Creation of D9-brane—anti-D9-brane Pairs from Hagedorn Transition of Closed Strings

~ Cylinder Amplitude and Sphere Amplitude

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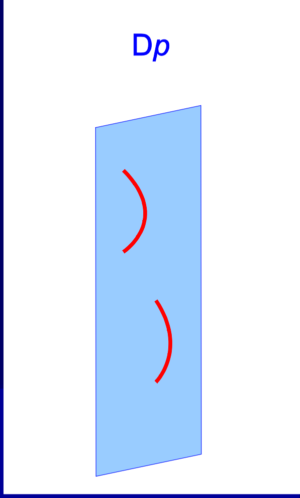
Hokkaido Univ.
Kenji Hotta

1. Introduction



- Hagedorn Temperature T_H

  Type II superstring in 10 dim.

  maximum temperature for perturbative strings

  A single energetic string captures most of the energy.

$$
\begin{aligned}
d_n &\sim e^{2\pi\sqrt{2n}} \\
\Omega(E) &\sim e^{\beta_H E} \\
Z(\beta) &= \int_0^\infty dE\ \Omega(E)\ e^{-\beta E}
\end{aligned}
$$

$$\beta_H \equiv \frac{1}{T_H} = 2\pi\sqrt{2\alpha'}$$

$$Z(\beta) \to \infty \quad \text{for} \quad \beta < \beta_H$$

- Hagedorn Transition of Closed Strings

  $Z(\beta) \to \infty$ for $T > T_H$ (Matsubara Method)

  winding tachyon in the Euclidean time direction

  Hagedorn Transition (Sathiapalan, Kogan, Atick-Witten)

  A phase transition takes place due to the condensation of tachyon fields. (stable minimum?)

- Brane-antibrane Pair Creation Transition

  Dp-$\overline{\text{D}p}$ Pairs are unstable at zero temperature

  finite temperature system of Dp-$\overline{\text{D}p}$ Pairs

  → D9-$\overline{\text{D9}}$ pairs become stable near the Hagedorn temperature.

Hotta

- Thermodynamic Balance on D9-$\overline{\text{D9}}$ Pair

  (open string ⟷ closed string) $\mathcal{T}_{open} = \mathcal{T}_{closed}$

  - open strings

    We need an infinite energy to reach $\mathcal{T}_H$.

  - closed strings

    We can reach $\mathcal{T}_H$ by supplying a finite energy.

  Energy flows from closed strings to open strings.

  → Open strings dominate the total energy

- Relation between two phase transitions?

  From above arguments we conjecture that

  D9-$\overline{\text{D9}}$ Pairs are created
  by the Hagedorn transition of closed strings.

2. Hagedorn Transition of Closed Strings

- Matsubara Method (type II)

  Euclidean time with period $\beta = \frac{1}{T}$

  $$Z = \mathrm{Tr}\ e^{-\beta H}$$

  ideal closed string gas → 1-loop (torus worldsheet)

  $Z(\beta) \to \infty$ for $T > T_H$.

- Winding Tachyon

  This divergence comes from `winding mode' in the Eucledean time direction

  $$w = \pm 1 \qquad M^2 = \frac{2}{\alpha'} \frac{\beta^2 - {\beta_H}^2}{{\beta_H}^2}$$

  winding tachyon in the Euclidean time direction

Hagedorn Transition (Sathiapalan, Kogan, Atick-Witten)

A phase transition takes place due to the

→ condensation of tachyon fields.

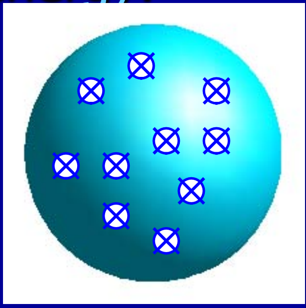
We have not known the stable minimum of potential of this winding tachyon.

Winding Tachyon Condensation Atick-Witten

For $\mathcal{T} < \mathcal{T}_H$ sphere worldsheet does not contribute to free energy.

