Direct observations of thermal fluctuations at sub-shot noise levels

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熱場の量子論とその応用 2012

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Contents

Thermal fluctuations

- Thermal fluctuation phenomena: Brownian motion, liquid surfaces, solid surfaces, $\cdots$
- Thermal fluctuations are ubiquitous!
- Directly observe thermal fluctuations of common materials in an everyday environment. $\Rightarrow$ Fluctuations are small! $\leftarrow$ noise!
- Measure at hitherto unseen low noise levels
  - □ Test theory to previously unachieved precision.
  - □ Observe new phenomena.

Liquid surface fluctuations — "Ripplons"

- Typical thermal fluctuation phenomena which is relatively well understood.
- Liquid surfaces fluctuate thermally. (von Schmoluchowski, 1908; Mandelstam, 1913)
- Experimentally observed effectively as gratings. (Katyl, Ingard, 1967) $\leftarrow$ Will not work for viscous liquids
- Size of fluctuations $\sim$ atomic scale

$$\langle z^2 \rangle \simeq \frac{k_B T}{4\pi\sigma} \log \frac{\sigma}{g\rho\ell^2} \sim (0.4\,\mathrm{nm})^2$$

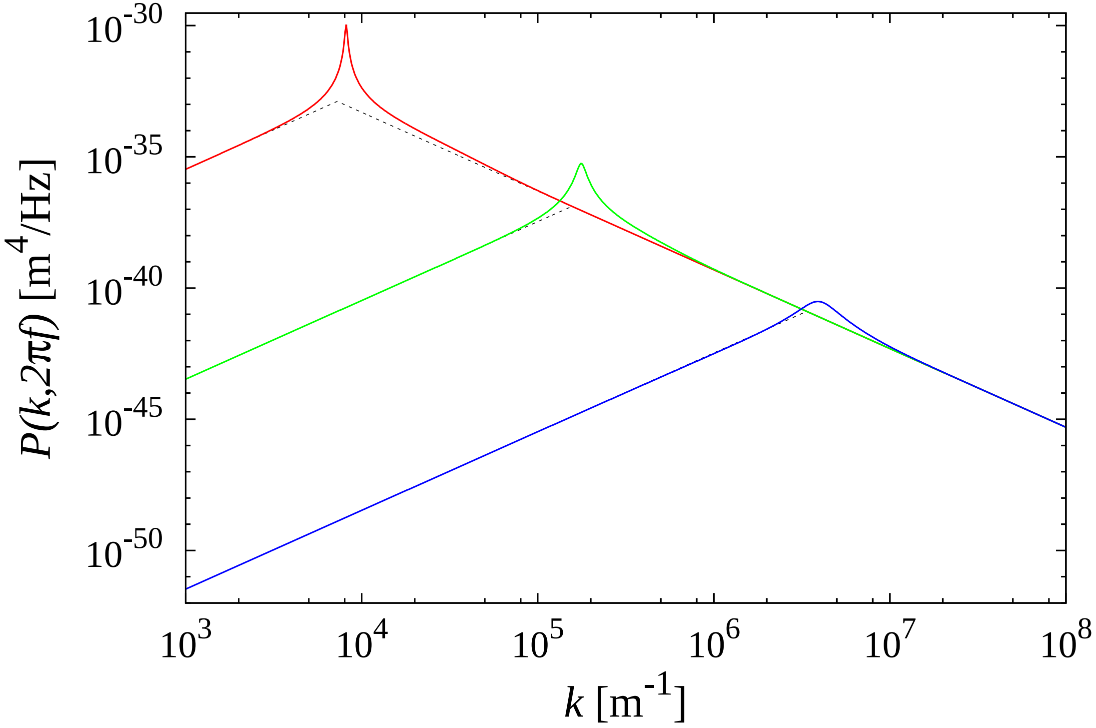
(σ : surface tension, ℓ: molecular scale)

