A study on charmonium spectral function with the variational method in lattice QCD

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Plan of this talk

- Introduction
- Spectral functions via the variational method
- Numerical results
  - **Test in free quark case**
  - **Results at zero temperature**
  - **Results at finite temperature**
- Conclusions

Introduction : motivation

- Charmonia dissociation in Quark Gluon Plasma (QGP)
  - **Sequential J/Ψ suppression**

    : Suppression of J/Ψ is one of the important signals of QGP formation in heavy ion collisions such as RHIC and LHC. T. Matsui and H. Satz (1986)

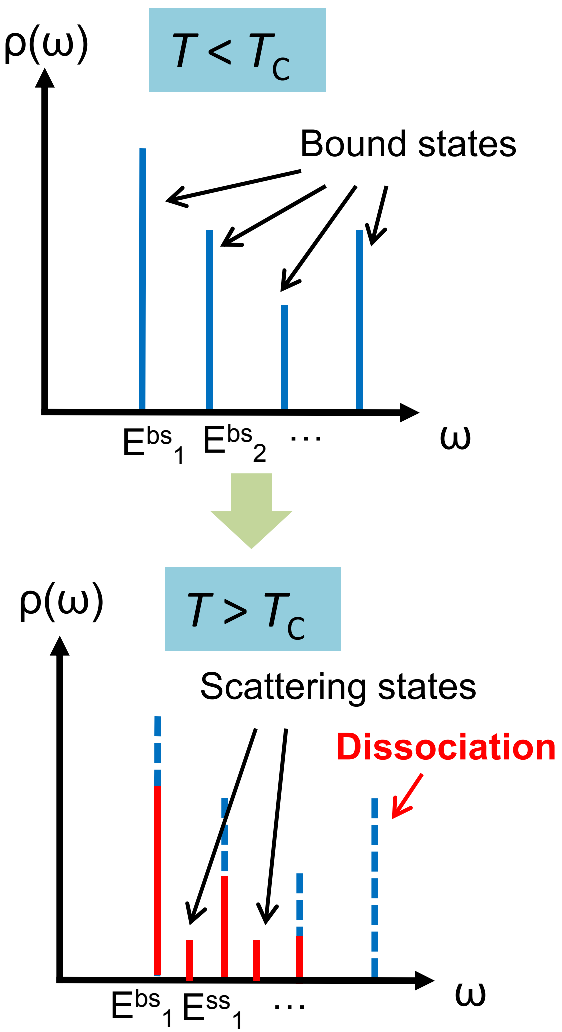
    : Ψ' → J/Ψ 10%, χ_c → J/Ψ 30% L. Antoniazzi et al. [E705 Collaboration] (1993)

    : Dissociation of excited states and P-wave states is also important.
- Meson spectral function (SPF) at finite temperature
  - has information about the meson properties in medium.
- In current lattice studies e.g. A. Jackovac et al (2007)
  - **Maximum Entropy Method (MEM) → meson SPFs**
  - **S-wave states (η_c, J/Ψ) : survive up to 1.5 T_C?**
  - **P-wave states (χ_c) : dissolve just above T_C**
  - **Excited states have NOT been investigated well yet.**

It is necessary to check the results and also investigate excited states with the other methods.

Introduction : our approach

spectral function



- On a finite volume lattice
  - $\rho(\omega_i) \neq 0$, i=1,2,… : discrete spectra only.
- Below T_c (the critical temperature)
  - $\rho(E^{bs}) \neq 0$ for E^{bs}: energy of bound state
- Above T_c
  - If bound states still survive,
    $\rho(E^{bs}) \neq 0$ but $\rho(E^{bs})_{T<Tc} \neq \rho(E^{bs})_{T>Tc}$
  - If a bound state dissolves,
    $\rho(E^{bs}) = 0$ and $\rho(E^{ss}) \neq 0$ for E^{ss}: energy of scattering state

- **We investigate temperature dependence of SPFs**
  - Not whole shape of $\rho(\omega)$ but just $\rho(E^{bs})$ or $\rho(E^{ss})$ are needed.
  - Excited states should be investigated to understand Ψ' property.
- **Variational method**
  - can extract the properties of some low-lying states.
  - is well-suited for discrete spectra.

