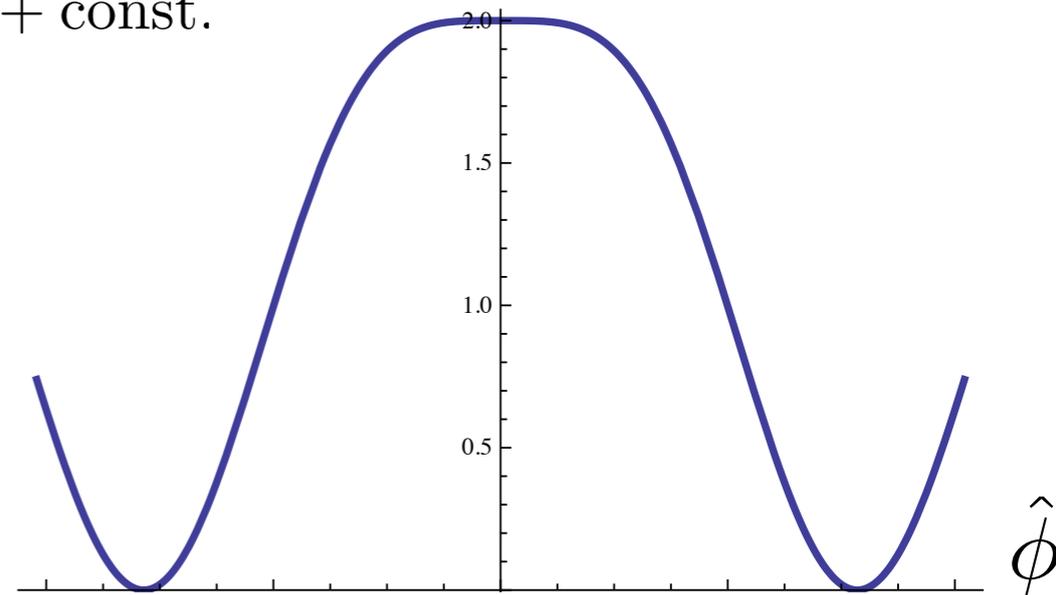
Czerny, Higaki FT 1403.0410

Hilltop quartic inflation (new inflation) can be realized by requiring a flat-top potential in multi-natural inflation.

$$V(\phi) = \Lambda_1^4 \left(1 - \cos \left(\frac{\phi}{f_1} \right) \right) + \Lambda_2^4 \left(1 - \cos \left(\frac{\phi}{f_2} + \theta \right) \right) + \text{const.}$$

$$\simeq V_0 - \lambda \hat{\phi}^4 + \dots \quad \hat{\phi} \equiv \phi - \pi f_1$$

$$\text{for } \frac{\Lambda_1^2}{f_1} = \frac{\Lambda_2^2}{f_2} \text{ and } \theta = -\pi \frac{f_1}{f_2}$$



Axion hilltop inflation is possible for $f < M_{\text{P}}$.

- Simple realization of hilltop inflation by axion.
- The potential shape is under control.
- Spectral index can give a better fit to the Planck data by a slight shift of the phase.

Axion hilltop inflation

(Small-field Multi-Natural inflation)

Czerny, FT 1401.5212

Czerny, Higaki FT 1403.0410

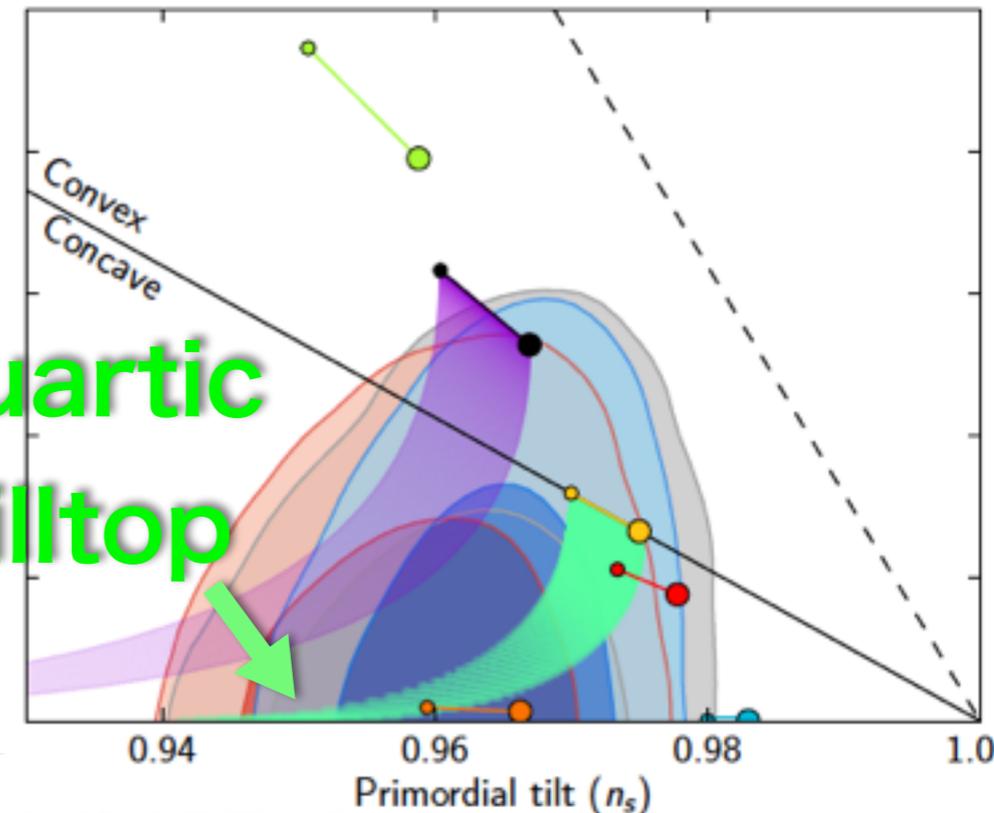
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Quartic
hilltop



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- Spectral index can give a better fit to the Planck data by a slight shift of the phase.

Aligned Natural Inflation

Kim, Nilles, Peloso, hep-ph/0409138

Czerny, Higaki, FT 1403.5883, Harigaya and Ibe 1404.3511, Choi, Kim, Yun, 1404.6209, Higaki, FT, 1404.6923, Tye, Won, 1404.6988, Kappl, Krippendorf, Nilles, 1404.7127, Bachlechner et al, 1404.7496, Ben-Dayan, Pedro, Westphal, 1404.7773, Long, McAllister, McGuirk 1404.7852

The effectively large decay constant can be realized by the alignment of two (or more) axion potentials.

• Two axions: $\phi_1 \rightarrow \phi_1 + 2\pi f_1$ $\phi_2 \rightarrow \phi_2 + 2\pi f_2$

