Keldysh形式による量子輸送における揺らぎの定理

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Fluctuation theorem (FT)

Evans, Cohen, Morris, PRL 1993, Gallavotti, Cohen, PRL 1995

$$\frac{P(-\Delta S)}{P(+\Delta S)} = \exp(-\Delta S)$$

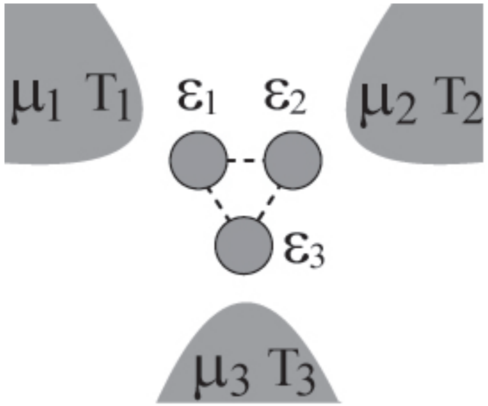
ΔS entropy production during τ

An exact relation in *far from* equilibrium --- microscopic reversibility

- Second law of thermodynamics [Jarzynski PRL 1997, Crooks PRE 1999, etc.]
- Landauer principle [Piechocinska, PRA 2000]
- Fluctuation-dissipation theorem & Onsager relations [Gallavotti PRL 1996]

Full counting statistics (FCS)

[Tobiska Nazarov PRB 2005, Saito YU, PRB 2008, Förster Büttiker PRL 2008, Andrieux Gaspardt Monnai Tasaki NJP 2009]



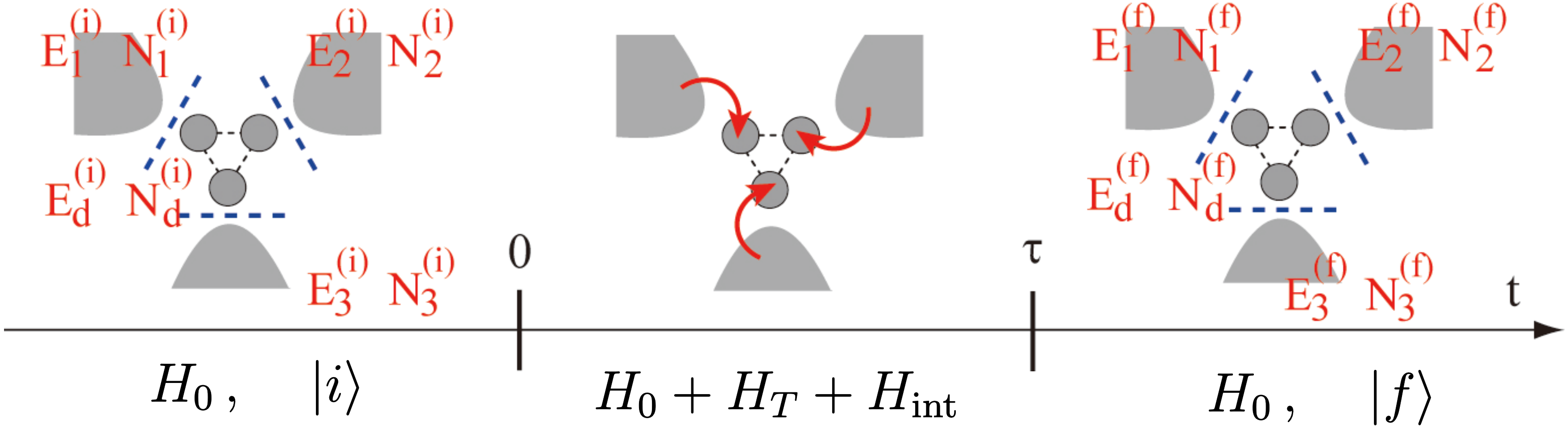
$$H_d = \sum_{i,,j,\sigma} |t_{ij}| \mathrm{e}^{i\,\phi_{ij}(B)}\, d^\dagger_{i\sigma} d_{j\sigma}$$

$$H_r = \sum_{k\sigma} \varepsilon_{rk} a^\dagger_{rk\sigma} a_{rk\sigma} \quad (r = 1, \cdots, m)$$

$$H_{\mathrm{int}} = \sum_{ij\sigma\sigma'} U_{i\sigma j\sigma'}\, d^\dagger_{i\sigma} d^\dagger_{j\sigma'} d_{j\sigma'} d_{i\sigma}/2$$

$$H_T = \sum_{rki\sigma} |t_{rki}| \mathrm{e}^{i\,\phi_{ri}(B)}\, d^\dagger_{i\sigma} a_{rk\sigma} + \mathrm{H.c.}$$

$$\phi(-B) = -\phi(B)$$



$$H_0 = \sum_{s=1,\cdots,m,d} H_s$$

At initial and final states, reservoirs and dots are decoupled
⇒ Energy & number of electrons are good quantum numbers

