非平衡相転移・非平衡臨界点のAdS/CFT対応による解析

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Refs. S.N. arXiv:1204.1971 (accepted to **PRL**),
arXiv:1006.4105 (PTP124(2010)1105).

多自由度系の物理学

- 非平衡物理学
- 相転移・相構造・臨界現象

クォーク・ハドロン物理学はもちろん、物性系を含めた
「多粒子系の物理学」の共通の興味

両方をあわせてみる。

非平衡相転移・非平衡臨界現象

この講演では

非平衡状態のみで観測可能な相転移と臨界現象

新奇な非平衡相転移の発見について報告する。

[S.N. arXiv:1204.1971 (accepted to **PRL**)]

非平衡物理学

現代物理学のフロンティアの一つ

非平衡状態の分類

	時間依存性なし（非平衡定常系）	時間依存性あり（非平衡かつ非定常）
線形応答領域（平衡近傍）	線形応答理論や流体力学などが一定の成功を収めている	
線形応答を超えた領域（平衡から離れた領域）	ここを考える	

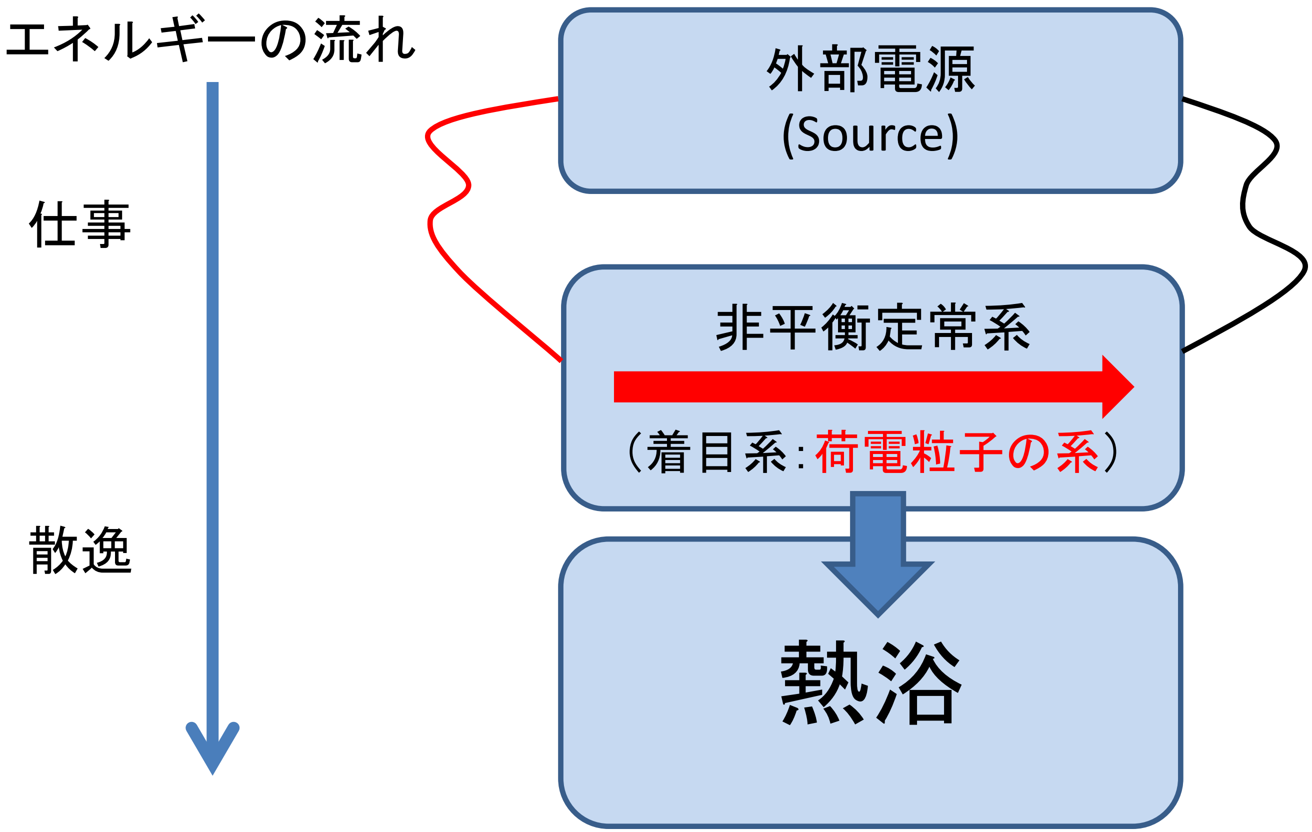
非平衡定常状態　例えば、「定常電流が流れるヒーター」

- エントロピーや熱が生成されており、散逸が存在する非平衡系。
- 巨視的な時間変化がない。

非平衡定常系の設定

非平衡定常系を準備するには、外力と熱浴が必要。

外力：非平衡にドライブする



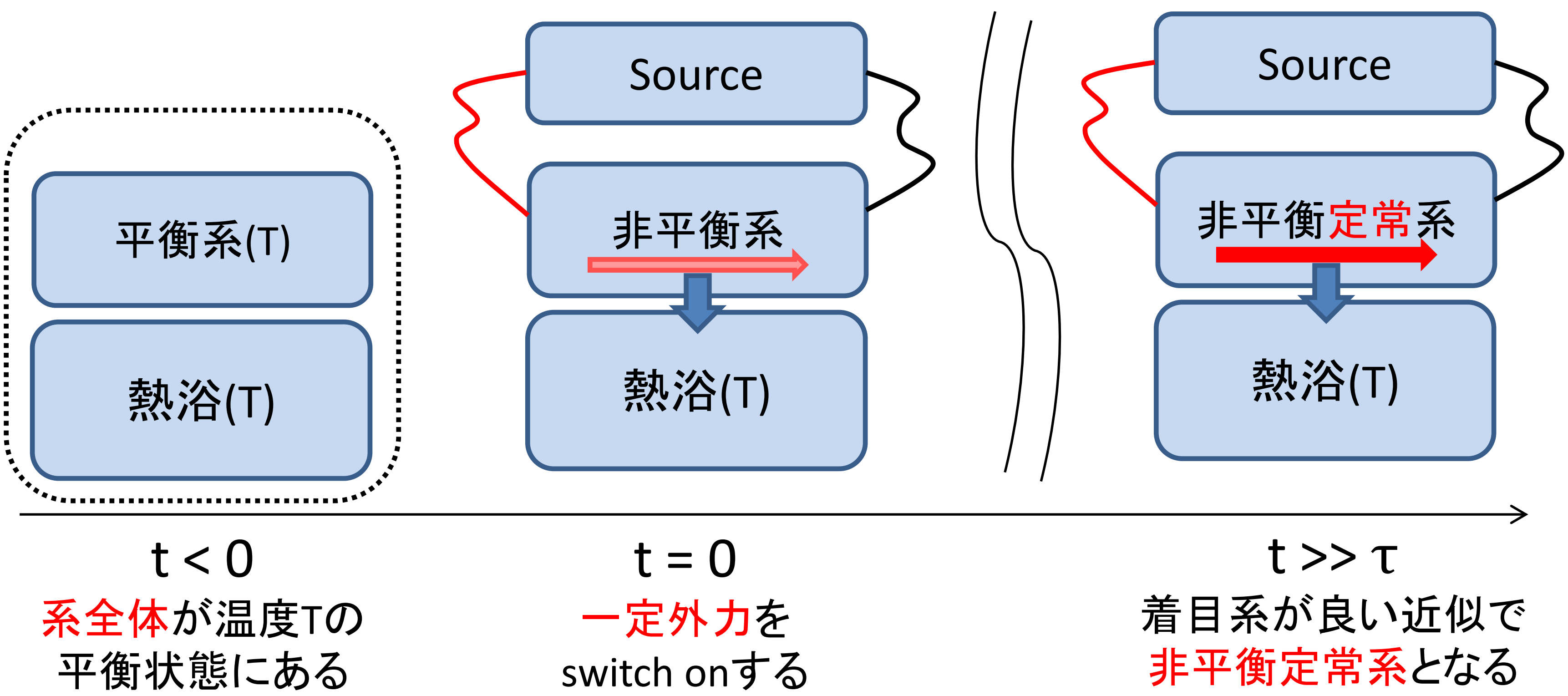
外力の行う仕事と熱浴への散逸がバランスしていれば、
着目系は（非平衡ながら）定常状態となる。

非平衡系の記述

実時間形式の場の理論では:

- 初期状態(典型的には平衡・有限温度)を時間発展させて記述。
- 問題は、この時間発展を相互作用や外力の意味で非摂動的に解けるか？

我々の場合の時間発展としては



非平衡系の記述

AdS/CFT対応 ：実時間の入った非摂動的解析手法

場の量子論の経路積分 ─置き換え→ 重力理論の古典力学

平衡系(T) 荷電粒子の系	“クォーク系” →	D-brane
熱浴(T)	“グルーオン系” →	高次元・負曲率時空内のブラックホール

強結合・Large-Nc
ゲージ理論では

これで熱平衡にある
初期状態を構成できる。

ブラックホール時空上のD-braneの古典力学！

これが、外力存在のもと、
古典的に時間発展する。

AdS/CFTにおける非平衡定常系

- 輸送係数(電気伝導度):
  外力(電場E_x)と応答(電流密度J_x)がわかれば良い。

AdS/CFT対応における標準的な辞書(GKP-W処方)により
D-brane上の5次元的な磁場 F_{xz} から読み取られる。
(Z: 5番目の座標)

- ブラックホール時空内のD-braneの解析力学・電磁気学:
  古典的運動方程式の定常解から得られる。

- ただし、D-braneのon-shell作用がrealとなる条件を課す。

[A. Karch and A. O'Bannon, JHEP0709(2007)024]

強相関電荷系の線形応答を超えた非線形電気伝導度の計算が可能となる。[A. Karch and A. O'Bannon, JHEP0709(2007)024]

AdS/CFTを用いた非平衡系の研究

AdS/CFT対応を適用できるゲージ理論は、Large-Nc、超対称性の名残の存在などにより、現実世界の微視的理論とは厳密には異なる理論である。

しかしながら、微視的理論の差異を超えた所に目標がある。

- 系の微視的な詳細によらず、広い範囲の非平衡定常系で共通に成立する巨視的な物理法則を見出すこと。
- AdS/CFT対応による新しいpictureから非平衡定常系の記述に新たな切り口を見出すこと。

ここでは非平衡定常系の相転移と臨界現象に焦点をあてる。

用いるゲージ理論: D3-D7 system

[A. Karch and E. Katz, JHEP 0206 (2002) 043]

熱浴の役割をはたす “gluon 系” “quark 系” 非平衡定常系

3+1 dim. SU(Nc) N=4 SYM + N=2 hypermultiplet

@ large-Nc , strong-coupling

⇕

ここは温度を入れる 非平衡定常系

AdS_5-BH × S^5 および probe D7-brane

微視的理論のレベルでは現実とは異なるが

・最も標準的なAdS/CFT対応で

・重力の視点で最も単純な

モデルを用いることで、非平衡定常系の

新奇な振る舞いを定性的に探る。

解析結果

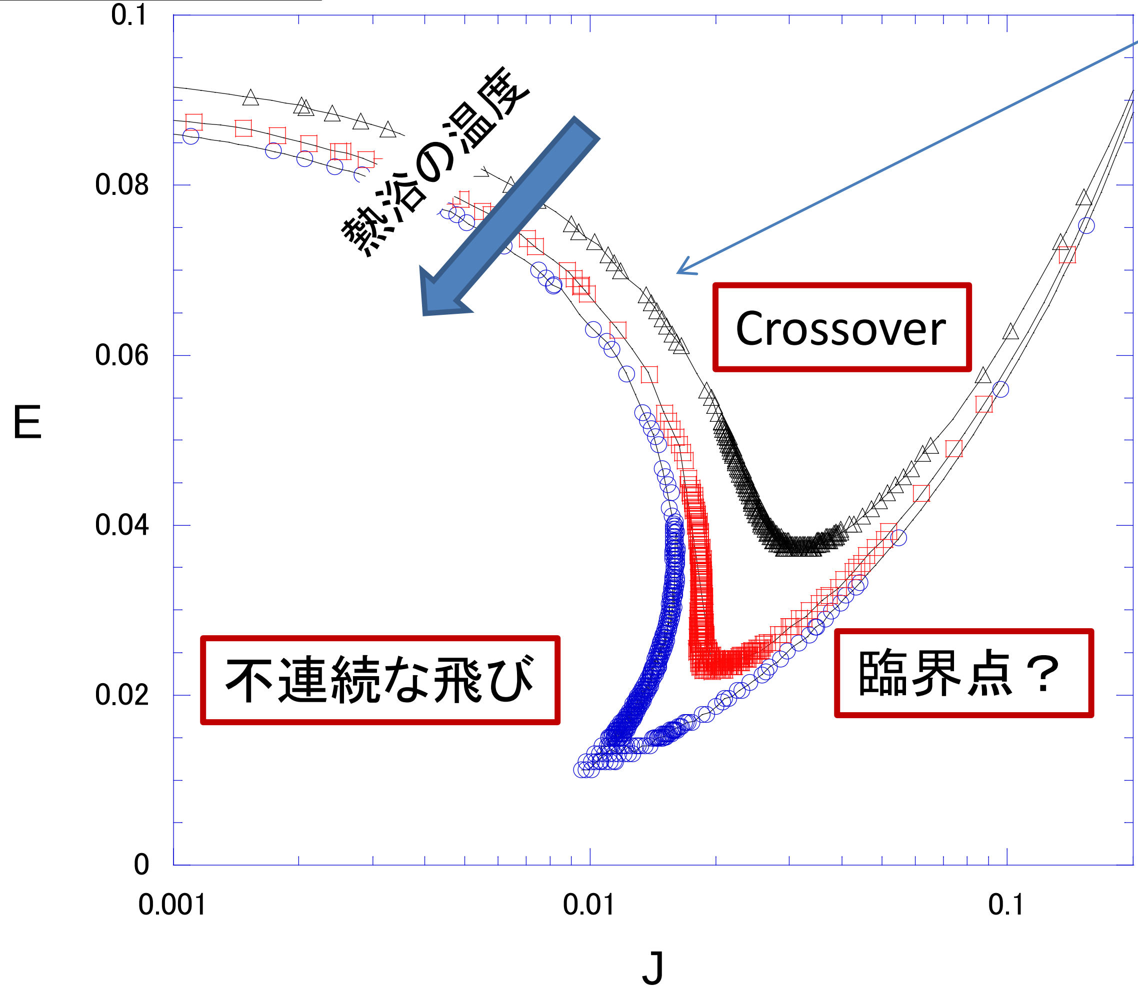
S.N. arXiv:1204.1971   (to appear in PRL)

J-E特性

熱浴の温度

△ : T=0.34337
□ : T=0.34365=Tc
○ : T=0.34379

S.N. PTP124(2010)1105.



負性微分抵抗/
負性微分伝導度

強相関電子系の
絶縁体で一般に
見られる。

(See, e.g.
[Oka, Aoki, arXiv:0803.0422])

$$\lambda = (2\pi)^2/2,$$

$$N_C = 40.$$

コントロールパラメータとして電流を選ぶ（電流駆動型の非平衡現象）

J-E特性

熱浴の温度

△ : T=0.34337
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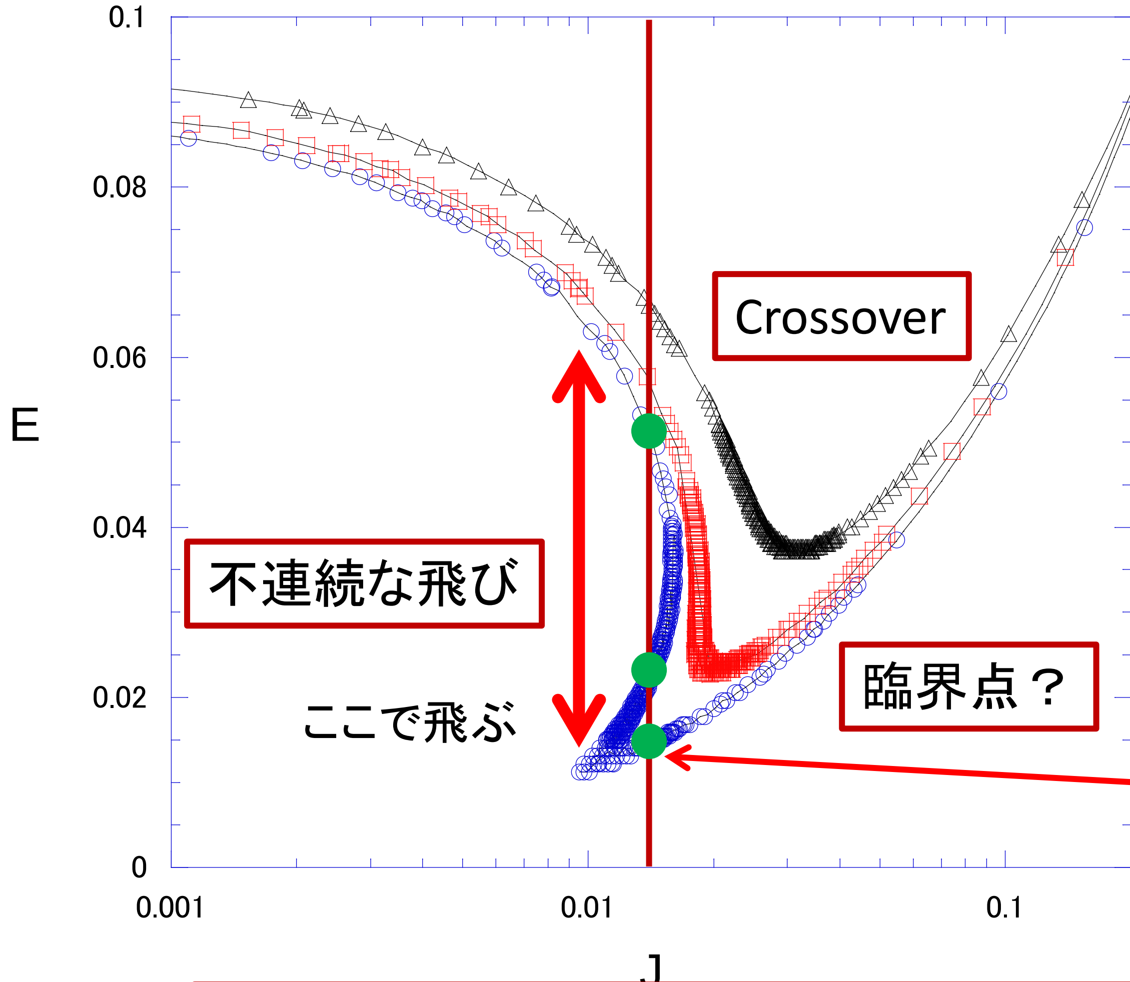


