The role of magnetic monopole in confinement/deconfinement phase

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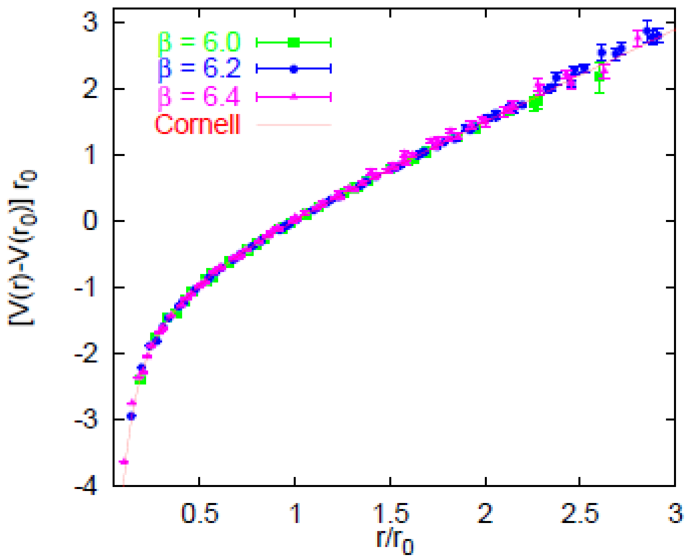
Introduction

- Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]

Non-Abelian Wilson loop $\left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\mathrm{YM}}^{\mathrm{no\ GF}} \sim e^{-\sigma_{NA}|S|}$,

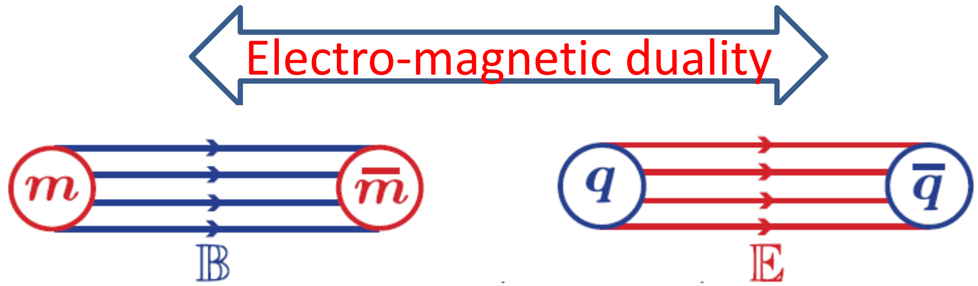


$$V(r) = -C\frac{g^2_{\mathrm{YM}}(r)}{r} + \sigma r$$

➔The dual superconductivity picture can be promising mechanism for quark confinement

Introduction (cont')

- Dual superconductivity is a promising mechanism for the quark confinement. [Y.Nambu (1974). G. 't Hooft, (1975). S. Mandelstam, (1976) A.M. Polyakov, (1975). Nucl. Phys. B 120, 429(1977).]



- Numerical simulations that support dual superconductor picture
  - Abelian dominance [Suzuki & Yotsuyanagi, 1990]
  - Monopole dominance[Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]
  - Center vortex dominance [e.g. Greensite (2007)]

SU(2) case

Abelian-projected Wilson loop $\left\langle \exp\left\{ ig \oint_C dx^\mu A_\mu^3(x) \right\} \right\rangle_{\text{YM}}^{\text{MAG}} \sim e^{-\sigma_{Abel}|S|}$!?

•**Problems** that these are only obtained by gauge fixings by the maximal Abelian (MA) gauge and the Laplacian Abelian gauge ,and the gauge fixing also breaks color symmetry(globale symmetery).

•We have given Cho-Faddev-Nniemi-Chabanov (CFNS) decomposition on a lattice, which can extract relevant modes for quark confinement in gauge independent way.

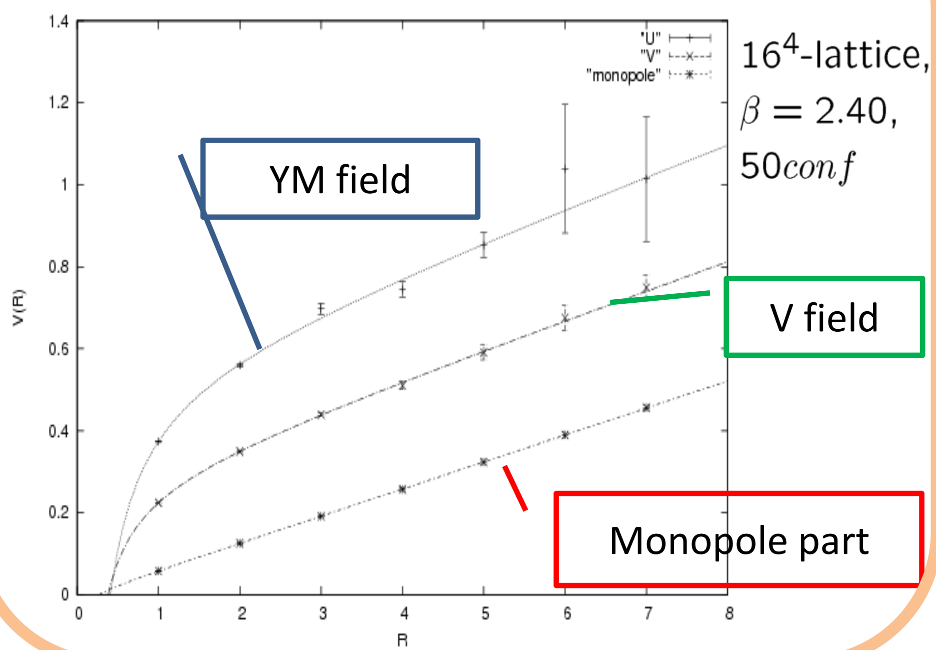
- quark-antiquark potential from Wilson loop operator
- *gauge-independent "Abelian" Dominance*
- The decomposed V field reproduced the potential of original YM field.

  $\sigma_{full} \sim \sigma_V \quad (93 \pm 16\%)$
- *gauge-independent monopole dominance*
- The string tension is reproduced by only magnetic monopole part.