Flowing out charge $q_{cs} = N_s^{(f)} - N_s^{(i)}$ Flowing out energy $q_{hs} = E_s^{(f)} - E_s^{(i)}$

Joint probability distribution

$$P(\{q_{cs}, q_{hs}\}; B) = \sum_{if} \left| \langle f | \mathrm{e}^{-iH\tau} | i \rangle \right|^2 \langle i | \rho_0 | i \rangle \prod_{s'} \delta_{q_{cs'},\, N_{s'}^{(f)} - N_{s'}^{(i)}} \, \delta\left(q_{hs'} - E_{s'}^{(f)} + E_{s'}^{(i)} \right)$$

FCS & FT

Fourier Transform ⇒ Characteristic Function

$$\mathcal{Z}(\{\lambda_{cs}, \lambda_{hs}\}; B) = \mathrm{Tr}\left[\rho_0 V^\dagger e^{iH\tau} V^2 e^{-iH\tau} V^\dagger\right] = \mathrm{Tr}\left[e^{iH_-\tau} e^{-iH_+\tau} \rho_0\right]$$

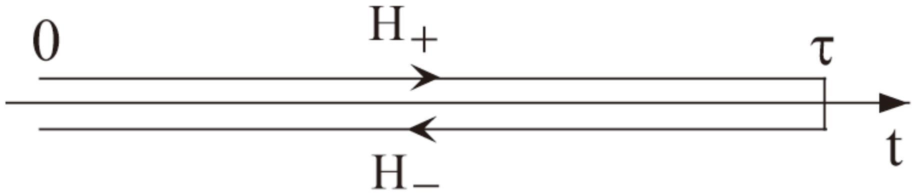
$$V(\{\lambda_{cs}, \lambda_{hs}\}) = \prod_s \exp[-i(\lambda_{hs} H_s + \lambda_{cs} N_s)/2]$$

↗ ↗

Time translation + Gauge transformation

$$H_+ = V H V^\dagger \qquad H_- = V^\dagger H V$$

Keldysh contour



Microscopic reversibility + $\rho_0 \propto \prod_{s=1,\cdots,m,d} \exp[-(H_s - \mu_s N_s)/T_s]$

$$\mathcal{F}(\{\chi_{\alpha s} \equiv \lambda_{\alpha s} - \lambda_{\alpha m}\}; B) \equiv \lim_{\tau\to\infty} \ln \mathcal{Z}(\{\lambda_{\alpha s}\}; B)/\tau$$

$$\mathcal{F}(\{\chi_{\alpha r}\}; B) = \mathcal{F}(\{-\chi_{\alpha r} + i\mathcal{A}_{\alpha r}\}; -B)$$

$\mathcal{A}_{cr} = \mu_r/T_r - \mu_m/T_m \qquad \mathcal{A}_{hr} = T_m^{-1} - T_r^{-1}$ Affinities

Universal Relations among Nonlinear Transport Coefficients

Two terminal isothermal system

$$\mathcal{F}(\lambda; B) = \mathcal{F}(-\lambda + i\mathcal{A}; -B) \qquad \mathcal{A} = (\mu_1 - \mu_2)/T$$

$$\frac{P(q;B)}{P(-q;-B)} \approx \mathrm{e}^{q\mathcal{A}}$$

Nonlinear transport coefficient $\langle\langle I^n \rangle\rangle = \left.\frac{\partial^n \mathcal{F}(\lambda;B)}{\partial (i\lambda)^n}\right|_{\lambda=0} = \sum_{m=0}^{\infty} L_m^n(B) \frac{\mathcal{A}^m}{m!}$

