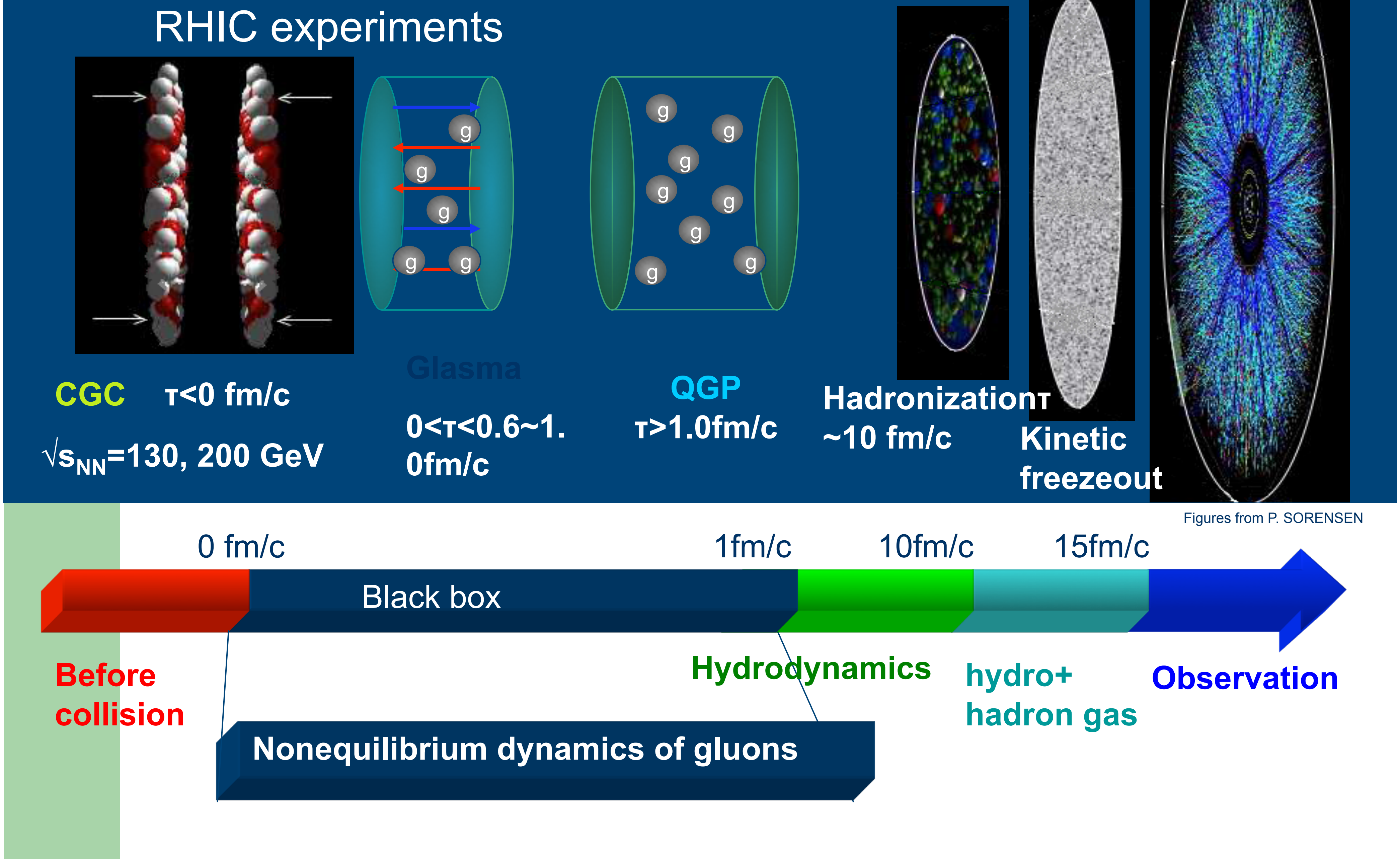
Entropy Production of Quantum Fields with Kadanoff-Baym equation

Akihiro Nishiyama
Yukawa Institute for Theoretical Physics, Kyoto University

Aug 31st in 2010.

Background


RHIC experiments
g
CGC τ<0 fm/c
√sNN=130, 200 GeV
Glasma
0<τ<0.6~1.0fm/c
QGP
τ>1.0fm/c
Hadronizationτ ~10 fm/c
Kinetic freezeout
Figures from P. SORENSEN
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before collision
Hydrodynamics
hydro+ hadron gas
Observation
Nonequilibrium dynamics of gluons

Topics in nonequilibrium gluodynamics

- **<u>Success of ideal Hydrodynamics</u>**
- **Assumption: <u>Early thermalization</u> for Partons**
  (τ_{eq}=0.6～1fm/c, (Boltzmann dynamics with gg⇔ggg →2-3fm/c))

Dense system (semi-classical Boltzmann eq. should not be applied)

No consideration of particle number changing process g→gg, g→ggg (Off-shell effect)

To take them into account, (and to give initial condition for hydrodynamics)

Based on Nonequilibrium Quantum field theory,
we apply Kadanoff-Baym eq. to gluonic system.

Purpose of this talk

Introduction for the Kadanoff-Baym equation and Application it to gluodynamics

To introduce kinetic entropy and show H-theorem in Quantum Field Theories

To show entropy production of Quantum Fields in Numerical Simulation

Rest of this talk

- Kadanoff-Baym equation
- Application to scalar and gauge theory, H-theorem, Numerical Simulation
- Towards 3+1 dimension
- Summary and Remaining Problems

Kadanoff-Baym equation

- Quantum evolution equation of 2-point Green's function (fluctuations).

statistical (distribution) and spectral functions

$$F(x,y) = \frac{1}{2}\left\langle\left\{\tilde{\phi}(x),\tilde{\phi}(y)\right\}\right\rangle$$

$$\rho(x,y) = \left\langle\left[\tilde{\phi}(x),\tilde{\phi}(y)\right]\right\rangle$$

$$F(p^0,p) = 2\pi\delta(p^2-m^2)\left(1+\frac{1}{e^{\beta|p^0|}-1}\right)$$

Boson

$\gamma \to 0$

$$\rho(p^0,p) = \frac{\gamma}{(p^0-\omega)^2+\gamma^2/4} \to 2i\pi\epsilon(p^0)\delta(p^2-m^2)$$

Breit-Wigner type

$$\left(-G_0^{-1}+\Sigma_{\rm loc}\right)F(x,y) = \int_0^{y^0} dz\Sigma_F(x,z)\rho(z,y) - \int_0^{x^0} dz\Sigma_\rho(x,z)F(z,y)$$

$$\left(-G_0^{-1}+\Sigma_{\rm loc}\right)\rho(x,y) = \int_{x^0}^{y^0} dz\Sigma_\rho(x,z)\rho(z,y)$$

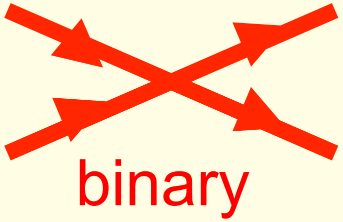
Memory integral

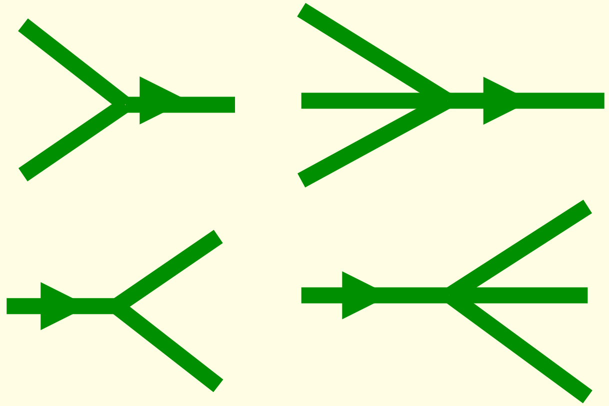
$G_0^{-1} = -\partial^2 - m^2$ **Σ=Self-energies**

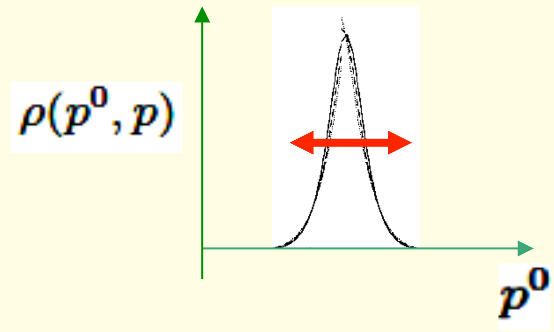
Self-energies: local $\Sigma_{\rm loc}$ mass shift, nonlocal real Σ_F and imaginary part Σ_ρ

Merit

- **Quantum evolution with conservation law**
- **Evolution of spectral function with decay width + distribution function**







Finite decay width

- **Off-shell effect**

Decay width ⇒ particle number changing process (gg⇔g (2-to-1) and ggg⇔g (3-to-1))+ binary collisions (gg⇔gg).

They are prohibited kinematically in Boltzmann simulation. This process might contribute to the early thermalization.

Demerit

Numerical simulation needs much memory of computers.

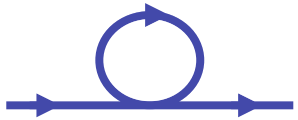
Application to scalar and gauge theory

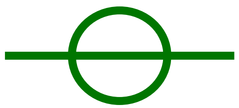
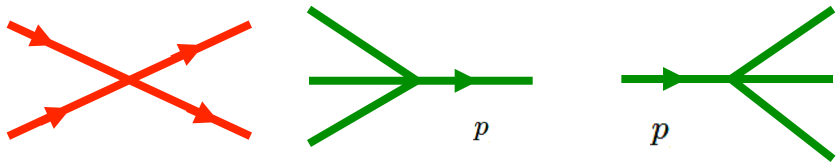
- ϕ^4 theory with no condensate $\langle\phi\rangle=0$

$$\mathcal{L}_{\mathrm{int}} = -\frac{1}{4!}\lambda\hat{\phi}^4$$

- Next Leading Order Self-Energy of coupling

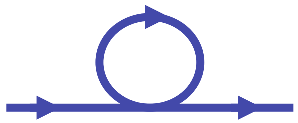
Full Green's function $G=1/[G_0^{-1}-\Sigma(G)]$

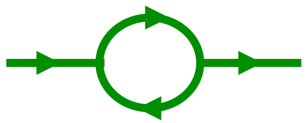
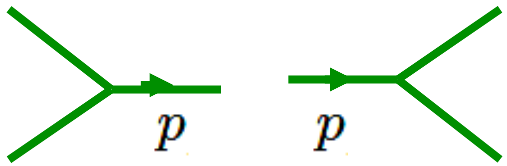
LO $\Sigma=$  $O(\lambda)$

NLO  ⇔ 

$O(\lambda^2)$

- No classical field $\langle A\rangle=0$
- Leading Order Self Energy of coupling

LO (local)  $O(g^2)$

LO (nonlocal)  ⇔ 

$O(g^2)$

However does the dynamics contribute to thermalization? To confirm it,

H-theorem for Scalar Theory (Φ^4, O(N))

- Introduction of kinetic entropy current based on relativistic Kadanoff-Baym eq. **A.N. Nucl. Phys. A 832:289-313, 2010.**
- 1st order gradient expansion of KB eq.

