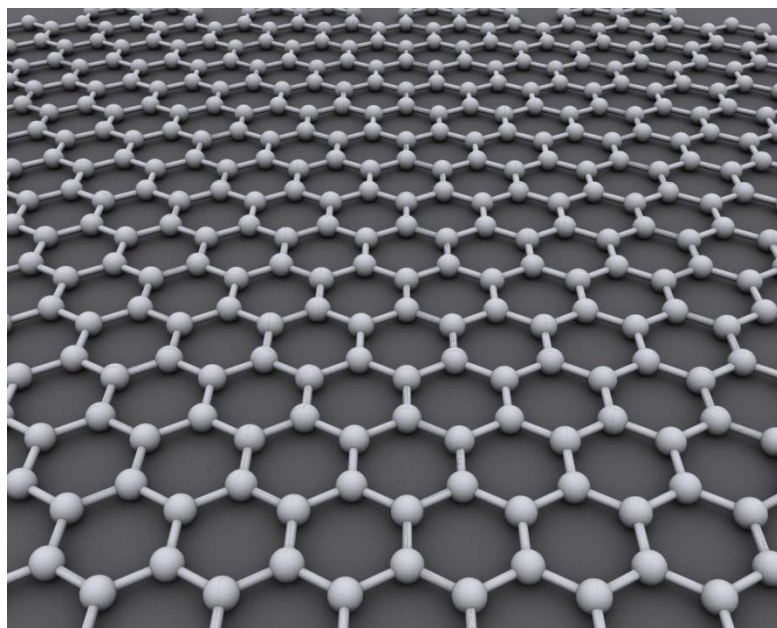


グラフェン状の系における 秩序現象と相構造

Competition of orders and phase structure in graphene-like systems



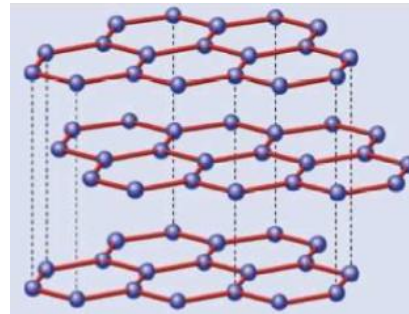
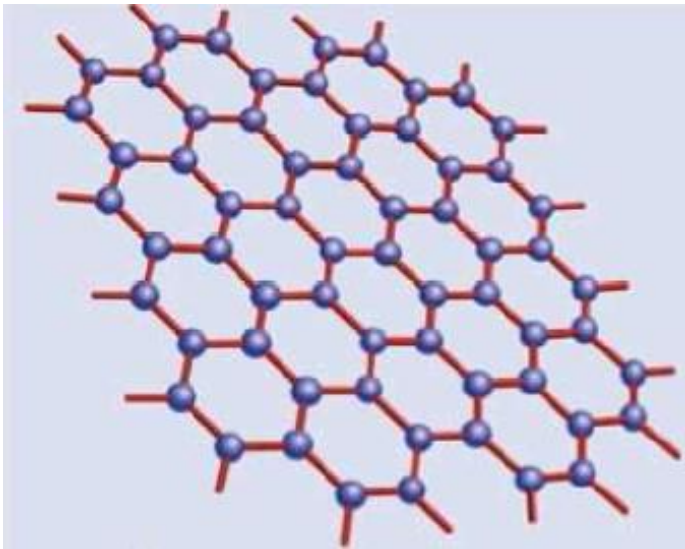
荒木 康史 (東大院理)
Yasufumi Araki



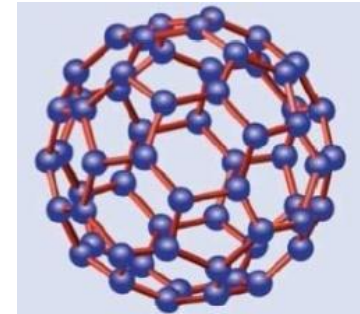
What is “graphene”?

