# Effects of fluctuations for QCD phase diagram with isospin chemical potential

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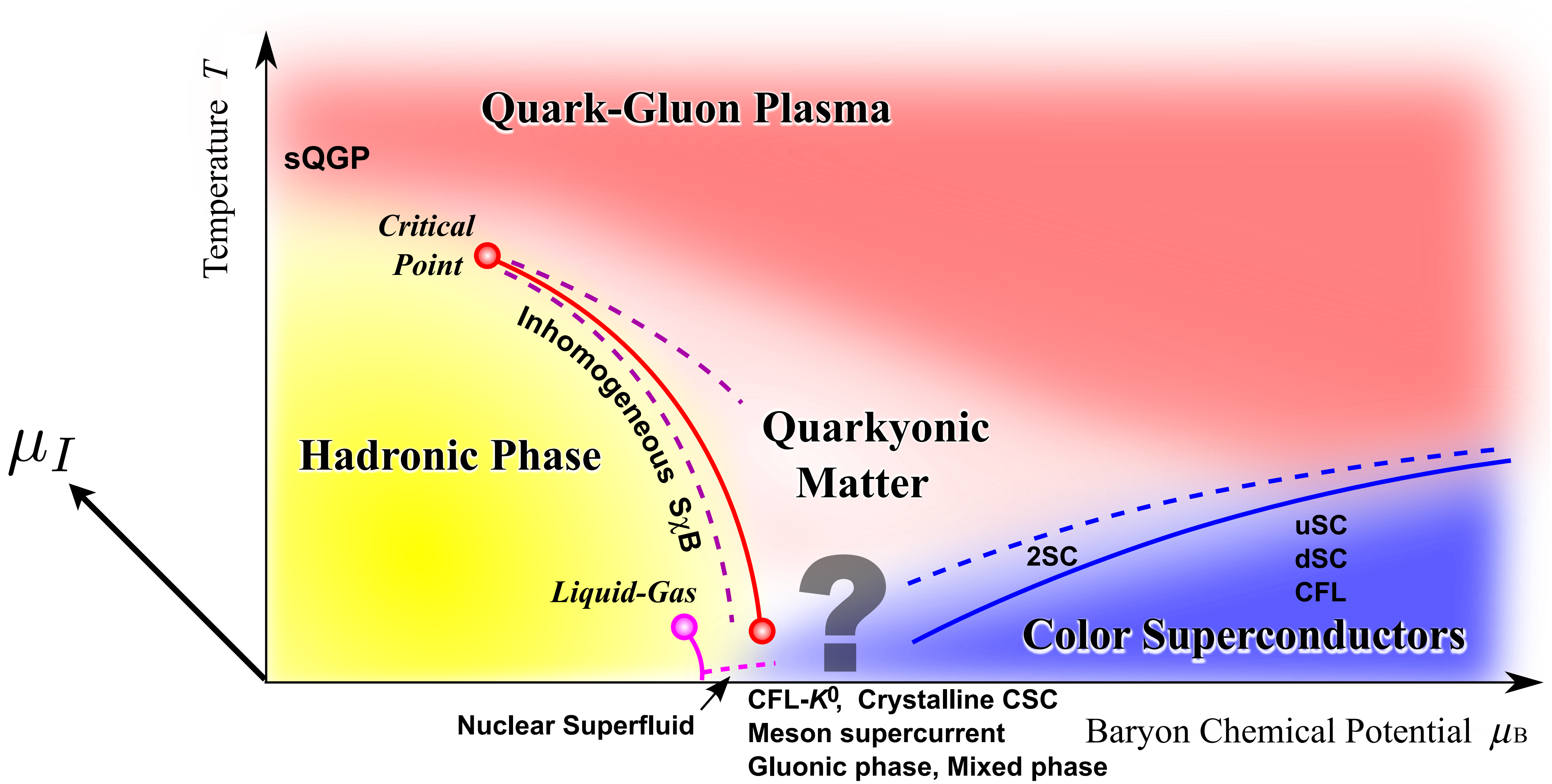
# QCD phase diagram




Fukushima and Hatsuda(2010)

- Rich structure is expected.
- The sign problem exists for finite baryon-chemical potential thus Lattice calculation is not available.

# Isospin chemical potential

- Three-dimensional phase diagram
  Temperature [T]
  Quark-chemical potential [$\mu$]
  Isospin-chemical potential [$\mu_I$]

