「熱場の量子論とその応用」研究会
２０１０年８月３０日-９月１日

有限密度QCDの臨界点近傍の特異性

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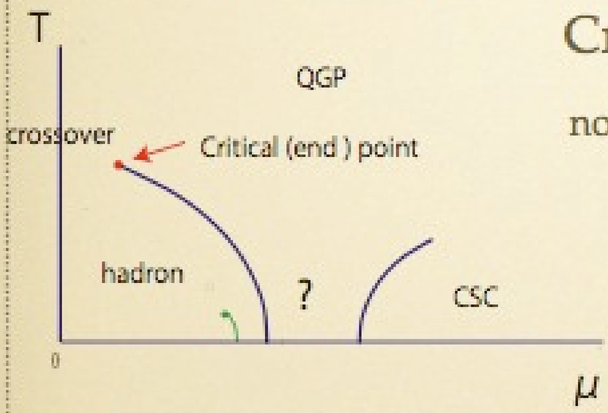
1. Introduction

QCD phase diagram



Critical (end) point ?

notorious sign problem for $\mu \neq 0$

- Taylor series
- imaginary chemical potential
- reweighing
- strong coupling expansion

etc

Taylor series $\langle O \rangle = \sum_n c_n \left(\frac{\mu}{T}\right)^n$

⟶ convergence radius

⟶ singularities in complex μ plane

Partition function zeros　Lee-Yang (1952)

Towards MC determination: Ejiri's calculations (2009)

Plan

- 1. Introduction (done)
- 2. Lee-Yang zeros and Fisher zeros
- 3. Edge singularities
- 4. QCD effective theory (Hatta-Ikeda 2003)
- 5. Convergence radius
- 6. Stokes line and MC simulations
- 7. summary

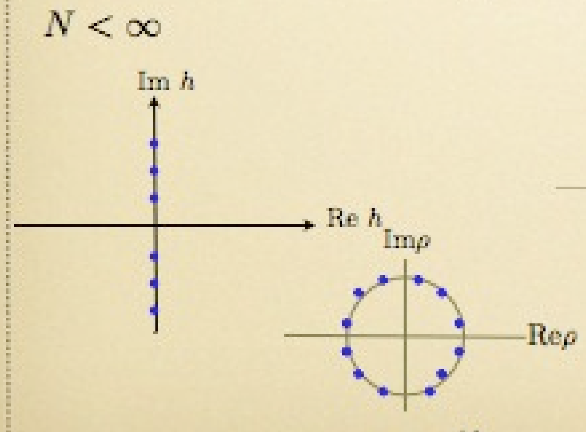
2. Lee-Yang zeros and Fisher zeros

- Lee-Yang zeros for complex h in d-dimension

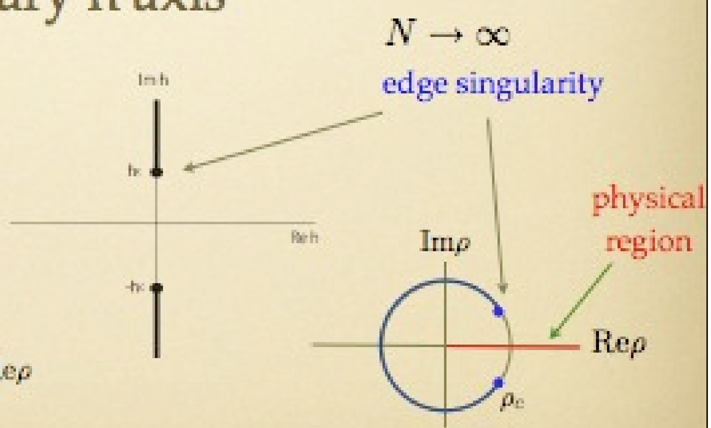
$$Z(h) = \sum_{\sigma} \exp\left(\sum_{ij} \beta \sigma_i \sigma_j + h \sum_i \sigma_i \right) = 0$$