Properties of thermal surface fluctuations

- Surface fluctuation properties determined from hydrodynamics.
- Interesting fundamental physics.
- Thermal fluctuation spectral properties known. $\leftarrow$ Important test!
- Spectral function: (Bouchiat, Meunier, 1971) $\leftarrow$ Analytic, but behavior not simple. (ρ: density, σ: surface tension, η: viscosity)

$$P(k,\omega) = \frac{k_B T}{\pi\omega}\frac{k\tau_0^2}{\rho}\,\mathrm{Im}\left[(1+s)^2 + y - \sqrt{1+2s}\right]^{-1}$$

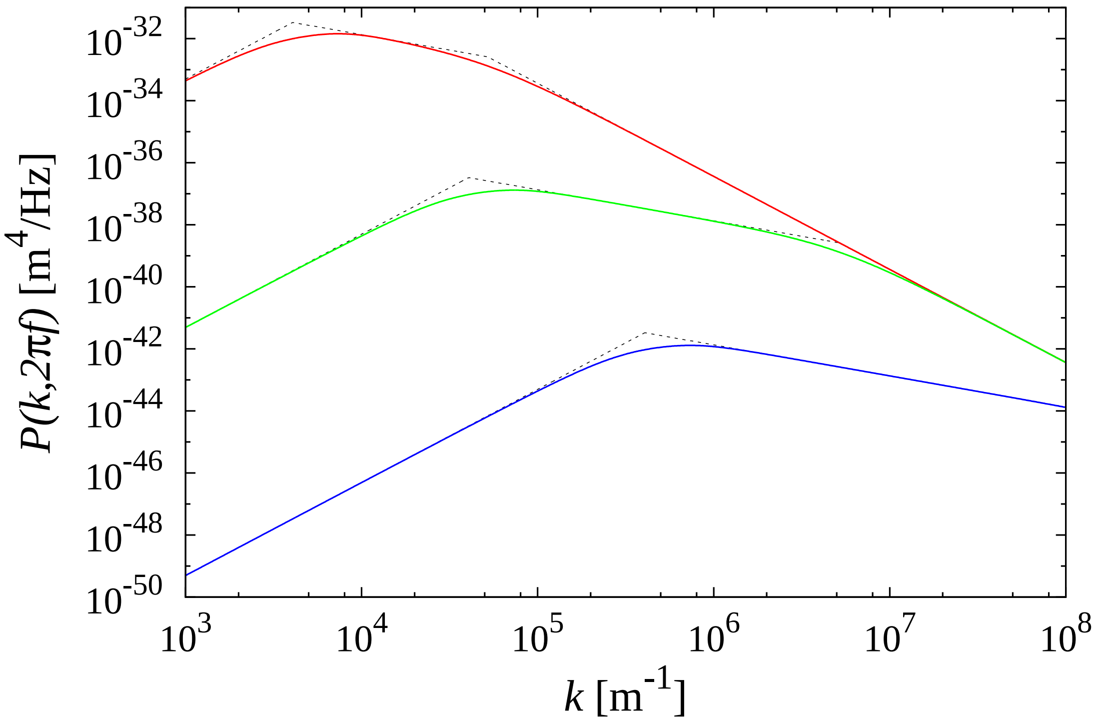
$$s \equiv -i\omega\tau_0, \qquad \tau_0 \equiv \frac{\rho}{2\eta k^2}, \qquad y \equiv \frac{\sigma\rho}{4\eta^2 k}$$



Low dissipation

- $\eta^3\omega/(\rho\sigma^2) < 1/(8\sqrt{2})$
- Compare with simple analytic behavior (black, dotted) – 2 regions.
- Peak behavior disappears for higher ω $\leftarrow$ "Viscous"
- $f = 10^3,\ 10^5,\ 10^7$ [s^{-1}]

$$P(k,\omega) \simeq \frac{k_B T}{\pi} \times \begin{cases} \dfrac{4\eta k^3}{\rho^2\omega^4} & k < 2^{-1/6} k_R(\omega), \quad k_R(\omega) \equiv \left(\dfrac{\rho\omega^2}{\sigma}\right)^{1/3} \\ \dfrac{2\eta}{\sigma^2 k^3} & k > 2^{-1/6} k_R(\omega) \end{cases}$$