$$\mu_{\rm u} = \mu + \mu_I$$

$$\mu_{\rm d} = \mu - \mu_I$$

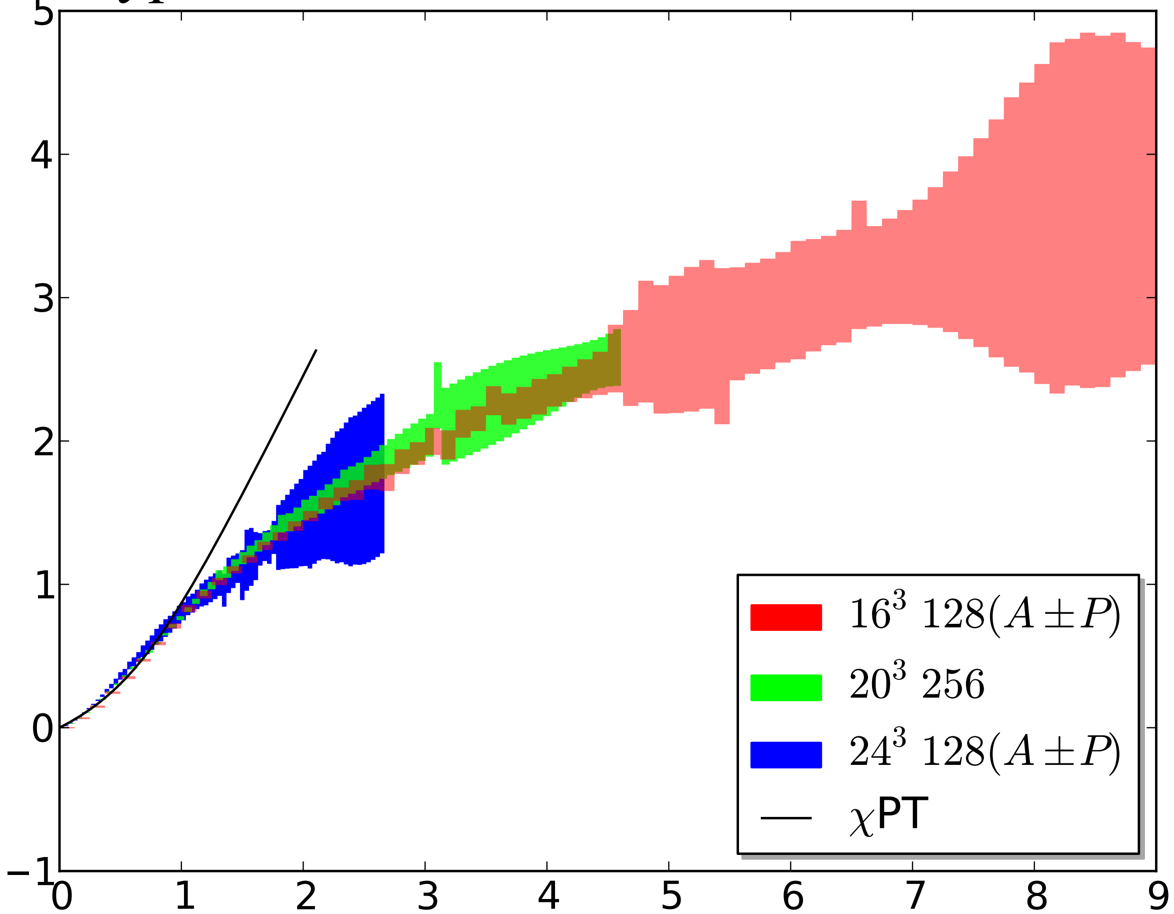
- Quark Determinant is real ($\mu$=0).
- The important sampling method is available .

M. Alford, A. Kapustin and F. Wilczek, Phys. Rev. D59, 054502 (1999).

# Property at T = 0 (Silver blaze)

a typical lattice result

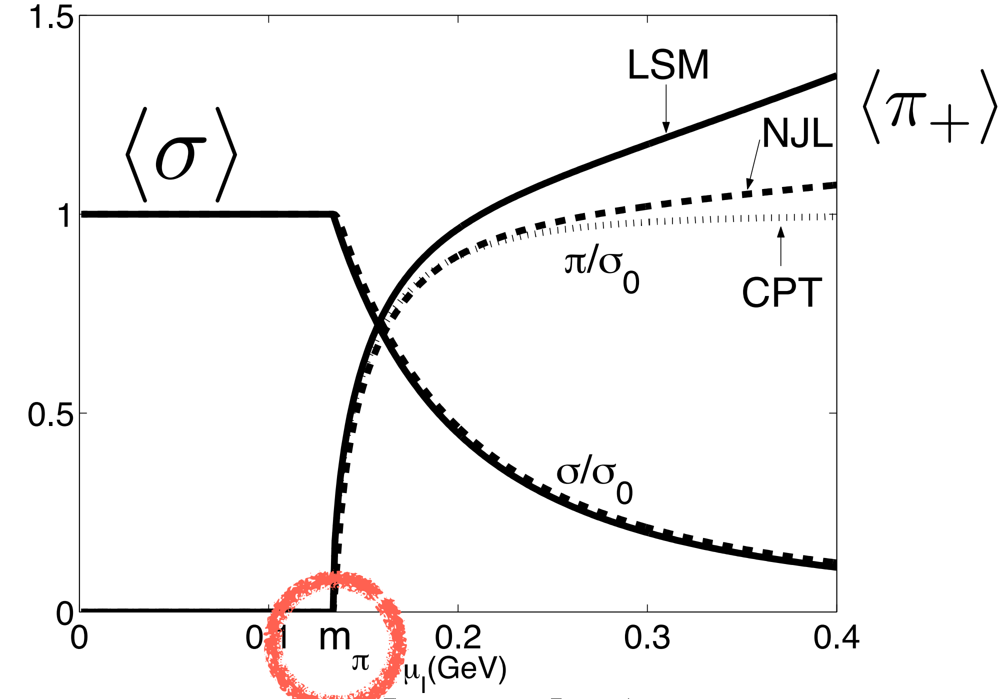
$$\frac{\mu_I - m_\pi}{m_\pi}$$




$$\rho_I = -\frac{\partial P}{\partial \mu_I}$$

W. Detmold, K. Orginos and Z. Shi, arXiv:1205.4224

order parameter




$\mu_I$ [GeV]

L. He, M. Jin, and P. Zhuang, Phys. Rev. D 71 (2005)

- At T = 0, nothing happens before $\mu_I$ reaches to the mass of the lightest charged particle (charged pion).

