クォーク閉じ込め･非閉じ込め相転移とノンアーベリアン双対超伝導描像

柴田章博

KEK　計算科学センター

In collaboration with:
近藤慶一 (千葉大･理)
篠原徹(千葉大･理)
加藤清孝(福井高専)

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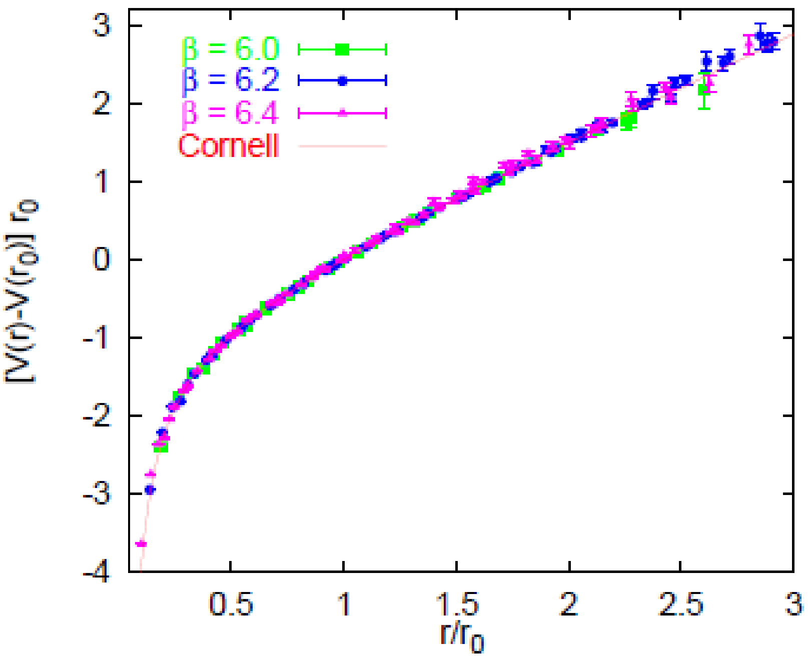
Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

- The **dual superconductivity** is a promising mechanism for quark confinement. [Y.Nambu (1974). G. 't Hooft, (1975). S. Mandelstam, (1976) A.M. Polyakov, (1975). Nucl. Phys. B 120, 429(1977).]



G.S. Bali, [hep-ph/0001312], Phys. Rept. **343, 1–136 (2001)**

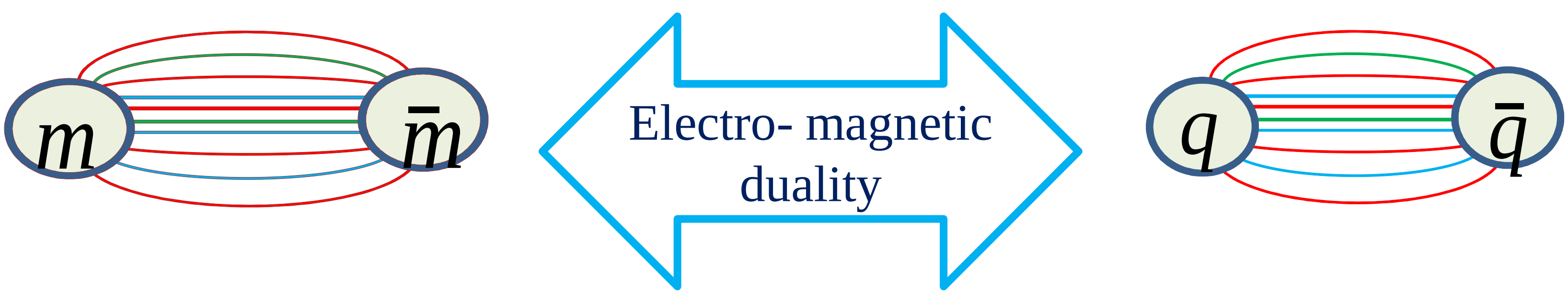
dual superconductivity

superconductor

- ➢ Condensation of electric charges (Cooper pairs)
- ☐ Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- ◆ Linear potential between monopoles

dual superconductor

- ➢ Condensation of magnetic monopoles
- ☐ Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- ◆ Linear potential between quarks



The evidence for dual superconductivity

To establish the dual superconductivity picture, we must show that **the magnetic monopole plays a dominant role for quark confinement:**

Many preceding studies based on the **Abelian projection**: $U_{x,\mu} = X_{x,\mu} V_{x,\mu}$

The gauge link is decomposed into the Abelian (diagonal) part V and the remainder (off-diagonal) part X

- ☐ Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- ☐ Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]
- ☐ Measurement of (Abelian) dual Meissner effect
- ◆ Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux[]
- ◆ Type the super conductor is the order between Type I and Type II [Y.Matsubara, et.al. 1994]

◆These are only obtained in the case of **special gauge** such as maximal Abelian gauge (MAG), and gauge fixing breaks the gauge symmetry as well as color symmetry (global symmetry).

A new lattice formulation

- *We have presented a new lattice formulation of Yang-Mills theory, that can establish "Abelian" dominance and magnetic monopole dominance in the gauge independent way (gauge-invariant way)*

We have proposed the decomposition of gauge link,

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

which can extract the relevant mode *V* for quark confinement.

- For SU(2) case, the decomposition is a lattice compact representation of the *Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.*
- For SU(N) case, the formulation is the extension of the SU(2) case.

The path integral formulation in continuum theory by Kondo-Murakami-Shinohara;
SU(2) case: Eur. Phys. J. C 42, 475 (2005), Prog. Theor. Phys. 115, 201 (2006).
SU(N) case: Prog.Theor. Phys. 120, 1 (2008)

Plan of the talk

- Introduction
- A new formulation of Yang-Mills theory on a lattice
- lattice measurement at zero temperature (quick review)
- lattice measurement at finite temperature
  - Restricted field dominance
  - Measurement of flux tube in deconfinement phase
- summary

A new formulation of Yang-Mills theory (on a lattice)

Decomposition of SU(N) gauge links

- The decomposition as the extension of the SU(2) case.
- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

□ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

□ SU(3) Yang-Mills link variables: Two options

maximal option : $U(1) \times U(1) \subset SU(3)$

✓ Maximal case is **a gauge invariant version of Abelian projection** in the maximal Abelian (MA) gauge. (the maximal torus group)

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

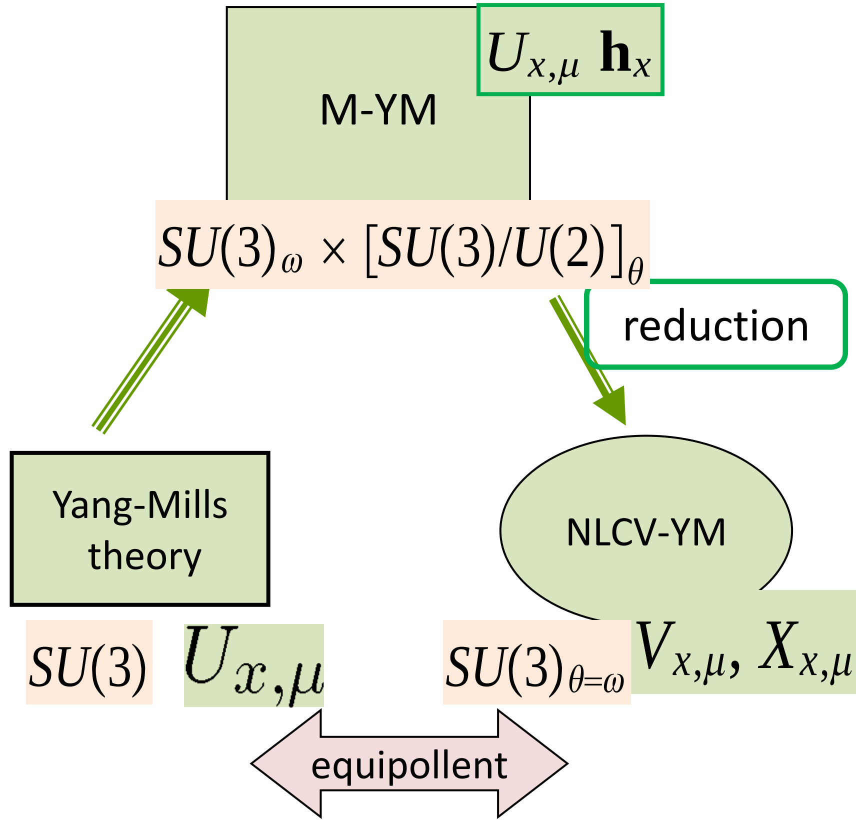
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^\dagger_x$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x,x+\mu\rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.} W_C[V] \quad !!$$

Defining equation for the decomposition

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D^\epsilon_\mu[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a^{(0)}_x\mathbf{h}_x - i\sum_{i=1}^{3} a^{(l)}_x u^{(i)}_x) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}^\dagger_{x,\mu}(\det\hat{L}_{x,\mu})^{1/N} g^{-1}_x \quad V_{x,\mu} = X^\dagger_{x,\mu}U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det\hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L^\dagger_{x,\mu}}\right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U^{-1}_{x,\mu}$$

continuum version by continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

Reduction Condition

- The decomposition is uniquely determined for a given set of link variables $U_{x,\mu}$ describing the original Yang-Mills theory and color fields.
- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- The configuration of the color fields $\mathbf{h}_x$ can be determined by the reduction condition such that the reduction functional is minimized for given $U_{x,\mu}$

$$F_{\mathrm{red}}[\mathbf{h}_x; U_{x,\mu}] = \sum_{x,\mu} \mathrm{tr}\left\{ (D_\mu^\epsilon[U]\mathbf{h}_x)^\dagger (D_\mu^\epsilon[U]\mathbf{h}_x) \right\}$$

$$SU(3)_\omega \times [SU(3)/U(2)]_\theta \to SU(3)_{\omega=\theta}$$

- This is invariant under the gauge transformation θ=ω
- The extended gauge symmetry is reduced to the same symmetry as the original YM theory.
- We choose a reduction condition of the same type as the SU(2) case

Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection

$$W_C[\mathcal{A}] = \int [d\mu(\xi)]_\Sigma \exp\left(-ig\int_{S:C=\partial\Sigma} dS^{\mu\nu}\sqrt{\frac{N-1}{2N}}\mathrm{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x))\right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp\left(ig\sqrt{\frac{N-1}{2N}}(k,\Xi_\Sigma) + ig\sqrt{\frac{N-1}{2N}}(j,N_\Sigma)\right)$$

magnetic current $k := \delta^* F = {}^* dF$, $\quad \Xi_\Sigma := \delta^*\Theta_\Sigma \Delta^{-1}$

electric current $j := \delta F$, $\quad N_\Sigma := \delta\Theta_\Sigma\Delta^{-1}$

$\Delta = d\delta + \delta d$, $\quad \Theta_\Sigma := \int_\Sigma d^2S^{\mu\nu}(\sigma(x))\delta^D(x - x(\sigma))$

k and j are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

K.-I. Kondo PRD77 085929(2008)

The lattice version is defined by using plaquette:

$$\Theta^8_{\mu\nu} := -\arg \mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x\right)V_{x,\mu}V_{x+\mu,\mu}V^\dagger_{x+\nu,\mu}V^\dagger_{x,\nu}\right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_\nu\Theta^8_{\alpha\beta},$$

LATTICE RESULTS IN ZERO TEMPERATURE

- SU(3) Yang-Mills theory
  - In confinement of fundamental quarks, a restricted non-Abelian variable ***V*** **,** and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

gauge independent "Abelian" dominance

$$\frac{\sigma_V}{\sigma_U} = 0.92$$

$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

Gauge independent non-Abalian monople dominance

$$\frac{\sigma_M}{\sigma_U} = 0.85$$

$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

U^* is from the table in R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B517, 377 (1998).

