強磁場中における真空複屈折の詳細解析とその応用に向けて

Vacuum birefringence by nonlinear QED effects in strong magnetic fields

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KH and K. Itakura in preparation

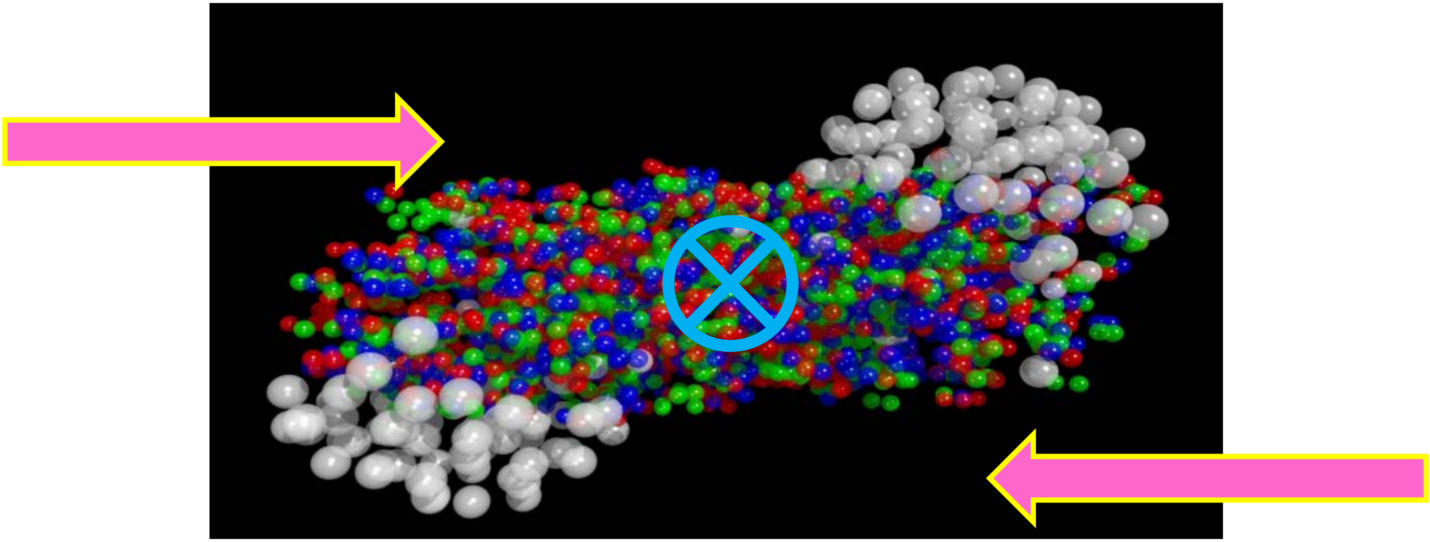
熱場の量子論とその応用
8月22-24日 2012

A magnetic field in Ultrarelativistic Heavy-Ion Collisions

An extremely strong B-field
induced by highly accelerated heavy nuclei

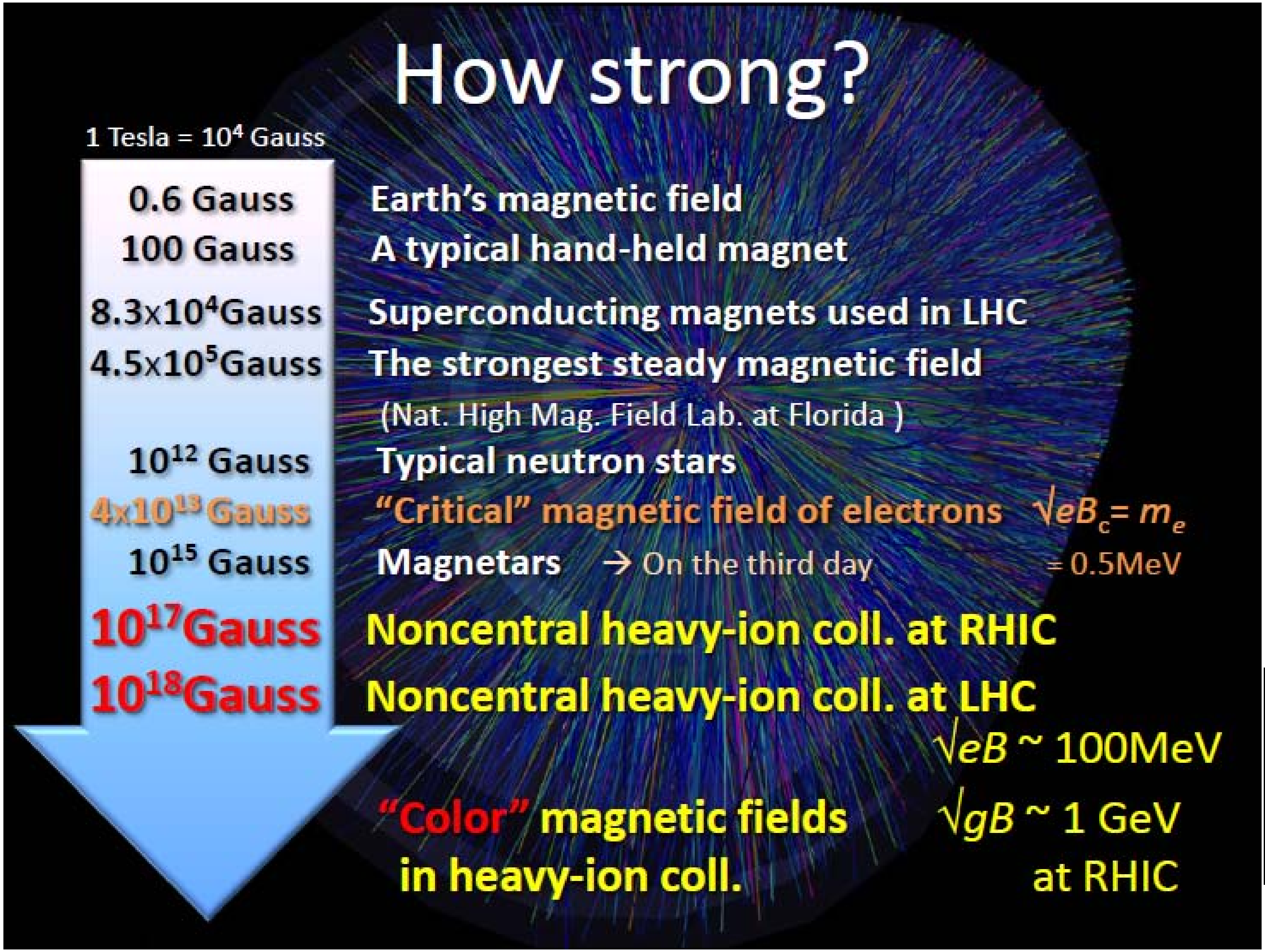
v

$Z =$

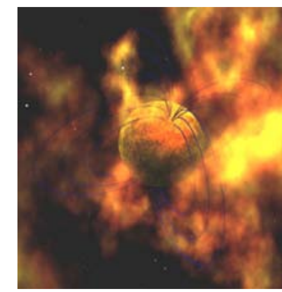


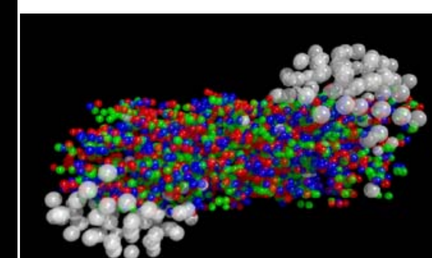
Pb+Pb peripheral, ALICE

Strong magnetic fields in nature and laboratory




Magnet in Lab.