Monoatomic layer material of carbon atoms

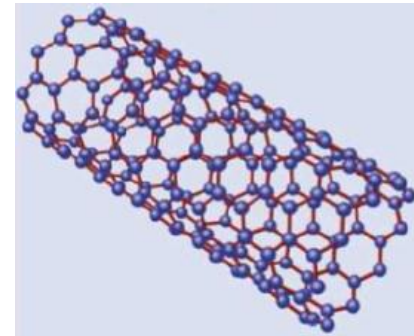
Honeycomb lattice structure in 2-dim plane.



graphite



fullerene



nanotube

Building block of carbon materials.

For review:

A. Castro Neto *et al.*, RMP **81**, 109 (2009).

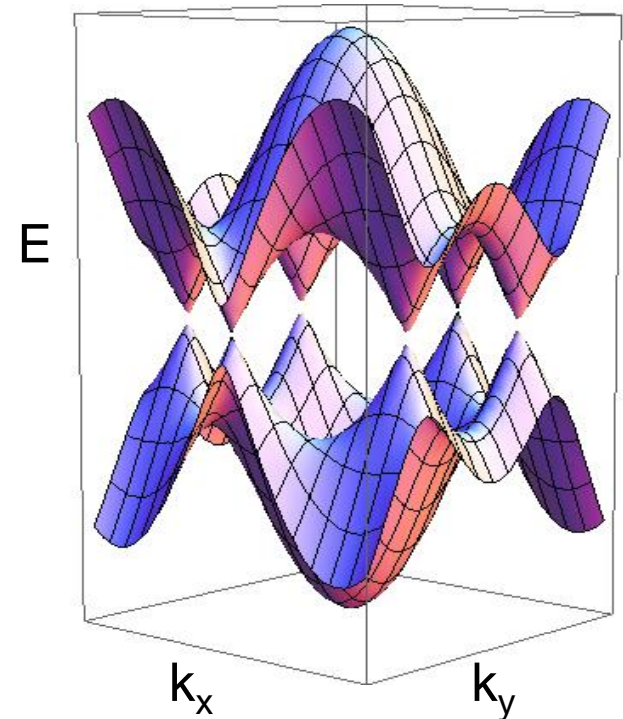
Interaction and gap formation in graphene

Electronic spectrum of monolayer graphene:
linear dispersion around two “Dirac points”.

$$E(\mathbf{K}_{\pm} + \mathbf{k}) \simeq v_F |\mathbf{k}|$$

(Fermi velocity $v_F = (3/2)ah \sim c/300$)

Wallace(1947)
Semenoff(1984)



► Effective Coulomb interaction strength:

“Fine structure const.”

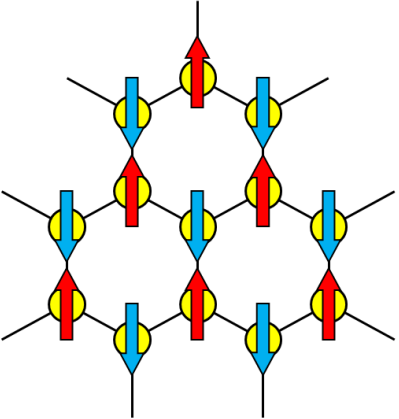
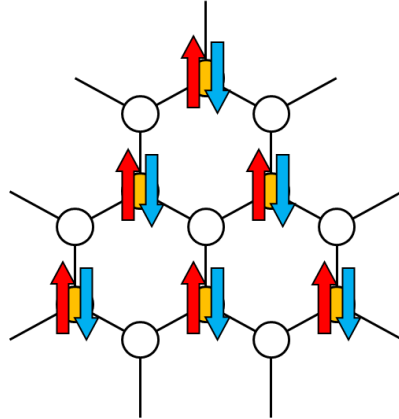
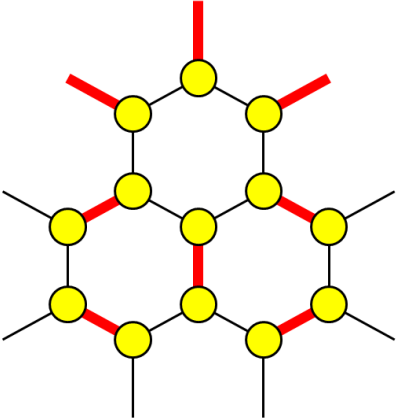
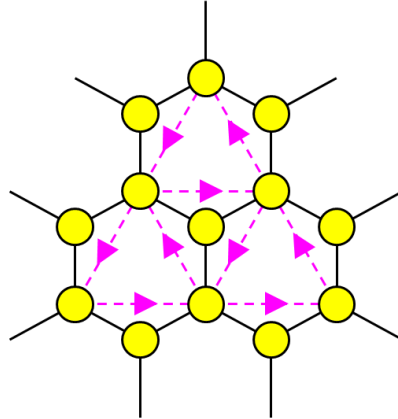
$$\alpha_{\text{eff}} = \frac{e^2}{4\pi\epsilon v_F} \quad (\gg \alpha_{\text{QED}})$$