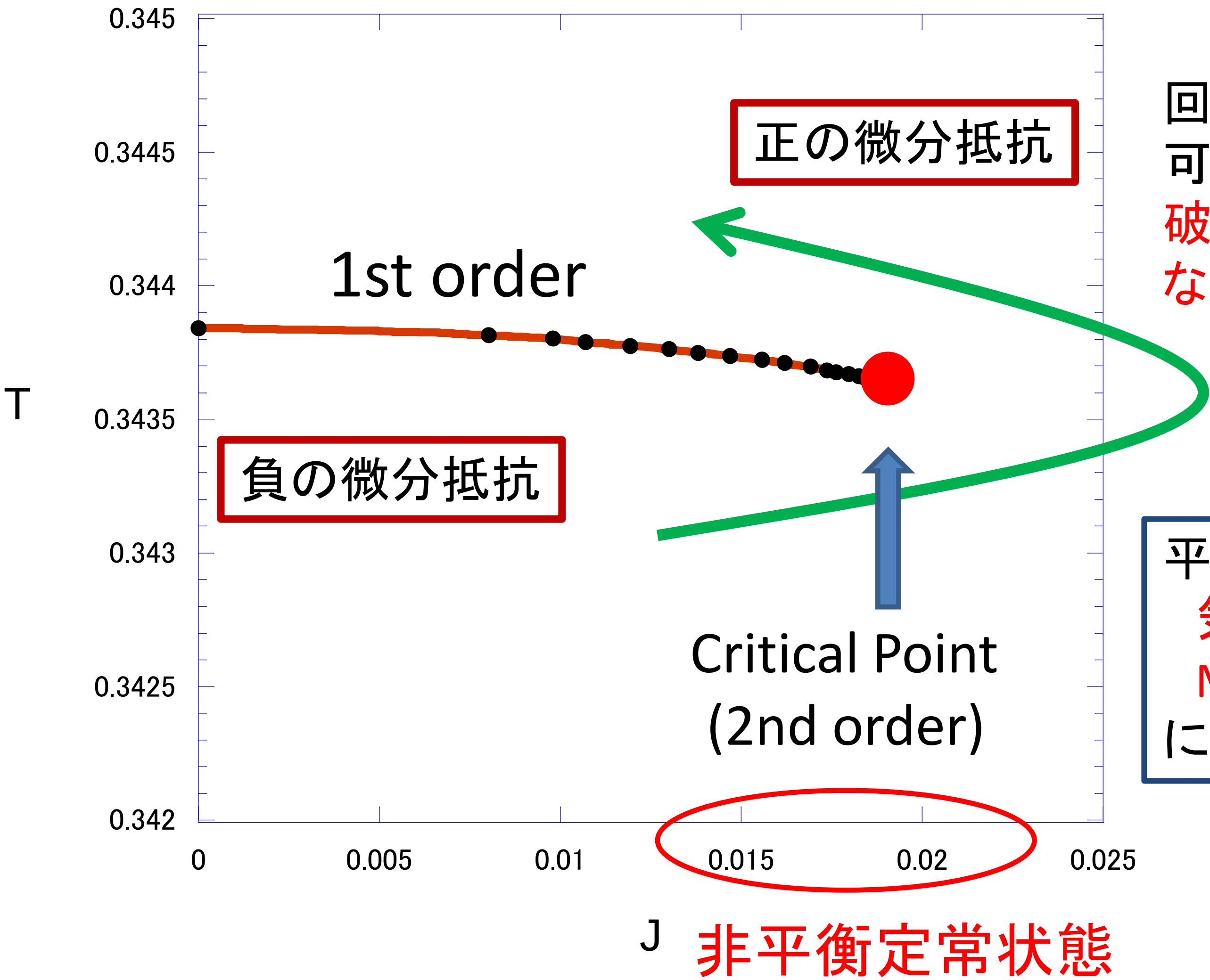
J を指定した
非平衡定常状態
において、どれが
最も安定に実現
されるかという問題。

D7-braneのHamiltonian
を「自由エネルギー」の
非平衡定常系への一般化
として用いることを提案。
[S.N. arXiv:1204.1971]

数値的に調べると
常にこれが最も小さい
Hamiltonianを与える。

少なくとも、この系は散逸が少ない方を好む。

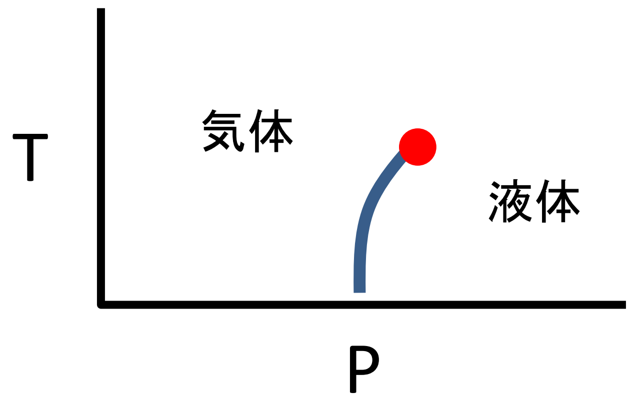
相図



回り込むことが可能：対称性の破れ（回復）はない。

平衡系の
気・液相転移
Mott転移
に似ている。

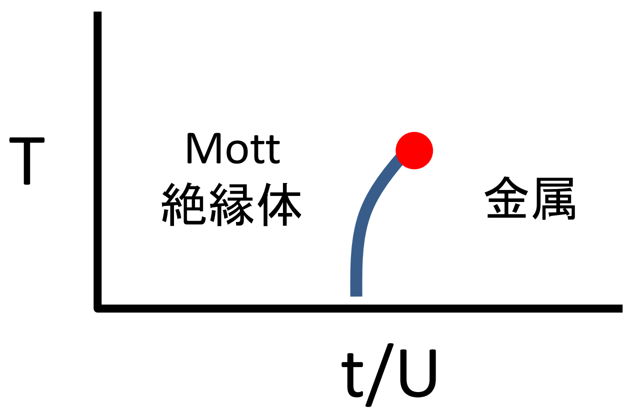
臨界現象（平衡系）



気体ー液体相転移（平衡系）

$$n_L - n_g \propto (T_C - T)^{\beta}$$

密度差



Mott 転移（平衡系）

Mott転移では「密度」のかわりに
伝導度 σ が用いられている。

P. Limelette et al., Science 302 (2003) 89.
F. Kagawa et al.,Nature 436 (2005) 534.

- 両者は同じuniversality class = Ising universality class
- 平均場近似では β=1/2

そこで、以下を提案する:

$$\sigma_{\mathrm{PDC}} - \sigma_{\mathrm{NDC}} \propto \left(T - T_C\right)^{\beta}$$

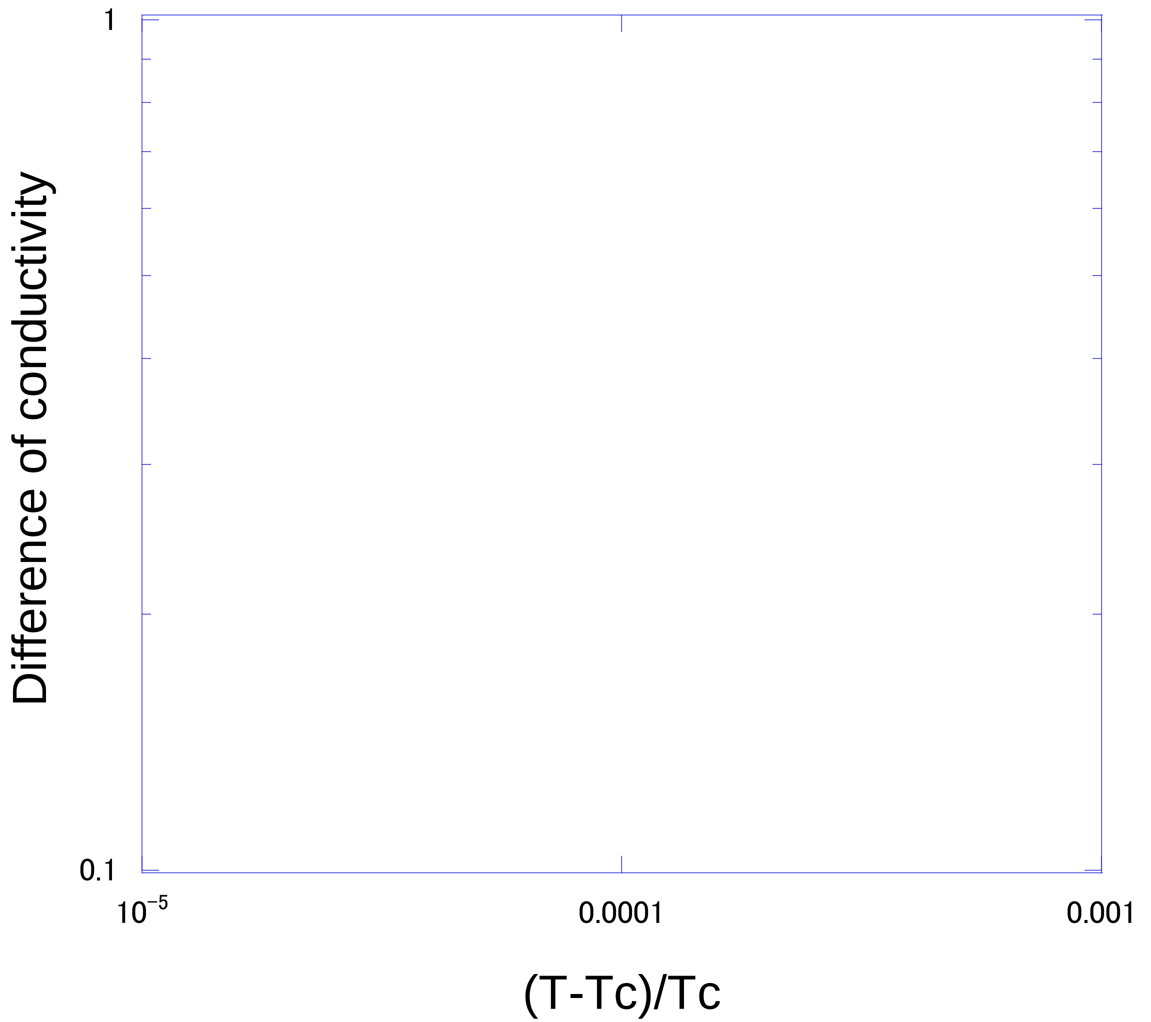
伝導度の差

熱浴の温度

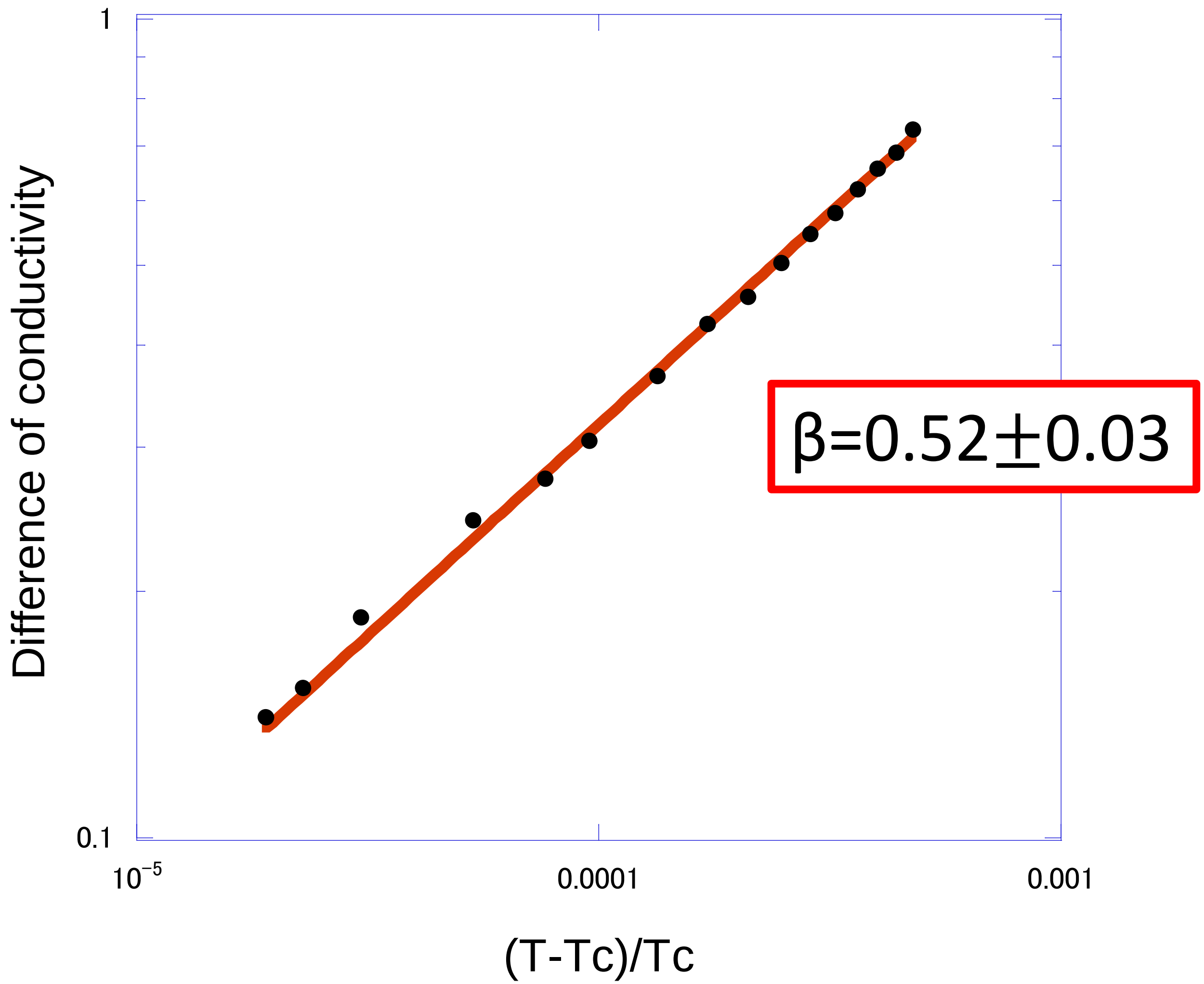
PDC: 正の微分伝導度
NDC: 負の微分伝導度

とりあえず、これを見てみよう。

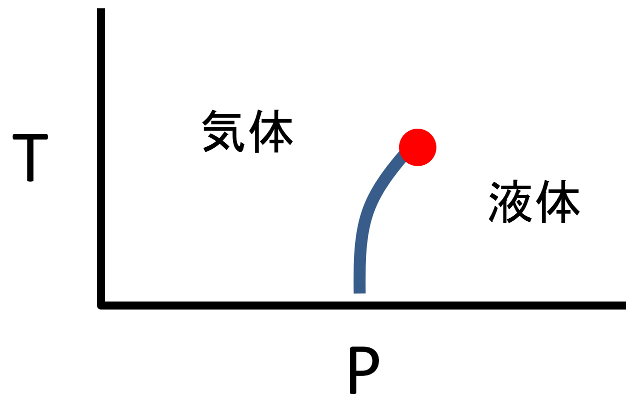
伝導度の飛びの振る舞い



伝導度の飛びの振る舞い



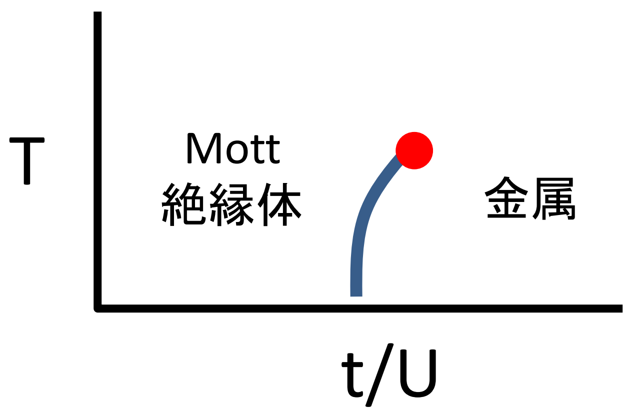
臨界現象（平衡系）



気体ー液体相転移（平衡系）

$$\left|n-n_C\right|_{T=T_C} \propto \left|P-P_C\right|^{1/\delta}$$

密度　　　　圧力



Mott 転移（平衡系）

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伝導度σが用いられている。

P. Limelette et al., Science 302 (2003) 89.
F. Kagawa et al.,Nature 436 (2005) 534.

- 両者は同じuniversality class = Ising universality class
- 平均場近似では $\delta=3$

新しい臨界指数を定義

我々のsetupにはコントロールパラメータとしての
圧力が存在しない。(無限体積系)

手持ちのコントロールパラメータで残っているもの:

J

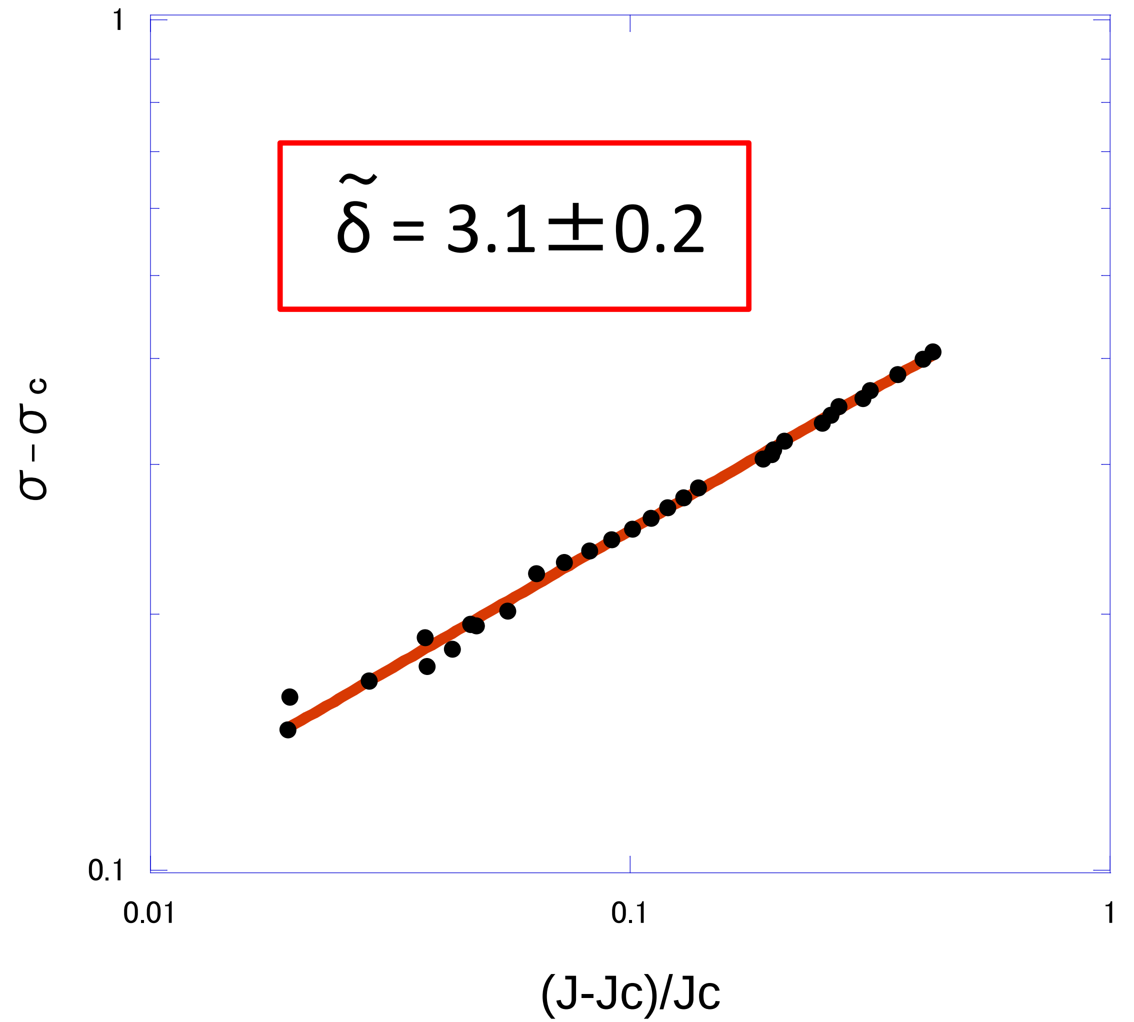
本質的に非平衡系の物理量

提　案

$$\left.\left(\sigma - \sigma_c\right)\right|_{T=Tc} \propto \left|J - J_c\right|^{1/\tilde{\delta}}$$

