Quark-Hadron Phase Transition in an Extended NJL Model with Scalar-Vector Eight-Point Interaction

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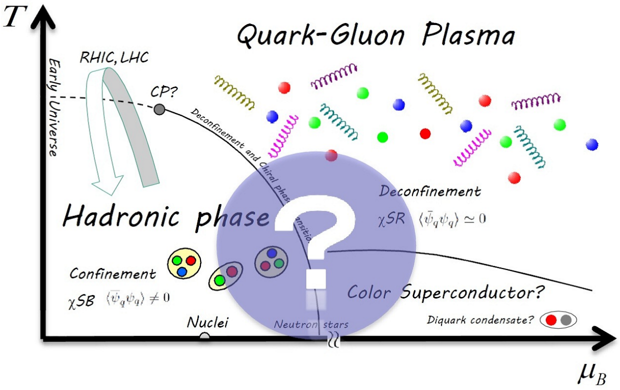
arXiv:1207.1499

TQFT2012 @ YITP

Schematic QCD phase diagram



Finite density systems

- Difficulty in the Lattice-QCD calculations (Sign problem)
- Effective model approach (e.g. NJL model [Nambu et al.,'61])
- Quark-Hadron phase transition (Confinement problem)

Objective and Methods

Objective and Methods

- To investigate the quark-hadron phase transition at finite temperature and density.
- We use the extended NJL model with the scalar-vecter interaction.

The extended NJL model

- Reproduction of nuclear saturation property
  - Introduction of G_v^N, G_{sv}^N
    G_v^N : 4-point vector-type interaction
    G_{sv}^N : 8-point scalar-vecter-type interaction [Koch et al.,'87]
- Influence on the chiral phase transition with G_{sv}^q
  - Introduction of G_{sv}^q
    Strengthening attractive quark-antiquark interaction
    (Role in pushing the chiral restoration point to higher density side)
    $\rightarrow$ Tuning parameter of the chiral symmetry restoration point
- Nucleon/Quark is treated as a fundamental fermion.
- Chiral symmetry is preserved.

Objective and Methods

Procedure for investigating

Assumption

- Hadronic phase side $\Rightarrow$ Symmetric nuclear matter
- Quark phase side $\Rightarrow$ Free quark phase (No quark-pair correlations)

Model application

- Symmetric nuclear matter
  $\Rightarrow$ The extended 2-flavor NJL model with G_v^N and G_{sv}^N
  $\Rightarrow$ Nucleon field is treated as a fundamental field with N_c^N=1.
- Quark matter
  $\Rightarrow$ The extended 2-flavor NJL model with G_{sv}^q
  $\Rightarrow$ Quark field is treated as a fundamental field with N_c^q=3.

Phase determination

- Comparison of the pressure of nuclear matter with that of quark matter
  $\Rightarrow$ The phase which has the largest pressure is physically realized.
  $\Rightarrow$ Phase diagram (Paying attention to the order of phase transition)

Outline

Formalism

Thermodynamics

(Extended NJL model + Mean field approximation)

Formalism

- Lagrangian density for nuclear and quark matters :

$$\mathcal{L}_i = \overline{\psi}_i i\gamma^\mu \partial_\mu \psi_i + G_s^i \left[(\overline{\psi}_i \psi_i)^2 + (\overline{\psi}_i i\gamma_5 \boldsymbol{\tau} \psi_i)^2 \right] \\ -G_v^i (\overline{\psi}_i \gamma^\mu \psi_i)^2 - G_{sv}^i \left[(\overline{\psi}_i \psi_i)^2 + (\overline{\psi}_i i\gamma_5 \boldsymbol{\tau} \psi_i)^2 \right] (\overline{\psi}_i \gamma^\mu \psi_i)^2$$

  - For nuclear matter $(i = N)$
    $\Rightarrow \ N_f^N = 2, \ N_c^N = 1, \ G_v^N \neq 0, \ G_{sv}^N \neq 0$
  - For quark matter $(i = q)$
    $\Rightarrow \ N_f^q = 2, \ N_c^q = 3, \ G_v^q = 0, \ G_{sv}^q \neq 0$

