Hadron-Quark Crossover and Massive Hybrid Stars

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Typical value of the observed mass for double NS binaries $\sim 1.4M_\odot$

In 2010, NS (PSR J1614-2230, NS-WD binary) with $M = (1.97 \pm 0.04)M_\odot$ was found

Key Questions:

- Any EOS which can explain 2 $M_\odot$ NS?
- The fate of the quark matter inside a heavy NS?
Hadronic EOSs

- Hyperons soften EOS
- Maximum mass is less than $1.44M_\odot$

**Graph**

- Hadronic EOS
- PSR J1614-2230
- PSR 1913+16

**Table**

<table>
<thead>
<tr>
<th>Method</th>
<th>(1) AV18+TBF</th>
<th>(2) TNI2</th>
<th>(3) SCL3ΛΣ</th>
</tr>
</thead>
<tbody>
<tr>
<td>2NF</td>
<td>AV18</td>
<td>Reid</td>
<td>RMF</td>
</tr>
<tr>
<td>3NF</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Hyperons</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
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</table>

(1) Baldo et al. (2000), Schulze et al. (2010)
(2) Nishizaki et al. (2001,2002)
(3) Tsubakihara et al. (2010)
• Universal 3-body force stiffens EOS $\rightarrow$ Maximum mass is larger than $1.44M_\odot$

• However maximum mass cannot exceed $2M_\odot$
This research seeks the possibility of crossover

Ref.)
Baym (1979)
Celik, Karsch and Satz (1980)
Fukushima (2004)
Hatsuda, Tachibana, Yamamoto and Baym (2006)
Method of Interpolation

Phenomenological interpolation: $P(\rho)$

\[
\begin{align*}
P &= p_H \times f_- + p_Q \times f_+ \\
\rho &= \rho^2 \frac{\partial(\varepsilon/\rho)}{\rho}
\end{align*}
\]

\[
f_\pm = \frac{1 \pm \tanh(\frac{\rho - \bar{\rho}}{1})}{2}
\]

Condition for $\bar{\rho}$: $f_+ < 0.1$ at $\rho_0 \quad \Rightarrow \quad \bar{\rho} > \rho_0 + 2\Gamma$
(2+1)-flavor NJL Lagrangian (u,d,s, e^-, \mu^-)

\[ L_{NJL} = \bar{q}(i\partial - m)q + \frac{G_s}{2} \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q}\gamma_5 \lambda^a q)^2] - \frac{g_v}{2} (\bar{q}\gamma \mu q)^2 + G_D[\text{det}q(1 + \gamma_5)q + \text{h.c.}] \]

Parameter set

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<th>cutoff (MeV)</th>
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<th>( m_{u,d}(\text{MeV}) )</th>
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0 \leq g_v \leq 1.5G_s

Hatsuda and Kunihiro (1994)

Conditions:
1. beta-equilibrium
2. charge neutrality

Recent estimate of \( g_V \)

\[ \kappa = -T_c \frac{d^2 T_c(\mu)}{d\mu^2} \bigg|_{\mu^2 = 0} \]

\[ \rightarrow g_V \sim G_S \]

\[ g_V \geq 0 : \text{repulsive} \]

\[ g_V / G_S = 0 : \text{no-repulsion} \]

\[ g_V / G_S = 1.0 : \text{medium repulsion} \]

\[ g_V / G_S = 1.5 : \text{strong repulsion} \]

In the crossover region, interpolated EOS is larger than H-EOS.
• Rapid stiffening of the EOS in the crossover region.
Results (1): Effects of Q-EOS

M-R relation $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$  

- Maximum mass exceeds 2 solar mass, no matter what kind of H-EOS is taken
## Results (2): Effects of parameters