で臨界指数 $\tilde{\delta}$ を定義する。

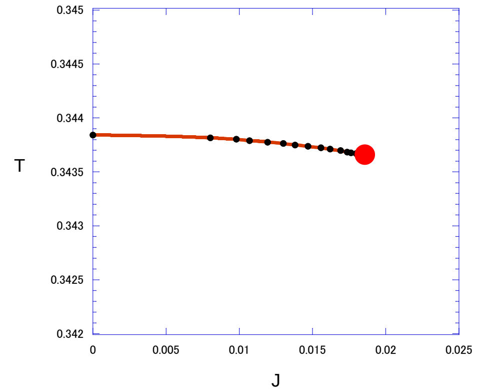
$\tilde{\delta}$ は意味を成すか？



新しい非平衡相転移

S.N. arXiv:1204.1971 (to appear in PRL)

ここで見られたタイプの非平衡相転移・非平衡臨界点
（電流駆動型・非バリスティックな非線形伝導で、
正・負の微分伝導度の間の転移）
は、理論的にも実験的にも知られていなかった。



新しい非平衡相転移・非平衡臨界点を発見したと言えるのではないか。

実験的検証に向けて

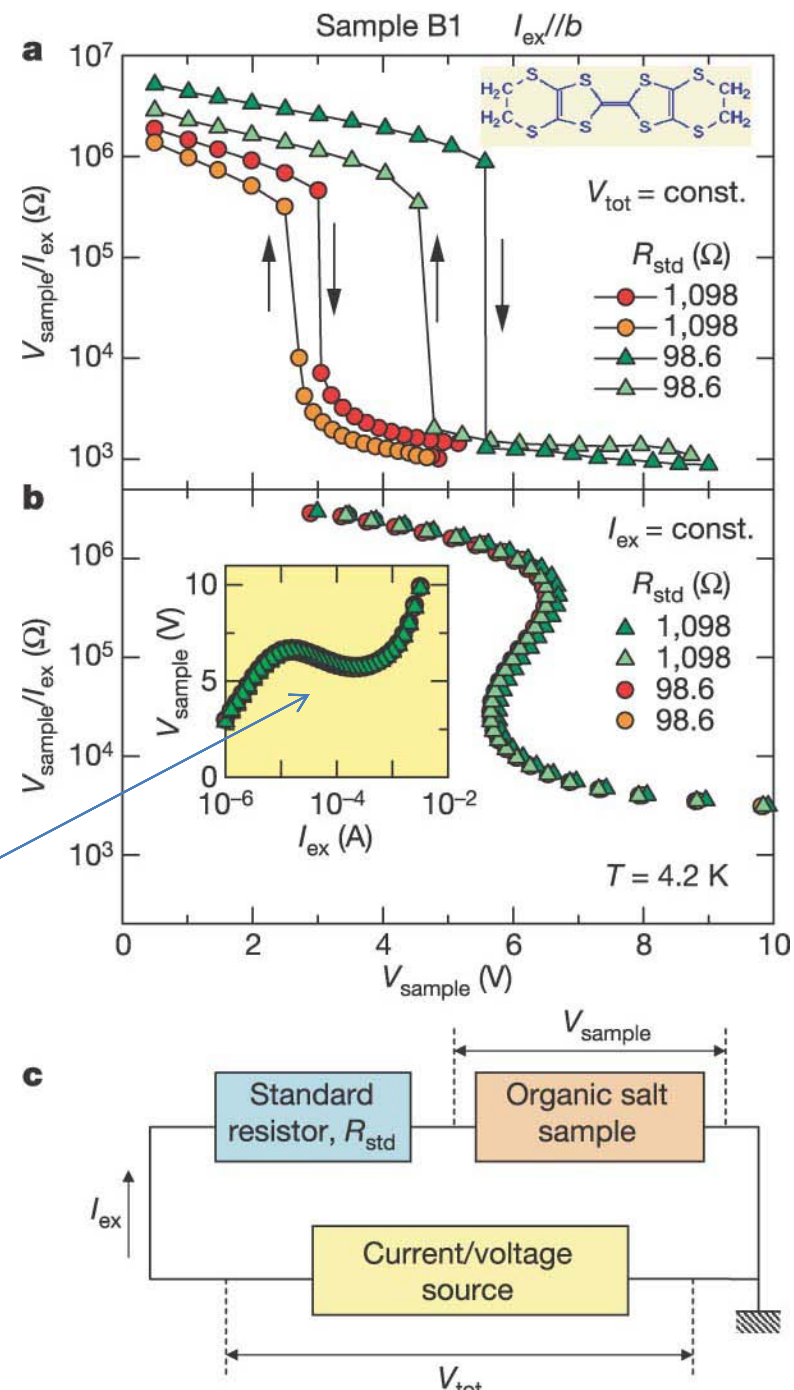
名古屋大学大学院理学研究科　物質理学専攻
機能性物質物性研究室（V研）（寺崎一郎先生）
にてセミナーと意見交換をさせて頂いた。（5/24）

寺崎研での研究内容

θ-(BEDT-TTF)$_2$CsCo(SCN)$_4$
crystal at 4.2 K.

Charge order insulator

F. Sawano, I. Terasaki, et al.,
Nature 437 (2005) 522.



さらに温度上げてどうなるか？

Back up

解析結果

S.N. arXiv:1204.1971 (to appear in PRL)

- 数値的にD7-braneの運動方程式を解いて、新たな発見を探る。

実験的発想：discoveryがあるかどうかが勝負

- Formalismとして新たな提案も行う。
  - 「自由エネルギー」の非平衡定常系への一般化
  - 新たな臨界指数を定義する

GKP-Witten 処方

D7上のU(1)ゲージ場

z=0: boundary

$$A_0(z) = \mu - \frac{(2\pi)^2}{2N_c}\left\langle J^0 \right\rangle z^2 + O(z^4)$$

$$A_x(z) = -Et + \frac{(2\pi)^2}{2N_c}\left\langle J^x \right\rangle z^2 + O(z^4)$$

- ただし、D7-braneのon-shell作用がrealとなる条件を課す。
  [Karch and O'Bannon, 2007]
- D7-braneの形状に関しても運動方程式を解く。

AdS/CFTを用いる利点

非線形伝導度の計算という、非平衡定常系の挑戦的な計算

曲がった時空上のD7-braneの古典力学という単純な問題

Nonequilibrium Phase Transitions and Nonequilibrium Critical Point from AdS/CFT

Shin Nakamura

(Dept. of Phys. Kyoto Univ.)

S.N. arXiv:1204.1971 [hep-th]

The natural units will be used: $\hbar = c = k_B = 1$

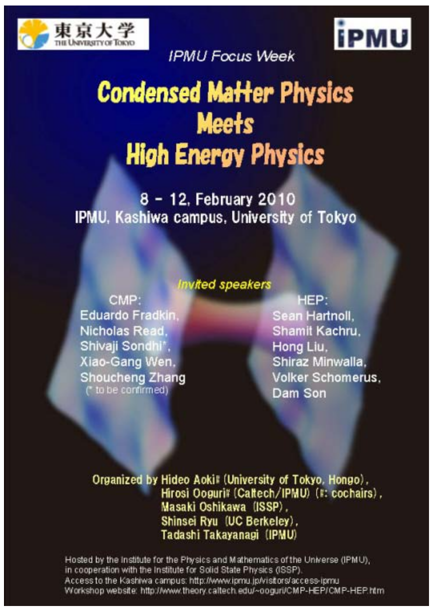
Prologue and Introduction

IPMU Focus Week

Condensed Matter Physics Meets High Energy Physics

hosted by the Institute for the Physics and Mathematics of the Universe (IPMU),
in cooperation with the Institute for Solid State Physics (ISSP).

February 8 - 12, 2010, in the main auditorium of IPMU



Organizers (*: co-chairs)

Hideo Aoki* (Department of Physics, University of Tokyo),
Hirosi Ooguri* (Caltech & IPMU, University of Tokyo),
Masaki Oshikawa (ISSP, University of Tokyo),
Shinsei Ryu (University of California at Berkeley),
Tadashi Takayanagi (IPMU, University of Tokyo).

Taken from http://www.theory.caltech.edu/~ooguri/CMP-HEP/CMP-HEP.htm

A comment from condensed matter physicists

青木秀夫さん　　　　　　　　岡隆史さん

Prof. H. Aoki and Prof. T. Oka (@Univ. of Tokyo) told me an interesting property of strongly-correlated insulators: negative differential conductivity (NDC).

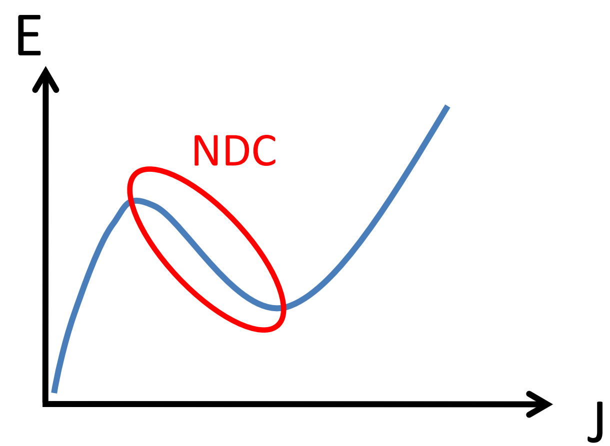
"In correlated insulators (Mott, charge order), negative differential conductivity is widely observed regardless of the dimensions....''

"It would be wonderful if you can reproduce NDC by using the AdS/CFT correspondence....."

What is NDC?

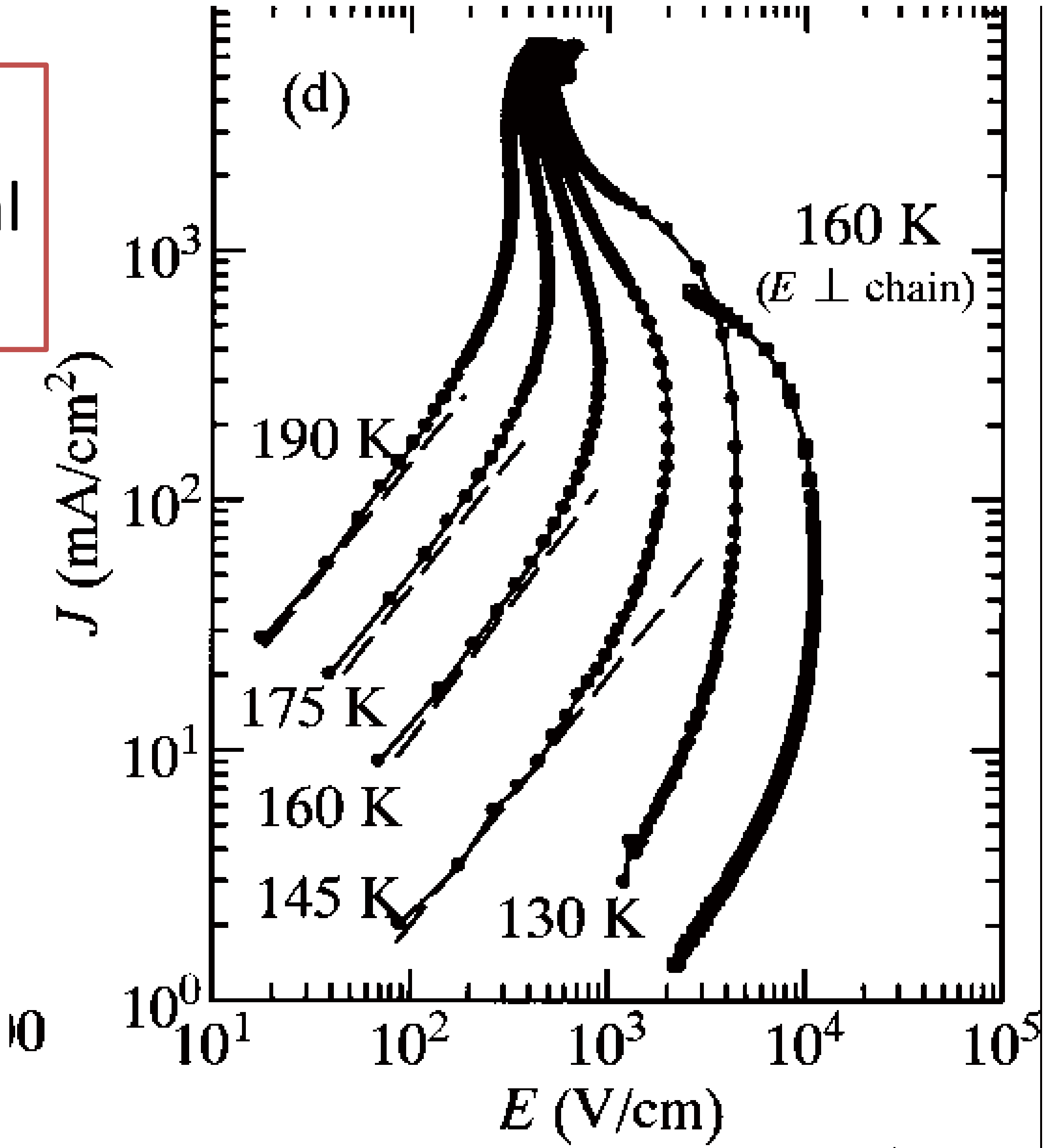
NDC: Negative Differential Conductivity（負性微分伝導度）

NDC: the voltage goes down when the current increases.
Typically, the current can be a multi-valued function of the electric field if we have NDC in correlated insulators.



Example of experimental data:

$SrCuO_2$
(1d Mott)



Y. Taguchi T. Matsumoto and Y. Tokura. *Phys. Rev. B, 62:7015, 2000.*

Example of experimental data:

θ-(BEDT-TTF)$_2$CsCo(SCN)$_4$ crystal at 4.2 K.

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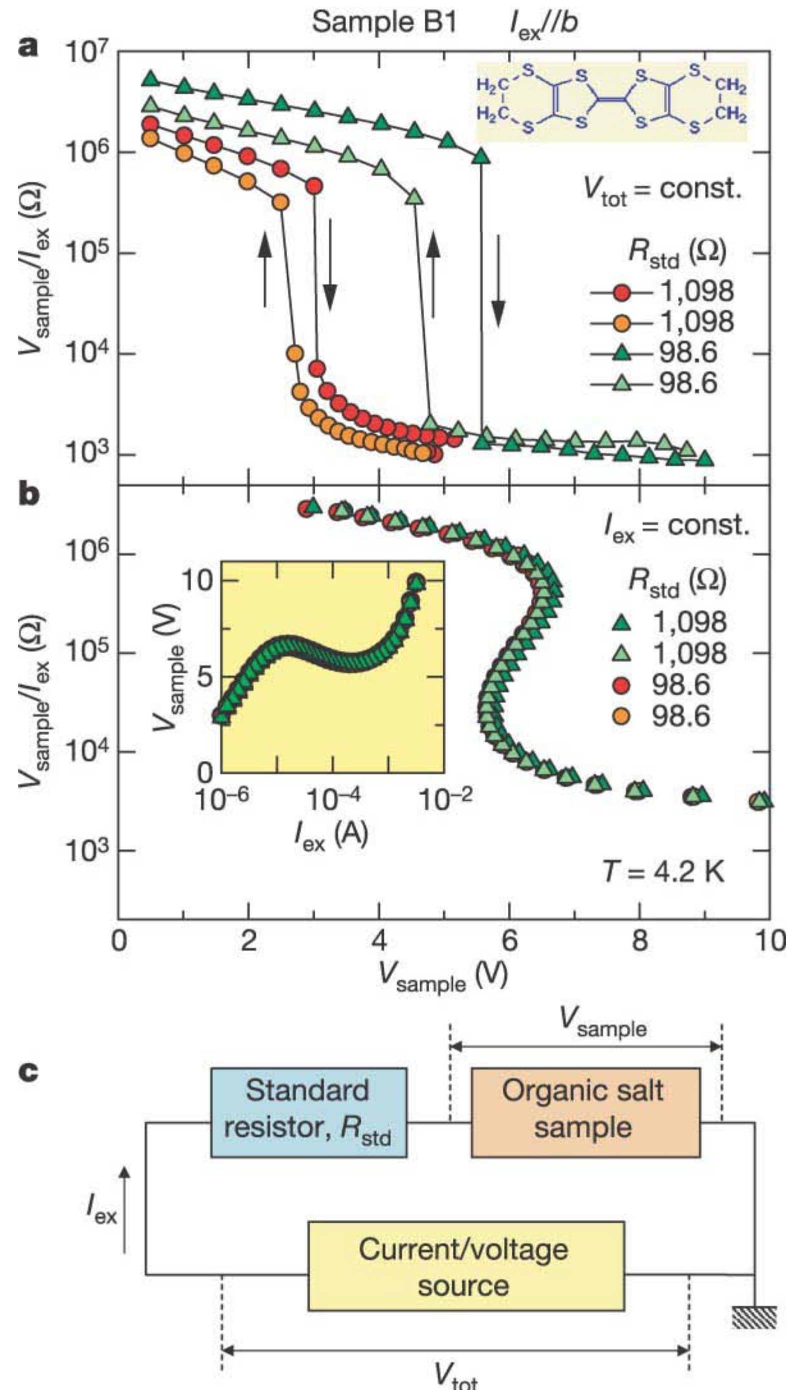


A homework I got from the condensed matter physicists:

"Formulate a non-equilibrium physics in the nonlinear regime non-perturbatively."

This homework is really deep.

- The system with a current along the electric field is out of equilibrium.
- NDC is a nonlinear phenomenon: we need to go beyond the linear response theory.
- The system is strongly correlated.

What I am going to talk:

- Actually, I have already reproduced NDC in the previous work: NDC itself is not the main issue in this talk. (S.N. PTP124(2010)1105.)

- The main subject in this talk is non-equilibrium phase transitions: I am going to show the presence of novel non-equilibrium phase transitions and non- equilibrium critical point associated with NDC in strongly-correlated insulators.

- My main tool is the AdS/CFT correspondence. This provides a new picture on non-equilibrium physics.

