Creation of D9-brane—anti-D9-brane Pairs from Hagedorn Transition of Closed Strings ~ Cylinder Amplitude and Sphere Amplitude

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1. Introduction

- **Hagedorn Temperature** $\mathcal{T}_H$

  Type II superstring in 10 dim.

  maximum temperature for perturbative strings

  A single energetic string captures most of the energy.

\[
d_n \sim e^{2\pi\sqrt{2n}}
\]

\[
\Omega(E) \sim e^{\beta_H E}
\]

\[
Z(\beta) = \int_0^\infty dE \, \Omega(E) \, e^{-\beta E}
\]

\[
\beta_H \equiv \frac{1}{\mathcal{T}_H} = 2\pi\sqrt{2\alpha'}
\]

\[
Z(\beta) \to \infty \quad \text{for} \quad \beta < \beta_H
\]
Hagedorn Transition of Closed Strings

\[ Z(\beta) \rightarrow \infty \text{ for } T > T_H \] (Matsubara Method)

winding tachyon in the Euclidean time direction

Hagedorn Transition (Sathiapalan, Kogan, Atick-Witten)

A phase transition takes place due to the condensation of tachyon fields. (stable minimum?)

Brane-antibrane Pair Creation Transition

\( D_p\overline{D_p} \) Pairs are unstable at zero temperature

finite temperature system of \( D_p\overline{D_p} \) Pairs

\( \rightarrow \) D9-\( \overline{D9} \) pairs become stable near the Hagedorn temperature. Hotta
Thermodynamic Balance on D9-D9 Pair

(open string ↔ closed string) \( \mathcal{T}_{open} = \mathcal{T}_{closed} \)

- **open strings**
  - We need an infinite energy to reach \( \mathcal{T}_H \).

- **closed strings**
  - We can reach \( \mathcal{T}_H \) by supplying a finite energy.

Energy flows from closed strings to open strings.

⇒ Open strings dominate the total energy

Relation between two phase transitions?

From above arguments we conjecture that

D9-D9 Pairs are created by the Hagedorn transition of closed strings.
2. Hagedorn Transition of Closed Strings

- **Matsubara Method** (type II)
  
  Euclidean time with period \( \beta = \frac{1}{T} \)
  
  Ideal closed string gas \( \rightarrow \) 1-loop \((\text{torus worldsheet})\)
  
  \( Z(\beta) \rightarrow \infty \) for \( T > T_H \).

- **Winding Tachyon**

  This divergence comes from `winding mode’
  in the Euclidean time direction

  \[
  w = \pm 1 \quad \quad M^2 = \frac{2 \beta^2 - \beta_H^2}{\alpha'} \frac{1}{\beta_H^2}
  \]

  Winding tachyon in the Euclidean time direction
- **Hagedorn Transition** (Sathiapalan, Kogan, Atick-Witten)
  
  A phase transition takes place due to the condensation of tachyon fields. We have not known the stable minimum of potential of this winding tachyon.

- **Winding Tachyon Condensation** Atick-Witten
  
  For $\mathcal{T} < \mathcal{T}_H$ sphere worldsheet does not contribute to free energy.
  
  For $\mathcal{T} > \mathcal{T}_H$ sphere worldsheet contributes to free energy.

  insertion of winding tachyon vertex

  $\rightarrow$ creation of a tiny hole in the worldsheet which wraps around Euclidean time

  boundary of a hole $\leftrightarrow$ boundary of open string on D9-D9 pair
3. Brane-anti-brane Creation Transition

- **Dp-Db Pair**
  - unstable at zero temperature
  - open string tachyon tachyon potential
  - Sen’s conjecture potential height=brane tension

- **Tachyon Potential of Dp-Db (BSFT)**
  - tree level tachyon potential (disk worldsheet)
  - $V(T) = 2\tau_p \nu_p \exp(-8|T|^2)$,
  - $\tau_p$ : tension of Dp-brane
  - $\nu_p$ : p-dim. volume
  - $g_s = e^\phi$ : coupling of strings
  - $\nu_p$ : $d$ of strings
  - $\nu_p$ : $d$ of strings
Free Energy of Open Strings on $Dp$-$\overline{Dp}$ Pair