  $\sigma_V \sim \sigma_{monopole} \quad (94 \pm 9\%)$

  $\sigma_{full} \sim \sigma_{monopole} \quad (88 \pm 13\%)$

$$V(R) = c + \frac{\alpha}{R} + \sigma R$$



arXiv:0911.0755 [hep-lat]

Introduction (cont')

➔The magnetic monopole plays a central role in quark confinement.

- It is important to Investigate the magnetic monopoles as a quark confiner.
- We study
  - the implication between the magnetic monopoles and the phase transition of confinement /deconfinement .
  - Implication between the magnetic monopoles and the topological configuration of Yang-Mills fields such as instantons.
- There are many pioneering studies of magnetic monopoles for SU(2) YM theory, which is done in the maximal Abelian gauge (MAG). ➔ Our CFNS decomposition enables ones the gauge independent study.
- For SU(3) case there are only naïve extention of magnetic monopoles based on the Abelian projection.

➔We extend of CFNS decomposition to SU(N) (N=3)YM theory, and derive the gauge independent non-Abelian magnetic monopoles based on the non-Abalian Stokes theorem.

Plan of talk

In this talk, we study the dual superconductivity picture of **SU(3) Yang-Mills** theory based on the extended Cho-Faddeev-Niemi-Shavanov (CFNS) decomposition for SU(N) Yang-mills theory and non-Abalian Stokes theorem. We demonstrate by lattice simulation the Gauge-independent U(2)-dominance and non-Abelian magnetic monopole dominance in SU(3) Yang-Mills theory.
Then, study non-Abelian magnetic monopole as quark confiner.

- CFNS decomposition (minimal-option) for SU(3) Yan-Mills theory.
- Gauge independent magnetic monopole from decomposed variables. For SU(3) case: non-Abelian magnetic monopoles are derived by using the non-Abalian Stokes theorem
- Lattice data

We give the Cho-Faddeev-Niemi-Shabanov (CFNS) decomposition for SU(N) Yang-Mills theory as the extension of SU(2) version, which can extract the relevant elements of gauge fields for confinement.

CFNS DECOMPOSITION FOR SU(3) YANG-MILLS

Decomposition of SU(3) Yang-Mills link variables

•KEK-PREPRINT-2008-36, CHIBA-EP-173, arXiv:0810.0956 [hep-lat]
•Phys.Lett.B669:107-118,2008.
•KEK-PREPRINT-2009-32, CHIBA-EP-181, arXiv:0911.5294 [hep-lat]

- The decomposition as the extension of the SU(2) case.
- Is there any possibility other than projecting to the maximal torus group? ➔ **Two options are possible corresponding to stability group**

$$\text{minimal case} \quad U(2) \cong SU(2) \times U(1) \in SU(3)$$
$$\text{maximal case} \quad U(1) \times U(1) \in SU(3)$$

✓ **Maximal case** is gauge invariant version of Abailan projection in the maximal Abelian (MA) gauge. (the maximal tours group)

POS(LATTICE-2007)331, arXiv:0710.3221 [hep-lat]

✓ **Minimal case** is derived for the Wilson loop, which gives the static potential of the quark and anti-quark for the fundamental representation. Kei-Ichi Kondo, Phys.Rev.D77:085029,2008.

The decomposition of link variables

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

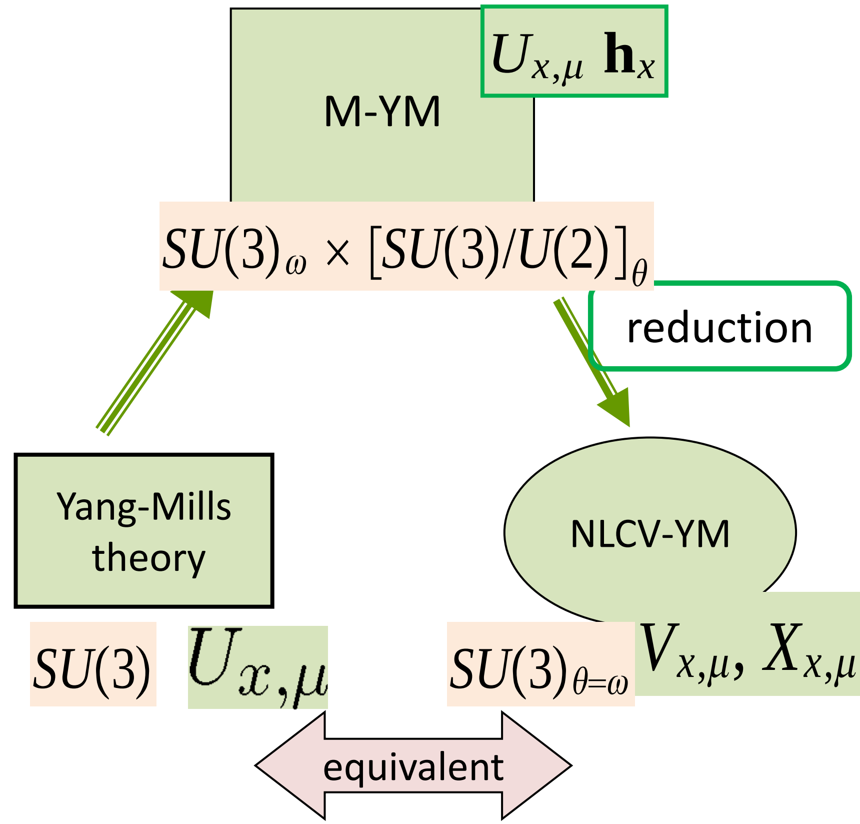
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^{\dagger}_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^{\dagger}_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega^{\dagger}_{x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.}\, W_C[V] \quad !!$$

Defining equation for the decomposition

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equastion of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-ia_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}\mathbf{u}_x^{(l)}) = 1$$

which correspod to the continume version of the decomposition $\mathsf{A}_\mu(x) = \mathsf{V}_\mu(x) + \mathsf{X}_\mu(x)$:

$$D_\mu[\mathsf{V}]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathbf{h}(x)\mathsf{X}_\mu(x)) = 0.$$

The solution is given by

Phys.Lett.B691:91-98,2010.