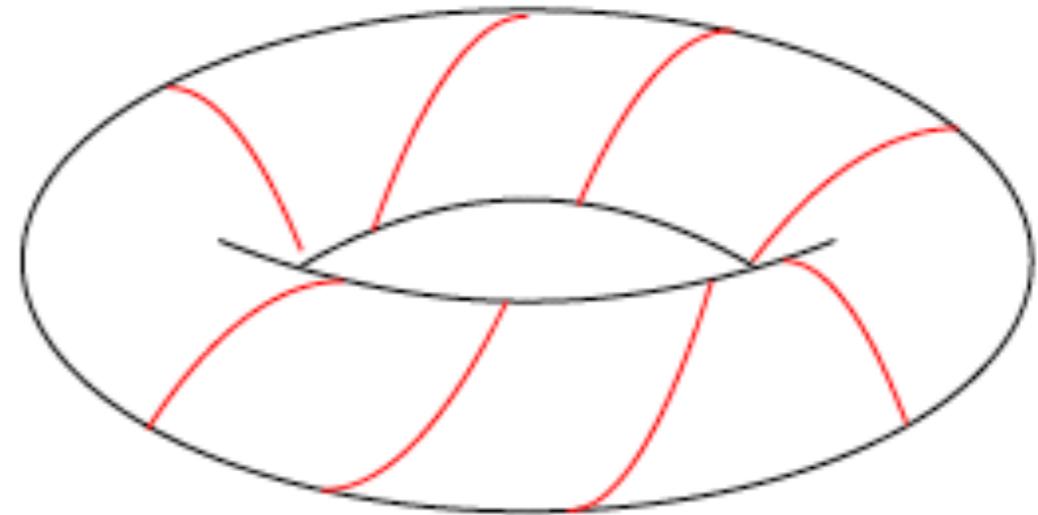
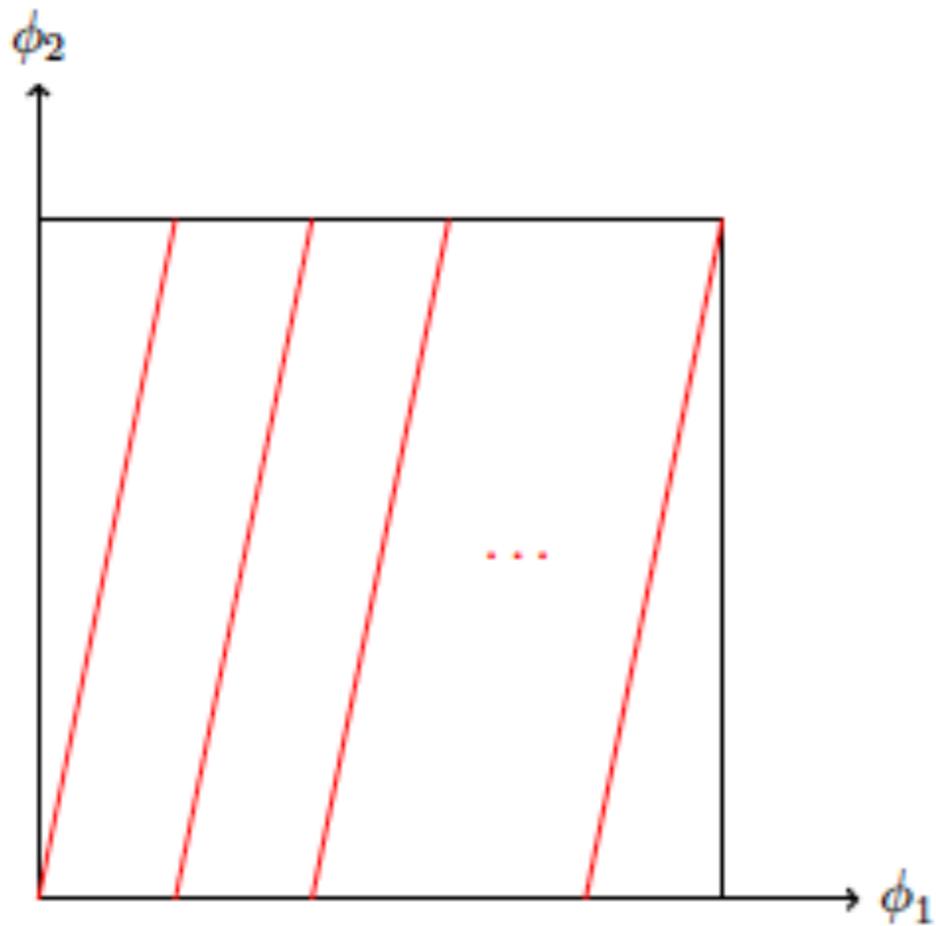
$$V(\phi_i) = \Lambda_1^4 \left[1 - \cos \left(n_1 \frac{\phi_1}{f_1} + n_2 \frac{\phi_2}{f_2} \right) \right] + \Lambda_2^4 \left[1 - \cos \left(m_1 \frac{\phi_1}{f_1} + m_2 \frac{\phi_2}{f_2} \right) \right]$$

If $n_1/n_2 = m_1/m_2$, there is a flat direction; the corresponding decay constant would be infinite.

If $n_1/n_2 \approx m_1/m_2$, there is a relatively light direction; the corresponding decay constant can be larger than f_1 or f_2 .

Aligned Natural Inflation

Kim, Nilles, Peloso, hep-ph/0409138



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For $\Lambda_1 \gg \Lambda_2$, the effective decay constant is

$$f_{\text{eff}} = \frac{\sqrt{n_1^2 f_2^2 + n_2^2 f_1^2}}{|n_1 m_2 - n_2 m_1|}$$

Some hierarchy among the anomaly coefficients are needed to realize a large enhancement of $O(100)$.

Aligned Natural Inflation

- Multiple axions: $\phi_i \equiv \phi_i + 2\pi f_i \quad (i = 1, \dots, N)$

$$V(\phi_i) = \sum_{i=1}^N \Lambda_i^4 \left[1 - \cos \left(\sum_{j=1}^N \frac{n_{ij} \phi_j}{f_j} \right) \right]$$

For a moderately large N (> 5 or so), the effective decay constant can be enhanced w/o hierarchy among the anomaly coefficients.

[Choi, Kim, Yun, 1404.6209](#)

Prob. dist. was studied in detail for various cases incl. $N_{\text{source}} \neq N_{\text{axion}}$

[Higaki, FT, 1404.6923](#)

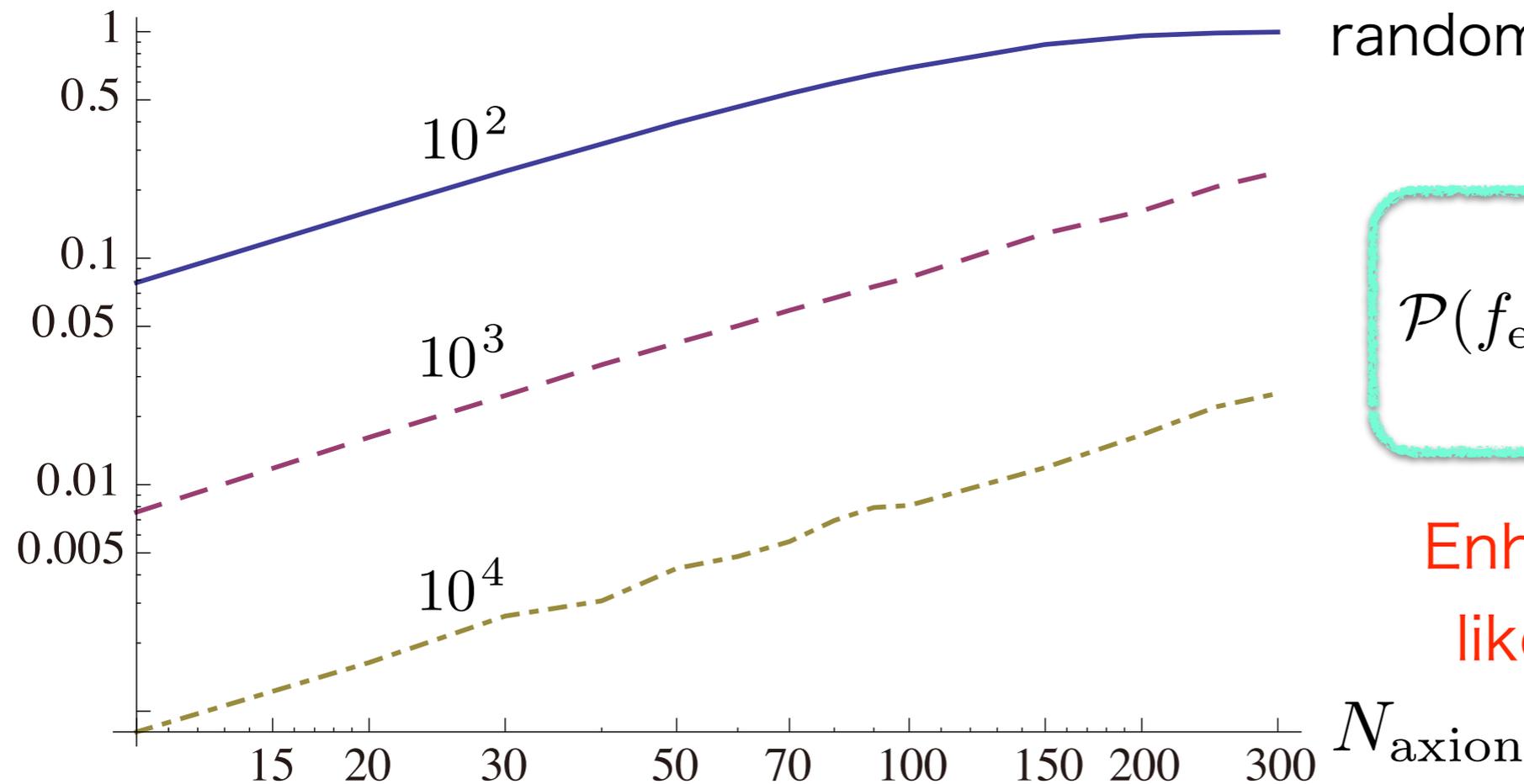
Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