Formalism

- Mean field approximation :

$$\mathcal{L}_i^{MF} = \overline{\psi}_i(i\gamma^\mu\partial_\mu - m_i)\psi_i + \widetilde{\mu}_i\overline{\psi}_i\gamma^0\psi_i + C_i$$
$$\mathcal{H}_i^{MF} = -i\overline{\psi}_i\boldsymbol{\gamma}\cdot\nabla\psi_i + m_i\overline{\psi}_i\psi_i + \widetilde{\mu}_i\overline{\psi}_i\gamma^0\psi_i - C_i$$

with

$$C_i \equiv -G_s^i\langle\langle\overline{\psi}_i\psi_i\rangle\rangle^2 + G_v^i\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle^2 + 3G_{sv}^i\langle\langle\overline{\psi}_i\psi_i\rangle\rangle^2\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle^2$$
$$m_i = -2\left[G_s^i + 2G_{sv}^i\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle^2\right]\langle\langle\overline{\psi}_i\psi_i\rangle\rangle$$
$$\widetilde{\mu}_i = 2\left[G_v^i + 2G_{sv}^i\langle\langle\overline{\psi}_i\psi_i\rangle\rangle^2\right]\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle$$

MFA : $\overline{\psi}\Gamma\psi \to \langle\langle\overline{\psi}\Gamma\psi\rangle\rangle + (\overline{\psi}\Gamma\psi - \langle\langle\overline{\psi}\Gamma\psi\rangle\rangle)$

$\langle\langle\overline{\psi}_i\psi_i\rangle\rangle \neq 0$, $\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle \neq 0$, others $= 0$

(Fermion number density : $\rho_i \equiv \langle\langle\psi_i^\dagger\psi_i\rangle\rangle = \langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle$)

$\langle\langle\cdots\rangle\rangle$: The finite-temperature expectation value which represents thermal average.

Formalism

- Introduce the chemical potential μ_i :

$$\begin{aligned}\mathcal{H}'_i &= \mathcal{H}_i^{MF} - \mu_i \psi_i^\dagger \psi_i \\ &= -i\overline{\psi}_i \boldsymbol{\gamma}\cdot\nabla\psi_i + m_i\overline{\psi}_i\psi_i - \mu_i^r\overline{\psi}_i\gamma^0\psi_i - C_i\end{aligned}$$

$\Rightarrow$ The effective chemical potential μ_i^r :

$$\begin{aligned}\mu_i^r &= \mu_i - \widetilde{\mu}_i \\ &= \mu_i - 2\left[G_v^i + G_{sv}^i \langle\!\langle\overline{\psi}_i\psi_i\rangle\!\rangle^2\right]\langle\!\langle\overline{\psi}_i\gamma^0\psi_i\rangle\!\rangle\end{aligned}$$

Formalism

- Gap equation (Self-consistent equation for m_i) :

$$m_i = -2G_s^i \left[1 - \frac{G_{sv}^i}{G_s^i} \langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle^2\right] \langle\langle\overline{\psi}_i\psi_i\rangle\rangle$$

where

$$\langle\langle\overline{\psi}_i\psi_i\rangle\rangle = \nu_i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{m_i}{\sqrt{\boldsymbol{p}^2 + m_i^2}} (n_+^i - n_-^i)$$

$$\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle = \nu_i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} (n_+^i + n_-^i - 1)$$

with

$\nu_i = 2N_f^i N_c^i$ Degeneracy factor

$n_\pm^i = \left[e^{\beta(\pm\sqrt{\boldsymbol{p}^2+m_i^2}-\mu_i^r)} + 1\right]^{-1}$ Fermion number distribution function

$\beta = 1/T$ Temperature

$\mu_i^r = \mu_i - 2\left[G_v^i + G_{sv}^i \langle\langle\overline{\psi}_i\psi_i\rangle\rangle^2\right] \langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle$ Effective chemical potential

Formalism

- Pressure of nuclear and quark matters :

$$p_i(T,\mu_i) \;=\; -\left[\langle\langle \mathcal{H}_i^{MF}\rangle\rangle_{(T,\mu_i)} - \langle\langle \mathcal{H}_i^{MF}\rangle\rangle_{(T=0,\ \mu_i=m_i(T=0))}\right] + \mu_i\langle\langle \mathcal{N}_i\rangle\rangle + T\langle\langle S_i\rangle\rangle$$

where

$$\begin{aligned}
\langle\langle \mathcal{H}_i^{MF}\rangle\rangle &= \langle\langle \overline{\psi}_i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\psi_i\rangle\rangle - G_s^i\langle\langle\overline{\psi}_i\psi_i\rangle\rangle^2 \\
&\quad + G_v^i\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle^2 + G_{sv}^i\langle\langle\overline{\psi}_i\psi_i\rangle\rangle^2\langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle^2 \\
\langle\langle \mathcal{N}_i\rangle\rangle &= \langle\langle\overline{\psi}_i\gamma^0\psi_i\rangle\rangle = \rho_i \\
\langle\langle S_i\rangle\rangle &= -\nu_i\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\,\big[\,n_+^i\ln n_+^i + (1-n_+^i)\ln(1-n_+^i) \\
&\quad + n_-^i\ln n_-^i + (1-n_-^i)\ln(1-n_-^i)\,\big]
\end{aligned}$$