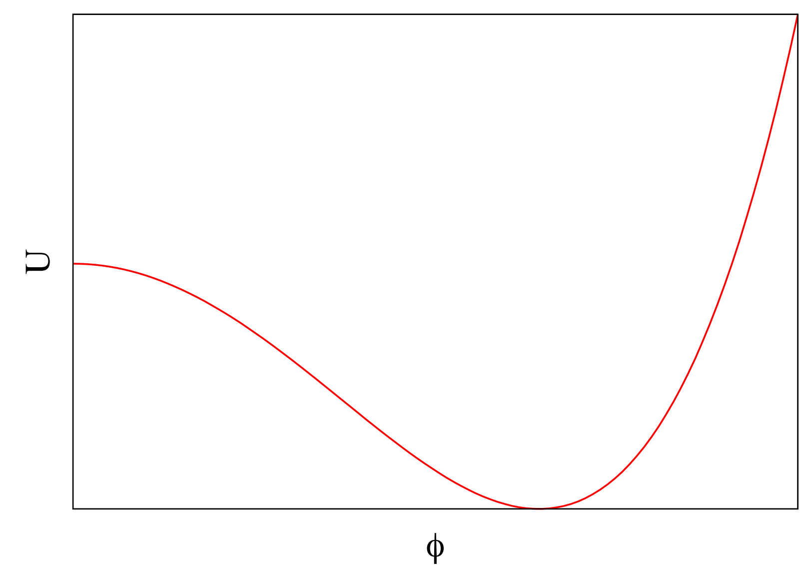
T. D. Cohen, Phys. Rev. Lett. 91, 222001 (2003) ***"Silver blaze" Arthur Conan Doyle***

# Fluctuations beyond mean-field

$$\phi' = \phi - \langle\phi\rangle \qquad \langle\phi\rangle = \frac{1}{2}\sqrt{\frac{\mu^2}{\lambda}}$$

$$U = -\frac{1}{2}\mu^2\phi^2 + \lambda\phi^4$$
$$= a\phi' + b\phi'^2 + c\phi'^3 + d\phi'^4$$

$$b \geq 0$$




- Neglect Φ'[3] and Φ'[4] terms (mean-field approximation.) breaks down in a critical region (b ~ 0)
- Include the effects of *c* or *d* by solving functional-RG

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quark-meson model with $\mu_I$ $\qquad \langle\sigma\rangle \neq 0 \qquad \langle\pi_+\rangle \neq 0$

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ + g(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}) + \mu_I\gamma_0\tau_3\right]\psi$$
$$+ \frac{1}{2}\partial\sigma\partial\sigma + \frac{1}{2}\partial\pi_0\partial\pi_0 + \vec{\partial}\pi_+\vec{\partial}\pi_+ + \vec{\partial}\pi_-\vec{\partial}\pi_- + (\partial_0 + 2\mu_I)(\pi_+ + i\pi_-)(\partial_0 - 2\mu_I)(\pi_+ - i\pi_-)$$
$$+ U(\sigma^2 + \vec{\pi}^2) - c\sigma$$

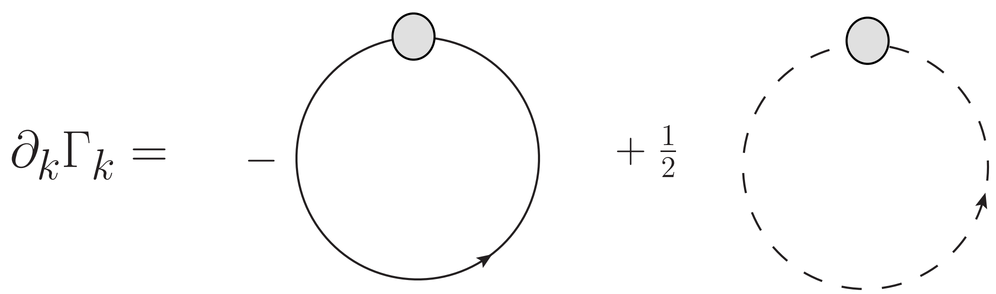
# Functional Renormalization Group (FRG)

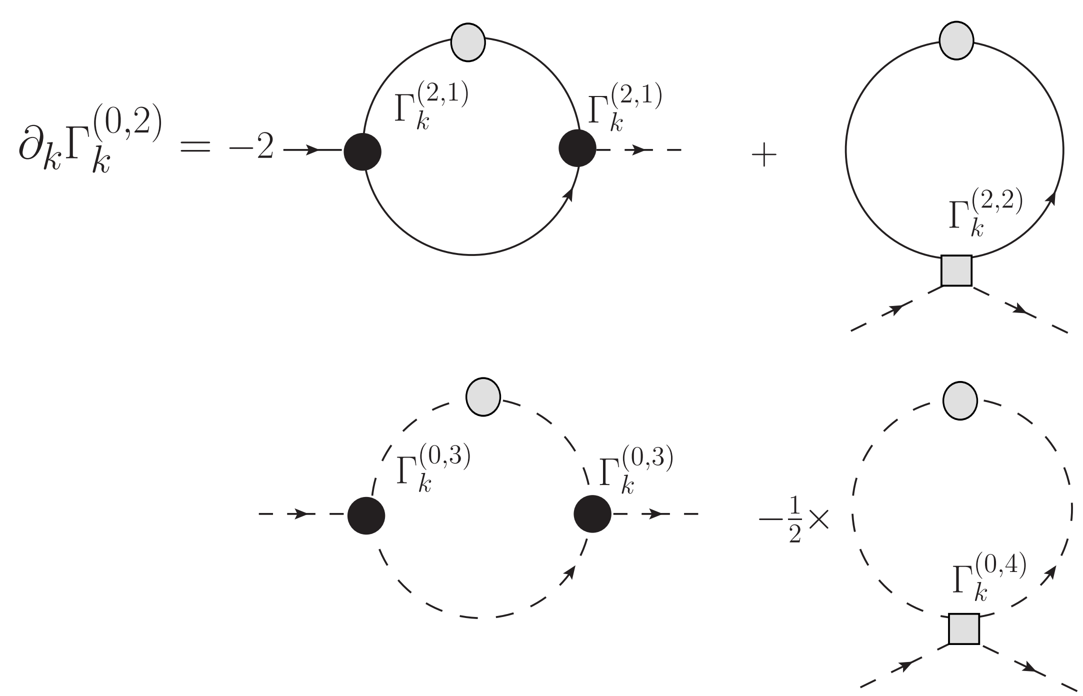
$$k\partial_k\Gamma_k[\varphi] = -\mathrm{Tr}\left[\frac{k\partial_k R_{kF}}{R_{kF}+\Gamma_k^{(2,0)}[\varphi]}\right]\frac{1}{2}\mathrm{Tr}\left[\frac{k\partial_k R_{kB}}{R_{kB}+\Gamma_k^{(0,2)}[\varphi]}\right]$$