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-1)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1})$$
$$+ 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1},$$

$$L_{x,\mu} = \sqrt{L_{x,\mu}L_{x,\mu}^\dagger}\,\hat{L}_{x,\mu} \Leftrightarrow \hat{L}_{x,\mu} = (\sqrt{L_{x,\mu}L_{x,\mu}^\dagger})^{-1}L_{x,\mu}.$$

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger(\det(\hat{L}_{x,\mu}))^{1/N}g_x^{-1}$$

$$V_{x,\mu} = X_{x,\mu}^\dagger U_{x,\mu} = g_x\hat{L}_{x,\mu}U_{x,\mu}(\det(\hat{L}_{x,\mu}))^{-1/N}$$

The defining equation and the Wilson loop for the fundamental representation

By inserting the complete set of the coherent state $|\xi_x, \Lambda\rangle$ at every site on the Wislon loop C, $1 = \int |\xi_x, \Lambda\rangle d\mu(\xi_x) \langle \Lambda, \xi_x|$ we obtain

$$W_C[U] = \mathrm{tr}\left(\prod_{<x>\in C} U_{x,\mu}\right) = \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x) \langle \Lambda, \xi_x | U_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle$$

$$= \prod_{<x,x+\mu>\in C} \int d\mu(\xi_x) \langle \Lambda, |(\xi_x^\dagger X_{x,\mu} \xi_x)(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu})|, \Lambda\rangle$$

where we have used $\xi_x \xi_x^\dagger = 1$.

For the stability group of $\tilde{H}$, the 1st defining equation

$$\xi V_{x,\mu} \xi^\dagger \in \tilde{H} \iff [\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}, \tilde{H}] \iff \mathbf{h}_x V_{x,\mu} - V_{x,\mu} \mathbf{h}_{x+\mu} = 0$$

implies that $|\Lambda\rangle$ is eigenstate of $\xi_x^\dagger V_{x,\mu} \xi_{x+\mu}$:

$$(\xi_x^\dagger V_{x,\mu} \xi_{x+\mu})|\Lambda\rangle = |\Lambda\rangle e^{i\phi}, \quad e^{i\phi} := \langle \Lambda | \xi_x^\dagger V_{x,\mu} \xi_{x+\mu} | \Lambda\rangle = \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle.$$

Then we have

$$W_C[U] = \int d\mu(\xi_x) \rho[X;\xi] \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle$$

$$\rho[X;\xi] := \prod_{<x>\in C} \langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda\rangle$$

The defining equation and the Wilson loop for the fundamental representation (2)

By using the expansion of $X_{x,\mu}$: the 2nd defining equaiton, $\mathrm{tr}(\mathsf{X}_\mu(x)\mathbf{h}(x)) = 0$, derives

$$\langle \Lambda, \xi_x | X_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = \mathrm{tr}(X_{x,\mu})/\mathrm{tr}(\mathbf{1}) + 2\mathrm{tr}(X_{x,\mu}\mathbf{h}_x)$$
$$= 1 + 2ig\epsilon tr(\mathsf{X}_\mu(x)\mathbf{h}(x)) + O(\epsilon^2).$$

Then we have $\rho[X;\xi] = 1 + O(\epsilon^2)$.

Therefore, we obtain

$$W_c[U] = \int d\mu(\xi_x) \prod_{<x,x+\mu>\in C} \langle \Lambda, \xi_x | V_{x,\mu} | \xi_{x+\mu}, \Lambda \rangle = W_C[V]$$

By using the non−Abalian Stokes theorem, Wilson loop along the path C is written to area integral on $\Sigma : C = \partial\Sigma$;

$$W_C[\mathsf{A}] := \mathrm{tr}\left[P \exp\left(-ig \oint_C dx^\mu \mathsf{A}_\mu(x) \right) \right] / \mathrm{tr}(\mathbf{1}) = \int d\mu_\Sigma(\xi) \exp\left(\int_{S:\, C=\partial\Sigma} dS^{\mu\nu} F_{\mu\nu}[\mathsf{V}] \right),$$

(no path ordering), and the decomposed $V_{x,\mu}$ corresponds to the Lie algebra value of $\mathsf{V}_{x,\mu}$ and the field strength on a lattice is given by plaquet of $V_{x,\mu}$

THE GAUGE INVARIANT (INDEPENDENT) NON-ABELIAM MAGNETIC MONOPOLES FROM THE DECOMPOSED VARIABLES

Non-Abelian magnetic monopole

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived without using the Abelian projection ['t Hooft 1981.]

$$W_C[\mathrm{A}] = \int d\mu_\Sigma(\xi) \exp\left(\int_{S:\, C=\partial\Sigma} dS^{\mu\nu} F_{\mu\nu}[\mathrm{V}] \right)$$

$$= \int d\mu_\Sigma(\xi) \exp\left[ig\sqrt{\frac{N-1}{N}}(k, \Xi_\Sigma) + ig\sqrt{\frac{N-1}{N}}(j, N_\Sigma) \right]$$

$$k \ := \delta^* F = {}^* dF, \qquad \Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}$$

$$j \ := \delta F, \qquad N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$$

$$\Delta \ := d\delta + \delta d$$

$$\Theta_\Sigma^{\mu\nu} \ := \int_\Sigma d^2 S^{\mu\nu}(x(\sigma)) \delta^D(x - x(\sigma))$$

k and j are gauge invariant and conserved current $\delta k = 0 = \delta j$.

K.-I. Kondo PRD77 085929(2008)

Note that the Wilson loop operator knows the non-Abaelian magnetic monopole ***k*** .

