

# Neutrino Spectral Density at Electroweak Scale Temperature

K. Miura<sup>A</sup>, Y. Hidaka<sup>B</sup>, D. Satow<sup>B</sup>, and T. Kunihiro<sup>C</sup>

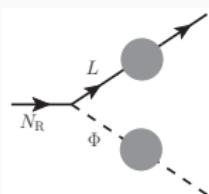
KMI<sup>A</sup>, LNF-INFN<sup>A</sup>, RIKEN<sup>B</sup>, Kyoto Univ.<sup>C</sup>

TQFT & Their Applications, August 26, 2013

## References

K. Miura, Y. Hidaka, D. Satow, and T. Kunihiro, arXiv:1306.1701 [hep-ph].

# Leptogenesis, Particularly in Low Energy Scale



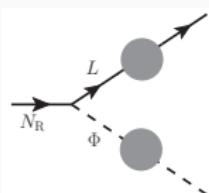
$N_B$  = Right-Handed Neutrino

$$L^T = (\nu, l)$$

$\Phi = \text{Higgs Doublet}$

- The standard model (SM) seems to fail to explain the observed baryon asymmetry of the universe (BAU):  $\eta_B = (n_B - \bar{n}_B)/n_\gamma \simeq 6.1 \times 10^{-10}$ .
  - An extension of the SM by adding right-handed Majorana neutrinos ( $N_R$ ) may have a chance to account for the BAU (Fukugita et.al. ('86)): A decay of  $N_R$  (e.g. Fig.) generates a net lepton number, which are partially converted into the baryon number via sphaleron process in the electroweak (EW) phase trans. (Kuzmin et.al. ('85), Klinkhamer et.al. ('84), Arnold et.al. ('87)).
  - If the mass difference between two  $N_R$ s is in the order of their CP-violating decay width, the CP asymmetry is dynamically enhanced (Pilaftsis ('97)), and the leptogenesis in the EW scale can be relevant (Pilaftsis et.al. ('05)).

## Leptogenesis, Particularly in Low Energy Scale



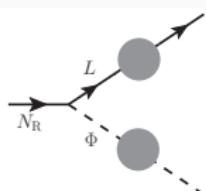
$N_R$  = Right-Handed Neutrino

$$L^T = (\nu, l)$$

$\Phi = \text{Higgs Doublet}$

- The standard model (SM) seems to fail to explain the observed baryon asymmetry of the universe (BAU):  $\eta_B = (n_B - \bar{n}_B)/n_\gamma \simeq 6.1 \times 10^{-10}$ .
  - An extension of the SM by adding right-handed Majorana neutrinos ( $N_R$ ) may have a chance to account for the BAU (Fukugita et.al. ('86)): A decay of  $N_R$  (e.g. Fig.) generates a net lepton number, which are partially converted into the baryon number via sphaleron process in the electroweak (EW) phase trans. (Kuzmin et.al. ('85), Klinkhamer et.al. ('84), Arnold et.al. ('87)).
  - If the mass difference between two  $N_R$ s is in the order of their CP-violating decay width, the CP asymmetry is dynamically enhanced (Pilaftsis ('97)), and the leptogenesis in the EW scale can be relevant (Pilaftsis et.al. ('05)).

# Leptogenesis, Particularly in Low Energy Scale



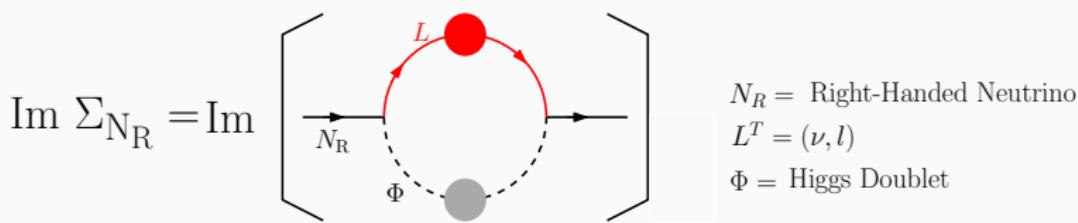
$N_R$  = Right-Handed Neutrino

$$L^T = (\nu, l)$$

$\Phi =$  Higgs Doublet

- The standard model (SM) seems to fail to explain the observed baryon asymmetry of the universe (BAU):  $\eta_B = (n_B - \bar{n}_B)/n_\gamma \simeq 6.1 \times 10^{-10}$ .
  - An extension of the SM by adding right-handed Majorana neutrinos ( $N_R$ ) may have a chance to account for the BAU (Fukugita et.al. ('86)): A decay of  $N_R$  (e.g. Fig.) generates a net lepton number, which are partially converted into the baryon number via sphaleron process in the electroweak (EW) phase trans. (Kuzmin et.al. ('85), Klinkhamer et.al. ('84), Arnold et.al. ('87)).
  - If the mass difference between two  $N_R$ s is in the order of their CP-violating decay width, the CP asymmetry is dynamically enhanced (Pilaftsis ('97)), and the leptogenesis in the EW scale can be relevant (Pilaftsis et.al. ('05)).

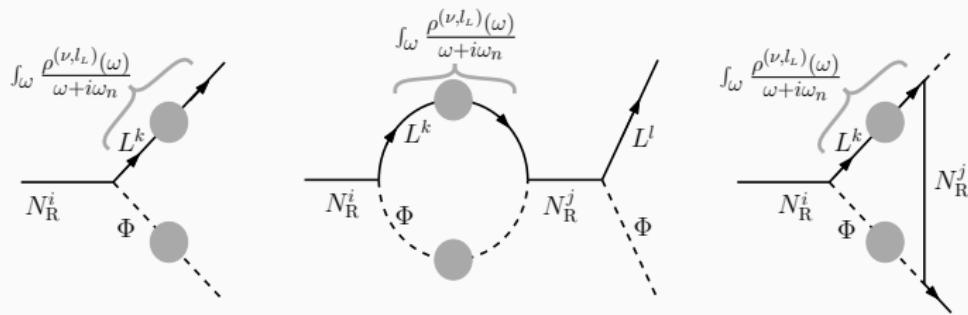
# Spectral Density of Leptons in Leptogenesis



$$\tilde{G}_{\text{Lepton}}(\mathbf{k}, i\omega_n; T) = \int_{-\infty}^{\infty} d\omega \frac{\rho_{\text{Left}}^{(\nu, l)}(\mathbf{k}, \omega; T)}{\omega + i\omega_n}. \quad (1)$$

If the standard-model leptons have non-trivial spectral properties in EW scale plasma, the lepton number creation via the  $N_R$  decay may be significantly modified (c.f. Kiessig et.al., PRD. 2010).