(Extension of nonrelativistic case, Ivanov, Knoll and Voskresenski (2000), Kita (2006))

I. $$s^\mu = \int \frac{d^{d+1}p}{(2\pi)^{d+1}} V^\mu \left[-f \log f + (1+f)\log(1+f)\right]$$

Offshell $$V^\mu = \frac{\rho}{i}\left(p^\mu - \frac{1}{2}\frac{\partial \mathrm{Re}\Sigma_R}{\partial p_\mu}\right) + \frac{\Sigma_\rho}{i}\frac{1}{2}\frac{\partial \mathrm{Re}G_R}{\partial p_\mu}$$

V^μ: Entropy flow spectral function

$$\partial_\mu s^\mu \geq 0$$

NLO of the coupling (Φ^4), (2⇔2, 3⇔1)

NLO of 1/N expansion (O(N)) (2⇔2, 3⇔1)

A.N. and A. Ohnishi (2010)

H-theorem is derived at the level of Green's function with off-shellness.

In the quasiparticle limit **We reproduce the entropy for the boson.**

II. $$s^\mu \to \int \frac{d^d p}{(2\pi)^d} v^\mu \left[-n_{\mathbf{p}} \ln n_{\mathbf{p}} + (1+n_{\mathbf{p}})\ln(1+n_{\mathbf{p}})\right]$$

v^μ **:velocity**

H-theorem for Non-Abelian Gauge Theory

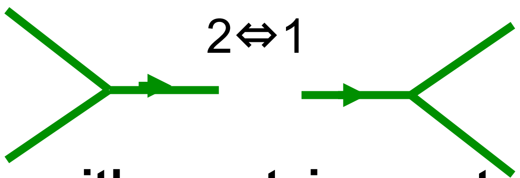
$$D^{-1}(x,y) = D_0^{-1}(x,y) - \Pi(x,y)$$

Green's function **Self-energy**

In Temporal Axial Gauge (TAG),
divide Green's function and self-energy into transverse (T) and longitudinal part (L),
take 1st order gradient expansion, then

$$\partial_\mu s^\mu = g^2N[\,(TTT) + (TTL) + (TLL)\,] \geq 0.$$

Each term is positive definite.



<u>Controlled gauge dependence</u> of our entropy density with a certain constant term is assured at thermal equilibrium.

(Blaiziot, Iancu and Rebhan (1999))
(Smit and Arrizabaraga (2002), Carrington et al (2005))

For gauge transformation $\delta s^0_{eq} \sim g^2 s^0_{eq}$

Gauge dependence is higher order of coupling.

Proof of controlled gauge dependence **out of equilibrium** is still remaining problem.

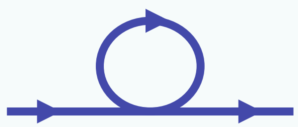
Boltzmann vs. KB (Non-Abelian)

Boltzmann eq in 2+1 and 3+1 dimension. (On-shell)

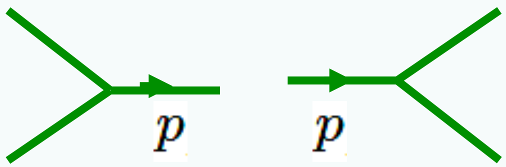
<u>No thermalization occurs due to on-shell g⇔gg.</u>

Energy momentum conservation ⇒ g⇔gg is prohibited.

Kadanoff-Baym eq. in 2+1 dimension. (Off-shell)