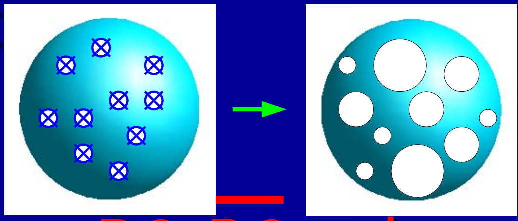
↕

For $\mathcal{T} > \mathcal{T}_H$ sphere worldsheet contributes to free energy.



insertion of winding tachyon vertex

→ creation of a tiny hole in the worldsheet which wraps around Euclidean time



boundary of a hole ↔ boundary of open string on D9-$\overline{D9}$ pair

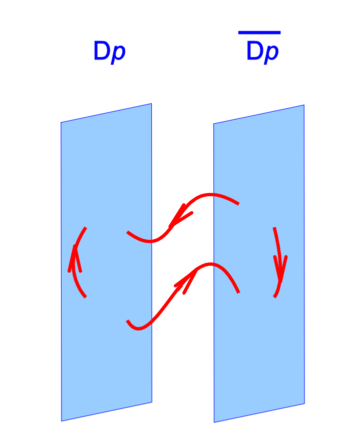
3. Brane-anti-brane Creation Transition

- Dp-$\overline{\mathrm{D}p}$ Pair

  unstable at zero temperature

  open string tachyon tachyon potential

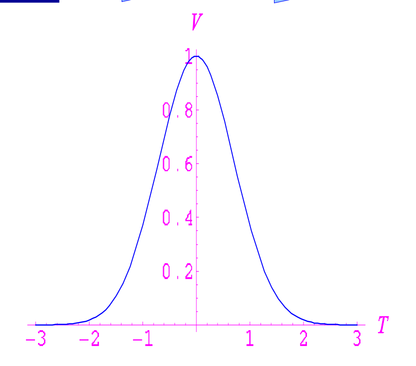
  Sen's conjecture potential height=brane tension



- Tachyon Potential of Dp-$\overline{\mathrm{D}p}$ (BSFT)

  tree level tachyon potential (disk worldsheet)

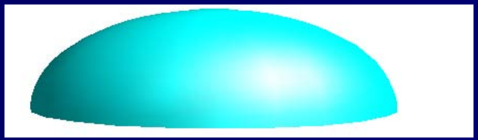
$$V(T) = 2\tau_p \mathcal{V}_p \exp(-8|T|^2),$$



T : complex scalar field τ_p : tension of Dp-brane

$g_s = e^{\phi}$: coupling of strings $\mathcal{V}_p$: p-dim. volume

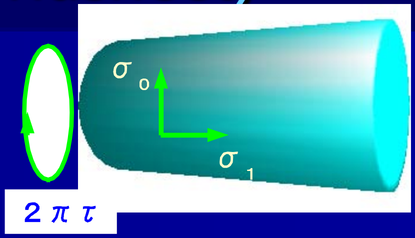
$$\tau_p = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}} g_s}$$



■ Free Energy of Open Strings on Dp-$\overline{\mathrm{D}p}$ Pair

Euclidean time with period $\beta = \frac{1}{T}$

ideal open string gas → 1-loop (cylinder worldsheet)



$$F(T,\beta) = -\frac{16\pi^4 \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau^{\frac{p+3}{2}}} e^{-4\pi|T|^2\tau} \times \left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left(\vartheta_3\left(0 \middle| \frac{i\beta^2}{{\beta_H}^2\tau}\right) - 1 \right) - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left(\vartheta_4\left(0 \middle| \frac{i\beta^2}{{\beta_H}^2\tau}\right) - 1 \right) \right]$$

■ Finite Temperature Effective Potential

$$V(T,\beta) = V(T) + F(T,\beta)$$

cf) We cannot trust the canonical ensemble method

→ microcanonical ensemble method

- Brane-anti-brane Pair Creation Transition
  - N D9-$\overline{\text{D9}}$ Pairs

$|T|^2$ term of finite temperature effective potential

$$\left[-16N\tau_9\mathcal{V}_9+\frac{8\pi N^2\mathcal{V}_9}{\beta_H{}^{10}}\ln\left(\frac{\pi\beta_H{}^{10}E}{2N^2\mathcal{V}_9}\right)\right]|T|^2.$$

Critical temperature

$$\mathcal{T}_c\simeq\beta_H{}^{-1}\left[1+\exp\left(-\frac{\beta_H{}^{10}\tau_9}{\pi N}\right)\right]^{-1}.$$

Above $\mathcal{T}_c$, $T=0$ becomes the potential minimum.