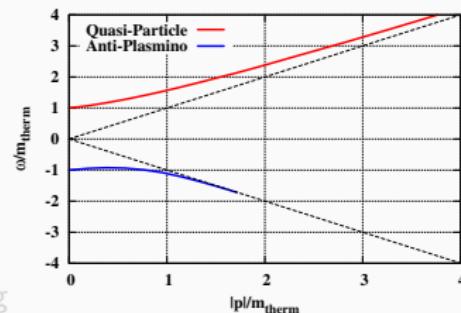
# Spectral Density of Leptons in Leptogenesis



If the standard-model leptons have non-trivial spectral properties in EW scale plasma, the CP asymmetry would be modified.

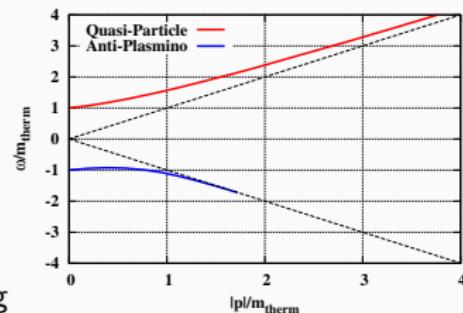
# Spectral Property of Finite $T$ Gauge Theory

- There is a growing interest in the collective nature of the fermions in the scenario of thermal leptogenesis (Drewes, arXiv:1303.6912).
  - In QED and QCD at extremely high  $T$ , the Hard Thermal-Loop (HTL) approx. indicates that a probe fermion interacting with thermally excited gauge bosons and anti-fermions admits a **collective excitation** mode (See, The text book by LeBellac).
  - In the neutrino dispersion relation in the electroweak scale plasma, the existence of a novel branch in the ultrasoft-energy region has been indicated by using the HTL and the unitary gauge (Boyanovsky, PRD. 2005).



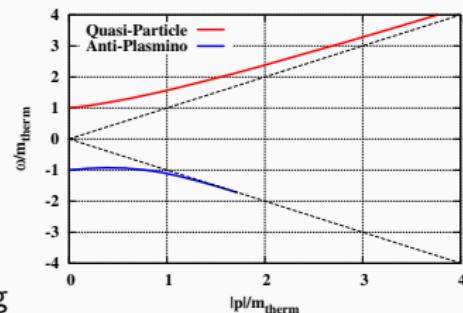
## Spectral Property of Finite $T$ Gauge Theory

- There is a growing interest in the collective nature of the fermions in the scenario of thermal leptogenesis (Drewes, arXiv:1303.6912).
  - In QED and QCD at extremely high  $T$ , the Hard Thermal-Loop (HTL) approx. indicates that a probe fermion interacting with thermally excited gauge bosons and anti-fermions admits a **collective excitation** mode (See, The text book by LeBellac).
  - In the neutrino dispersion relation in the electroweak scale plasma, the existence of a novel branch in the ultrasoft-energy region has been indicated by using the HTL and the unitary gauge (Boyanovsky, PRD. 2005).



# Spectral Property of Finite $T$ Gauge Theory

- There is a growing interest in the collective nature of the fermions in the scenario of thermal leptogenesis (Drewes, arXiv:1303.6912).
  - In QED and QCD at extremely high  $T$ , the Hard Thermal-Loop (HTL) approx. indicates that a probe fermion interacting with thermally excited gauge bosons and anti-fermions admits a **collective excitation** mode (See, The text book by LeBellac).
  - In the neutrino dispersion relation in the electroweak scale plasma, the existence of a novel branch in the ultrasoft-energy region has been indicated by using the HTL and the unitary gauge (Boyanovsky, PRD. 2005).



# From QGP to Particle Cosmology

**Goal: We investigate Neutrino Spectral Density at  $T \gtrsim M_{w,z}$**

- ① Without restricting ourselves to the dispersion,
- ② In  $R_\xi$  Gauge (Fujikawa et.al. PRD. 1972),
- ③ And discuss a possible implication to the leptogenesis.

## Hints in QGP Physics

- The Spectral Density of massless fermion coupled with the massive mesonic mode in plasma (an effective description of QGP) has been investigated and shown to have a Three-Peak Structure with a Ultrasoft Mode. (Kitazawa et.al. ('05-'06), Harada et.al. ('08), w.o. HTL).
- In particular, when the fermion is coupled with the massive *vectorial* meson, the Gauge Independent Nature of the three-peak structure has been confirmed (Satow et.al. ('10)) by using the Stueckelberg formalism.

# From QGP to Particle Cosmology

**Goal:** We investigate Neutrino Spectral Density at  $T \gtrsim M_{w,z}$

- ① Without restricting ourselves to the dispersion,
- ② In  $R_\xi$  Gauge (Fujikawa et.al. PRD. 1972),
- ③ And discuss a possible implication to the leptogenesis.

## Hints in QGP Physics

- The **Spectral Density** of massless fermion coupled with the massive mesonic mode in plasma (an effective description of QGP) has been investigated and shown to have a **Three-Peak Structure with a Ultrasoft Mode**. (Kitazawa et.al. ('05-'06), Harada et.al. ('08), w.o. HTL).
- In particular, when the fermion is coupled with the massive *vectorial* meson, the **Gauge Independent Nature** of the three-peak structure has been confirmed (Satow et.al. ('10)) by using the Stueckelberg formalism.