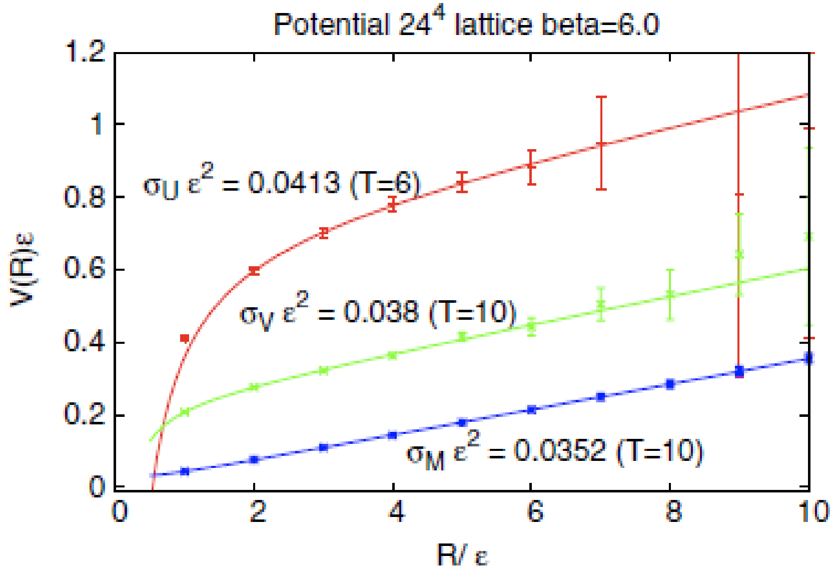


FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance R: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on 24^4 using 500 configurations where ϵ is the lattice spacing.

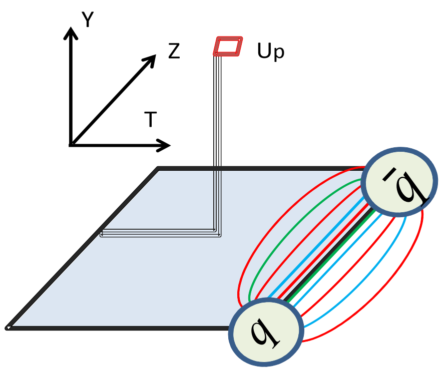
PRD 83, 114016 (2011)

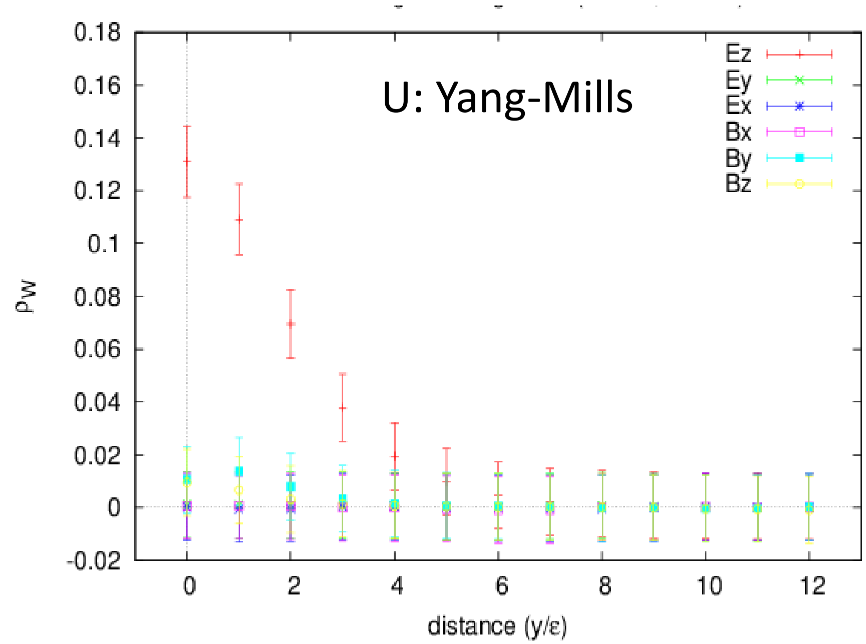
Chromo flux

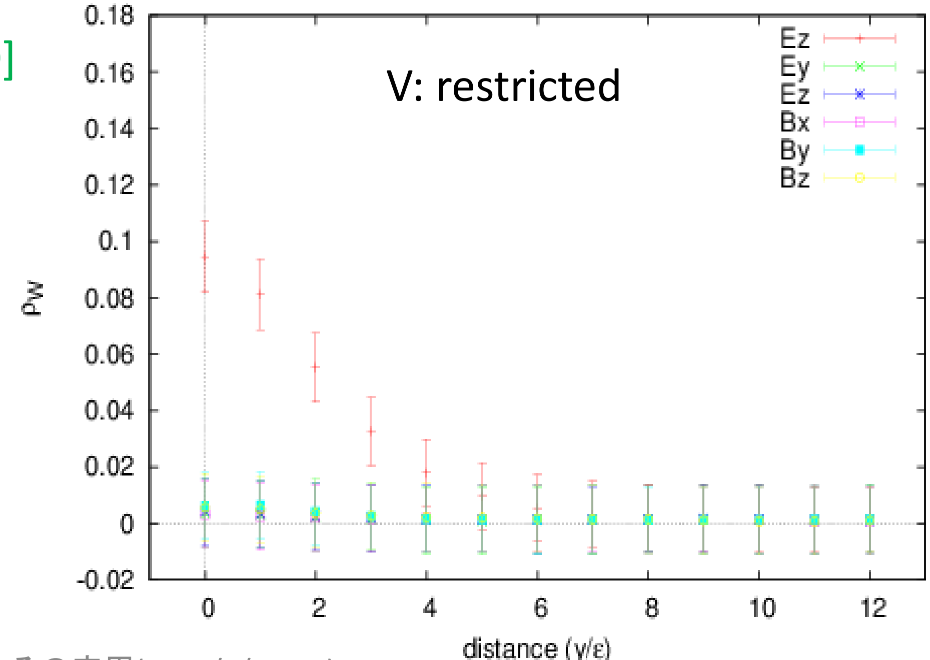
$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger) \rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(W) \mathrm{tr}(U_p) \rangle}{\langle \mathrm{tr}(W) \rangle}$$

Gauge invariant correlation function: This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3) [Adriano Di Giacomo et.al. PLB236:199,1990 NPBB347:441-460,1990]

$\mathrm{tr}(U_p L W L^\dagger)$

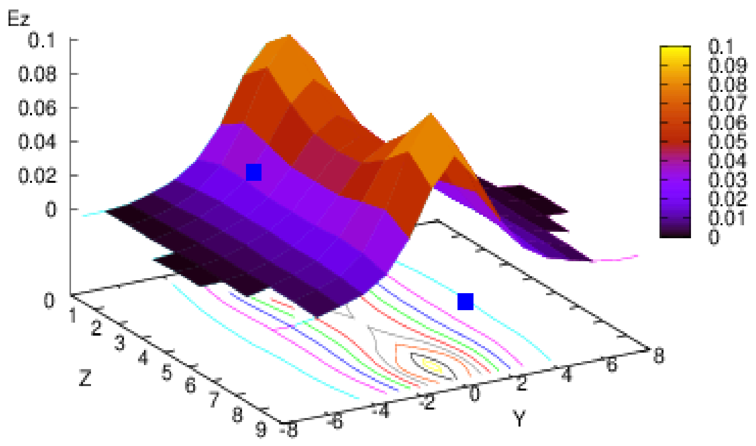




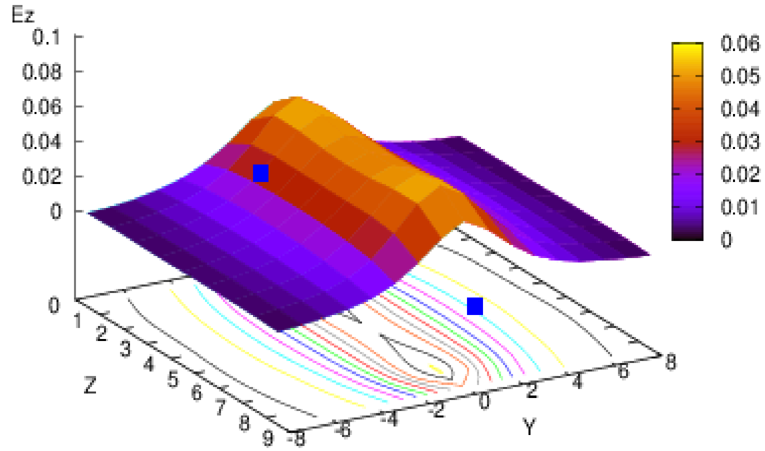


Chromo-electric (color flux) Flux Tube

Original YM filed



Restricted field



A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

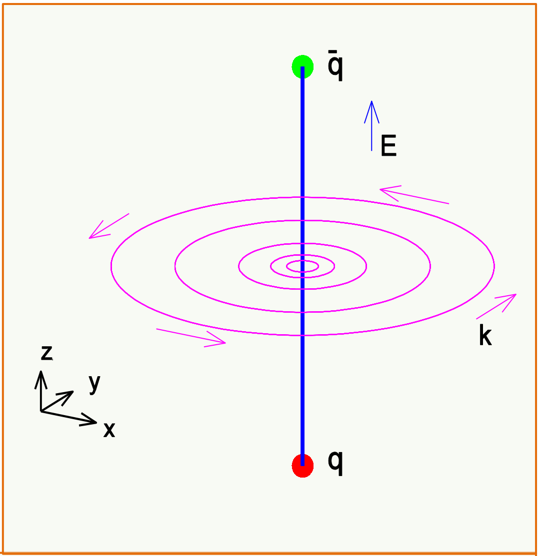
Flux tube is observed for the restricted U(2) field case.