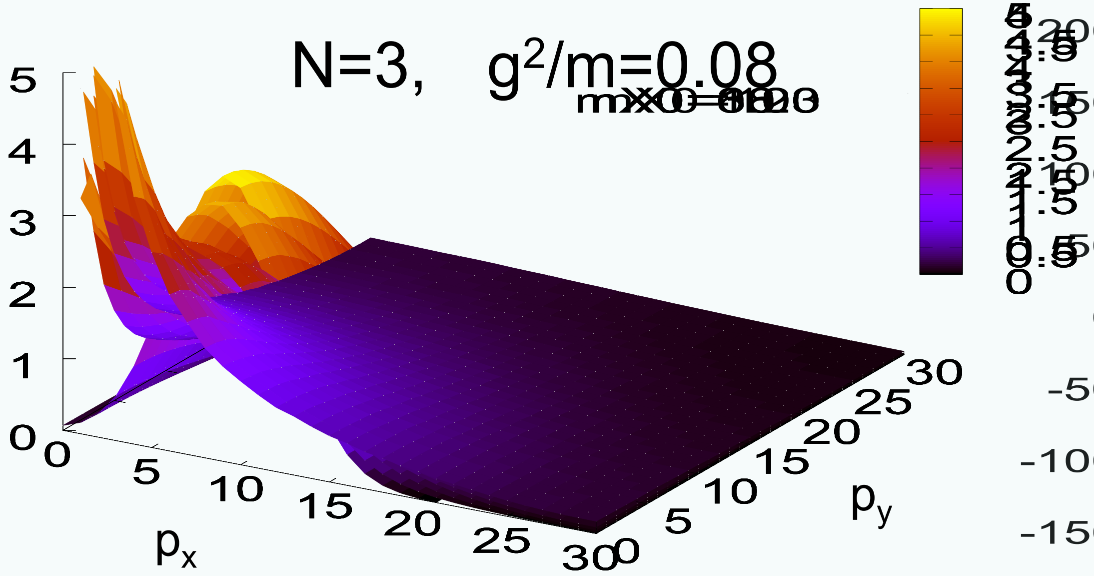
→ m^2

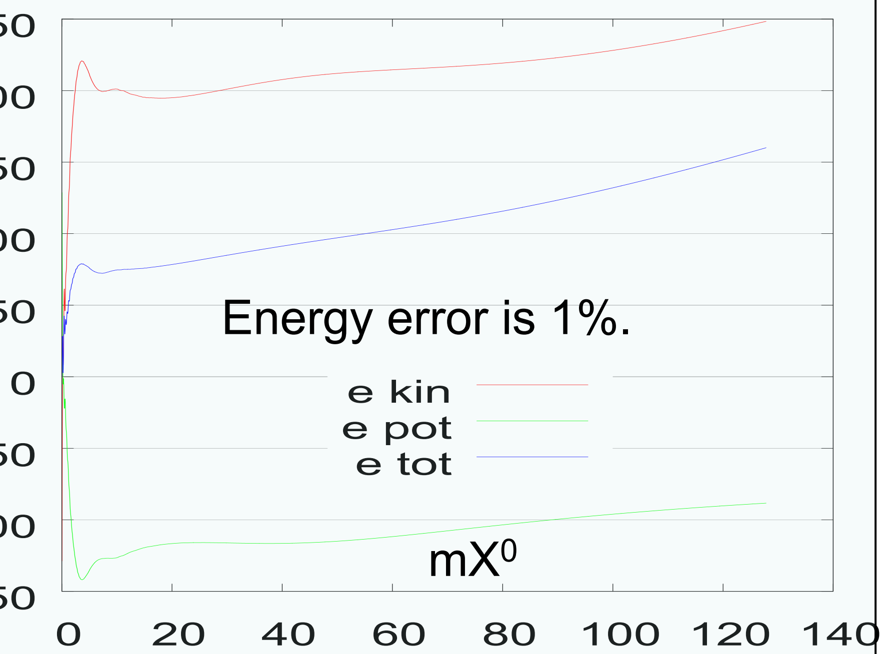


→ Only transverse Part

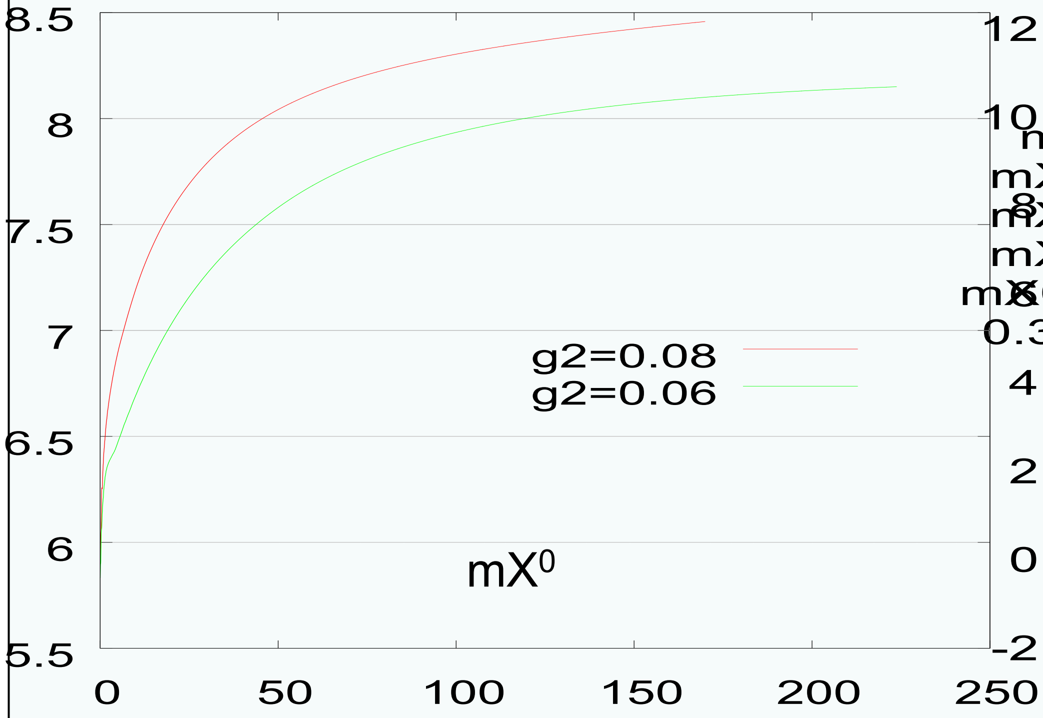
Number distribution function

N=3, $g^2/m=0.08$

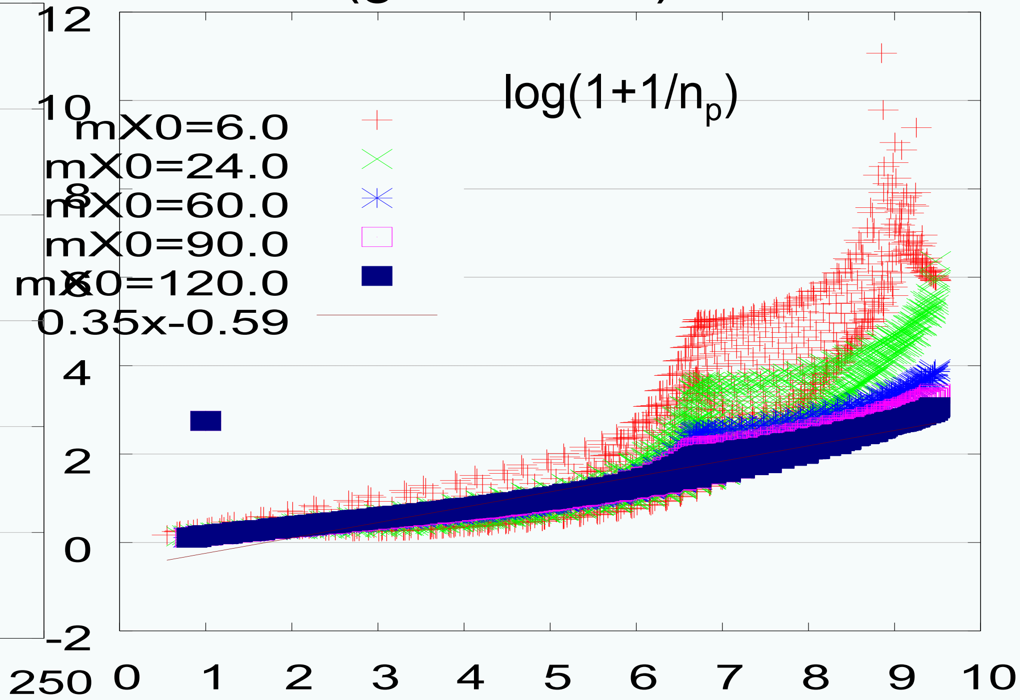




Entropy density



Logarithmic plot of distribution function ($g^2/m=0.08$)



Entropy production occurs due to offshell g⇔gg, which is consistent with the proof of H-theorem.

Logarithmic plot seems to approach straight line with slope 1/T. (Thermalization ?)

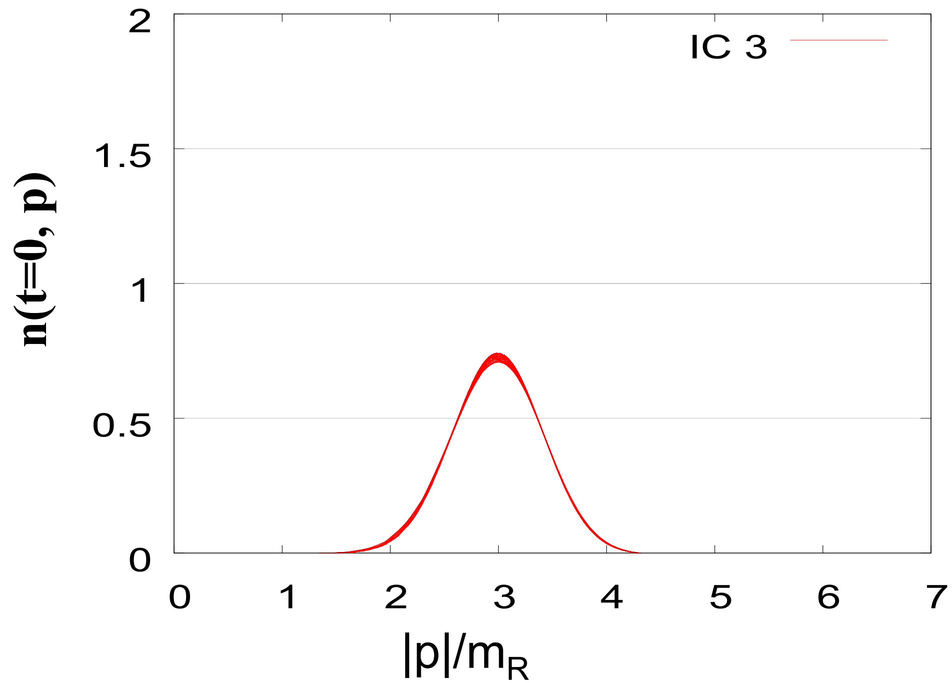
Towards 3+1 dimension

Initial condition (φ^4 in 3+1 dim)

Reproduction of Lindner and Mueller (2006).

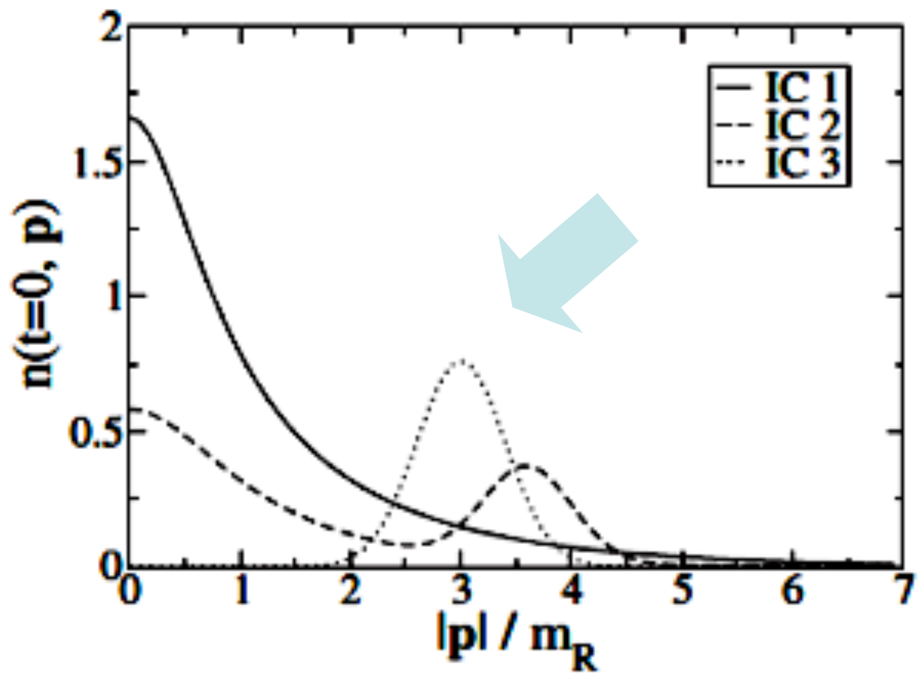
We shall reproduce the results of case with initial condition (IC) 3.

Number distribution function in momentum space



My result

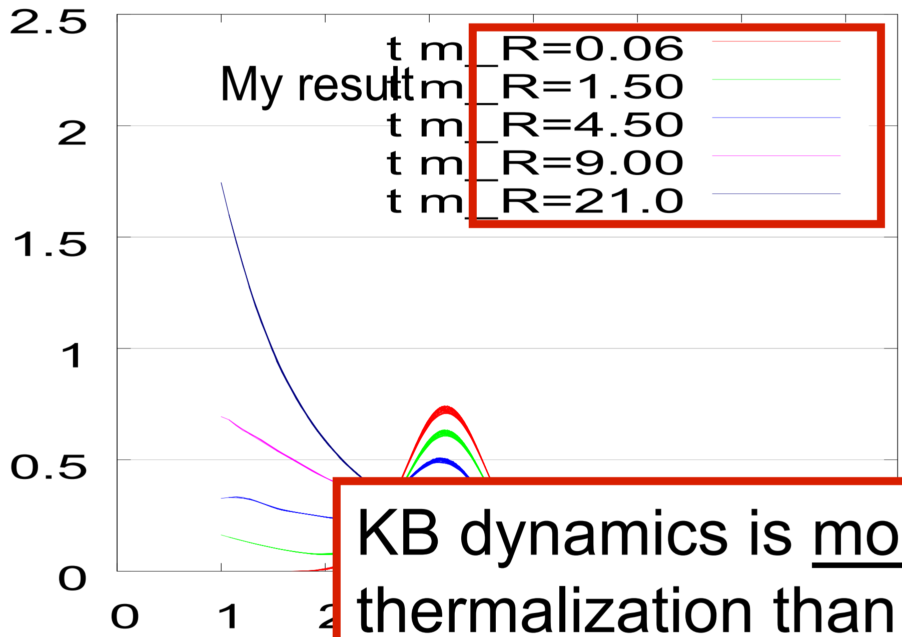
Starting with λ= 18.

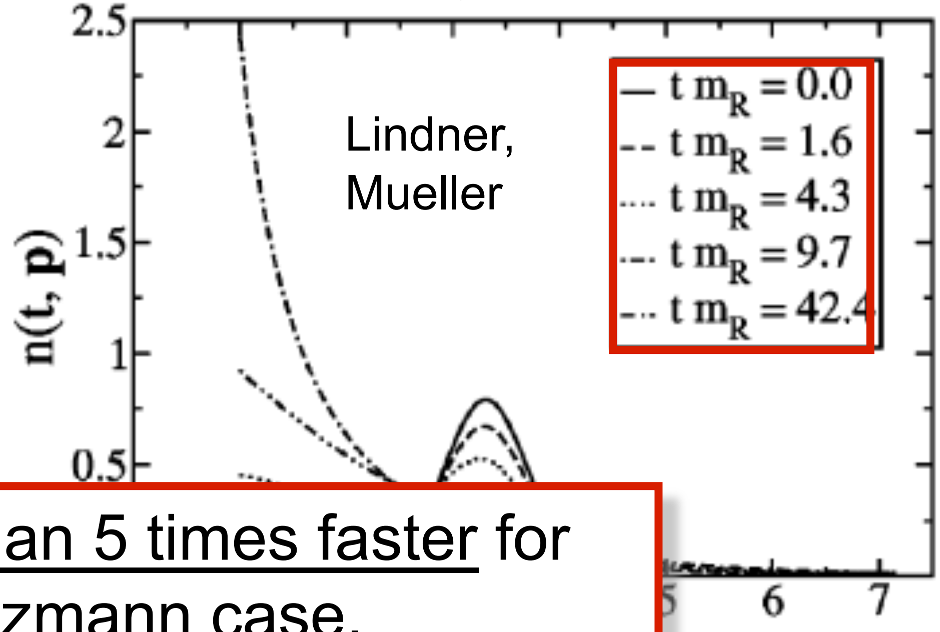


Lindner and Mueller (2006)

