Brane-Antibrane at Finite Temperature in the Framework of Thermo Field Dynamics

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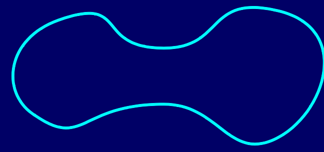
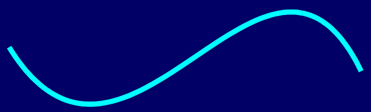
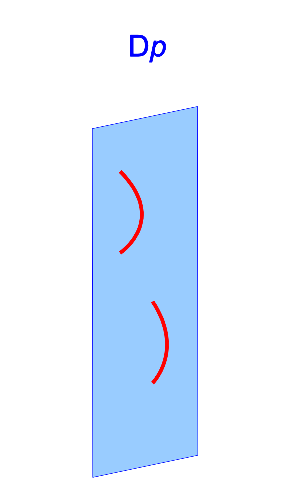
1. Introduction







- Hagedorn Temperature T_H

  Type II superstring in 10 dim.

  maximum temperature for perturbative strings

  A single energetic string captures most of the energy.

$$
\begin{aligned}
d_n &\sim e^{2\pi\sqrt{2n}} \\
\Omega(E) &\sim e^{\beta_H E} \\
Z(\beta) &= \int_0^\infty dE\ \Omega(E)\ e^{-\beta E}
\end{aligned}
$$

$$\beta_H \equiv \frac{1}{T_H} = 2\pi\sqrt{2\alpha'}$$

$Z(\beta) \to \infty$ for $\beta < \beta_H$

- Brane-antibrane Pair Creation Transition

  Dp-$\overline{\text{D}p}$ Pairs are unstable at zero temperature

  finite temperature system of Dp-$\overline{\text{D}p}$

  based on Matsubara formalism

  → D9-$\overline{\text{D9}}$ pairs become stable

  near the Hagedorn temperature. Hotta

- Thermo Field Dynamics (TFD) Takahashi-Umezawa

  expectation value

$$\langle A \rangle = Z^{-1}(\beta) \sum_n \langle n|\hat{A}|n\rangle e^{-\beta E_n}$$

  We can represent it as

$$\langle A \rangle = \left\langle 0(\beta) \middle| \hat{A} \middle| 0(\beta) \right\rangle$$

  by introducing a fictitious copy of the system.

$$|0(\beta)\rangle = Z^{-\frac{1}{2}}(\beta) \sum_n e^{-\frac{\beta E_n}{2}} |n, \tilde{n}\rangle$$

$$|n, \tilde{n}\rangle = |n\rangle \otimes |\tilde{n}\rangle$$

  thermal vacuum state

  → finite temperature system of Dp-$\overline{\text{D}p}$ based on TFD?

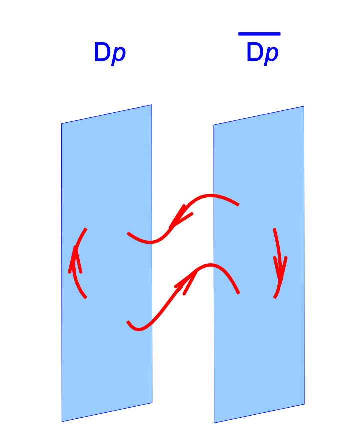
2. Brane-anti-brane Creation Transition

- Dp-$\overline{\mathrm{D}p}$ Pair

  unstable at zero temperature

  open string tachyon tachyon potential

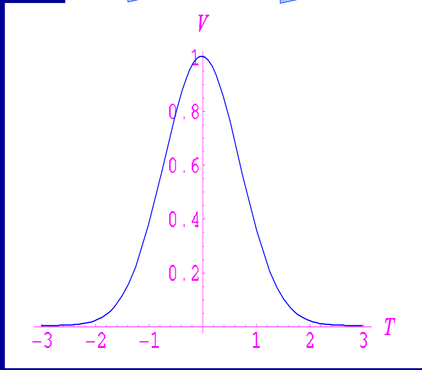
  Sen's conjecture potential height=brane tension