and

$$\langle\langle\overline{\psi}_i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\psi_i\rangle\rangle = \nu_i\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{\boldsymbol{p}^2}{\sqrt{\boldsymbol{p}^2+m_i^2}}(n_+^i - n_-^i)$$

Numerical Results

Parameter set, Results at finite temperature and density

Parameter set

- The parameter set for nuclear matter($i = N$) and quark matter ($i = q$)

[Y. Tsue, J. da Providência, C Providência and M. Yamamura, Prog. Theor. Phys. 123, (2010), 1013]

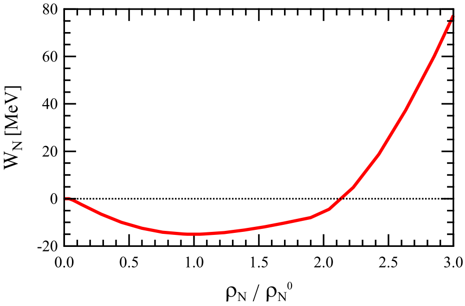
Λ_N	377.8 MeV	Λ_q	653.961 MeV
$G_s^N \Lambda_N^2$	19.2596	$G_s^q \Lambda_q^2$	2.13922
$G_v^N \Lambda_N^2$	-1069.89	$G_v^q \Lambda_q^2$	0
$G_{sv}^N \Lambda_N^8$	17.9824	$G_{sv}^q \Lambda_q^8$	free*)

$*)$ G_{sv}^q=0, $G_{sv}^q \Lambda_q^8$=-68.4, $G_{sv}^q \Lambda_q^8$=-81.9 [T.-G. Lee, et al., arXiv:1207.1499]

Reproduction of nuclear saturation property

- Energy density per single nucleon W_N

vs Normal nuclear density ρ_N/ρ_N^0



- Incompressibility of nuclear matter

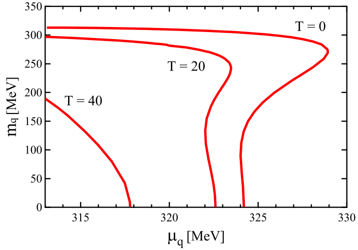
$$K = 9\rho_N^0 \frac{\partial^2 W_N(\rho_N)}{\partial \rho_N^2}\Bigg|_{\rho_N=\rho_N^0} \simeq 260 \text{ MeV}$$

Numerical Results

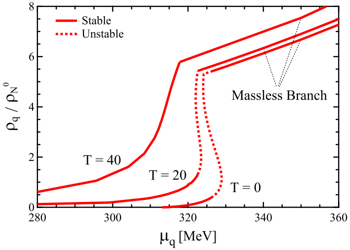
with $G_{sv}^{q}\Lambda_{q}^{8} = -68.4$

Numerical Results with $G_{sv}^{q}\Lambda_{q}^{8} = -68.4$

- Dynamical quark mass m_q vs Quark chemical potential μ_q



- Quark number density ρ_q vs Quark chemical potential μ_q



▷ Unphysical regions which have unstable solutions

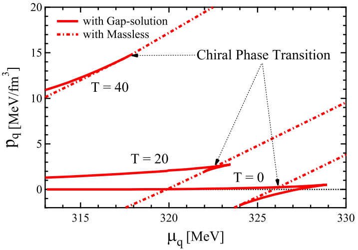
Comparison of pressure

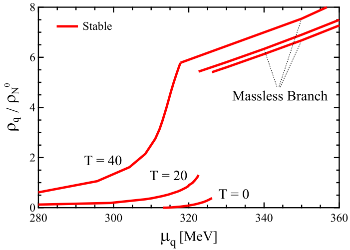
$\Longrightarrow$ Determine the physically realized solution (stable solution)

▷ The solution with largest pressure = The physically realized solution

Numerical Results with $G_{sv}^q \Lambda_q^8 = -68.4$

- Pressure of quark matter p_q vs Quark chemical potential μ_q
- Quark number density ρ_q vs Quark chemical potential μ_q