How maximum mass depends on $\bar{\rho}, \Gamma$

<table>
<thead>
<tr>
<th>$\bar{\rho}$</th>
<th>$\Gamma/\rho_0 = 1$</th>
<th>$\Gamma/\rho_0 = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$g_v = G_s$</td>
<td>$g_v = 1.5G_s$</td>
</tr>
<tr>
<td>$3\rho_0$</td>
<td>2.05</td>
<td>2.17</td>
</tr>
<tr>
<td>$4\rho_0$</td>
<td>1.89</td>
<td>1.97</td>
</tr>
<tr>
<td>$5\rho_0$</td>
<td>1.73</td>
<td>1.79</td>
</tr>
<tr>
<td>$6\rho_0$</td>
<td>1.60</td>
<td>1.64</td>
</tr>
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Crossover occurs at relatively low densities and quarks are strongly interacting $\rightarrow 2M_\odot$
M-R relation \((\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)\) \(g_v = G_S\)

- The emergence of strangeness softens EOS
- Due to the interpolation, the sound velocity increases rapidly in the crossover region
Results (4): Strangeness Core

\[ \rho - r \text{ relation } (\bar{\rho}, \Gamma) = (3\rho_0, \rho_0) \quad g_v = G_S \]

Typical NSs with universal 3-body force do not include strangeness inside themselves

\[ g_v = G_S \]

\[ \frac{\rho}{\rho_0} \]

possibility of solving cooling problem
Color Superconductivity (CSC)

- Chiral Condensate

\[ L_{\text{CSC}} = L_{\text{NJL}} + \frac{H}{2} \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{q} i \gamma_5 \tau_A \lambda_{A'} C \bar{q}^T) (q^T C i \gamma_5 \tau_A \lambda_{A'} q) \]

- Diquark Condensate

\[ H = \frac{3}{4} G_s \] (Fierz)

- NJL model

- Chiral Condensate

- Diquark Condensate
$g_v = 0$

**Results (5): Gap parameter**

![Graph showing the gap parameter for different quark chemical potentials. The graph plots the constituent mass M (MeV) against the quark chemical potential (MeV) for s quark, d quark, and u quark. The y-axis shows the constituent mass in MeV, while the x-axis shows the quark chemical potential in MeV. The graph also includes annotations for different gaps ($\Delta_1$, $\Delta_2$, $\Delta_3$).]
Results (5): Gap parameter

\[ g_v = 0 \]

Dispersion relation (bd-gs)

gapless phase
Results (6): Effects of CSC

Diquark condensation with $J^P = 0^+$

$$L_{CSC} = L_{NJL} + \frac{H}{2} \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{q} i \gamma_5 \tau_A \lambda_{A'} C \bar{q}^T) (q^T C i \gamma_5 \tau_A \lambda_{A'} q)$$

M-R relation $$(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0) \quad g_v = G_s \quad H = \frac{3}{4} G_s$$

- CSC softens EOS, but the effects of CSC is very small
Proto-NS with 2 solar mass puzzle

At the birth stage, NSs are composed of so-called supernova matter:

- constant lepton fraction \( Y_l = 0.3 - 0.4 \)
- constant entropy per baryon \( s = 1 - 1.5 \)

Finite temperature extension of our model

\[
P = f_-(\rho) \times p_H(\rho, T)|_{Y_l, s} + f_+(\rho) \times p_Q(\rho, T)|_{Y_l, s}
\]
Results (7): Finite T

\[ g_v = G_s \quad (\bar{\rho}, \Gamma) = (3\rho_0, \rho_0) \]

\[ N' = 2.49 \times 10^{57} \]

\[ M' = 1.92M_\odot < 1.97M_\odot \]

\[ \rightarrow \text{neutron star merger} \]
(1) Crossover occurs at relatively low densities
(2) Quarks are strongly interacting at and above the crossover region

Interpolated EOS can become stiffer due to the presence of quark matter

Observation of very massive neutron star cannot exclude the existence of the quark matter core

Conventional belief

* Other Characteristics:
  Interpolated EOS with the repulsive 3-body force among nucleons and hyperons have an impact on the cooling problem of neutron star with hyperon core

* Perspective
  1. Finite temperature extension of the present model (proto-neutron star)
  2. Constraints on the EOS from other observables such as neutron star radius and cooling