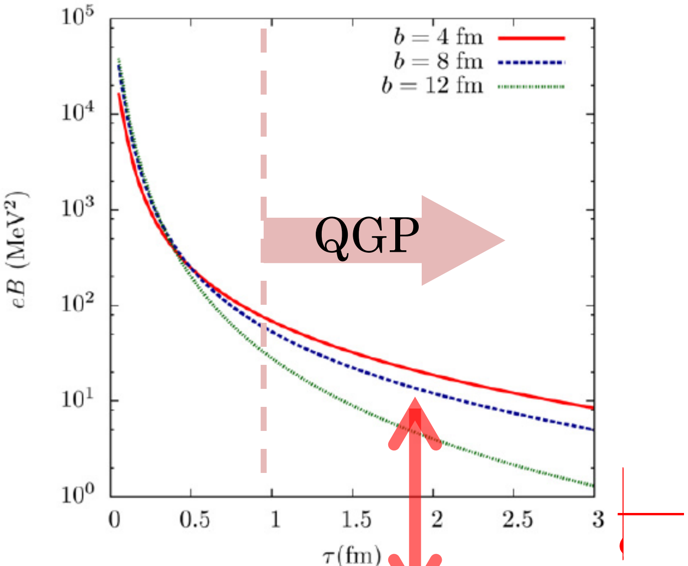

Magnetar

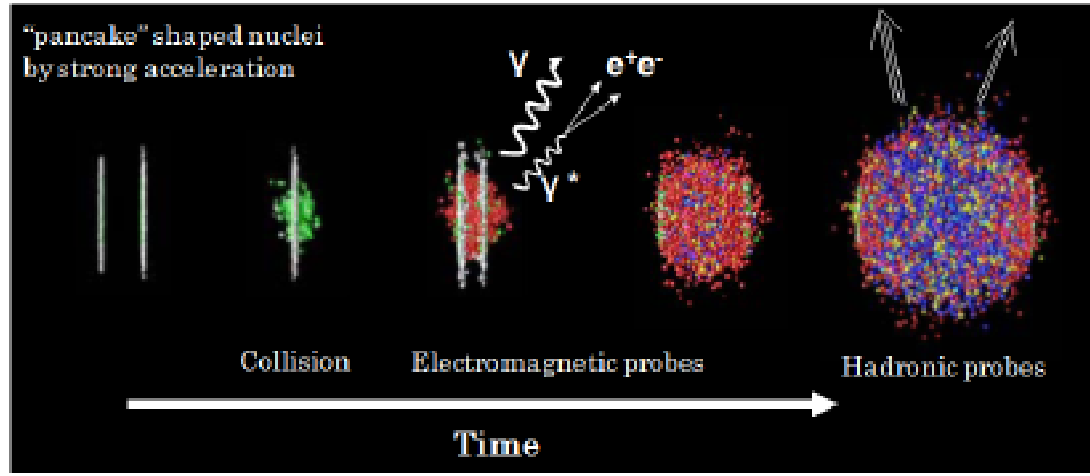

Heavy ion collisions

From Itakura-san's talk in *International conference on physics in intense field 2010 @ KEK*

Time evolution of the magnetic field in UrHIC

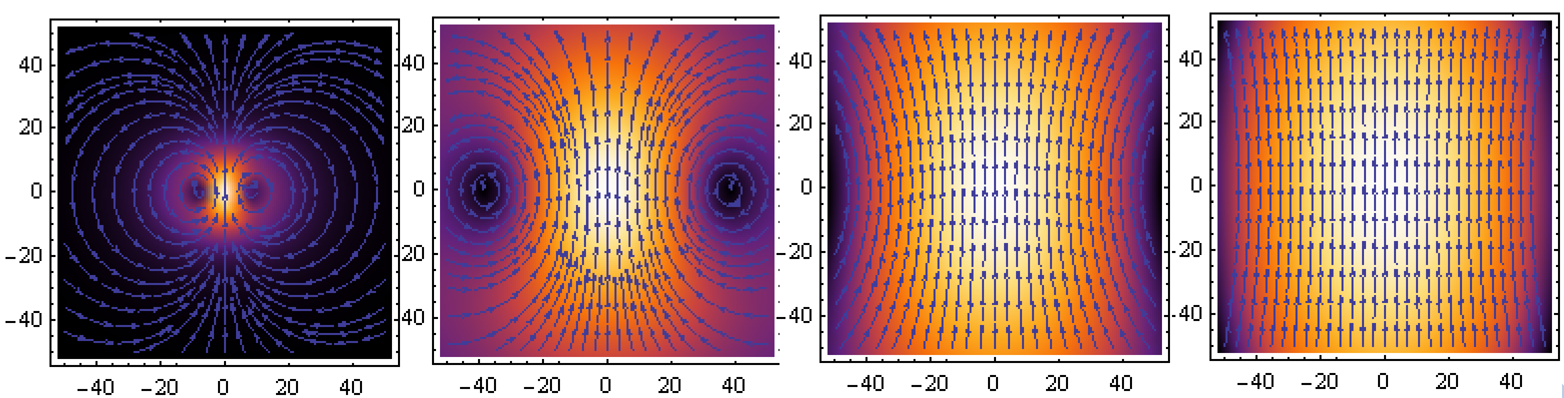
A simple estimate by Lienard-Wiechert potential





$eB_c =$

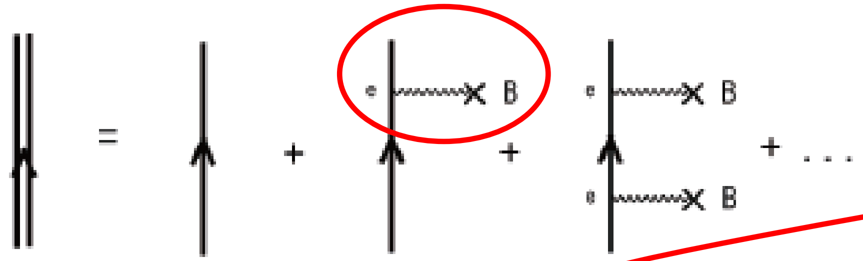
E_{cm} = 200 GeV (RHIC)
Z = 79 (Au), b = 6 fm



t = 0.1 fm/c   0.5 fm/c   1 fm/c   2 fm/c

Break-down of a perturbation in strong magnetic fields

Dressed fermion propagator



$1 + O(eB/m_e^2) + O((eB/m_e^2)^2) + \ldots$

Critical field strength
$B_c = m_e^2 / e$

Naïve perturbation breaks down when $B > B_c$

→ Need to take into account all-order diagrams

In heavy ion collisions,
$B/B_c \sim O(10^4) >> 1$

Resummation w.r.t external legs by "proper-time method " Schwinger

$$G(p|A) = \frac{i\,(\not p - e\not A}{(\not p - e\not A)^2}$$

$$= i\,(\not p - e\not A +$$

τ : proper-time

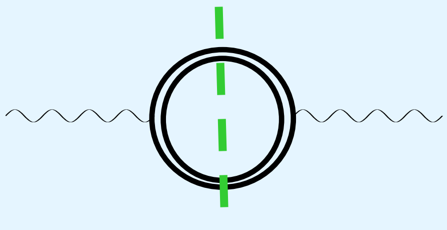
Employing Fock-Schwinger gauge $x^\mu A_\mu = 0$,

$$G(p|A) = \int_0^\infty d\tau\,[\not p - e\gamma^\mu F_\mu{}^\nu$$

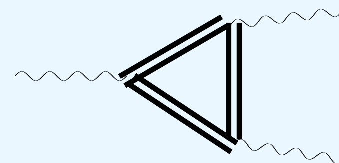
What happens in strong magnetic fields ?