$$L_0^2 = 2L_1^1 \qquad L_{1,-}^1 = 0$$

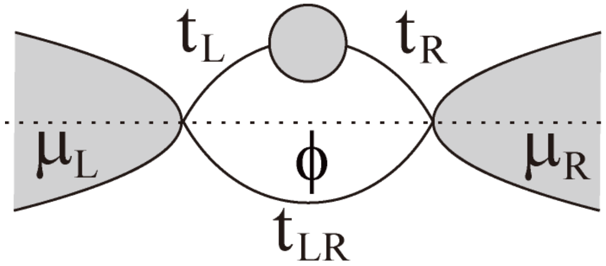
Fluctuation-dissipation theorem
& Onsager relation

$$L_{2,+}^1 = L_{1,+}^2 \qquad L_{0,+}^3 = 0 \qquad L_{2,-}^1 = L_{1,-}^2/3 = L_{0,-}^3/6$$

[Saito YU PRB 2008]

$$L_\pm = L(B) \pm L(-B)$$

QD Aharonov-Bohm interferometer



$$H = \sum_{r=L,R} H_r + H_D + H_{\mathsf{T}} + H_{\mathsf{ref}}$$

$$H_r = \sum_{k\sigma} \varepsilon_{rk\sigma}\, a^\dagger_{rk\sigma}\, a_{rk\sigma}$$

$$H_D = \sum_{\sigma=\uparrow,\downarrow} \epsilon_D\, d^\dagger_\sigma\, d_\sigma \; + U n_\uparrow n_\downarrow$$

$$H_{\mathsf{T}} = \sum_{rk\sigma} t_r\, d^\dagger_\sigma a_{rk\sigma} + \text{H.c.}$$

Anderson model

$$H_{\mathsf{ref}} = \sum_{kk'\sigma} t_{LR}\, \mathrm{e}^{i\phi}\, a^\dagger_{Rk\sigma} a_{Lk'\sigma} + \text{H.c.}$$

$$\phi(-B) = -\phi(B)$$

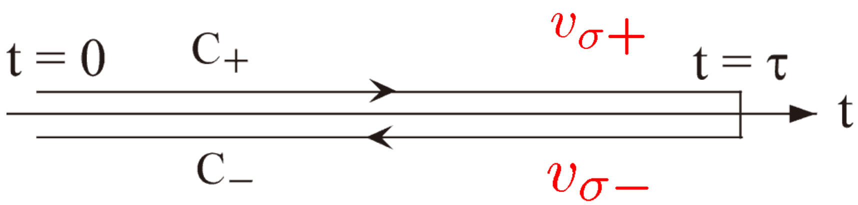
[YU Saito PRB 2009]

Keldysh path-integral

[Kamenev, in "Nanophysics: Coherence and Transport" (Elsevier 2005)]

$$\mathcal{Z} = \int \mathcal{D}[v_\sigma, a^*_{rk\sigma}, d^*_\sigma, a_{rk\sigma}, d_\sigma,] \exp\left[i \int_C dt \left(\mathcal{L}_0 + \frac{i}{U} v_\uparrow(t) v_\downarrow(t)\right)\right]$$

Stratonovich-Hubbard transformation $\quad U d^*_\uparrow d_\uparrow d^*_\downarrow d_\downarrow \to \sum_\sigma v_\sigma d^*_\sigma d_\sigma - v_\uparrow v_\downarrow / U$



Saddle-point approximation $\quad v_{\sigma,\pm}(t) = v_\pm$

$$\frac{v_+ + v_-}{2} = v_c \qquad v_+ - v_- = i\, v_q$$

$$\mathcal{F} = \mathcal{F}_0 - 2 v_c v_q / U \qquad v_q = \frac{U}{2}\frac{\partial \mathcal{F}_0}{\partial v_c} \qquad v_c = \frac{U}{2}\frac{\partial \mathcal{F}_0}{\partial v_q}$$