Non-Abelian Magnetic monopole (2)

The lattice version of the magnetic monopoles is given by as follows

The magnetic monopole currents are calculated from decomposed variable $V_{x,\mu}$ as

$$V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}_{x,\nu} = \exp(-ig\mathsf{F}[\mathbf{V}_{\mu}(x)]_{\mu\nu}) = \exp(-ig\Theta^{8}_{\mu\nu}\mathbf{h}_{x'}),$$

$$\Theta^{8}_{\mu\nu} = -\arg\mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_{x}\right)V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}_{x,\nu}\right],$$

$$k_{x,\mu} := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_{\nu}\Theta^{8}_{\alpha\beta}.$$

Integer valued monopole charge is defined by $n_{x,\mu} = k_{x,\mu}/(2\pi)$.

Note that:

Since the current *k* is defined by the field strength *F[V],* it is the non-Abalian magnetic monopole *defined in the gauge invariant (independent) way,*

Lattice data

Numerical simulation

Quark and anti-quark potential for the fundamental representation

- Wilson loop by decomposed variables V
- Non-Abalian monopoles Monopole and static potential

Correlation functions of decomposed variables

- Correlation function of color fields

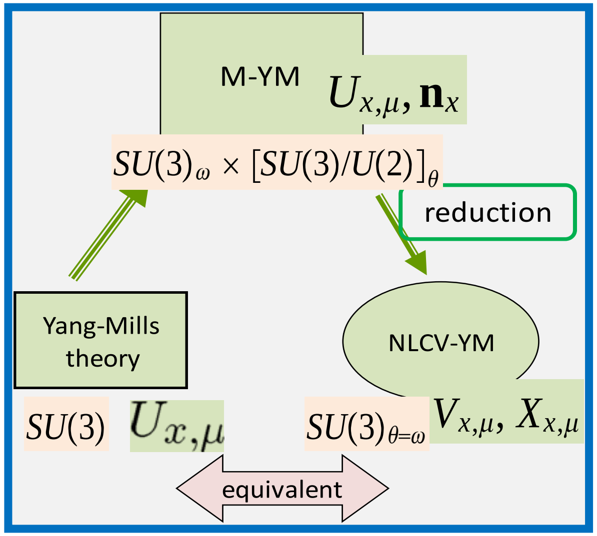
Numerical simulation

- The configurations of YM field are generated by using the standard Wilson action and pseudo heat-bath method.
- The color fields are determined by using the reduction condition such that the theory in terms of new variables (V,X,h) is equaipolet to the original Yang-Mills theory $SU(3)_{\omega} \times [SU(3)/U(2)]_{\theta} \rightarrow SU(3)_{\omega=\theta}$

Determining $\mathbf{h}_x$ to minimize the reduction function for given $U_{x,\mu}$

$$F_{\mathrm{red}}[\mathbf{h}_x; U_{x,\mu}] = \sum_{x,\mu} \mathrm{tr}\left[(D_{\mu}^{\epsilon}[U_{x,\mu}]\mathbf{h}_x)(D_{\mu}^{\epsilon}[U_{x,\mu}]\mathbf{h}_x)^{\dagger} \right]$$

- The decomposition $U=XV$ is obtained for arbitrary YM field U (and the color field h) by using the formula (U,h ➔ L ➔ V,X)



global SU(3) (color) symmetry

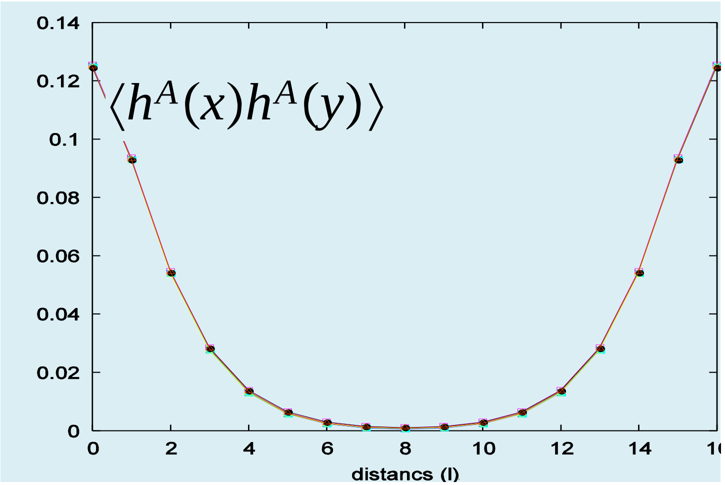
- **VEV of color field**

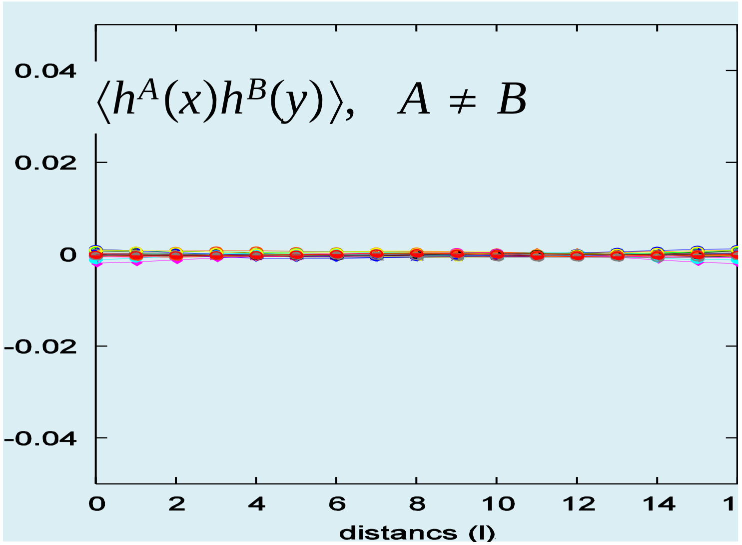
$$\langle h^A(x) \rangle = 0 \pm 0.002$$

- **Two point correlation function** of color vector fields. (right figures)

$$\langle h_x^A h_y^B \rangle = \delta^{AB} D(x-y)$$

Color symmetry is preserved.





Static potential

- Wilson loop by the decomposed variable V
- Dose Wilson of V loop reproduces the original one?

$W_C[U] = \mathrm{const.} W_C[V]$!!