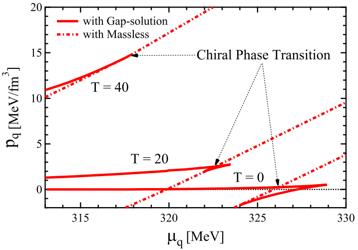


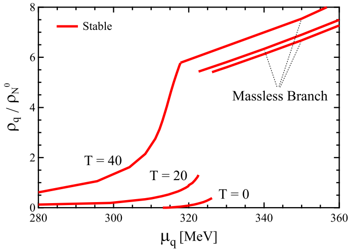


- $T = 0$ MeV
  - $\mu_q^{\text{chiral}} \approx 326$ MeV : Chiral phase transition
  - $\rho_q^{\text{coex}} = 0.38\rho_N^0 \sim 5.41\rho_N^0$: 1$^{\text{st}}$-order phase transition
    $(\rho_B = 0.13\rho_N^0 \sim 1.80\rho_N^0)$

Numerical Results with $G_{sv}^{q}\Lambda_q^8 = -68.4$

- Pressure of quark matter p_q vs Quark chemical potential μ_q
- Quark number density ρ_q vs Quark chemical potential μ_q





▷ $T = 20$ MeV

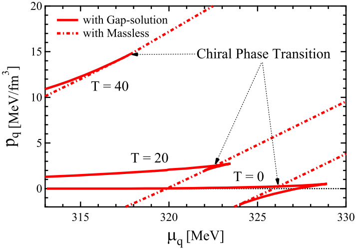
- $\mu_q^{\text{chiral}} \approx 323$ MeV : Chiral phase transition
- $\rho_q^{\text{coex}} = 1.30\rho_N^0 \sim 5.41\rho_N^0$: 1$^{\text{st}}$-order phase transition

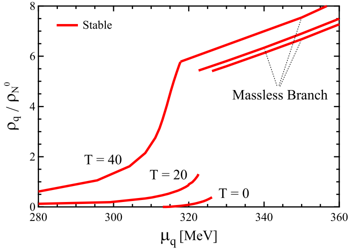
  $(\rho_B = 0.43\rho_N^0 \sim 1.80\rho_N^0)$

Numerical Results with $G_{sv}^q \Lambda_q^8 = -68.4$

▶ Pressure of quark matter p_q vs Quark chemical potential μ_q

▷ Quark number density ρ_q vs Quark chemical potential μ_q





▷ $T = 40$ MeV

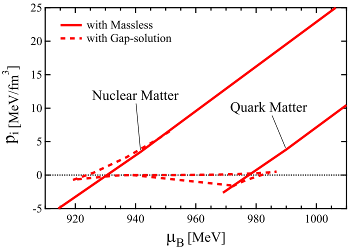
- $\mu_q^{\text{chiral}} \approx 318$ MeV : Chiral phase transition
- $\rho_q^{\text{chiral}} \sim 5.78\rho_N^0$: 2nd-order phase transition
  $(\rho_B \sim 1.93\rho_N^0)$

Quark-Hadron phase transition

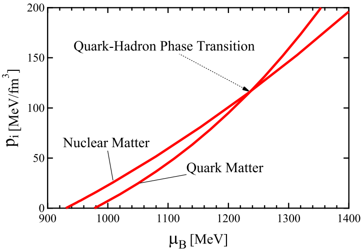
Numerical Results with $G^q_{sv}\Lambda^8_q = -68.4$

Numerical Results with $G^q_{sv}\Lambda^8_q = -68.4$

- Pressure p_i [Lower-μ_B side] vs Baryon number density μ_B



- Pressure p_i [Higher-μ_B side] vs Baryon number density μ_B



▷ The condition for thermodynamic equilibrium between the hadron and quark phases*) :

$$p_N(T,\mu_N) \;=\; p_q(T,3\mu_q)$$

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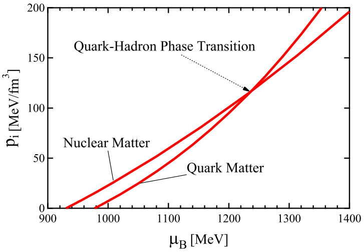
*) The condition for chemical equilibrium (conservation of baryon number) : $\mu_N(T) = 3\mu_q(T) \;\Leftrightarrow\; \mu_B(T)$