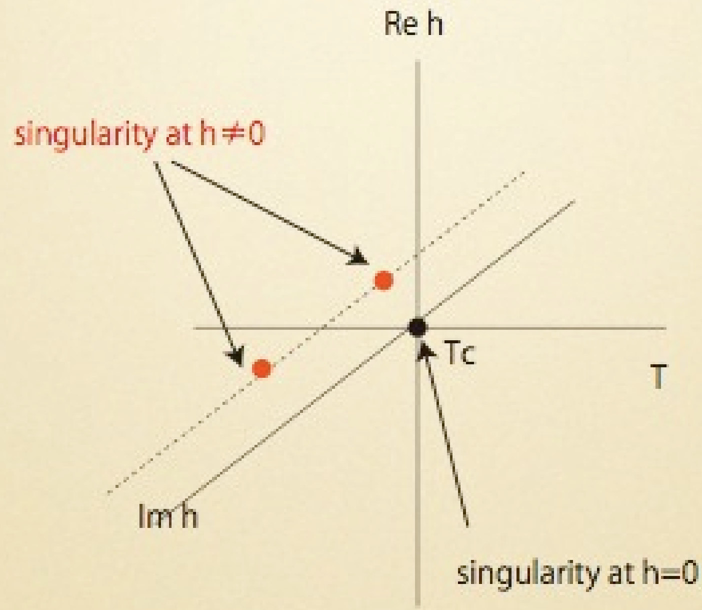
- Lee-Yang theorem:

zeros on the imaginary h axis



$\rho = e^{-2h}$





3. Lee-Yang edge singularities

Ising model (complex h)

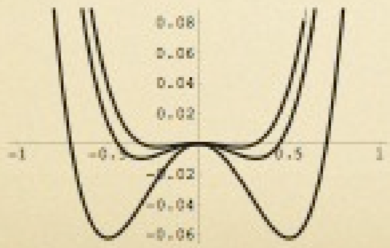
$$\Omega(\varphi) = a\varphi^2 + b\varphi^4 + h\varphi$$

$$\frac{\partial \Omega(\varphi)}{\partial \varphi} = 2a\varphi + 4b\varphi^3 + h = 0 \quad (1)$$

onset of a multivalued solution for minimization equation (1)

$$\frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} = 2a + 12b\varphi^2 = 0. \quad \text{for } \varphi \text{ satisfying (1)}$$

$h = 0$



$$\frac{\partial \Omega(\varphi)}{\partial \varphi} = \frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} = 0.$$

$$\Rightarrow \quad a = 0, \varphi = 0$$

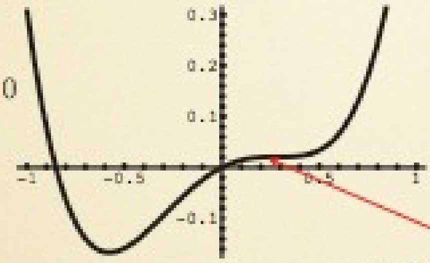
$h \neq 0$

$a \propto t < 0$

$$\frac{\partial \Omega(\varphi)}{\partial \varphi} = 2a\varphi + 4b\varphi^3 + h = 0$$

$$\frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} = 2a + 12b\varphi^2 = 0.$$

$$t = \frac{T - T_c}{T_c} > 0$$

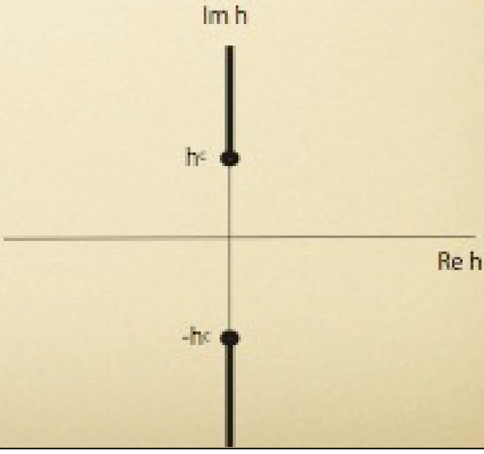


$$\Omega = a\varphi^2 + b\varphi^4 + h\varphi$$

$a = -0.5, b = 1, h = 0.19245008972$

$\varphi = 0.288675$ satifying $\frac{\partial \Omega(\varphi)}{\partial \varphi} = \frac{\partial^2 \Omega(\varphi)}{\partial \varphi^2} = 0.$