SPFs via variational method

- Smeared meson operator

$$\mathcal{O}_\Gamma(\vec{x},t)_i \equiv \sum_{\vec{y},\vec{z}} \omega_i(\vec{y})\omega_i(\vec{z})\bar{q}(\vec{x}+\vec{y},t)\Gamma q(\vec{x}+\vec{z},t)$$

A_1	A_2	A_3	A_4	A_5	A_6	A_7
∞	0.25	0.20	0.15	0.10	0.05	0.02

↑ point operator

  – **Gaussian smearing function**

$$\omega_i(\vec{x}) \equiv e^{-A_i||\vec{x}||^2} \qquad i = 1, 2, \cdots, n$$

- Meson correlator matrix

$$\mathcal{C}_\Gamma(t) = \left[\sum_{\vec{x}} \langle \mathcal{O}_\Gamma(\vec{x},t)_i \mathcal{O}^\dagger_\Gamma(\vec{0},0)_j\rangle\right]^n_{i,j=1}$$

  – **solve a generalized eigenproblem**

$$\mathcal{C}_\Gamma(t)\boldsymbol{v}_k = \lambda_k(t;t_0)\mathcal{C}_\Gamma(t_0)\boldsymbol{v}_k$$

$$\begin{cases} \Lambda(t;t_0) = \mathrm{diag}\{\lambda_1(t;t_0), \lambda_2(t;t_0), \cdots, \lambda_n(t;t_0)\} \\ V = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n] \\ \mathcal{C}_\Gamma(t) = \mathcal{C}_\Gamma(t_0)V\Lambda(t;t_0)V^{-1} \end{cases}$$

  – **point-point component**

$$\mathcal{C}_\Gamma(t)_{11} = \sum_k \rho_\Gamma(m_k)\frac{\cosh[m_k(t-N_t/2)]}{\sinh[m_kN_t/2]}$$