Magnetic current induced by quark and antiquark pair

Yang-Mills equation (Maxwell equation) for $\mathbf{V}_\mu$ field,
the magnetic monopole (current) can be calculated as

$$\mathbf{k} = {}^*dF[\mathbf{V}] \ ,$$

$F[\mathbf{V}]$ is the field strength 2-form of V_μ field
d the exterior derivative and $*$ denotes the Hodge dual.



$\mathbf{k} \neq \mathbf{0} \Rightarrow$
signal of the monopole condensation
the field strength is given by $F[\mathbf{V}] = d\mathbf{V}$
the Bianchi identity : $\mathbf{k} = {}^*d^2\mathbf{V} = 0$

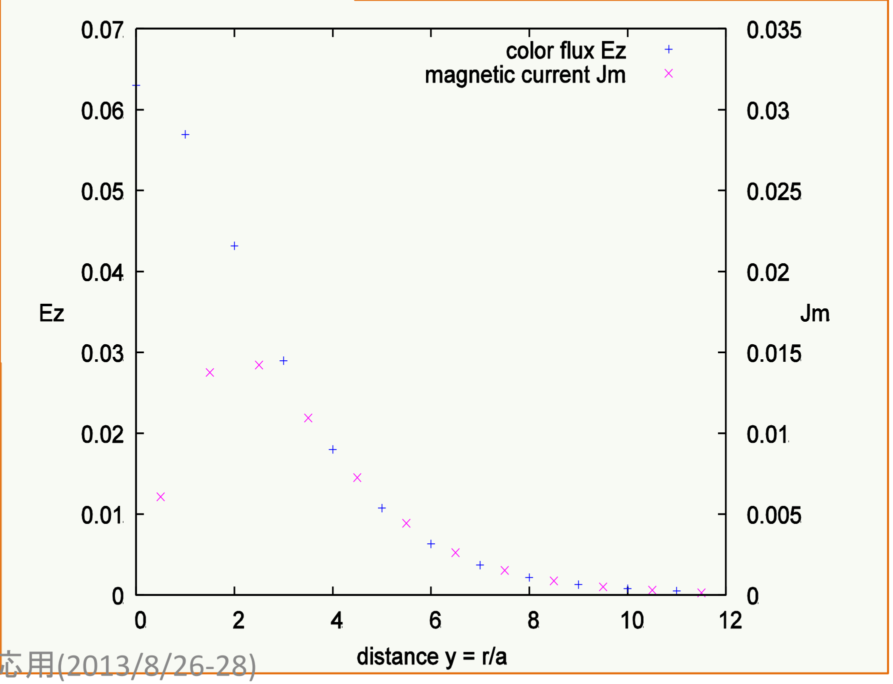


Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current(right scale).

Type of dual superconductivity (Ginzburg-Landau parameter)

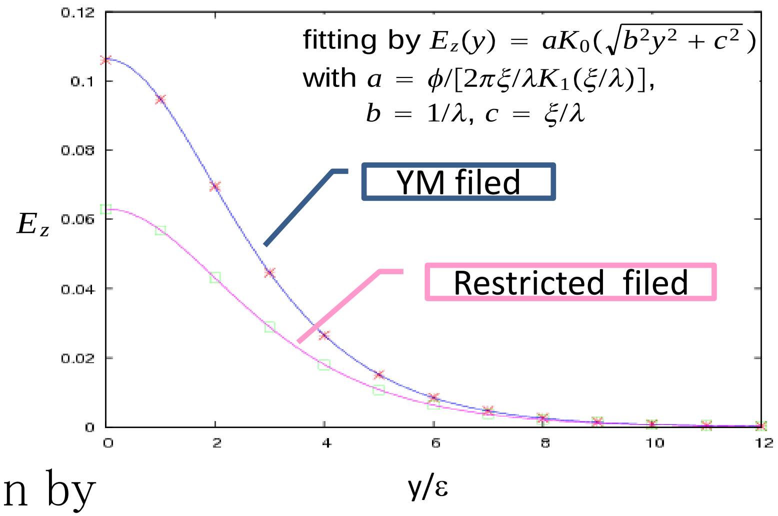
Phys. Rev. D 87, 054011 (2013)

Ginzburg–Landau equation

$$D_\mu D^\mu \phi - \lambda(\phi^* \phi - \mu^2/\lambda^2)\phi = 0$$

Ampere equation

$$\partial^\nu F_{\mu\nu} + iq[\phi^*(D_\mu \phi) - (D_\mu \phi)^* \phi] = 0$$



The shape of the chromoelectric field is given by

$$E_x[y] = \frac{\phi}{2\pi}\frac{1}{\lambda\xi}\frac{K_0(R/\lambda)}{K_1(\xi/\lambda)},\ R = \sqrt{y^2 + \xi^2}$$

where K_ν is the modefied bessel function of the ν–th order, λ the London penetration length, ξ a variational core radius parameter, and ϕ external flux, respectively. **J.R.Clem J. low Temp. Phys. 18 427 (1975)**

	λ/ϵ	ξ/ϵ	$a\epsilon^2$	Φ_0	κ
Yang-Mills	1.65	3.24	1.09	2.00	0.43
restricted U(2)	1.81	3.36	0.567	1.33	0.45

Ginzburg-Landau (GL) parameter

$$\kappa = \sqrt{2}/(\xi/\lambda)\sqrt{1 - K_0^2(\xi/\lambda)/K_1^2(\xi/\lambda)}.$$

Type I $\kappa < \kappa_c = 1/\sqrt{2} \simeq 0.707$

Type || $\kappa > \kappa_c$

LATTICE RESULTS
IN NON-ZERO TEMPERATURE

Polyakov loops

- Polyakov loops

$$P_U(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} U_{(x,t),4}\right) \text{ for original Yang-Mills filed}$$
$$P_V(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} V_{(x,t),4}\right) \text{ for restricted field}$$

- Distribution of space-averaged Polyakov loops for each configurations

$$\langle P_U \rangle := 1/V \sum_x P_U(x), \quad \langle P_V \rangle := 1/V \sum_x P_V(x)$$

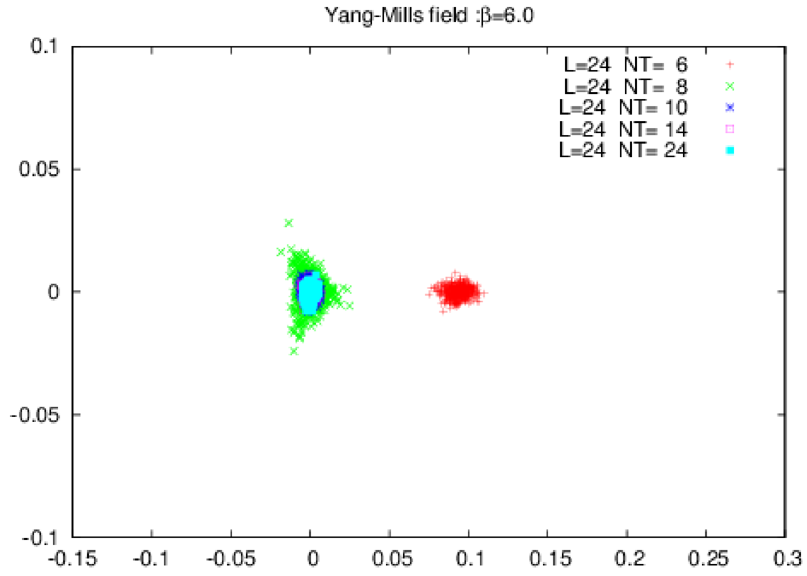
- Vacuum expectation value of space-averaged Polyakov loop and variances

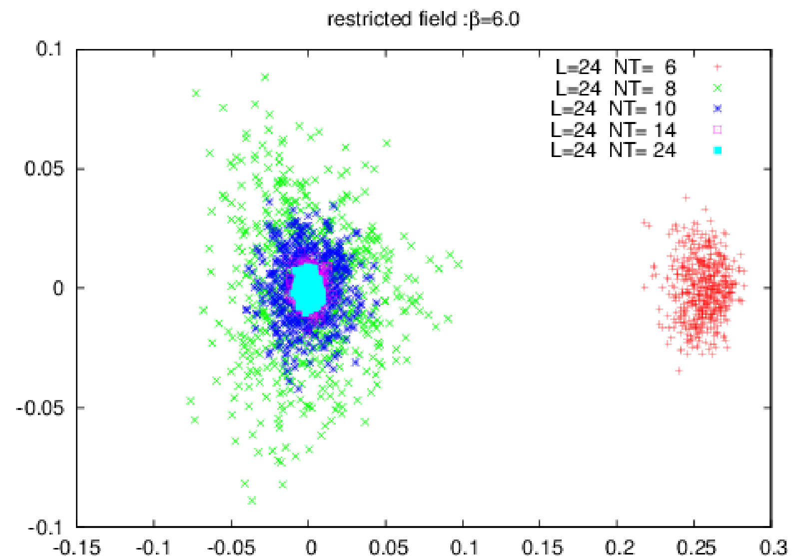
$$\langle\langle P_U \rangle\rangle, \langle\langle P_V \rangle\rangle$$

Distribution of space-averaged Polyakov loops β=6.0

$$\langle P_U \rangle := 1/V \sum_x P_U(x)$$

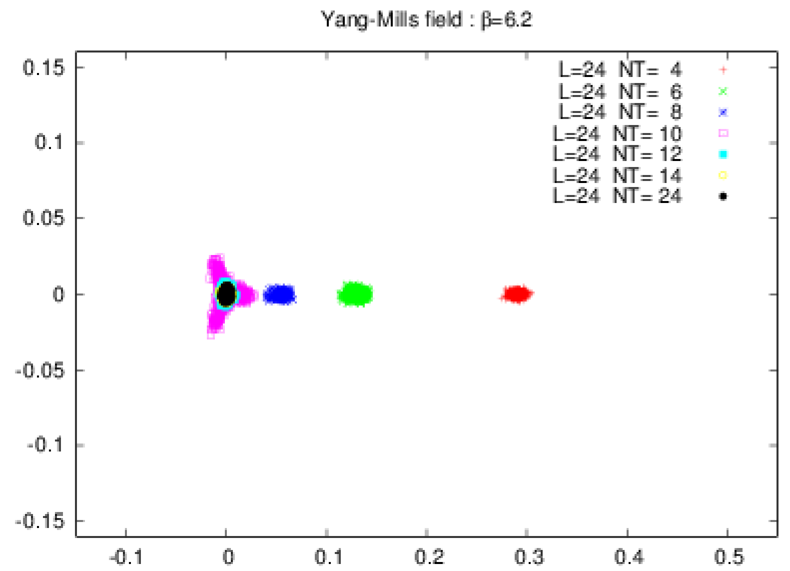
$$\langle P_V \rangle := 1/V \sum_x P_V(x)$$