$a \propto t > 0$



$$\Rightarrow \quad h = -2\varphi(a + 2b\varphi^2) = \pm\frac{4}{3}\sqrt{-\frac{a}{6b}}\,a \equiv \pm i h_c,$$

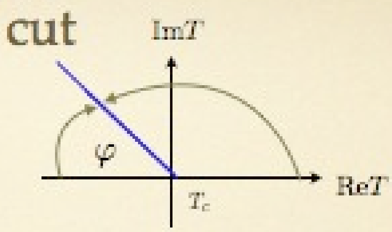
$$h_c \sim t^{\Delta}, \quad \Delta = 3/2$$

$$\Omega_{\text{sing}} \sim (h \pm i h_c)^{1+\sigma}, \quad \sigma = 1/2$$

Fisher zeros complex T

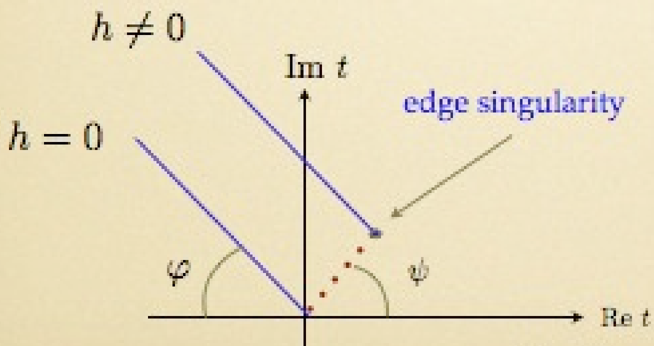
$$F_{\text{sing}}^{(+)} \approx A_+ \left(\frac{T - T_c}{T_c}\right)^{2-\alpha}$$

$$F_{\text{sing}}^{(-)} \approx A_- \left(\frac{T_c - T}{T_c}\right)^{2-\alpha}$$



$$\tan\left[(2-\alpha)\,\varphi\right] = \frac{\cos\alpha\pi - \frac{A_-}{A_+}}{\sin\alpha\pi}$$

universal relationship



$$t = \frac{T - T_c}{T_c}$$

$$t \approx e^{i\psi}\left(\frac{h}{\sqrt{c}}\right)^{1/\beta\delta}$$

$$\psi = \frac{\pi}{2\beta\delta}$$

4. Hatta-Ikeda model

Y. Hatta and T.Ikeda, Phy. Rev. D 67, 014028(2003).

$$\Omega = -m\sigma + \frac{a}{2}\sigma^2 + \frac{b}{4}\sigma^4 + \frac{c}{6}\sigma^6,$$

expands around the tricritical point (TCP) $a = 0, b = m = 0$

$$\begin{aligned} a(T,\mu) &= C_a(T - T_t) + D_a(\mu - \mu_t) \\ b(T,\mu) &= C_b(T - T_t) + D_b(\mu - \mu_t), \end{aligned} \qquad C_b D_a - C_a D_b > 0.$$

By switching on m , the condition for the critical end point

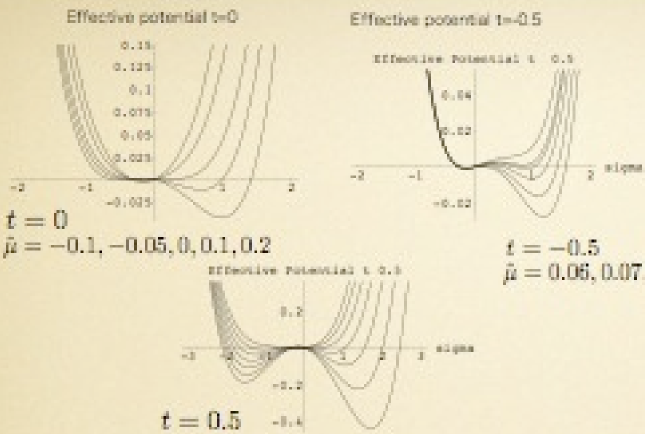
$$\frac{\partial\Omega(T_{CE}, \mu_{CE}, \sigma_0)}{\partial\sigma} = \frac{\partial^2\Omega(T_{CE}, \mu_{CE}, \sigma_0)}{\partial\sigma^2} = \frac{\partial^3\Omega(T_{CE}, \mu_{CE}, \sigma_0)}{\partial\sigma^3} = 0$$