Extension of the Onsager relation

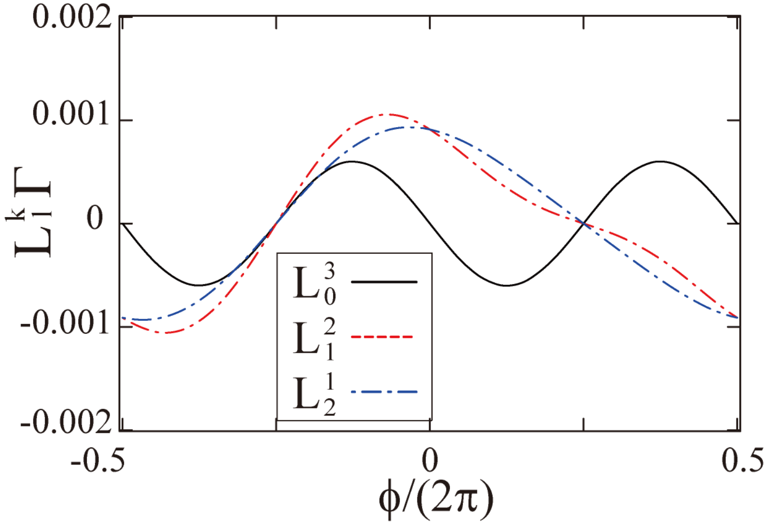
$$
\begin{cases}
L_0^3 = \langle\langle I^3 \rangle\rangle\big|_{\mathcal{A}=0} = 6\,\tilde{U}^{eq.} S_{IN}^{eq.}\,\chi_{II,N}^{eq.} \\
L_1^2 = d\langle\langle I^2 \rangle\rangle/d\mathcal{A}\big|_{\mathcal{A}=0} = 2\,\tilde{U}^{eq.}\,\chi_{NI}^{eq.}\,\chi_{II,N}^{eq.} + 4\,\tilde{U}^{eq.}\,S_{IN}^{eq.}\,\chi_{I,IN}^{eq.} \\
L_2^1 = d^2\langle\langle I \rangle\rangle/d\mathcal{A}^2\big|_{\mathcal{A}=0} = 4\,\tilde{U}^{eq.}\,\chi_{NI}^{eq.}\,\chi_{I,IN}^{eq.}
\end{cases}
$$

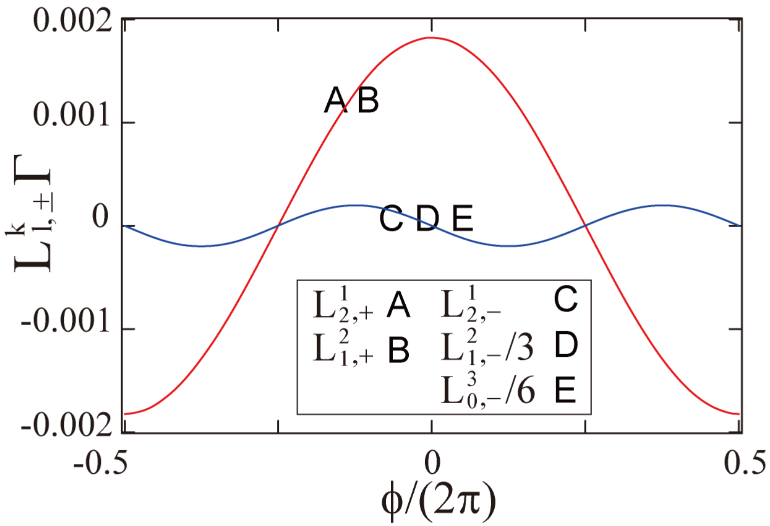
$$\chi_{II,N} = \frac{\partial S_{II}}{\partial \epsilon_D}$$

$$\chi_{I,IN} = \frac{\partial \chi_{IN}}{\partial \mathcal{A}}$$

$$\chi_{IN} = \frac{1}{2}\frac{\partial \langle\langle I \rangle\rangle}{\partial \epsilon_D}$$