Highly dissipative case

- $\eta^3\omega/(\rho\sigma^2) > 1/(8\sqrt{2})$
- No peaks $\leftarrow$ viscous
- Leading order behavior — 3 regions, simple.
- $f = 10^3,\ 10^5,\ 10^7$ [s^{-1}]

$$P_0(k,\omega) = \frac{k_B T}{\pi} \times \begin{cases} \dfrac{4\eta k^3}{\rho^2\omega^4} & k < 2^{-1/4}\sqrt{\dfrac{\rho\omega}{2\eta}} \\ \dfrac{1}{2\eta k\omega^2} & 2^{-1/4}\sqrt{\dfrac{\rho\omega}{2\eta}} < k < \dfrac{2\eta\omega}{\sigma} \\ \dfrac{2\eta}{\sigma^2 k^3} & k > \dfrac{2\eta\omega}{\sigma} \end{cases}$$

- Measure surface fluctuations directly.
- Characterize the fluctuations $\Rightarrow$ spectra.
- Fluctuations too small to examine precisely. $\leftarrow$ Noise, esp. shot noise

Shot noise

- Essentially quantum noise. Randomness of photon arrival times.
- Photoconversion $\Rightarrow$ noise in current

$$\langle(\Delta I)^2\rangle = 2eI\,\Delta f$$

- White noise (f independent).
- Unavoidable in photoconversion (*cf.* gravitational wave detection).
- Reduce shot noise $\Rightarrow$ Squeezed light ($\sim 1/2$ so far), quantum non-demolition(?).

Noise reduction

- A measurement: $D_1 = S + N_1$, S: signal, N_1: noise
- S has no known periodicity ($eg.$ thermal fluctuations).
- Power spectrum: ($\tilde{D}_1$ Fourier transform of D_1)

$\langle|\tilde{D}_1|^2\rangle = \langle|\tilde{S}|^2\rangle + \langle|\tilde{N}_1|^2\rangle$ Impossible to separate S, N_1 even in theory

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- Make multiple measurements: $D_{1,2} = S + N_{1,2}$

$$\langle\overline{\tilde{D}_1}\tilde{D}_2\rangle \to \langle|\tilde{S}|^2\rangle \quad , \qquad \mathcal{N} \to \infty$$

relative error $\sim 1/\sqrt{\mathcal{N}}$

Uncorrelated noise statistically reduced.