Non-equilibrium Steady States and AdS/CFT

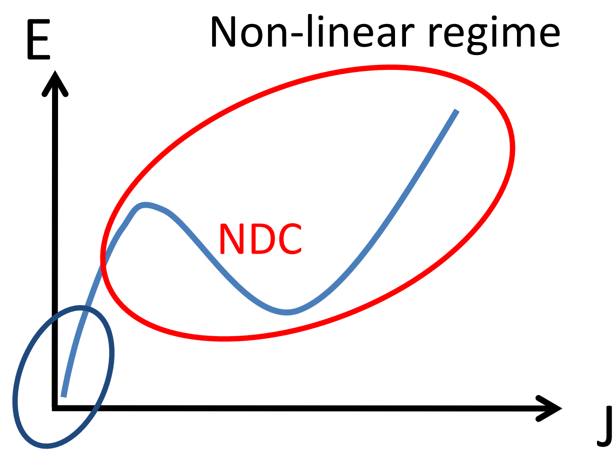
Non-equilibrium physics:

A challenge in modern physics

Classification of Non-equilibrium States

	Time independent	Time dependent
Linear-response regime (Near equilibrium)	Linear-response theory, hydrodynamics,……	
Out of the linear-response regime (Far from equilibrium)	We attack here.	

Non-equilibrium Steady States (NESS)



Non-equilibrium steady state (NESS)

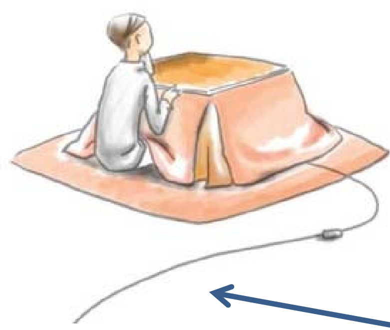
Non-equilibrium, but time-independent.

A typical example:

A system with a constant current along the electric field.

- It is non-equilibrium, because heat and entropy are produced.
- The macroscopic variables can be time independent.

Air



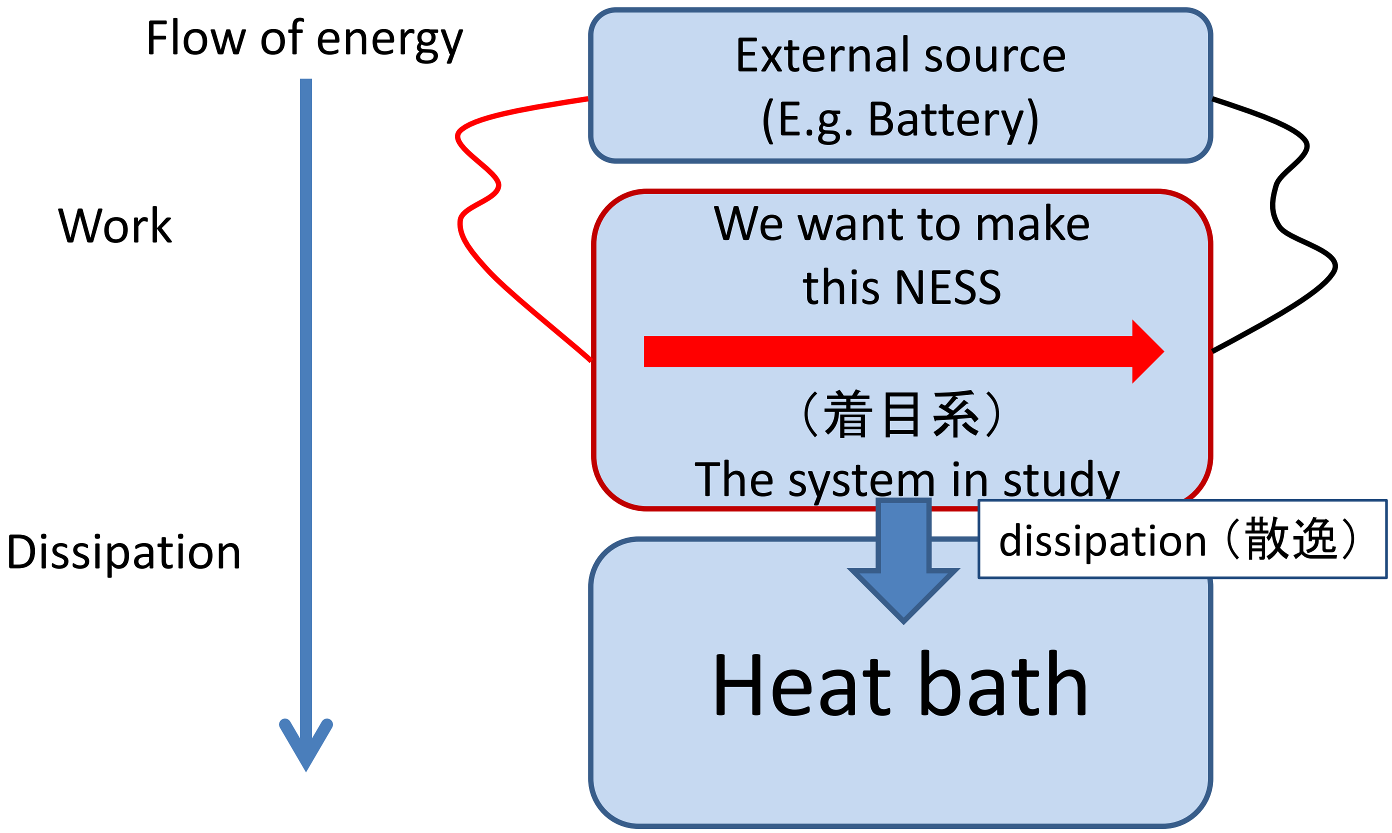
Power supply

In order to realize a NESS, we need
a heat bath and an external force.

Setup for NESS

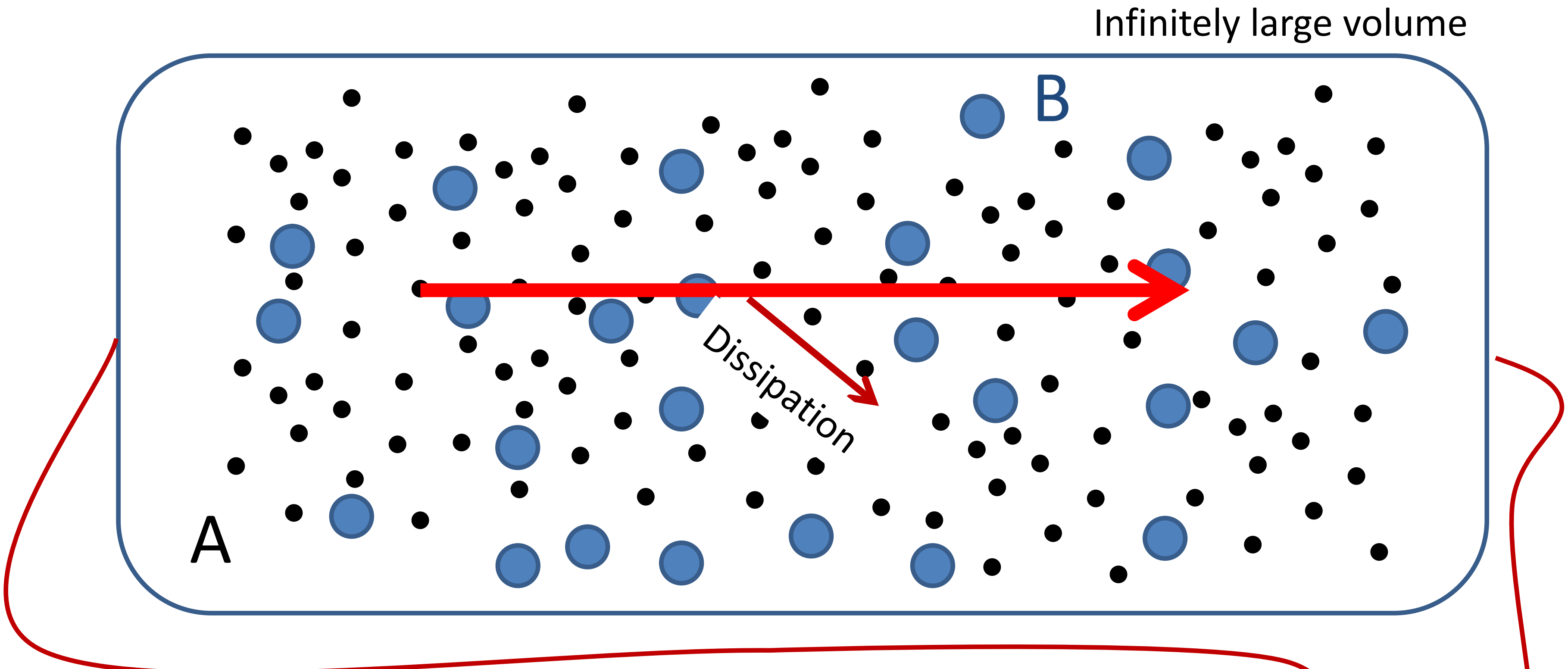
External force and heat bath are necessary.

Power supply drives the system our of equilibrium.



The subsystem can be NESS if the work of the source and the energy dissipated into the heat bath are in balance.

How to prepare the heat bath?



- Neutral particles A:

  Equilibrium at T.

- Charged particles B:

  Driven to out of equilibrium,

  not necessarily at thermal equilibrium.

Heat bath

NESS

Degree of freedom

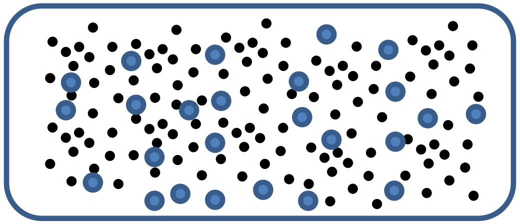
Very large

$\vee$

$\vee$

large

We can realize this situation.



SU(Nc) gauge theory

Degree of freedom

particle A	→ gluon	$\sim Nc^2-1$	Heat bath
		$\vee\vee$	
particle B	→ quark/antiquark	$\sim Nc$	NESS

- If we take the large-Nc (Nc>>1) limit:
  The gluon subsystem can be a heat bath.
- We can apply an external electric field acting on the quark charge ($U(1)_B$ charge).

External force

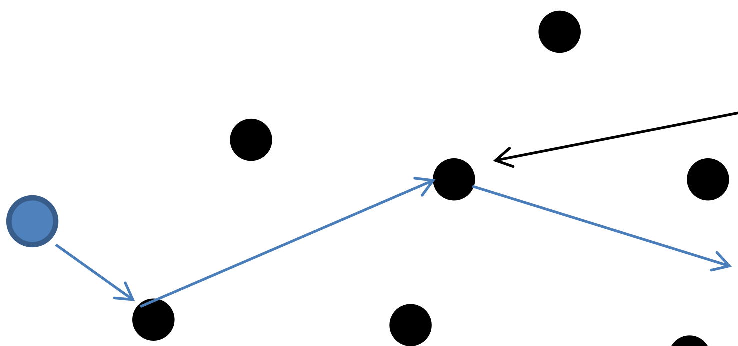
Now, the interaction among the charged particles are given by the large-Nc gauge theory.

The large-Nc gauge theory is not realized in Nature.
Does it make sense to use the large-Nc gauge theory?

Yes.

Conventional models in condensed matter physics

Drude model:



We do not ask the origin of the interaction between the ion and the electron.

These successful models do not ask the origin of the interaction.

Hubbard model:

$$H = \sum_{i \neq j, \sigma} t_{ij} c_{i\sigma}^{+} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

We do not ask the origin of the interaction among the electrons.

Statistical physics

The game is to extract the macroscopic physics that is common to wide range of different systems, regardless of the microscopic details.

Example:

$\boxed{\text{Phase transitions and critical phenomena.}}$

Important thing is the fact that the charged particles are interacting.

We focus on

- Non-equilibrium phase transitions and non-equilibrium critical phenomena associated with the non-linear conductivity of strongly-correlated insulators.

- We make qualitative predictions, but not quantitative predictions.

You may still worry:

The interaction of the SU(Nc>1) gauge theory can be qualitatively very different from that of QED (Nc=1). (e.g. confinement,......)

Do not worry. We are going to employ

N=4 Supersymmetric SU(Nc) Yang-Mills theory,
(N=4 SYM)

that produces Coulomb interaction (at T=0).

Why large-Nc?

- Heat bath is naturally prepared within the theory.
- We can employ the AdS/CFT correspondence, easily.

The AdS/CFT correspondence

J. Maldacena 1997

Some strongly-interacting quantum gauge theory = (equivalent) Some gravitational theory (General Relativity + matter)

Large-Nc

Classical theory

Advantages in the gravity dual:
the problem becomes much simpler.

“Many-body physics” in the gravity

Particles A: gluons (heat bath) ⇨ single black hole

(Hawking and Bekenstein said that black hole has the notion of temperature and entropy. We still have real time.)

E. Witten, Adv. Theor. Math. Phys. 2 (1998) 505.

Particles B: quark/antiquarks ⇨ single D-brane (a brane-like object)

A. Karch and E. Katz, JHEP 0206 (2002) 043.

The complicated many-body problem of strongly interacting system is reduced to just a “two-body” problem of classical mechanics.

More about AdS/CFT

We have employed large-Nc N=4 SYM

This is good for us: the most standard and the simplest example of AdS/CFT is that for N=4 SYM:

The most standard example of AdS/CFT:

$\boxed{\text{N=4 SYM}} \Longleftrightarrow \boxed{AdS_5 \times S^5}$

However, this describes only the gluon sector.

D3-D7 system

We can add the flavor degree of freedom (quarks and anti-quarks) by adding the D7-brane to the system.

(Karch and Katz, JHEP0206(2002)043)

The F1 string between the D3 and the D7 acts as a quark (or antiquark) from the viewpoint of the D3-branes.

The gauge theory realized on the D3-branes is
N=4 SYM + N=2 hyper-multiplet

AdS/CFT based on D3-D7

SU(Nc) N=4 Supersymmetric Yang-Mills (SYM) theory at large-Nc with $\lambda = g_{YM}^2 Nc >> 1$.
(Quantum field theory) Finite T

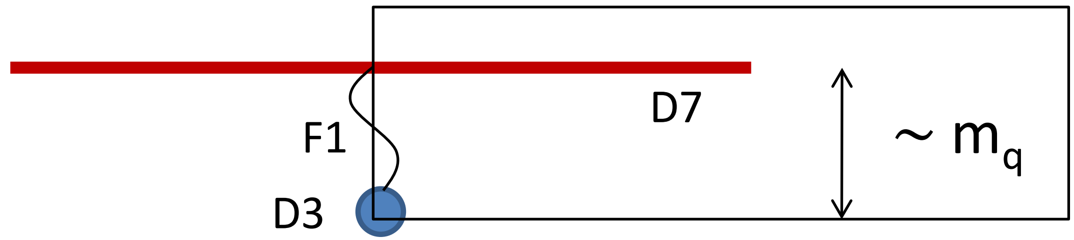
+ quark sector (N=2 hyper-multiplets)

Equivalent

Type IIBsupergravity at the classical level on weakly curved AdS-BH $\times S^5$

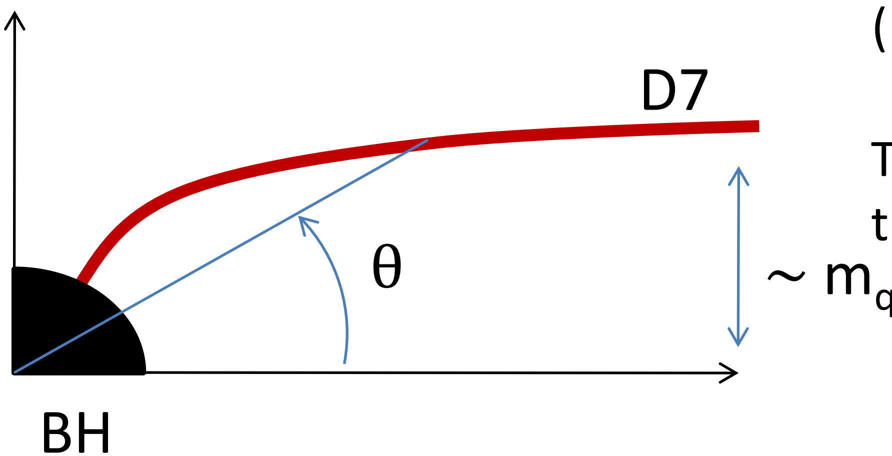
+ D7-brane on this curved spacetime

Gravity Dual



We draw only this part.

- The D3 is replaced with an AdS-BH in the gravity dual.



"z" represents the radial direction.
(But, the boundary is at z=0.)

The shape of the D7 is described by the function $\theta(z)$.

$$\theta(z) = m_q z + \text{const.}\, z^3 + \ldots\ldots$$

$$\left.\frac{1}{z}\sin\theta(z)\right|_{z\to 0} = m_q$$

The dual geometry

	$R^{3,1}$	z	S^3	S^2
Horizon	○			
D7	○	○	○	

$\longleftarrow$ AdS$_5$-BH $\longrightarrow$ $\longleftarrow$ S^5 $\longrightarrow$

$$ds^2{}_{\text{AdS-BH}} = -\frac{1}{z^2}\frac{\left(1 - z^4/z_H^4\right)^2}{1 + z^4/z_H^4}dt^2 + \frac{1 + z^4/z_H^4}{z^2}d\vec{x}^2 + \frac{dz^2}{z^2}$$

$$ds^2{}_{S^5} = d\theta^2 + \sin^2\theta\, d\varphi^2 + \cos^2\theta\, d\Omega_3^2$$

$$\left(0 \le \theta \le \frac{\pi}{2}\right)$$

The D7 is located at $\varphi = 0$ (our choice).

The D7 configuration is given by θ(z).

Physics of D7-brane

Black hole geometry plays the role of heat bath for the D7-brane.

- The D7-brane is affected by the black hole
- The black hole is not affected by the D7.

