Towards thermalization in heavy-ion collisions

Yoshitaka Hatta
(U. Tsukuba)

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Early stage of HIC

Hydro works at late times

Earliest times Color Glass Condensate (CGC)

Intermediate times difficult.

Very fast equilibration?
Gluodynamics in the $\tau - \eta$ coordinates

Proper time $\tau = \sqrt{t^2 - (x^3)^2}$, Rapidity $\eta = \tanh^{-1} \frac{x^3}{t}$

Solve the classical Yang—Mills equation in the gauge $A^\tau = 0$

\[ -\frac{1}{\tau} \partial_\tau \left( \frac{1}{\tau} \partial_\tau A_\eta \right) + \frac{1}{\tau^2} D_i F_{i\eta} = 0, \]
\[ -\frac{1}{\tau} \partial_\tau (\tau \partial_\tau A_i) + \left( \frac{1}{\tau^2} D_\eta F_{\eta i} + D_j F_{j i} \right) = 0 \]

The initial condition

\[ A_i = A^1_i + A^2_i, \quad A_\eta = 0, \]
\[ \tau \partial_\tau A_i = 0, \quad \frac{1}{\tau} \partial_\tau A^a_\eta = -g f_{abc} A^{1b}_i A^{2c}_i \]

Thermalization in CGC picture

Classical YM eq. alone is insufficient for the problem of thermalization. Need quantum fluctuations

- Romatschuke, Venugopalan (2006);
- Fukushima, Gelis, McLerran (2007)

Present status: Classical statistical approach
- Quantum fluctuations resummed to all orders (see later)
  - Dusling, Epelbaum, Gelis, Venugopalan, (2010)

In this work we propose to apply the 2PI formalism
to the problem of thermalization in HIC
2PI formalism

• First principle calculation in field theories out of equilibrium.
• Based on the CJT effective action

\[ \Gamma[A, G] = S_{YM}[A] + \frac{i}{2} \text{tr} \ln G^{-1} + \frac{i}{2} \text{tr} G_0^{-1}[A]G + \Gamma_2[A, G] \]

Classical field \quad Quantum fluctuation \quad G = \langle aa \rangle \quad 2PI diagrams

• Achieves quantum thermal equilibrium (Bose-Einstein distribution) starting from far-from-equilibrium initial conditions
Effective action to three-loops

Equation of motion

\[
\frac{\delta \Gamma}{\delta A} = 0 \quad \text{and} \quad \frac{\delta \Gamma}{\delta G} = 0 \quad (G = G_0 + G_0 \Pi G)
\]

\[
G(x, x') = \langle T_C \{a(x)a(x')\} \rangle = \mathcal{F} - \frac{i}{2}(\theta_C(\tau - \tau') - \theta_C(\tau' - \tau)) \rho
\]

\[
D = \partial - igA
\]
Getting rid of the metric factors

Spatial metric
\[ \gamma_{\alpha\beta} \equiv \text{diag} (\tau^2, 1, 1) \quad x^\alpha = (\eta, x_\perp) \]

\[
S_{YM} = \int d\tau d\eta d^2 x_\perp \sqrt{\gamma} \left[ \frac{1}{2} \gamma^{\alpha\beta} \partial_\tau A_\alpha \partial_\tau A_\beta - \frac{1}{4} \gamma^{\alpha\beta} \gamma^{\gamma\delta} F_{\alpha\gamma} F_{\beta\delta} \right]
\]

\[
\mathcal{L}_3 + \mathcal{L}_4 = -g_{abc} \gamma^{\alpha\gamma} \gamma^{\beta\delta} (D_\alpha a_\beta)^a b c d e f - \frac{g^2}{4} f_{abc} f_{ab'c'} \gamma^{\alpha\gamma} \gamma^{\beta\delta} a_\alpha a_\beta a_\gamma a_\delta
\]

Rescale
\[ \zeta \equiv \tau \eta \quad A_\eta = \tau A_\zeta , \quad a_\eta = \tau a_\zeta , \quad \partial_\eta = \tau \partial_\zeta \]

in the interaction terms, but not in the kinetic term.