Numerical Results with $G_{sv}^q \Lambda_q^8 = -68.4$

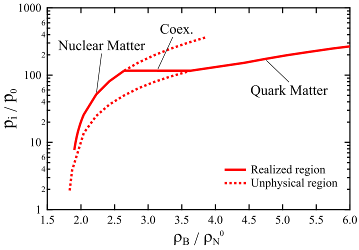
- ▶ Pressure p_i

  vs Baryon number density μ_B



- ▷ Pressure p_i/p_0

  vs Baryon number density μ_B/ρ_N^0



▷ $T = 0$ MeV

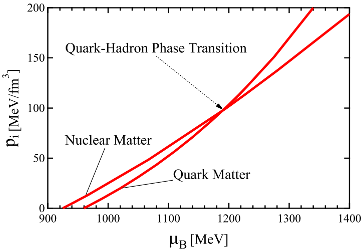
- $\mu_B^{\rm QH} \approx 1236$ MeV : Quark-Hadron phase transition
- $\rho_B^{\rm coex} = 2.64\rho_N^0 \sim 3.63\rho_N^0$: 1st-order phase transition

  $(\rho_N = 2.64\rho_N^0 \sim \rho_q = 10.9\rho_N^0)$

Numerical Results with $G_{sv}^q \Lambda_q^8 = -68.4$

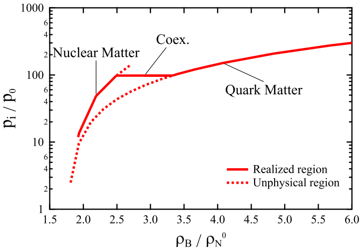
▶ Pressure p_i

vs Baryon number density μ_B



▷ Pressure p_i/p_0

vs Baryon number density μ_B/ρ_N^0



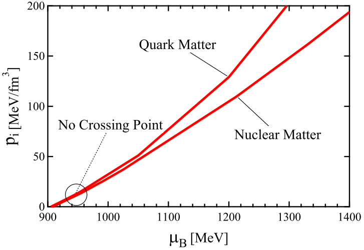
▷ $T = 20$ MeV

- $\mu_B^{\mathrm{QH}} \approx$ 1190 MeV : Quark-Hadron phase transition
- $\rho_B^{\mathrm{coex}} = 2.49\rho_N^0 \sim 9.99\rho_N^0$: 1[st]-order phase transition

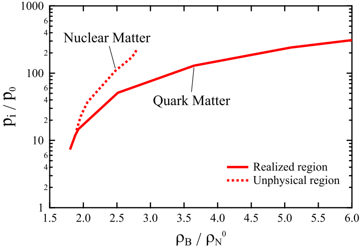
  $(\rho_N = 2.49\rho_N^0 \sim \rho_q = 3.33\rho_N^0)$

Numerical Results with $G_{sv}^q \Lambda_q^8 = -68.4$

- Pressure p_i vs Baryon number density μ_B



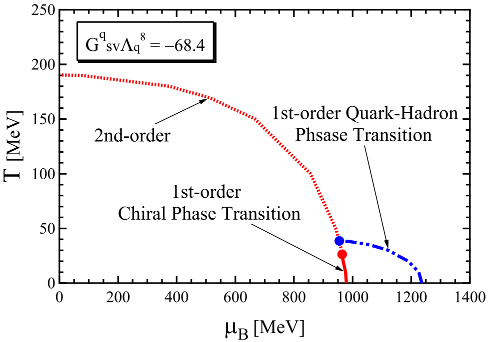
- Pressure p_i/p_0 vs Baryon number density μ_B/ρ_N^0



- $T = 40$ MeV
  - There is no crossing point.
    - $\Rightarrow$ 1$^{\text{st}}$-order quark-hadron phase transition disappears.

Phase diagram (μ_B, T)

- Phase diagram with $G_{sv}^q \Lambda_q^8 = -68.4$

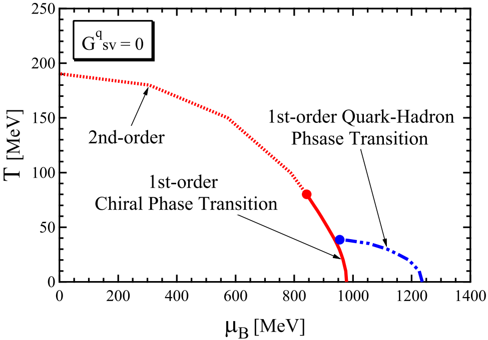


▷ Low-T : 1st-order chiral phase transition → 1st-order quark-hadron phase transition
▷ High-T : 2nd-order chiral phase transition
▷ Moderate-μ_B : Chiral restoration + Nucleonic/Hadronic excitation
(Chiral symmetric nuclear phase ⇒ Quarkyonic-like phase ?)