# Table of Contents

## 1 Introduction

## 2 Preliminaries

## 3 Results

- Neutrino Spectral Density: Overview
- Three Peak Structure: In Details
- Gauge Parameter  $\xi$  Dependence of Spectral Property
- Implication to Low Energy Scale Leptogenesis

## 4 Summary

# Table of Contents

## 1 Introduction

## 2 Preliminaries

## 3 Results

- Neutrino Spectral Density: Overview
- Three Peak Structure: In Details
- Gauge Parameter  $\xi$  Dependence of Spectral Property
- Implication to Low Energy Scale Leptogenesis

## 4 Summary

# Setups

- Massless Lepton Sector:

$$\begin{aligned}\mathcal{L}_L = & \sum_{i=e,\mu,\tau} \left[ (\bar{\nu}^i, \bar{l}_L^i) i \not{\partial} \begin{pmatrix} \nu^i \\ l_L^i \end{pmatrix} + \bar{l}_R^i i \not{\partial} l_R^i \right] \\ & + \left[ W_\mu^\dagger J_W^\mu + J_W^{\mu\dagger} W_\mu + Z_\mu J_Z^\mu + A_\mu^{\text{EM}} J_{\text{EM}}^\mu \right],\end{aligned}\quad (2)$$

- Weak Bosons in  $R_\xi$  Gauge:

$$G_{\mu\nu}(q, T) = -\frac{(g_{\mu\nu} - q_\mu q_\nu / M_{W,Z}^2(T))}{q^2 - M_{W,Z}^2(T)} + \frac{q_\mu q_\nu / M_{W,Z}^2(T)}{q^2 - \xi M_{W,Z}^2(T)}. \quad (3)$$

- Weak Boson Masses  $T \ll v(T)$  (c.f. Manuel, PRD. 1998):

$$M_W(T) = \frac{gv(T)}{2} + \mathcal{O}(gT), \quad M_Z(T) = \frac{\sqrt{g^2 + g'^2} v(T)}{2} + \mathcal{O}(gT). \quad (4)$$

# Higgs Effective Potential ( $R_\xi$ Gauge)

$$V_{\text{eff}} = -\frac{\mu_0^2}{2} \left[ 1 - \frac{T^2(2\lambda + 3g^2/4 + g'^2/4)}{4\mu_0^2} \right] v^2(T) + \frac{\lambda}{4} v^4(T), \quad (5)$$

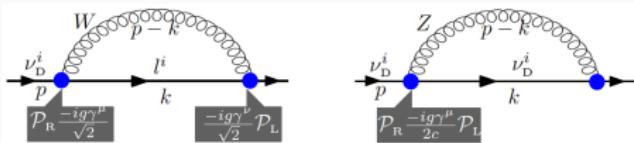
- The  $\xi$  dependences cancel out between the Nambu-Goldstone modes and the ghost contributions (Text Book by Kapusta).
- The effective potential leads to the second-order phase transition. Note that in reality the possibility of the strong first-order transition has been ruled out within the standard model (Kajantie et.al.('96), Y.Aoki et.al.('99), Csikor et.al.('99, '00)).
- The temperature region satisfying  $M_{W,Z}(T) \lesssim T \ll v(T)$  should exist, and a non-trivial spectral property is anticipated there.

# Higgs Effective Potential ( $R_\xi$ Gauge)

$$V_{\text{eff}} = -\frac{\mu_0^2}{2} \left[ 1 - \frac{T^2(2\lambda + 3g^2/4 + g'^2/4)}{4\mu_0^2} \right] v^2(T) + \frac{\lambda}{4} v^4(T), \quad (5)$$

- The  $\xi$  dependences cancel out between the Nambu-Goldstone modes and the ghost contributions (Text Book by Kapusta).
- The effective potential leads to the second-order phase transition. Note that in reality the possibility of the strong first-order transition has been ruled out within the standard model (Kajantie et.al.('96), Y.Aoki et.al.('99), Csikor et.al.('99, '00)).
- The temperature region satisfying  $M_{W,Z}(T) \lesssim T \ll v(T)$  should exist, and a non-trivial spectral property is anticipated there.

# Neutrino Self-Energy and Spectral Density



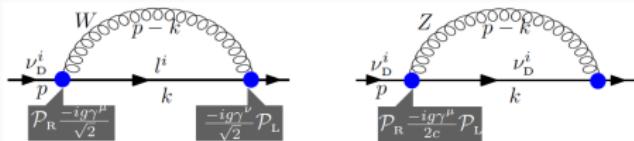
- For the massless left-handed neutrinos, the finite- $T$  effects are solely encoded in the coefficients in the decomposition

$$\Sigma_{\text{ret}}^{(\nu)}(\mathbf{p}, \omega; T) = \sum_{s=\pm} [\mathcal{P}_{\text{R}} \Lambda_{s,\mathbf{p}} \gamma^0 \mathcal{P}_{\text{L}}] \Sigma_s^{(\nu)}(|\mathbf{p}|, \omega; T), \quad (6)$$

$$\mathcal{P}_{L/R} = (1 \mp \gamma_5)/2, \quad \Lambda_{\pm, p} = (1 \pm \gamma^0 \gamma \cdot p / |p|)/2. \quad (7)$$

- For the spectral density, similarly,

# Neutrino Self-Energy and Spectral Density



- For the massless left-handed neutrinos, the finite- $T$  effects are solely encoded in the coefficients in the decomposition

$$\Sigma_{\text{ret}}^{(\nu)}(\mathbf{p}, \omega; T) = \sum_{s=\pm} [\mathcal{P}_{\text{R}} \Lambda_{s,\mathbf{p}} \gamma^0 \mathcal{P}_{\text{L}}] \Sigma_s^{(\nu)}(|\mathbf{p}|, \omega; T), \quad (6)$$

$$\mathcal{P}_{\text{L/R}} = (1 \mp \gamma_5)/2, \quad \Lambda_{\pm, \mathbf{p}} = (1 \pm \gamma^0 \boldsymbol{\gamma} \cdot \mathbf{p}/|\mathbf{p}|)/2. \quad (7)$$