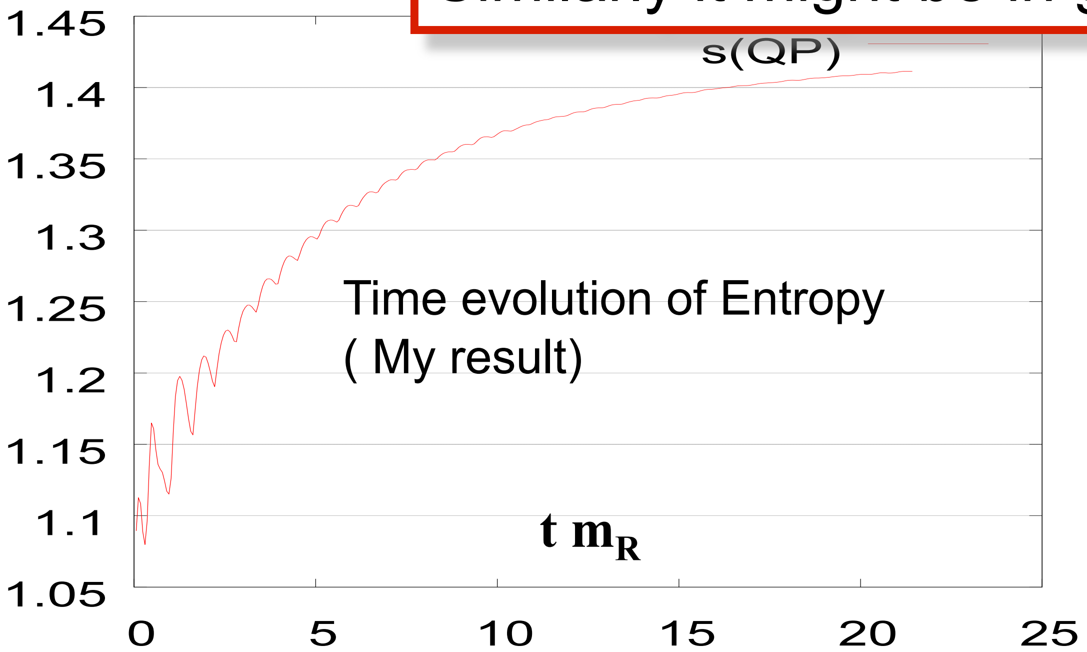
λ= 18

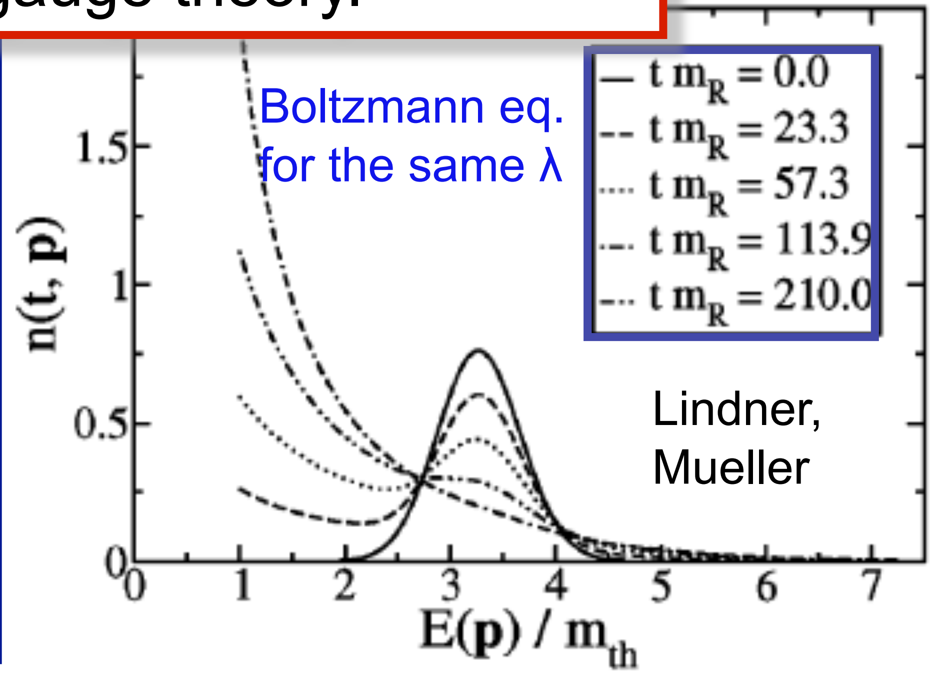
Time evolution of distribution function with Kadanoff-Baym eq.





KB dynamics is <u>more than 5 times faster</u> for thermalization than Boltzmann case.
Similarly it might be in gauge theory.





Summary

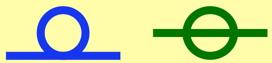
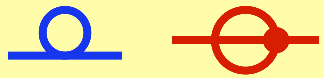
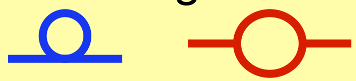
- We have considered the Kadanoff-Baym approach to thermalization of dense nonequilibrium gluonic system.
- We have introduced the kinetic entropy based on the Kadanoff-Baym equation. (Criteria for thermalization)
- The kinetic entropy satisfies H-theorem for NLO of $\lambda(\Phi^4)$ and 1/N (O(N)). It may do for LO of coupling in SU(N).
- Entropy production occurs with the Kadanoff-Baym dynamics with off-shell effects even in situations where it does not occur in on-shell Boltzmann dynamics. This property may help the understanding of the early thermalization.
- It will become possible to perform calculation in 3+1 dimension in scalar theory by my codes. Then we notice that thermalization time scale in KB eq is remarkably faster than Boltzmann dynamics.

Remaining Problems

- **Solution for the KB eq. in and out of equilibrium for the LO of g^2 for the gauge theory with longitudinal part (2+1 and 3+1dimensions)**
- **Gauge invariance of the entropy far from equilibrium, Infrared singularity of longitudinal part in Green's function.**
- **Coupling dependence of entropy saturation**
- **Background classical field in gauge theory**
- **Effect of expansion**

Time irreversibility

Symmetric phase ⟨Φ⟩=0

	λΦ⁴	**O(N)**	**SU(N)**
Exact 2PI (no truncation)	**×**	**×**	**×**
Truncation	NLO of λ  △	NLO of 1/N  △	LO of g^2  **?**
LO of Gradient expansion **H-theorem**	○	○	△ **(TAG)**

Numerical Simulation for KB eq.

Symmetric phase $\langle\Phi\rangle=0$

	$\lambda\Phi^4$	O(N)	SU(N)
Truncation	NLO of λ 	NLO of 1/N 	LO of g^2 
Others' Code	1+1 dim 2+1 dim 3+1 dim	1+1 dim 3+1 dim	?
Our Code	1+1 dim 2+1 dim 3+1 dim	1+1 dim	Part of 2+1 dim

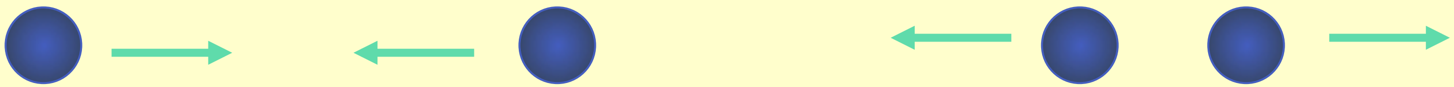
Boltzmann vs. KB

(φ^4 in 1+1 dimension)

Boltzmann eq in 1+1 dimension. (On-shell)

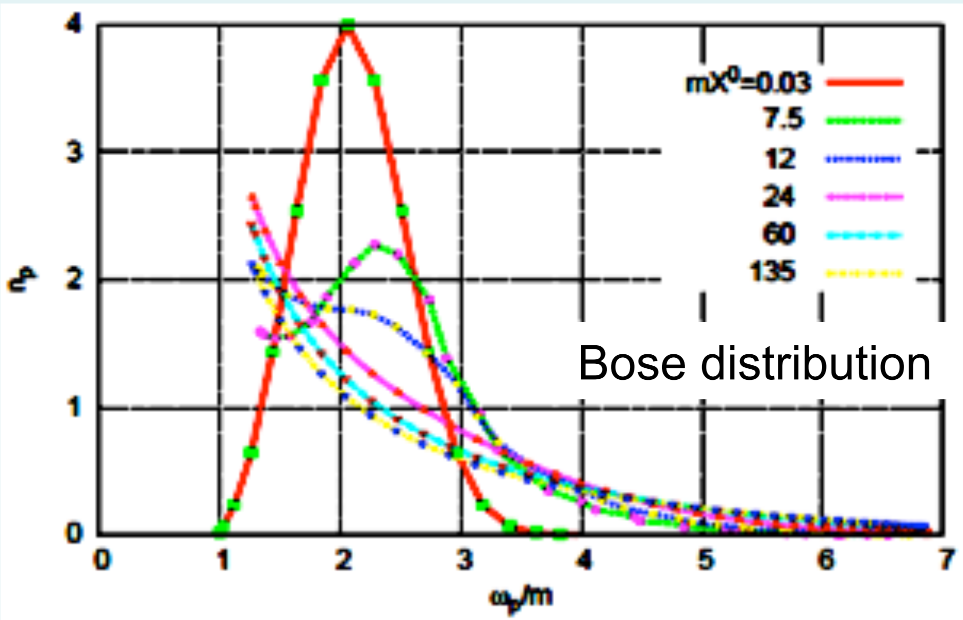
<u>No thermalization occurs.</u>

Energy momentum conservation ⇒No change of momentum.

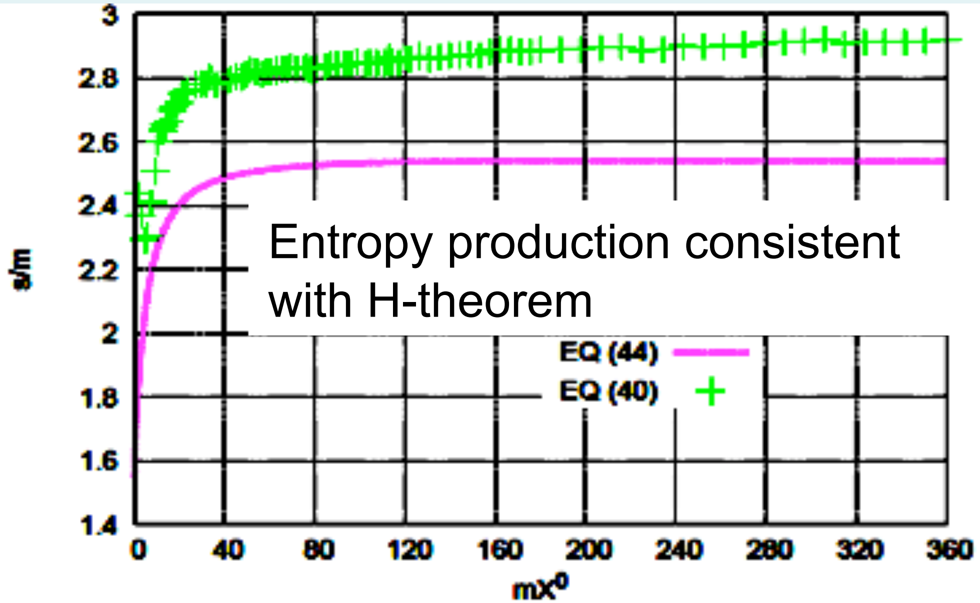


$(\epsilon_{\mathbf{p}}, p)$ $(\epsilon_{\mathbf{p}}, -p)$ $(\epsilon_{\mathbf{p}}, -p)$ $(\epsilon_{\mathbf{p}}, p)$

Kadanoff-Baym eq. in 1+1 dimension. (Off-shell) (Numerical results)



A. Aarts and J. Berges (2001), J. Berges (2002)



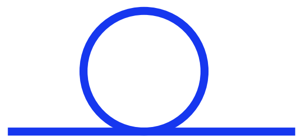
A.N. Nucl. Phys. A 832 (2010) 289.