Quantum and Nonlinear modification of photon propagations

Refraction & real photon decay



Photon splitting

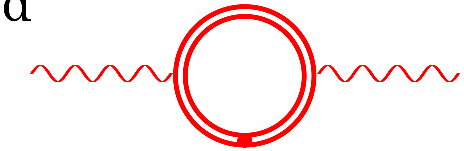


Photon propagation is studied by incorporating a vacuum polarization tensor from the dressed fermion propagator :

$$i\Pi^{\mu\nu}_{\rm ex}(q) = -(-ie)^2 \int$$

Modified Maxwell eq. : $\left(q^2\eta^{\mu\nu} - q^\mu q^\nu - \Pi^{\mu\nu}_{\rm ex}(q^2)\right) A_\nu(q) = 0$

Photon propagation in a constant external magnetic field



Lorentz and gauge symmetries lead to a tensor structure,

$$\Pi^{\mu\nu}_{\rm ex}(q^2) = -\left\{\chi_0 P_0^{\mu\nu} + \chi_1 P_1^{\mu\nu} + \chi_2 P_2^{\mu\nu}\right\}$$

q

$P_0^{\mu\nu} = q^2\eta^{\mu\nu} - q^\mu q^\nu$

$P_1^{\mu\nu} = q_\parallel^2 \eta_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu$

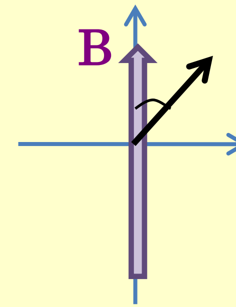
$P_2^{\mu\nu}$

θ: angle btw B-field and propagation

q^μ

$q_\parallel^\mu$

$q_\perp^\mu$



Eigen-equations from the modified Maxwell eq.

$$\begin{cases} \{\ (1+\chi_0)\ q^2\ \}\ \pi_0^\mu = 0 \\ \{\ (1+\chi_0)\ q^2 + \chi_2\ q_\perp^2\ \}\ \pi_1^\mu = 0 \\ \{\ (1+\chi_0)\ q^2 + \chi_1\ q_\parallel^2\ \}\ \pi_2^\mu = 0 \\ \xi_g^{-1}\ q^2\ \pi_3^\mu = 0\ . \end{cases}$$

"Vacuum birefringence" Dielectric

Following from the tensor structure, we obtain distinct eigenmodes!!

$$\begin{cases} \epsilon_\parallel = \\ \epsilon_\perp = \end{cases}$$

with eigenvectors,

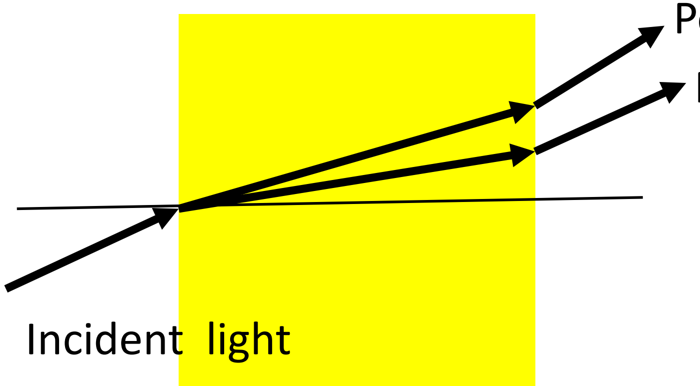
$$\begin{cases} \boldsymbol{E}_\parallel \propto ((1+\chi_0+\chi_1)\cos\theta, 0, -(1+\chi_0)\sin\theta) \\ \boldsymbol{E}_\perp \propto (0,1,0) \end{cases}$$

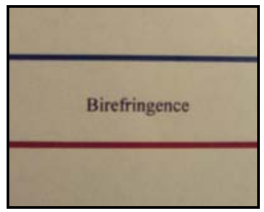
Melrose and Stoneham

When $\theta = 0$, we find $\epsilon_\parallel = \epsilon_\perp =$
owing to a boost invariance al

“Birefringence” in a dielectric substance

Doubled image by a splitting in a birefringent dielectric medium



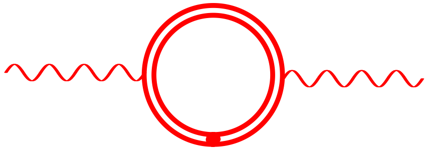


“Calcite” (方解石)

An birefringence has been led by the tensor structure.

What dynamics is encoded in the scalar functions, χ_i ?

Scalar coefficients χ in the proper-time method



Given by double integral associated with two fermion lines

$$\chi_i(r_\parallel^2, r_\perp^2, B_\mathrm{r}) = \frac{\alpha B_\mathrm{r}}{4\pi}$$

q^μ

$q_\parallel^\mu$

$q_\perp^\mu$

Dimesionless variables

$$B_\mathrm{r} = \frac{B}{B_c} \ , \quad r_\parallel^2 = \frac{q_\parallel^2}{4m^2} \ , \quad r_\perp^2 =$$

$$\begin{cases} \Gamma_0(\tau, \beta) = \mathrm{c} \\ \Gamma_1(\tau, \beta) = (\\ \Gamma_2(\tau, \beta) = 2 \end{cases}$$

Schwinger, Adler, Shabad, Urrutia,
Tsai and Eber, Dittrich and Gies

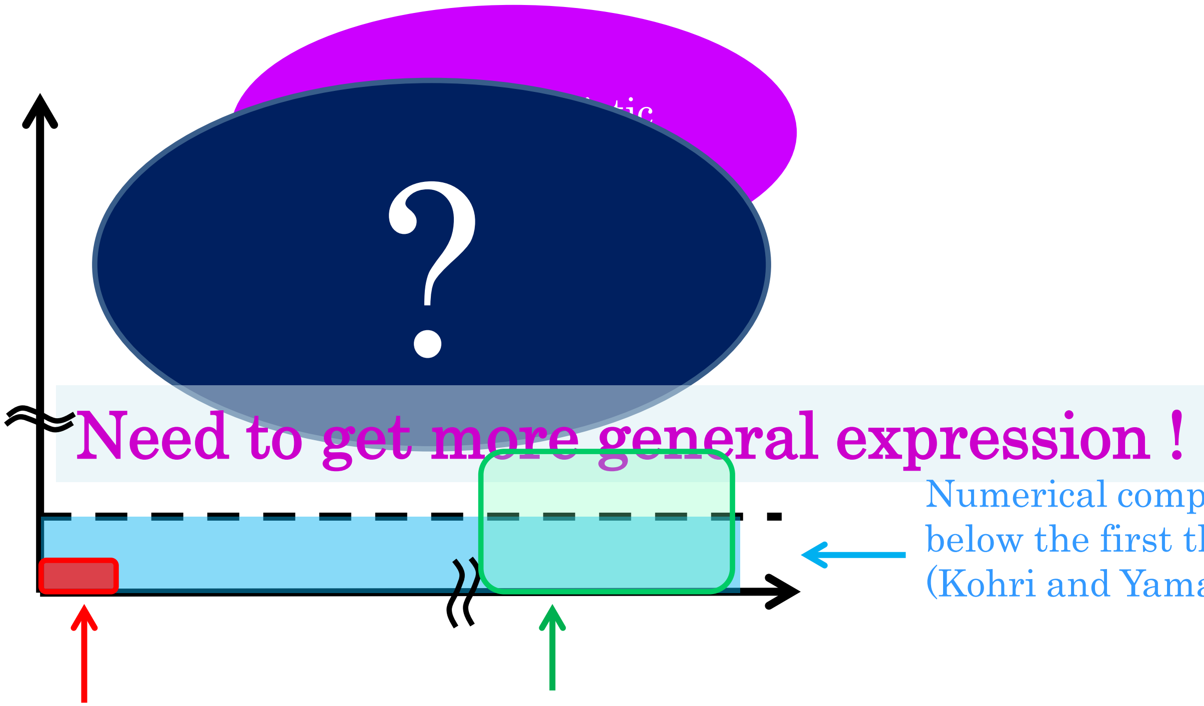
$$\phi_\parallel(r_\parallel^2, B_\mathrm{r}) =$$