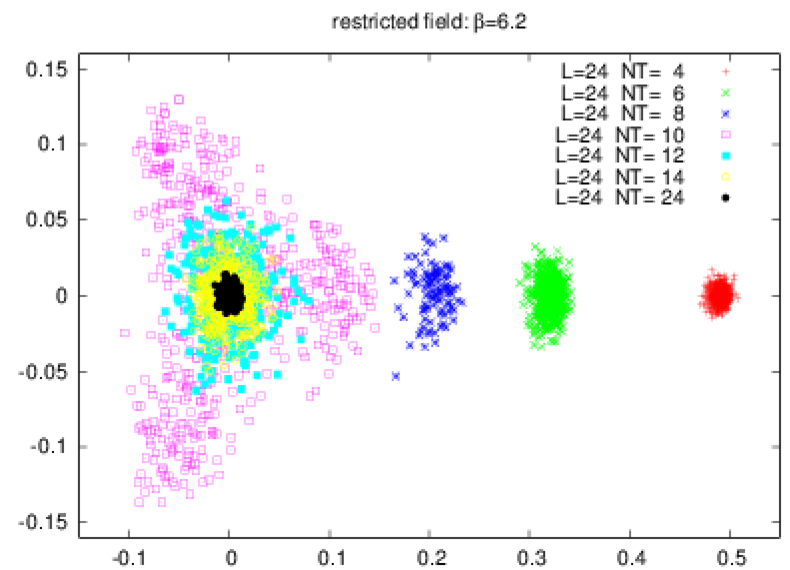


Distribution of space-averaged Polyakov loops β=6.2

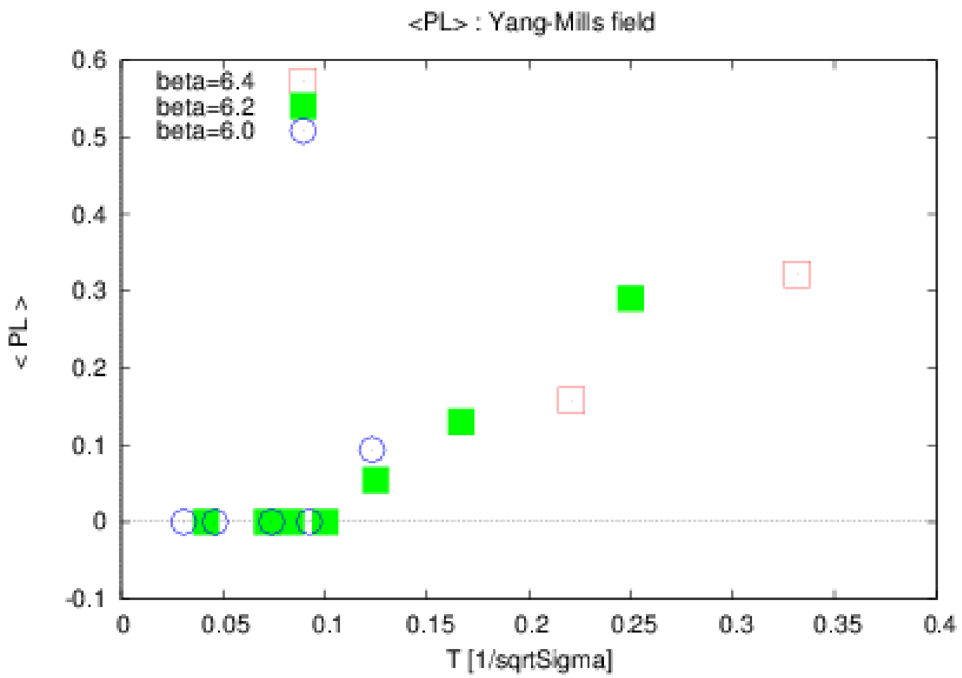
$$\langle P_U \rangle := 1/V \sum_x P_U(x)$$

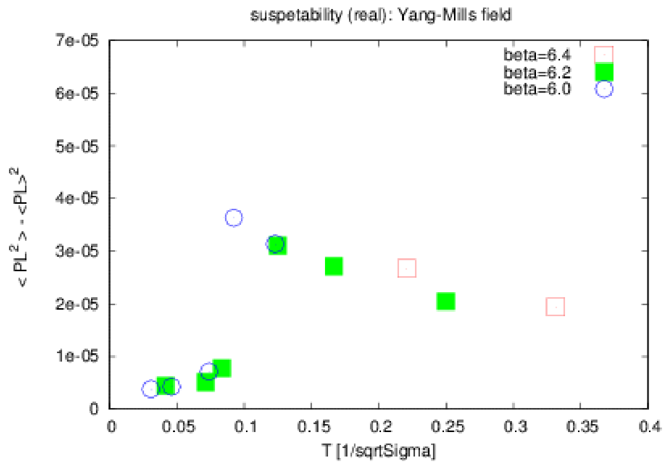


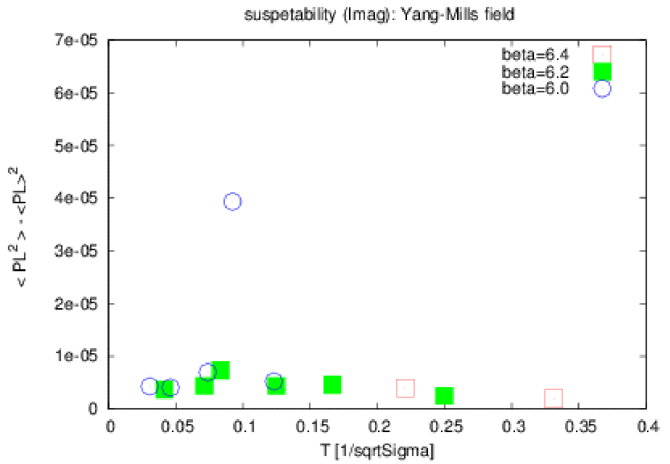
$$\langle P_V \rangle := 1/V \sum_x P_V(x)$$