Thank you!
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Introduction: QCD Phase Diagram

**Equation of state (EOS):**

\[ P = f(\varepsilon) \]

**Tolman-Oppenheimer-Volkov equation (TOV eq.):**

\[
\frac{dP}{dr} = - \left( M + 4\pi Pr^3 \right) (\varepsilon + P) \left( \frac{r^2}{G} - 2Mr \right)^{-1}
\]

\[ M = \int 4\pi r^2 \varepsilon(x) \, dr \]

**Observation**

Mass of Neutron stars (NSs)

**Theory**

Fukushima, Hatsuda (2010)
Introduction: Hadronic EOSs

- Method: Variational
- 2NF: AV18
- 3NF: Yes
- Hyperons: No

Akmal et al. (1998)
Crossover at finite temperature

- Phenomenological Interpolation: $s(T)$

$s$: entropy density, $T$: temperature

Asakawa, Hatsuda (1995)

\[
s(T) = s_h(T)w_h(T) + s_q(T)w_q(T)
\]

\[
w_q(T) = \frac{n\left(1 + \tanh\left(\frac{T-T_c}{T}\right)\right)}{m\left(1 - \tanh\left(\frac{T-T_c}{T}\right)\right) + n\left(1 + \tanh\left(\frac{T-T_c}{T}\right)\right)}
\]

Phenomenological Interpolation

lattice QCD

Karsch (1995)
(2+1)-flavor NJL Lagrangian (u,d,s, e⁻, μ⁻)

\[ L_{NJL} = \bar{q}(i\slashed{\partial} - m)q + \frac{G_s}{2} \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q}\gamma_5\lambda^a q)^2] - \frac{g_v}{2}(\bar{q}\gamma^\mu q)^2 + G_D[\det\bar{q}(1 + \gamma_5)q + \text{h.c.}] \]

\[ \Omega = -\frac{T}{V}\ln Z \]

\[ \begin{aligned} M_i &= m_i - 2G_s\langle \bar{q}_i q_i \rangle - 2G_D\langle \bar{q}_j q_j \rangle\langle \bar{q}_k q_k \rangle \\ \mu_i &\rightarrow \mu_i^{\text{eff}} \equiv \mu_i - g_v \sum_i \langle q_i^\dagger q_i \rangle \end{aligned} \]

\[ \Omega_q(M, \mu^{\text{eff}}) + \Omega_l + G_s \sum \langle \bar{q}_i q_i \rangle^2 + 4G_D\langle \bar{q}_i q_i \rangle\langle \bar{q}_j q_j \rangle\langle \bar{q}_k q_k \rangle - \frac{1}{2}g_v \left( \sum_i \langle q_i^\dagger q_i \rangle \right)^2 \]

\[ \Omega_q(\mu^{\text{eff}}) = -T \sum_i \sum_l \int \frac{d^3p}{(2\pi)^3} \text{Tr} \ln \left( \frac{1}{T}S_i^{-1}(i\omega_l, \vec{p}) \right) , \]

\[ S_i^{-1} = p - \mu^{\text{eff}}\gamma^0 - M_i, \quad p^0 = i\omega_l = (2l + 1)\pi T \]

Gap equations: \[ \frac{\partial \Omega}{\partial \langle \bar{q}_i q_i \rangle} = 0 \]

Parameter sets

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<td>(Fierz: ( G_V = 0.5G_s ))</td>
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Hatsuda and Kunihiro (1994)

Bratovic et al. (2012)

Conditions:
1. beta-equilibrium
2. charge neutrality
Chiral restoration
- $u,d$ quark: low densities
- $s$ quark: $4\rho_0$

- $s$ quark starts to appear above $4\rho_0$
- SU(3) flavor symmetric matter at high densities
- Muon does not appear due to charge neutrality

- Figures do not depend on the magnitude of vector interaction
EOS at $\rho \gg \bar{\rho}$

Pressure $P$

- EOS becomes stiffer as $g_v$ increases due to the universal repulsion
Results (2): Effects of Crossover Density ($\bar{\rho}$)