- Prob dist for the enhancement of the decay constant

$$\mathcal{P}(f_{\text{eff}}/f_i > 10^x)$$

$$N_{\text{axion}} = N_{\text{source}}, \quad n = 2$$



We generated integer-valued random matrix $-n \leq a_{ij} \leq n$

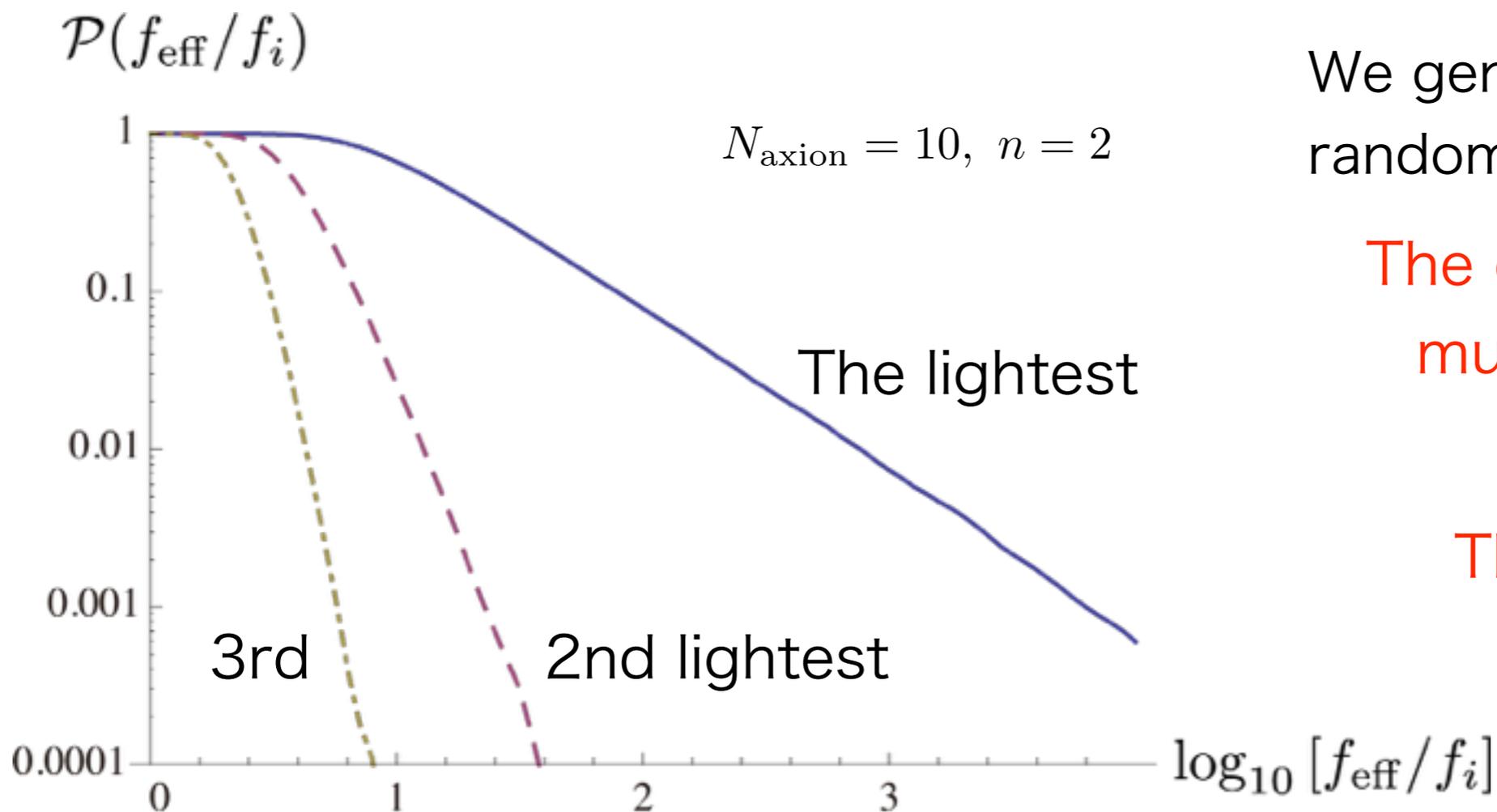
$$\mathcal{P}(f_{\text{eff}}/f_i) \sim N_{\text{axion}} \left(\frac{f_i}{f_{\text{eff}}} \right)$$

Enhancement becomes likely for larger N_{axion} .

Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

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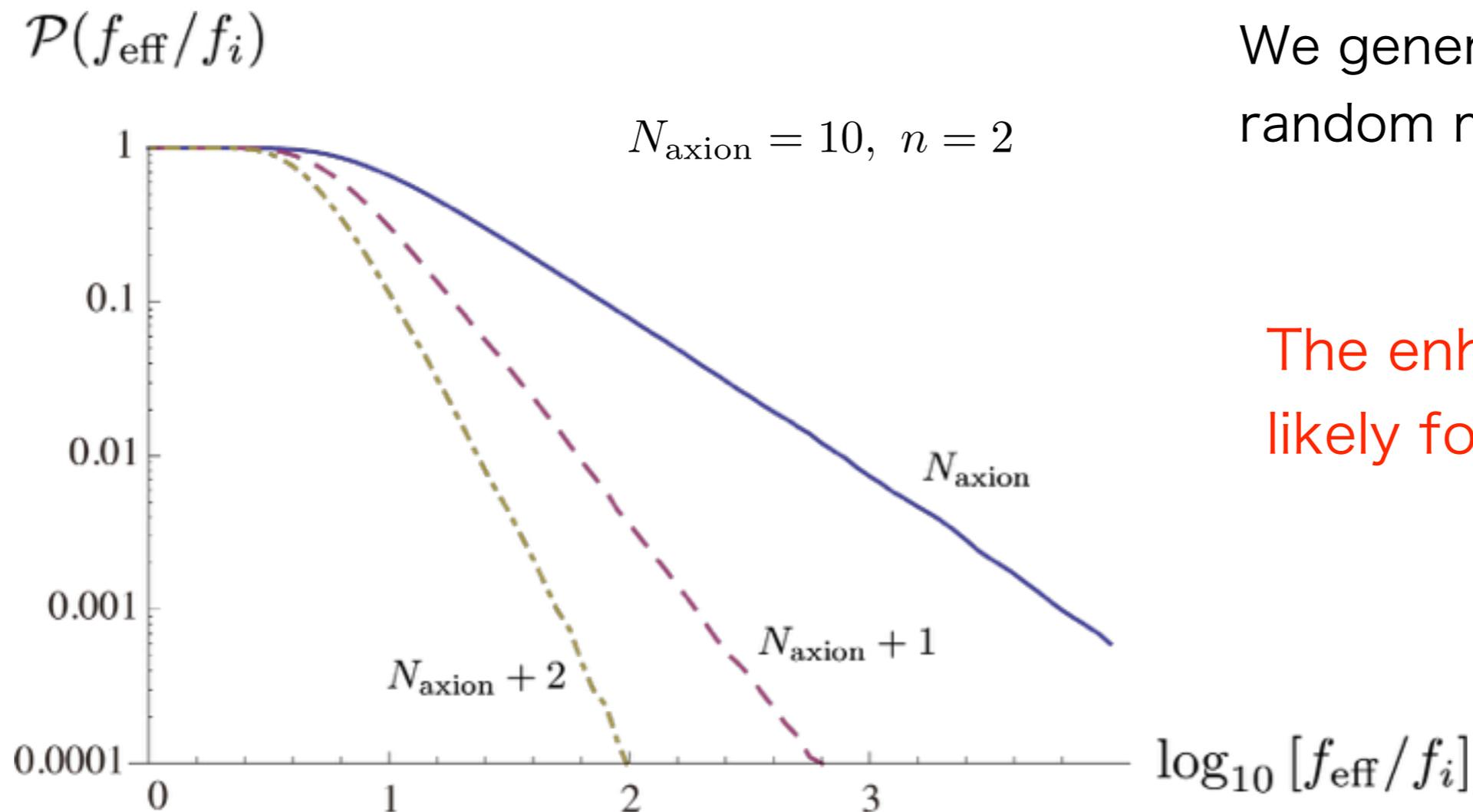
The enhancement along multiple directions is highly unlikely.