$$\left(\alpha_{\text{QED}} = \frac{e^2}{4\pi\epsilon_0 c} = \frac{1}{137} \right)$$

⇒ **Effectively strong** coupling (in **vacuum-suspended** graphene).

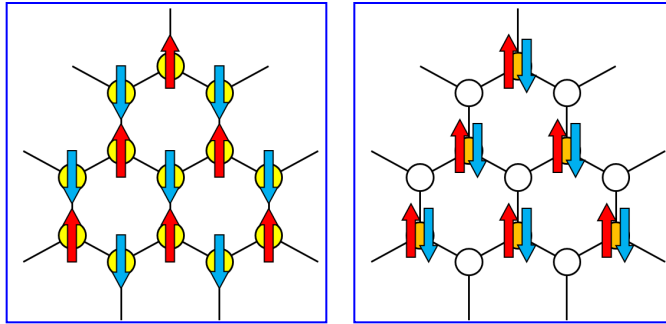
Spontaneous symmetry breaking (ordering) → Gap generation?

Possible orders on honeycomb lattice

Spin density wave (SDW)	Charge density wave (CDW)	Kekulé distortion (KD)	Quantum anomalous Hall
			
Sublattice (inversion) symmetry is broken.		Translational symmetry is broken.	(Time-reversal symm. is broken)
	Induced <u>explicitly</u> by substrates (BN, SiC, ...) "mass term"	Induced <u>explicitly</u> by adatoms (Ca, Al, ...)	(Model of topological insulators) Haldane(1988)