Renormalization (O(N) model)

A. Arrizabalaga, J. Smit and A. Tranberg (2004)

Mass

$$\mu_0^2 = \pm\mu^2 - \lambda\frac{N+2}{6N}c_1(a_s\mu)\frac{1}{a_s^2}$$



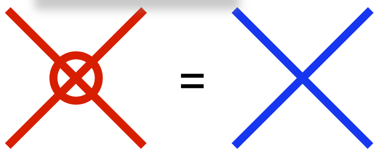
Quadratic divergent

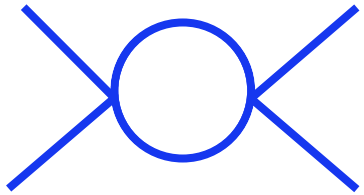
→ Subtraction

Plus and minus in front of μ means symmetric and broken phase, respectively.

$$c_1(a_s\mu) = \int_{-\pi}^{\pi}\frac{d^3k}{(2\pi)^3}\frac{1}{2\sqrt{a_s^2\mu^2 + k_{\mathrm{lat}}^2}}$$

Vertex





Logarithmic divergent

→ Bethe-Salpeter eq.

$$\lambda_0 = \lambda - \lambda^2\frac{N+8}{6N}\left(\frac{1}{16\pi^2}\ln(a_s^2\mu^2) + \frac{1}{16\pi^2} - C_2\right) + O(a_s^2\mu^2)$$

For $0.5 \leqq \mu a_s \leqq 1$ **the difference between λ_0 and λ is less than 10%**

We can use bare coupling as if it were renormalized coupling when the relevant length scale is larger than a_s in numerical simulation. ($a_s<\mu^{-1}$)

The above analysis holds at φ4 model with coupling expansion.

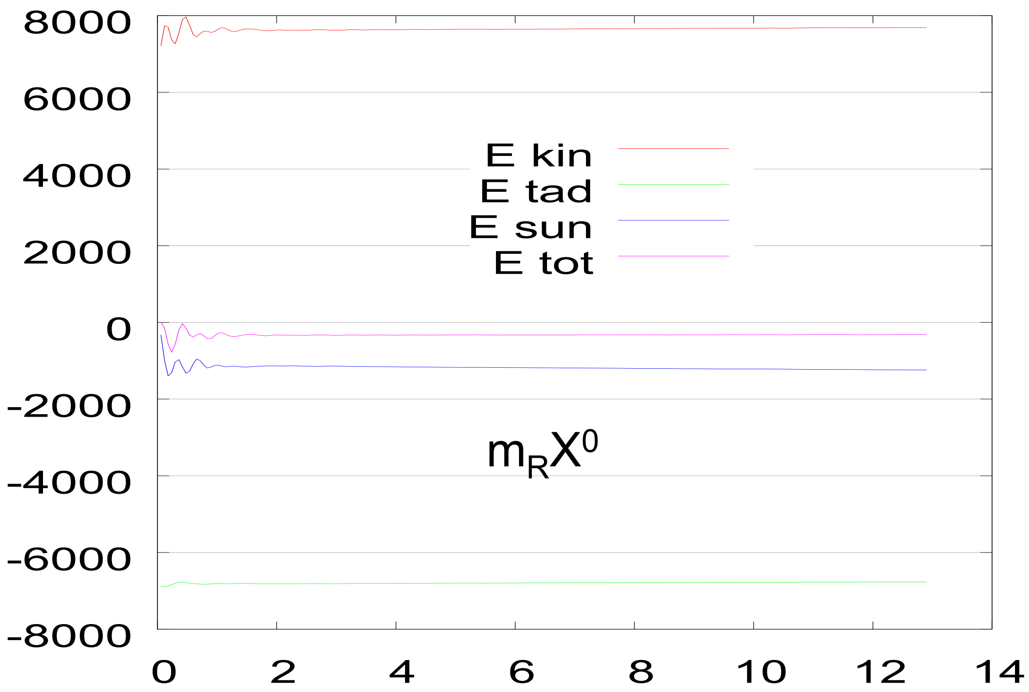
Φ^4 model in 3+1 dim.

- Reinvestigation of preceding research

  (Lindner and Mueller (2006))
- Without condensate
- Initial condition

  Nonthermal distribution (Gaussian configuration)

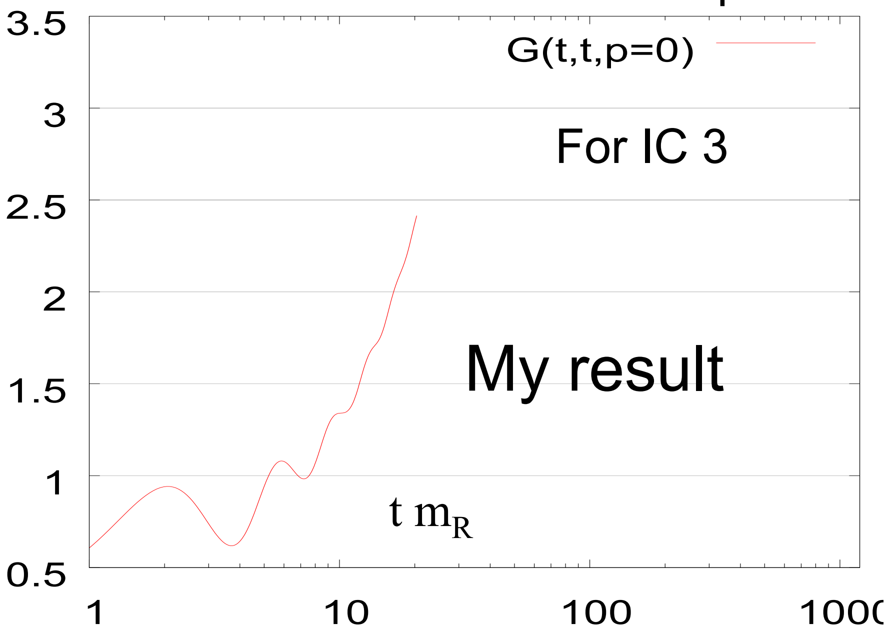
  Uniform Space
- Without expansion

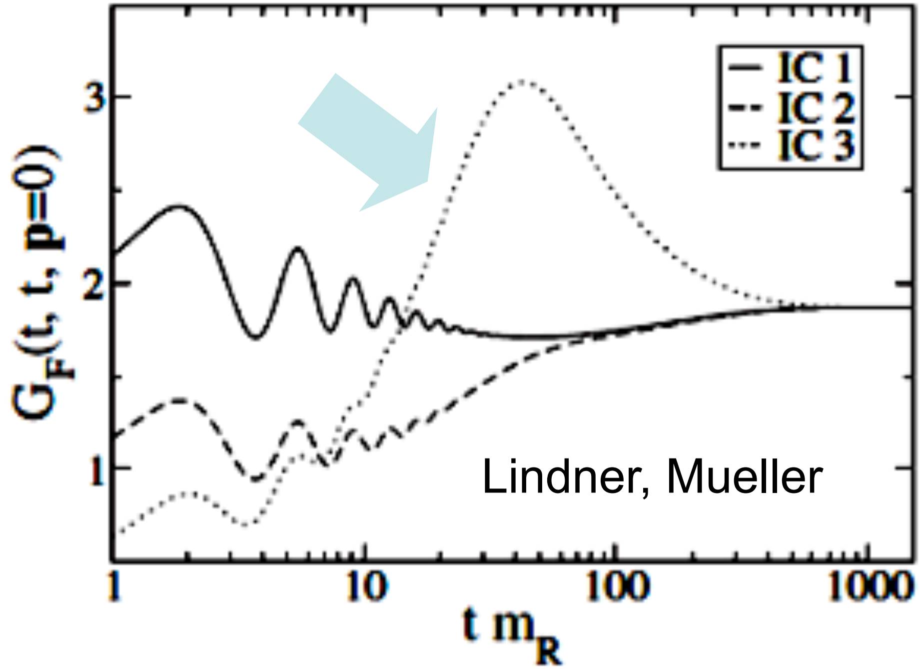


Kinetic and potential energy (kinetic part is subtracted by total energy).

Energy Error is 1%

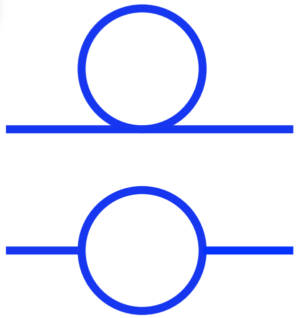
Plot of equal-time Green's function





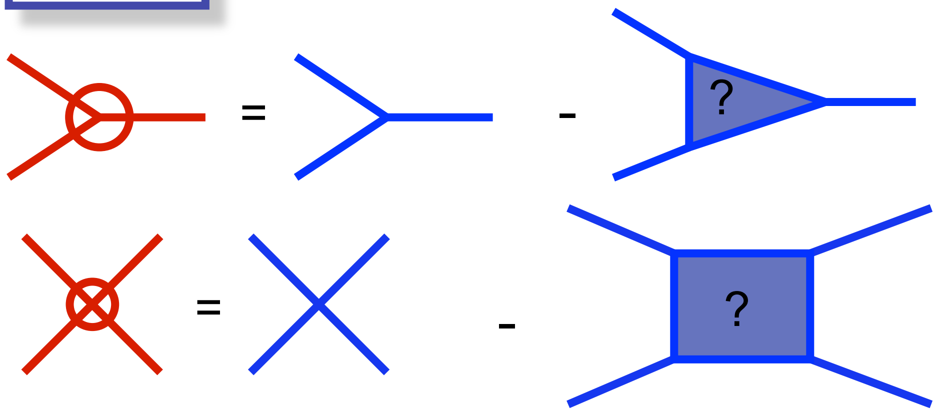
Renormalization (gauge theory)

Mass



Quadratic divergent → Subtraction

Vertex



Logarithmic divergent

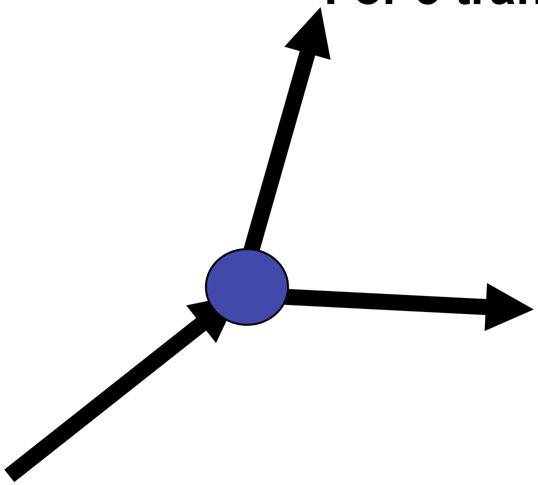
→ Bethe-Salpeter eq.