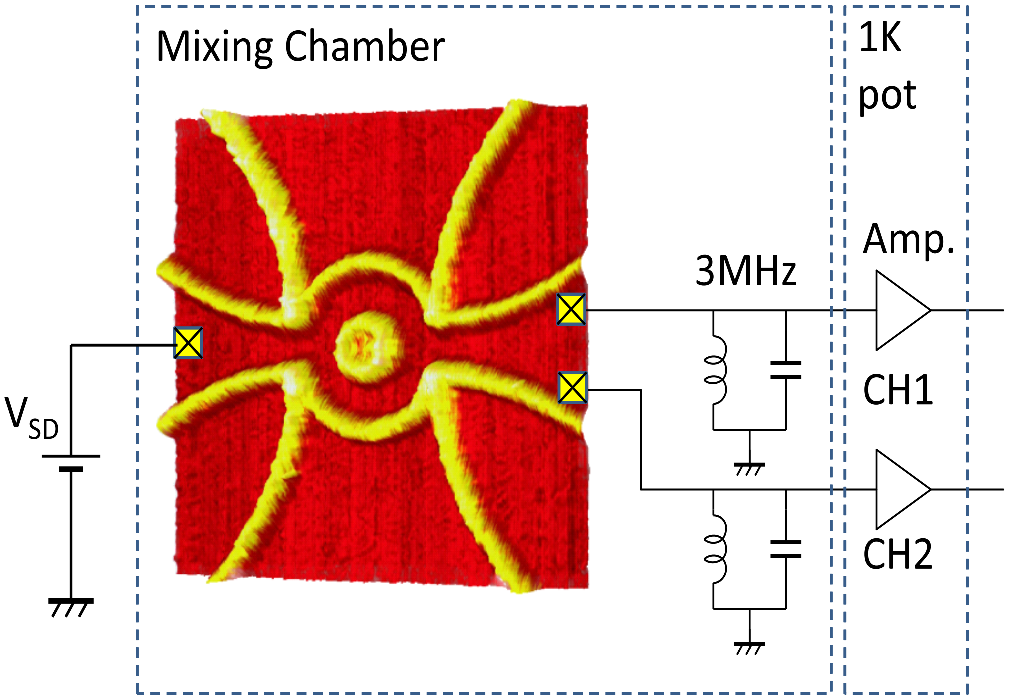
Coulomb interaction induces non-zero nonlinear transport coefficients