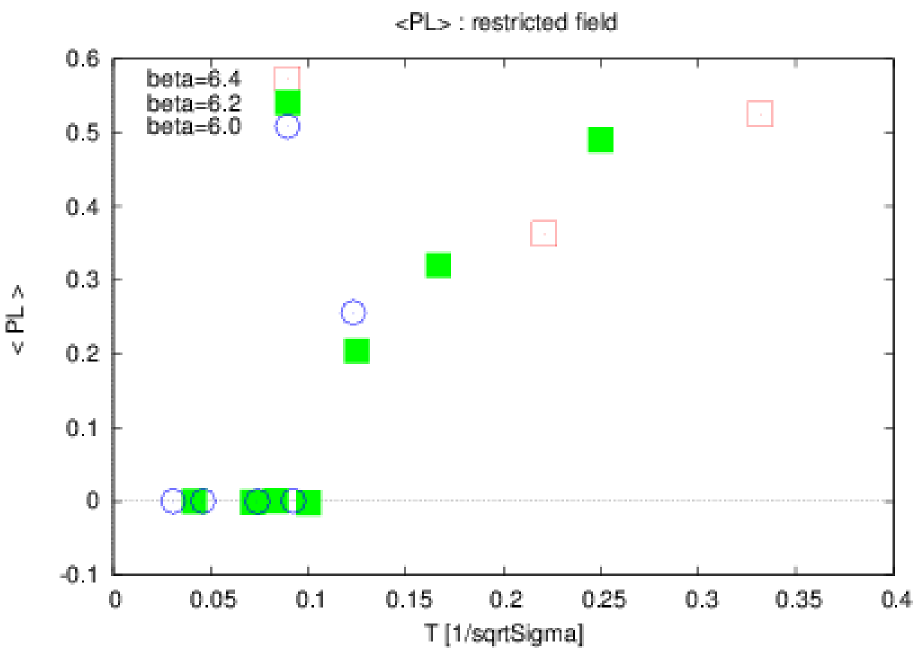
Average of Polyakov loops : YM field

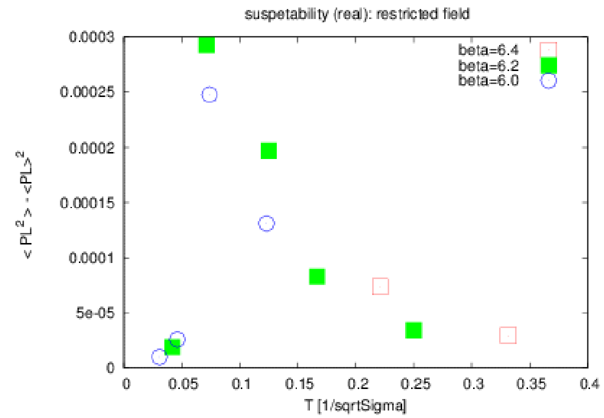


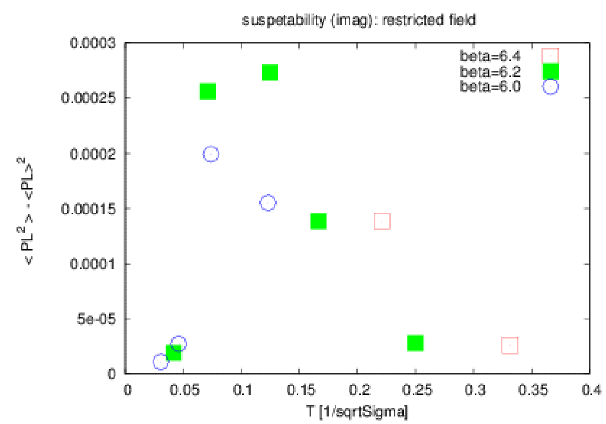




Average of Polyakov loops : restricted field







Comparison of average of Polyakov loops YM field v.s. restricted field

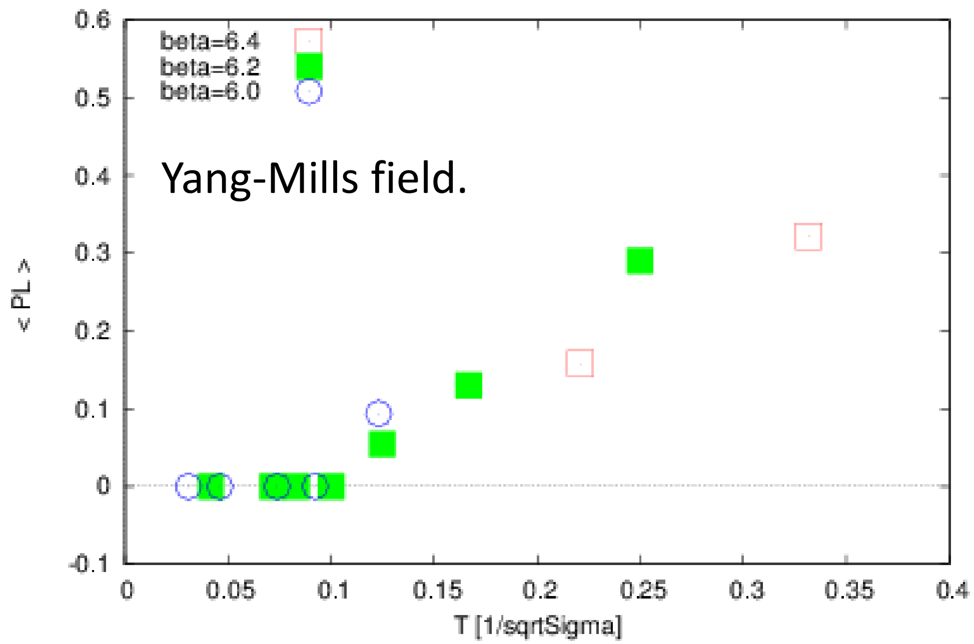
Confinement phase

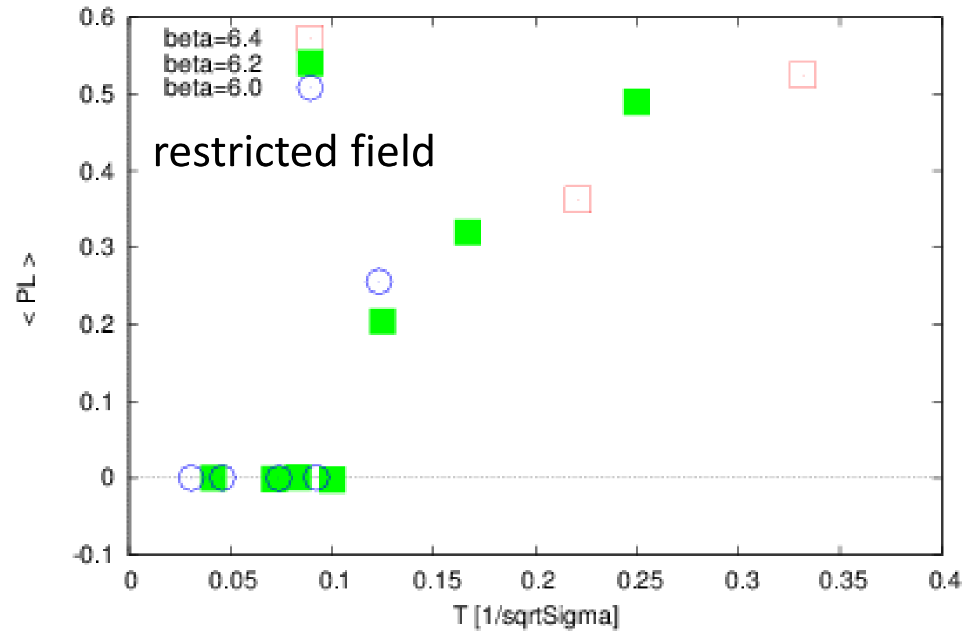
$$\langle\langle P_U(x) \rangle\rangle = 0$$
$$\langle\langle P_V(x) \rangle\rangle = 0$$

Deconfiement phase

$$\langle\langle P_U(x) \rangle\rangle \neq 0$$
$$\langle\langle P_V(x) \rangle\rangle \neq 0$$

The same critical temperature from YM field and restricted fileds





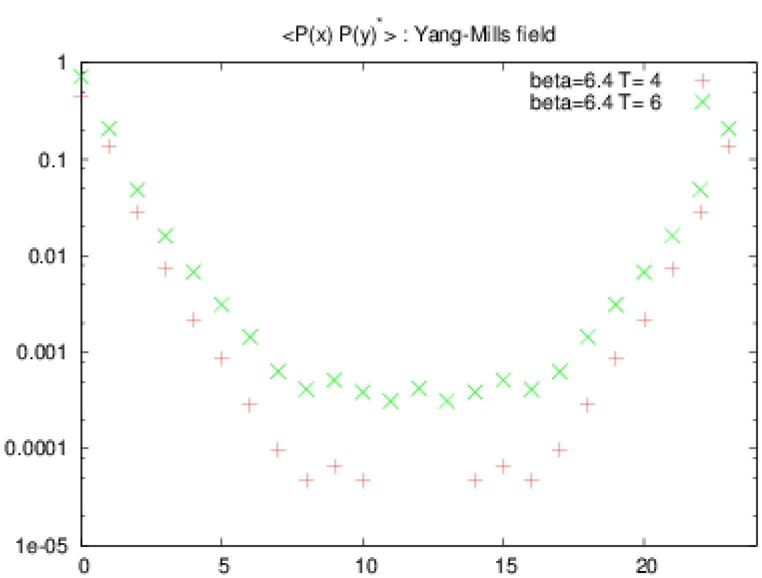
Correlation functions

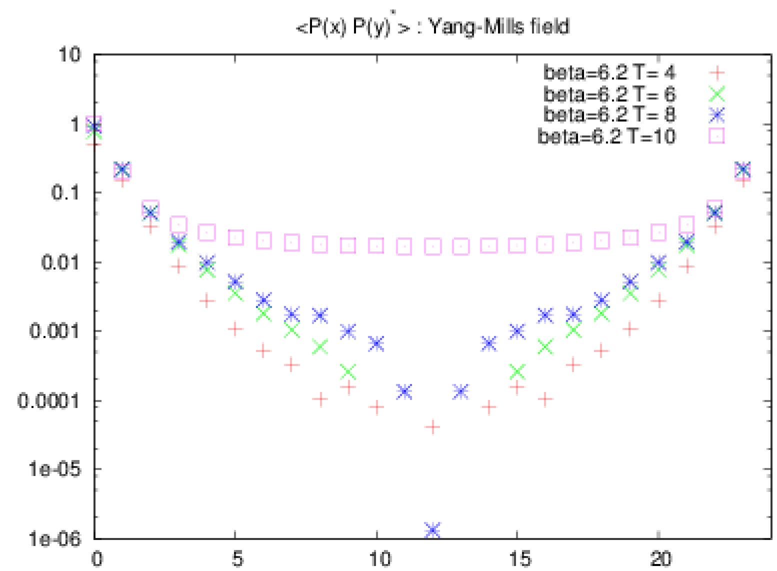
- Correlation function of Polyakov loop
  $\langle P(x)P(y)^* \rangle - |\langle P \rangle|^2$

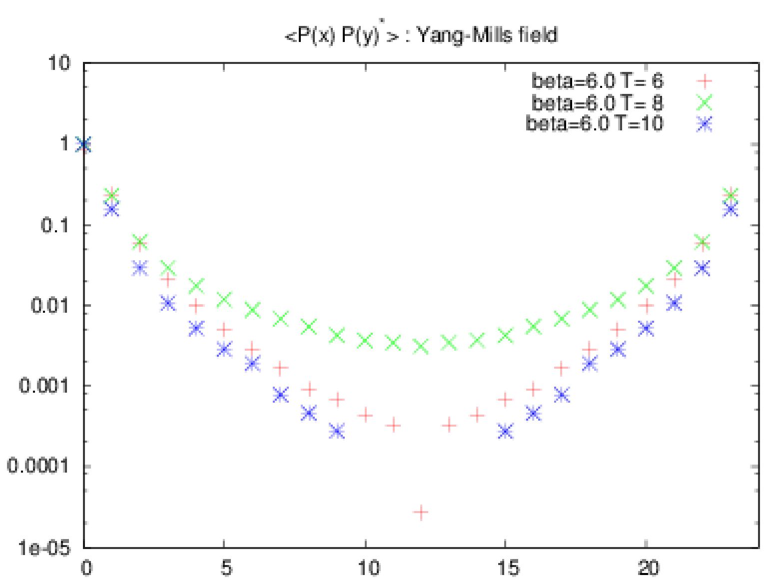
for Yang-Mills field and restricted field (extracted relevant mode for confinement)

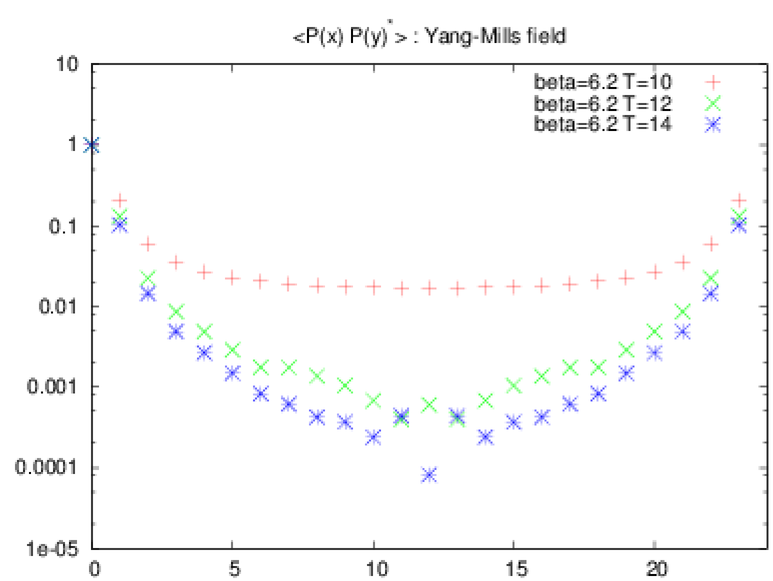
- Check of the restricted field dominance (what is called, “Abelian” dominance) for the correlation functions in both confinement and deconfinement phase.