The inflaton is the lightest axion!

Aligned Natural Inflation

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

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The enhancement is less likely for larger N_{source} .

Axion Landscape

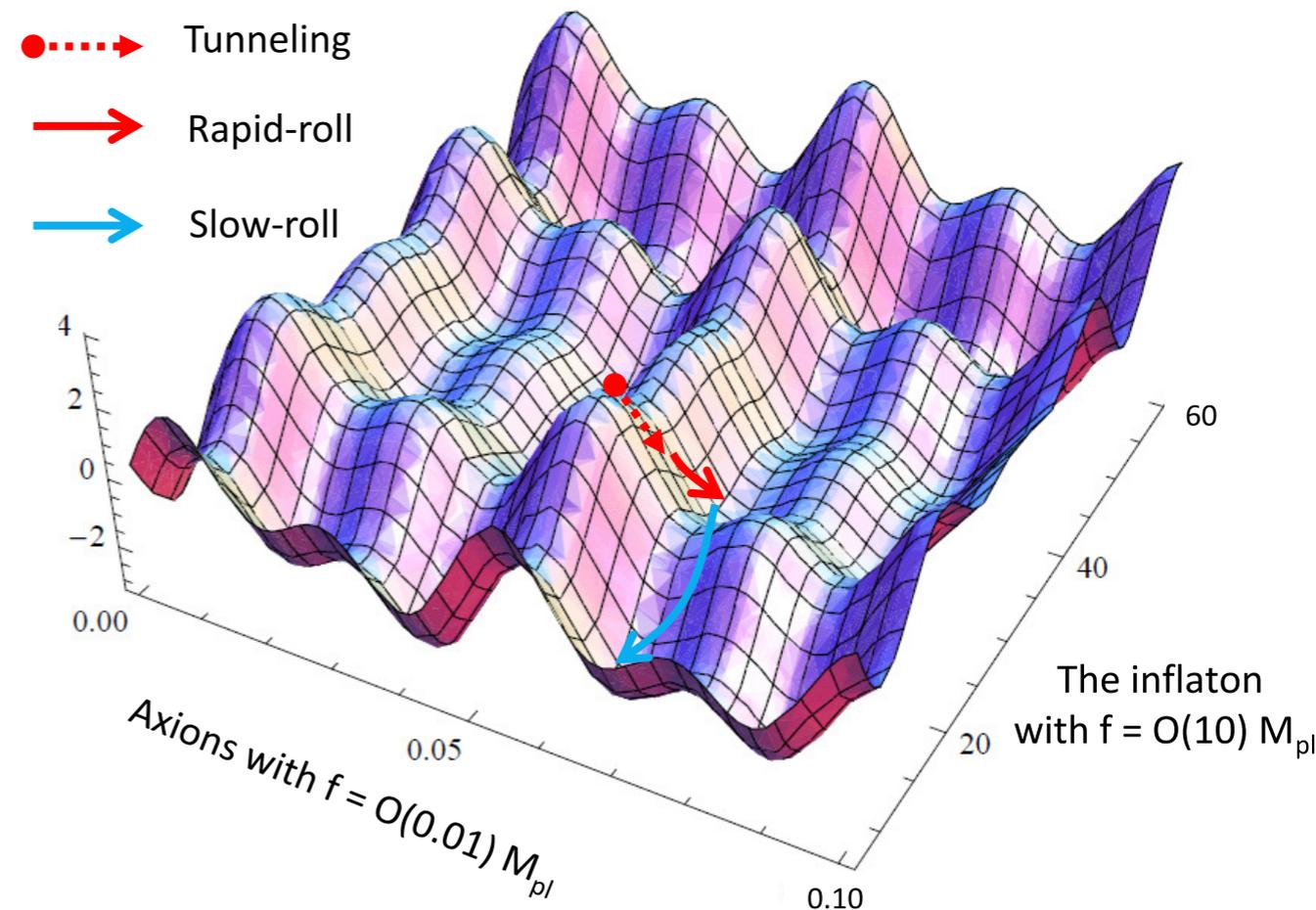
Higaki, FT 1404.6923

For $N_{\text{source}} > N_{\text{axion}}$, many axions may form a mini-landscape.

$$V(\phi_i) = \sum_{i=1}^{N_{\text{source}}} \Lambda_i^4 \cos \left(\sum_{j=1}^{N_{\text{axion}}} a_{ij} \frac{\phi_j}{f_j} + \theta_i \right) + V_0$$

- Eternal inflation takes place in local minima.
- A flat direction arises by the KNP mechanism.
- Slow-roll inflation starts along the flat direction after the tunneling event.
- Negative curvature/suppression at large scales if the total e-folding is just 50-60.

Linde '95, Freivogel et al '05,
Yamauchi et al '11, Bousso et al '13

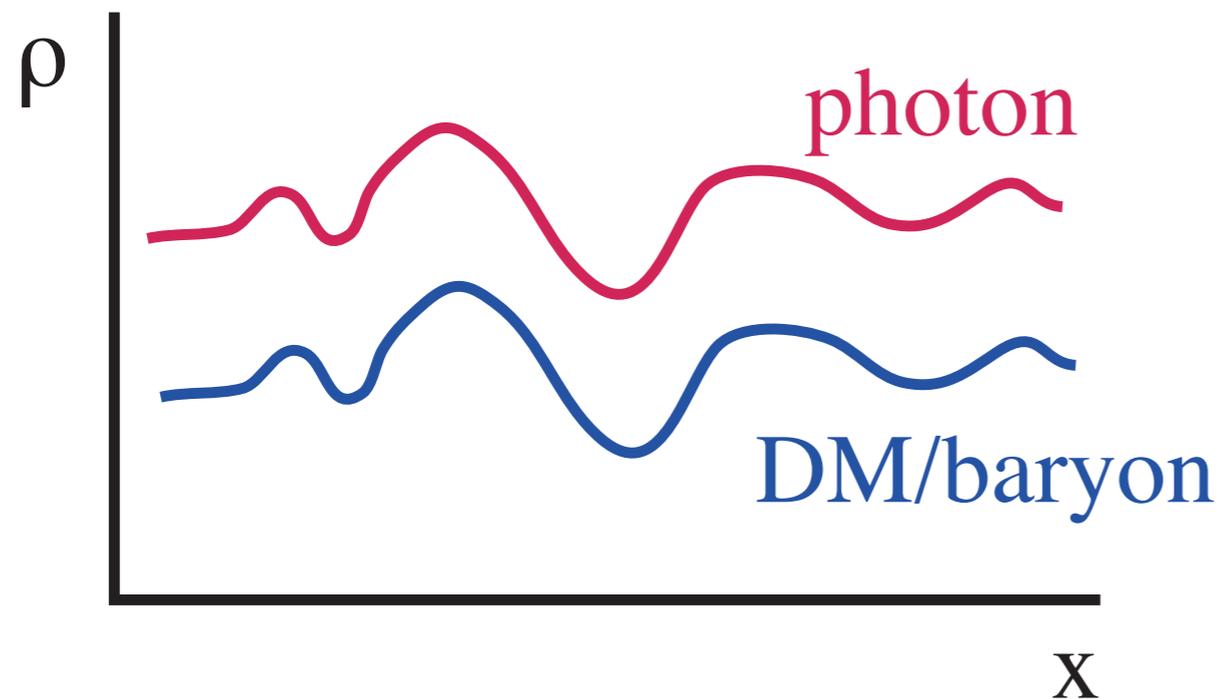


Any little something extra?