We must confirm whether we can use bare coupling as if it were renormalized coupling in numerical simulation.

Microscopic process (Non-Abelian)

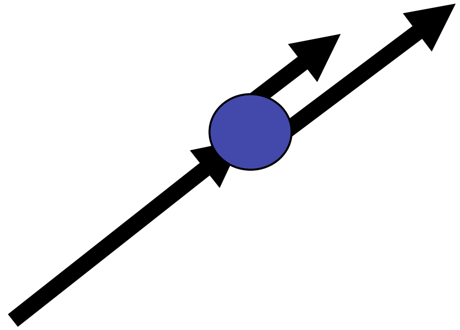
Each microscopic process is possible in 2+1 and 3+1 dimensions.

For 3 transverse fluctuations,



$C \neq 0$

Entropy production



$C = 0$

No entropy production

The 0-to-3 and 1-to-2 might contribute to isotropization with entropy production. These processes are prohibited in Boltzmann limit without spectral width and memory integral.

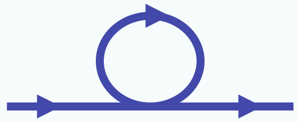
Boltzmann vs. KB (Non-Abelian)

Boltzmann eq in 2+1 and 3+1 dimension. (On-shell)

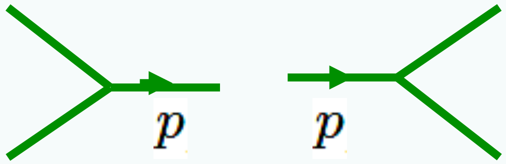
No thermalization occurs due to on-shell g⇔gg.

Energy momentum conservation ⇒ g⇔gg is prohibited.

Kadanoff-Baym eq. in 2+1 dimension. (Off-shell)