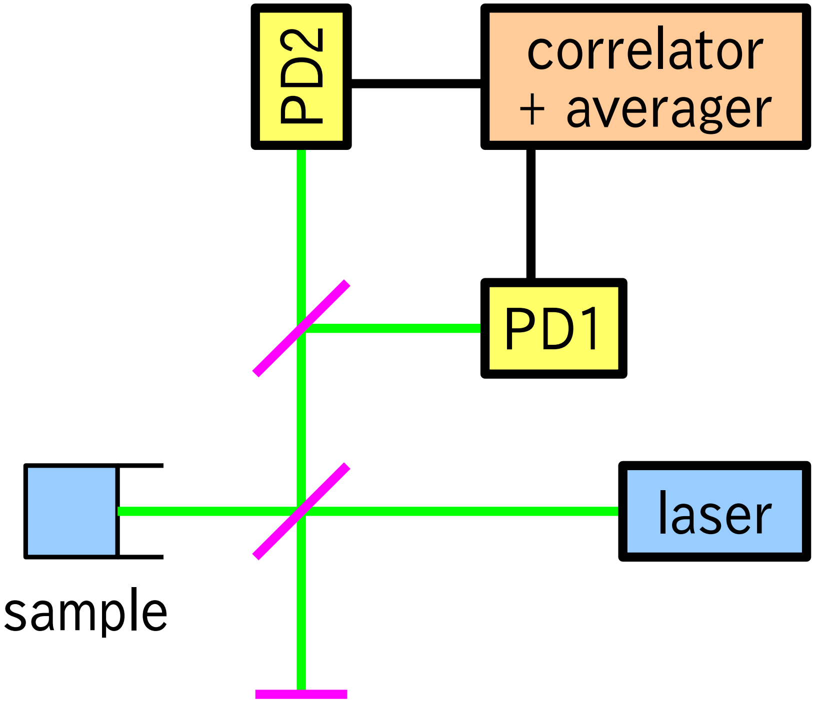
Comments on the noise reduction method

- Simple! — at least in theory
- Principle *not* limited to optical or surface measurements.
- Noise needs to be uncorrelated. $\leftarrow$ crucial
- *Any* uncorrelated noise eliminated: Shot noise, AM noise, amplification noise, FM noise, directional fluctuations, $\cdots$.
- In practice, 10^{-3} reduction achieved.

Practical limitations of the noise reduction method

- Always applicable, but efficacy depends on the properties of the signal.
- Noise elimination statistical.
- Reltative reduction: $\sim 1/\sqrt{\mathcal{N}}$.
- Given Δf (frequency resolution), $\mathcal{N} = \Delta f T$. (T: total measurement time).
- $\Delta f = 1\,\mathrm{MHz}$, noise reduction factor $10^{-3} \Rightarrow T = 1\,\mathrm{s}$.
- Roughly speaking, need many periods of the "wave component" for the method to be effective.