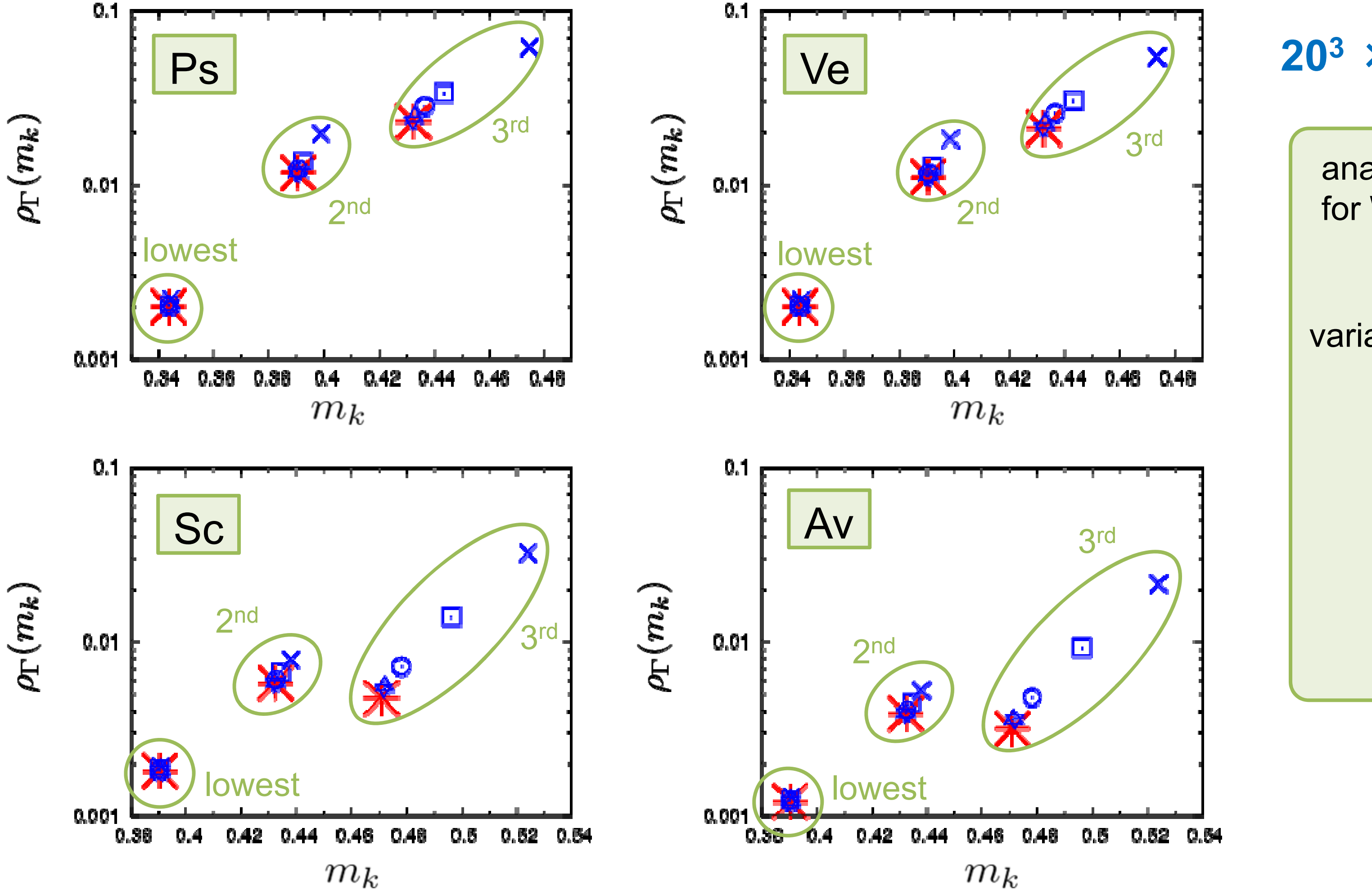
- Effective mass

$$\lambda_k(t;t0) = \frac{\cosh[m_k(t;t_0)(t-N_t/2)]}{\cosh[m_k(t;t_0)(t_0-N_t/2)]}$$

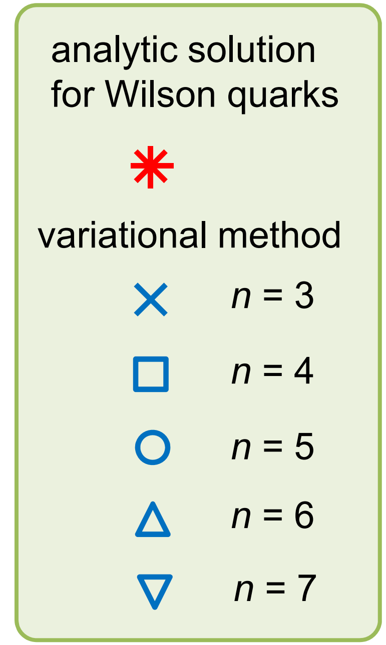
- SPF

$$\rho_\Gamma(m_k) = (\mathcal{C}_\Gamma(t_0)V)_{1k}(V^{-1})_{k1}\frac{\sinh[m_kN_t/2]}{\cosh[m_k(t_0-N_t/2)]}$$

Numerical results (1) : Test in free quark case

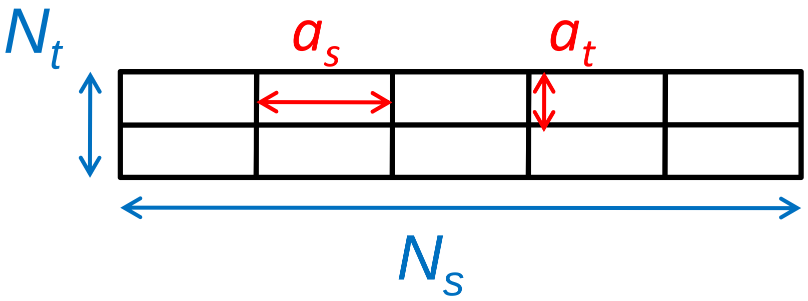


$20^3 \times 128$ lattice



Lowest state → well consistent with analytic solution for all n
2nd, 3rd lowest state → **improved as n increases**