G_{sv}^{q}-dependence of phase diagram

G_{sv}^q-dependence of phase diagram

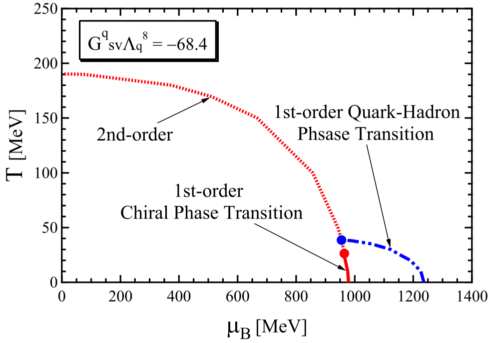
- Phase diagram without scalar-vector interaction



▷ 1[st]-order chiral phase transition : $(\mu_B, T) \simeq (978, 0) \rightarrow (842, 80)$ MeV
▷ 2[nd]-order chiral phase transition : $(\mu_B, T) \simeq (842, 80) \rightarrow (0, 190)$ MeV
▷ 1[st]-order quark-hadron transition : $(\mu_B, T) \simeq (1236, 0) \rightarrow (955, 39)$ MeV

G^q_{sv}-dependence of phase diagram

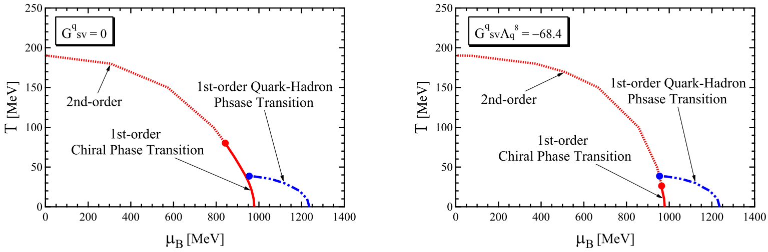
- Phase diagram with scalar-vector interaction



- ▷ 1^{st}-order chiral phase transition : $(\mu_B, T) \simeq (979, 0) \rightarrow (964, 26)$ MeV
- ▷ 2^{nd}-order chiral phase transition : $(\mu_B, T) \simeq (964, 26) \rightarrow (0, 190)$ MeV
- ▷ 1^{st}-order quark-hadron transition : $(\mu_B, T) \simeq (1236, 0) \rightarrow (955, 39)$ MeV

G^q_{sv}-dependence of phase diagram

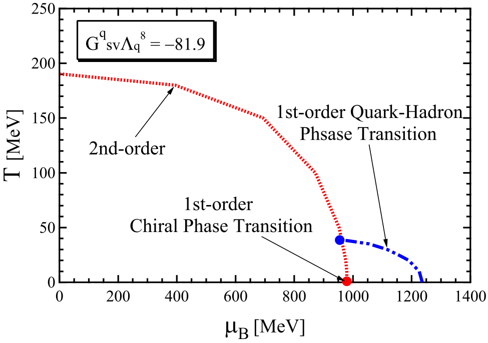
- Phase diagram without and with scalar-vector interaction



▷ 1st-order chiral phase transition : Shrinks with increasing G^q_{sv}.
▷ 2nd-order chiral phase transition : Tends to bloat outward.
▷ 1st-order quark-hadron transition : There is no change.

G^q_{sv}-dependence of phase diagram

- Phase diagram with stronger scalar-vector interaction



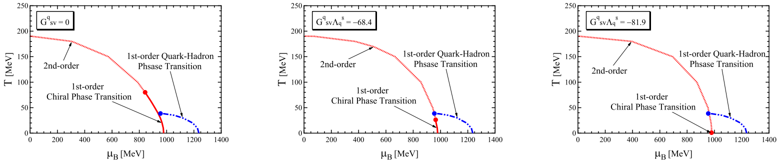
- ▷ 1^{st}-order chiral phase transition : $(\mu_B, T) \simeq (979, 0) \rightarrow (979, 1)$ MeV
- ▷ 2^{nd}-order chiral phase transition : $(\mu_B, T) \simeq (979, 1) \rightarrow (0, 190)$ MeV
- ▷ 1^{st}-order quark-hadron transition : $(\mu_B, T) \simeq (1236, 0) \rightarrow (955, 39)$ MeV