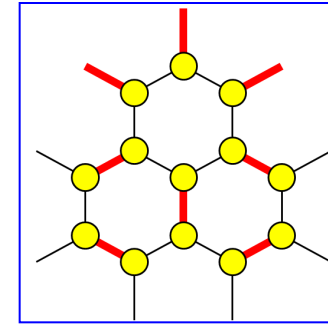
Competition of orders

e.g.) Sublattice symmetry breaking

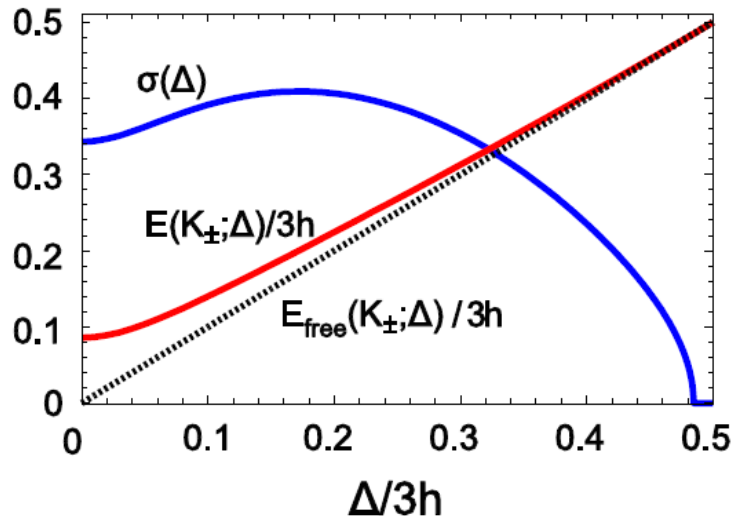


Kekulé distortion

VS

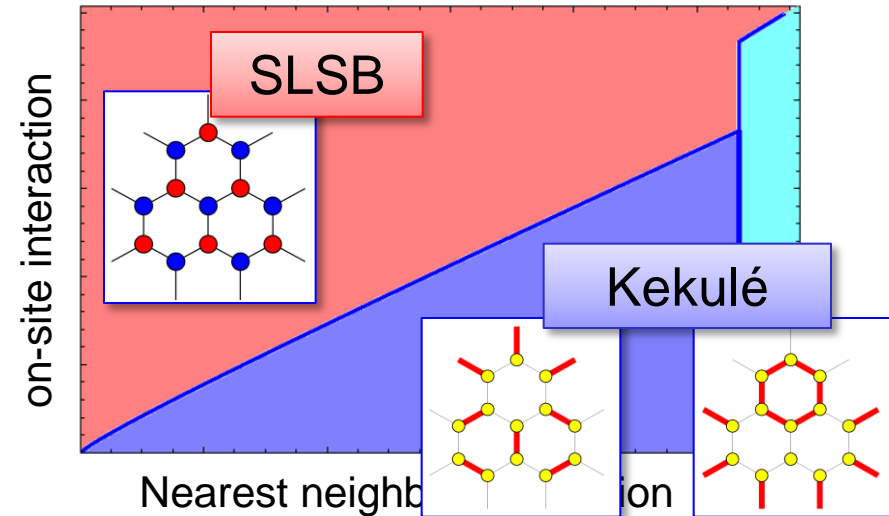


Spontaneous SLSB VS External Kekulé



YA, Phys. Rev. B **84**, 113402 (2011)

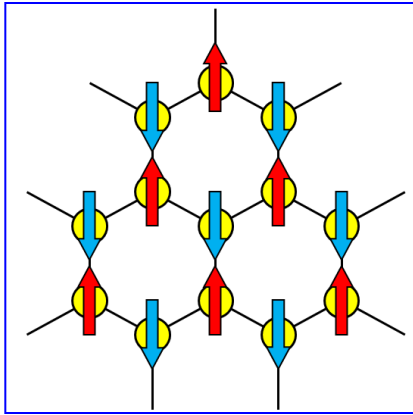
Spontaneous SLSB VS Spontaneous Kekulé



YA, Phys. Rev. B **85**, 125436 (2012)

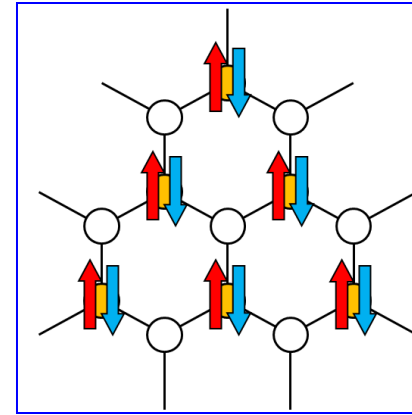
This talk

Spontaneous/External **CDW**

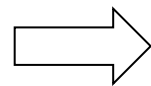


VS

Spontaneous **SDW**



- ▶ Is there any **interplay effect** between these two orders?
- ▶ What may happen to the **band structure** and **electronic properties** of the system?