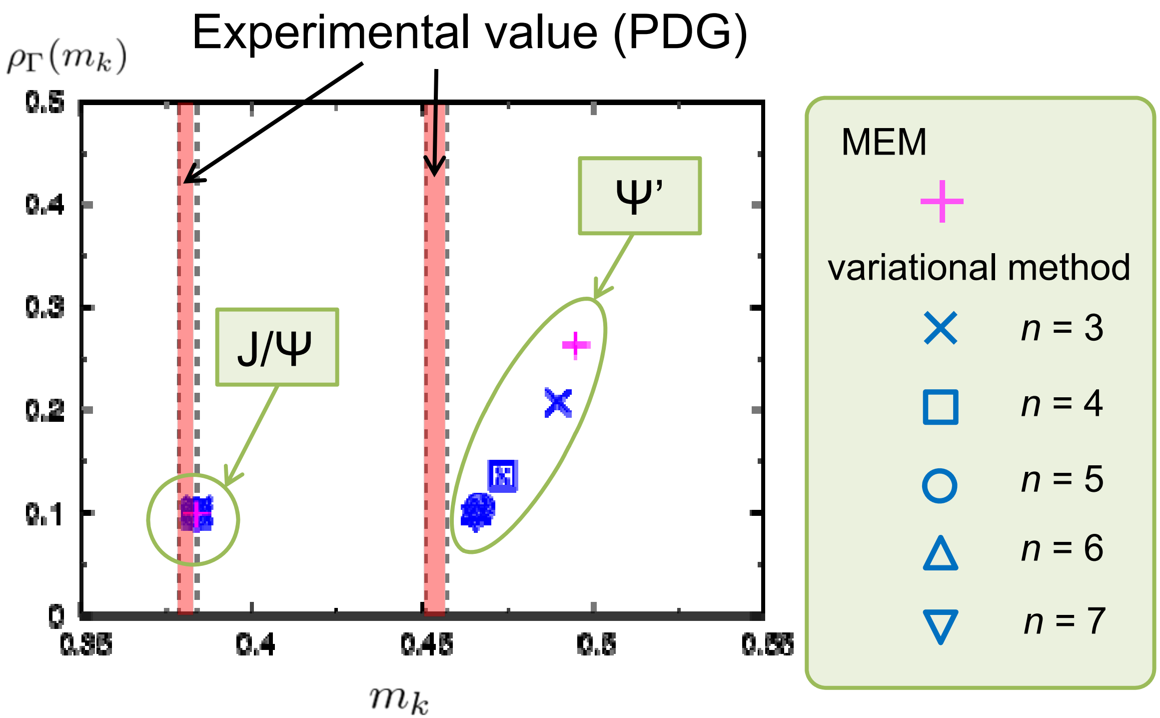
Lattice setup

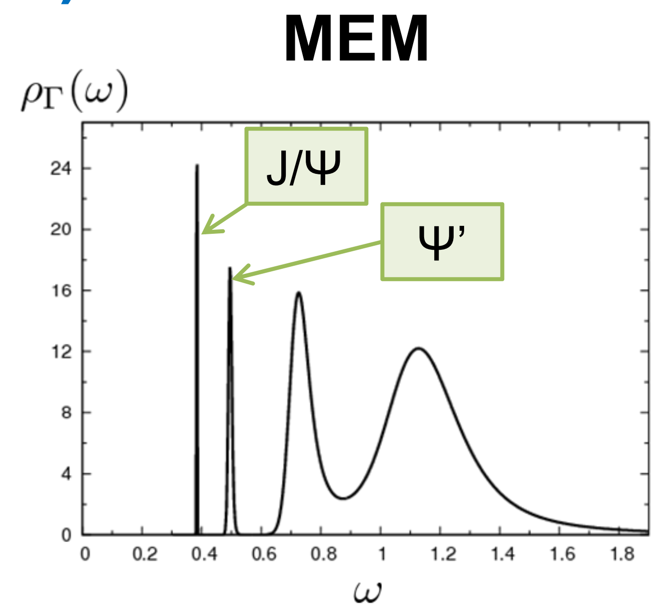


- Action
  - Standard plaquette gauge action
  - $O(a)$-improved Wilson fermion action
  - Quenched approximation
- Lattice
  - Anisotropic lattice : anisotropy $a_s/a_t = 4$
  - $a_s = 0.0970(5)$ fm ($a^{-1}_s = 2.030(13)$ GeV)
  - $N_s = 20$
  - $N_t = 160$ (zero temperature),
    32 ($0.88T_c$), 26 ($1.1T_c$), 20 ($1.4T_c$)
- Number of gauge configurations
  - for zero temperature : 299
  - for finite temperature : 800