Euclidean time with period $\beta = \frac{1}{T}$

Ideal open string gas $\rightarrow$ 1-loop (cylinder worldsheet)

$$F(T, \beta) = -\frac{16\pi^4 V_p}{\beta_H^{p+1}} \int_0^{\infty} \frac{d\tau}{\tau^{p+3}} e^{-4\pi |T|^2 \tau} \left[ \left( \frac{\vartheta_3(0|i\tau)}{\vartheta_1'(0|i\tau)} \right)^4 \left( \vartheta_3 \left( 0 \left| \frac{i\beta^2}{\beta_H^2 \tau} \right. \right) - 1 \right) \right. \\
- \left. \left( \frac{\vartheta_2(0|i\tau)}{\vartheta_1'(0|i\tau)} \right)^4 \left( \vartheta_4 \left( 0 \left| \frac{i\beta^2}{\beta_H^2 \tau} \right. \right) - 1 \right) \right]$$

Finite Temperature Effective Potential

$$V(T, \beta) = V(T) + F(T, \beta)$$

cf) We cannot trust the canonical ensemble method

$\rightarrow$ microcanonical ensemble method
Brane-anti-brane Pair Creation Transition

- $N$ D9-\( \overline{\text{D9}} \) Pairs
  - $|T|^2$ term of finite temperature effective potential
  
  $$-16NT_9\nu_9 + \frac{8\pi N^2\nu_9}{\beta_H^{10}} \ln \left( \frac{\pi \beta_H^{10} E}{2N^2\nu_9} \right) |T|^2.$$  

Critical temperature

$$T_c \simeq \beta_H^{-1} \left[ 1 + \exp \left( -\frac{\beta_H^{10}T_9}{\pi N} \right) \right]^{-1}.$$  

Above $T_c$, $T = 0$ becomes the potential minimum.

- A phase transition occurs and D9-\( \overline{\text{D9}} \) pairs become stable.
- $T_c$ is a decreasing function of $N$.

- Multiple D9-\( \overline{\text{D9}} \) pairs are created simultaneously.

- $N$ D$p$-\( \overline{\text{D}p} \) Pairs with $p \leq 8$
  - No phase transition occurs.

- D9-\( \overline{\text{D}p} \) Pairs with $p > 8$
4. Correspondence in the Closed String Vacuum Limit

Free Energy near the Closed String Vacuum

large $|T|$ limit $\Rightarrow$ small $\tau$ limit $\Rightarrow$ small hole limit

$$F(T, \beta) = -\frac{16\pi^4 \nu_9}{\beta_H^{10}} \int_0^\infty dt \, \exp\left(-\frac{4\pi |T|^2}{t}\right)$$

$$\times \left[ \left( \frac{\varphi_3(0|it)}{\varphi_1'(0|it)} \right)^4 \left( \varphi_3 \left( 0 \left| \frac{i\beta^2 t}{\beta_H^2} \right. \right) - 1 \right) - \left( \frac{\varphi_4(0|it)}{\varphi_1'(0|it)} \right)^4 \left( \varphi_4 \left( 0 \left| \frac{i\beta^2 t}{\beta_H^2} \right. \right) - 1 \right) \right]$$

leading term for large $t$ (small $\tau$)

$$F(T, \beta) \simeq -\frac{4 \nu_9}{\beta_H^{10}} \int_0^\infty dt \, \exp \left[ -\frac{4\pi |T|^2}{t} - \pi \frac{\beta^2 - \beta_H^2}{\beta_H^2} \frac{t}{t} \right]$$

propagator of winding tachyon if we ignore $|T|^2$ part

momentum 0 $\leftarrow$ momentum conservation for Neumann direction
Cylinder with a Massless Closed String Insertion

\[ S^1_{C_2} = \int_0^\infty \frac{dt}{2t} \left< cb \nu_{e \mu \nu} e_{00} \right> C_2 \]

\[ \tau \rightarrow \tau_H \]

\[ |T| \rightarrow \infty \]

\[ = \left< c \bar{c} \nu_{w=+1} (z_1) c \bar{c} \nu_{w=-1} (z_2) c \bar{c} \nu_{e \mu \nu} (z_3) \right> S_2 \]

closed string massless boson

\[ \nu_{e \mu \nu} = \frac{2g_s}{\alpha'} e_{\mu \nu} : \partial X^\mu_L \partial X^\nu_R : \]

winding tachyon

\[ \nu_{w=\pm 1} = g_s e^{-\phi - \bar{\phi}} e^{\pm i \sqrt{\frac{2}{\alpha'}} X_0^L(z) \mp i \sqrt{\frac{2}{\alpha'}} X_0^R(\bar{z})} \]