→ Probe approximation

D7-brane action : Dirac-Born-Infeld action

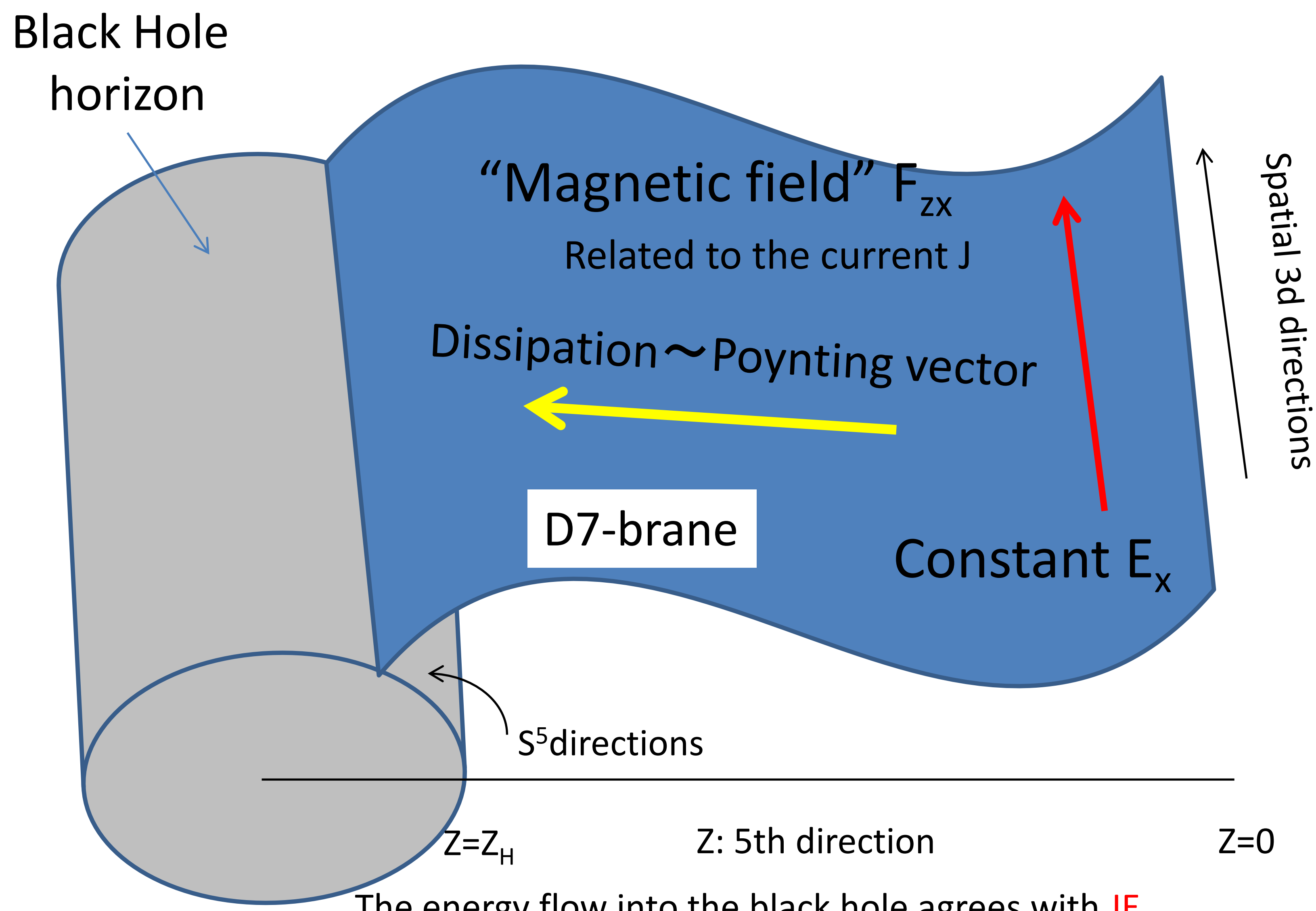
$$S_{D7} = -T_{\mathrm{D7}} \int d^{7+1}x \sqrt{-\det\left(\partial_a x^\mu \partial_b x^\nu g_{\mu\nu} + F_{ab}\right)}$$

Spacetime metric.

$$F_{ab} = \partial_a A_b - \partial_b A_a \quad (2\pi\alpha' = 1)$$

Field strength of the U(1) gauge field on the D7-brane.

Cartoon of D-brane configuration

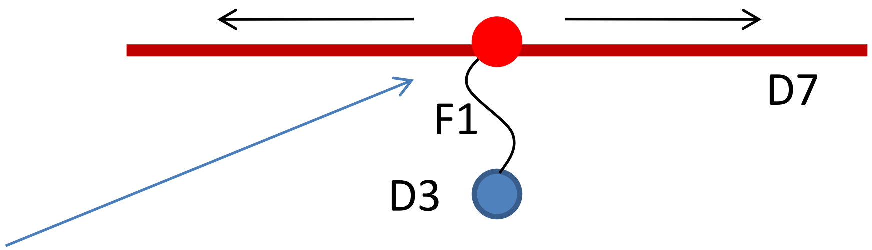


The energy flow into the black hole agrees with JE.
(Karch, O'Bannon, Thompson, 2008 and 2009)

The U(1) on the D7

The U(1) gauge field on the D7 is linked to the $U(1)_B$ charge ($U(1)_B$ current) in the YM side.

The U(1) electric field will be induced.



This endpoint acts as a unit charge of the U(1) gauge field on the D7.

A_0 at the boundary will be lifted.

If the quark moves (in x-direction), the “magnetic” field will be induced on the D7 and A_x at the boundary will be lifted as well.

AdS/CFT dictionary: GKP-Witten relation

z=0: boundary

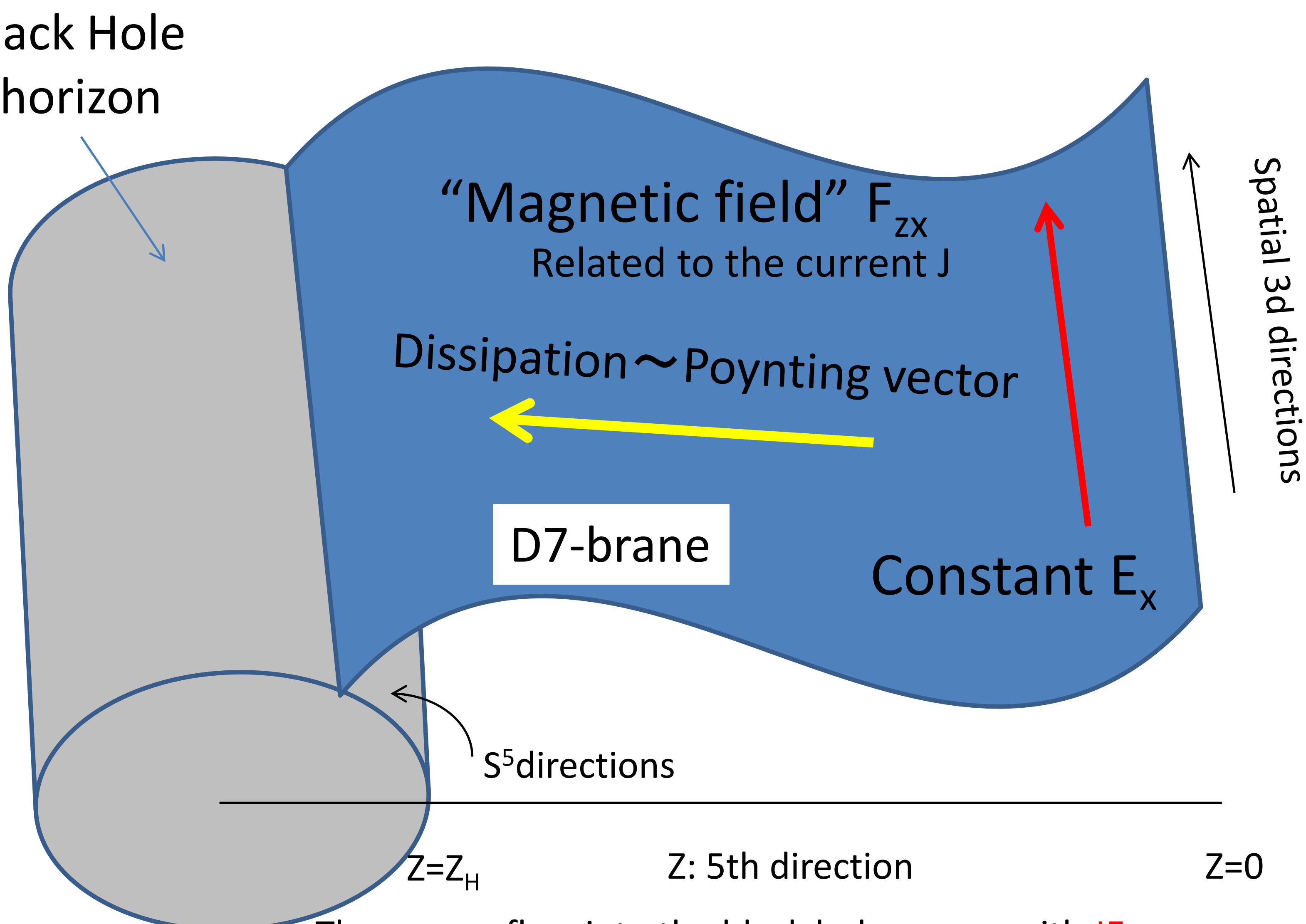
$$A_0(z) = \mu - \frac{(2\pi)^2}{2N_c}\left\langle J^0 \right\rangle z^2 + O(z^4)$$

$$A_x(z) = -Et + \frac{(2\pi)^2}{2N_c}\left\langle J^x \right\rangle z^2 + O(z^4)$$

If the configuration of $A_x(z)$ on the D7-brane is specified as a function of z, the relationship between E and J^x can be read from it.

⇒ We obtain the (non-linear) conductivity.

Cartoon of D-brane configuration



The energy flow into the black hole agrees with JE.
(Karch, O’Bannon, Thompson, 2008 and 2009)

However,

$$A_x(z) = -Et + \frac{(2\pi)^2}{2N_c}\left\langle J^x \right\rangle z^2 + O(z^4)$$

A_x obeys a second-order differential equation.

We need two boundary conditions to fix the solution.

The first and the second terms are the input conditions we need to specify by hand!

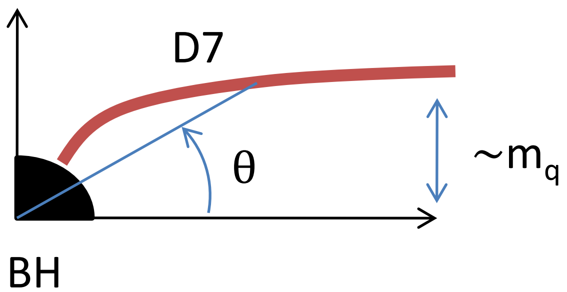
However, if we specify them as we like,
the on-shell D7-brane action will be complex in general.

The reality of the D7-brane action (the stability of the system) constrains the relationship between the first and the second terms.

[Karch, O'Bannon JHEP0709(2007)024]

The on-shell D7-brane action

$$S_{\mathrm{D7}} = -N \int dz dt \cos^6\theta \; g_{xx}^{5/2} \left| g_{tt} \right|^{1/2} \sqrt{W}$$



$$W = \frac{g_{zz}\left(\left| g_{tt} \right| g_{xx} - E^2 \right)}{\left| g_{tt} \right| g_{xx}^3 \cos^6\theta - \frac{g_{xx} \left\langle J_x \right\rangle^2}{N^2}}$$

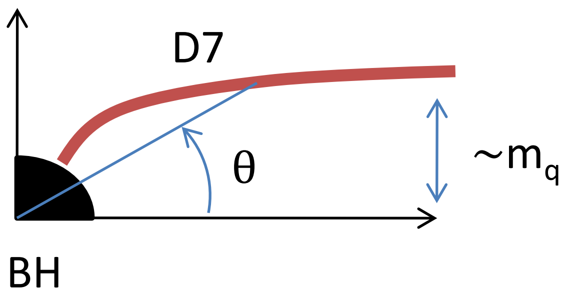
The metric of the AdS-BH

$$ds^2{}_{\text{AdS-BH}} = -\frac{1}{z^2}\frac{\left(1-{z^4}\big/{z_H^4}\right)^2}{1+{z^4}\big/{z_H^4}}dt^2 + \frac{1+{z^4}\big/{z_H^4}}{z^2}d\vec{x}^2 + \frac{dz^2}{z^2}$$

- The horizon is located at $z=z_H$.
- The boundary is at $z=0$.

On-shell D7-brane action

$$S_{\mathrm{D7}} = -N\int dzdt \cos^6\theta \; g_{xx}^{5/2}\left|g_{tt}\right|^{1/2}\sqrt{W}$$



$$W = \frac{g_{zz}\left(\left|g_{tt}\right|g_{xx} - E^2\right)}{\left|g_{tt}\right|g_{xx}^3\cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}}$$

Both the numerator and the denominator go across zero somewhere between the boundary and the horizon.

Only the way to make the action real is to make them go across zero at the same point.
(We define this point as $z=z_*$.)

The conditions for reality

$$\left(-g_{tt}\right)g_{xx}\Big|_{z=z_*} - E^2 = 0$$ ➡ z_* in terms of E

$$\left(-g_{tt}\right)g_{xx}^3\cos^6\theta - \frac{g_{xx}\langle J_x\rangle^2}{N^2}\Bigg|_{z=z_*} = 0$$

➡ J_x is given in terms of E and $\theta(z_*)$.

$$\frac{1}{z}\sin\theta(z)\Big|_{z\to 0} = m_q$$

➡ J_x is given by using E and m_q.

The conductivity is given as a function of m_q (at given T, λ).

The non-linear conductivity

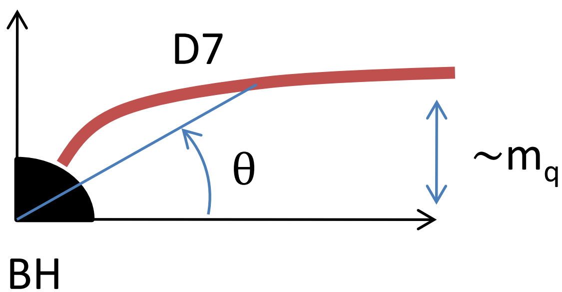
The charge density is also taken into account.
[Karch, O'Bannon JHEP0709(2007)024]

Normal conduction

$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Pair-creation

$$d \equiv \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{2\lambda}T^2}, \quad e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2},$$



- $\cos\theta(z_*)$ goes to 1 at $m_q \to 0$.
- $\cos\theta(z_*)$ goes to zero at $m_q \to$ infinity.

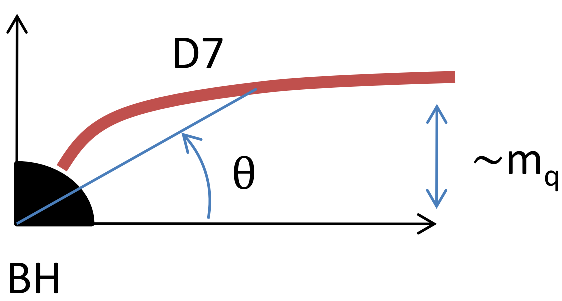
We have no way to reproduce NDC from the normal-conduction part.

$$J_x\big|_{m_q\to\infty} = \frac{d}{\sqrt{1+e^2}}E \to \begin{cases} \approx & d\cdot E & (e \ll 1) \\ \approx & \text{saturate} & (e \gg 1) \\ \approx & 0 & (T \gg 1) \\ & (e \leftrightarrow -e \text{ symmetric}) & \end{cases}$$

We consider the neutral case: the contribution of the pair-creation

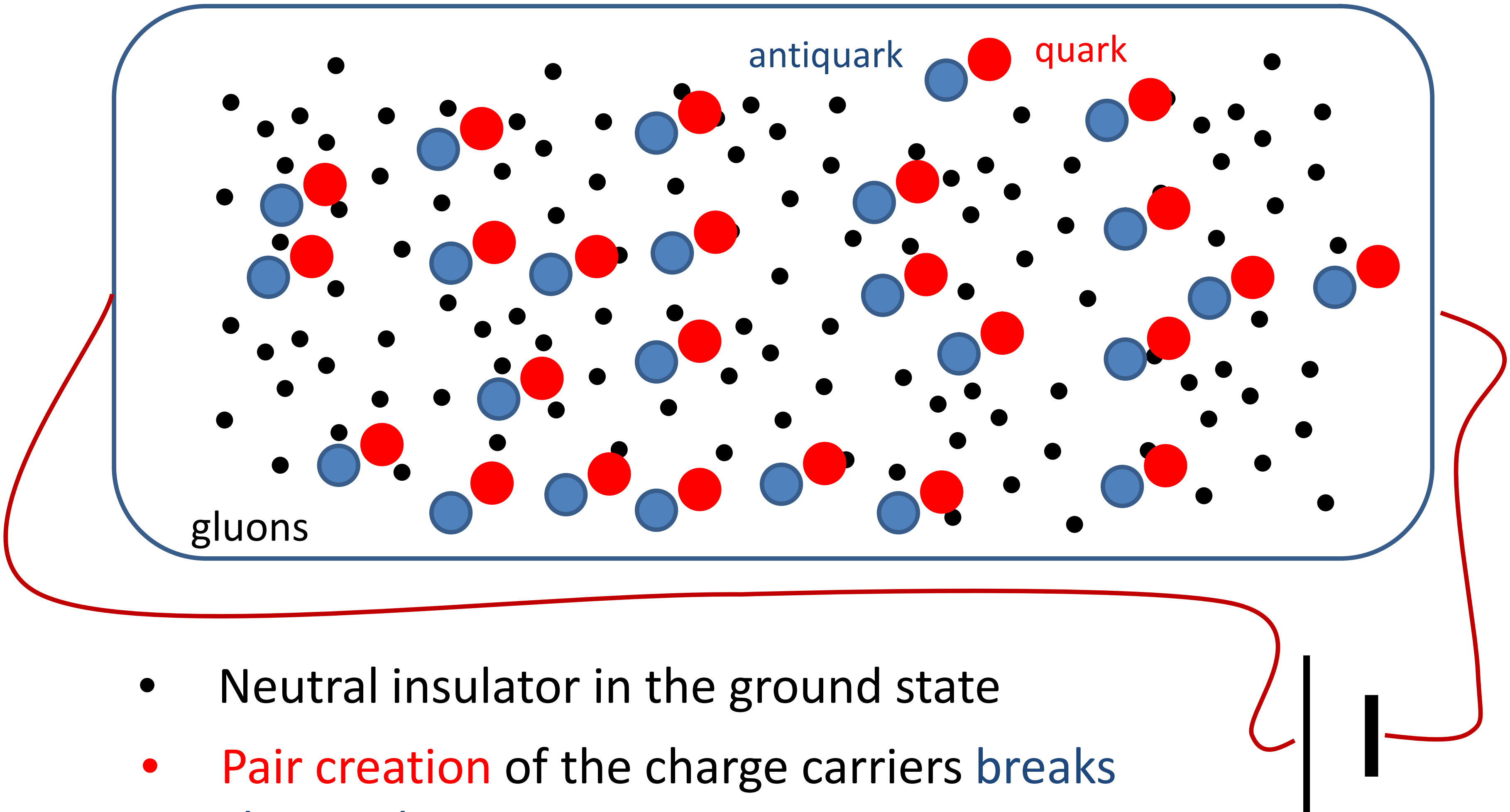
$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*)}$$

$$e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2}$$



This function can be given by solving a non-linear differential equation, numerically.

Insulation breaking



- Neutral insulator in the ground state
- Pair creation of the charge carriers breaks the insulation.

⇨ Insulation breaking.

Our setup

- We consider a neutral system.
- The volume of the system is infinite.
- Strongly interacting system.
- Our current is non-ballistic.
- Insulator in the ground state.
- Strong-enough electric field breaks the insulation.

Interaction among the charged particles, and the pair creation of the charges are taken into account.