T is varied by changing N_t (fixed-scale approach)

Comparison with MEM (Ve channel)





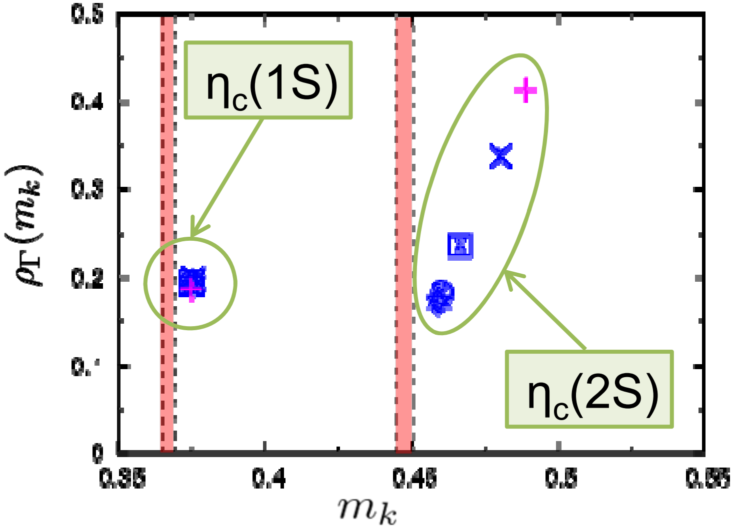
m_k ← location of each peak
$\rho_\Gamma(m_k)$ ← area of each peak

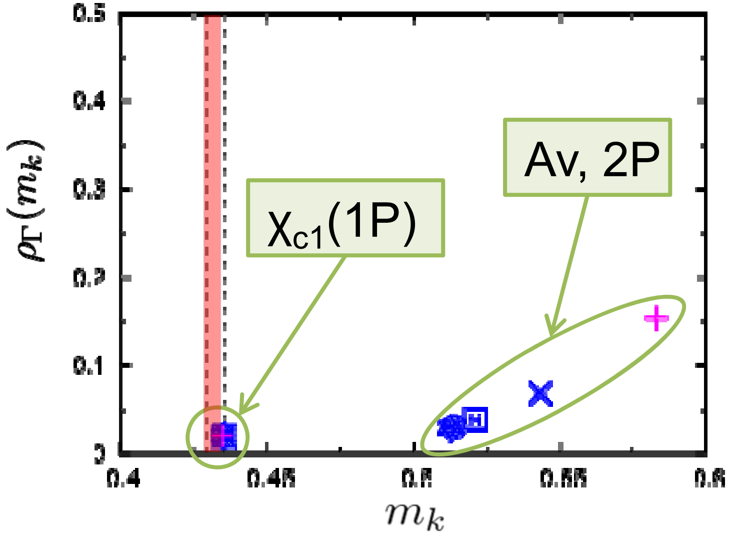
Ground state → all data almost consistent with experimental value
1st excited state → there is difference between variational method data and MEM one
→ variational method data get closer to experimental value as n increases

