Quark spectrum near the critical point of chiral transition

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Our previous study on the quark spectrum

How do the fluctuations of the chiral condensate affect the quark spectrum near $T_c$?

- model: Nambu-Jona-Lasinio model (2-flavor, chiral limit)
- phase diagram of the chiral transition
  - 2nd order in the low density region
  - 1st order in the high density region

- spectrum of the fluctuations of the chiral condensate
  (Hatsuda-Kunihiro 85)
quark spectrum

quark self-energy: $\Sigma(p_0, p)$:

quark spectral function:

The scattering off the fluctuations forms the three-peak structure.

$E = 1.05T_C, \mu = 0$

Contour of the spectral function

red lines:

$\omega - |p| - \text{Re}\Sigma_+ = 0$
This study: **FINITE current quark mass**

- phase diagram of the chiral phase transition

  - current quark mass: 5.5 MeV

  ![Graph showing phase diagram](image)

  - The pseudo-critical line is determined from a maximum of the spectral function for $p=10$ MeV (dynamic chiral susceptibility).

- masses of the sigma, pion, and dynamical quark

  ![Graph showing masses](image)
The soft mode is not the sigma mode, but appears in the space-like region. (Fujii 03, Fujii-Ohtani 04)

What is the soft mode at CP?

The soft mode is not the sigma mode, but appears in the space-like region. (Fujii 03, Fujii-Ohtani 04)

scalar density fluc.
\[ \langle \bar{q}q(p, p_0)\bar{q}q(0,0) \rangle \]

pseudo-scalar density fluc.
\[ \langle \bar{q}i\gamma_5\tau q(p, p_0)\bar{q}i\gamma_5\tau q(0,0) \rangle \]

**softneing**

\[ \mu = \mu_{CP}, T \sim T_{CP} \]

**NOT softneing**

sigma meson, \( m_\sigma \sim 2m_q \)

pion pole: not shown
pion dispersion relations in medium

\[ \mu = \mu_{CP}, T \sim T_{CP} \]

\[ \mu = 0, T \sim T_{PC} \text{ (pseudo-critical)} \]

\[ m_q: \text{dynamically generated (constituent) quark mass (mean field)} \]

\[ E_\pi(p) \neq \sqrt{p^2 + E_\pi(0)^2} \]

\[ \text{c.f: fermion and gauge boson in HTL} \]
Quark spectrum near CP

• Quark spectral function $\rho_\pm$ for $p=0$

$$\rho_\pm(p_0,0) = -\frac{1}{\pi} \text{Im} \frac{1}{p_0 + \mu \mp m_q - \Sigma_\pm(p_0,0)}$$

self-energy

$$\Sigma = \text{soft mode} + \text{pi mode}$$

- scalar fluc.
- space-like
- sigma mode
- time-like (cont.)

- pseudo-scalar fluc.
- time-like (pole or cont.)

- s, ps fluctuations

ex. pion pole contribution

$$\text{Im } \Sigma_+(0,p_0) \sim \int^\Lambda dq \left(1 - \frac{m}{E_q}\right) Z(E_\pi(q)) \delta(p_0 - E_q + \mu - E_\pi(q)) \left(1 + n(E_\pi(q)) - f(E_q - \mu)\right)$$

$$+ \left(1 - \frac{m}{E_q}\right) Z(E_\pi(q)) \delta(p_0 - E_q + \mu + E_\pi(q)) \left(n(E_\pi(q)) + f(E_q - \mu)\right)$$

$$+ \left(1 + \frac{m}{E_q}\right) Z(E_\pi(q)) \delta(p_0 + E_q + \mu - E_\pi(q)) \left(n(E_\pi(q)) + f(E_q + \mu)\right)$$

$$+ \left(1 + \frac{m}{E_q}\right) Z(E_\pi(q)) \delta(p_0 + E_q + \mu + E_\pi(q)) \left(1 + n(E_\pi(q)) - f(E_q + \mu)\right)$$

pion pole residue

BE dist. Func.

FD dist. Func.
Quark spectrum near the critical point

- **quark spectrum:**
  - one peak at 120 MeV
    - shift by coupling with the soft mode
  - the other peak at 80 MeV
    - but small residue $\sim 0.01$

- **self-energy**
  - large imaginary around 200 MeV
    - through the below process
      - $(0, \vec{p}_0)$
      - $(\vec{q}, E_q)$
        - on-shell
        - soft
      - $(-\vec{q}, p_0 - E_q)$
        - space-like

- **divergence at ±80 MeV**
  - van Hove singularities

\[ \mu = \mu_{CP}, T \sim T_{CP} \]

\[
\begin{align*}
\rho_+ & \\
\text{residue: } 0.01 & \\
m_q & = 185
\end{align*}
\]
van Hove singularity

van Hove singularity = divergence of density of states

density of states $D(E)$

$$D(E)dE = \int_{E<E(p)<E+dE} d^3p \sim \frac{1}{|\nabla_p E(p)|} dE$$

$D(E)$ diverges when $E(p)$ has a maximum.

ex: plasmino in HTL

joint density of states

$$D(E)dE = \int_{E<E_f-E_i<E+dE} d^3p \sim \frac{1}{|\nabla_p (E_f - E_i)|} dE$$

$D(E)$ diverges when $E_f(p) - E_i(p)$ has a maximum.

present case