Surface height fluctuation measurements



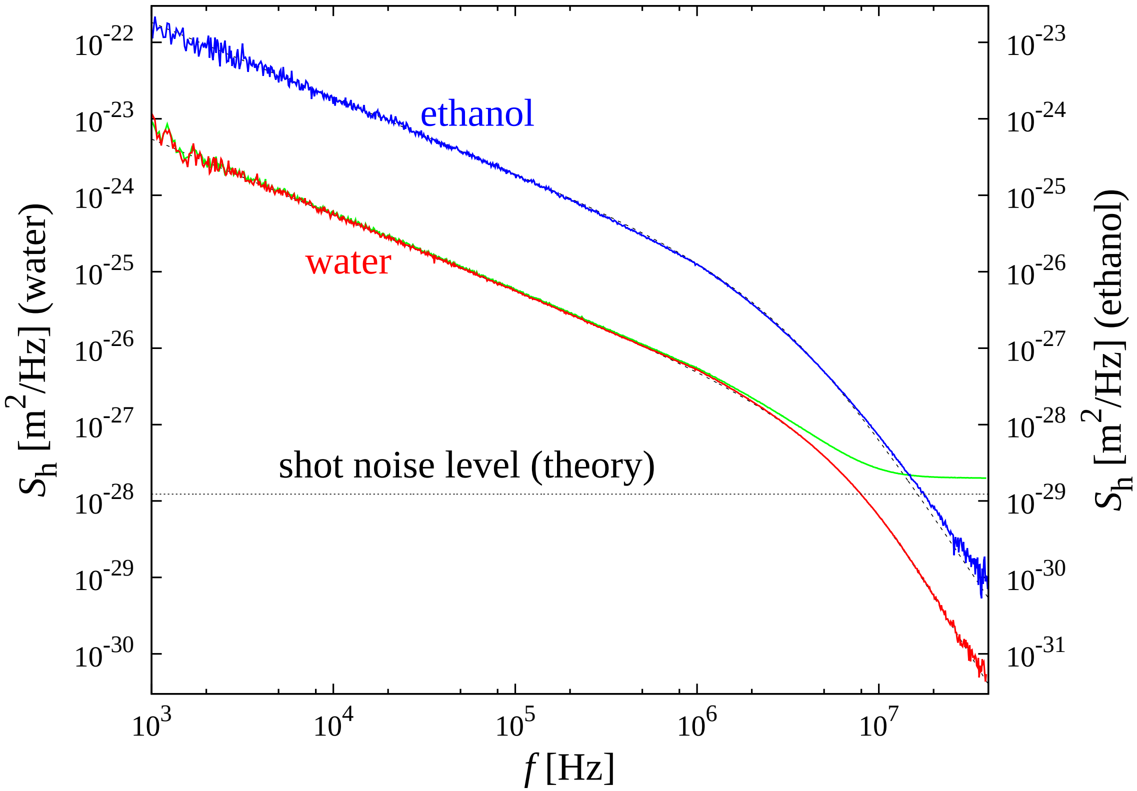
Basic setup

- Michelson interferometer, except for photodetection and data processing. ← applicable to any Michelson measurement
- Laser: 532 nm wavelength, 0.5 mW at sample, reference.
- Sample size: 2.2 mm, beam diameter: 0.96 μ m.
- Sample surface reflectivity∼few %. (water: 2 %, oil: 4 %)

Observed spectrum (summed over k, b: beam diameter):

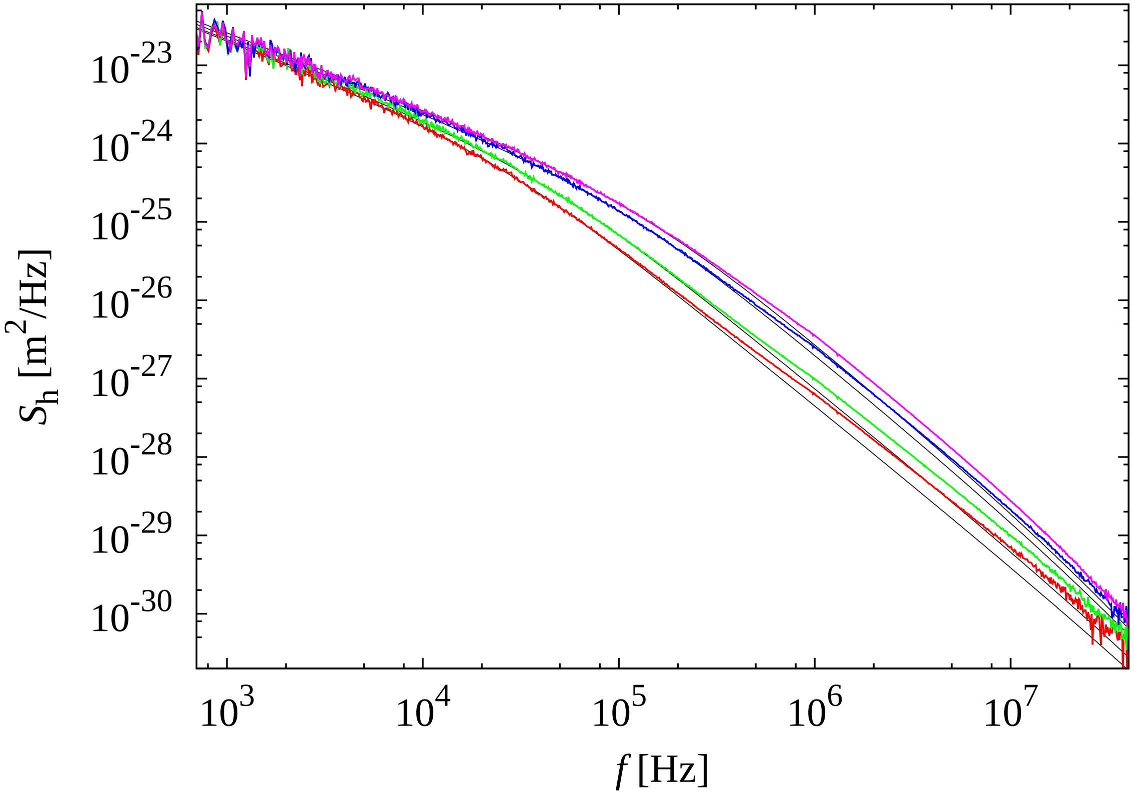
$$S(f) = 2\langle \tilde{a}_0(\omega)^2 \rangle = 2\int_0^\infty dk\, k e^{-b^2k^2/8} P(k, 2\pi f), \quad a_0\text{: average height}$$