- For the spectral density, similarly,

$$\rho^{(\nu)}(\mathbf{p}, \omega; T) = \sum_{s=\pm} [\mathcal{P}_{\text{R}} \Lambda_{s,\mathbf{p}} \gamma^0 \mathcal{P}_{\text{L}}] \rho_s^{(\nu)}(|\mathbf{p}|, \omega; T)$$

$$\rho_{\pm}^{(\nu)}(|\mathbf{p}|, \omega; T) = \frac{-\text{Im } \Sigma_{\pm}^{(\nu)}(|\mathbf{p}|, \omega; T)/\pi}{\{\omega - |\mathbf{p}| \mp \text{Re} \Sigma_{\pm}^{(\nu)}(|\mathbf{p}|, \omega; T)\}^2 + \{\text{Im} \Sigma_{\pm}^{(\nu)}(|\mathbf{p}|, \omega; T)\}^2}. \quad .$$

# Table of Contents

## 1 Introduction

## 2 Preliminaries

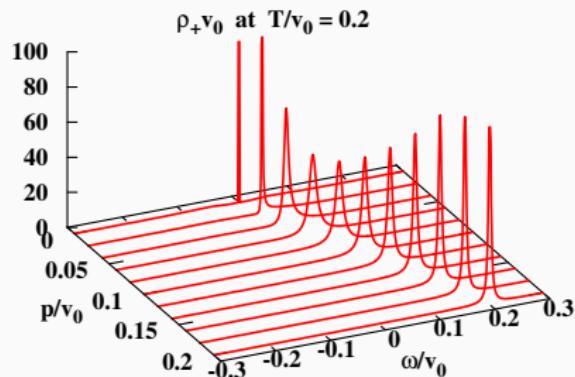
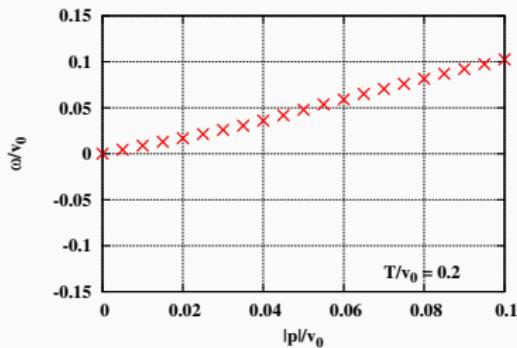
## 3 Results

- Neutrino Spectral Density: Overview
- Three Peak Structure: In Details
- Gauge Parameter  $\xi$  Dependence of Spectral Property
- Implication to Low Energy Scale Leptogenesis

## 4 Summary

## Low Temperature Region

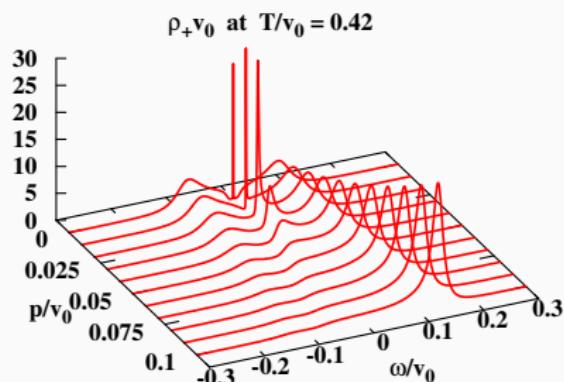
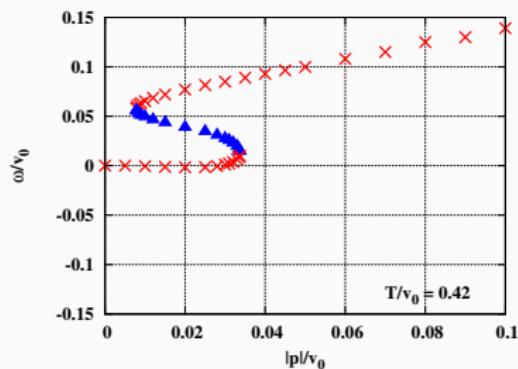
$$T/v_0 = 0.2, \quad T/M_W(T) \simeq 0.63, \quad (8)$$



$$\omega - |\mathbf{p}| - \text{Re } \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

# Intermediate Temperature Region I

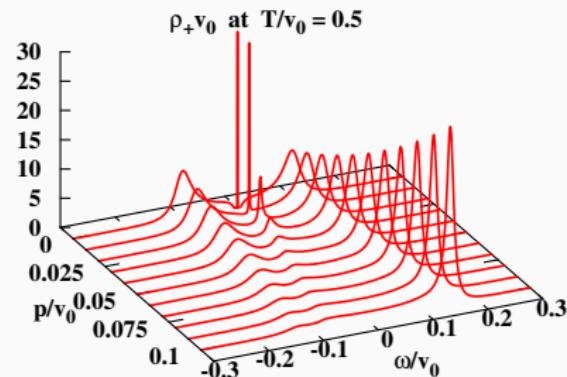
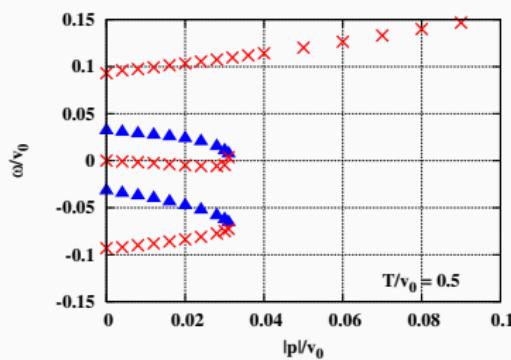
$$T/v_0 = 0.42 , \quad T/M_W(T) \simeq 1.45 , \quad (9)$$



$$\omega - |\mathbf{p}| - \text{Re } \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