- To get the static potential

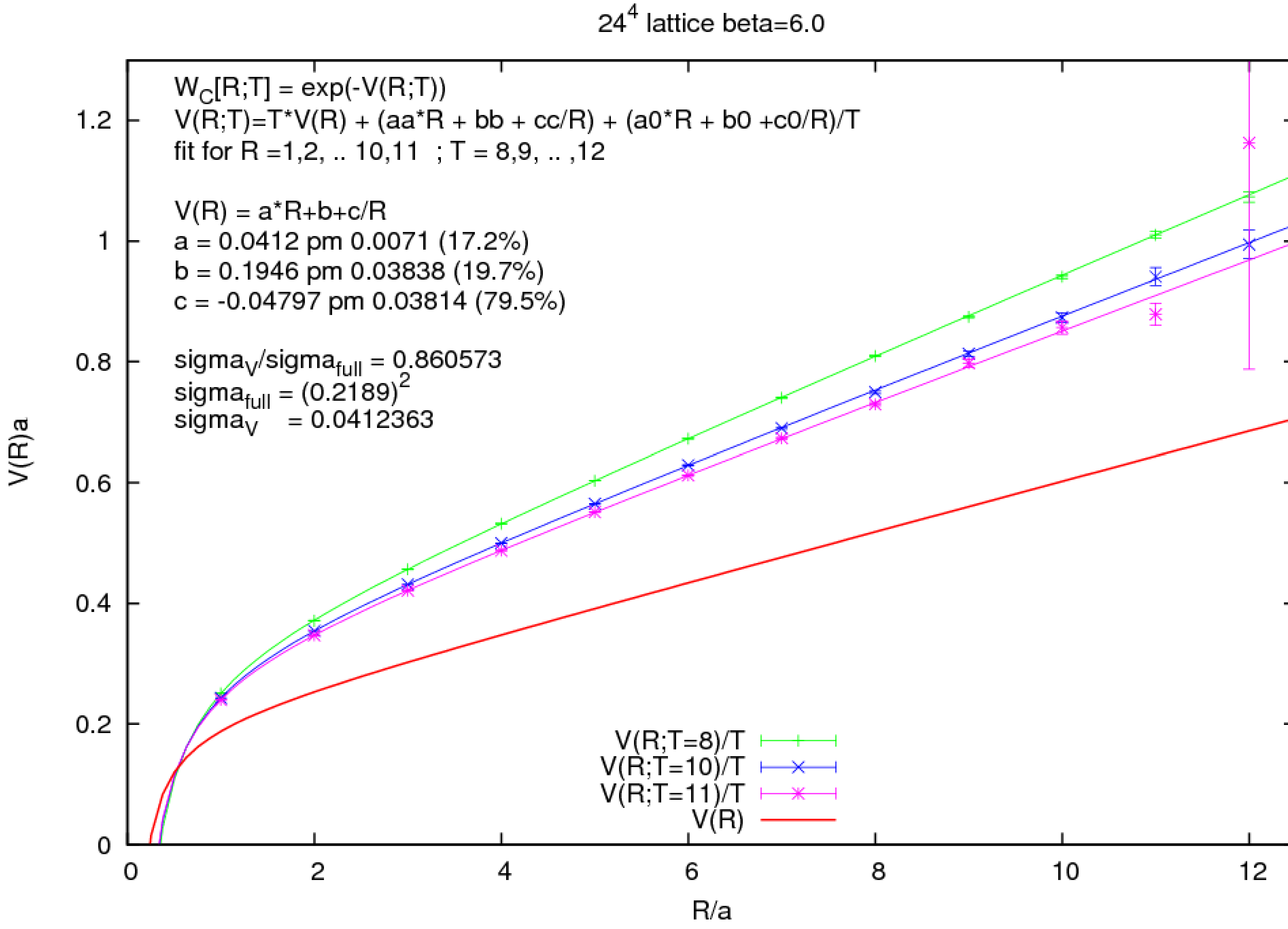
$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log\langle W_{(R,T)}[V]\rangle$$

- We fit the Wilson loop *$W_C[V]$* by the function *V(R,T)*

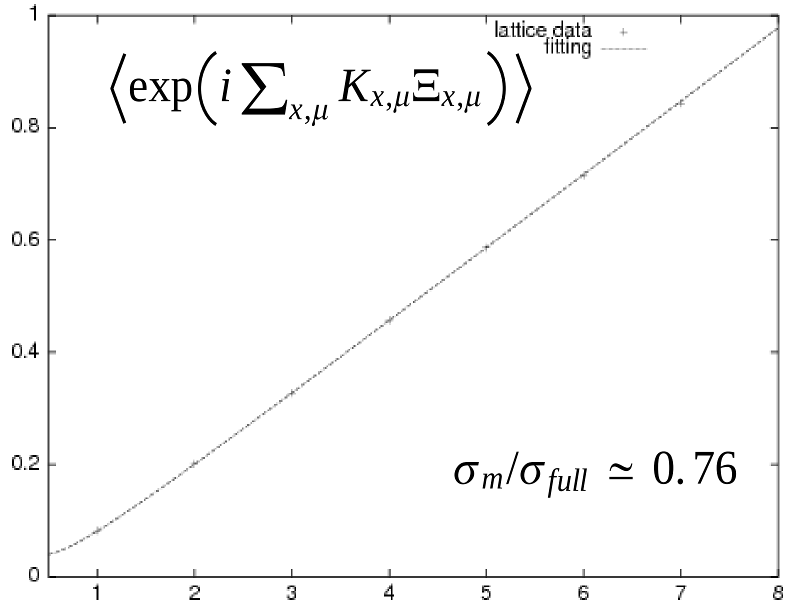
$$\langle W_{(R,T)}[V]\rangle = \exp(-V(R,T))$$

$$V(R,T) := T \times V(R) + (a'R + b' + c'/R) + (a''R + b'' + C''/R)/T$$

$$V(R) = \sigma R + b + c/R$$


24^4 lattice beta=6.0
W_C[R;T] = exp(-V(R;T))
V(R;T)=T*V(R) + (aa*R + bb + cc/R) + (a0*R + b0 +c0/R)/T
fit for R =1,2, .. 10,11 ; T = 8,9, .. ,12
V(R) = a*R+b+c/R
a = 0.0412 pm 0.0071 (17.2%)
b = 0.1946 pm 0.03838 (19.7%)
c = -0.04797 pm 0.03814 (79.5%)
sigma_V/sigma_full = 0.860573
sigma_full = (0.2189)^2
sigma_V = 0.0412363
V(R)a
R/a
V(R;T=8)/T
V(R;T=10)/T
V(R;T=11)/T
V(R)
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10
12