- Tachyon Potential of Dp-$\overline{\mathrm{D}p}$ (BSFT)

  tree level tachyon potential (disk worldsheet)

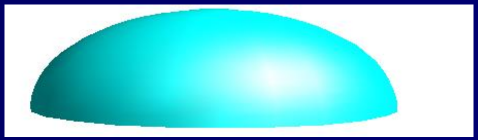
$$V(T) = 2\tau_p \mathcal{V}_p \exp(-8|T|^2),$$



T : complex scalar field τ_p : tension of Dp-brane

$g_s = e^{\phi}$: coupling of strings $\mathcal{V}_p$: p-dim. volume

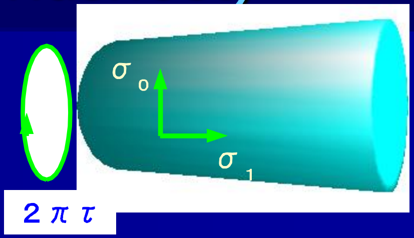
$$\tau_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}$$



Free Energy of Open Strings on Dp-$\overline{\mathrm{D}p}$ Pair

Euclidean time with period $\beta = \frac{1}{T}$

ideal open string gas → 1-loop (cylinder worldsheet)



$$
\begin{aligned}
F(T,\beta) \;=\; & -\frac{16\pi^4 \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau^{\frac{p+3}{2}}}\, e^{-4\pi|T|^2\tau} \\
& \times \left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left(\vartheta_3 \left(0 \left| \frac{i\beta^2}{{\beta_H}^2\tau} \right. \right) - 1 \right) \right. \\
& \quad \left. - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left(\vartheta_4 \left(0 \left| \frac{i\beta^2}{{\beta_H}^2\tau} \right. \right) - 1 \right) \right]
\end{aligned}
$$

Finite Temperature Effective Potential

$$V(T,\beta) = V(T) + F(T,\beta)$$

cf) We cannot trust the canonical ensemble method

→ microcanonical ensemble method

- Brane-anti-brane Pair Creation Transition
  - N D9-$\overline{\text{D9}}$ Pairs

$|T|^2$ term of finite temperature effective potential

$$\left[-16N\tau_9\mathcal{V}_9+\frac{8\pi N^2\mathcal{V}_9}{{\beta_H}^{10}}\ln\left(\frac{\pi{\beta_H}^{10}E}{2N^2\mathcal{V}_9}\right)\right]|T|^2.$$

Critical temperature

$$\mathcal{T}_c\simeq{\beta_H}^{-1}\left[1+\exp\left(-\frac{{\beta_H}^{10}\tau_9}{\pi N}\right)\right]^{-1}.$$

Above $\mathcal{T}_c$, $T=0$ becomes the potential minimum.

➡ A phase transition occurs and D9-$\overline{\text{D9}}$ pairs become stable.

$\mathcal{T}_c$ is a decreasing function of N.

➡ Multiple D9-$\overline{\text{D9}}$ pairs are created simultaneously.

- N Dp-$\overline{\text{D}p}$ Pairs with $p\leq 8$

No phase transition occurs.

3. Brane Antibrane Pair in TFD

- Light-Cone Momentum

We consider a single first quantized string.

light-cone momentum

$$p^0 = \frac{1}{2}(p^+ + p^-)$$

$$p^+ p^- - |\boldsymbol{p}|^2 - M^2 = 0$$

$$p^- = \frac{|\boldsymbol{p}|^2 + M^2}{p^+}$$

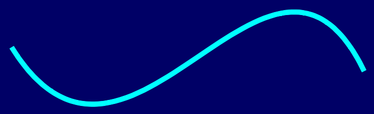
light-cone Hamiltonian

$$H = |\boldsymbol{p}|^2 + M^2$$

partition function

$$\begin{aligned} Z(\beta) &= \mathrm{Tr}\ \exp\left(-\beta p^0\right) = \mathrm{Tr}\ \exp\left[-\ \frac{1}{2}\ \beta(p^+ + p^-)\right] \\ &= \mathrm{Tr}\ \exp\left[-\ \frac{1}{2}\ \beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{p^+}\right)\right] \\ &= \mathrm{Tr}\ \exp\left[-\ \frac{1}{2}\ \beta\left(p^+ + \frac{H}{p^+}\right)\right] \end{aligned}$$