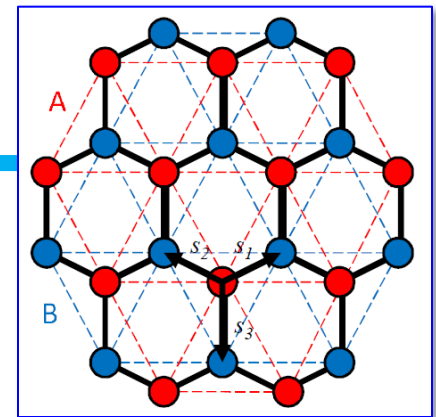


Solve by **Hamiltonian variational method**.

*“Spin Versus Charge Density Wave Order in Graphene-like Systems”,
Y. A. and G. W. Semenoff, arXiv:1204.4531[cond-mat.str-el].*

Effective model

“Extended Hubbard model”



- ▶ Electron hopping (tight-binding model):

$$H_T = -t \sum_{\langle \mathbf{r}_A, \mathbf{r}_B \rangle} [a_{\sigma}^{\dagger}(\mathbf{r}_A)b_{\sigma}(\mathbf{r}_B) + b_{\sigma}^{\dagger}(\mathbf{r}_B)a_{\sigma}(\mathbf{r}_A)]$$

- ▶ On-site (Hubbard) repulsion:

$$H_U = U \left[\sum_{\mathbf{r}_A} n_{\uparrow}(\mathbf{r}_A)n_{\downarrow}(\mathbf{r}_A) + \sum_{\mathbf{r}_B} n_{\uparrow}(\mathbf{r}_B)n_{\downarrow}(\mathbf{r}_B) \right]$$

Favors **SDW**

- ▶ Nearest neighbor (NN) repulsion:

$$H_V = V \sum_{\langle \mathbf{r}_A, \mathbf{r}_B \rangle} n(\mathbf{r}_A)n(\mathbf{r}_B)$$

Favors **CDW**

- ▶ Staggered potential:

$$H_M = m \left[\sum_{\mathbf{r}_A} a_{\sigma}^{\dagger}(\mathbf{r}_A)a_{\sigma}(\mathbf{r}_A) - \sum_{\mathbf{r}_B} b_{\sigma}^{\dagger}(\mathbf{r}_B)b_{\sigma}(\mathbf{r}_B) \right]$$

External **CDW**

Hamiltonian variational method

(1) Take a **reference Hamiltonian** H_0 .

$$H_0^T = - \sum_{\sigma} Z_{\sigma} t \sum_{\mathbf{r} \in A, i} [a_{\sigma}^{\dagger}(\mathbf{r}) b_{\sigma}(\mathbf{r} + \mathbf{s}_i) + \text{H.c.}]$$

$$H_0^M = \sum_{\sigma} \Delta_{\sigma} \sum_{\mathbf{r}} [a_{\sigma}^{\dagger} a_{\sigma} - b_{\sigma}^{\dagger} b_{\sigma}]$$

\Rightarrow dispersion: $E_{\sigma}(\mathbf{K}_{\pm} + \mathbf{k}) \simeq \sqrt{|Z_{\sigma} v_F \mathbf{k}|^2 + \Delta_{\sigma}^2}$

velocity renormalization \rightarrow $Z_{\sigma} v_F \mathbf{k}$
 quasiparticle gap \rightarrow Δ_{σ}

(2) **Vary** H_0 to approximate the original Hamiltonian H as much as possible.

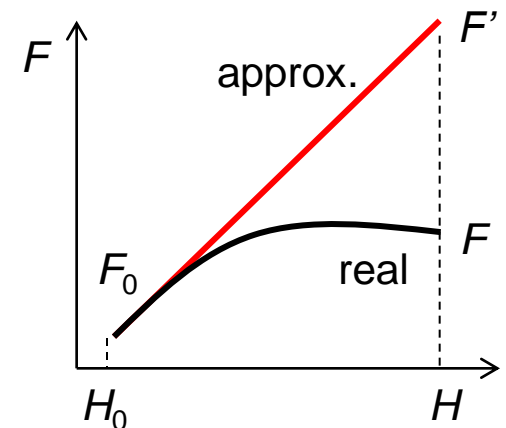
$$F \leq F' \equiv F_0 + \langle H - H_0 \rangle_0 \quad (\langle A \rangle_0 \equiv \text{Tr} A e^{-\beta H_0})$$