Correlation function of Polyakov loop : YM field

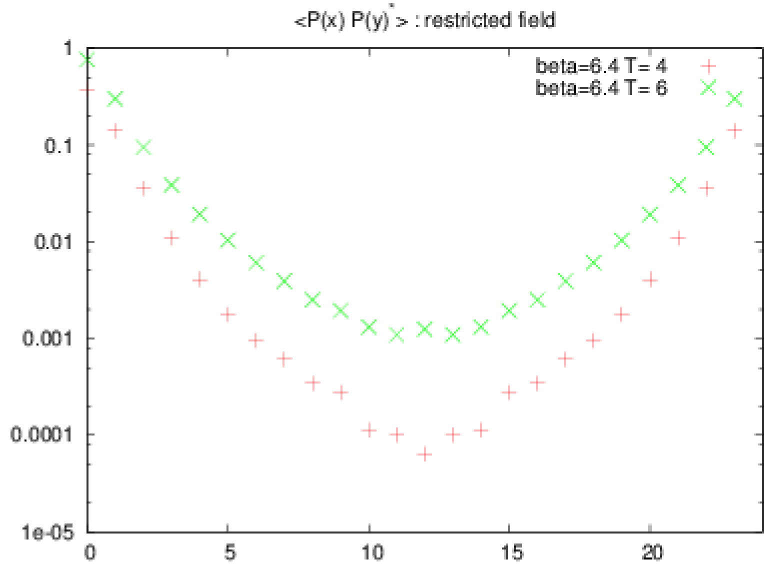


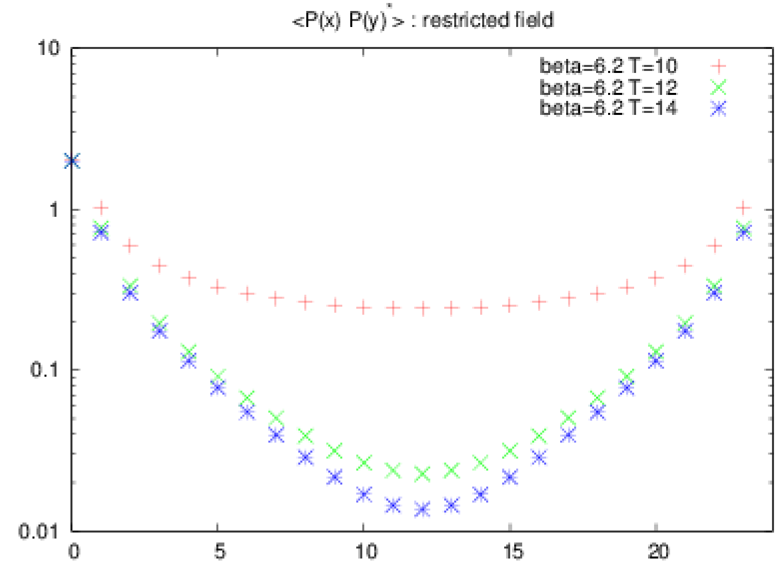


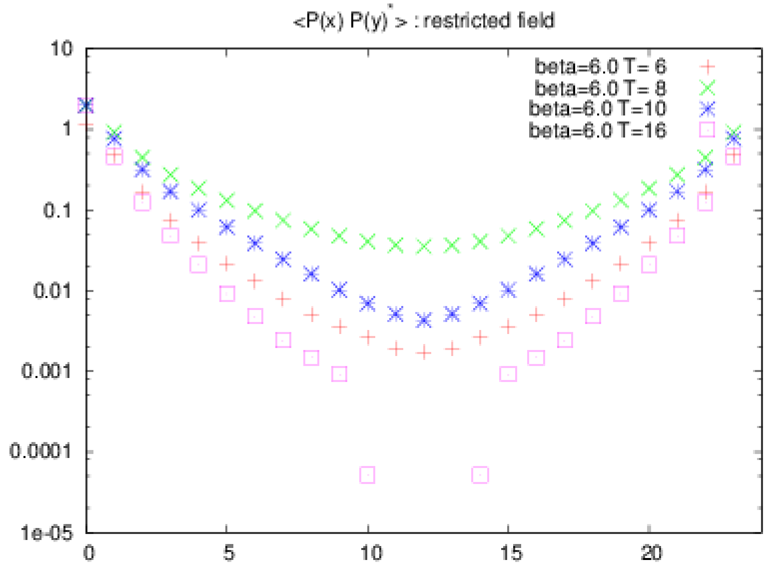


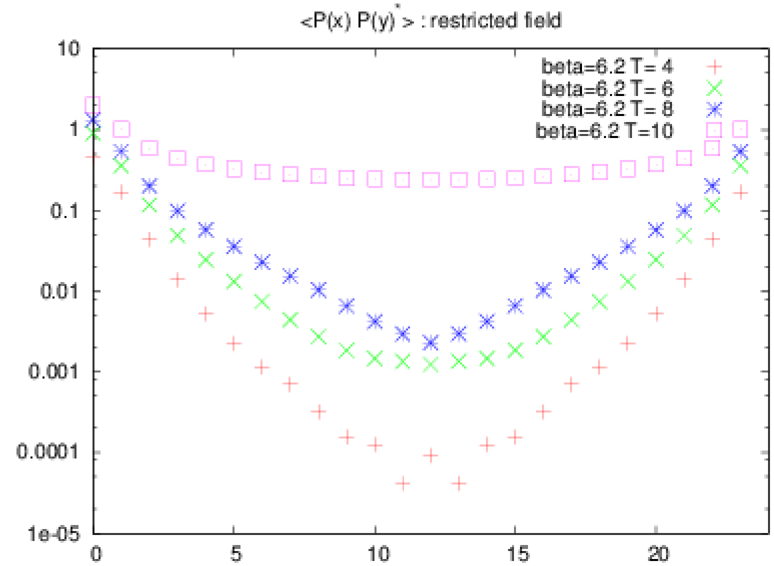


Correlation function of Polyakov loop : restricted field



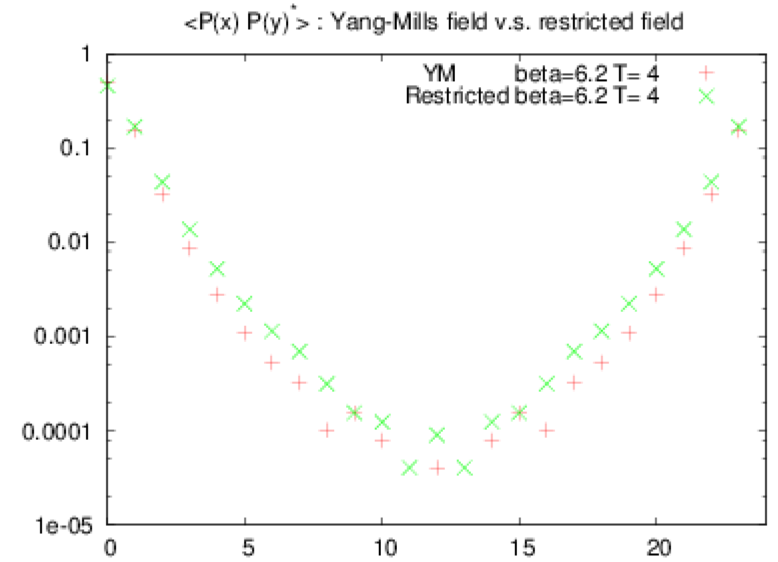


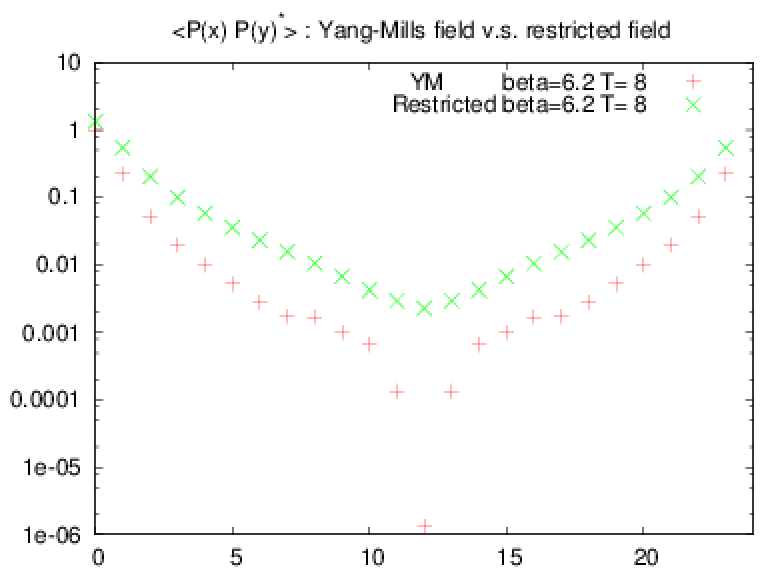


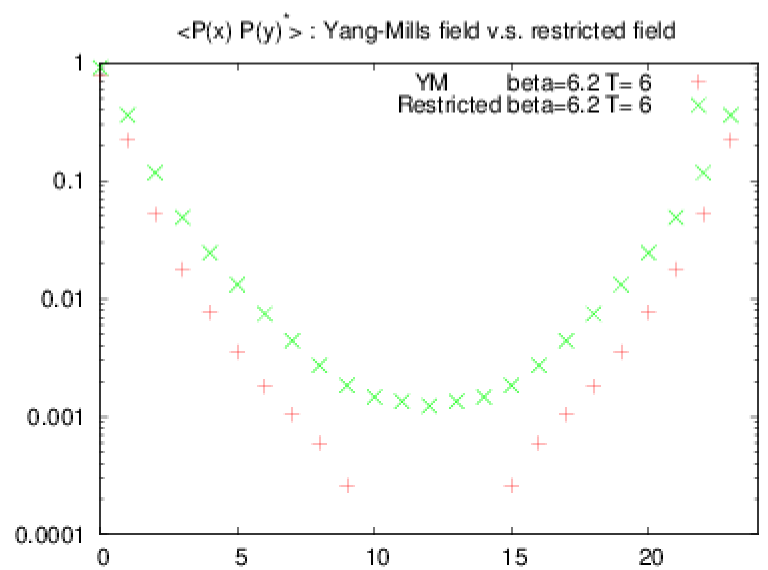


Comparison of the correlation function between the original Yang-Mills, U and the restricted filed, V