Mass Spectrum



We consider an open string

on a Brane-antibrane pair with $T = 0$.

mass spectrum

$$M_{NS}{}^2 = \frac{1}{\alpha'}\left(N_B + N_{NS} - \frac{1}{2}\right) \qquad \text{space time boson}$$

$$M_R{}^2 = \frac{1}{\alpha'}(N_B + N_R) \qquad \text{space time fermion}$$

number ops.

$$N_B = \sum_{l=1}^{\infty}\sum_{I=1}^{8} \alpha^I_{-l}\alpha^I_l \qquad \text{oscillation mode of world sheet boson}$$

$$N_{NS} = \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=1}^{8} b^I_{-r}b^I_r \qquad \text{oscillation mode of world sheet fermion (NS b. c)}$$

$$N_R = \sum_{m=1}^{\infty}\sum_{I=1}^{8} d^I_{-m}d^I_m \qquad \text{oscillation mode of world sheet fermion (R b. c)}$$

Thermal Vacuum State

generator of Bogoliubov tr.

$$
\begin{aligned}
G_b &= G_B + G_{NS} \\
G_f &= G_B + G_R
\end{aligned}
$$

$$
\begin{aligned}
\tanh(\theta_l) &= \exp\left(-\frac{\beta l}{4\alpha' p^+}\right) \\
\tan(\theta_r) &= \exp\left(-\frac{\beta r}{4\alpha' p^+}\right) \\
\tan(\theta_m) &= \exp\left(-\frac{\beta m}{4\alpha' p^+}\right)
\end{aligned}
$$

$$
\begin{aligned}
G_B &= i\sum_{l=1}^{\infty}\frac{1}{l}\,\theta_l\left(\alpha_{-l}\cdot\tilde{\alpha}_{-l}-\tilde{\alpha}_l\cdot\alpha_l\right) \\
G_{NS} &= i\sum_{r=\frac{1}{2}}^{\infty}\theta_r\left(b_{-r}\cdot\tilde{b}_{-r}-\tilde{b}_r b_r\cdot\right) \\
G_R &= i\sum_{m=1}^{\infty}\theta_m\left(d_{-m}\cdot\tilde{d}_{-m}-\tilde{d}_m\cdot d_m\right)
\end{aligned}
$$

thermal vacuum state for a single string

$$
\begin{aligned}
|0_1(\beta)\rangle\rangle &= \mathcal{N}\int dp^+\int d^{p-1}p \\
&\quad\times\prod_{l=1}^{\infty}\left\{\left(\frac{1}{\cosh(\theta_l)}\right)^8\exp\left[\frac{1}{l}\,\tanh(\theta_l)\alpha_{-l}\cdot\tilde{\alpha}_{-l}\right]\right\} \\
&\quad\times\left\{\prod_{r=\frac{1}{2}}^{\infty}(\cos(\theta_r))^8\exp\left[\,\tan(\theta_r)b_{-r}\cdot\tilde{b}_{-r}\right]\right. \\
&\qquad\left.+\prod_{m=1}^{\infty}(\cos(\theta_m))^8\exp\left[\,\tan(\theta_m)d_{-m}\cdot\tilde{d}_{-m}\right]\right\}|0\rangle\rangle
\end{aligned}
$$

cf) D-brane in bosonic string theory Vancea et al., Cantcheff

Partition Function

partition function for a single string

$$
\begin{aligned}
K &= -\sum_l \left\{ \frac{1}{l}\,\alpha_{-l}\cdot\alpha_l \ln\sinh^2\theta_l - \frac{1}{l}\,\alpha_l\cdot\alpha_{-l}\ln\cosh^2\theta_l \right\} \\
&\quad -\sum_r \left\{ b_{-r}\cdot b_r \ln\sin^2\theta_r - b_r\cdot b_{-r}\ln\cos^2\theta_r \right\} \\
&\quad -\sum_m \left\{ d_{-m}\cdot d_m \ln\sin^2\theta_m - d_m\cdot d_{-m}\ln\cos^2\theta_m \right\}
\end{aligned}
$$