Results of analysis

S.N. arXiv:1204.1971

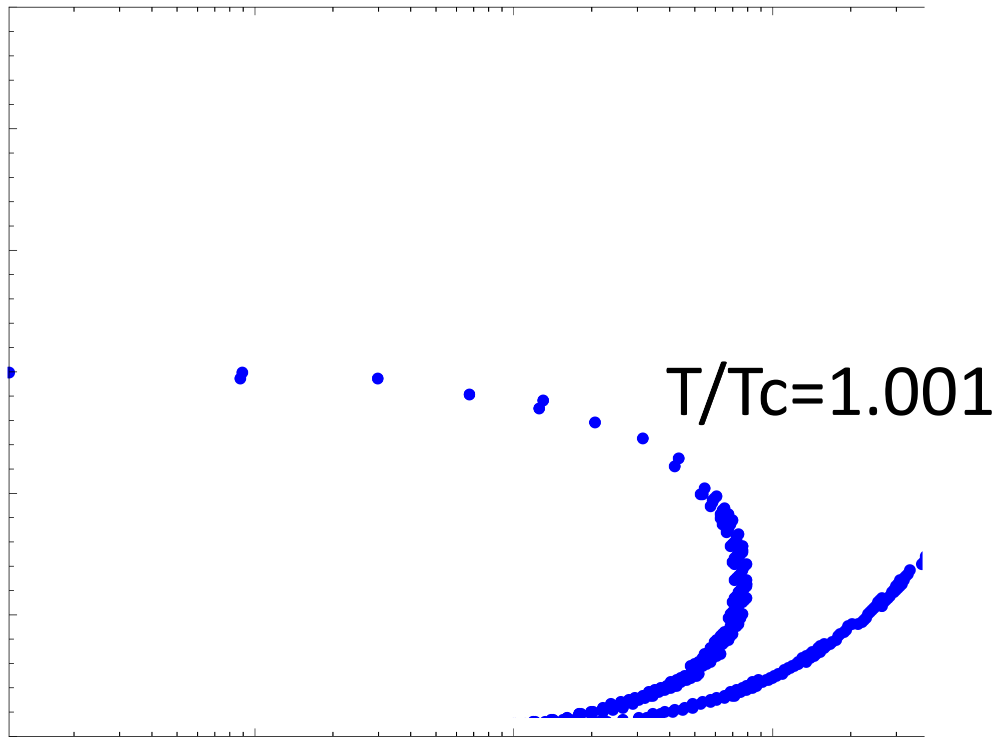
（See also, S.N. PTP124(2010)1105.）

$$\lambda = (2\pi)^2/2, \quad N_C = 40,$$

We solve a non-linear differential equation numerically to obtain the conductivity.

Non-linear conductivity at various T

(T: temperature of the heat bath)



Temp. of heat bath

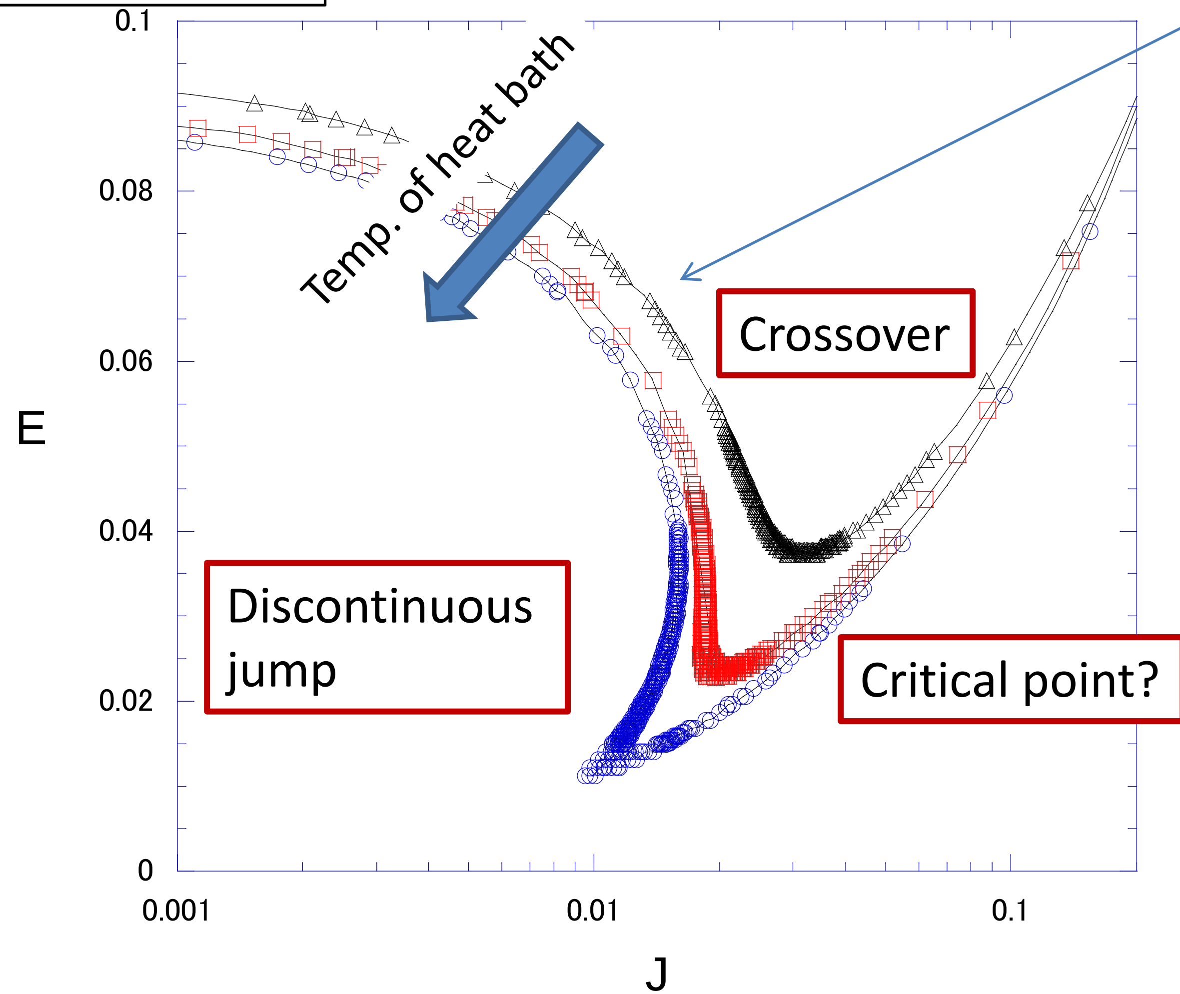
△ : T=0.34337

□ : T=0.34365=Tc

○ : T=0.34379

J-E characteristics

S.N. PTP124(2010)1105.



Negative Differential conductivity (NDC)

It is widely observed in strongly-correlated insulators

(See, e.g. [Oka, Aoki, arXiv:0803.0422])

How to determine the transition point?

(S.N. arXiv:1204.1971)

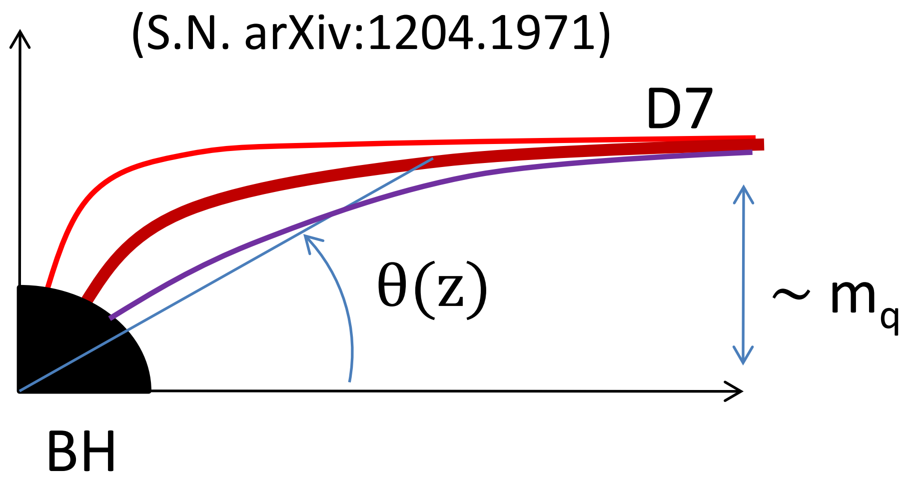
Equilibrium cases:

We compute the free energy and compare which branch (phase) is most economic.

In the non-equilibrium systems:

- Are there a non-equilibrium generalization of free energy?
- If yes, how to compute it?

The things are classical mechanics



The question is which D-brane configuration is most stable.
This is a problem of classical mechanics of membrane with electro-magnetic flux on a curved geometry.

The most natural way is to compare the Hamiltonian.

Hamiltonian of D7

Renormalized Hamiltonian

"Bare Hamiltonian"

$$H = V \lim_{\varepsilon \to 0} \int_{\varepsilon}^{z_H} dz \left[\dot{A}_x \frac{\partial L_{\mathrm{D7}}}{\partial \dot{A}_x} - \left(L_{\mathrm{D7}} - A'_x \frac{\partial L_{\mathrm{D7}}}{\partial A'_x} \right) - L_{\mathrm{count}}(\varepsilon) \right]$$

"Routhian"

E

J

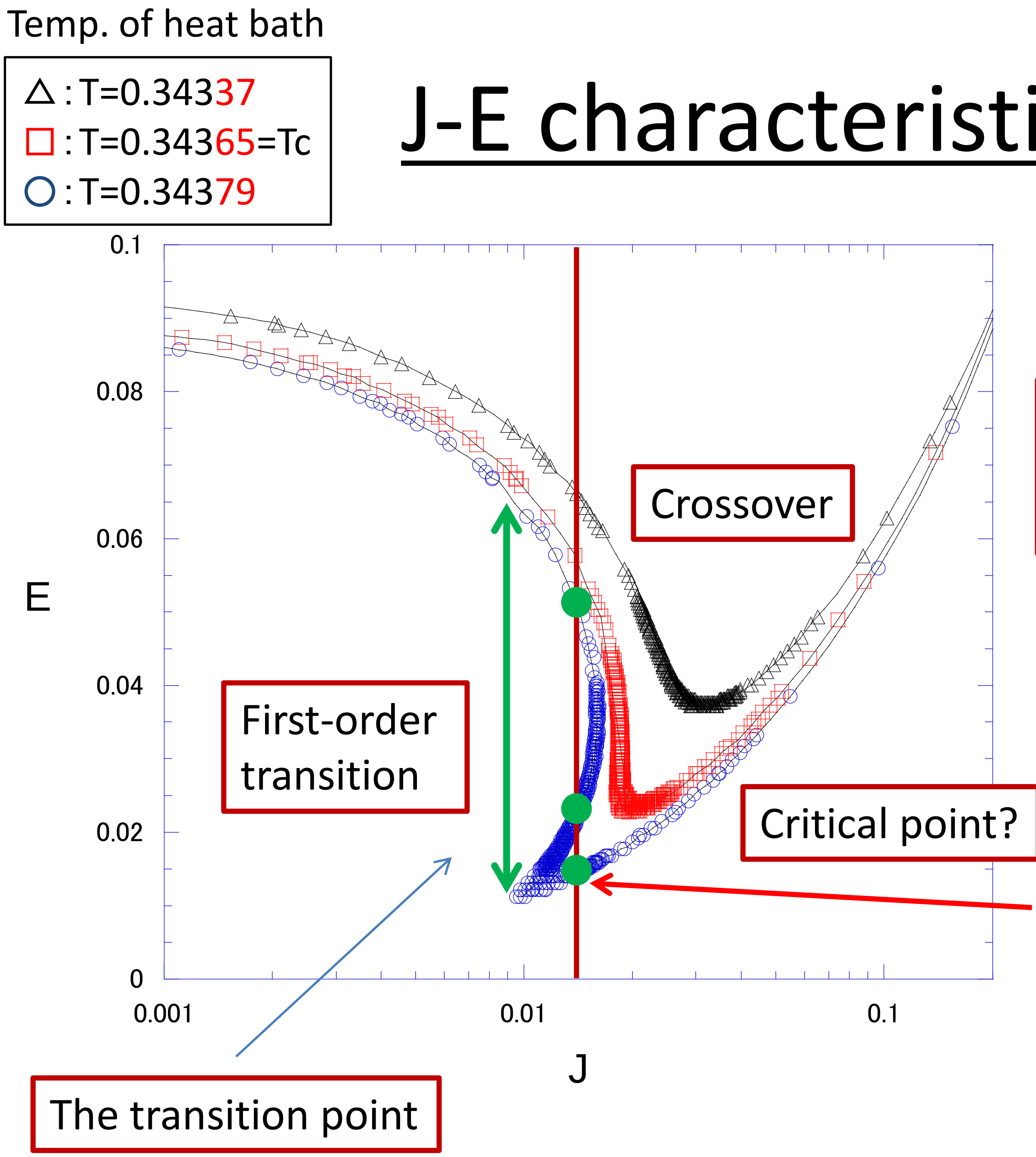
- The UV divergence is renormalized by the counter terms.
- The IR divergence is canceled within the Legendre transformation.

We propose to define this Hamiltonian as a NESS generalization of the free energy.

Any relationship to the steady state thermodynamics?

(Y. Oono and M. Paniconi, PTP.Suppl. **130** (1998), 29.)

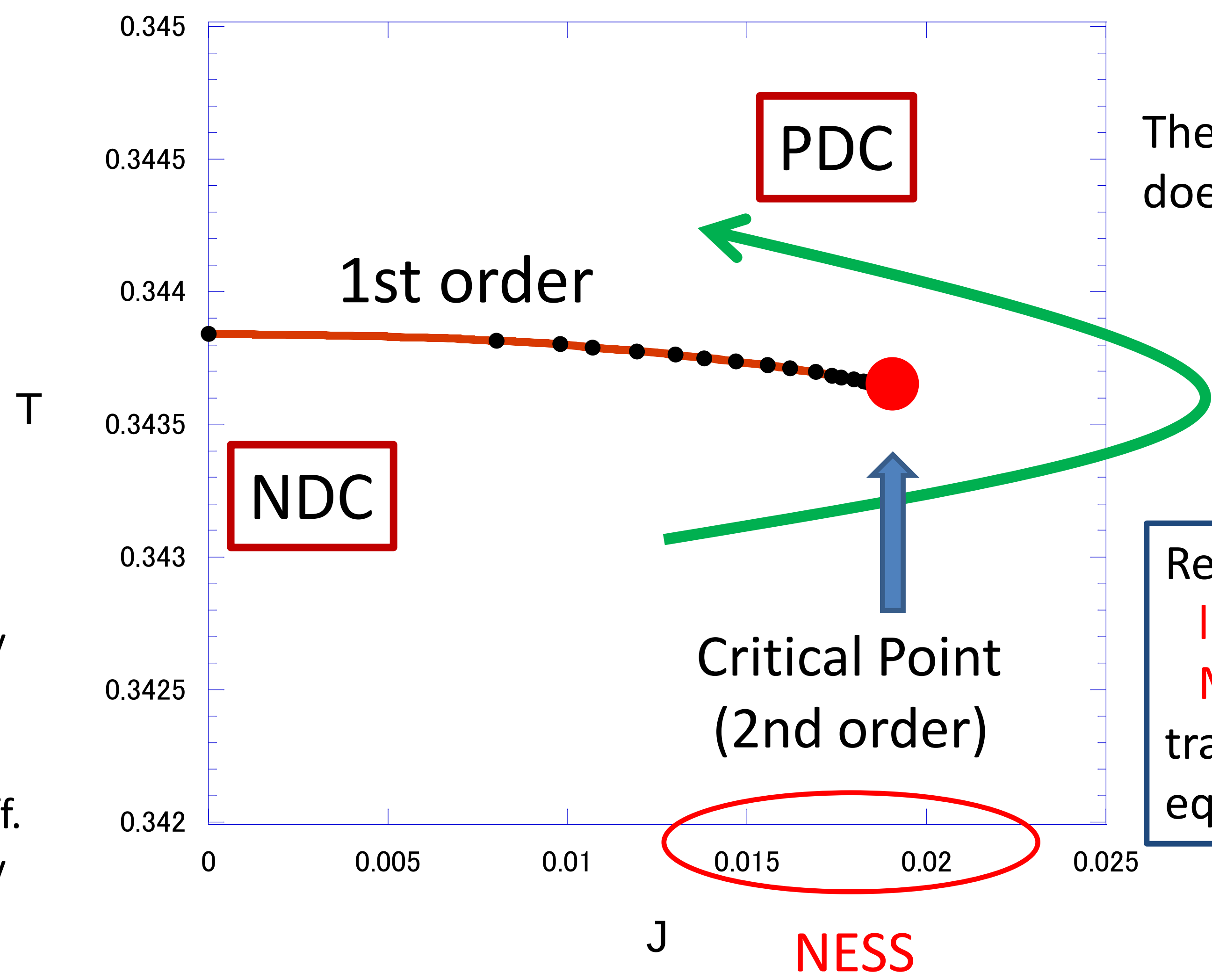
J-E characteristics



The system prefers smaller dissipation.

This has the smallest Hamiltonian in the gravity side.

Phase diagram



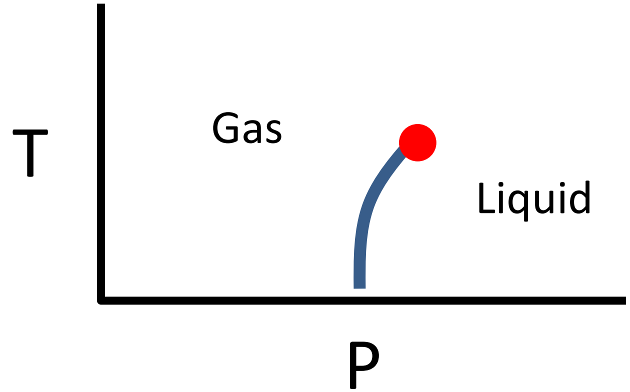
The symmetry does not change.

PDC:
Positive Diff.
Conductivity

NDC:
Negative Diff.
Conductivity

Resembles of
liquid-gas
Mott
transitions at
equilibrium.

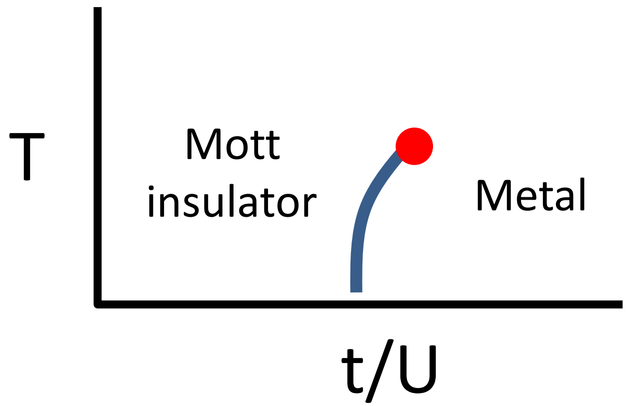
Critical Phenomena（equilibrium）



Liquid-gas transition (equilibrium)

$$n_L - n_g \propto \left(T_C - T\right)^{\beta}$$

Difference of density



Mott transition (equilibrium)

Experimentally detected by using the conductivity σ, instead of the density.

P. Limelette et al., Science 302 (2003) 89.
F. Kagawa et al.,Nature 436 (2005) 534.

- Universality class = Ising universality class
- β=1/2 within the mean-field theory.