Summary and Future Work

Summary and Future Work

Summary

- ▷ The quark-hadron phase transition at finite temperature and density is investigated in an Extended NJL model.
- ▷ The extended NJL phase diagram with G^q_{sv} is obtained in the T-μ_B plane.
  - Phase diagram with $G^q_{sv}\Lambda^q = -68.4$
    - 1st-order quark-hadron phase transition
    - 1st and 2nd-order chiral phase transition
    - The phase in which chiral symmetry is restored but the elementary excitation modes are nucleonic appears just before deconfinement.
      $\Rightarrow$ Quarkyonic phase? [McLerran et al.,'07]



Summary and Future Work

Summary

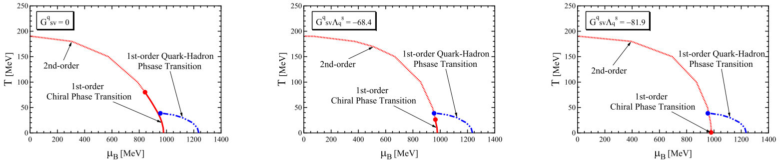
- Influence on the phase diagram with G^q_{sv}
  - Does not affect to the 1st-order quark-hadron phase transition.

    $\Rightarrow$ The phase boundary is not changed. (G^q_{sv}-independence)
  - Affects the chiral phase transition. (G^q_{sv}-dependence)

    $\Rightarrow$ Critical line of 1st-order shrinks with increasing G^q_{sv}

    $\Rightarrow$ Moves the tricritical point toward a larger μ_B and a lower T.

    $\Rightarrow$ The endpoint of 1st-order quark-hadron phase transition is located on the critical line of chiral phase transition in the case of $G^q_{sv}\Lambda^q = -68.4$.



Summary and Future Work

Future Work

- Consideration of the color-superconducting phase
  $\Rightarrow$ Pairing interaction (Nuclear superfluidity in nuclear side, CSC in quark side)
- Consideration of the neutron star matter
  $\Rightarrow$ Phase transition between neutron matter and quark matter.

($\rightarrow$ Physics of neutron star)

Thank you for your attention!

Back up

Parameter set

- Nuclear matter
  - Model parameters : G_s^N, G_v^N, G_{sv}^N, Λ_N 3-momentum cutoff : Λ_i
  - Conditions :

$m_N(\rho_N{=}0) = 939$ MeV

$\rho_N^0{=}0.17$ fm^{-3} Normal nuclear density

$m_N(\rho_N{=}\rho_N^0) = 0.6 m_N(\rho_N{=}0)$ MeV

$W_N(\rho_N{=}\rho_N^0) = -15$ MeV

Binding energy per single nucleon : $W_N(\rho_N) = \frac{\langle \mathcal{H}_i^{MF}\rangle(\rho_N) - \langle \mathcal{H}_i^{MF}\rangle(\rho_N{=}0)}{\rho_N} - m_N(\rho_N{=}0)$

- Quark matter
  - Model parameters : G_s^q, Λ_q
  - Conditions :

$m_q(\rho_q{=}0) = 313$ MeV Dynamical quark mass

$f_\pi^2 = 93$ MeV Pion decay constant

  - Free parameter : G_{sv}^q
  - Conditions :

$m_q(\rho_q/3{=}\rho_N^0) = 0.625 m_q(\rho_q{=}0)$ MeV $\rightarrow G_{sv}^q \Lambda_q^8 = -68.4$

$m_q(\rho_q/3{=}\rho_N^0) = 0.63 m_q(\rho_q{=}0)$ MeV $\rightarrow G_{sv}^q \Lambda_q^8 = -81.9$

G_{sv}^{q}-independence of the quark-hadron phase transition

G_{sv}^{q}-independence

- There is no influence on the 1st-order quark-hadron phase transition.
  - 1st-order quark-hadron phase transition occurs after the chiral restoration.

    $$\Rightarrow \ m_q = 0, \ \langle\langle \overline{\psi}_q \psi_q \rangle\rangle = 0$$