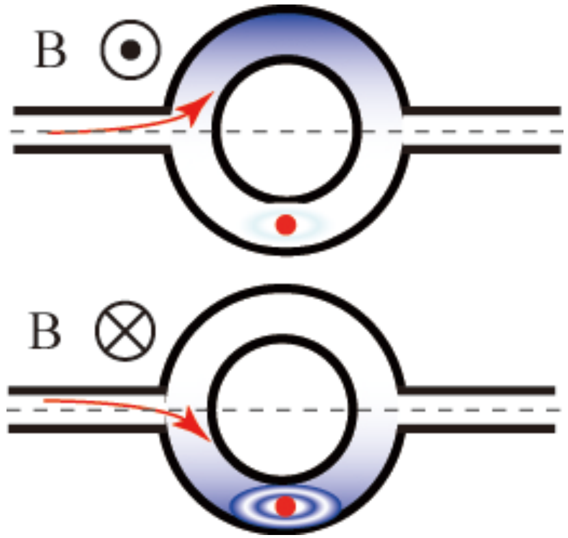




Magnetic field asymmetry in nonlinear conductance

Nakamura, Kobayashi, Leturcq, YU, et al. PRL 2010

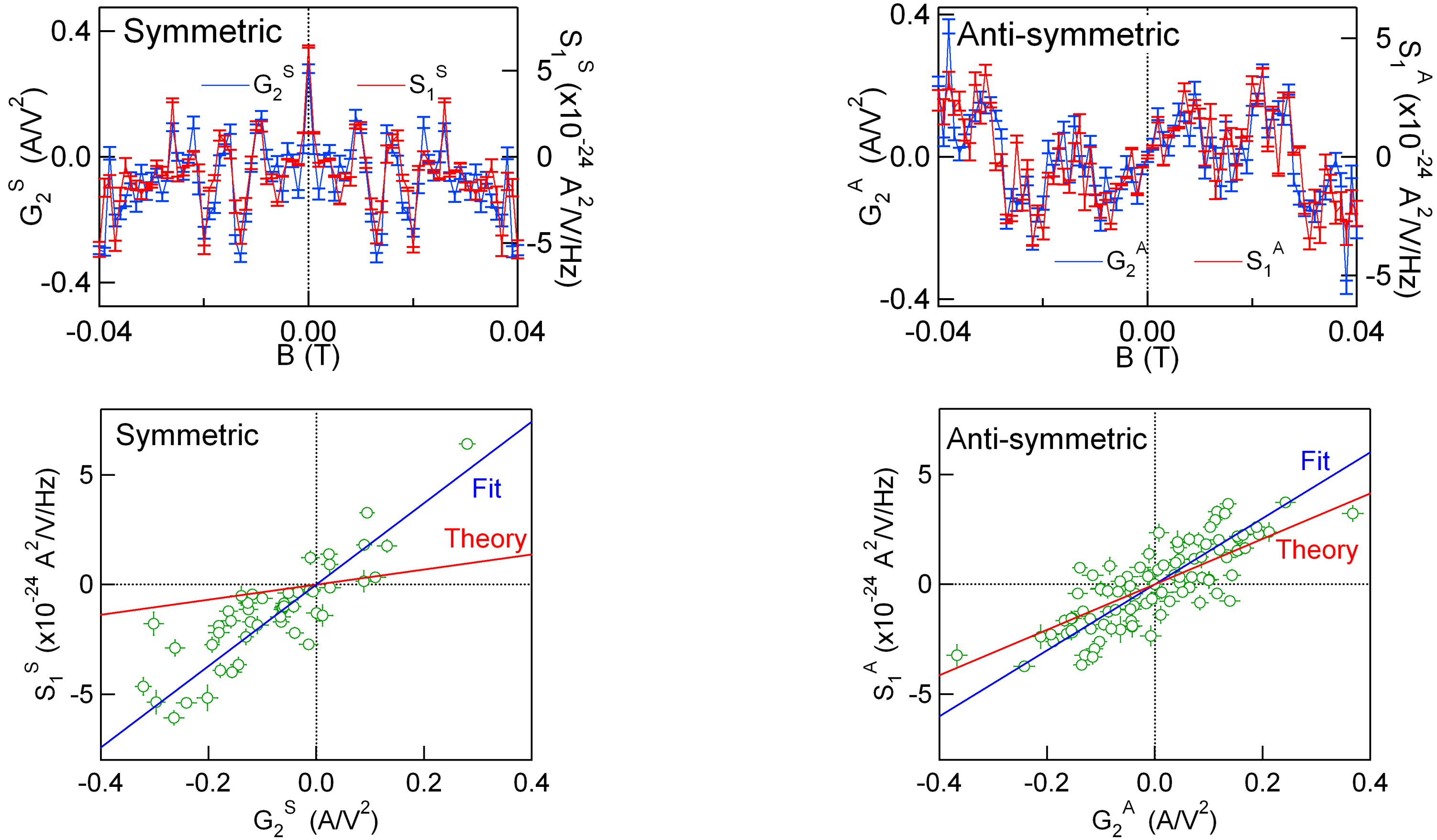




FT for lower order cumulants

$$G_2^S(B) \equiv G_2(B) + G_2(-B)$$
$$S_1^S(B) \equiv S_1(B) + S_1(-B)$$

$$G_2^A(B) \equiv G_2(B) - G_2(-B)$$
$$S_1^A(B) \equiv S_1(B) - S_1(-B)$$

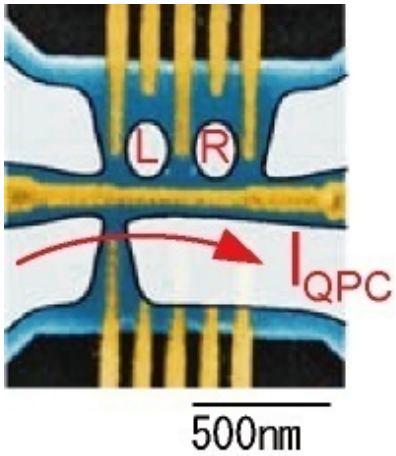


$$S_1^S(B) = 2k_B T G_2^S(B)\alpha^S$$
$$\alpha^S = 5.38^{+1.18}_{-0.70} \quad 1\text{(theory)}$$

$$S_1^A(B) = 6k_B T G_2^A(B)\alpha^A$$
$$\alpha^A = 1.45^{+0.21}_{-0.12} \quad 1\text{(theory)}$$