$$\phi_\perp(r_\parallel^2, B_\mathrm{r}) =$$

Exponentiated trig-functions generate strongly oscillating behavior with arbitrarily high frequency.

Relevant scales and available calculations for χ's

UrHIC
$\sim 100^2 - 300^2$ MeV^2 (therm
$\sim$ GeV^2 (prompt)



The first threshold:
$q_\parallel^2 = 4m^2 = 1$ MeV^2

Numerical comp.
below the first threshold
(Kohri and Yamada)

Weak field & soft photon limit
(Adler)

Strong field limit: the lowest-Landau-level approximation
(Tsai and Eber, Shabad, Fukushima)

Analytic calculation of the double integral

$$\chi_i(r_\parallel^2, r_\perp^2, B_\mathrm{r}) = \frac{\alpha B_\mathrm{r}}{4\pi}$$

$\phi_\perp(r_\parallel^2, B_\mathrm{r})$

$\eta =$

Two important relations

★ $$e^{-iu\cos(\beta\tau)} = \sum_{n=0}^{\infty}(2-\delta_{n0})I_n(-iu)\ e^{in\beta\tau}$$

★ $$e^{i\eta\cot\tau}\ I_n(-iu) = \ \eta^n\ e^{-\frac{|\boldsymbol{q}_\perp|^2}{2|eB|}}$$

Wave function of LLL

Associated Laguerre polynomial

Any term reduces to either of elementary integrals.

$$F_\ell^n(r_\parallel^2, B_\mathrm{r}) = \frac{i}{B_\mathrm{r}}\int_{-1}^{1}d\beta\int_0^\infty d\tau\ e^{-i\left(\phi_\parallel+2\ell-n\beta+n\right)\tau}$$

$$G_\ell^n(r_\parallel^2, B_\mathrm{r}) = \frac{i}{B_\mathrm{r}}\int_{-1}^{1}d\beta\int_0^\infty d\tau\ \beta\, e^{-i\left(\phi_\parallel+2\ell-n\beta+n\right)\tau}$$

$$H_\ell^n(r_\parallel^2, B_\mathrm{r}) = \frac{i}{B_\mathrm{r}}\int_{-1}^{1}d\beta\int_0^\infty d\tau\ \beta^2\, e^{-i\left(\phi_\parallel+2\ell-n\beta+n\right)\tau}$$

Analytic results!

Applicable to any momentum regime and field strength !

$$\chi_i = \frac{\alpha B_{\rm r}}{4\pi} e^{-\eta} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty}$$

Sum wrt Landau levels

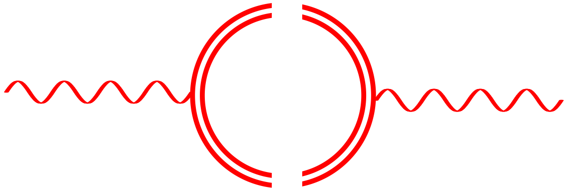
Combination of known functions

Polarization tensor has an imaginary part above

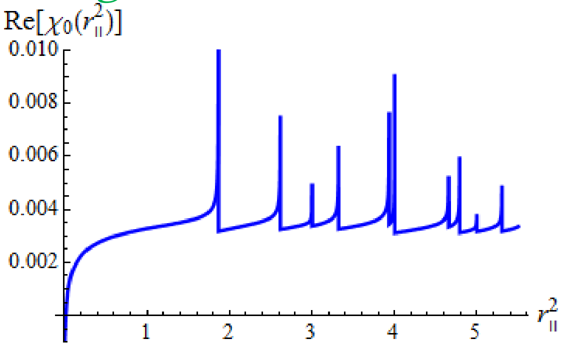
$$q_{\parallel}^2 = \Big[\sqrt{m^2 + 2\ell eB} + \sqrt{m^2}$$

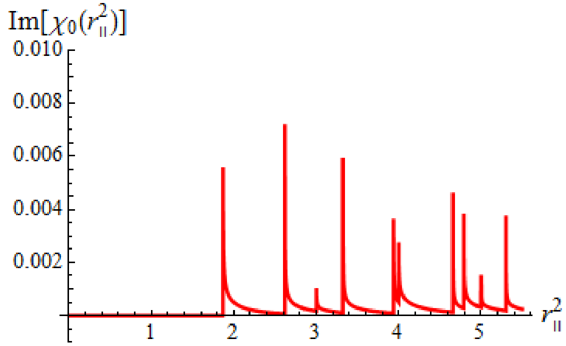
ℓ and n: "Landau levels" of

Photon decay into a fermion pair



Divergence at infinite number of the thresholds





Dielectric constants stay finite at the thresholds.

$$\begin{cases} \epsilon_\parallel = \\ \epsilon_\perp = \end{cases}$$

We've got a general expression of the vacuum polarization tensor

Dielectric constant from the vacuum polarization tensor in strong magnetic fields

Dielectric constant at the lowest-Landau-level

The first term $(\ell, n) = (0, 0)$

$$\chi_1 = \frac{\alpha B_{\mathrm{r}}}{4\pi} \, e$$

ArcTan : source of an imaginary part above LLL

K. Fukushima

Dielectric constant in the LLL

$\epsilon_{\parallel}$ Excitation only along a magnetic field

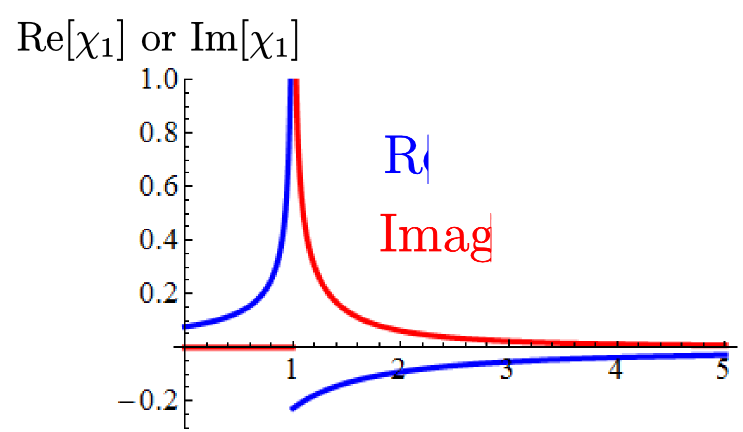
$\epsilon_{\perp}$

Limiting behavior of dielectric constant

As $r_{\parallel}^2 \to 1 \pm 0$, $\mathrm{Re}[\chi_1] \to +\infty$

$\mathrm{Im}[\chi_1] \to +\infty$

$$\lim_{r_{\parallel}^2 \to 1} \epsilon_{\parallel}(r_{\parallel}^2) = \frac{1}{\cos^2\theta}$$