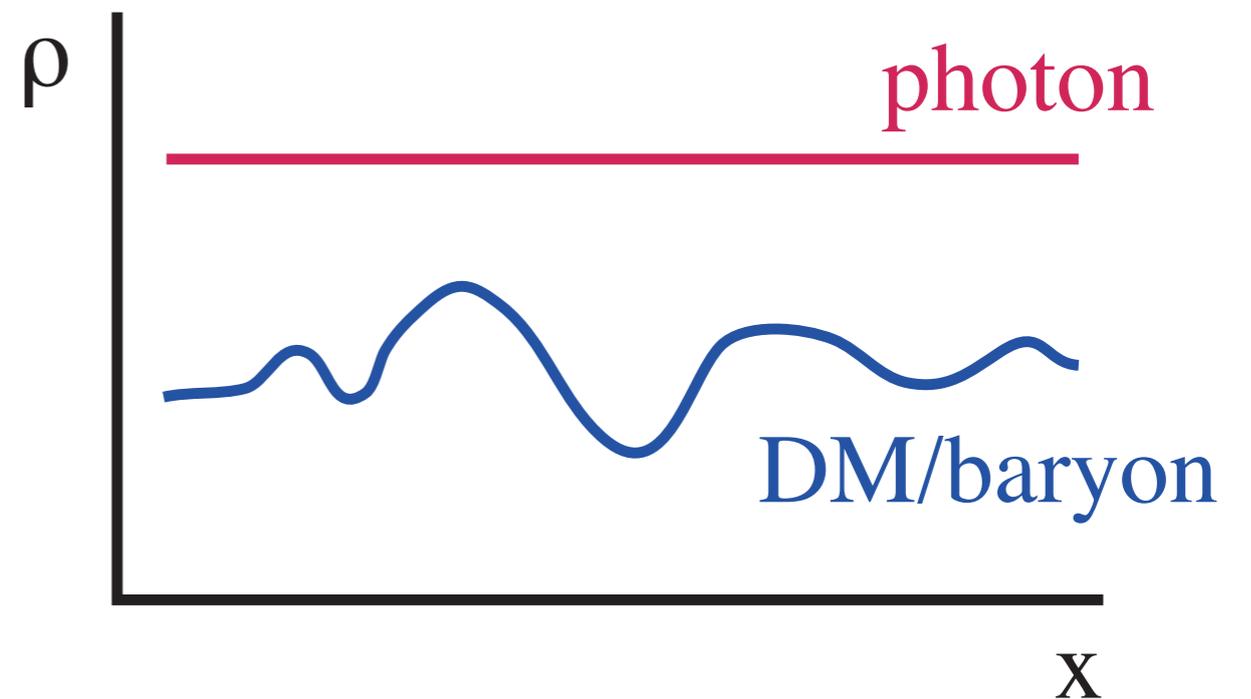
- Isocurvature perturbations
- Running spectral index

Axion isocurvature perturbations

Adiabatic perturbation



Isocurvature perturbation

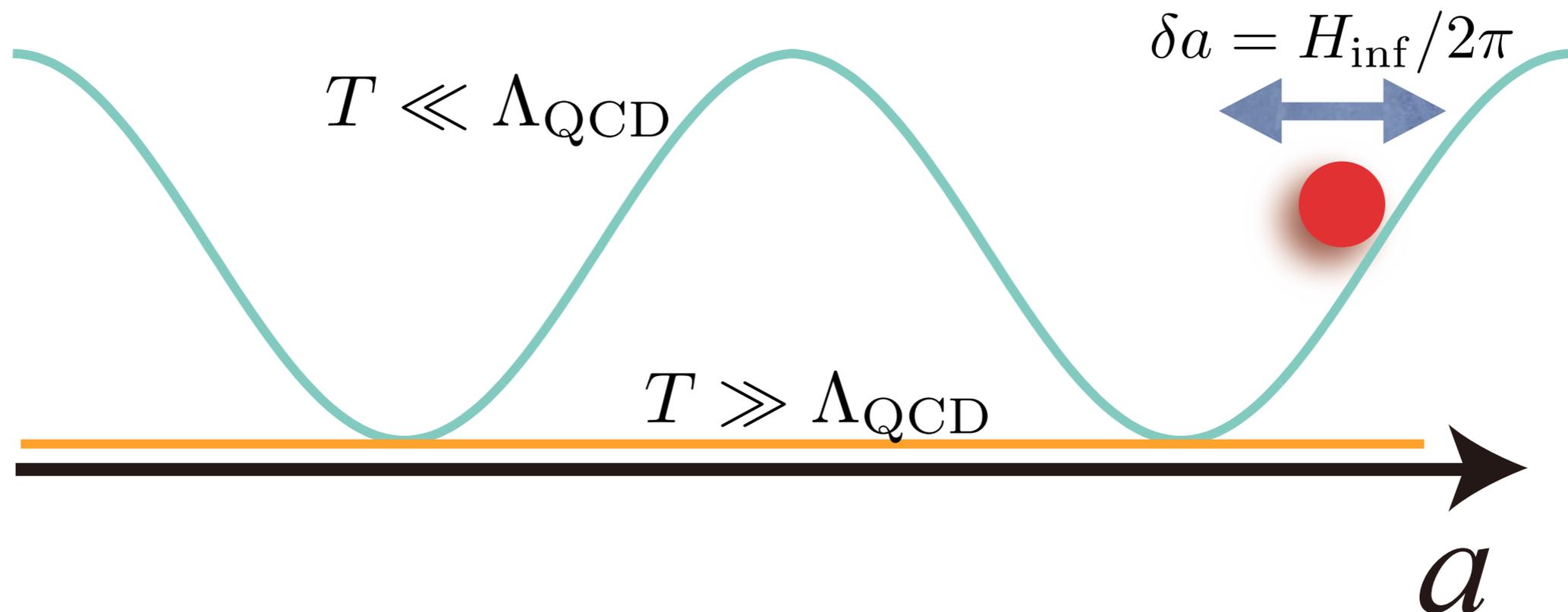


$$\alpha \equiv \frac{P_S}{P_{\mathcal{R}}} \lesssim 0.041 \quad (95\% \text{CL})$$