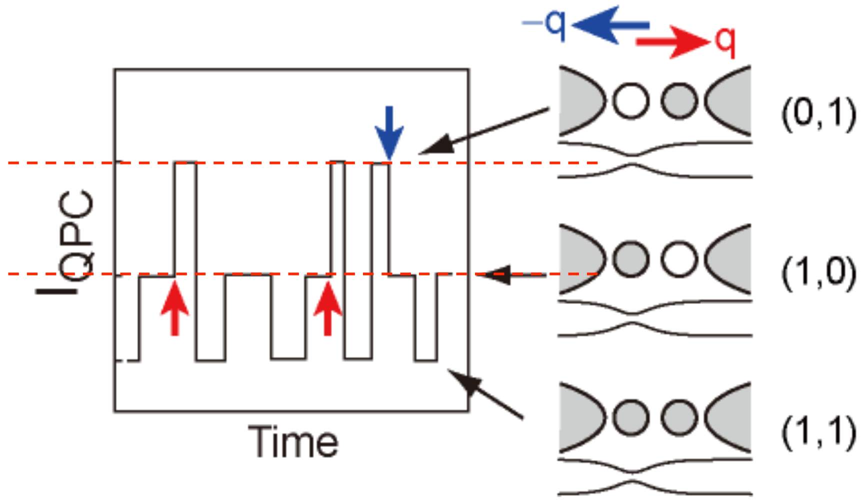
Bidirectional single electron counting

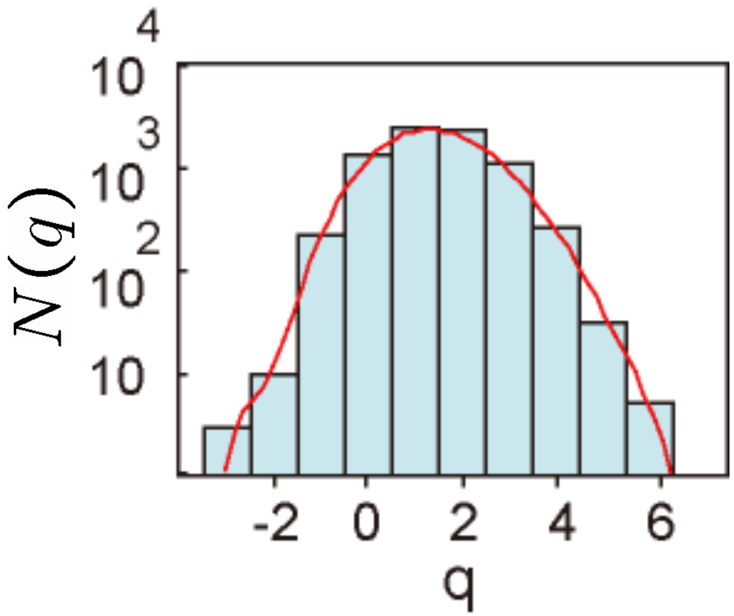
T. Fujisawa, T. Hayashi, R. Tomita, Y. Hirayama, Science 2006



Double quantum dots (sample)