## Intermediate Temperature Region II

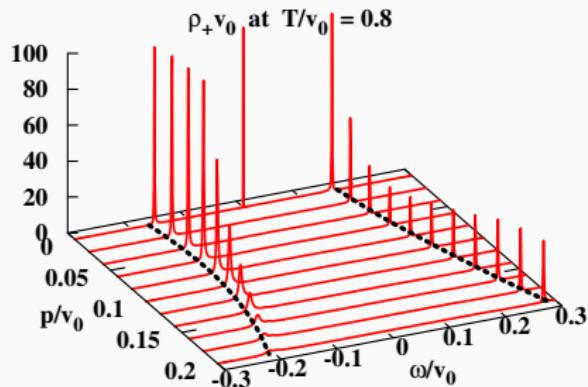
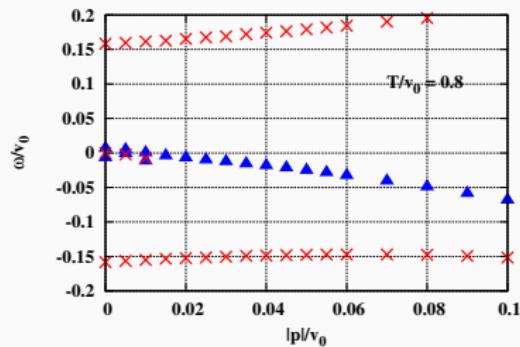
$$T/v_0 = 0.5 , \quad T/M_W(T) \simeq 1.83 , \quad (10)$$



$$\omega - |\mathbf{p}| - \text{Re } \Sigma_+(\omega, |\mathbf{p}|, T) = 0$$

# High Temperature Region

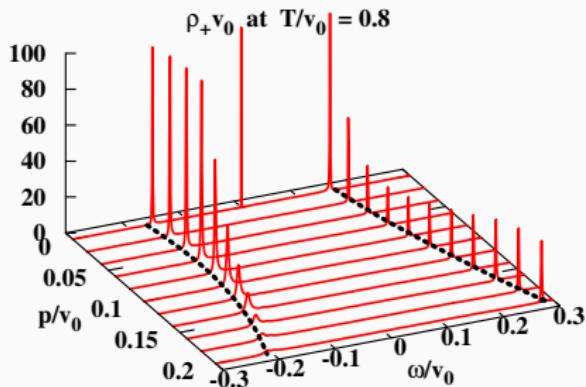
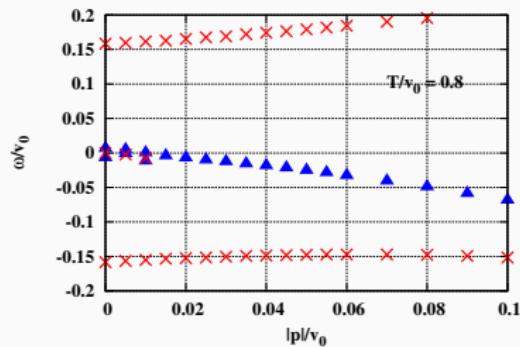
$$T/v_0 = 0.8 , \quad T/M_W(T) \simeq 4.9 .$$



- The spectral property becomes closer to the HTL result.
- $T/v(T) \simeq 1.59 > 1$ : The additional thermal-loop corrections may modify the spectral property (Hidaka-Satow-Kunihiro, Nucl.Phys.A, 2012).

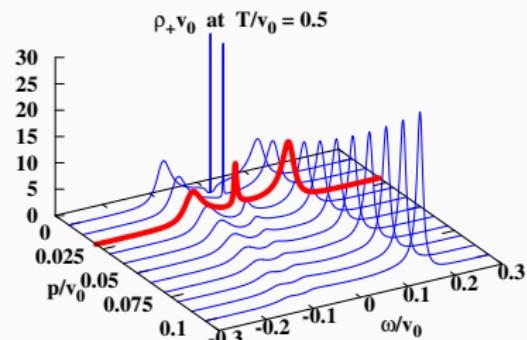
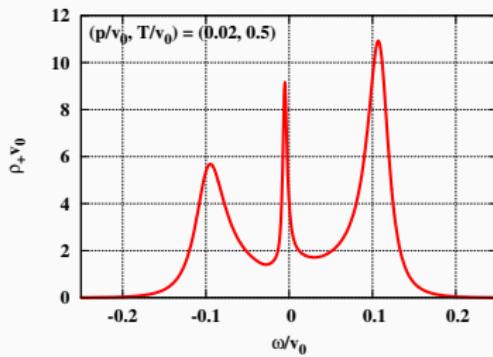
# High Temperature Region

$$T/v_0 = 0.8, \quad T/M_W(T) \simeq 4.9.$$

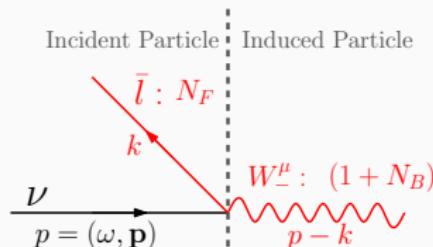


- The spectral property becomes closer to the HTL result.
- $T/v(T) \simeq 1.59 > 1$ : The additional thermal-loop corrections may modify the spectral property (Hidaka-Satow-Kunihiro, Nucl.Phys.A, 2012).

# Spectral Density at $(|p|/v_0, T/v_0) = (0.02, 0.5)$



# Landau Damping

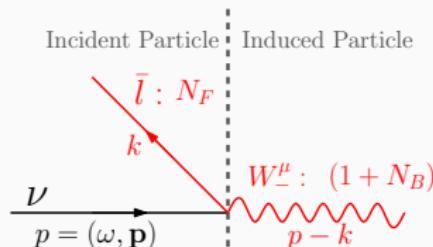


- Landau Damping (Fig.) makes the imaginary part being finite in the spacelike region:

$$\text{Im}\Sigma_+^{(\nu)} \ni \int_k \delta\left[\omega + |\mathbf{k}| - \sqrt{|\mathbf{p} - \mathbf{k}|^2 + M_{W,Z}^2}\right] \times [N_F(1 + N_B) + N_B(1 - N_F)] \cdot [\dots]. \quad (11)$$

- For a small external momentum  $(\omega, \mathbf{p})$  and a not small  $M_{W,Z}$ , the phase space in  $\int_k$  admitting the Landau Damping will be restricted.