$$Z_1(\beta) = \langle\langle 0_1(\beta)\,|\exp\left(-\beta\mathcal{H} + K\right)|\,0_1(\beta)\rangle\rangle \qquad \mathcal{H} = \frac{1}{2}\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{p^+}\right)$$

$$
\begin{aligned}
Z_1(\beta) &= \frac{16\pi^4\beta\mathcal{V}_p}{{\beta_H}^{p+1}}\int_0^\infty \frac{d\tau}{\tau}\,\tau^{-\frac{p+1}{2}} \\
&\quad\times\left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4 \exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau}\right) + \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4 \exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau}\right)\right]
\end{aligned}
$$

Free energy of multiple strings can be obtained from the following eq.

$$F(\beta) = -\sum_{r=1}^\infty \frac{1}{\beta r}\left\{Z_{1b}(\beta r) - (-1)^r Z_{1f}(\beta r)\right\}$$

$$
\begin{aligned}
F(\beta) &= -\frac{16\pi^4\mathcal{V}_p}{{\beta_H}^{p+1}}\int_0^\infty \frac{d\tau}{\tau}\,\tau^{-\frac{p+1}{2}} \\
&\quad\times\left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_3\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right) - 1\right\} - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)}\right)^4\left\{\vartheta_4\left(0\left|\frac{i\beta^2}{{\beta_H}^2\tau}\right.\right) - 1\right\}\right]
\end{aligned}
$$

This equals to the free energy with $T = 0$

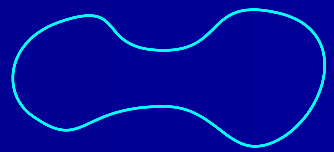
based on Matsubara formalism.

4. Conclusion and Discussion

- Brane-antibrane in TFD

  We computed thermal vacuum state and partition function of a single string on a Brane-antibrane pair based on TFD.

- D-brane boundary state of closed string

  This thermal vacuum state is reminiscent of the D-brane boundary state of a closed string.

$$|B9_{mat},\eta\rangle_{NSNS} = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\,\alpha_{-n}\cdot\tilde{\alpha}_{-n} + i\eta\sum_{u>0}\psi_{-u}\cdot\tilde{\psi}_{-u}\right]|B9_{mat},\eta\rangle^{(0)}_{NSNS}$$

- String Field Theory

  We need to use second quantized string field theory in order to obtain the thermal vacuum state for multiple strings.

- Open String Tachyon

  We need to introduce open string tachyon in order to see the phase structure of the system.

ϑ-functions

$$\vartheta_1{}'(0|\tau) = \pi \sum_{n=-\infty}^{\infty} (-1)^n (2n-1) q^{(n-\frac{1}{2})^2}$$

$$\vartheta_2(0|\tau) = \sum_{n=-\infty}^{\infty} q^{(n-\frac{1}{2})^2}$$

$$\vartheta_3(0|\tau) = \sum_{n=-\infty}^{\infty} q^{n^2}$$

$$\vartheta_4(0|\tau) = \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2}$$

$$q = e^{i\pi\tau}$$

modular transformation

$$\vartheta_1{}'\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{3}{2}}\vartheta_1{}'(0|\tau), \quad \vartheta_3\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}}\vartheta_3(0|\tau)$$

$$\vartheta_2\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}}\vartheta_4(0|\tau), \quad \vartheta_4\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}}\vartheta_2(0|\tau)$$

$$\vartheta_1{}'(0|\tau+1) = e^{\frac{i\pi}{4}}\vartheta_1{}'(0|\tau), \quad \vartheta_2(0|\tau+1) = e^{\frac{i\pi}{4}}\vartheta_2(0|\tau)$$

$$\vartheta_3(0|\tau+1) = \vartheta_4(0|\tau), \quad \vartheta_4(0|\tau+1) = \vartheta_3(0|\tau)$$