C. Wetterich, Phys. Lett. B301, 90 (1993)

$\Gamma_k[\phi]$: effective potential at scale $k$

$$\begin{cases}\Gamma_{k=\Lambda}[\phi] = S[\phi] \text{ classical} \\ \downarrow \\ \Gamma_{k=0}[\phi] = \Gamma[\phi] \text{ quantum}\end{cases}$$




local potential approximation

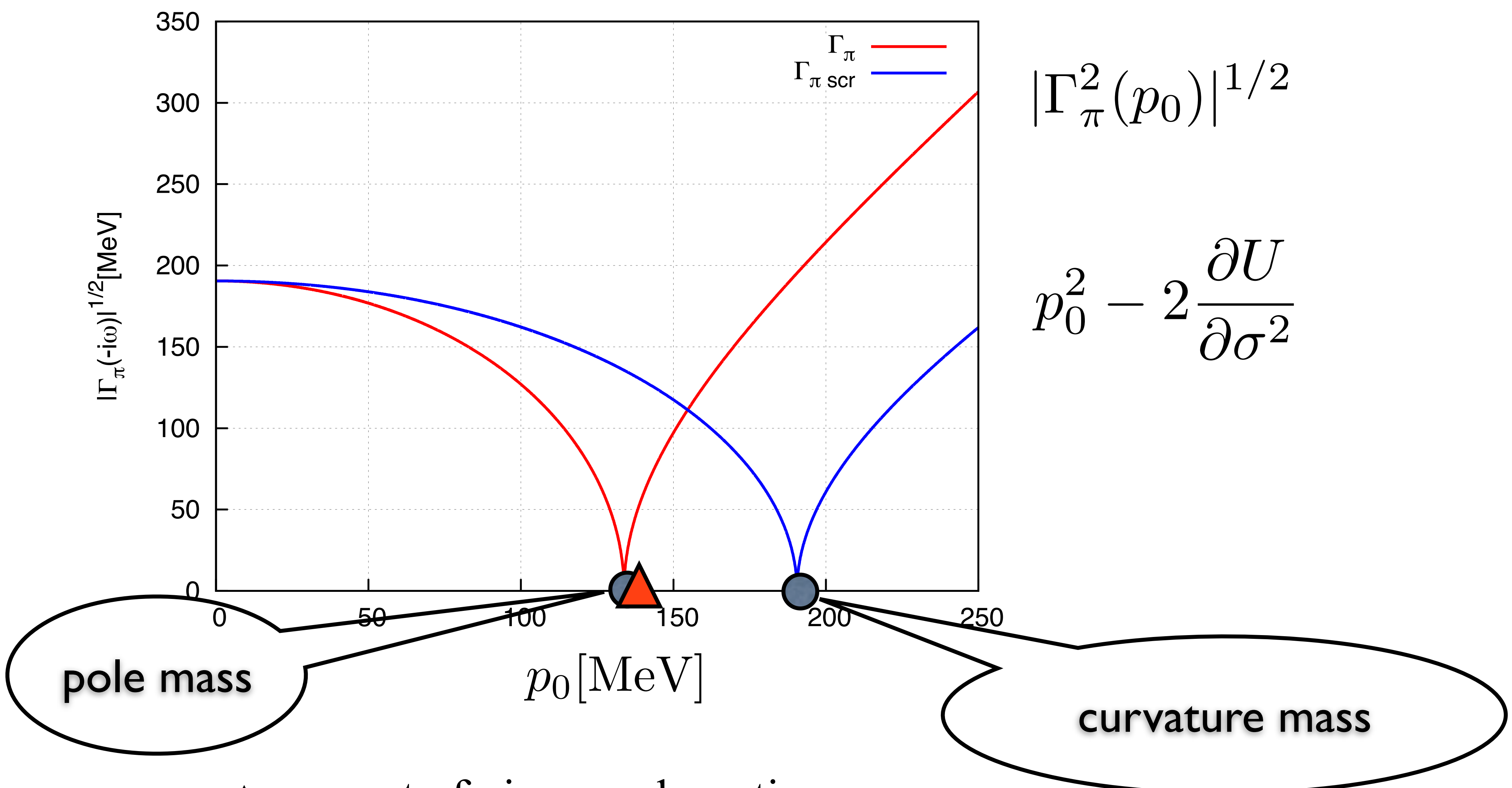
$$\Gamma_k^{LPA} = \text{Kinetic part} + \mathrm{U_k}(\sigma^2+\pi_0^2, \pi_+^2+\pi_-^2) - c\sigma$$

$$\Gamma_{abi}^{(0,3)} \to \frac{\partial^3\Gamma_{\mathrm{LPA}}}{\partial\phi_i\partial\phi_a\partial\phi_b}, \quad \Gamma_{abij}^{(0,4)} \to \frac{\partial^4\Gamma_{\mathrm{LPA}}}{\partial\phi_i\partial\phi_j\partial\phi_a\partial\phi_b},$$

$$\Gamma_i^{(2,1)} \to \frac{\partial^3\Gamma_{\mathrm{LPA}}}{\partial\bar\psi\partial\psi\partial\phi_i}, \quad \Gamma_{ij}^{(2,2)} \to \frac{\partial^4\Gamma_{\mathrm{LPA}}}{\partial\bar\psi\partial\psi\partial\phi_i\partial\phi_j}.$$