$$\Rightarrow \; a(T_{CE}, \mu_{CE}) = \frac{9b_m^2}{20c}, \quad -b(T_{CE}, \mu_{CE}) = \frac{5}{54^{1/5}} c^{3/5} m^{2/5}, \quad \sigma_0 = \sqrt{\frac{-3\, b(T_{CE}, \mu_{CE})}{10c}}.$$

thermodynamic potential around the critical end point

$$\Omega(T, \mu, \sigma) = \Omega(T_{CE}, \mu_{CE}, \sigma_0) + A_1\hat{\sigma} + A_2\hat{\sigma}^2 + A_3\hat{\sigma}^3 + A_4\hat{\sigma}^4,$$

$$\begin{aligned} A_1 &= \left(C_a\sigma_0 + C_b\sigma_0^3\right)(T - T_{CE}) + \left(D_a\sigma_0 + D_b\sigma_0^3\right)(\mu - \mu_{CE}) \\ A_2 &= \frac{1}{2}\left(C_a + 3C_b\sigma_0^3\right)(T - T_{CE}) + \frac{1}{2}\left(D_a + 3D_b\sigma_0^3\right)(\mu - \mu_{CE}) \\ A_3 &= \{C_b(T - T_{CE}) + D_b(\mu - \mu_{CE})\}\sigma_0 \\ A_4 &= -\frac{b_m}{2} + \frac{1}{4}C_b(T - T_{CE}) + D_b(\mu - \mu_{CE}.) \end{aligned}$$



$t = 0$
$\hat{\mu} = -0.1, -0.05, 0, 0.1, 0.2$

$t = -0.5$
$\hat{\mu} = 0.06, 0.07, 0.08, 0.087, 0.09, 1.0$

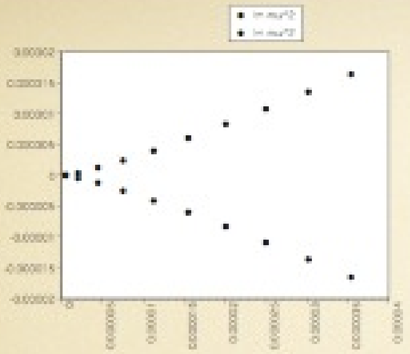
$t = 0.5$
$\hat{\mu} = -0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1$

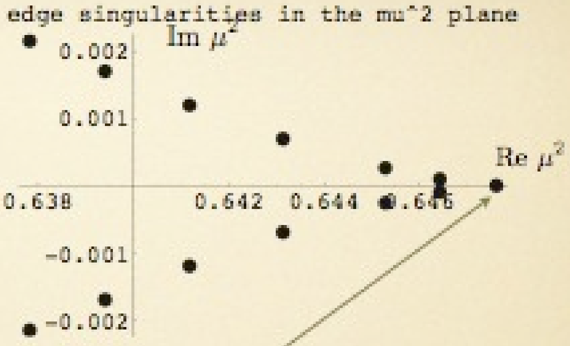
$t = T - T_{CE}$

$\hat{\mu} = \mu - \mu_{CE}$

$$\frac{\partial\Omega}{\partial\sigma} = 0, \quad \frac{\partial^2\Omega}{\partial\sigma^2} = 0$$

$$A_1 + 2A_2\sigma + 3A_3\sigma^2 + 4A_4\sigma^3 = 0, \quad 2A_2 + 6A_3\sigma + 12A_4\sigma^2 = 0.$$