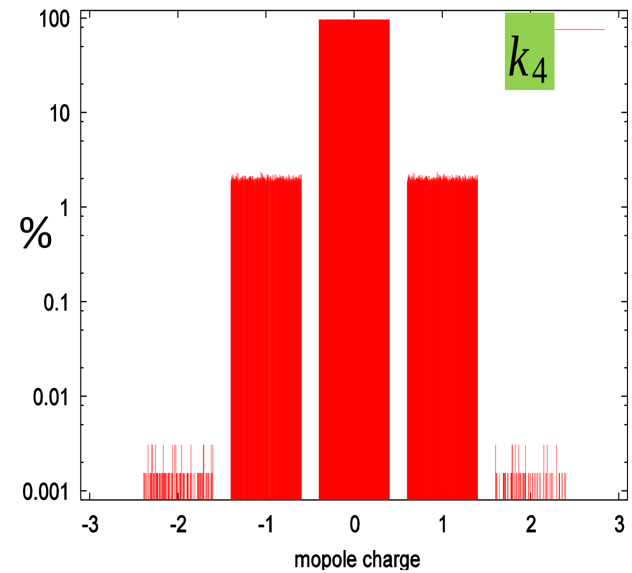
String tension from non-Abelian monopoles



$$V_m(R) = -\alpha_m/R + \sigma_m R + \beta_m$$

$$k_{x,\mu} = -\frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\nu \Theta^8_{x,\rho\sigma}$$

$$\Theta^8_{x,\mu\nu} \equiv -argTr[(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{h}_x)V_{x,\mu}V_{x+\hat{\mu},\nu}V^\dagger_{x+\hat{\nu},\mu}V^\dagger_{x,\nu}]$$

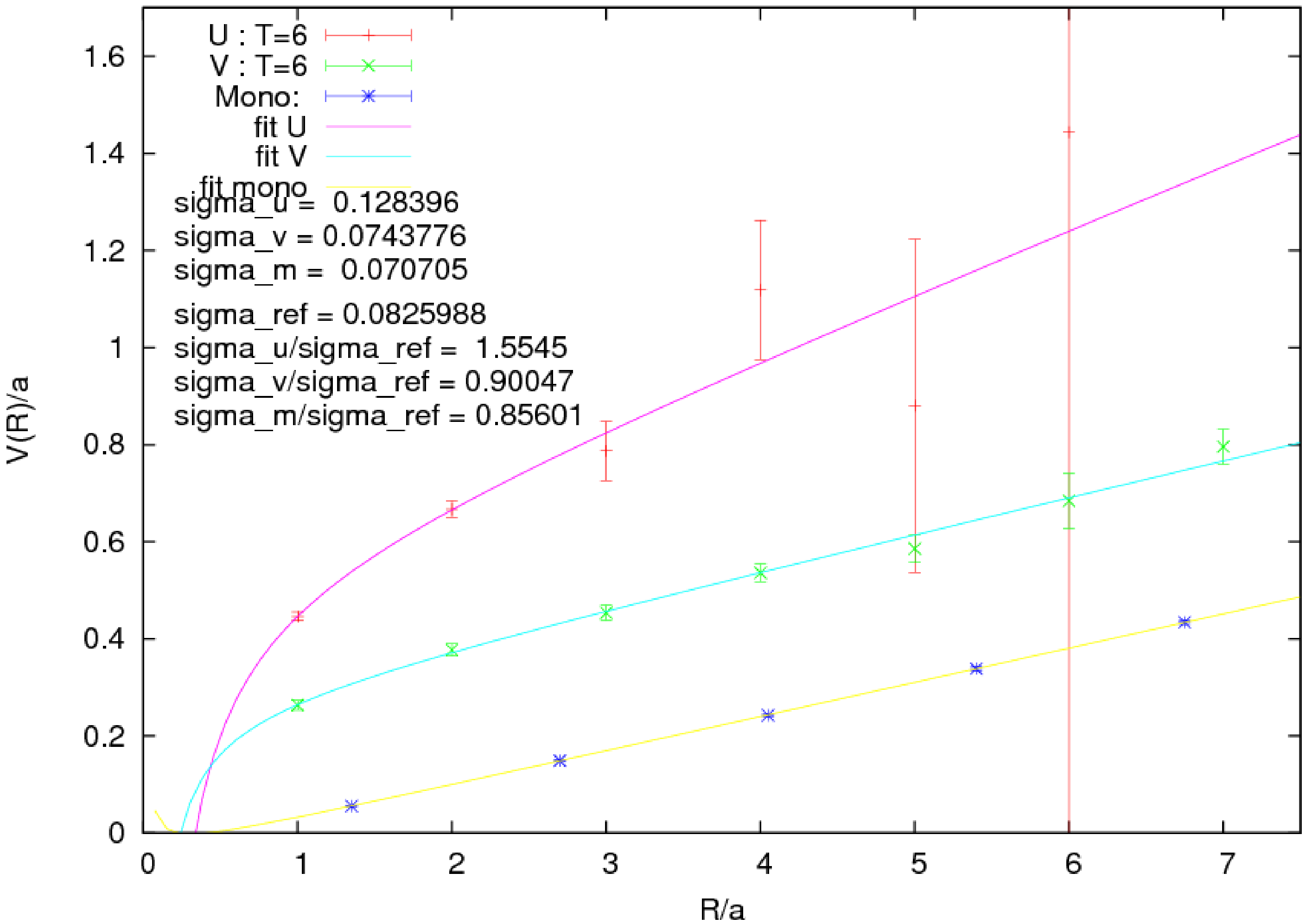


β	$a\sqrt{\sigma}$	r_0/a	Volume	Reference
5.7	0.3879(39)	2.9990(24)	$16^3 \cdot 32$	EHK
6.0	0.2189(9)	5.369(9)	$16^3 \cdot 32$	EHK
	0.2184(19)	5.34(+2)(-3)	16^4	SESAM
	0.2209(23)		32^4	Bali/Schilling/Hoeber
	0.2154(50)	5.47(11)	$16^3 \cdot 48$	UKQCD

From R.G.Edward et.al , Nucl.Phys. N617 377-392 (1988)

•The distribution of the monopole charges for 16^4 lattice β=5.7 400 configurations. The distribution of each configuration is shown by thin bar chart.