# Landau Damping

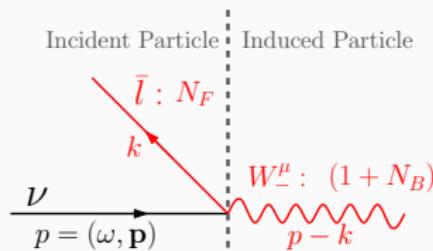


- Landau Damping (Fig.) makes the imaginary part being finite in the spacelike region:

$$\text{Im}\Sigma_+^{(\nu)} \ni \int_k \delta\left[\omega + |\mathbf{k}| - \sqrt{|\mathbf{p} - \mathbf{k}|^2 + M_{W,Z}^2}\right] \times [N_F(1 + N_B) + N_B(1 - N_F)] \cdot [\dots]. \quad (11)$$

- For a small external momentum  $(\omega, \mathbf{p})$  and a not small  $M_{W,Z}$ , the phase space in  $\int_k$  admitting the Landau Damping will be restricted.

# Landau Damping Suppression

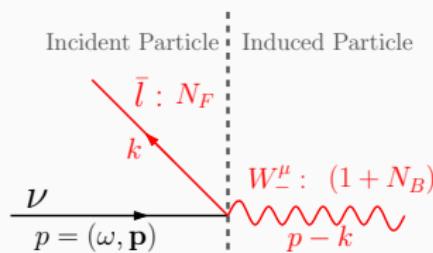


$$G(x_0) = \int_{x_0}^{\infty} dx x N_B(x) = \sum_{n=1}^{\infty} \frac{e^{-nx_0}}{n^2} [1 + nx_0], x_0 = \frac{\omega^2 - |\mathbf{p}|^2 - M_{W,Z}^2}{2T(\omega - |\mathbf{p}|)} > 0.$$

c.f. HTL Limit  $T \gg M_{W,Z}, \omega, |\mathbf{p}|$ :  $x_0 \rightarrow 0$  and  $G(x_0) \rightarrow \zeta(2) \gg 0$ .

The condition  $T \sim M_{W,Z}$  and the resultant finite  $x_0$  leads to  $G(x) \ll \zeta(2)$   
→ the suppression of the Landau damping.

# Landau Damping Suppression



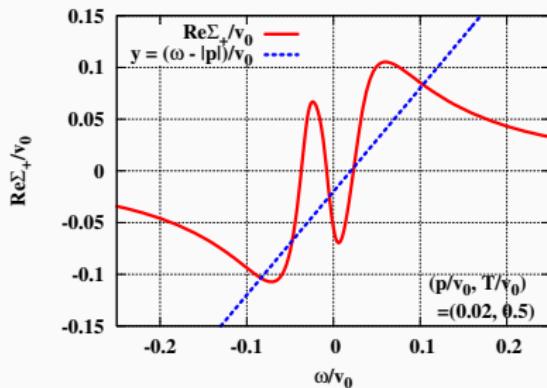
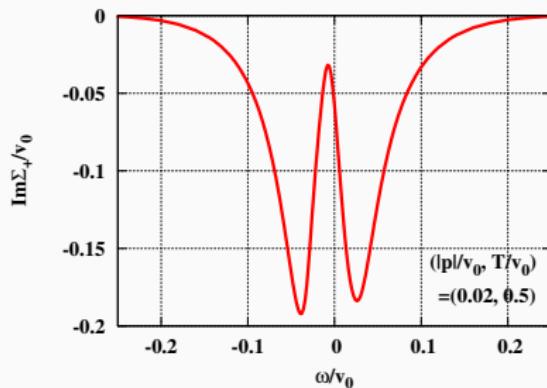
$$G(x_0) = \int_{x_0}^{\infty} dx \ x N_B(x) = \sum_{n=1}^{\infty} \frac{e^{-nx_0}}{n^2} [1 + nx_0] , x_0 = \frac{\omega^2 - |\mathbf{p}|^2 - M_{W,Z}^2}{2T(\omega - |\mathbf{p}|)} > 0 .$$

c.f. HTL Limit  $T \gg M_{W,Z}, \omega, |\mathbf{p}|$ :  $x_0 \rightarrow 0$  and  $G(x_0) \rightarrow \zeta(2) \gg 0$ .

The condition  $T \sim M_{W,Z}$  and the resultant finite  $x_0$  leads to  $G(x) \ll \zeta(2)$   
→ the suppression of the Landau damping.