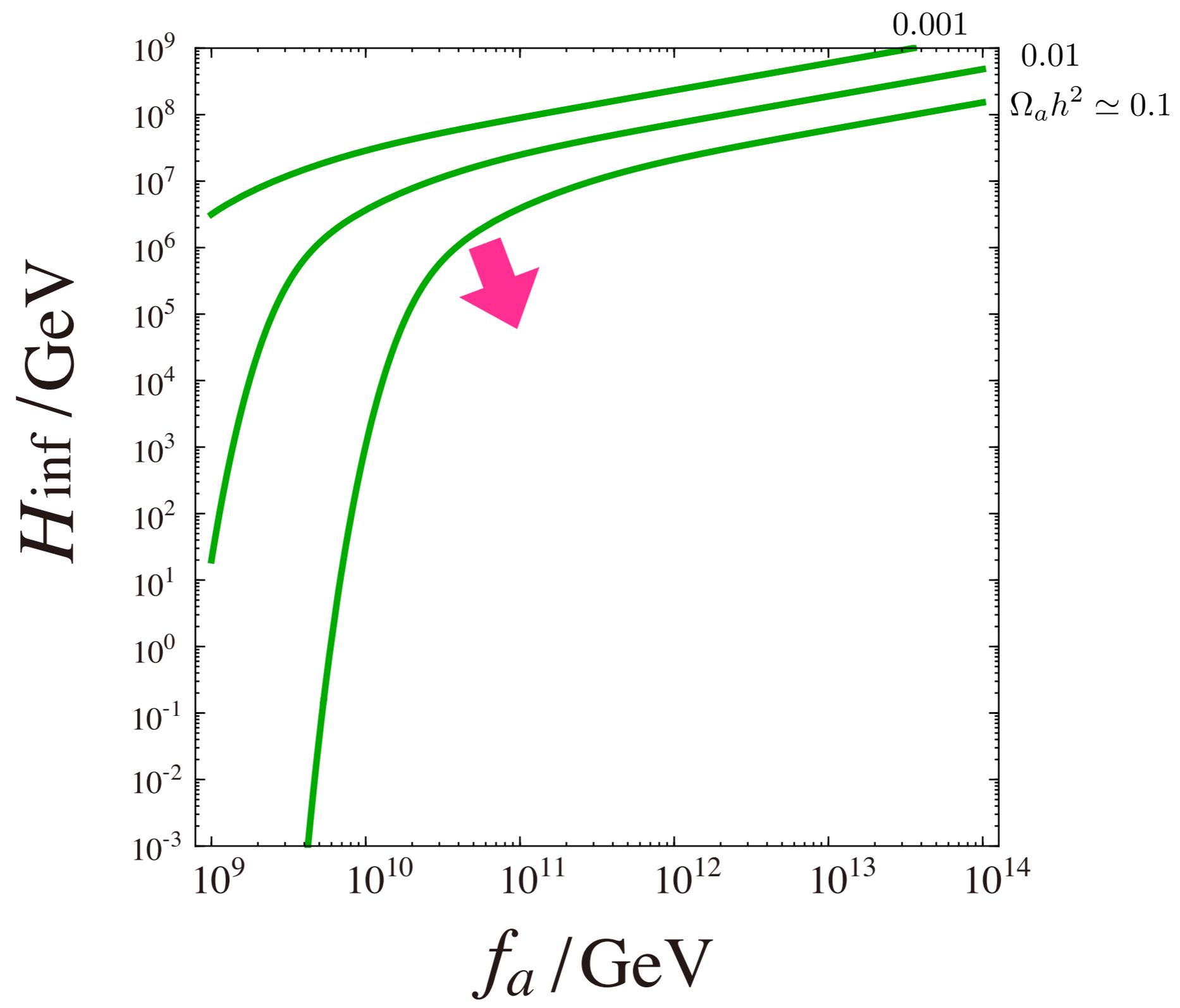
(Planck+WMAP polarization)

- The QCD axion is a plausible candidate for DM with isocurvature perturbations.

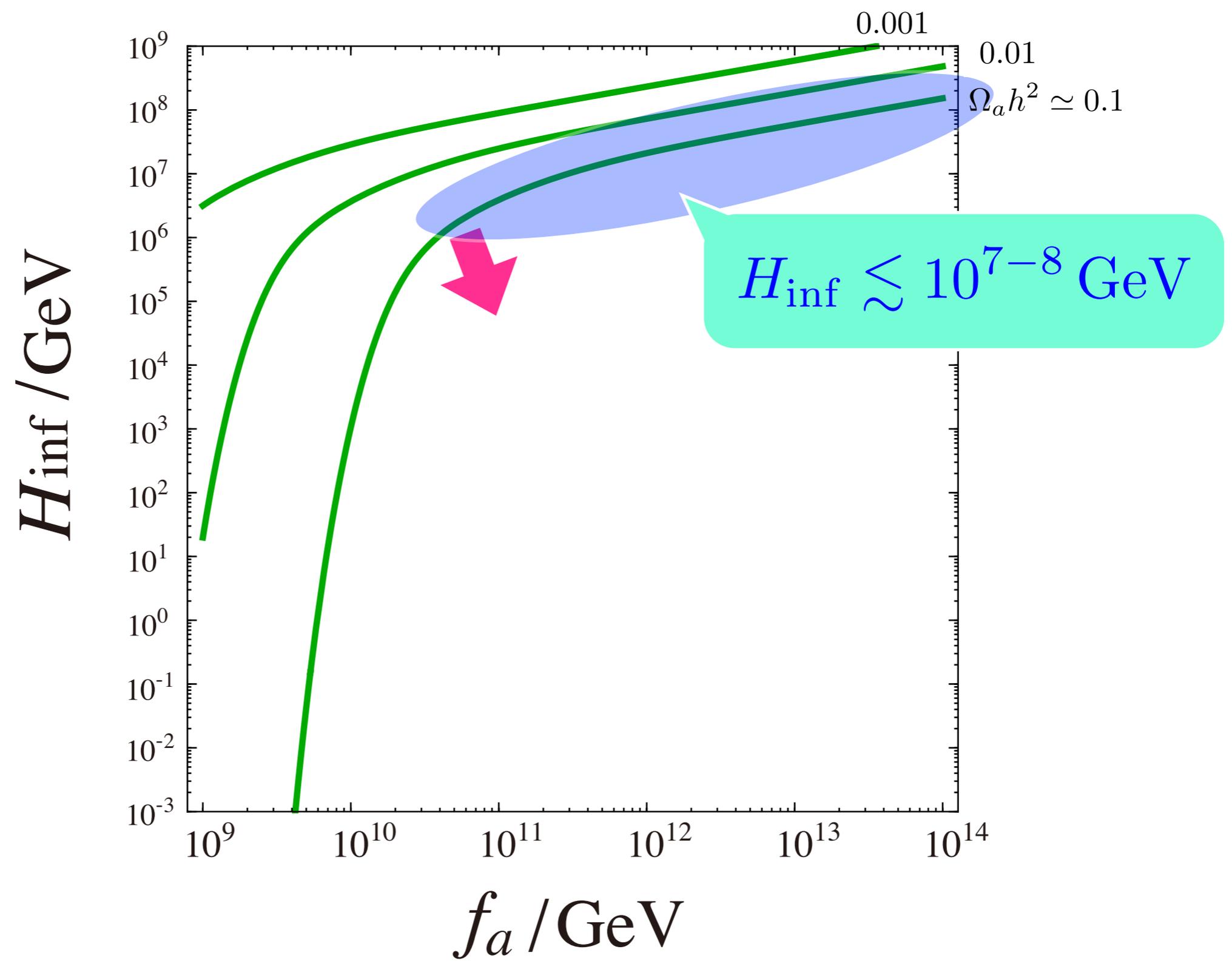
$$\mathcal{L} = \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$$