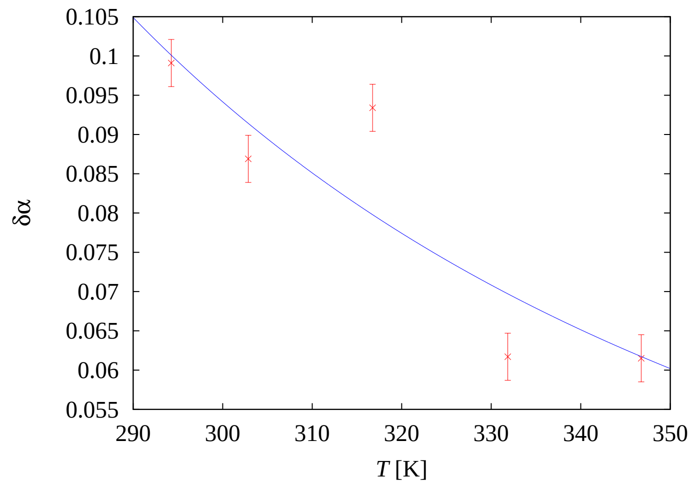
- Uniquely determined, given the liquid properties (ρ, σ, η) and b.
- Sample size: long distance cutoff.
- Beam size: short distance cutoff (averages within the beam spot).



- water surface spectrum
- ethanol surface spectrum
- Single signal ← shot noise
- Theoretical prediction!
- Theoretical shot noise level
  Good agreement, some lens aberration effects.
- 10^{-3} below shot noise level!

- Perfect agreement for water. Almost perfect for ethanol, very slight deviations at high frequencies.





Oil surface fluctuations: $T = 21,\ 30,\ 59,\ 74\,°$C. Good agreement, slight deviation at high frequencies. Mismatch larger for smaller T.

Deviation: $S_{\rm h}(f) \sim f^\alpha$, $\delta\alpha = C\exp(U/k_BT)$, $U = 7$ kJ/mol $\sim$ latent heat.

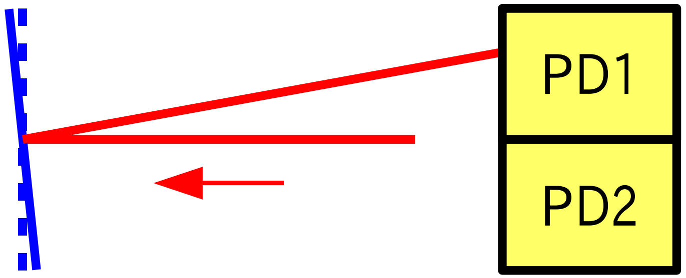
Quantum Optics

- Single light source:
  - ☐ Classically, difficult to imagine uncorrelated noise from a single source.
  - ☐ Photon arrival times are random *in the coherent state*.

- Photons partitioned at the mirror $\Rightarrow$ shot noise in the 2 detectors uncorrelated.

- Will *not* work with squeezed light (shot noise in two measurements correlated).