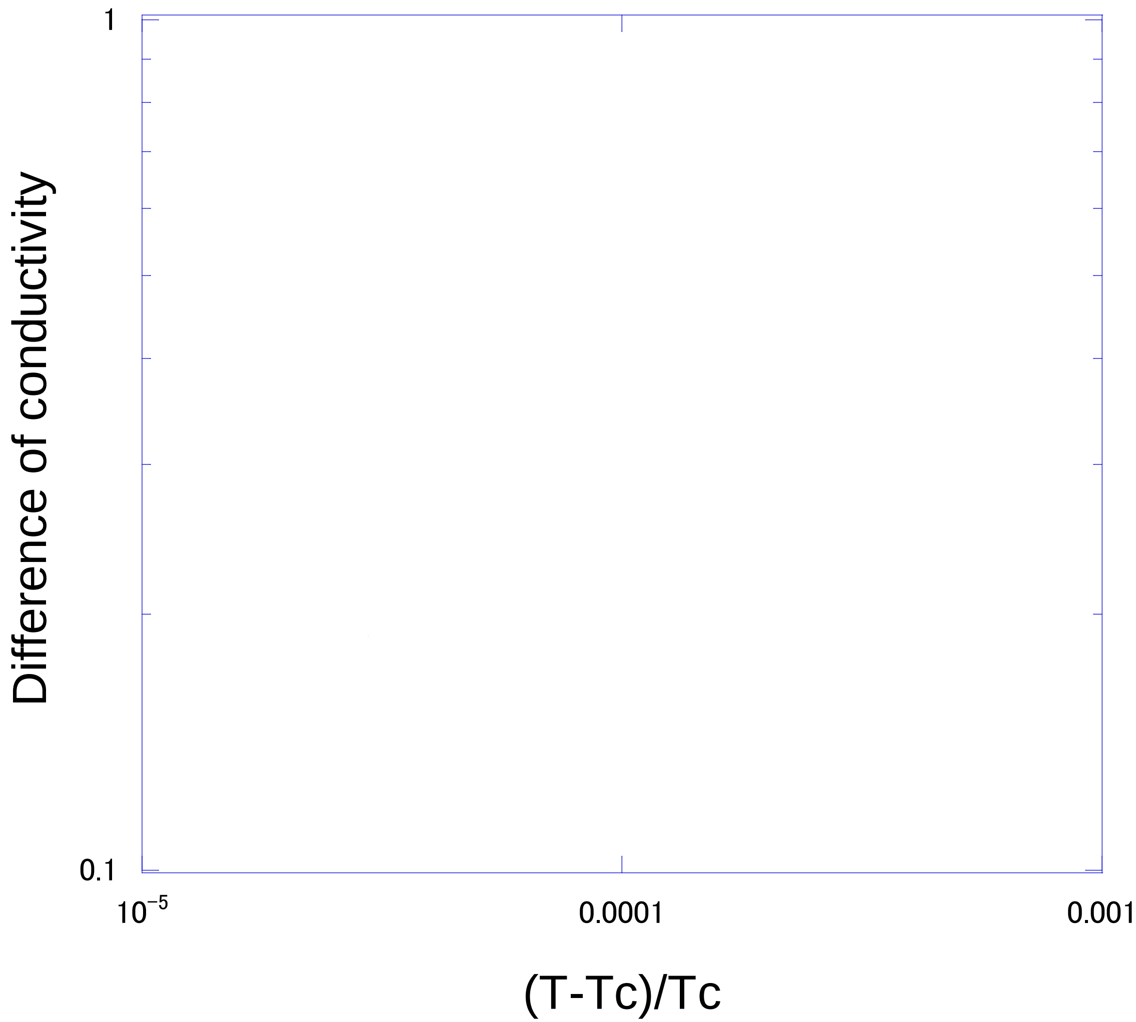
We propose to see

$$\sigma_{\mathrm{PDC}} - \sigma_{\mathrm{NDC}} \propto \left(T - T_C\right)^{\beta}$$

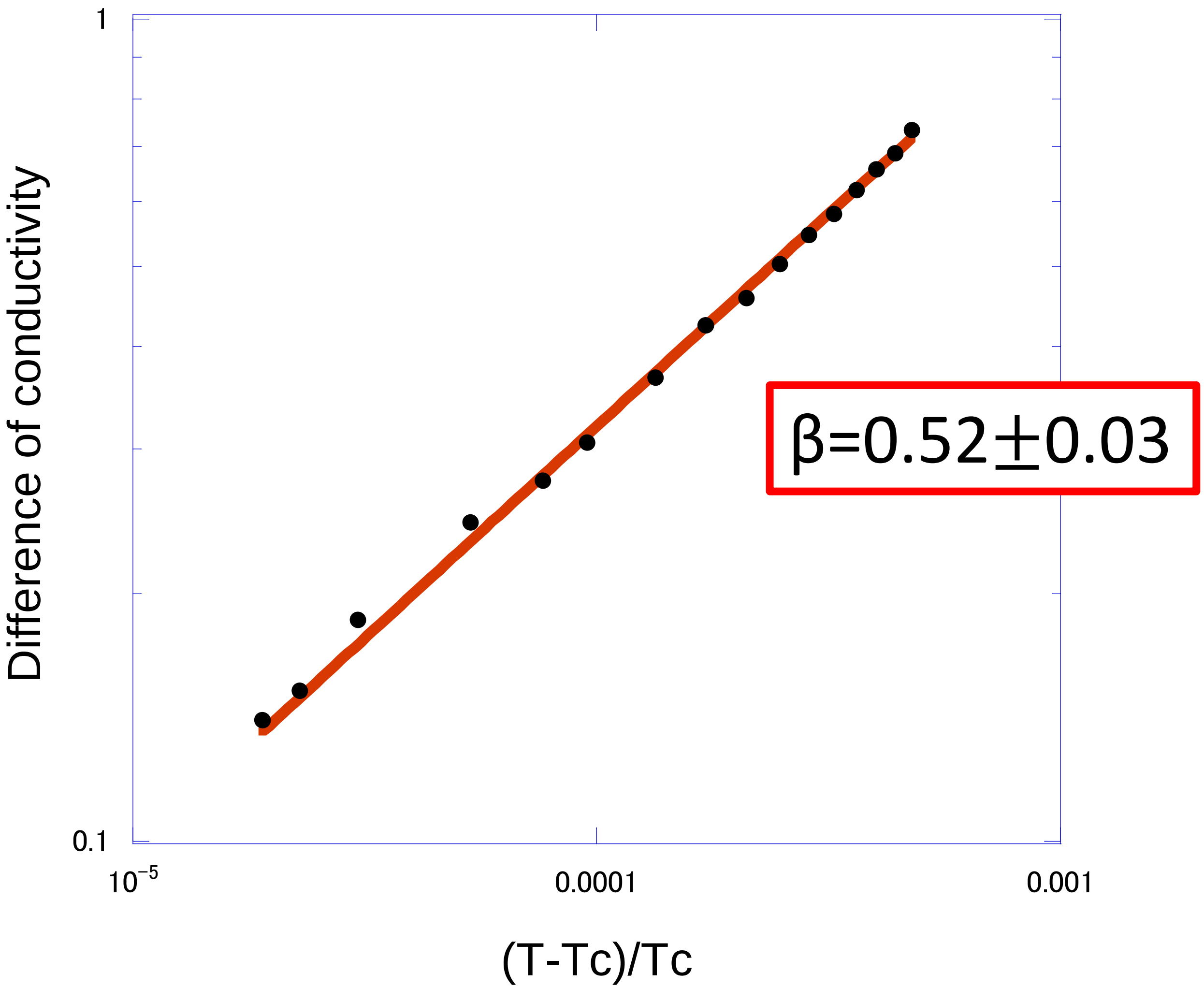
Difference of conductivity　　　Temperature of heat bath

Let us see what is going on.

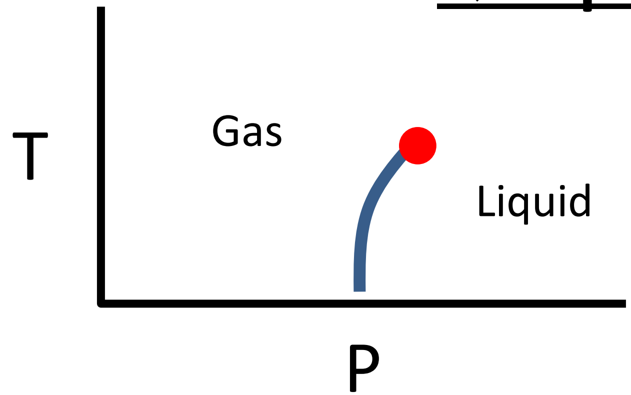
Behavior of diff. of conductivity



Behavior of diff. of conductivity

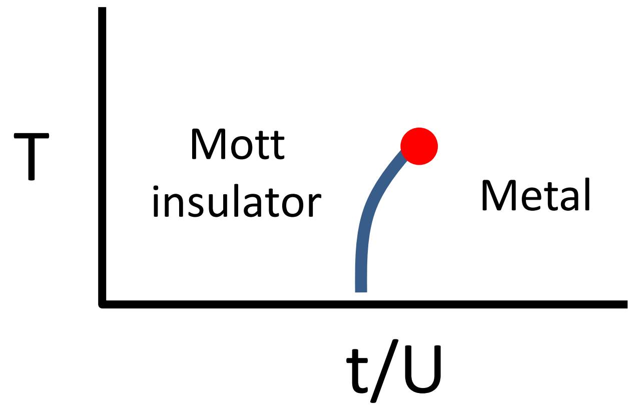


Another critical exponent (equilibrium)



Liquid-gas transition (equilibrium)

$$\left.(n-n_c)\right|_{T=Tc} \propto |P-P_c|^{1/\delta}$$



Mott transition (equilibrium)

$$\left.(\sigma-\sigma_c)\right|_{T=Tc} \propto |P-P_c|^{1/\delta}$$

Pressure

- δ=3 for the mean-field theory.

However, the pressure is not a control parameter in our system.
(Our system has infinite volume.)

LETTERS

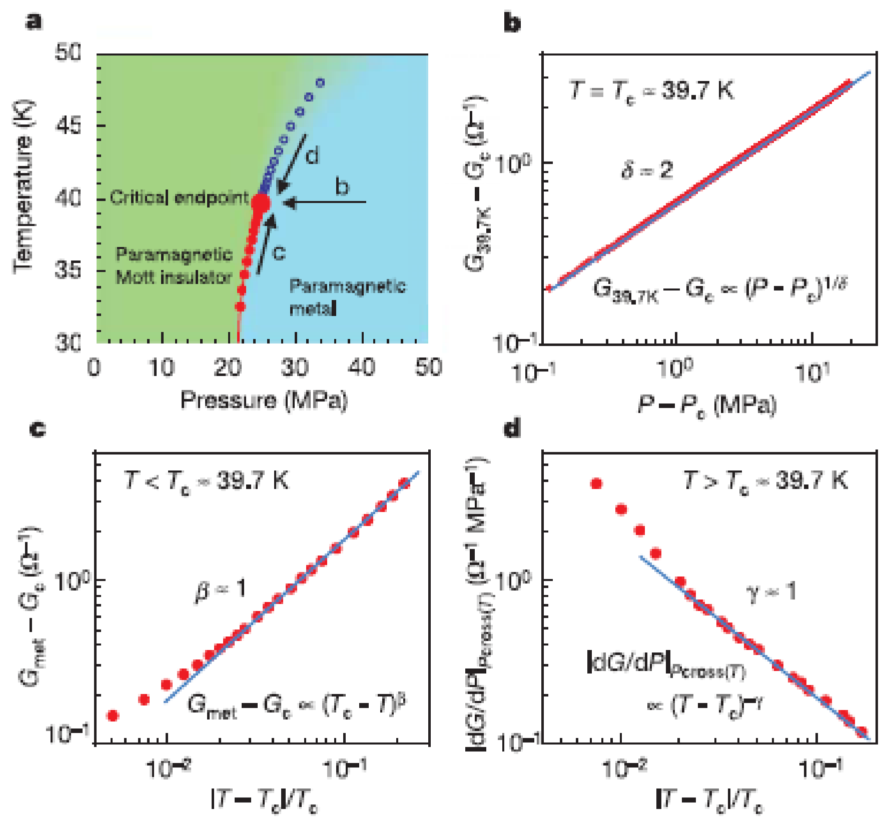
Unconventional critical behaviour in a quasi-two-dimensional organic conductor

F. Kagawa[1], K. Miyagawa[1,2] & K. Kanoda[1,2]

Fig. 2

Nature 436 (2005) 534.

Metal-insulator transition at equilibrium (J~0)



Definition of a new critical exponent

Our pressure is not a control parameter.

The remaining available control parameter :

J

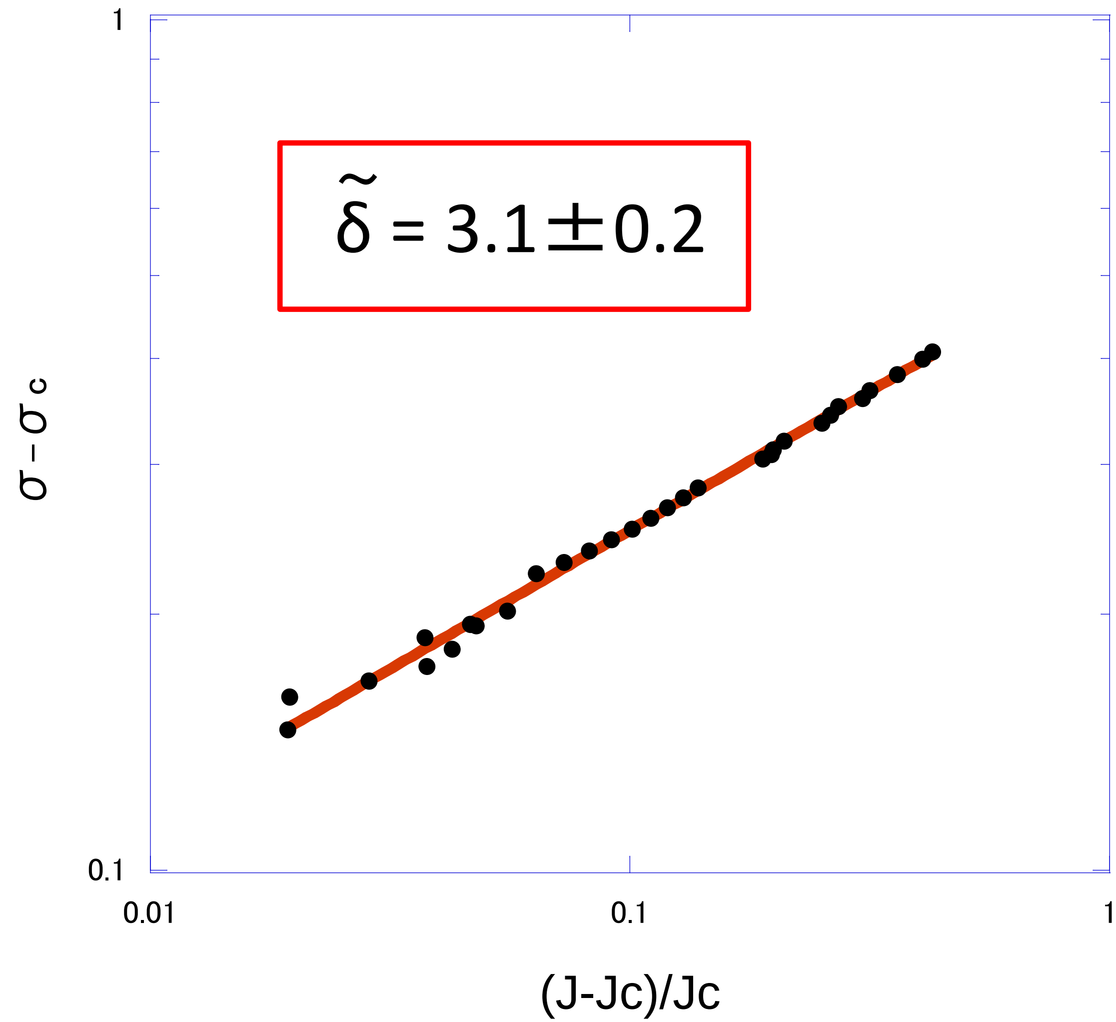
Essentially a non-equilibrium quantity

Proposal:

We define $\tilde{\delta}$ by

$$\left.\left(\sigma-\sigma_c\right)\right|_{T=Tc} \propto \left|J-J_c\right|^{1/\tilde{\delta}}$$

Does $\tilde{\delta}$ make sense?



Presence of mean-field theory for non-equilibrium phase transitions?

$\beta = 0.52 \pm 0.03 \sim 0.5$

$\tilde{\delta} = 3.1 \pm 0.2 \sim 3$

Suggest mean-field values?

It is interesting to see whether we can construct a non-equilibrium version of Landau-Ginzburg theory.

By the way, it is natural to have mean-field values in large-Nc theories.

Large-Nc as mean-field approximation

SU(Nc)

Internal degree of freedom

Large-Nc:

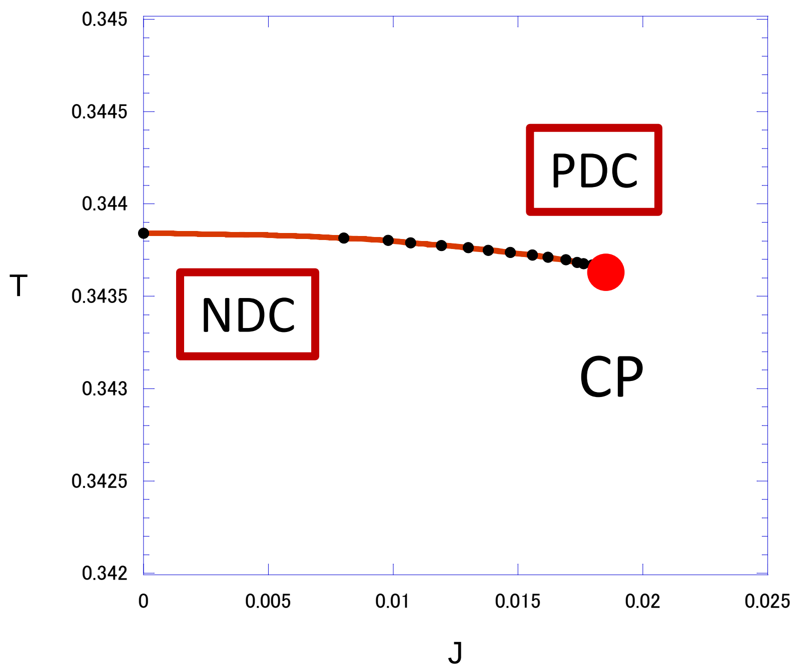
We are taking the internal degree of freedom of the particles very large: the mean-field behavior appears.