- "Jensen-Peierls inequality"

\Rightarrow **Minimize** F' as much as possible.

$$\text{i.e. } \frac{\delta F'}{\delta H_0} = 0$$

(corresponds to the MF gap equation)

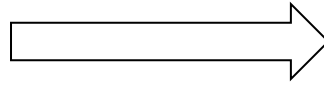


Phase structure

Input

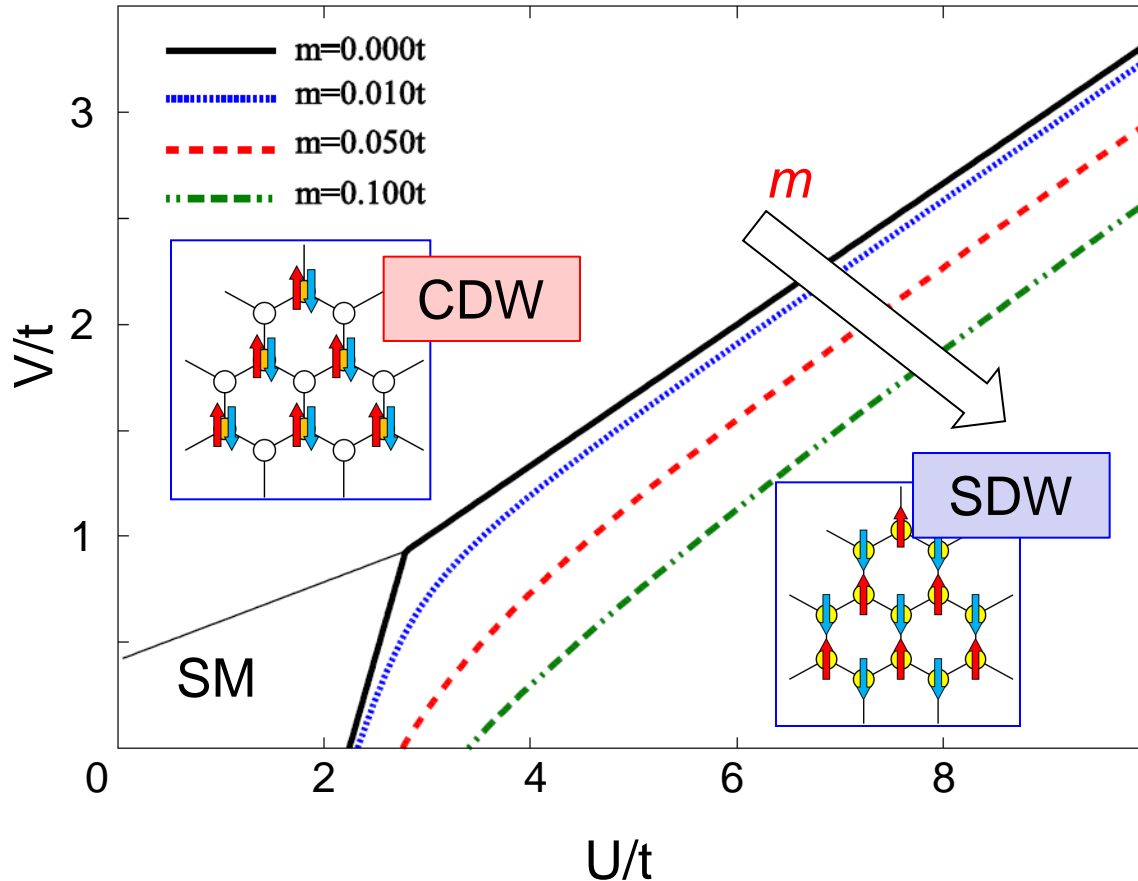
U, V, m

variational gap eq.