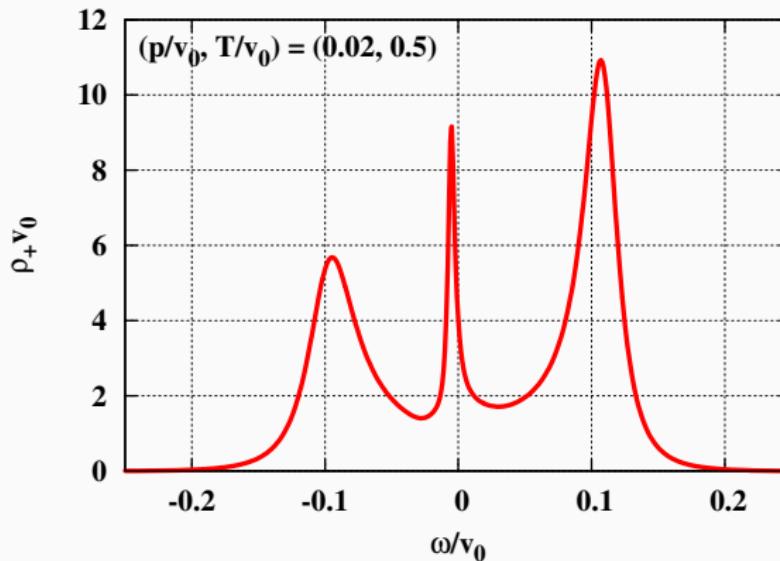
# Self-Energy at Three-Peak Region

$$\frac{T}{v_0} = 0.5, \quad \frac{|\mathbf{p}|}{v_0} = 0.02. \quad (12)$$

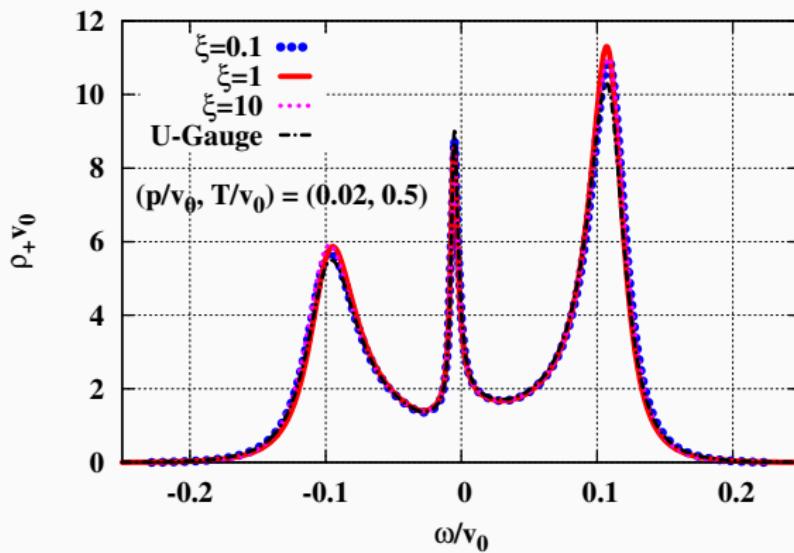


In the right panel, the crossing points corresponds to the solutions of  
 $\omega - |\mathbf{p}| - \text{Re } \Sigma_+(\omega, |\mathbf{p}|, T) = 0$   
(c.f. Kitazawa-Kunihiro-Nemoto, PTP 2007).

# Spectral Density at $(|p|/v_0, T/v_0) = (0.02, 0.5)$



# $\xi$ Dependence of Three-Peak Spectral Density



# Sphaleron Freeze-out Temperature

The net baryon number  $N_b$  is produced in the sphaleron process when the changing rate of  $N_b$  is larger than the expanding rate of the universe,

$$\left| \frac{1}{N_b} \frac{dN_b}{dt} \right| \geq H(T) , \quad (13)$$

where,

$$H(T) = 1.66 \sqrt{N_{\text{dof}}} \frac{T^2}{M_{\text{PL}}} \simeq T^2 \times 1.41 \times 10^{-18} \text{ (GeV)} , \quad (14)$$

$$\frac{1}{N_b} \frac{dN_b}{dt} = -1023 \cdot g^7 v(T) \exp \left[ -1.89 \frac{4\pi v(T)}{gT} \right] , \quad (15)$$

and we obtain

$$T \geq T_* \simeq 160 \text{ GeV} , \quad T_*/v_0 \simeq 0.65 . \quad (16)$$

# Sphaleron Freeze-out Temperature

The net baryon number  $N_b$  is produced in the sphaleron process when the changing rate of  $N_b$  is larger than the expanding rate of the universe,

$$\left| \frac{1}{N_b} \frac{dN_b}{dt} \right| \geq H(T) , \quad (13)$$

where,

$$H(T) = 1.66 \sqrt{N_{\text{dof}}} \frac{T^2}{M_{\text{PL}}} \simeq T^2 \times 1.41 \times 10^{-18} \text{ (GeV)} , \quad (14)$$

$$\frac{1}{N_b} \frac{dN_b}{dt} = -1023 \cdot g^7 v(T) \exp \left[ -1.89 \frac{4\pi v(T)}{gT} \right] , \quad (15)$$

and we obtain

$$T \geq T_* \simeq 160 \text{ GeV} , \quad T_*/v_0 \simeq 0.65 . \quad (16)$$

# Sphaleron Freeze-out Temperature

The net baryon number  $N_b$  is produced in the sphaleron process when the changing rate of  $N_b$  is larger than the expanding rate of the universe,

$$\left| \frac{1}{N_b} \frac{dN_b}{dt} \right| \geq H(T) , \quad (13)$$

where,

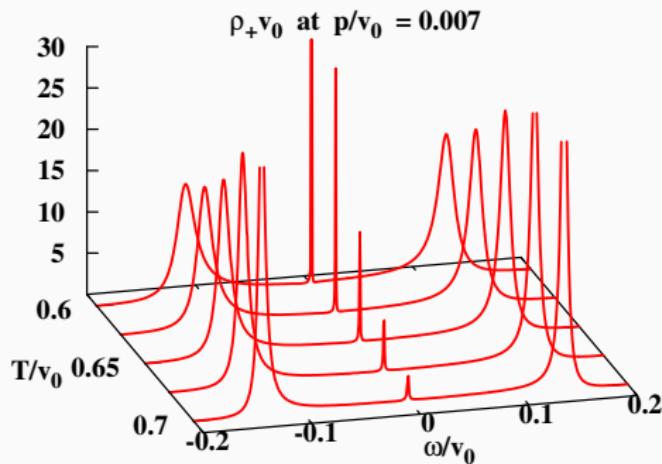
$$H(T) = 1.66 \sqrt{N_{\text{dof}}} \frac{T^2}{M_{\text{PL}}} \simeq T^2 \times 1.41 \times 10^{-18} \text{ (GeV)} , \quad (14)$$

$$\frac{1}{N_b} \frac{dN_b}{dt} = -1023 \cdot g^7 v(T) \exp \left[ -1.89 \frac{4\pi v(T)}{gT} \right] , \quad (15)$$

and we obtain

$$T \geq T_* \simeq 160 \text{ GeV} , \quad T_*/v_0 \simeq 0.65 . \quad (16)$$

# Neutrino Spectral Density around $T = T_*$



$$\frac{T_*}{v_0} \simeq 0.65 , \quad \frac{T_*}{v(T)} \sim 1 . \quad (17)$$

# Table of Contents

## 1 Introduction

## 2 Preliminaries

## 3 Results

- Neutrino Spectral Density: Overview
- Three Peak Structure: In Details
- Gauge Parameter  $\xi$  Dependence of Spectral Property
- Implication to Low Energy Scale Leptogenesis

## 4 Summary

## Summary

- We have investigated the spectral properties of standard-model left-handed neutrinos at finite  $T$  around the electroweak scale in a way where the gauge invariance is manifestly checked ( $R_\xi$  gauge).
- The spectral density of SM neutrino has the three-peak structure with the ultrasoft mode with a physical significance when  $T/M_{W,Z} \gtrsim 1$ .
- The collective excitation which involves the ultrasoft mode appears at temperature comparable to  $T_*$  within the present approximation. The three-peak collective modes could affect the leptogenesis at  $T \gtrsim T_*$ .
- Future Work: It is desirable to estimate how large the effects of two-loop or higher-order diagrams are on the neutrino spectral density.