  - G_{sv}^{q}-independence of pressure p_q

$$p_q(T, \ \mu_q) = -\left[\langle\langle \mathcal{H}_q^{MF} \rangle\rangle_{(T,\mu_q)} - \langle\langle \mathcal{H}_q^{MF} \rangle\rangle_{(T=0, \ \rho_q=0)}\right] + \mu_q \langle\langle \mathcal{N}_q \rangle\rangle + T \langle\langle S_q \rangle\rangle$$

$$\langle\langle \mathcal{H}_q^{MF} \rangle\rangle = \langle\langle \overline{\psi}_q (\boldsymbol{\gamma} \cdot \boldsymbol{p}) \psi_q \rangle\rangle - G_s^q \langle\langle \overline{\psi}_q \psi_q \rangle\rangle^2 + G_v^q \langle\langle \overline{\psi}_q \gamma^0 \psi_q \rangle\rangle^2 + G_{sv}^q \langle\langle \overline{\psi}_q \psi_q \rangle\rangle^2 \langle\langle \overline{\psi}_q \gamma^0 \psi_q \rangle\rangle^2$$

$$\mu_q^r = \mu_q - 2\left[G_v^q + G_{sv}^q \langle\langle \overline{\psi}_q \psi_q \rangle\rangle^2\right] \langle\langle \overline{\psi}_q \gamma^0 \psi_q \rangle\rangle$$

$\Rightarrow \langle\langle \mathcal{H}_i^{MF} \rangle\rangle$ and μ_q^r <u>do not depend on G_{sv}^q</u> due to $\langle\langle \overline{\psi}_q \psi_i \rangle\rangle = 0$.

Themodynamic potential density

- Thermodynamic potential density :

$$\omega_i \;=\; \langle\langle \mathcal{H}_i^{MF} \rangle\rangle - \mu_i \langle\langle \mathcal{N}_i \rangle\rangle - T \langle\langle S_i \rangle\rangle$$

where

$$\begin{aligned}\langle\langle \mathcal{H}_i^{MF} \rangle\rangle \;&=\; \langle\langle \overline{\psi}_i (\boldsymbol{\gamma}\cdot\boldsymbol{p})\psi_i \rangle\rangle - G_s^i \langle\langle \overline{\psi}_i \psi_i \rangle\rangle^2 \\ &\quad + G_v^i \langle\langle \overline{\psi}_i \gamma^0 \psi_i \rangle\rangle^2 + G_{sv}^i \langle\langle \overline{\psi}_i \psi_i \rangle\rangle^2 \langle\langle \overline{\psi}_i \gamma^0 \psi_i \rangle\rangle^2 \\ \langle\langle \mathcal{N}_i \rangle\rangle \;&=\; \langle\langle \overline{\psi}_i \gamma^0 \psi_i \rangle\rangle \;=\; \rho_i \\ \langle\langle S_i \rangle\rangle &= -\nu_i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left[\, n_+^i \ln n_+^i + (1-n_+^i)\ln(1-n_+^i) \right. \\ &\quad \left. + n_-^i \ln n_-^i + (1-n_-^i)\ln(1-n_-^i) \,\right]\end{aligned}$$

and

$$\langle\langle \overline{\psi}_i (\boldsymbol{\gamma}\cdot\boldsymbol{p})\psi_i \rangle\rangle = \nu_i \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\boldsymbol{p}^2}{\sqrt{\boldsymbol{p}^2 + m_i^2}} (n_+^i - n_-^i)$$

Minimize ω_i w.r.t m_i and $n_i^{\pm}$ $\Rightarrow$ Gap eq. and Fermion number distribution func.

Pressure

- Pressure of nuclear and quark matters :

$$p_i(T,\mu_i) = -\left[\langle\!\langle \mathcal{H}_i^{MF} \rangle\!\rangle_{(T,\mu_i)} - \langle\!\langle \mathcal{H}_i^{MF} \rangle\!\rangle_{(T=0,\ \mu_i=m_i(T=0))}\right] + \mu_i \langle\!\langle \mathcal{N}_i \rangle\!\rangle + T\langle\!\langle S_i \rangle\!\rangle$$

where

$$\langle\!\langle \mathcal{H}_i^{MF} \rangle\!\rangle_{(T=0,\ \mu_i=m_i(T=0))} = \langle \overline{\psi}_i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\psi_i \rangle - G_s \langle \overline{\psi}_i \psi_i \rangle^2$$

$\langle \cdots \rangle$: Zero-temperature expectation value

$$n_+^i(T=0) = \theta(\mu_i^r - \sqrt{\boldsymbol{p}^2 + m_i^2}) \qquad \text{Heaviside step function}$$

$$= \begin{cases} 1 & (\boldsymbol{p} < \sqrt{\mu_i^{r\,2} - m_i^2} \equiv \boldsymbol{p}_F^i) \\ 0 & (\boldsymbol{p} > \sqrt{\mu_i^{r\,2} - m_i^2} \equiv \boldsymbol{p}_F^i) \end{cases}$$

$\boldsymbol{p}_F^i$: Fermi momentum

$$n_-^i(T=0) = 1$$