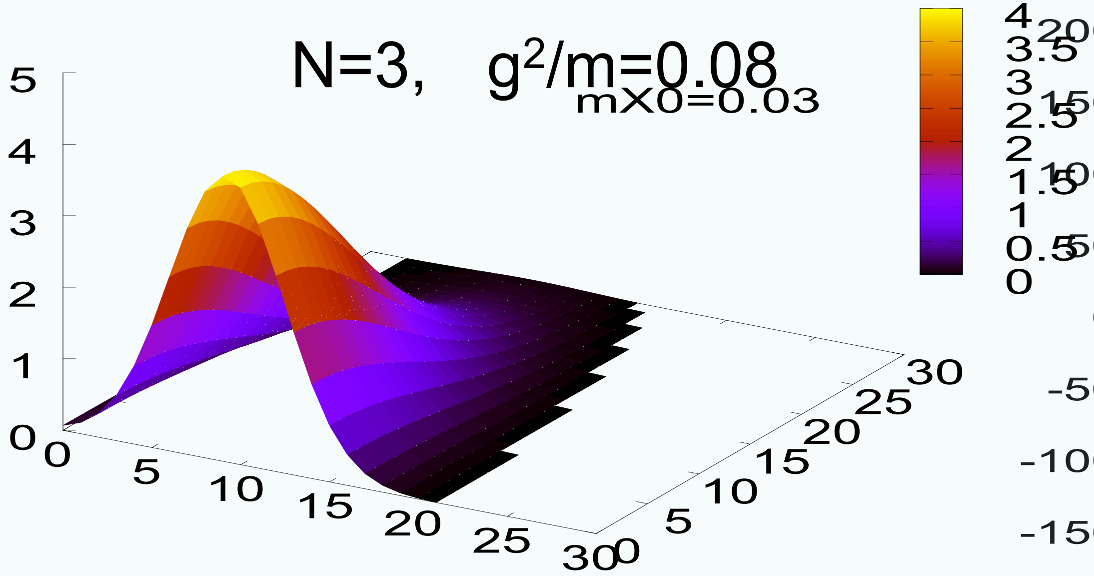
→ m^2

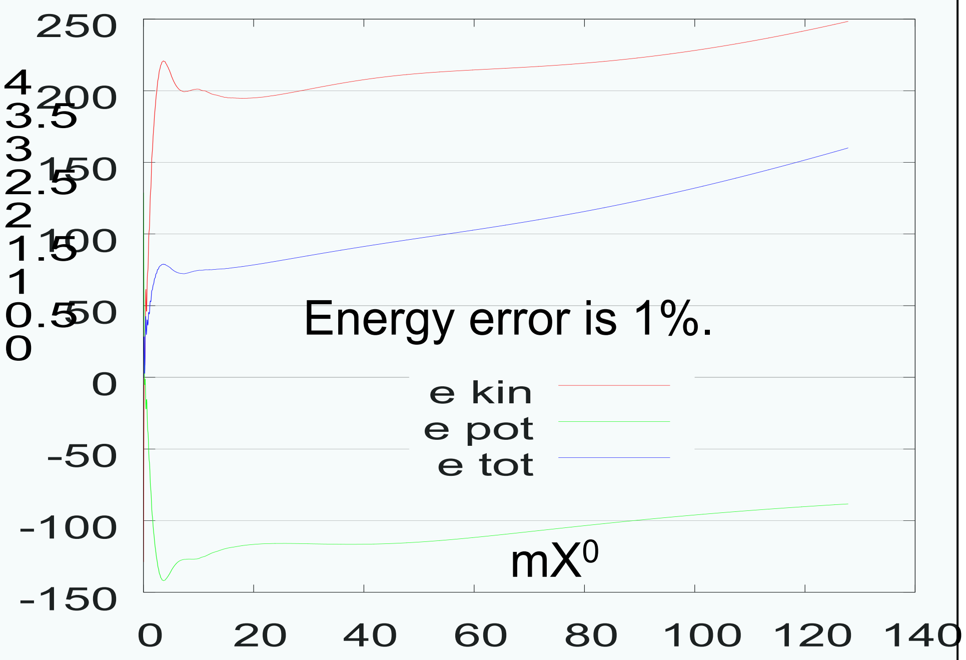


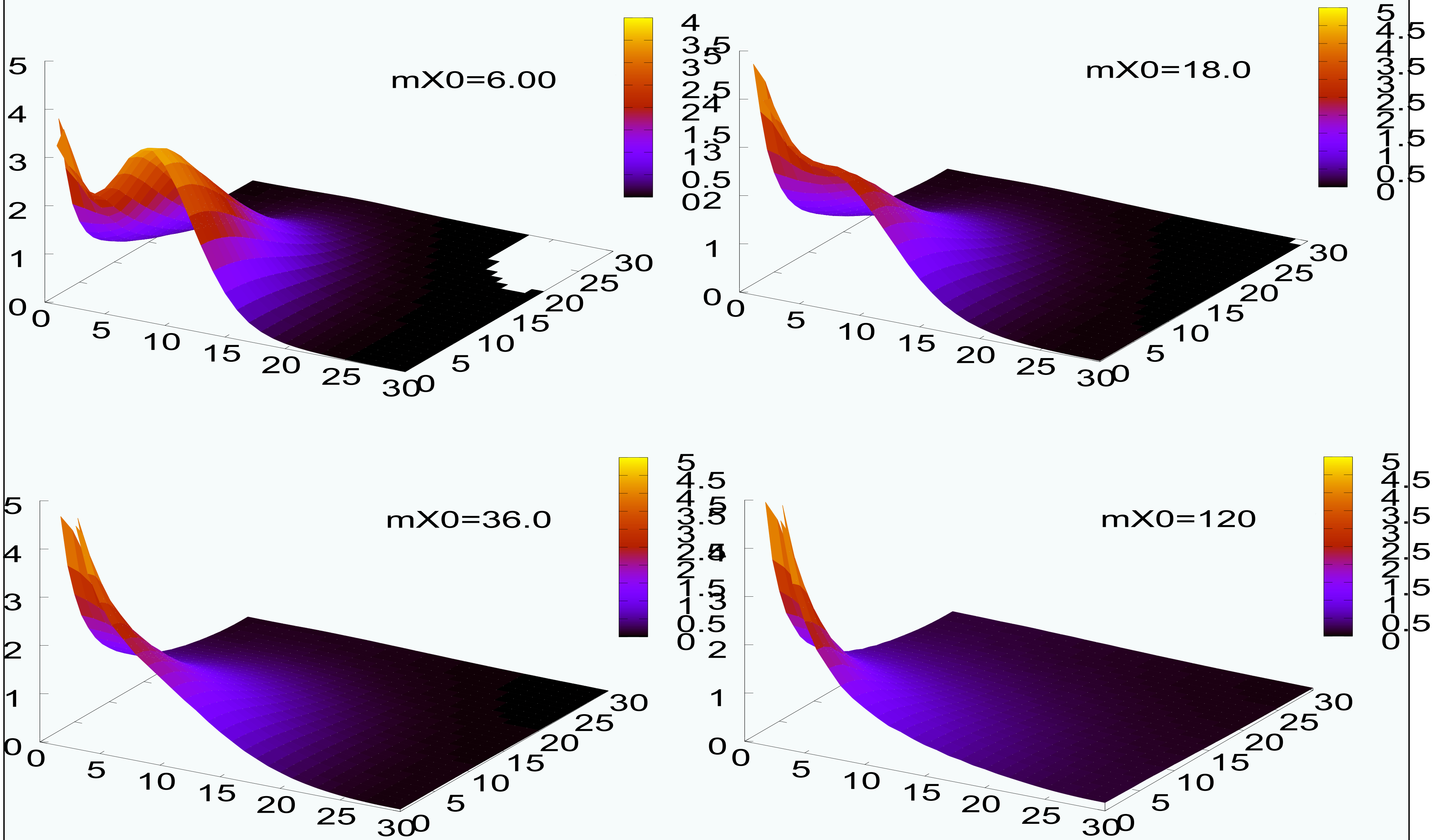
→ Only transverse Part

Number distribution function

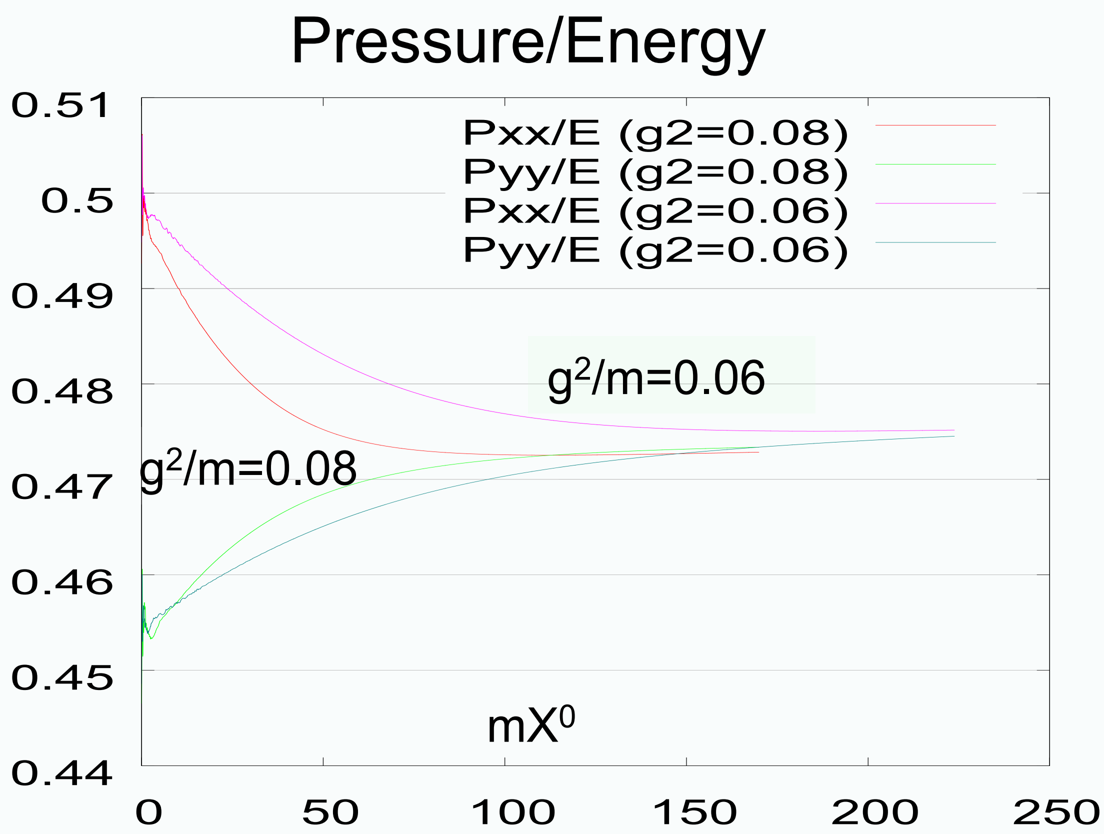
N=3, $g^2/m=0.08$






mX0=6.00
mX0=18.0
mX0=36.0
mX0=120

Isotropization

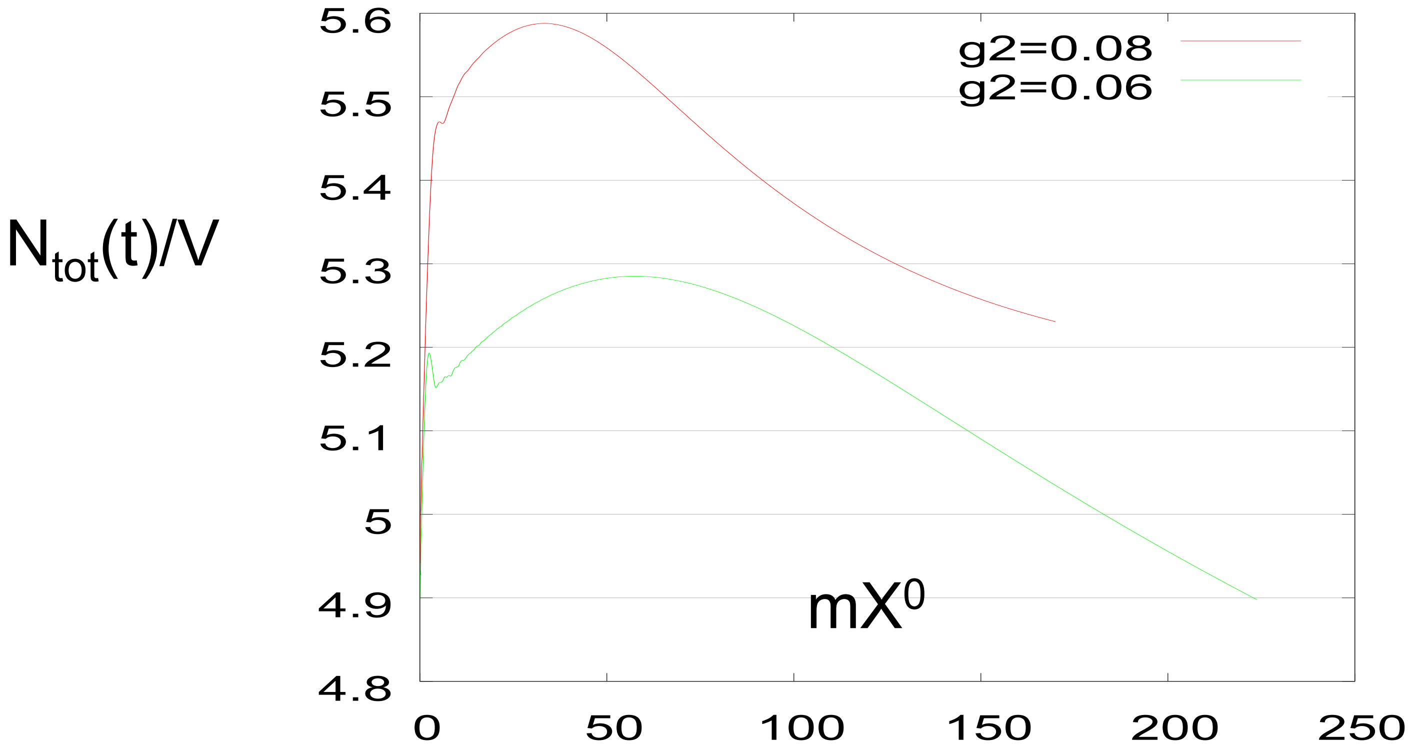


Time scale of isotropization is nearly the scale of entropy saturation.

Thermalization Time ≒ Isotropization Time ? for g⇔gg

(Initial condition dependent)

Chemical equilibrium ?



Particle production due to 1-to-2 is dominant at early time.

2-to-1 processes contribute at the late time.

It takes longer time to achieve chemical equilibrium.

If condensate or classical field remains,

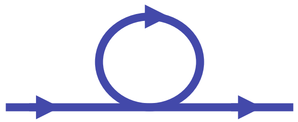
- **ϕ^4 theory with condensate $\langle\phi\rangle \neq 0$**

$$\mathcal{L}_{\rm int} = -\frac{1}{4!}\lambda\hat{\phi}^4$$

- **Next Leading Order Self-Energy of coupling**

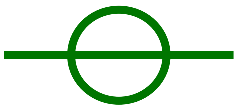
Full Green's function $G=1/[G_0^{-1}-\Sigma(G)]$

LO **$\Sigma=$**

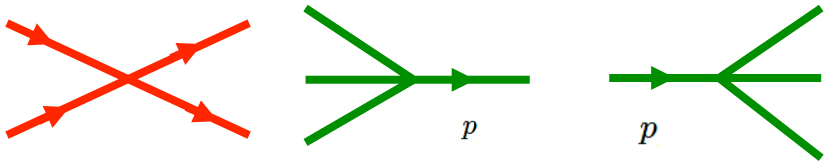


$O(\lambda)$

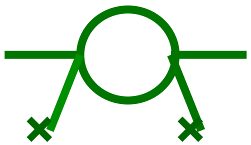
NLO



⇔



$O(\lambda^2)$



: Source Term of particles from the classical field <φ>
0⇔3 and 1⇔2

E.O.M of classical field
+ KB eq with <φ>

$$\left[\partial_x^2 + m^2 + \frac{\lambda}{6}\phi(x)^2 + \Sigma_{\rm loc}(x)\right]\phi(x) = \int_{t_0}^{x^0} dz^0 \int d^dz\, \tilde{\Sigma}_\rho(x,z)\phi(z),$$

$$\partial_\mu s^\mu(X) \quad = \quad ?$$

Definition of entropy with the condensate and H-theorem are needed.

Then some coarse graining procedure is necessary.

Particle Production in Reheating

Big Bang

10^{-36}s Inflation $H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$

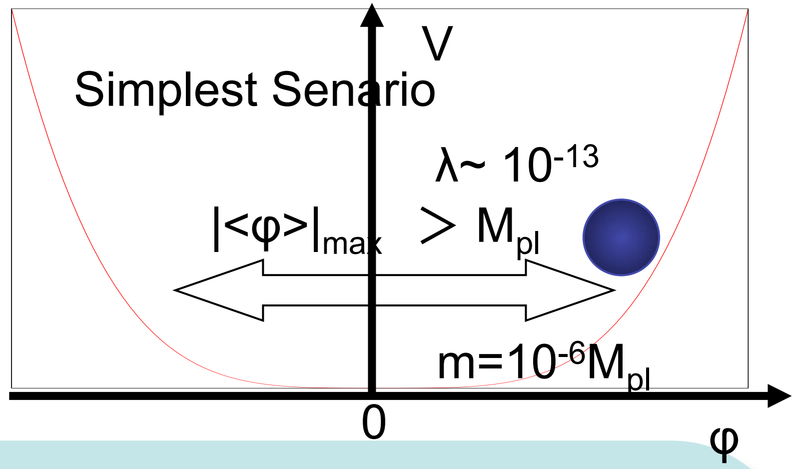
Friedmann eq. ($H=a'/a$)

At the end of inflation
Decay of scalar inflaton field <φ>≠0
Production of Standard Model particles

10^{-32}s

A.D. Linde (1982)

A. Albrecht, P.J. Steinhardt, M.S. Turner and F. Wilczek (1982)



Leptogenesis, Baryogenesis

ElectroWeak phase transition (~ 100GeV)

10^{-5}s QCD phase transition (~ 150MeV)

Nucleosynthesis (~ O(1) MeV)

Decoupling of Photons (~ 0.3 eV)

10^{17}s Today (2.725K)