# Pion masses

Real part of pion 2-point function

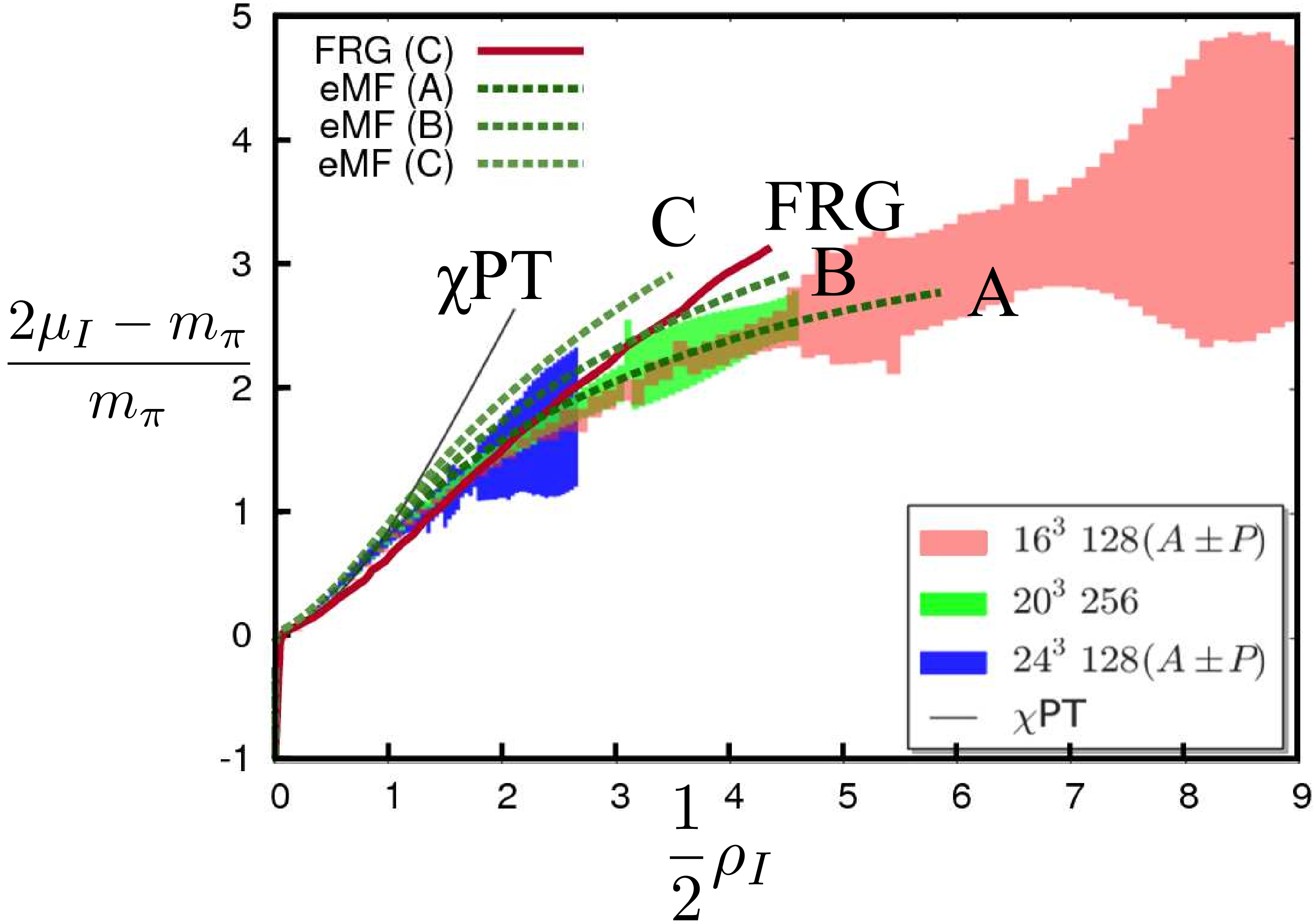



▲ :onset of pion condensation

- Pion pole mass and pion curvature mass are difference at 20%.
- The pole mass well agree the onset of pion condensation (the difference is just at 3%).

# Isospin density (T=0, μ=0)

the lattice data is from W. Detmold, K. Orginos and Z. Shi, arXiv:1205.4224 [hep-lat]
χPT calculation is from D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **86**, 592 (2001)




| | A | B | C | FRG |
|---|---|---|---|---|
| $M_\sigma$ [MeV] | 457 | 504 | 698 | 524 |

analytic form for linear sigma model

$$\rho_I(x,y) = 2f_\pi^2 m_\pi\, x \left(\frac{y^2-3}{y^2-1} - \frac{1}{x^4} + \frac{2}{y^2-1}x^2\right)$$