→ A phase transition occurs and D9-$\overline{\text{D9}}$ pairs become stable.

$\mathcal{T}_c$ is a decreasing function of N.

→ Multiple D9-$\overline{\text{D9}}$ pairs are created simultaneously.

- N Dp-$\overline{\text{D}p}$ Pairs with $p\leq 8$

No phase transition occurs.

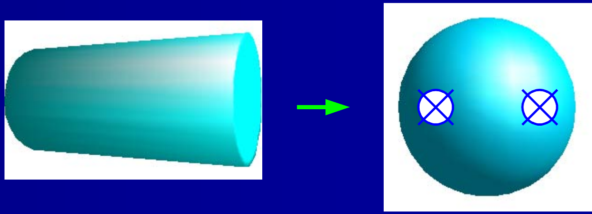
4. Correspondence in the Closed String Vacuum Limit

- Free Energy near the Closed String Vacuum

large $|T|$ limit → small τ limit → small hole limit

$$F(T,\beta) = -\frac{16\pi^4\mathcal{V}_9}{{\beta_H}^{10}}\int_0^\infty dt\ \exp\left(-\frac{4\pi|T|^2}{t}\right)$$
$$\times\left[\left(\frac{\vartheta_3(0|it)}{{\vartheta_1}'(0|it)}\right)^4\left(\vartheta_3\left(0\left|\frac{i\beta^2 t}{{\beta_H}^2}\right.\right)-1\right)\right.$$
$$\left.-\left(\frac{\vartheta_4(0|it)}{{\vartheta_1}'(0|it)}\right)^4\left(\vartheta_4\left(0\left|\frac{i\beta^2 t}{{\beta_H}^2}\right.\right)-1\right)\right]$$

$$\tau = \frac{1}{t}$$



leading term for large t (small τ)

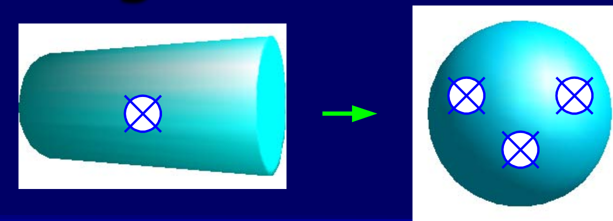
$$F(T,\beta) \simeq -\frac{4\mathcal{V}_9}{{\beta_H}^{10}}\int_0^\infty dt\ \exp\left[-\frac{4\pi|T|^2}{t}-\pi\frac{\beta^2-\beta_H^2}{{\beta_H}^2}\,t\right]$$

propagator of winding tachyon if we ignore $|T|^2$ part

momentum 0 ← momentum conservation for Neumann direction

Cylinder with a Massless Closed String Insertion

$$S^1_{C_2} = \int_0^\infty \frac{dt}{2t} \left\langle cb\mathcal{V}_{e_{\mu\nu}} \right\rangle_{C_2}$$



$$\Rightarrow \; - \frac{8\pi(2\pi)^{10} g_s}{\alpha'} e_{00}$$

$\mathcal{T} \to \mathcal{T}_H$

$|T| \to \infty$ $$= \left\langle c\tilde{c}\mathcal{V}_{w=+1}(z_1) c\tilde{c}\mathcal{V}_{w=-1}(z_2) c\tilde{c}\mathcal{V}_{e_{\mu\nu}}(z_3) \right\rangle_{S_2}$$

closed string massless boson $$\mathcal{V}_{e_{\mu\nu}} = \frac{2g_s}{\alpha'} \, e_{\mu\nu} : \partial X^\mu_L \bar{\partial} X^\nu_R :$$

winding tachyon $$\mathcal{V}_{w=\pm 1} = g_s e^{-\phi-\tilde{\phi}} : e^{\pm i\sqrt{\frac{2}{\alpha'}} X^0_L(z) \mp i \sqrt{\frac{2}{\alpha'}} X^0_R(\bar{z})}$$

Cylinder with Two Winding Tachyon Insertion

$$S^2_{C_2} = \left\langle D9 - \overline{D9} \right| \Delta(t_1) \mathcal{V}^{0,0}_{w=+1} \Delta(t_2, \phi) \mathcal{V}^{0,0}_{w=-1} \Delta(t_3) \left| D9 - \overline{D9} \right\rangle$$