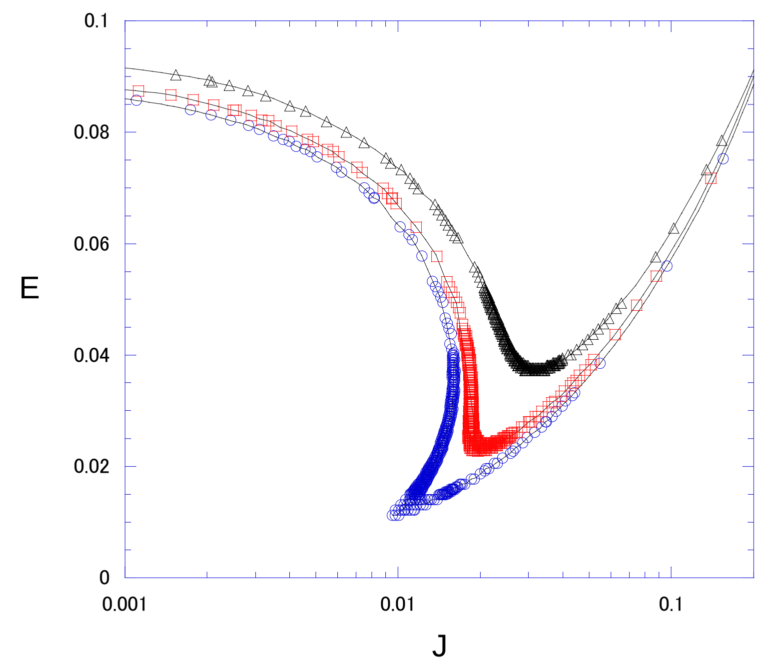
Similar situations can be found in the condensed matter physics.

- O(N) model, at $N \to \infty$.
- Mean-field theory: $d \to \infty$.
- ………

Summary

An AdS/CFT analysis of non-linear conductivity of a "strongly-correlated insulator" indicates the unknown non-equilibrium phase transitions and non-equilibrium critical point.



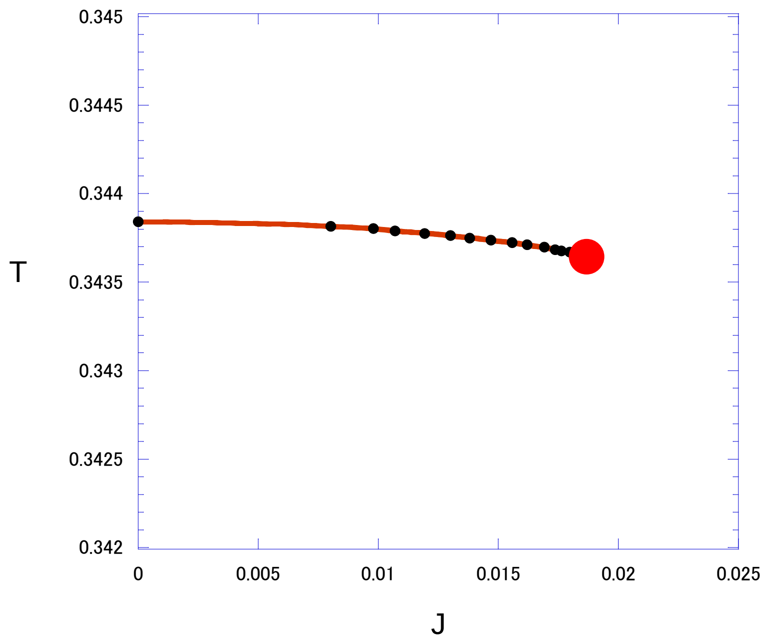


A prediction from superstring theory:

Prediction

In light of possible universality, the current-driven non-equilibrium critical point found in the present work may exist even in real materials of strongly-correlated insulators.

The precise values of the critical exponent can be different (from the mean-field values.)



Towards experimental verification

I have visited the experimental physicists at Nagoya University on May 24th.

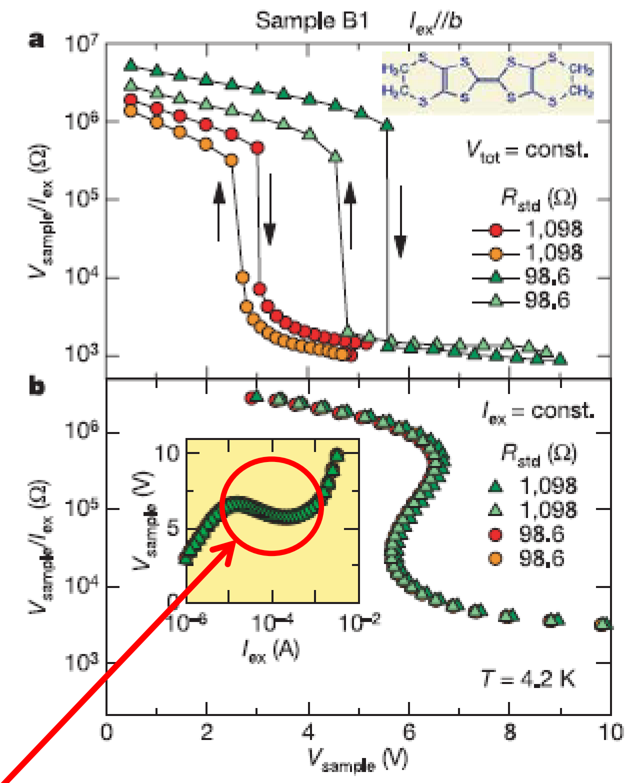
Question to the experimental physicists:

"Organic thyristor"

F. Sawano, I. Terasaki, Y. Nogami et al., Nature 437 (2005) 522.

θ-$(BEDT\text{-}TTF)_2CsCo(SCN)_4$ crystal at 4.2 K.

Charge order insulator



What happens if we raise the temperature?

要となる(と思われる)物理量:

有効温度

$z=z_*$の点はD7-brane理論の微小揺らぎ自由度に対して「horizon」の役割を果たしている。

(D7-brane上のopenstring metricを見ると、$z=z_*$にhorizonが存在する。)

M. S. Alam, V. S. Kaplunovsky and A. Kundu, arXiv:1202.3488 [hep-th]

ここでの「Hawking温度」をT_{eff}とすると　$T_{eff} > T$

「クォーク系」のnoiseは、この有効温度に従っている。

J. Sonner and A. G. Green, arXiv:1203.4908 [cond-mat.str-el]

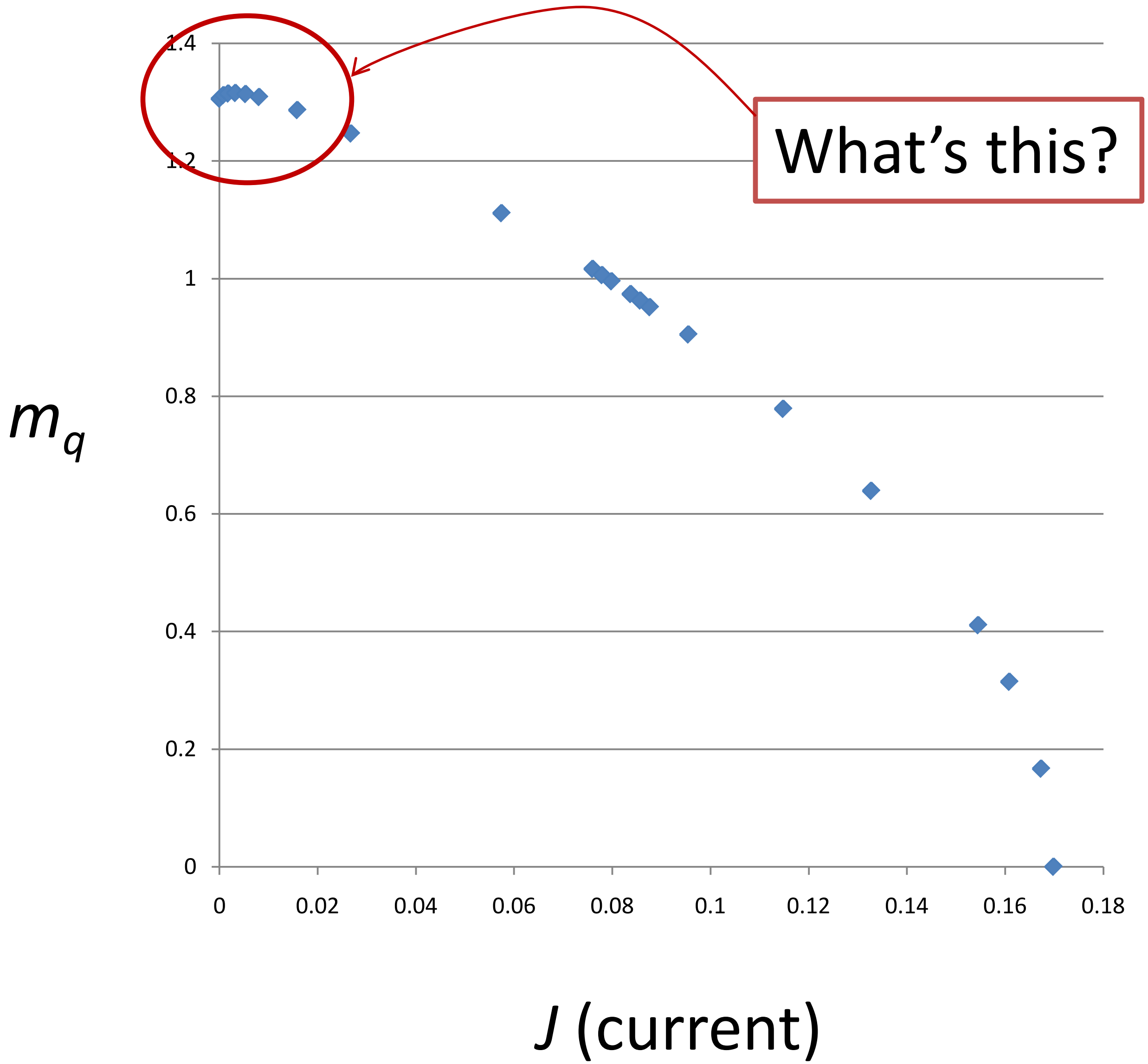
非平衡定常系における有効温度の概念:

例えば、原田崇広　日本物理学会誌64-6 (2009)445.

⇨ T_{eff}を基準とした解析を計画中。

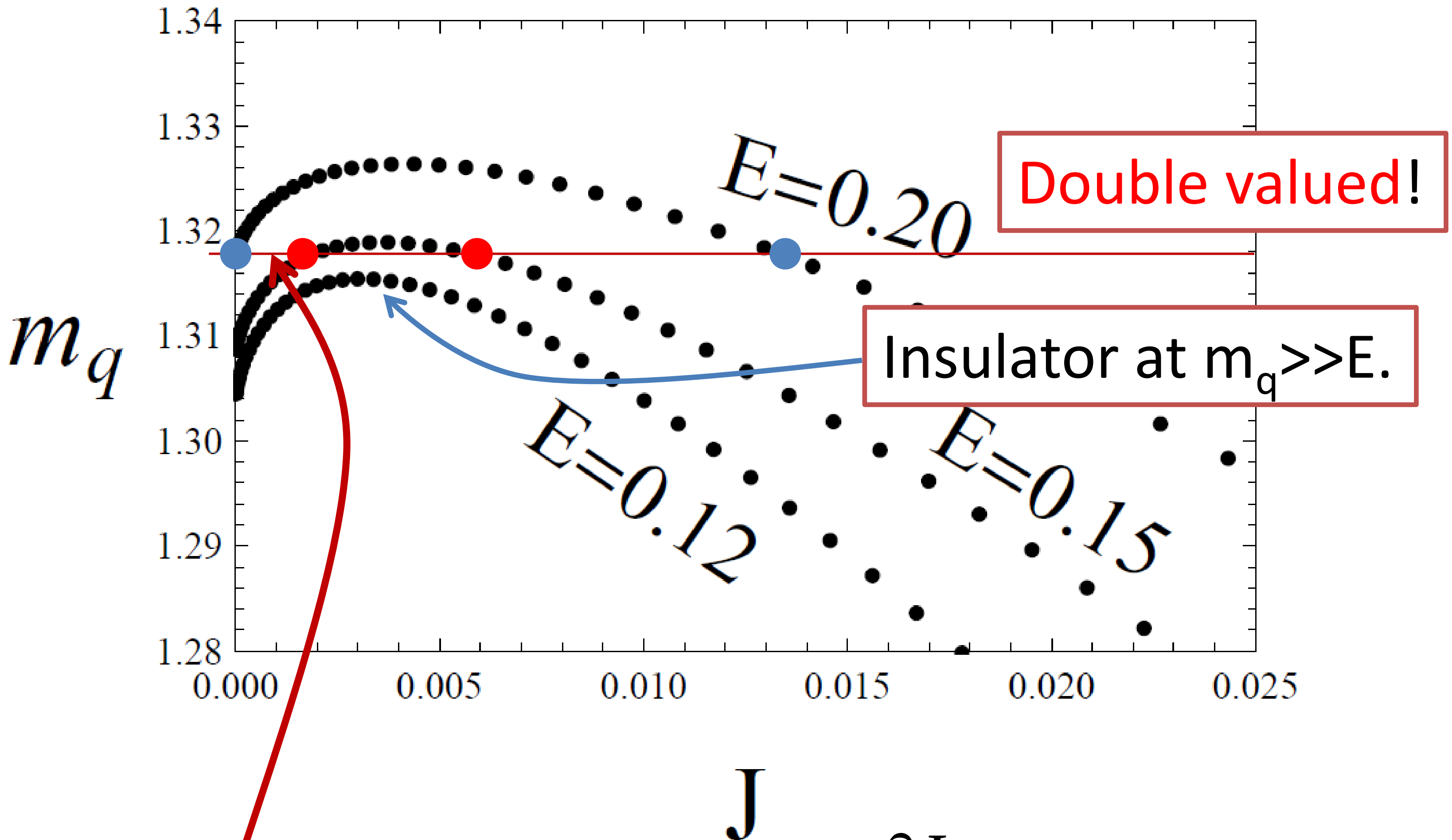
Backup

J-m_q chracteristics



E=0.13 in our convention.

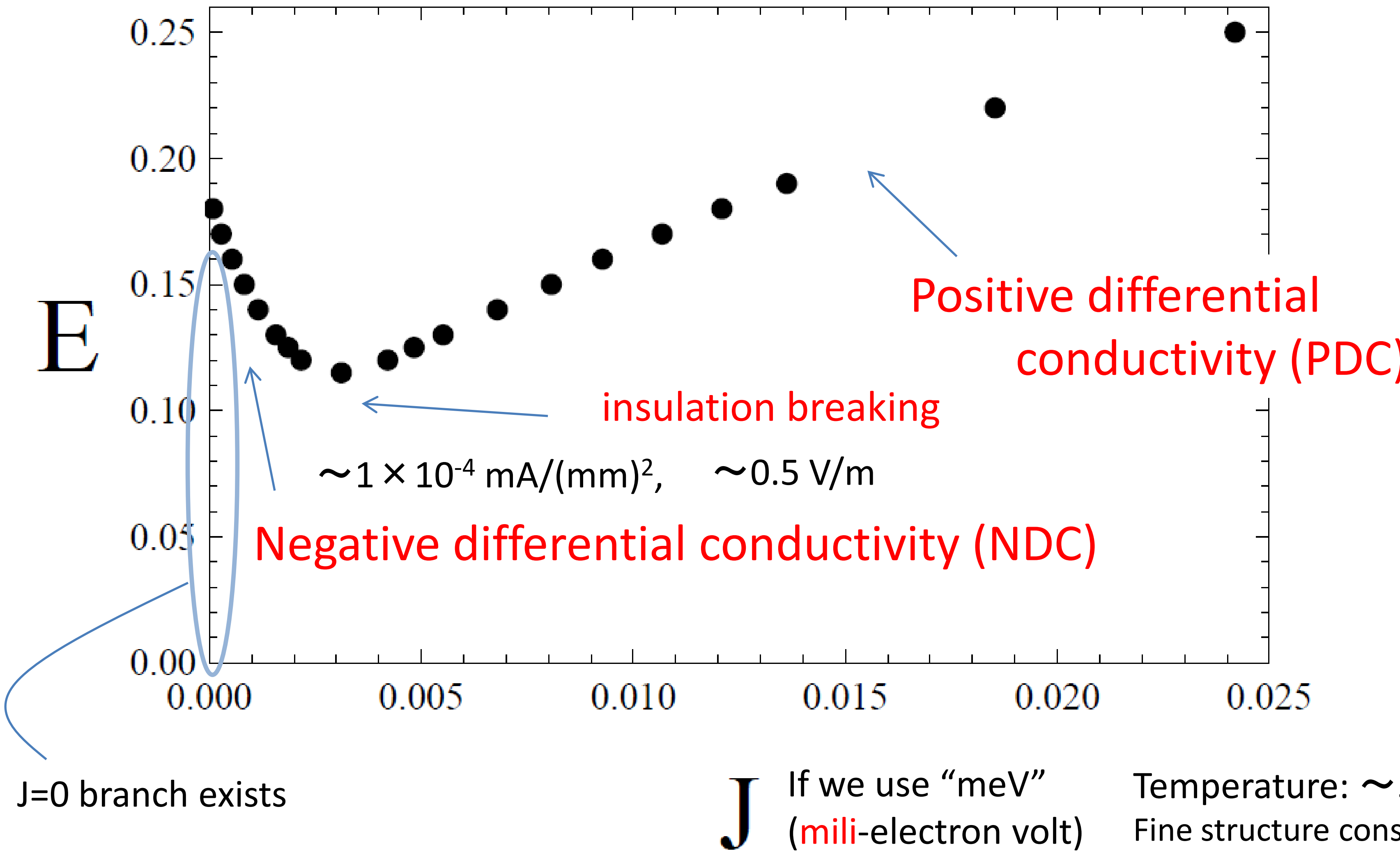
J-m_q chracteristics



Negative Differential Conductivity: $\frac{\partial J}{\partial E} < 0$

An example of J-E characteristics

S.N. Prog. Theor. Phys. 124(2010)1105.



If we use “meV” (mili-electron volt) as the unit.

Temperature: ∼5 K

Fine structure constant read form the Coulomb interaction : ∼O(1)

m_q=1.315

The non-linear conductivity

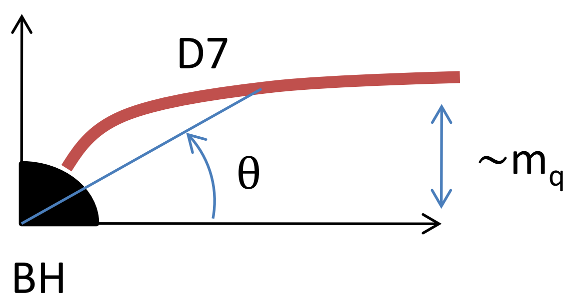
The charge density is also taken into account.
[Karch, O'Bannon JHEP0709(2007)024]

Normal conduction

$$\sigma_{xx} = \sqrt{\frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1}\cos^6\theta(z_*) + \frac{d^2}{e^2+1}}$$

Pair-creation

$$d \equiv \frac{\langle J^t \rangle}{\frac{\pi}{2}\sqrt{2\lambda}T^2}, \quad e \equiv \frac{E}{\frac{\pi}{2}\sqrt{2\lambda}T^2},$$



- $\cos\theta(z_*)$ goes to 1 at $m_q \to 0$.
- $\cos\theta(z_*)$ goes to zero at $m_q \to$ infinity.

Seems to be reasonable?

$$J_x\Big|_{m_q\to\infty} = \frac{d}{\sqrt{1+e^2}}E \to \begin{cases} \approx \quad d\cdot E & (e \ll 1) \\ \approx \quad \text{saturate} & (e \gg 1) \\ \approx \quad 0 & (T \gg 1) \\ (e \leftrightarrow -e \text{ symmetric}) & \end{cases}$$