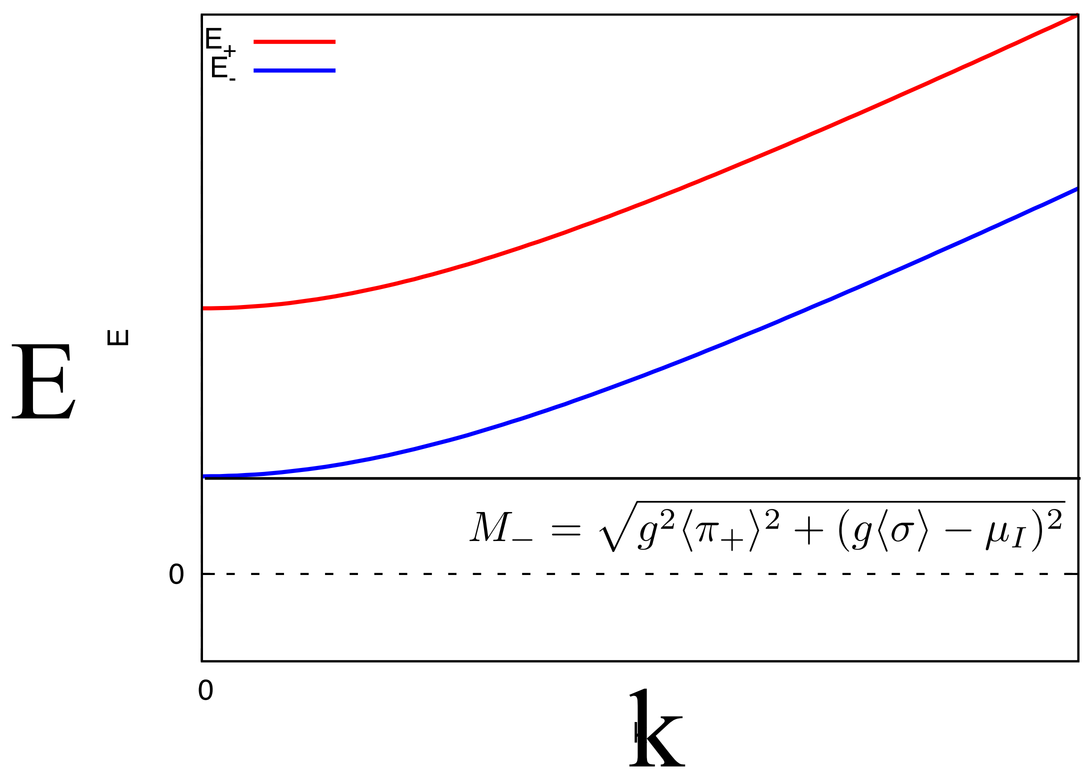
$$x = 2\mu_I/m_\pi, \quad y = m_\sigma/m_\pi$$

- Both QM and χPT models reproduce the charge density of the LQCD near the onset of the pion condensation.
- The difference comes from the mass of sigma.

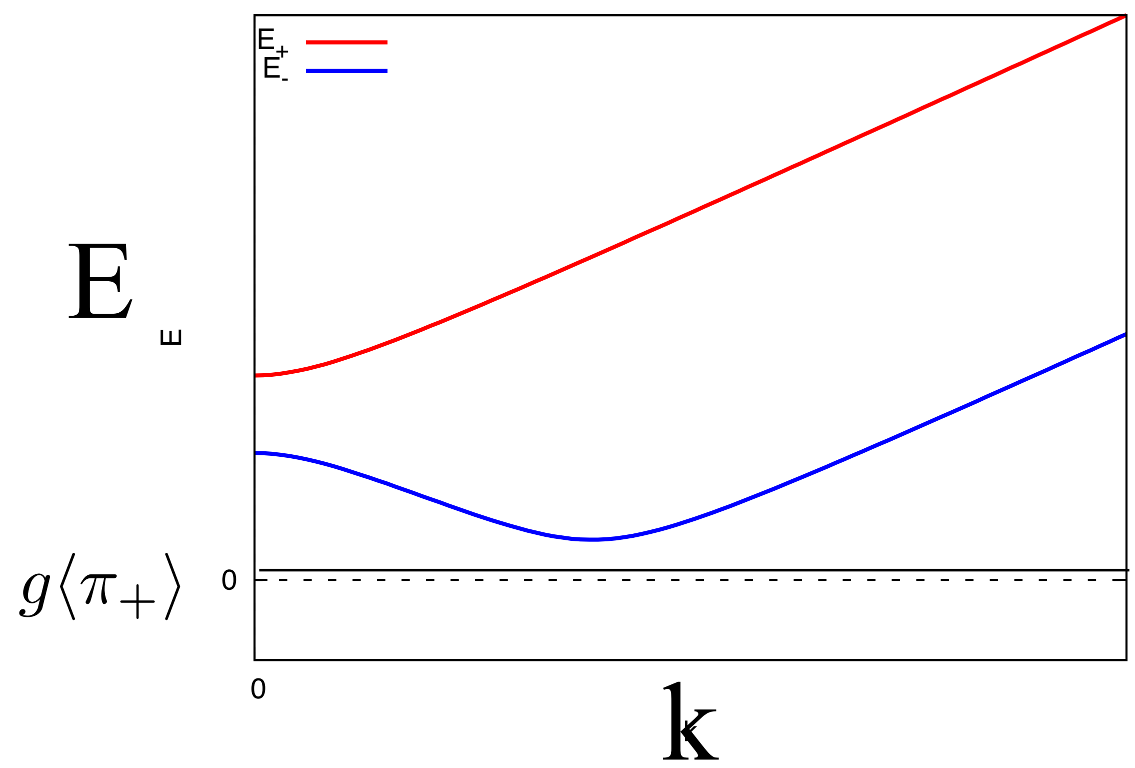
# Quarks in pion condensation phase

$$E_\pm(\vec{k}) = \sqrt{g^2\langle\pi_+\rangle^2 + (\sqrt{\vec{k}^2 + g^2\langle\sigma\rangle^2} \pm \mu_I)^2}$$