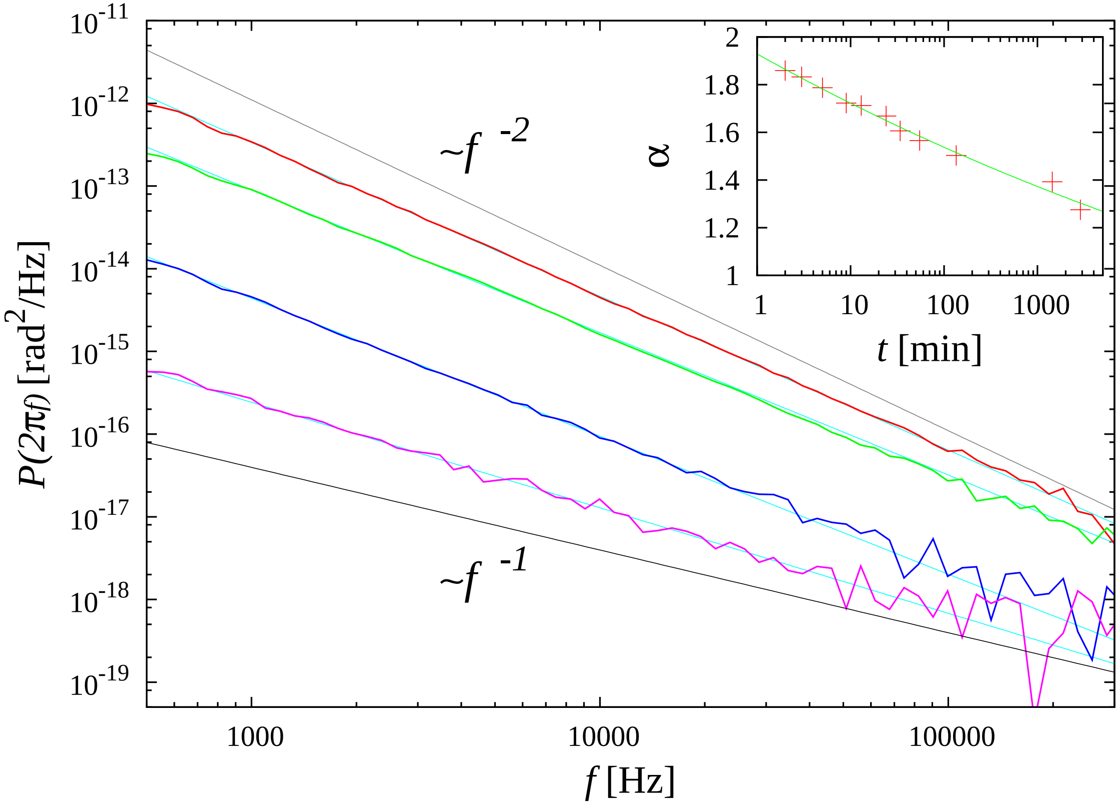
Surface inclination fluctuation measurements



- Basic concept: Surface = optical lever.
- Measure the fluctuation spectrum of the inclination.

- Dual PD for an inclination measurement.
- 2×Dual PD for correlation measurement ⇒ Noise reduction

Observed spectrum: $$S(f) = \int_0^\infty dk\, k^3 e^{-b^2k^2/8} P(k, 2\pi f)$$



Time dependent spectra

- Epoxy adhesive example.
  ← Gradually hardens
- Spectra

  $S(f) \sim f^{-\alpha}$, α: time dep.

  - α : $2 \longrightarrow 1$
  - $\sim f^{-2}$: Viscous liquid surface
  - $\sim f^{-1}$: Solid surface
- Why? $\alpha(t)$?

Discussion

- Directly measured surface thermal fluctuation spectra of ordinary materials at ordinary temperatures (simple liquids, biological materials, complex fluids, polymers, $\cdots$)
- Modification to standard measurements minimal $\Rightarrow$ broad area of application
- Noise reduction: not limited to optical, surfaces, thermal fluctuations.
- Achieved 10^{-3} reduction or more.
- Small power $\sim 0.5\,\mathrm{mW}$, small sample size $\sim$ few μm, short measurement time $\sim$ few sec. $\leftarrow$ wide applicability

- To do:
  - □ Understand surface fluctuation properties of various materials.
  - □ Applications in other areas, other than optical, surfaces measurements.
  - □ More to come$\cdots$