Output

Δ_σ, Z_σ



$\Delta_\uparrow = \Delta_\downarrow = 0$:

Semimetallic (SM) phase
(only at $m=0$)

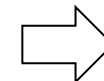
$\Delta_\uparrow = \Delta_\downarrow \neq 0$:

CDW phase

$\Delta_\uparrow \neq \Delta_\downarrow$:

SDW phase

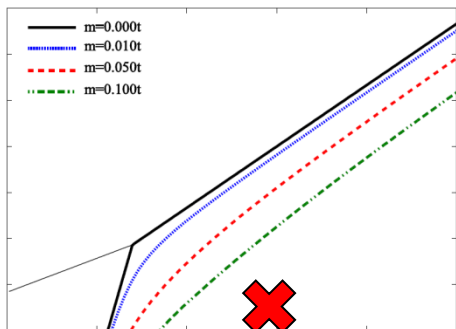
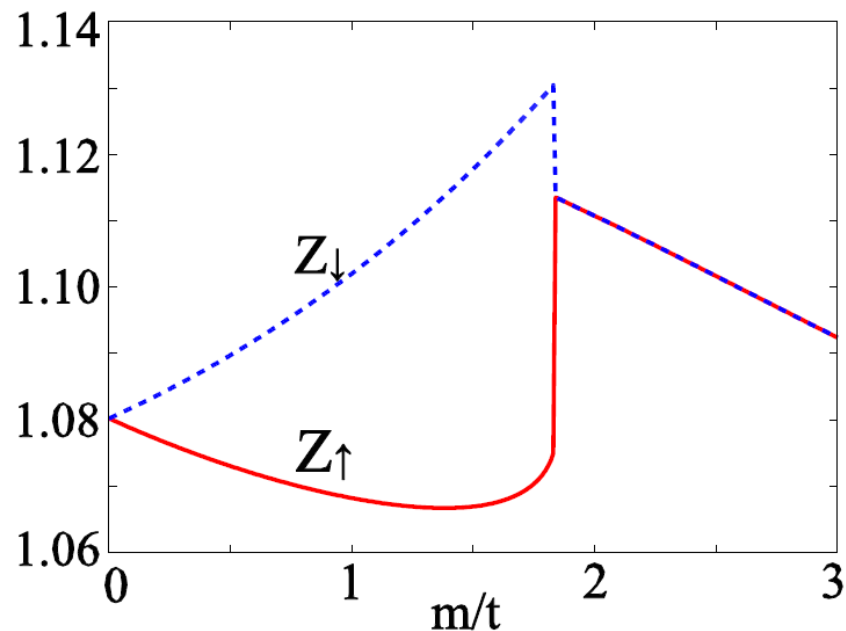
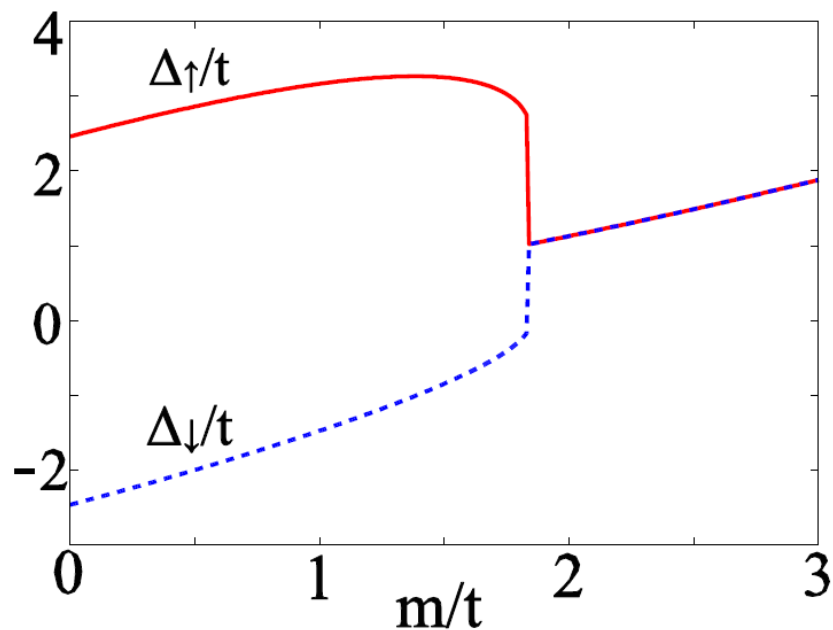
Introduction of m