Thanks for Your Attention!

## Summary

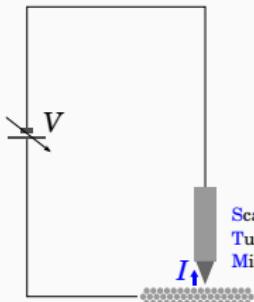
- We have investigated the spectral properties of standard-model left-handed neutrinos at finite  $T$  around the electroweak scale in a way where the gauge invariance is manifestly checked ( $R_\xi$  gauge).
- The spectral density of SM neutrino has the three-peak structure with the ultrasoft mode with a physical significance when  $T/M_{W,Z} \gtrsim 1$ .
- The collective excitation which involves the ultrasoft mode appears at temperature comparable to  $T_*$  within the present approximation. The three-peak collective modes could affect the leptogenesis at  $T \gtrsim T_*$ .
- Future Work: It is desirable to estimate how large the effects of two-loop or higher-order diagrams are on the neutrino spectral density.

Thanks for Your Attention!

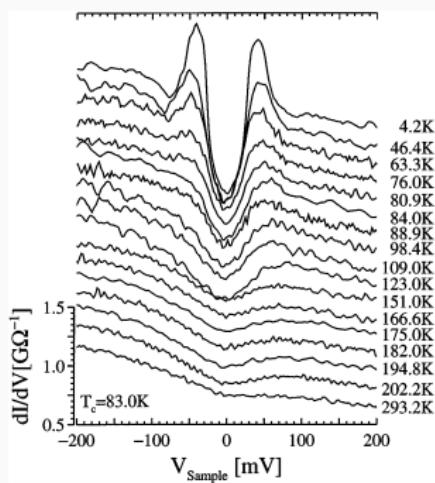
# Table of Contents

## 5 Buckups

## Spectral Density Example: Superconductivity



## Scanning Tunneling Microscopy

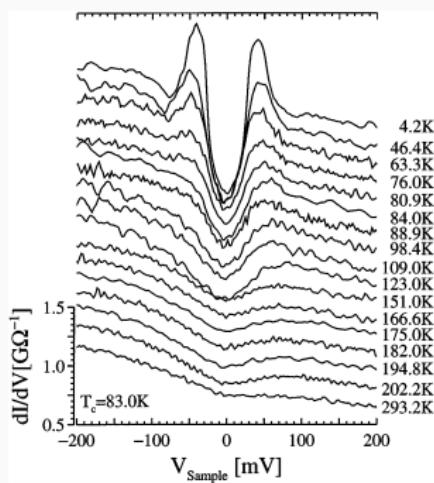
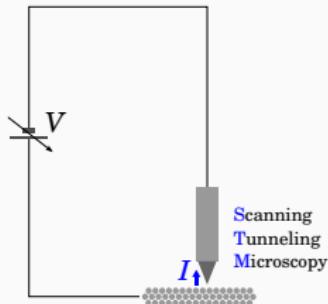


## Slightly Underdoped Bi2212 STM Superconductor Scan

Renner et. al.,  
Phys.Rev.Lett., 80  
149, (2008).

$$\frac{dI}{dV} \sim \frac{d}{dV} \left[ \int_{\epsilon_f}^{\omega=\epsilon_f+eV} d\omega \text{ Dos}(\omega) \right] \sim \text{Dos}(\omega) \sim \int_p \rho(\omega, p) , \quad (18)$$

# Spectral Density Example: Superconductivity

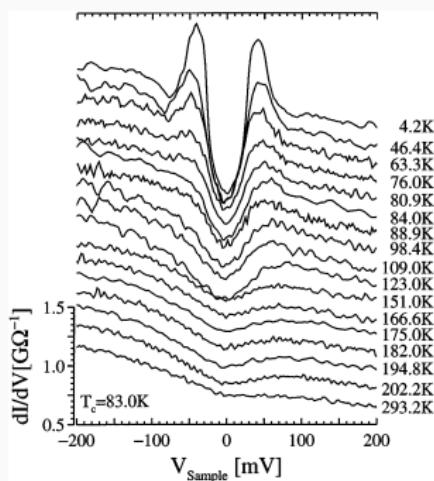
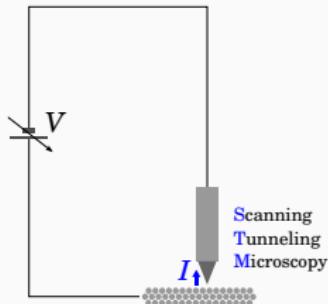


Slightly Underdoped Bi2212  
STM Superconductor Scan

Renner et. al.,  
Phys.Rev.Lett., 80  
149, (2008).

$$\frac{dI}{dV} \sim \frac{d}{dV} \left[ \int_{\epsilon_f}^{\omega=\epsilon_f+eV} d\omega \text{ Dos}(\omega) \right] \sim \text{Dos}(\omega) \sim \int_p \rho(\omega, p) , \quad (18)$$

# Spectral Density Example: Superconductivity



Slightly Underdoped Bi2212  
STM Superconductor Scan

Renner et. al.,  
Phys.Rev.Lett., 80  
149, (2008).

$$\frac{dI}{dV} \sim \frac{d}{dV} \left[ \int_{\epsilon_f}^{\omega=\epsilon_f+eV} d\omega \text{ Dos}(\omega) \right] \sim \text{Dos}(\omega) \sim \int_p \rho(\omega, p) , \quad (18)$$