Dielectric constant at the LLL

λ

$\epsilon_{\rm real} + i\epsilon_{\rm im}$

$q_{\parallel}^2 = \omega^2 -$

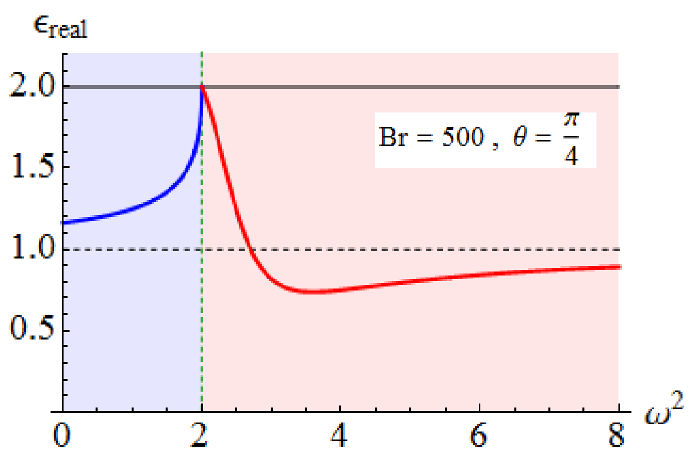
$q_{\perp}^2 = -$

Step 0

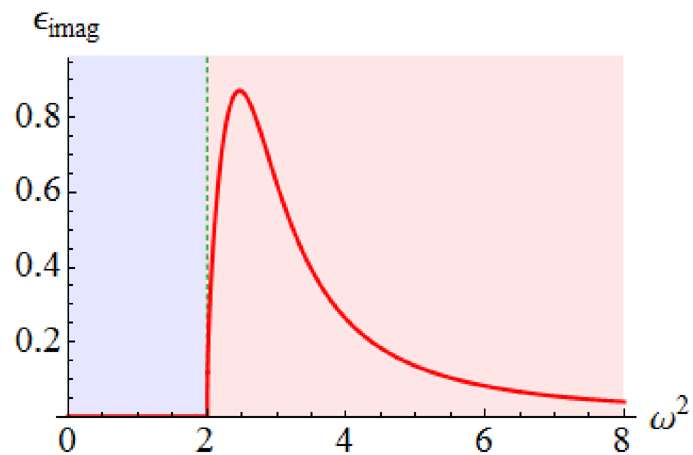
Without back-reaction to incident photon field from induced vacuum polarization

Assuming $\varepsilon = 1$ in χ_1 , we get a naïve profile of the dielectric constant.

ϵ



$\omega_{\rm tl}^2$



$\omega_{\rm tl}^2$

As $q_{\parallel}^2 \to 4m^2$, $\epsilon \to 1/\cos^2\theta$

$q_{\parallel}^2 = \omega^2(1 - \epsilon\cos^2\theta) \to 0.$

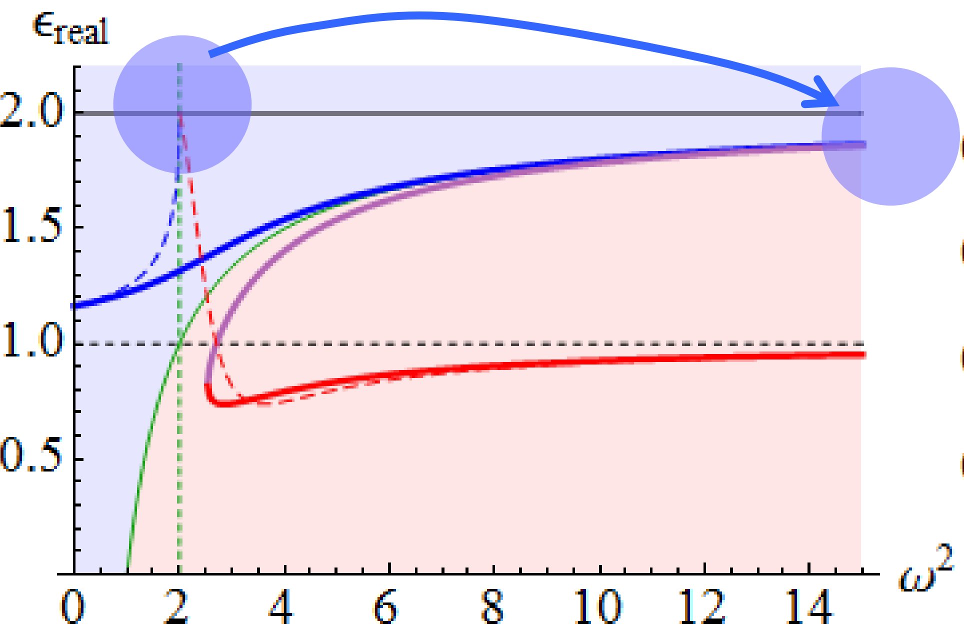
Implies a requirement of self-consistent treatment

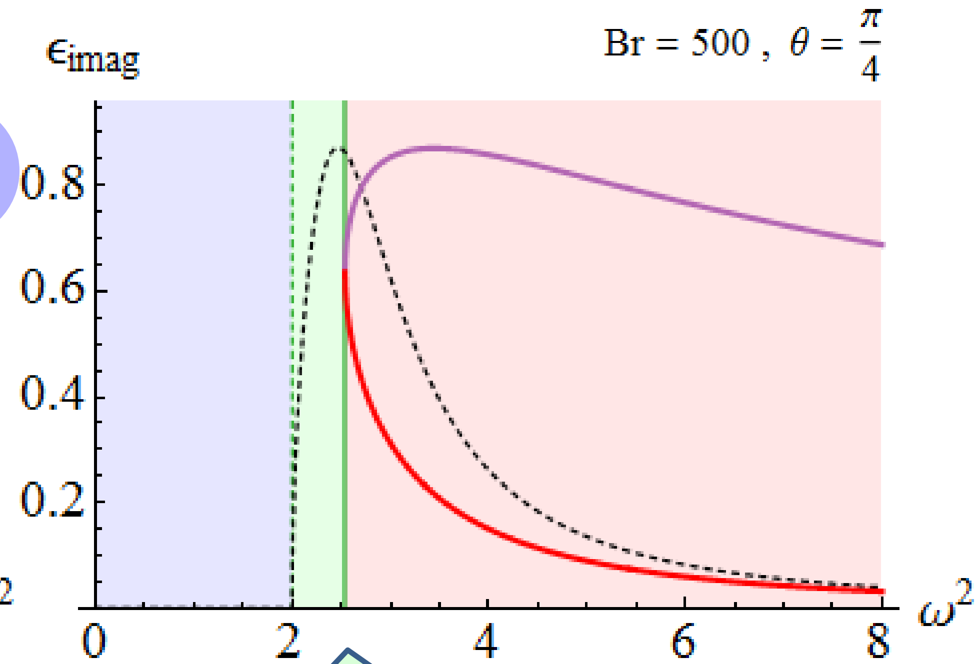
Step 1

Partially consistent solution

Screening of the incident photon field by an induced vacuum polarization

Taking into account $\varepsilon_{real} \neq 1$ and assuming $\varepsilon_{imag} = 1$ in χ_1 ,





Limiting point has gone to the asymptotic regime.