Quantum point-contact electrometer





Short measurement time
& Low temperature

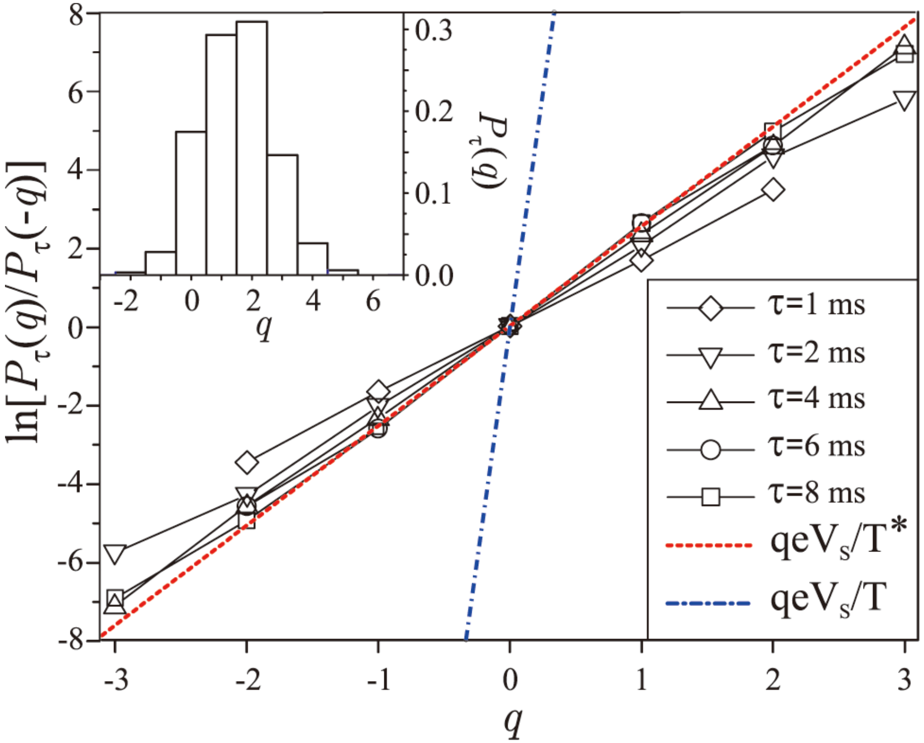
$\tau = 4\text{ms}$

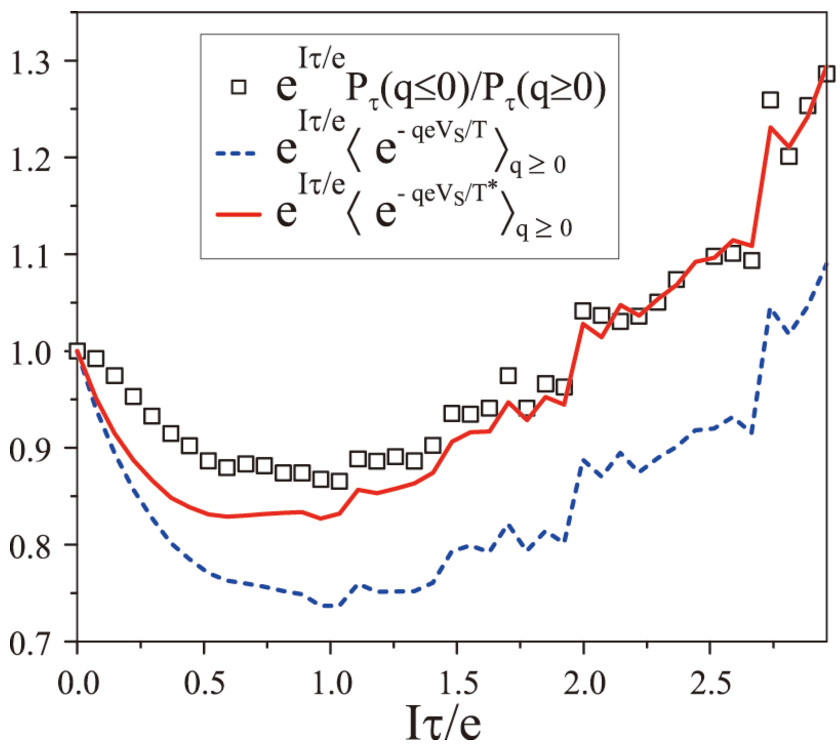
Experimental Test of FT

YU, D.Golubev, M.Marhtaler, K.Saito, T.Fujisawa, G.Schön, Phys. Rev. B **81**, 125331 (2010)

$$\frac{P_\tau(q)}{P_\tau(-q)} = \exp(qeV_S/T)$$

$$\frac{P_\tau(q \le 0)}{P_\tau(q \ge 0)} = \langle \exp(-qeV_S/T) \rangle_{q\ge 0}$$





I : average current

Fit to experiment with $T^* = 1.37\mathrm{K} \gg T = 130\mathrm{mK}$ ⇔ enhanced “temperature”

Summary

1. We combine the fluctuation theorem and the Keldysh formalism.
2. We experimentally confirmed the universal relations among nonlinear transport coefficients.

[1] K.Saito, YU, PRB **78**, 115429 (2008).
[2] YU, K. Saito, PRB 79, 23511 (2009).
[3] YU, D.Golubev, M.Marhtaler, K.Saito, T.Fujisawa, G.Schön, Phys. Rev. B 81, 125331 (2010)
[4] YU, D.Golubev, M.Marhtaler, T.Fujisawa, G..Schön, arXiv0908.0229.
[5] S.Nakamura, Y.Yamauchi, M.Hashisaka, K.Chiba, K.Kobayashi, T.Ono, R.Leturcq, K.Ensslin, K.Saito, YU, A.C.Gossard, arXiv:0911.3470, hys. Rev. Lett. 104, 080602 (2010).

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