$$T_c < T$$



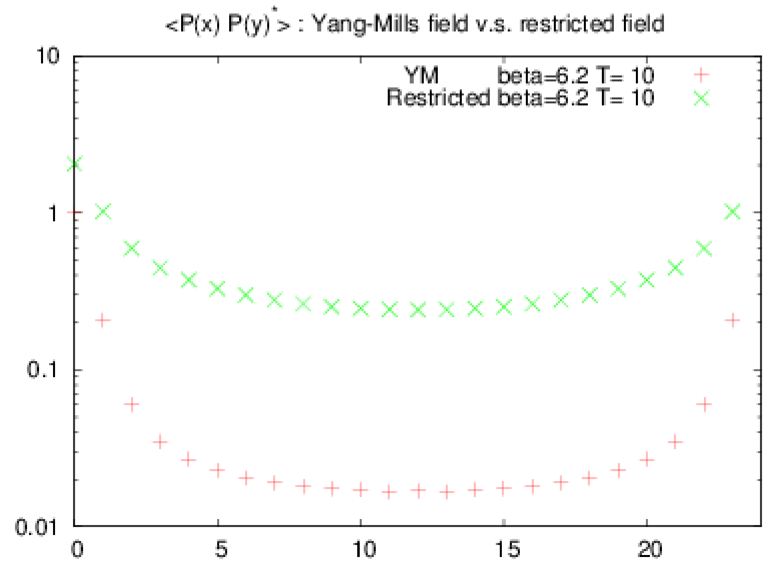


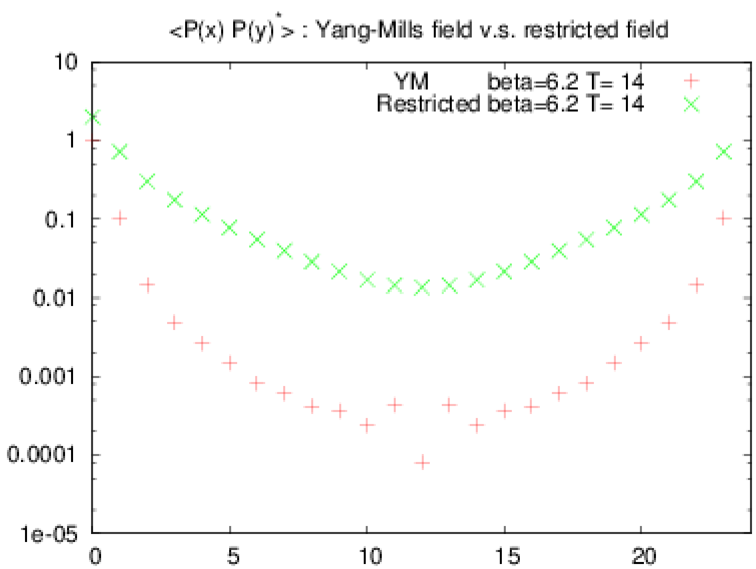


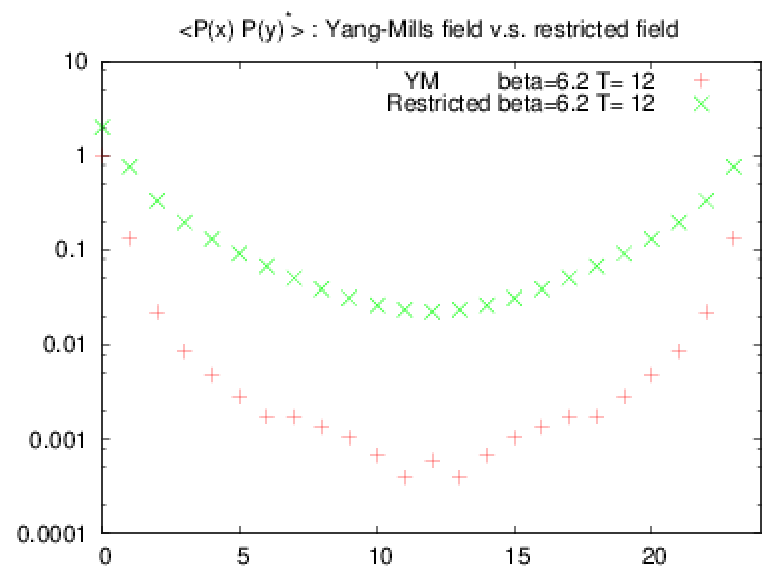
Comparison of the correlation function between the original Yang-Mills, U and the restricted filed, V

$T < T_c$ (right panels)

$T \simeq T_c$ (lower panel)







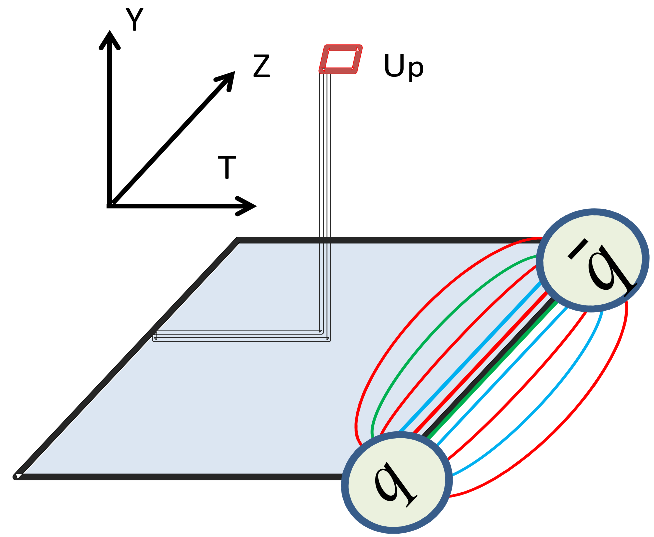
Chromo-electric flux

$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

By Adriano Di Giacomo et.al.

[Phys.Lett.B236:199,1990] [Nucl.Phys.B347:441-460,1990]

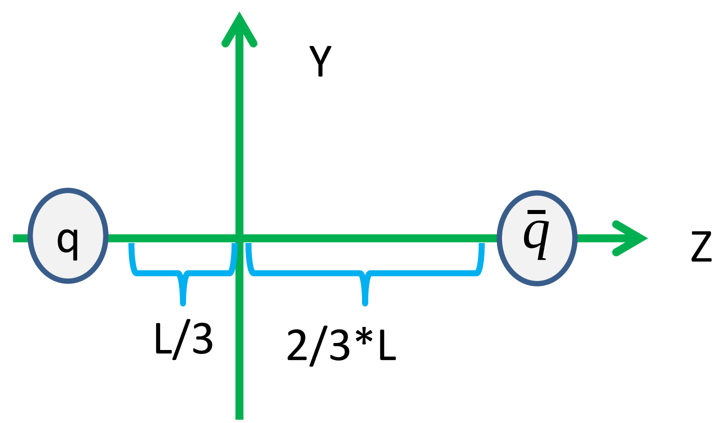
$$\mathrm{tr}(U_p L W L^\dagger)$$



Gauge invariant correlation function: This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3)

$$\rho_W \overset{\epsilon\to 0}{\simeq} \frac{\mathrm{tr}(ig\epsilon\mathcal{F}_{\mu\nu}LWL^\dagger)}{\mathrm{tr}(LWL^\dagger)} =: \langle g\epsilon\mathcal{F}_{\mu\nu}\rangle_{q\bar{q}}$$

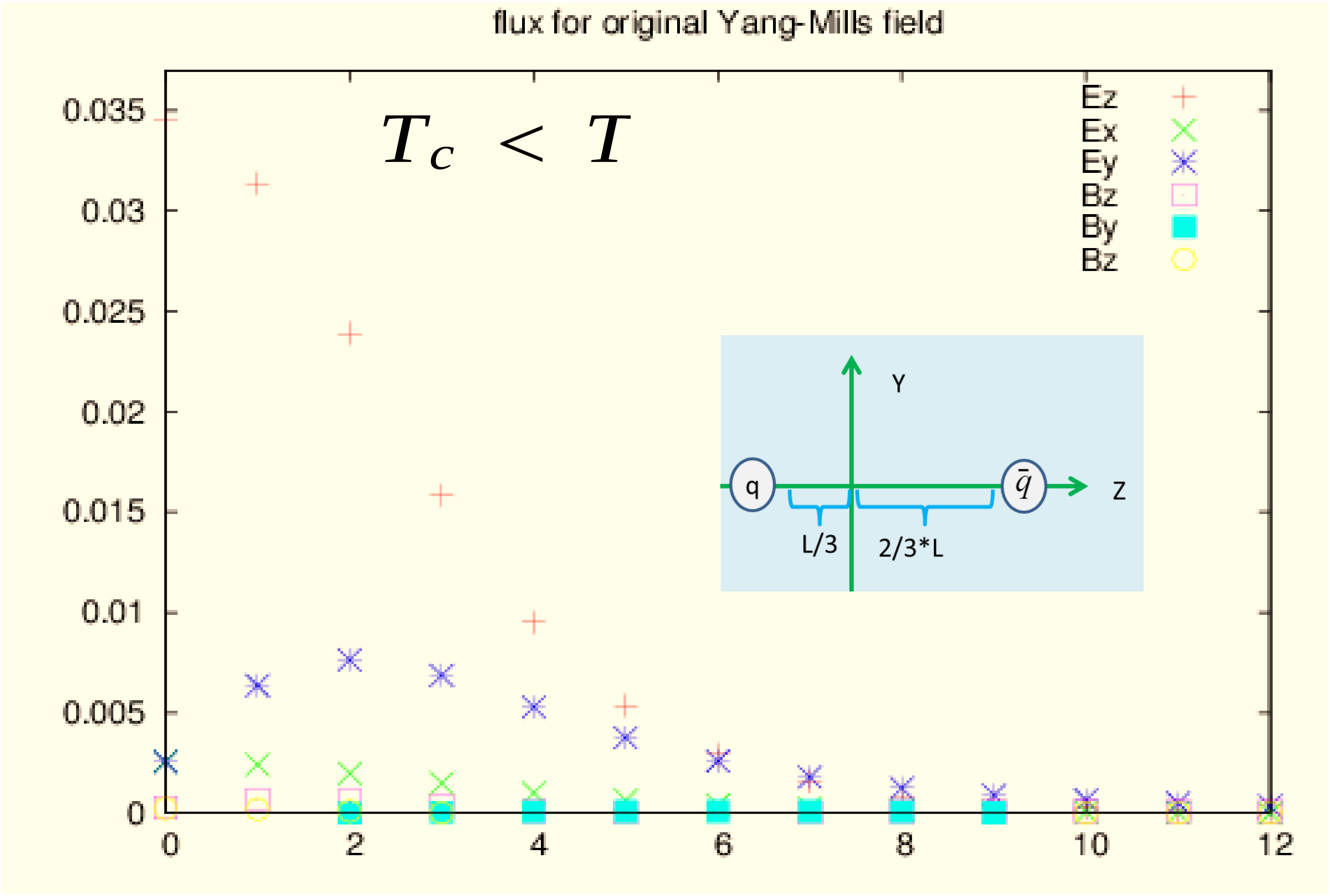
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W(x)$$