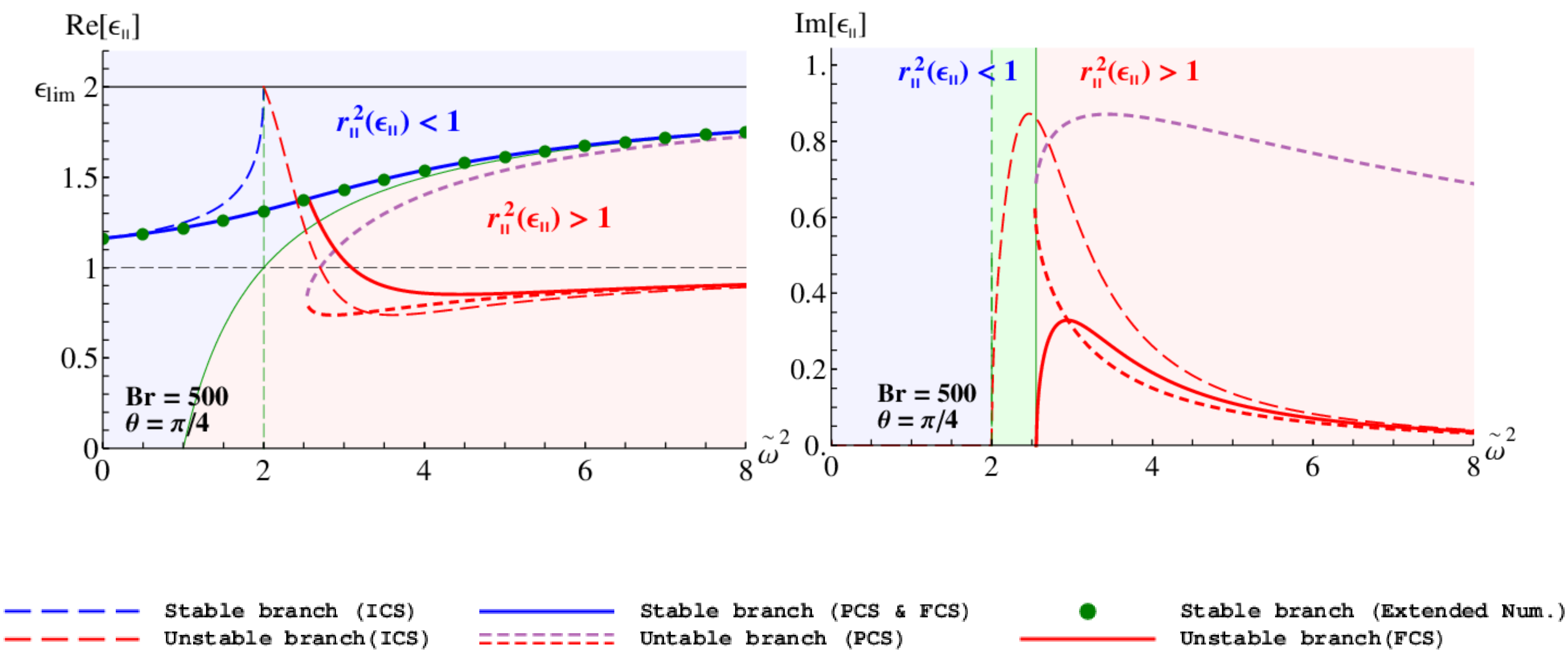
Shift of the threshold due to a screening caused by nonlinear response

Step 2

Fully consistent solution

Damping of the incident photon field due to a decay into a fermion pair

Taking into account both $\varepsilon_{real} \neq 1$ and $\varepsilon_{imag} \neq 1$ in χ_1 ,



Summary

- We performed an analytical evaluation of the vacuum polarization tensor in an external magnetic field, within constant field case.

- We inspected the dielectric constant around the LLL with a self-consistent treatment .

KH and K. Itakura, in preparation.

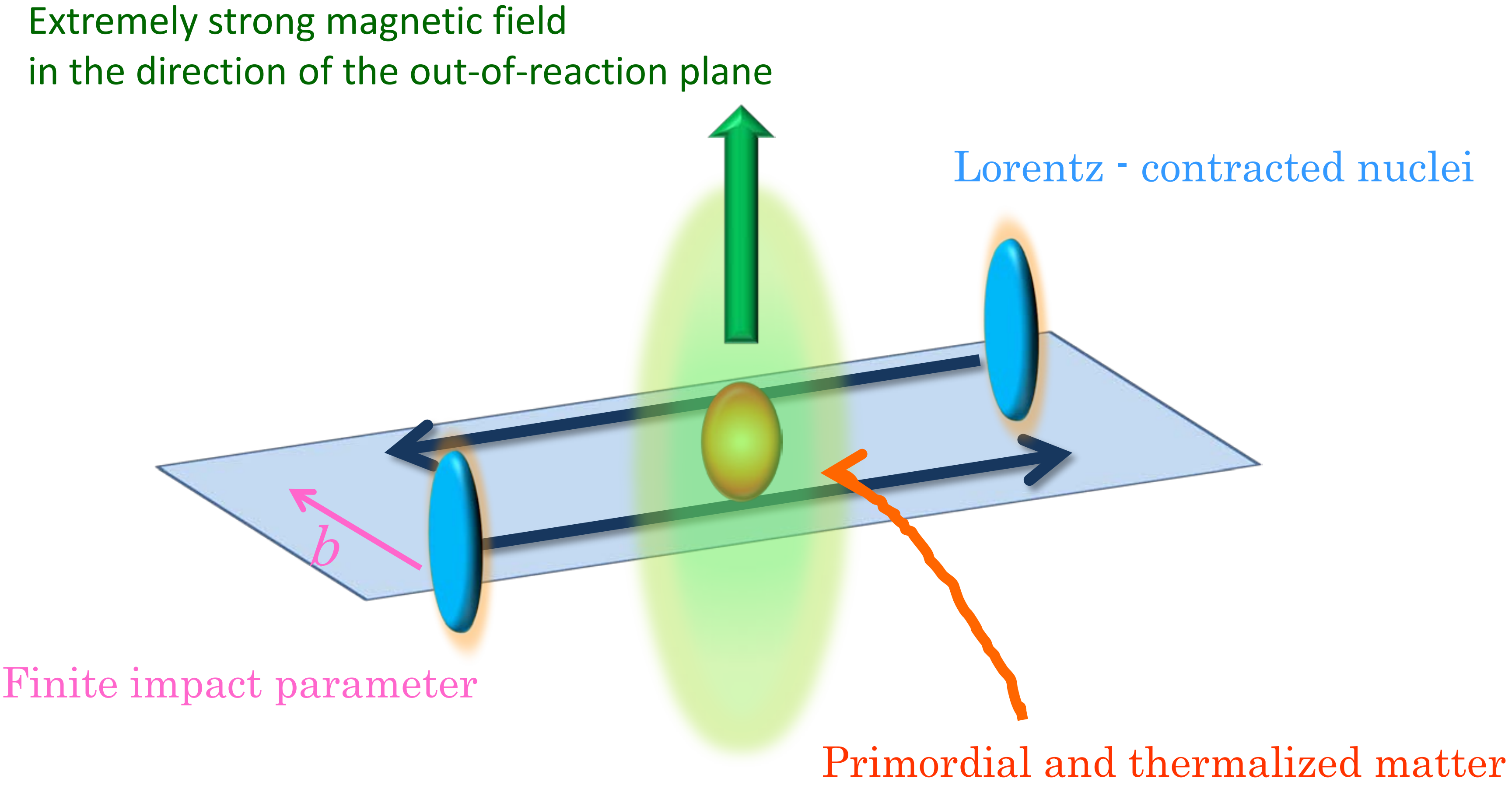
Prospects

- Application to photon spectrum in heavy-ion collisions. In progress

- Extension to strong “color” electromagnetic fields
  - What happens if a quarkonium is dropped into strong fields?
    In progress, in collaboration with S. H. Lee