$$\mathrm{Re}\mu = \mathrm{Re}\hat{\mu} + \mu_0, \quad \mathrm{Im}\mu = \mathrm{Im}\hat{\mu}$$

$$t \equiv T - T_{CEP} = 0, 0.01, 0.02, 0.04, 0.06, 0.08, 0.1$$

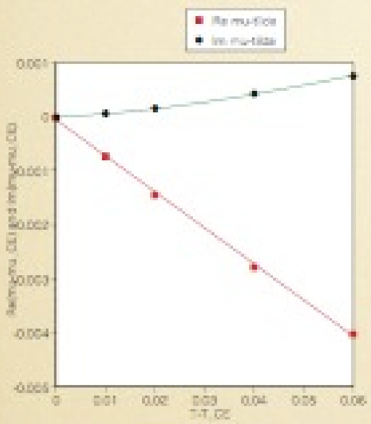
$$\mu_0 = \frac{5C_a c^{3/5}}{(54)^{1/5}(C_b D_a - C_a D_b)} m^{2/5} + \mu_t$$

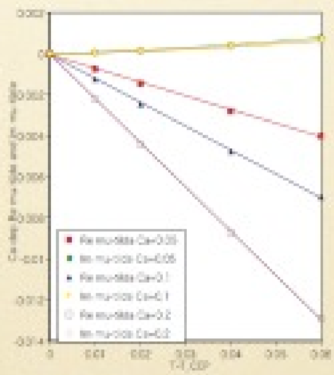
$$C_a = 0.05, x = m^{1/5} = 0.2$$

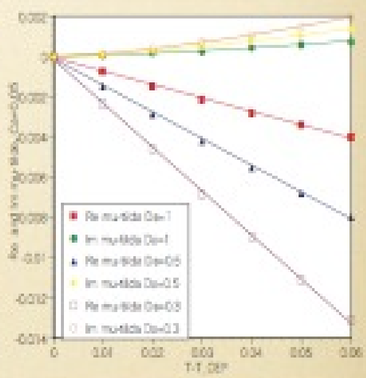
$$\mu_t = 0.8$$

$$C_b = D_a = D_b = c = 1.0$$

$$\mu(T) - \mu_{CEP} \approx c_1(T - T_{CEP}) + ic_2(T - T_{CEP})^{\beta\delta} \qquad \beta\delta = 3/2$$



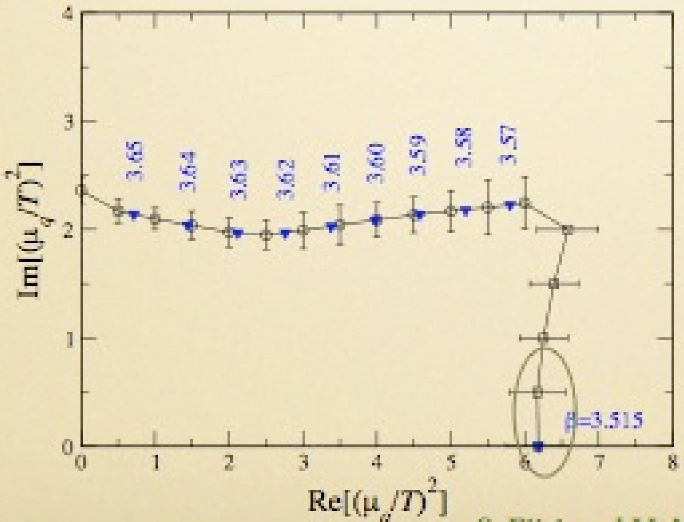




towards determination of convergence radius in MC

Ejiri: Taylor's expansion in $\frac{\mu}{T}$

→ Taylor's expansion in the complex μ plane



S. Ejiri and H. Y, arXiv:0911.2257[hep-lat] (2009).

7. Summary

Singularities around the QCD critical (end) point

· edge singularities

in terms of Hatta-Ikeda model

convergence radius

→ · investigation of more realistic models

· further study of MC simulations