M-R relation \( \Gamma = \rho_0 \quad g_v = G_s \)

- Crossover occurs at relatively low densities \( \rightarrow 2M_\odot \)
M-R relation \((\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)\)

- The maximum mass exceeds \(2M_\odot\) only if the vector type repulsion is as strong as the scalar interaction

- Radius: about 11 km
Another Interpolation

Phenomenological interpolation: $\varepsilon(\rho)$

\[
\begin{align*}
\varepsilon &= \varepsilon_H \times f_- + \varepsilon_Q \times f_+ \\
P &= \rho^2 \frac{\partial(\varepsilon/\rho)}{\rho} \\
f_{\pm} &= \frac{1 \pm \tanh\left(\frac{\rho - \bar{\rho}}{\Gamma}\right)}{2}
\end{align*}
\]

H-EOS: TNI2u, Q-EOS: NJL

$g_v = 0.5G_s$ \quad $(\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)$
Results (7): Effects of Method

M-R relation \((\bar{\rho}, \Gamma) = (3\rho_0, \rho_0)\)

- The \(\varepsilon\)-interpolation makes EOS stiff more drastically than the P-interpolation.
- Even for \((g_v, \bar{\rho}) = (0, 3\rho_0)\), the maximum mass can exceed \(1.97M_\odot\)
CSC Lagrangian

\[ L_{\text{CSC}} = L_{\text{NJL}} + \frac{H}{2} \sum_{A=2,5,7} \sum_{A'=2,5,7} (\bar{q}_i \gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i \gamma_5 \tau_A \lambda_{A'} q) \]

\[ H = \frac{3}{4} G_s \]

by Fierz

\[ q^C = C \bar{q}^T \quad \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ q^C \end{pmatrix} \]

\[ \Delta_1 = -H s_{55}, \quad \Delta_2 = -H s_{77}, \quad \Delta_3 = -H s_{22} \]

\[ \Omega(T, \mu_{u,d,s}) = -\frac{T}{2} \sum_{\ell} \int \frac{d^3p}{(2\pi)^3} \text{Trln} \left( \frac{S^{-1}(i\omega, p)}{T} \right) + G_s \sum_i \sigma_i^2 \]

\[ + 4 G_D \sigma_u \sigma_d \sigma_s - \frac{1}{2} g_V \left( \sum_i n_i \right)^2 + \frac{1}{2H} \sum_{\text{color}} |\Delta_c|^2 \]

\[ S^{-1} = \begin{pmatrix} S_{0+}^{-1} & \Phi^- \\ \Phi^+ & S_{0-}^{-1} \end{pmatrix} \]

\[ (\Phi^-)^{\alpha \beta}_{ab} = -\sum_{\text{color}} \epsilon^{\alpha \beta \epsilon} \epsilon_{abc} \Delta_c \gamma_5, \quad \Phi^+ = \gamma^0 (\Phi^-)^\dagger \gamma^0 \]

\[ S_{0\pm}^{-1} = p - M \pm \bar{\mu} \gamma^0 \]

\[ \tilde{\mu} = \mu - \frac{1}{2} \mu_3 - \frac{1}{2\sqrt{3}} \mu_8 \]
\[
p = \frac{1}{4\pi^2} \sum_{i=1,36} \int_0^\Lambda dpp^2 \left( |\varepsilon_i| + 2T \ln \left( 1 + e^{-|\varepsilon_i|/T} \right) \right) - G_s \sum_i \sigma_i^2
\]

\[
-4G_D \sigma_u \sigma_d \sigma_s + \frac{1}{2} g_V \left( \sum_i n_i^2 \right)^2 - \frac{1}{2H} \sum_{\text{color}} |\Delta_c|^2
\]

Gap equations:
\[
\frac{\partial p}{\partial \sigma_i} = -\frac{\partial p}{\partial \Delta_i} = \frac{\partial p}{\partial \mu_i} = 0
\]
Results (5): Case 1 \( H = \frac{3}{4} G_s \)