$g\langle\sigma\rangle > \mu_I$



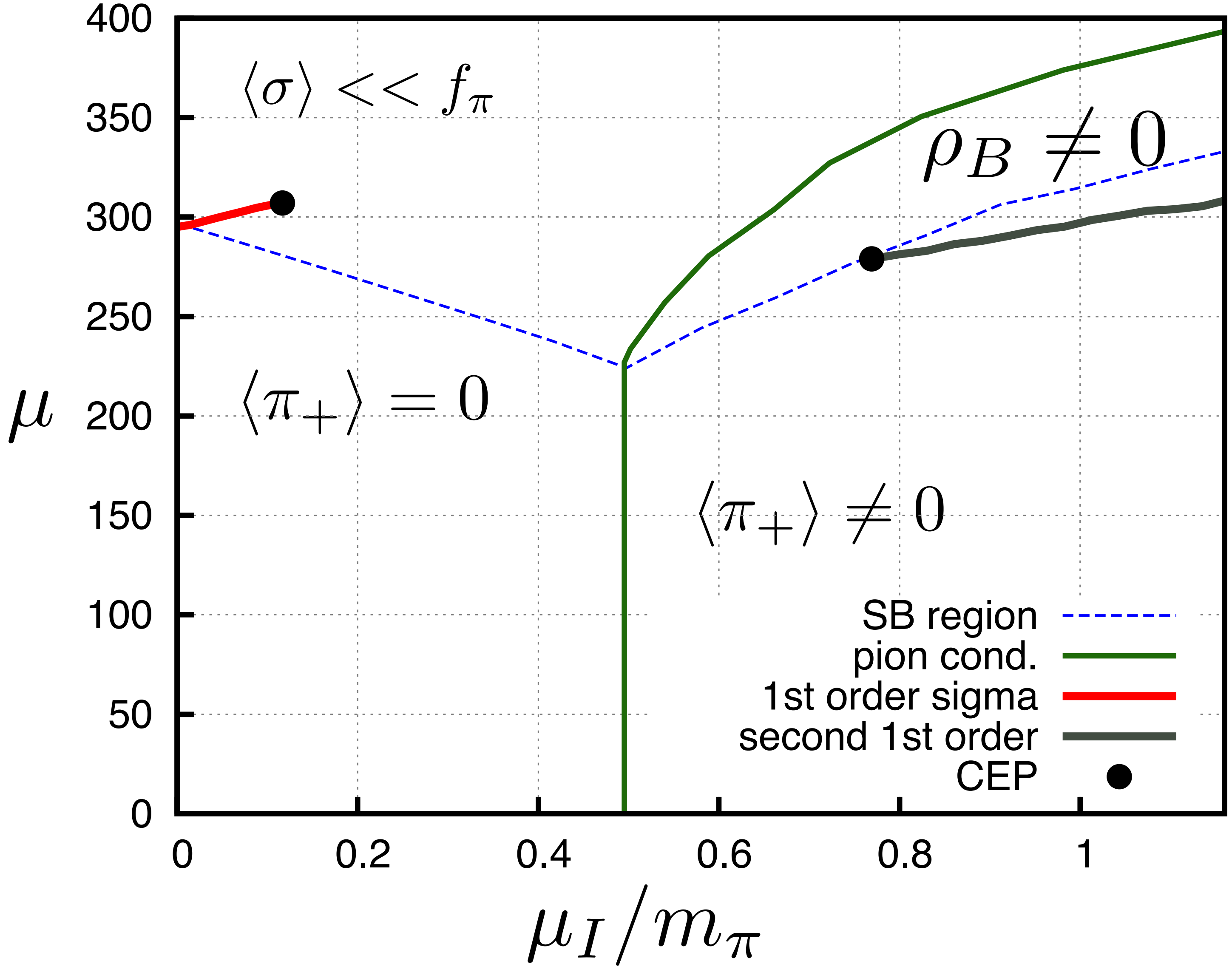

$g\langle\sigma\rangle < \mu_I$




- quark dispersion relation in pion condensation phase
- up and down quarks are mixed by the charged pion condensation.