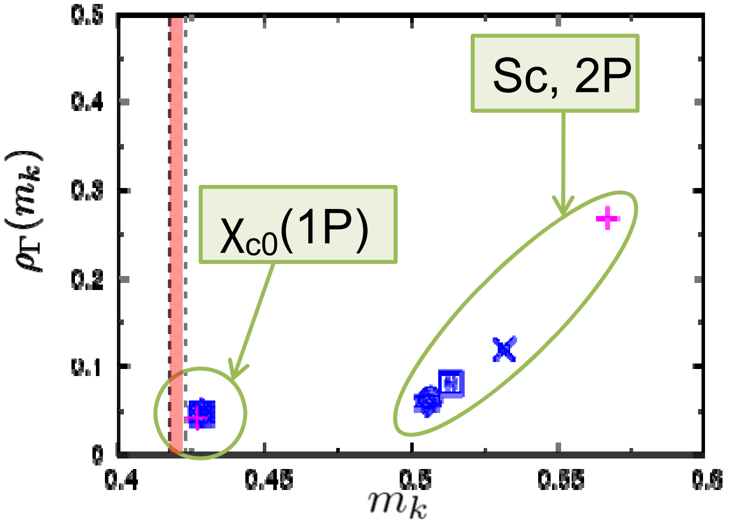
It seems that variational method can improve data accuracy for excited states.

Numerical results (2) : at zero temperature 2

Comparison with MEM (Ps, Sc, Av channel)







MEM
+

variational method

× $n = 3$

□ $n = 4$

○ $n = 5$

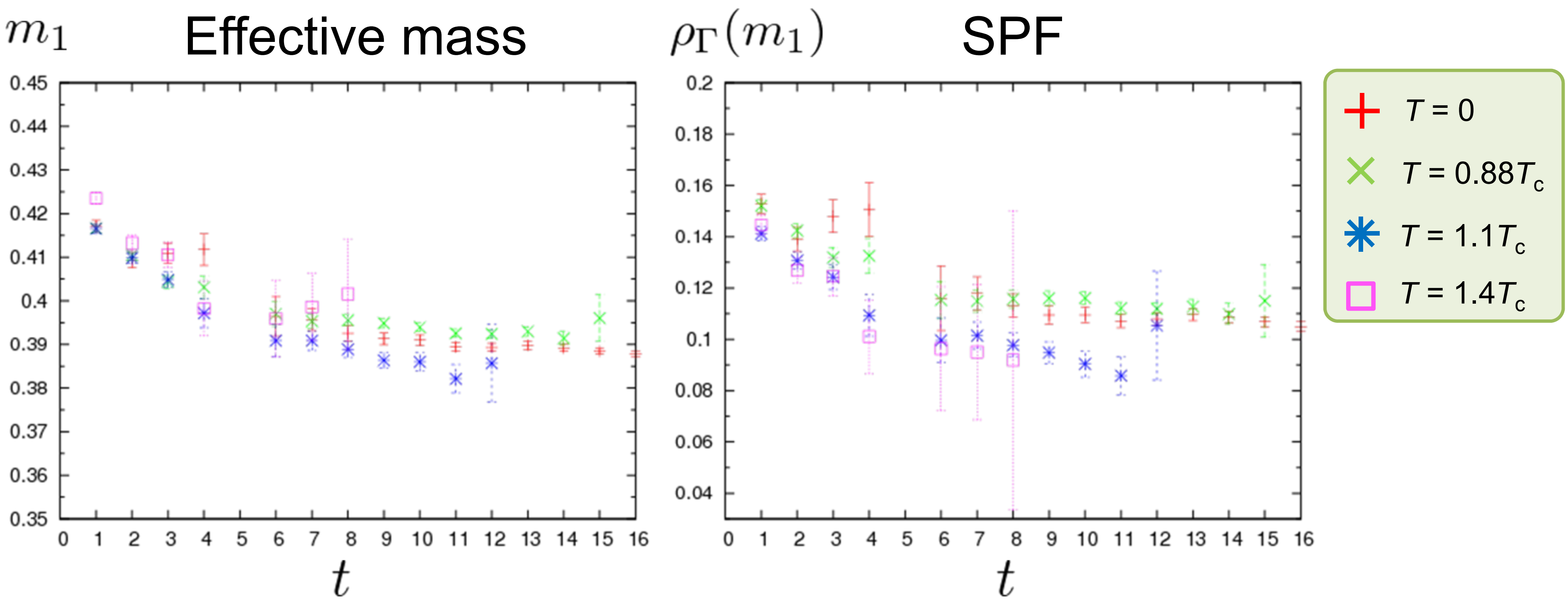
△ $n = 6$

▽ $n = 7$

Similarly to Ve case,
it seems that variational method can improve data accuracy for excited states.