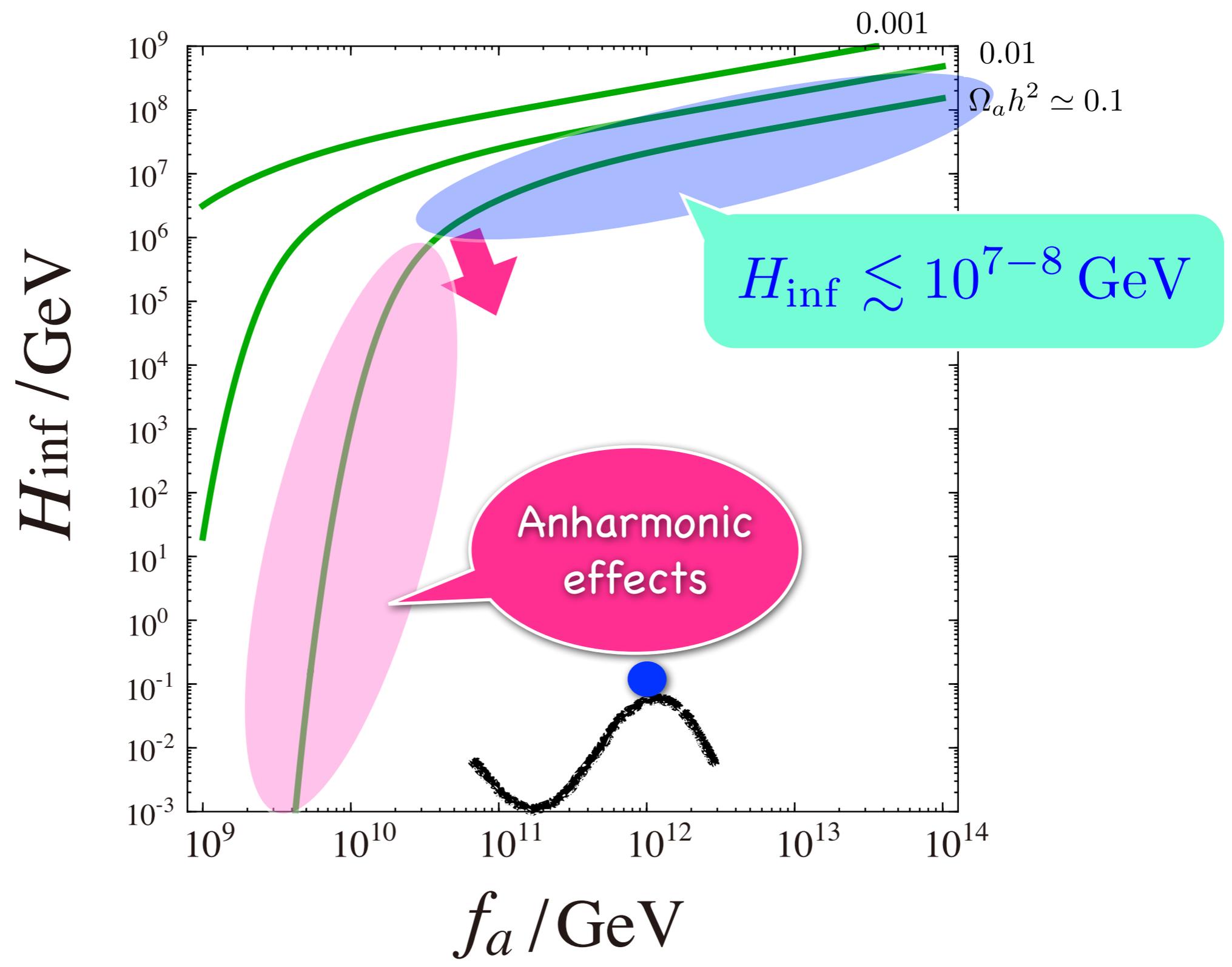
Isocurvature constraint on H_{inf}



Isocurvature constraint on H_{inf}

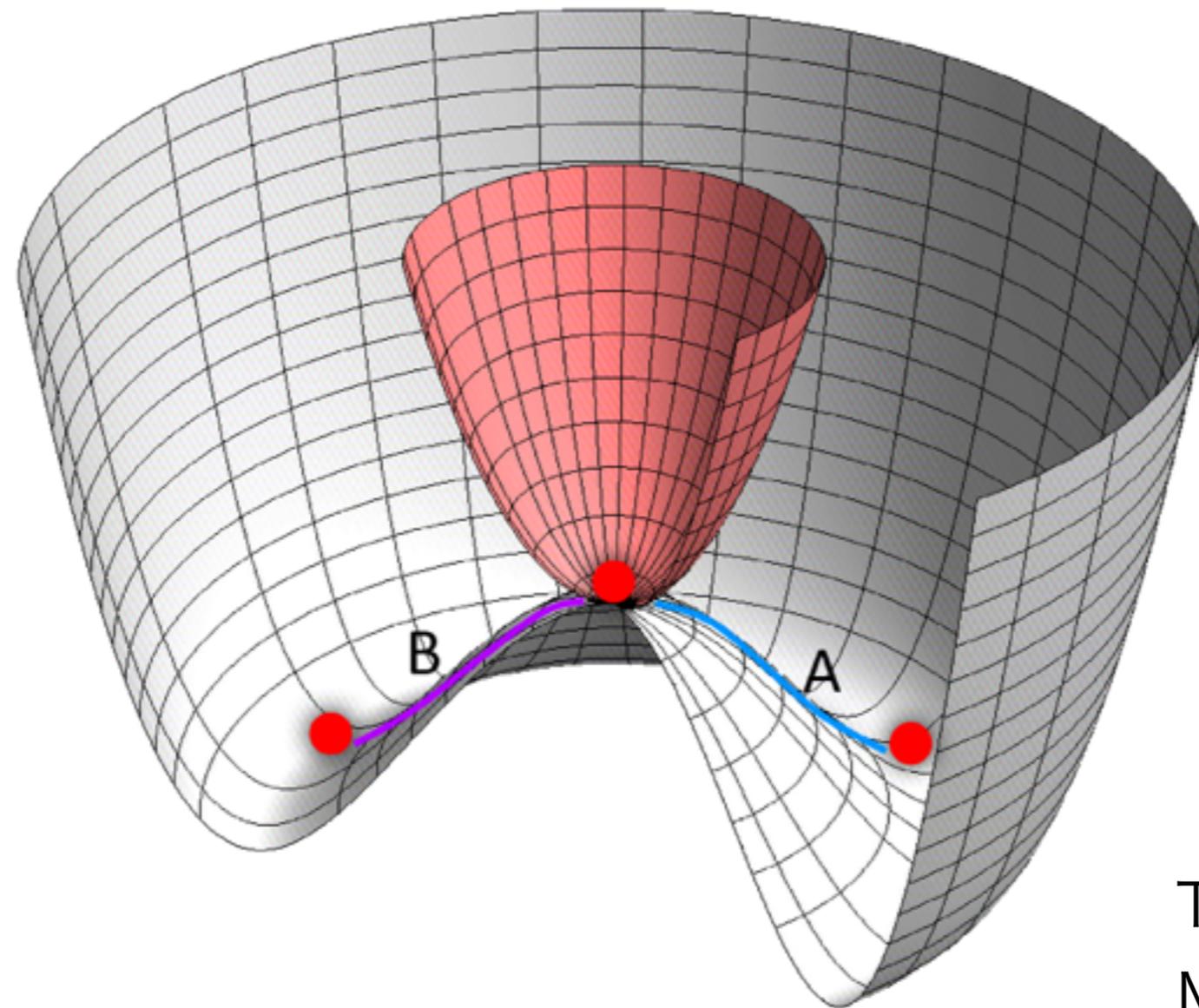


Isocurvature constraint on H_{inf}



Solutions

- Restoration of Peccei-Quinn symmetry during inflation.