# Results (T = 0)

## $\mu$-$\mu_I$ phase diagram

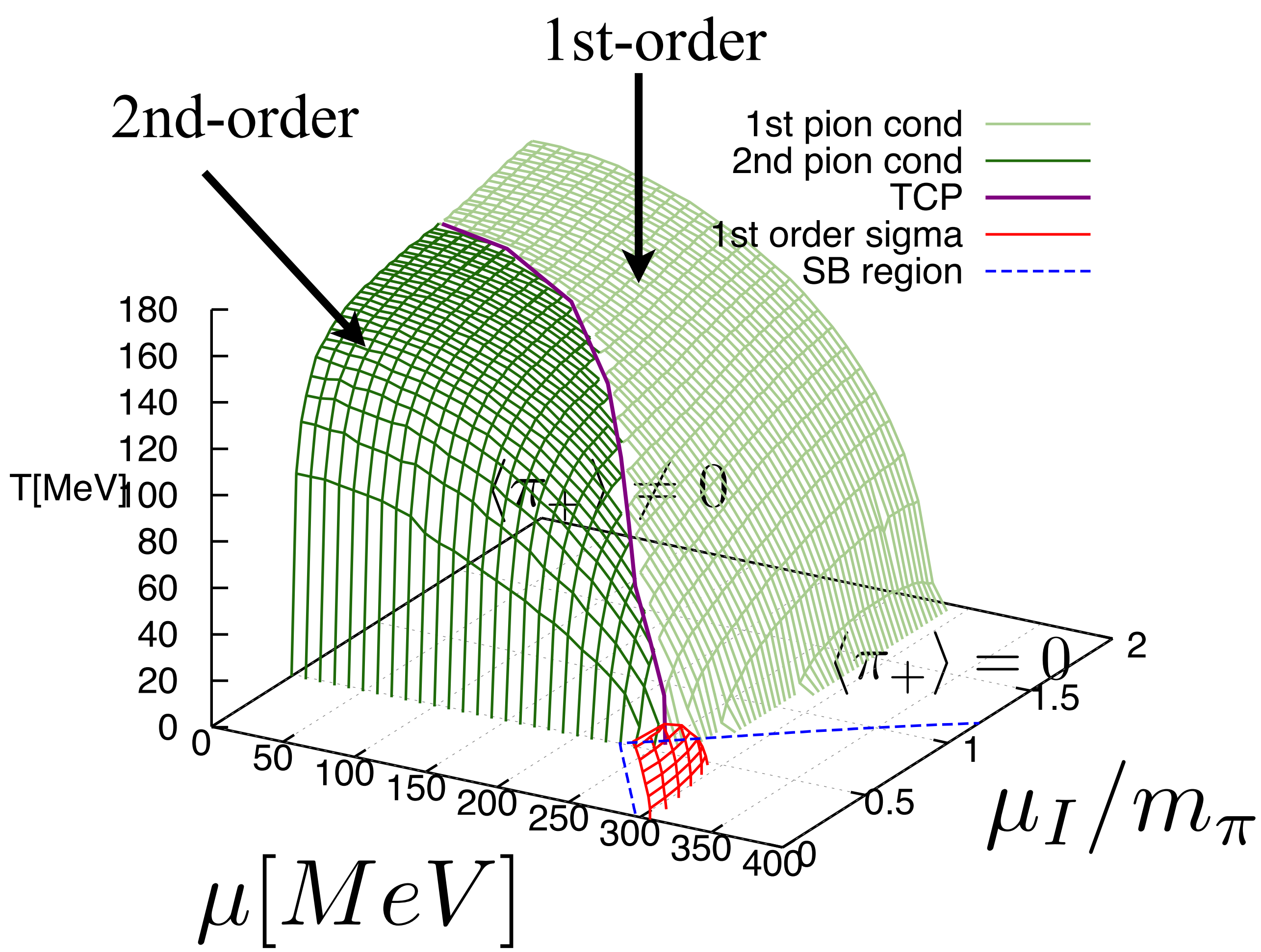



$\mu = \sqrt{g^2\langle\pi_+\rangle^2 + (g\langle\sigma\rangle - \mu_I)^2}$ or $g\langle\pi_+\rangle$

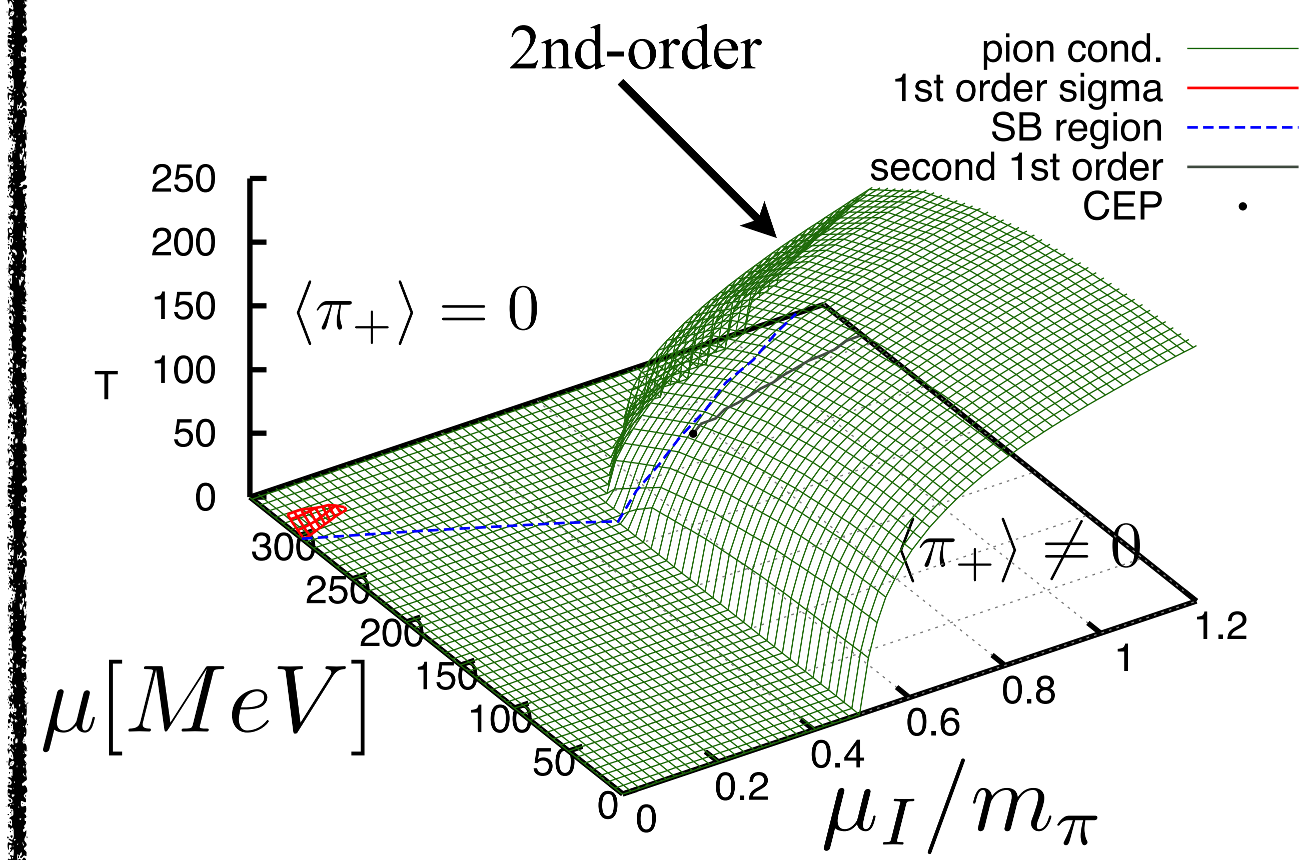
- $\mu_I^c = 1/2\, M_\pi$ is satisfied.
- Another 1st-order transition appear.
  Baryon-density jumps at the boundary.

# Phase diagram

## Mean-field




## FRG




- Meson fluctuation hide TCP line.
- Ordinary chiral 1st-order phase boundary (red surface) is shrunken by the fluctuations.

# Summary

- Silver Blaze relation is satisfied by the pion pole mass.
- The result of QM model agree with the Lattice QCD calculation.
- The result for higher $\mu_I$ depends on the sigma meson mass. We need to a light sigma mode.
- At low T, We have found the extra 1st-order phase transition at which quark density jumps.
- Meson fluctuations hide the TCP line which exists on the pion condensation surface.