Numerical results (3) : at finite temperature 1

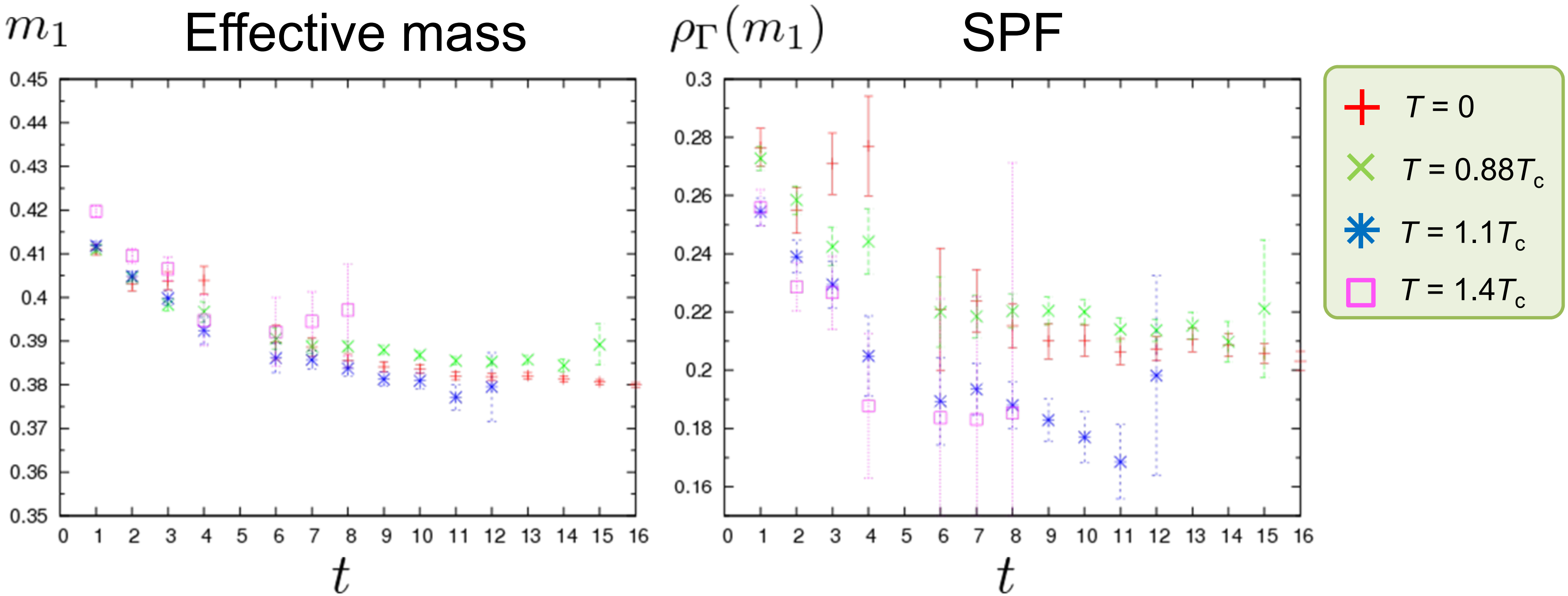
Temperature dependence (Ve channel, ground state) *n* = 7



No clear temperature dependence for the effective masses.
The value of SPF may change but does NOT become zero.
There is no clear evidence of dissociation for J/Ψ up to 1.4T_c

Numerical results (3) : at finite temperature 2

Temperature dependence (Ps channel, ground state) $n = 7$



No clear temperature dependence for the effective masses.

The value of SPF may change but does NOT become zero.

There is no clear evidence of dissociation for η_c up to $1.4T_c$

Conclusions

- Meson SPFs are calculated with the variational method.

- At zero temperature,
  - ground state → well extracted
  - excited state → improved by increasing the number of basis op.

- At finite temperature,
  - S-wave ground state charmonia (J/Ψ, η_c)
  - up to 1.4 T_c
  - no clear temperature dependence for the effective masses
  - value of SPF may change but it is still nonzero
  - **no clear evidence of dissociation**