Combination plot for SU(3) minimal



ANALYSIS OF MONOPOLES

Property of monopoles on lattice

$$n_{x,\mu} = \frac{1}{2\pi} k_\mu = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\nu \Theta_{x,\rho\sigma}$$

$$\mathcal{F}_{x,\mu\nu} \equiv argTr[(1 + \mathrm{n}_x) V_{x,\mu} V_{x+\hat{\mu},\nu} V^\dagger_{x+\hat{\nu},\mu} V^\dagger_{x,\nu}]$$

- Invariant under SU(2) gauge transformation.
- Monopole currents are define as link variables on the deal lattice (shifted by a half integer for each direction.)
- They take integer values

  $n_{x,\mu}$ ={-2,-1,0,1,2}
- Current conservation:

$$\epsilon \partial_\mu n_{x,\mu} = \sum_\mu (n_{x,\mu} - n_{x-\mu,\mu}) = \sum_{\mu=\pm 1,..,\pm 4} n_{x,\mu} = 0$$

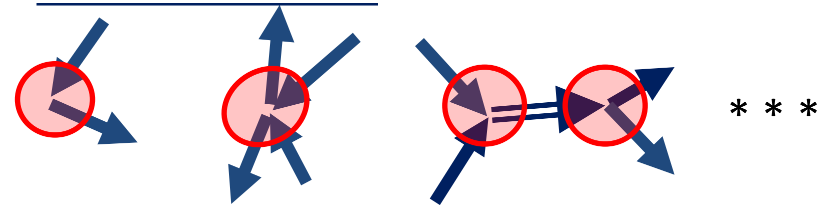
with beein $n_{x,-\mu} = n_{x-\mu,\mu}$

- Non-zero Monopole currents can be identified with geometrical objects.
  - Nonzero current ⬄ edge
  - end points (dual lattice site) ⬄ vertices
  - Sign (strength) of current ⬄ direction (waite)
- Current conservation

⬄ The same number of Incoming and outgoing links

➔ =monopole current construct loops

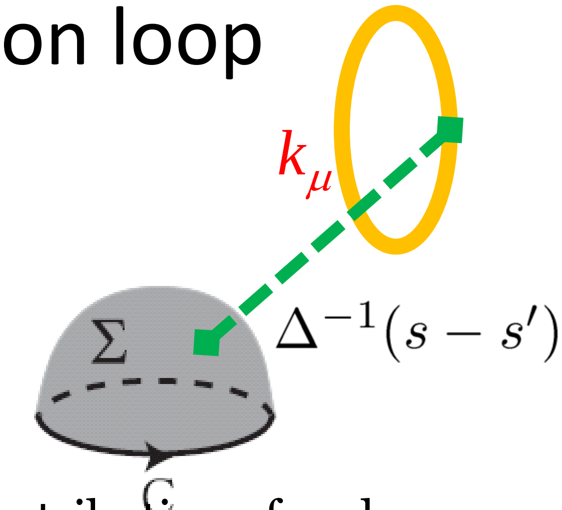
Possible verteces



Monopole contribution to the Wilson loop

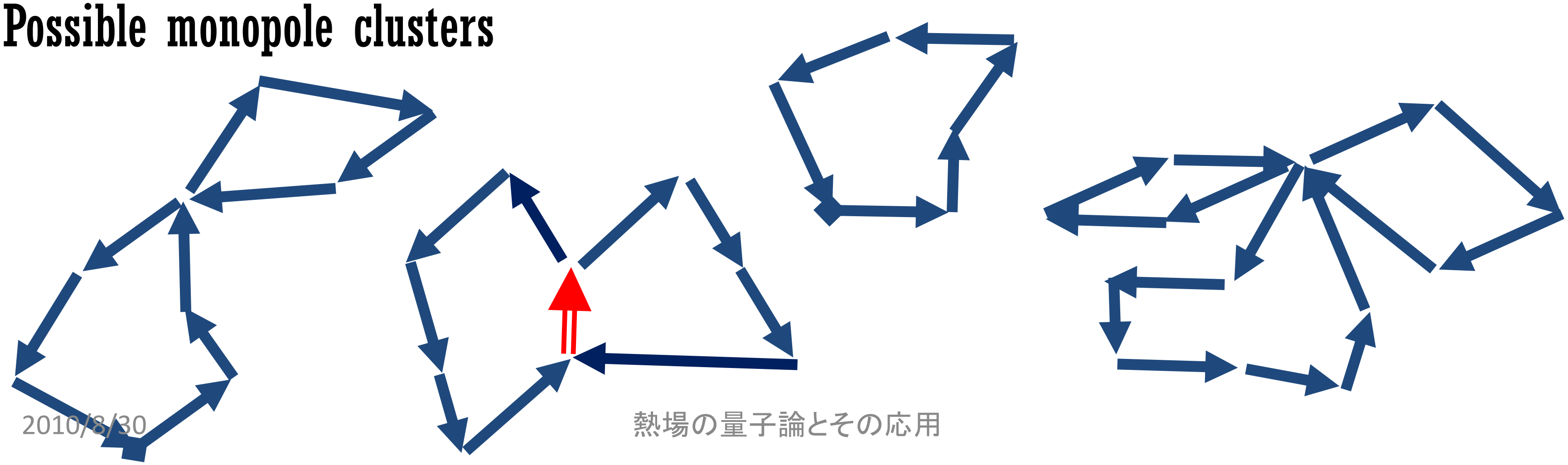
$$\langle W_C[V]\rangle \simeq \langle W_C[Mono]\rangle = \left\langle \exp\left\{ i\sum_{x,\mu} k_{x,\mu}\Xi_{x,\mu}\right\}\right\rangle$$

$$\Xi_{x,\mu} = \sum_{\sigma(y)\in\Sigma} \epsilon^{\mu\nu\alpha\beta}\partial_\nu \Delta^{-1}(x-y)\sigma^{\alpha\beta}(y)$$