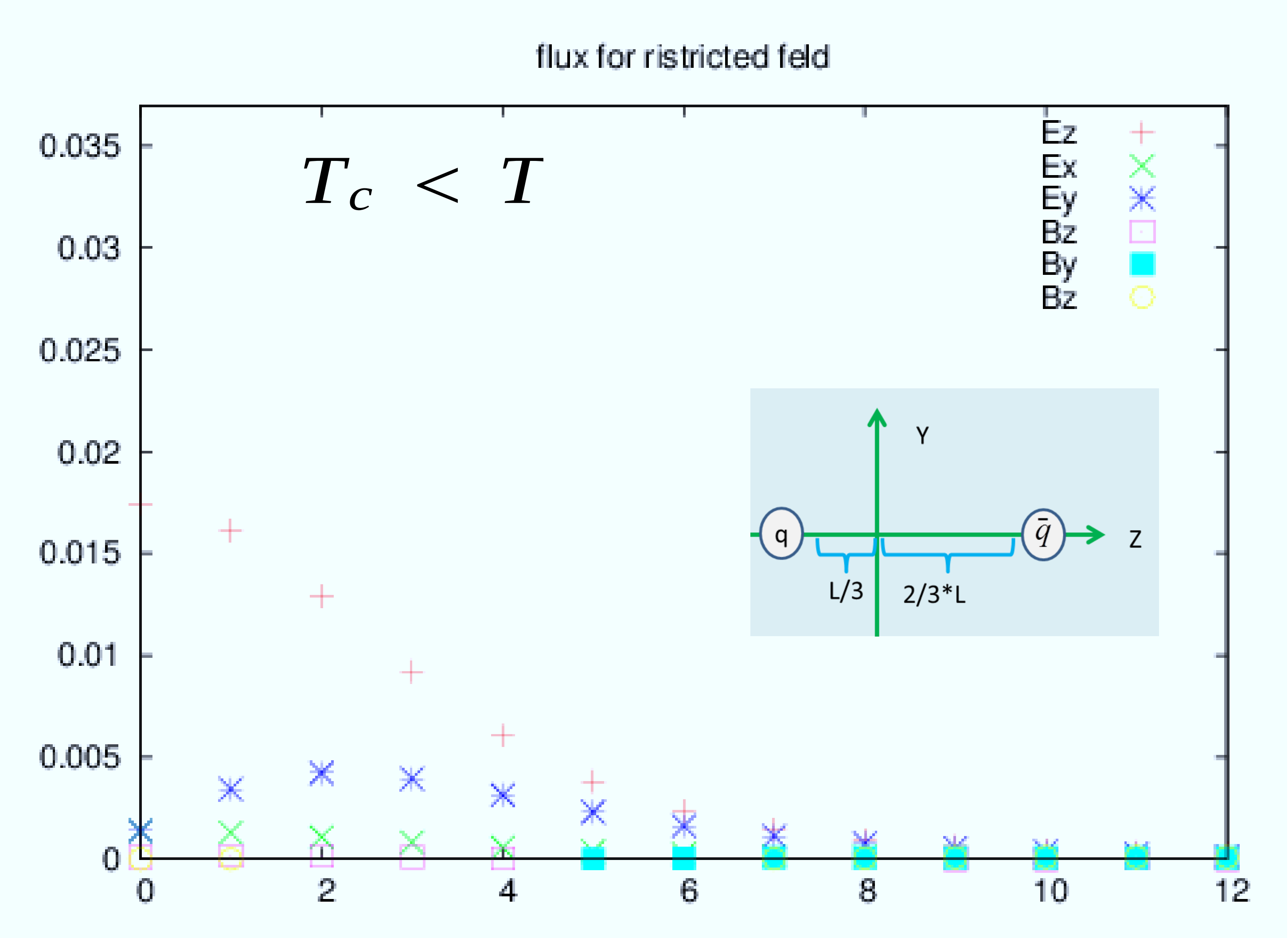
Size of Wilson loop T-direction = Nt

➔The quark and antiquark sources are given by **Plyakov loop**.

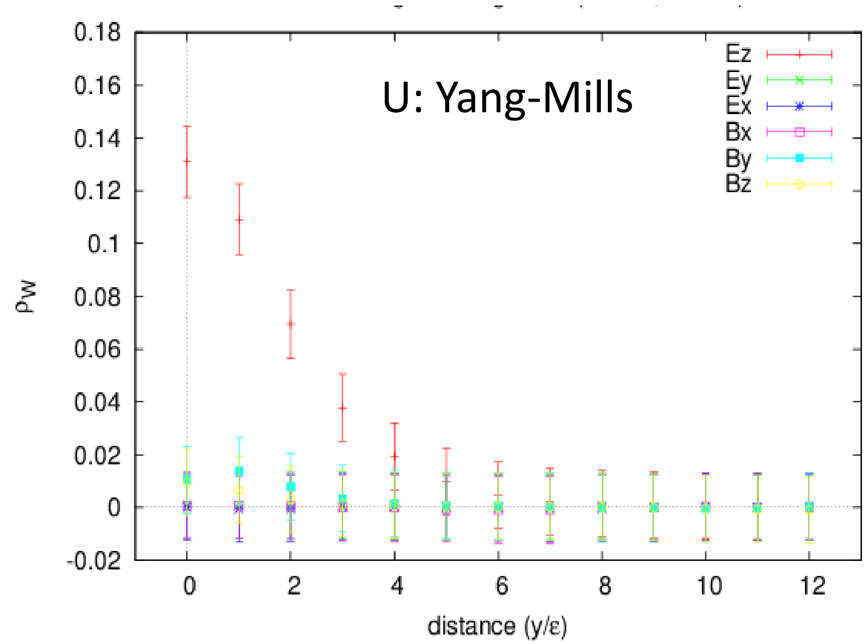
Chromo-flux measurements for YM source



Chromo-flux for restricted field source

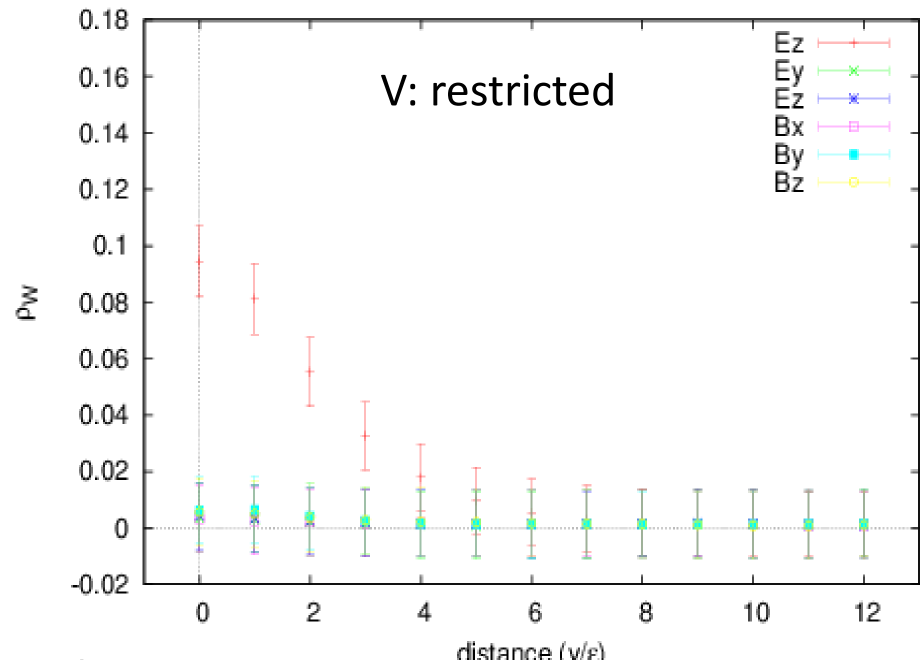


Chromo flux in confinement phase (T=0)



Flux tube is obtained
Ez only get non-zero value.

Ex=Ey=0
Bx=By=Bz=0



Chromo-electric flux in deconfinement phase

- $E_y \neq 0$ for deconfinemnte phase

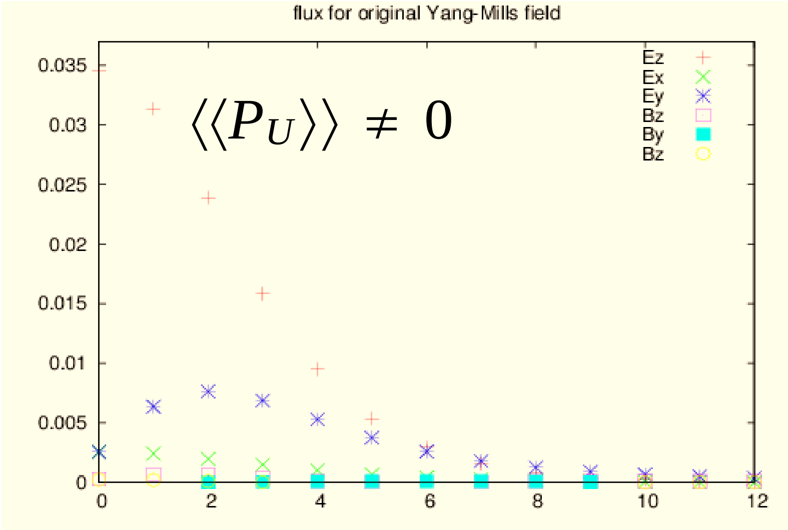
c.f., Ey = 0 (confinement phase)

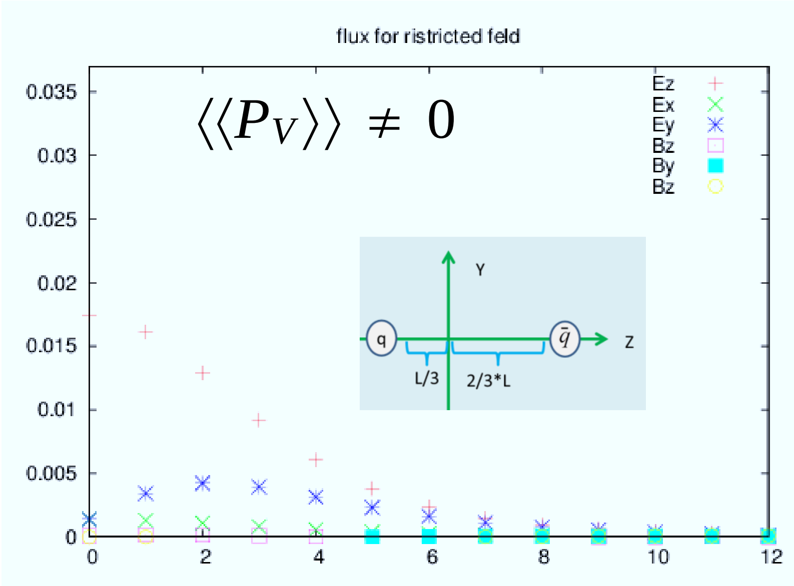
i.e., No sharp chromo-flux tube exists

➔ Disappearance of dual superconductivity.

- To know relation to the monopole condensation, we further need the measurement of magnetic current in Maxell equation for V field.

$k = {}^*dF[V]$ (under investigation)





Summary & outlook

- We investigate non-Abelian dual Meissner effects at finite temperature, applying our new formulation of Yang-Mills theory on the lattice.
- The restricted field play the dominant role in both confinement and deconfinement phase.
- We measure the chromo-electric flux and find the flux tube is broken in the deconfinement phase.
- This is first observation on quark confinement / deconfinement phase transition in terms of flux tube based on the ``non-Abelian'' dual superconductivity picture we have proposed in the previous work