SDW is suppressed
over CDW.

Splitting of gap and velocity

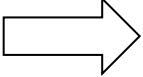
► Fix $U=6t$, $V=0.5t$ / Vary m :



$$|\Delta_{\uparrow}| \neq |\Delta_{\downarrow}| \quad \text{and} \quad Z_{\uparrow} \neq Z_{\downarrow}$$

in the SDW phase (if $m, V \neq 0$).

Summary

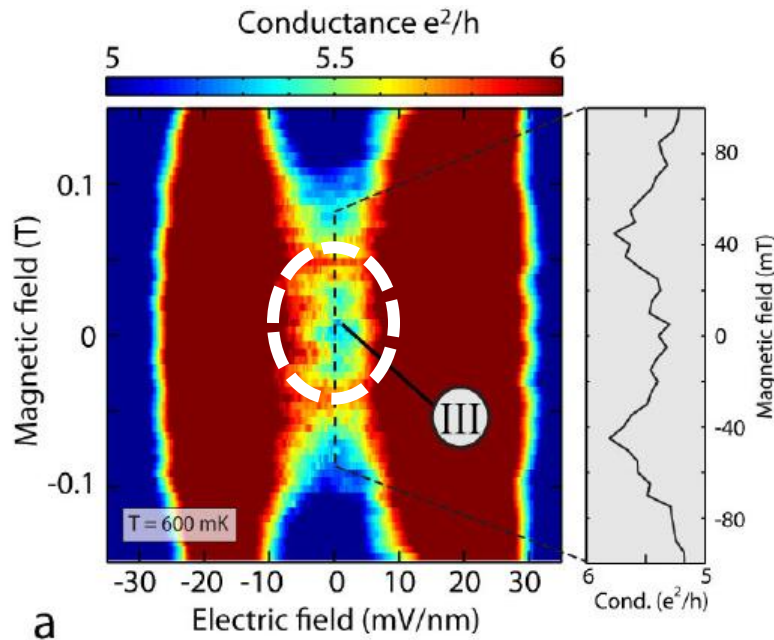
- ▶ Phase structure of **extended Hubbard model** with **explicit staggered potential** is investigated by the Hamiltonian variational method.
- ▶ There is an interplay between **CDW** and **SDW**.
- ▶ **SDW is suppressed** over CDW in the presence of staggered potential.
- ▶ Staggered potential (bare fermion “mass”) **splits the degeneracy of quasiparticle gap and Fermi velocity** between spin states.
- ▶ Extended Hubbard model  **Coulomb interaction?**
Momentum-dependent Fermi velocity?
cf. continuum limit: **Sabio, Sols, Guinea(2010)**
Derivation from QED-related model...?

Recent topics

► Extension to **bilayer graphene**:

Spontaneously gapped phase(?) is experimentally observed.

Thomas Weitz *et al.* (2010)



Bilayer:

dispersion relation becomes quadratic.

What kind of symmetry is broken?

- sublattice (chiral)
- Dirac point (pseudospin)
- spin
- layer

► Effect of **next-to-NN interaction** (~NNLO): “Anomalous QH” state?

► **Finite-size** sample: edge states (zigzag edge) vs. bulk properties

Various phases are expected ...