An extremely strong magnetic field in Ultrarelativistic Heavy-Ion Collision

Geometry of the peripheral collision and strong B-field



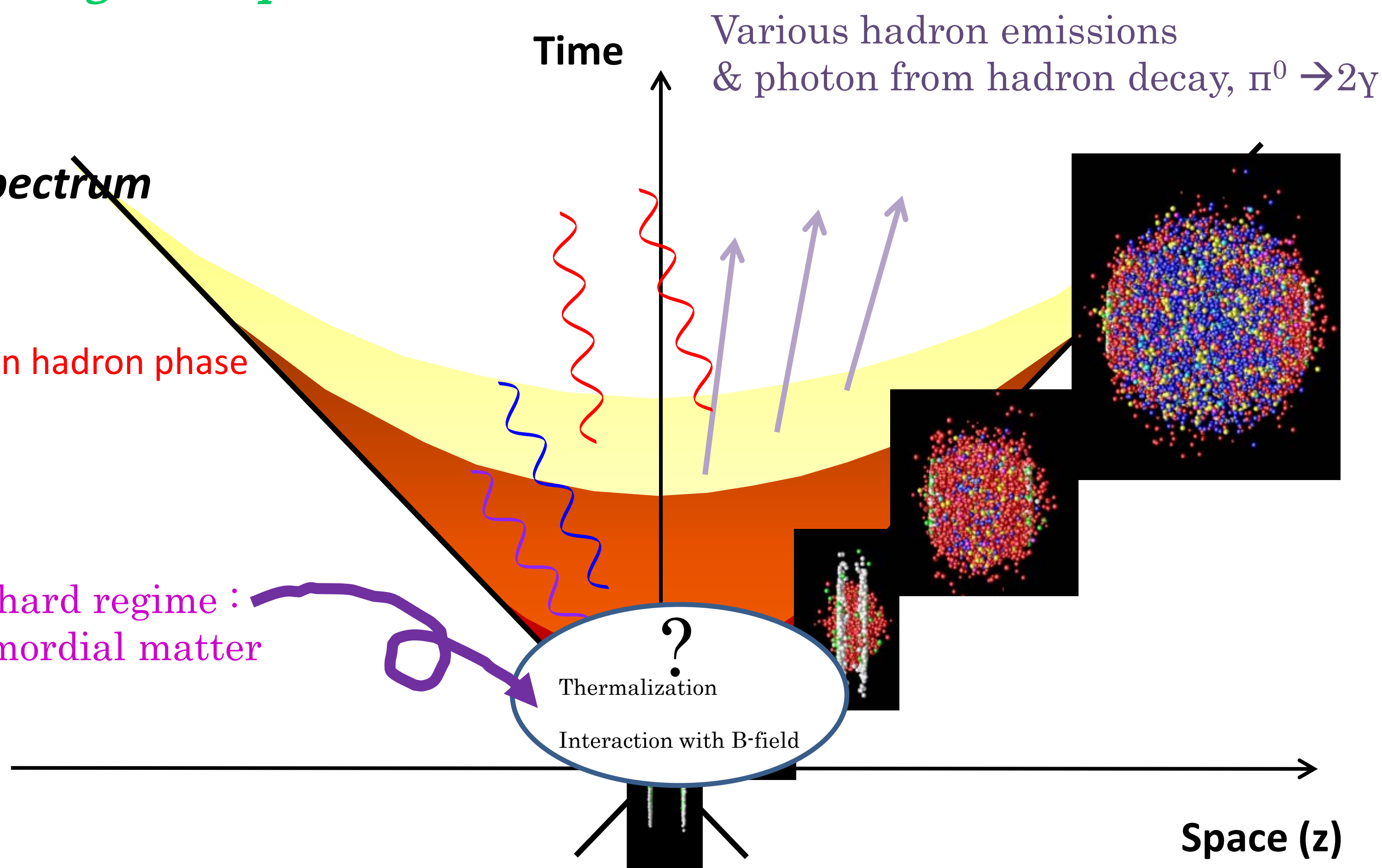
Direct photon from initial stage in UrHIC

Initial and background photons

Direct photon spectrum

Low Pt regime:
dominant emission in hadron phase

Intermediate and hard regime :
emission from primordial matter



Directly accessible to the initial stage and QGP

Detecting photon from the initial stage
⇔ Detecting effects of B-field

Close look at the integrals

What dynamics is encoded in the scalar functions ?

$$F_\ell^n(r_\parallel^2, B_{\rm r})$$
$$G_\ell^n(r_\parallel^2, B_{\rm r})$$
$$H_\ell^n(r_\parallel^2, B_{\rm r})$$

$$I_{\ell\Delta}^n(r_\parallel^2) = \frac{2}{\sqrt{4ac-b^2}} \Big[\arctan \Big($$

$$a = r_\parallel^2 \ , \quad b = -nB_{\rm r} \ , \quad c = (1$$

An imaginary part representing a real photon decay

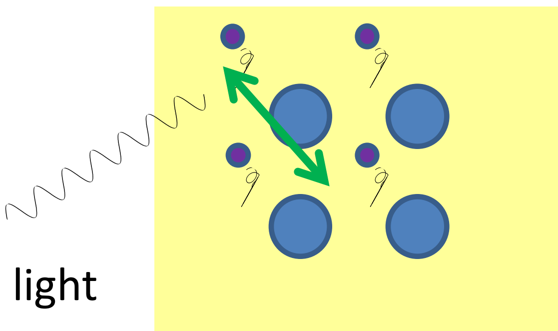
$$b^2 - 4ac = 0 \quad \Leftrightarrow \quad (-nB_{\rm r})^2 - 4r_\parallel^2$$

$$\Leftrightarrow \quad q_\parallel^2 = \Big[\sqrt{m^2 + 2\ell eB} + \sqrt{m^2}$$

Invariant mass of a fermion-pair in the Landau levels

Schematic picture of the birefringence

Polarization in dielectric medium :
a classical argument



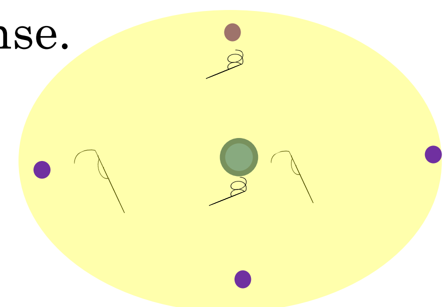
Incident light

Lorentz-type dispersion :

$\chi \propto$

with characteri

Anisotropic constants result
in an anisotropic response.



$\begin{cases} q : \mathrm{c} \\ N : \\ \boldsymbol{x} : \mathrm{c} \end{cases}$

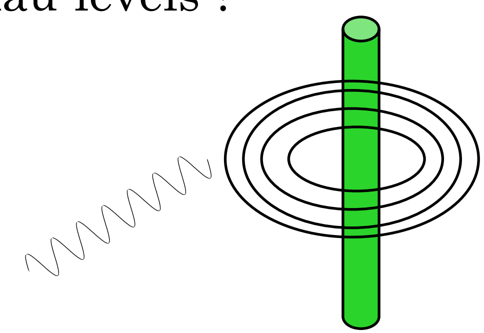
What happens with the anisotropic (discretized)
spectrum by the Landau-levels ?