In the new coordinates \[ x^I = (\zeta, x_\perp) \]

Feynman rules are formally the same as in the flat metric case.
Results at two-loops

\[
\left( \partial_\tau^2 A_I + \frac{1}{\tau} \partial_\tau A_I - \delta_I \zeta \frac{A_I}{\tau^2} - D_J F_{JI} \right)^a = g f_{abc} \left(D_{xI}^{be} F_{Jj}^{ec}(x, y) + D_{xJ}^{be} \left(F_{JI}^{ec}(x, y) - 2F_{JI}^{ec}(x, y)\right)\right)_{y=x} + \frac{ig^2}{2} C_{ab,cd} \int_{\tau_0}^{\tau} d^4 y \ V_{lmn,LMN}^y \left[ \rho^{bl}_{IL}(x, y) F_{JM}^{cm}(x, y) F_{IN}^{dn}(x, y) \right. \\
\left. + (F \rho F) + (F F \rho) - \frac{1}{4} (\rho \rho \rho) \right]
\]

\[
\left( \partial_\tau^2 + \frac{1}{\tau} \partial_\tau - \frac{1}{\tau^2} \delta_I \zeta \right) \delta_I J - (D^2 \delta_I J - D_I D_J - 2i g F_{IJ}) \right)^{ab} F_{JK}^{bc}(x, y) + g^2 \left(C_{ad,be} F_{IJ}^{de}(x, x) + \frac{1}{2} C_{ab,de} F_{MM}^{de}(x, x) \delta_{IJ} \right) F_{JK}^{bc}(x, y) \\
= - \int_{\tau_0}^{\tau'} d^4 z \ \Pi_\rho(x, z)^{ab}_{IJ} F_{JK}^{bc}(z, y) + \int_{\tau_0}^{\tau'} d^4 z \ \Pi_\mathcal{F}(x, z)^{ab}_{IJ} \rho_{JK}^{bc}(z, y),
\]

\[
\Pi_\mathcal{F}(x, y)^{ab}_{IJ} = \frac{1}{2} \tilde{V}_{x}^{al_1,l_2,LM,LM'} \left( F_{LL}^{ll'} F_{MM}^{mm'} - \frac{1}{4} \rho_{LL}^{ll'} \rho_{MM}^{mm'} \right)_{x}^{y} \tilde{V}_{bl'lm',JM,LM'}^{y}
\]

\[
\Pi_\rho(x, y)^{ab}_{IJ} = \frac{1}{2} \tilde{V}_{x}^{al_1,l_2,LM,LM'} \left( F_{LL}^{ll'} \rho_{MM}^{mm'} + \rho_{LL}^{ll'} F_{MM}^{mm'} \right)_{x}^{y} \tilde{V}_{bl'lm',JM,LM'}^{y}
\]
Remarks

• Transform covariantly under the residual (\(T\)-independent) gauge transformation

\[
A_I \rightarrow U A_I U^\dagger + \frac{i}{g} U \partial_t U^\dagger \quad a_I^a \rightarrow (U a_I U^\dagger)^a = U^{ab} a_I^b
\]

• Challenging to solve numerically if the background is inhomogeneous. First try the homogeneous case

• The initial condition for \(\mathcal{F}\) nontrivial

Dusling, Gelis, Venugopalan (2011)
Comparison with previous approaches


Solve the YM equation perturbatively

\[ A_I = A_I^{(0)} + A_I^{(1)} + A_I^{(2)} + \cdots \]

classical YM linearized around YM

with the initial condition

\[ A(\tau_0) = A^{(0)}(\tau_0) + A^{(1)}(\tau_0) \]

Identify

\[ F^{(0)}(x, y) \equiv A^{(1)}(x)A^{(1)}(y) \]
Diagrammatic interpretation
Classical statistical approximation

\[ \mathcal{F} \mathcal{F} \gg \rho \rho \rightarrow 0 \]

In free scalar theory,

\[ \mathcal{F}(t - t', p) = \frac{\cos(t - t')p}{p} \left( n(p) + \frac{1}{2} \right) \quad \rho(t - t', p) = \frac{\sin(t - t')p}{p} \]

Valid only at low momentum \( \rho \ll T \)

Classical thermal equilibrium \( n(p) = \frac{T}{p} - \frac{1}{2} \)

In the 2PI formalism the Bose-Einstein distribution is guaranteed.

Boltzmann (exponential) distribution is an unmistakable feature in HIC!
Conclusions

• CGC meets the 2PI formalism
• Complementary to the classical statistical approximation.
• In principle, 2PI can describe the evolution from right after the collision to the late quantum regime in a single framework.