\( g_v = 0 \)

\[ H = \frac{3}{4} G_s \]

\[ g_v = 0 \]

\[ M_s^2/\mu \text{[MeV]} \]

\[ \Delta_1, \Delta_2, \Delta_3 \]

\[ \text{CFL, gCFL} \]

\[ M_s^2/（\mu \Delta）\]

Alford (2004)
Results (6): Case 2  \( H = G_s \)

\[ g_v = 0 \]

2SC phase

![Graphs showing quark chemical potential vs. constituent mass and gap](image-url)
In the case of $g_V = 1.0, 1.5 G_S$, H-EOS and Q-EOS do not cross at all densities.
Crossover vs. 1st order Transition

Crossover

``QM`` stiffens EOS

``QM`` softens EOS

``H``

 pressure (P)

 baryon density (ρ)

 M > 2M⊙

1st order Transition

``H``

 pressure (P)

 baryon density (ρ)

 M < 2M⊙
Neutron Star Observation

Observables:

- binary period $P_b$
- projection of the pulsar’s semimajor axis on the line of sight $a_{\sin i}$
- eccentricity $e$
- time of periastron $T_0$
- longitude of periastron $\omega_0$

Mass function:

$$f = \frac{(m_2 \sin i)^3}{M^2}$$

General relativity effects:

- the advance of periastron of the orbit $\dot{\omega}$
- Doppler + gravitational redshift $\gamma$
- the orbital decay $\dot{P}_b$
- range parameter $r$
- shape parameter $s$

Shapiro delay:

$$\Delta = 2r \log \frac{1 + e \cos \nu}{1 - s \sin(\omega + \nu)}$$

Mass estimation

$$f + 2 \text{ general relativity effects}$$
Universal 3-body force

**TNI model**

\[
v_{TNI} = v_{TNA} + v_{TNR}
\]

\[= v_2 e^{-\left(\frac{r}{\lambda_a}\right)^2} \rho e^{-\eta_2 \rho} (\mathbf{\tau}_1 \cdot \mathbf{\tau}_2)^2 + v_1 e\left(-\frac{r}{\lambda_r}\right)^2 (1 - e^{-\eta_1 \rho})\]

**Urbana UIX model**

\[
v_{ijk} = v_{ijk}^{2\pi} + v_{ijk}^R
\]

\[= A \sum_{\text{cyc}} \left( \{X_{ij}, X_{jk}\}\{\mathbf{\tau}_i \cdot \mathbf{\tau}_j, \mathbf{\tau}_j \cdot \mathbf{\tau}_k\} + \frac{1}{4} [X_{ij}, X_{jk}] [\mathbf{\tau}_i \cdot \mathbf{\tau}_j, \mathbf{\tau}_j \cdot \mathbf{\tau}_k] \right) + U \sum_{\text{cyc}} T^2(r_{ij})T^2(r_{jk})\]

\[X_{ij} = Y(r_{ij})\sigma_i \cdot \sigma_j + T(r_{ij})S_{ij}\]
H-EOS: Universal 3-body force

3-body force is needed for saturation property

From the point of view of NS observation, 3-body force is needed for the stiffness of EOS

→ 3-body force between YN and YY can delay the appearance of the exotic components

Universal 3-body force

• TNI model:
  G-matrix
  NN : Reid soft-core potential
  YN,YY: Nijmegen type-D hard-core potential

TNI2(3):
  $\kappa=250(300)$MeV

Akmal et al. (1998)

→ From the point of view of NS observation, 3-body force is needed for the stiffness of EOS

Nishizaki et al. (2002)
Rapid cooling is occurred by hyperons (Y-Durca)

\[
\begin{align*}
\Lambda & \rightarrow p + l + \bar{\nu}_l, \ p + l \rightarrow \Lambda + \nu_l \\
\Sigma^- & \rightarrow \Lambda + l + \bar{\nu}_l, \ \Lambda + l \rightarrow \Sigma^- + \nu_l
\end{align*}
\]

Tsuruta et al. (2009)