$m\ddot{\boldsymbol{x}} +$

Dissipation

Linear bound force

Incident light field



Analytic results!

Applicable to any momentum regime and field strength !
Applicable to both on-shell and off-shell photon!

$$\chi_i = \frac{\alpha B_{\rm r}}{4\pi} e^{-\eta} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \Big|$$

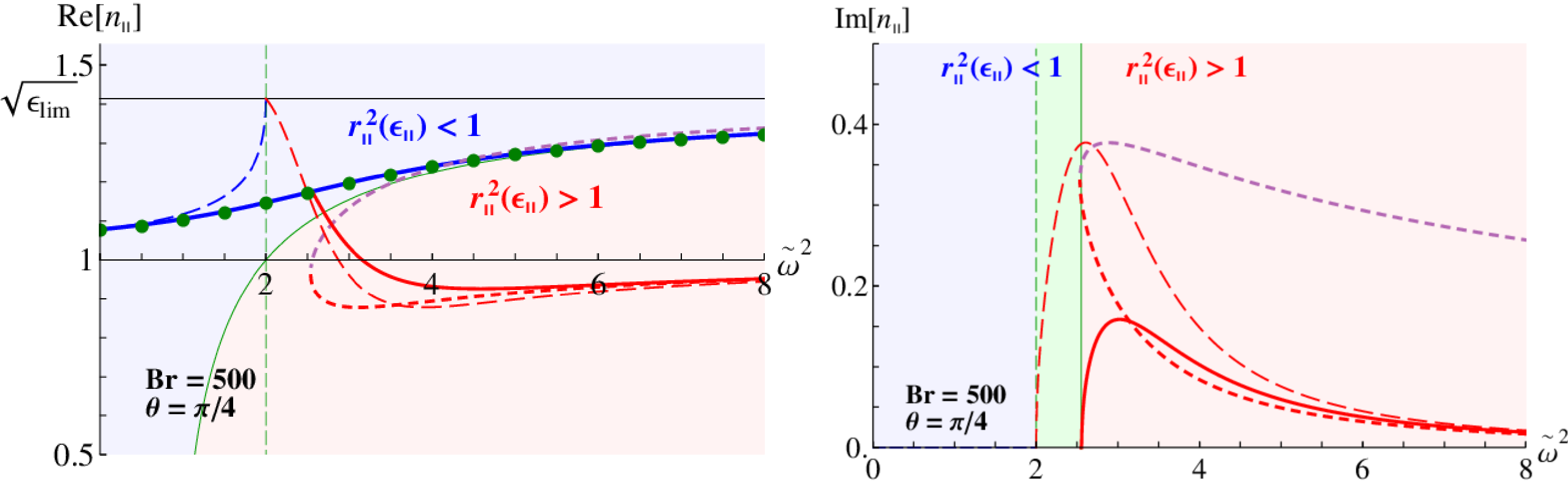
Sum wrt Landau levels

Combination of known functions

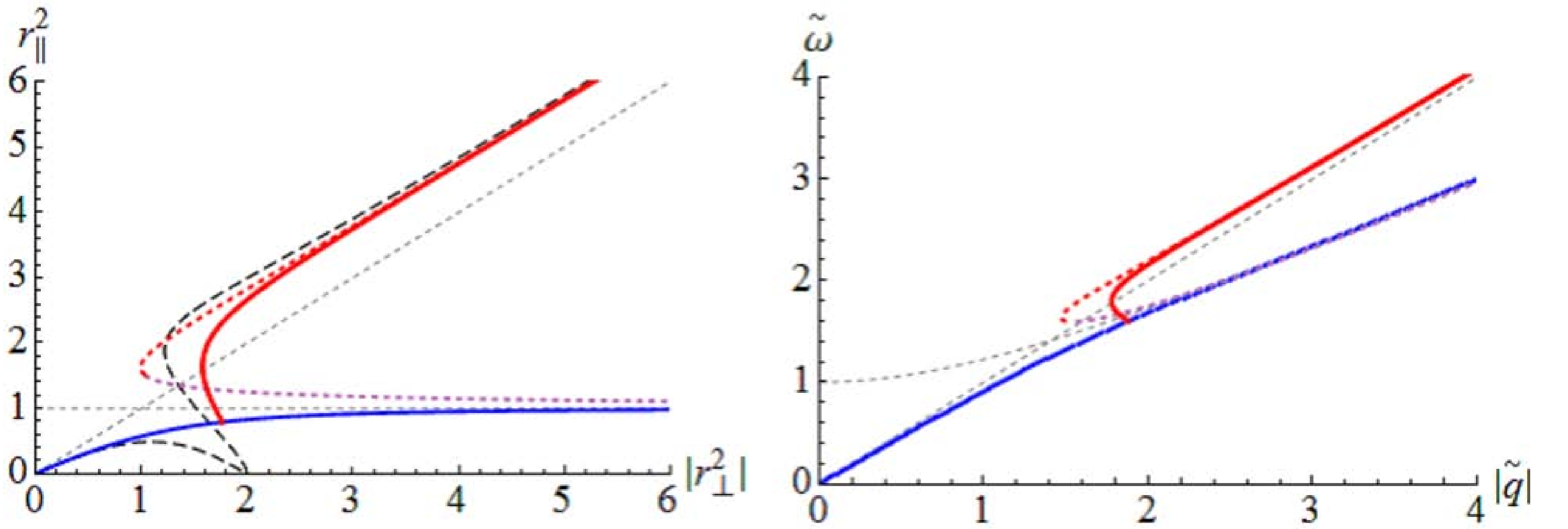
Photon decay channel opens at every Landau level

$$I_{\ell\Delta}^{n}(r_\parallel^2) = \begin{cases} \frac{1}{2\sqrt{(r_\parallel^2 - s_-)(r_\parallel^2 - s_+)}} \log \\ \frac{1}{\sqrt{|(r_\parallel^2 - s_-)(r_\parallel^2 - s_+)|}} \Big[\\ \frac{1}{2\sqrt{(r_\parallel^2 - s_-)(r_\parallel^2 - s_+)}} \Big[\end{cases}$$

Refraction index from the complex dielectric constant



Photon dispersion relation in a strong magnetic field



Real and imaginary parts of refraction index from dielectric constant

$$(n_r + i n_i)^2 = \epsilon_r + i\epsilon_i \quad (n^2 = \epsilon)$$

$$\begin{cases} n_r = \frac{1}{\sqrt{2}}\sqrt{|\epsilon| + \epsilon_r} \\ n_i = \frac{1}{\sqrt{2}}\sqrt{|\epsilon| - \epsilon_r} \end{cases} .$$

Electromagnetic field of propagating photon

$$\begin{cases} \boldsymbol{e} = \omega N \sqrt{\frac{\epsilon_{\|}(\theta)}{\epsilon_{\|}(\frac{\pi}{2})}} \left(-\epsilon_{\|}\left(\frac{\pi}{2}\right)\cos\theta, 0, \sin\theta\right) \ \Psi_{\|}(t, \boldsymbol{x}) \\ \boldsymbol{b} = \omega N \sqrt{\epsilon_{\|}\left(\frac{\pi}{2}\right)} \ (0, -1, 0) \ \Psi_{\|}(t, \boldsymbol{x}) \end{cases} .$$

Phase and damping factors provided by a complex refraction index

$$\Psi_{\|}(t, \boldsymbol{x}) = e^{-i\omega(t - n_{\|r}\hat{\boldsymbol{q}}\cdot\boldsymbol{x})} e^{-\omega n_{\|i}\hat{\boldsymbol{q}}\cdot\boldsymbol{x}}$$