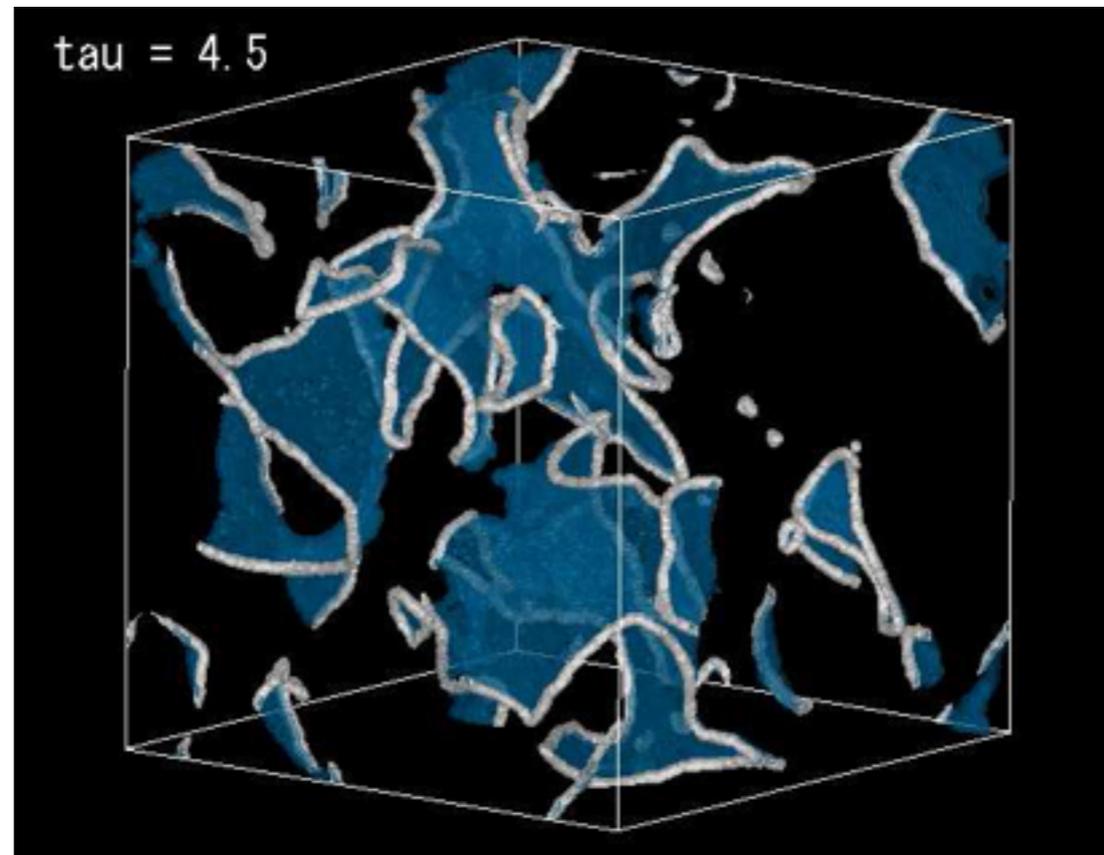


Taken from
M. Kawasaki's slide

Solutions

- Restoration of Peccei-Quinn symmetry during inflation.
- Axions are produced from domain walls and axion DM is possible for $f_a = 10^{10}\text{GeV}$.

[Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851,1207.3166](#)

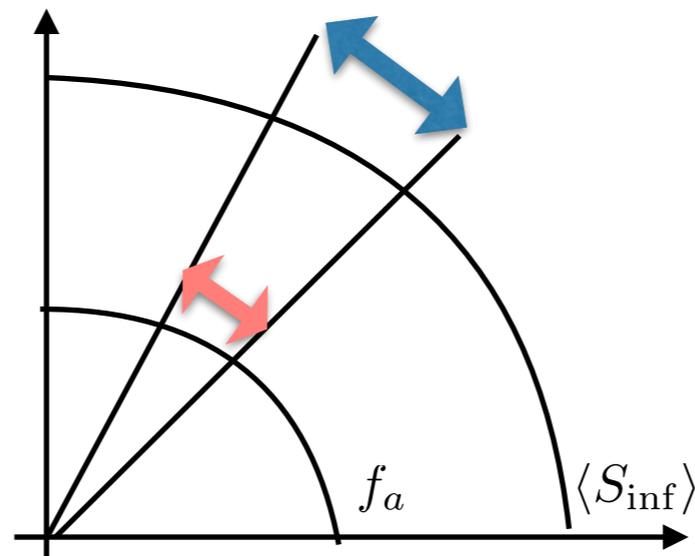


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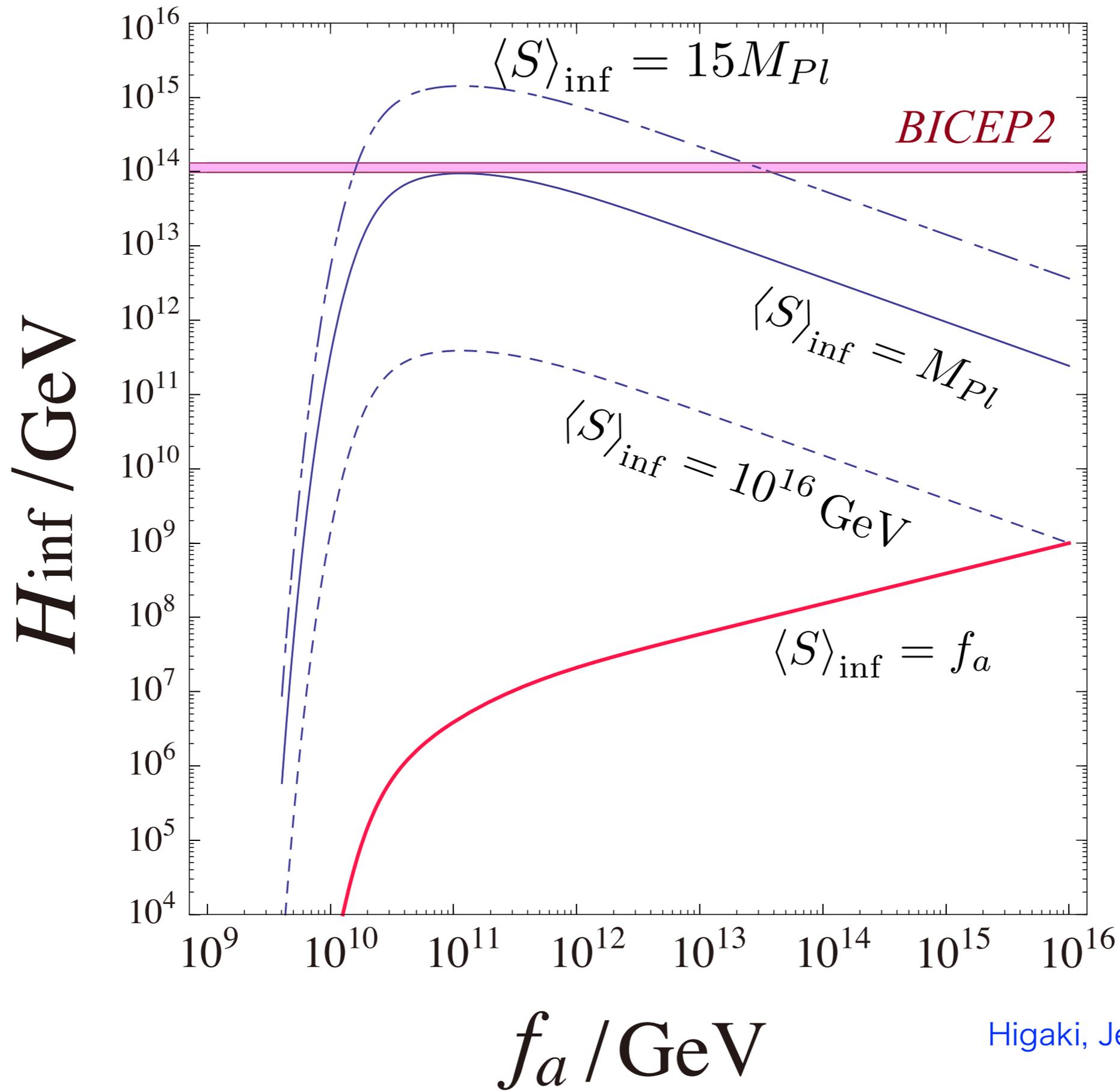
Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851, 1207.3166

- (Super-)Planckian saxion field value during inflation. (Saxion could be the inflaton)



Axion: phase component
Saxion: radial component

$$S = \frac{f_a + \sigma}{\sqrt{2}} e^{ia/f_a}$$



axion CDM
is assumed.

Solutions

- Restoration of Peccei-Quinn symmetry during inflation.
- Axions are produced from domain walls and axion DM is possible for $f_a = 10^{10}\text{GeV}$.

Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851, 1207.3166

- Super-Planckian saxion field value during inflation. (Saxion could be the inflaton)
- Heavy axions during inflation. $m_a^2 \gtrsim H_{\text{inf}}^2$

- Stronger QCD during inflation

Dvali, hep-ph/9505253

Jeong, FT 1304.8131

- Enhanced explicit PQ breaking

Dine, Anisimov hep-ph/0405256

Higaki, Jeong, FT, 1403.4186

Conclusions

- **If $r = 0(0.001-0.1)$** , we can get information of the very early Universe at the GUT-scale.
- **Large-field inflation** realized by shift symmetry.
 - Chaotic inflation/(multi-)natural inflation lead to various values of (n_s, r) .
- **Axion landscape**
 - Eternal inflation and subsequent slow-roll inflation realized in a unified manner.
 - (Multi-)natural inflation by the KNP mechanism
 - Just 50-60 e-foldings may lead to negative curvature.
- **Anything extra ?**
 - Axion CDM Isocurvature perturbations.
 - Running spectral index /spatial curvature, etc.