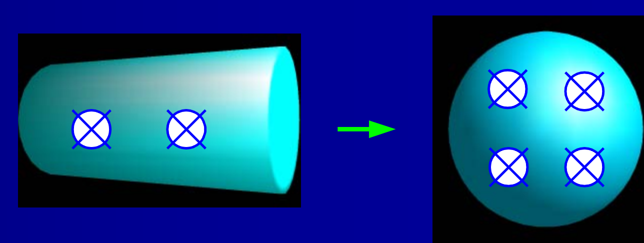
$$\Rightarrow \; g_s^2 \int d^2 z_4 \, \frac{|1-z_4|^2}{|z_4|^2}$$

$\mathcal{T} \to \mathcal{T}_H$

$|T| \to \infty$ $$= g_s^{-2} \int dz_4 \left\langle \tilde{c}c\mathcal{V}^{-1,-1}_{w=+1}(z_1.\bar{z}_1) \tilde{c}c\mathcal{V}^{-1,-1}_{w=-1}(z_2.\bar{z}_2) \tilde{c}c\mathcal{V}^{0,0}_{w=+1}(z_3.\bar{z}_3) \mathcal{V}^{0,0}_{w=-1}(z_4.\bar{z}_4) \right\rangle_{S_2}$$

closed string propagator

$$\Delta(t,\phi) = \frac{1}{4\pi} \int_0^\infty dt \int_0^{2\pi} d\phi \; e^{-t(L_0 + \tilde{L}_0 - 2)} e^{i\phi(L_0 - \tilde{L}_0)}$$



5. Stable Minimum near the Hagedorn Temperature

- Atick-Witten

Atick and Witten have looked for the potential minimum at closed string vacuum $|T| \to \infty$.

We now aware the space of open string tachyon field.

It is reasonable to look for the stable minimum in entire space of open string tachyon field.

- Potential Energy at Open String Vacuum $|T| = 0$

$$V_{o,eff}\left(\mathbf{T}=\mathbf{0},\beta\right) \simeq F_o\left(\mathbf{T}=\mathbf{0},\beta\right) \simeq -\frac{2N^2\mathcal{V}_9}{\pi\beta_H{}^9\left(\beta-\beta_H\right)}.$$

This potential energy decreases limitlessly as $\beta \to \beta_H$.

It is natural to think that the open string vacuum becomes the global minimum near the Hagedorn temperature.

6. Conclusion and Discussion

- Conjecture

  $D9\text{-}\overline{D9}$ Pairs are created

  by the Hagedorn transition of closed strings.

- Correspondence in the closed string vacuum limit

  Cylinder amplitude with some vertex insertion corresponds to sphere amplitude with two more winding tachyon insertion.

- Stable Minimum near the Hagedorn Temperature

  It is natural to think that the open string vacuum becomes the global minimum near the Hagedorn temperature.

- Non-perturbative Calculation?

  Matrix model (USp, IIB, BFSS), K-matrix Model

ϑ-functions

$$\vartheta_1{}'(0|\tau) = \pi \sum_{n=-\infty}^{\infty} (-1)^n (2n-1) q^{(n-\frac{1}{2})^2}$$

$$\vartheta_2(0|\tau) = \sum_{n=-\infty}^{\infty} q^{(n-\frac{1}{2})^2}$$

$$\vartheta_3(0|\tau) = \sum_{n=-\infty}^{\infty} q^{n^2}$$

$$\vartheta_4(0|\tau) = \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2}$$

$$q = e^{i\pi\tau}$$

modular transformation

$$\vartheta_1{}'\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{3}{2}}\vartheta_1{}'(0|\tau), \quad \vartheta_3\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}}\vartheta_3(0|\tau)$$

$$\vartheta_2\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}}\vartheta_4(0|\tau), \quad \vartheta_4\left(0\middle|-\frac{1}{\tau}\right) = (-i\tau)^{\frac{1}{2}}\vartheta_2(0|\tau)$$

$$\vartheta_1{}'(0|\tau+1) = e^{\frac{i\pi}{4}}\vartheta_1{}'(0|\tau), \quad \vartheta_2(0|\tau+1) = e^{\frac{i\pi}{4}}\vartheta_2(0|\tau)$$

$$\vartheta_3(0|\tau+1) = \vartheta_4(0|\tau), \quad \vartheta_4(0|\tau+1) = \vartheta_3(0|\tau)$$