Cylinder with Two Winding Tachyon Insertion

\[ S^2_{C_2} = \left< D9 - \bar{D9} \right| \Delta(t_1) \nu_{w=+1}^{0,0} \Delta(t_2, \phi) \nu_{w=-1}^{0,0} \Delta(t_3) \left| D9 - \bar{D9} \right> \]

\[ g_s^2 \int d^2 z_4 \frac{|1 - z_4|^2}{|z_4|^2} \]

\[ \tau \rightarrow \tau_H \]

\[ |T| \rightarrow \infty \]

\[ = g_s^{-2} \int dz_4 \left< \bar{c} c \nu_{w=+1}^{0,0} (z_1, \bar{z}_1) \bar{c} c \nu_{w=-1}^{0,0} (z_2, \bar{z}_2) \bar{c} c \nu_{w=+1}^{0,0} (z_3, \bar{z}_3) \nu_{w=-1}^{0,0} (z_4, \bar{z}_4) \right> S_2 \]

closed string propagator

\[ \Delta(t, \phi) = \frac{1}{4\pi} \int_0^\infty dt \int_0^{2\pi} d\phi \quad e^{-t(L_0 + \bar{L}_0 - 2)} e^{i\phi(L_0 - \bar{L}_0)} \]
5. Stable Minimum near the Hagedorn Temperature

• Atick-Witten

Atick and Witten have looked for the potential minimum at closed string vacuum $|T| \to \infty$. We now aware the space of open string tachyon field. It is reasonable to look for the stable minimum in entire space of open string tachyon field.

• Potential Energy at Open String Vacuum $|T| = 0$

$$V_{o,eff}(T = 0, \beta) \simeq F_o(T = 0, \beta) \simeq -\frac{2N^2 \nu_9}{\pi \beta_H^9 (\beta - \beta_H)}.$$ 

This potential energy decreases limitlessly as $\beta \to \beta_H$. It is natural to think that the open string vacuum becomes the global minimum near the Hagedorn temperature.
6. Conclusion and Discussion

- **Conjecture**
  
  D9-D9 Pairs are created by the Hagedorn transition of closed strings.

- **Correspondence in the closed string vacuum limit**
  
  Cylinder amplitude with some vertex insertion corresponds to sphere amplitude with two more winding tachyon insertion.

- **Stable Minimum near the Hagedorn Temperature**
  
  It is natural to think that the open string vacuum becomes the global minimum near the Hagedorn temperature.

- **Non-perturbative Calculation?**
  
  Matrix model (USp, IIB, BFSS), K-matrix Model
\( \psi \)-functions

\[ \psi_1'(0|\tau) = \pi \sum_{n=-\infty}^{\infty} (-1)^n (2n - 1) q^{(n-\frac{1}{2})^2} \]

\[ \psi_2(0|\tau) = \sum_{n=-\infty}^{\infty} q^{(n-\frac{1}{2})^2} \]

\[ \psi_3(0|\tau) = \sum_{n=-\infty}^{\infty} q^{n^2} \]

\[ \psi_4(0|\tau) = \sum_{n=-\infty}^{\infty} (-1)^n q^n \]

modular transformation

\[ \psi_1'(0 | - \frac{1}{\tau}) = (-i\tau)^{\frac{3}{2}} \psi_1'(0|\tau), \quad \psi_3(0 | - \frac{1}{\tau}) = (-i\tau)^{\frac{1}{2}} \psi_3(0|\tau) \]

\[ \psi_2(0 | - \frac{1}{\tau}) = (-i\tau)^{\frac{1}{2}} \psi_4(0|\tau), \quad \psi_4(0 | - \frac{1}{\tau}) = (-i\tau)^{\frac{1}{2}} \psi_2(0|\tau) \]

\[ \psi_1'(0|\tau + 1) = e^{\frac{i\pi}{4}} \psi_1'(0|\tau), \quad \psi_2(0|\tau + 1) = e^{\frac{i\pi}{4}} \psi_2(0|\tau) \]

\[ \psi_3(0|\tau + 1) = \psi_4(0|\tau), \quad \psi_4(0|\tau + 1) = \psi_3(0|\tau) \]

\[ q = e^{i\pi \tau} \]