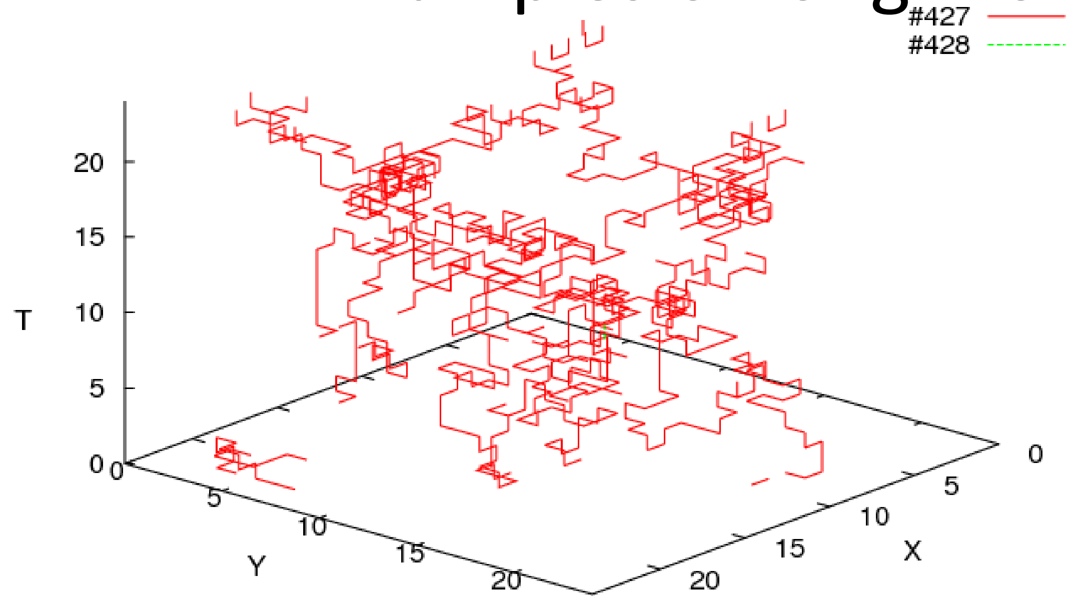


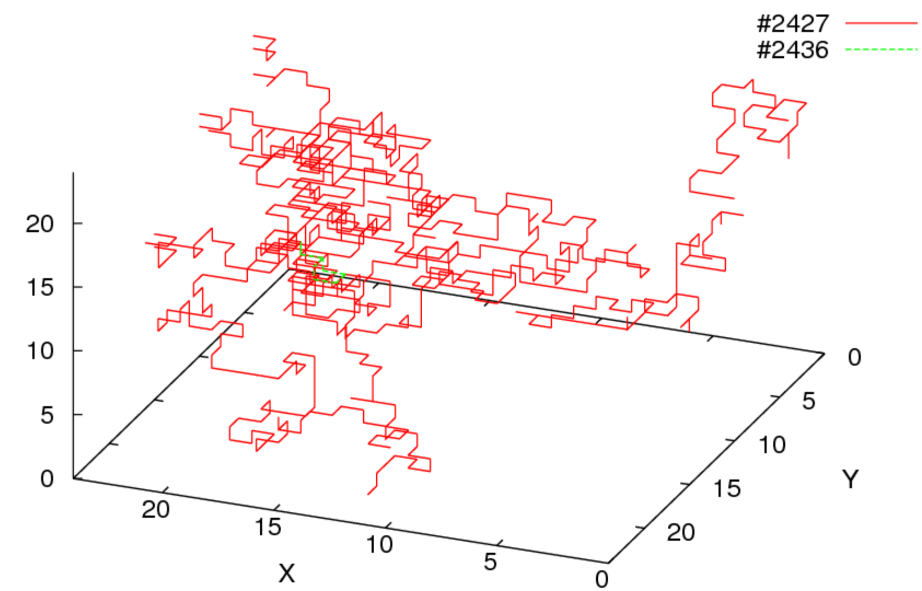
- Wilson loop of the monopole part decomposed into the contribution of each monopole loop, since monopole currents are decomposed into loops.
- The small monopole give zero contribution, since integral by opposite direction of current canceled each other. ➔The large cluster of monopole loops contribute to the Wilson loop.

Possible monopole clusters



Examples of long monopole loops





Monopole loops are plotted in 3-dimensional space (24^3 lattice with periodic boundary condition) by projection from 4D space (x,y,z,t) to 3D ^{T} space (x,y,t) .

Summary & outlook

- We have given the decomposition in the gauge independent way for SU(N) Yang-Mills fields, $U_{x,\mu}=X_{x,\mu}V_{x,\mu}$, as the extension of CFNS decomposition for the SU(2) YM theory.
- As the result of **non-Abelian stokes theorem**, we have shown that the Wilson loop **for the fundamental representation** is represented by field of minimal option, not the maximal option (Abailan projection in MAG)
- We have define non-Abelian magnetic monopole in gauge independent way.
- We have performed the numerical simulation in the minimal option of the SU(3) lattice Yang-Mills theory and shown:
  - V-dominance (say, U(2)-dominance) in the string tension (85-95%)
  - Non-Abalian magnetic monopole dominance in string tension (75%)
  - color symmetry preservation, infrared V-dominance (U(2)-dominance)
- The monopole configuration can be analyzed by using computational algebra.

➔Study the phase transition of confine/deconfine in terms of monopoles. (In progress.)