During oscillation with damping of <φ>,

Energy is transferred to SM particles due to decay of inflaton field
Thermalization of particles

Without this,

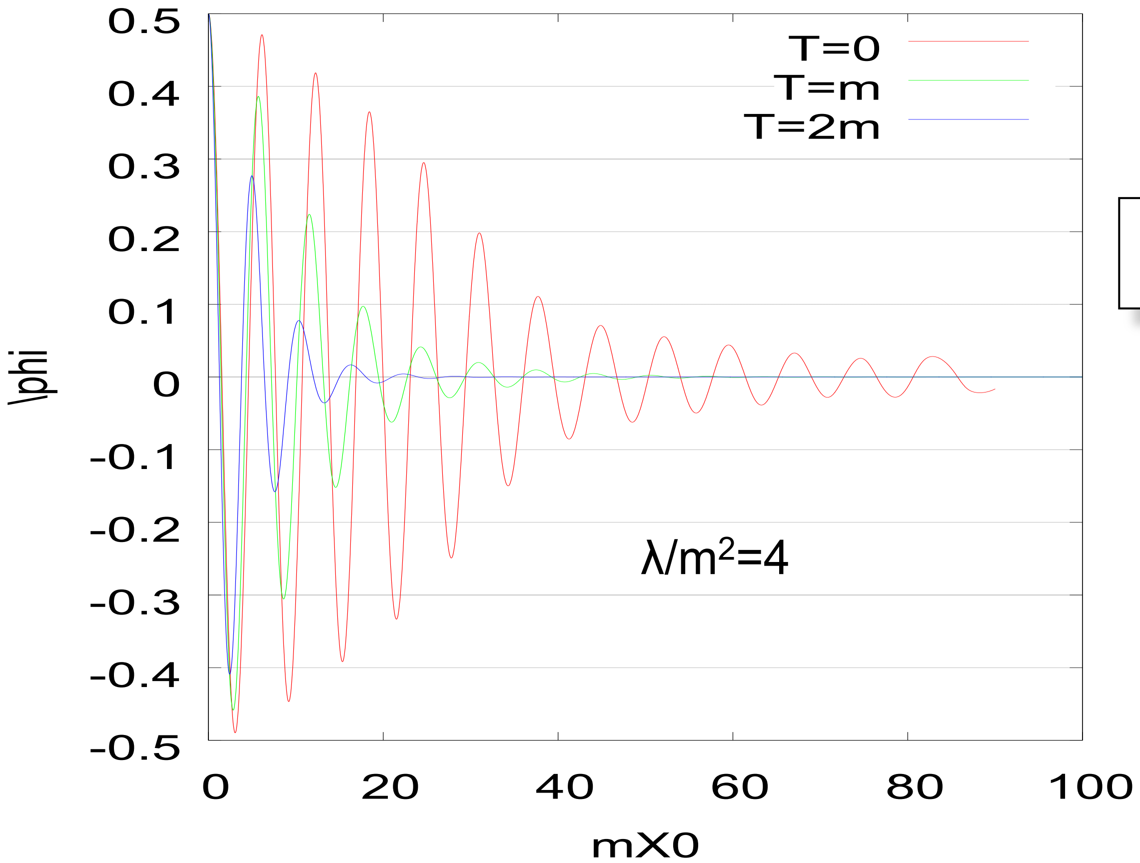
Parametric resonance
slow reheating
thermalization

<u>Inflation would leave behind a universe empty of matter today</u>.

(From Review by J. Smit (2005) and Talk by A. Tranberg (2010))

Numerical Analysis

φ^4 model in 1+1 dimension with $\langle\varphi\rangle \neq 0$ in the symmetric phase without expansion (H=0)



Initial distribution is Bose distribution.
Energy transfer from inflaton field with oscillation and damping to φ particles.

$$\phi(t) \approx \phi_i Z\, e^{-\gamma t} \cos\left(M_{\text{eff}} t - \alpha\right),$$

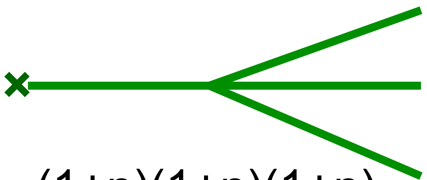
$$M^2 = m^2 + \frac{\lambda T^2}{24} - \frac{\lambda}{8\pi} M\, T$$

$$\gamma = Z \frac{\text{Im}\tilde{\Sigma}_0^R(M_{\text{eff}})}{M_{\text{eff}}}$$

$$\gamma\Big|_{T \gg m,\, \lambda \ll 1} = \frac{\lambda^2 T^2}{768\pi M}$$

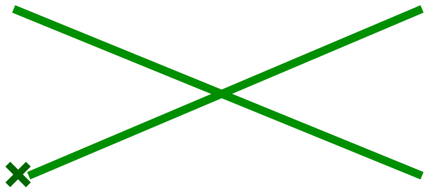
Arrizabaraga, Smit, Tranberg (2005)

For T << m (dilute), 0→3 contributes.



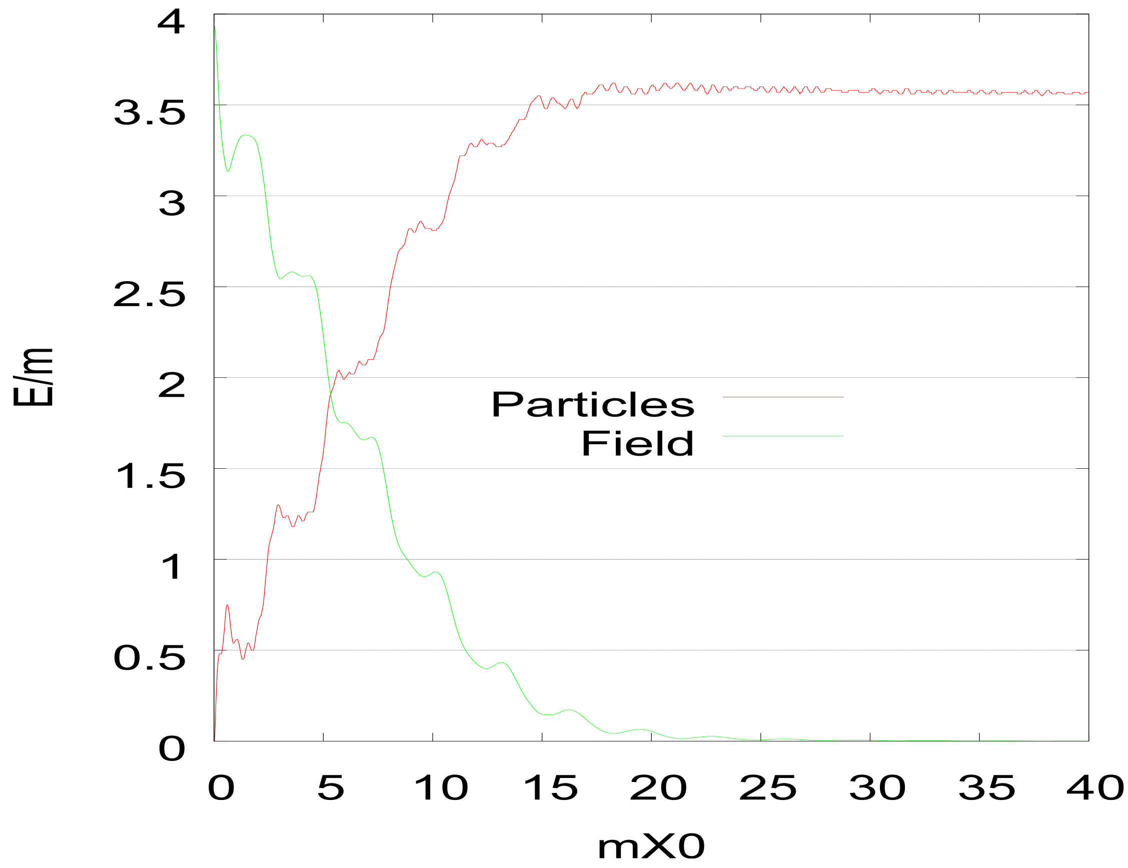
δn~ (1+n)(1+n)(1+n)

For T≠0, 1⇔2 also contributes



For larger T, rapider damping of the field.

Energy transfer from field to particles



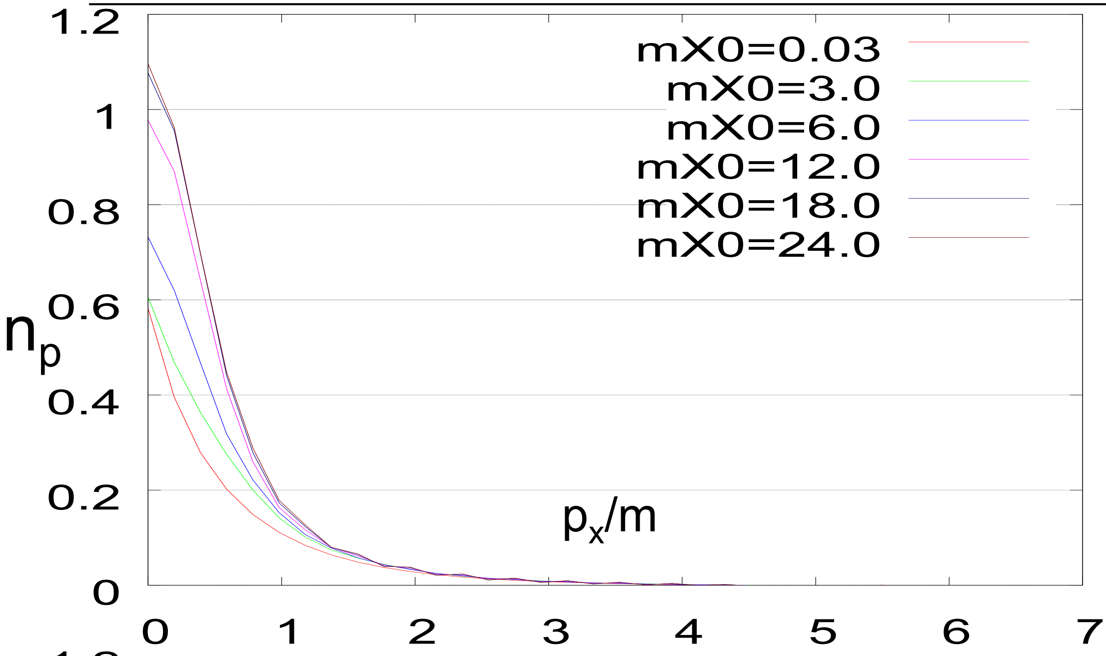
Energy error is 0.2% for $0 \leqq mX^0 \leqq 360$

Energy of particles is subtracted by initial value.

T=m

Particle production occurs due to decay of field.

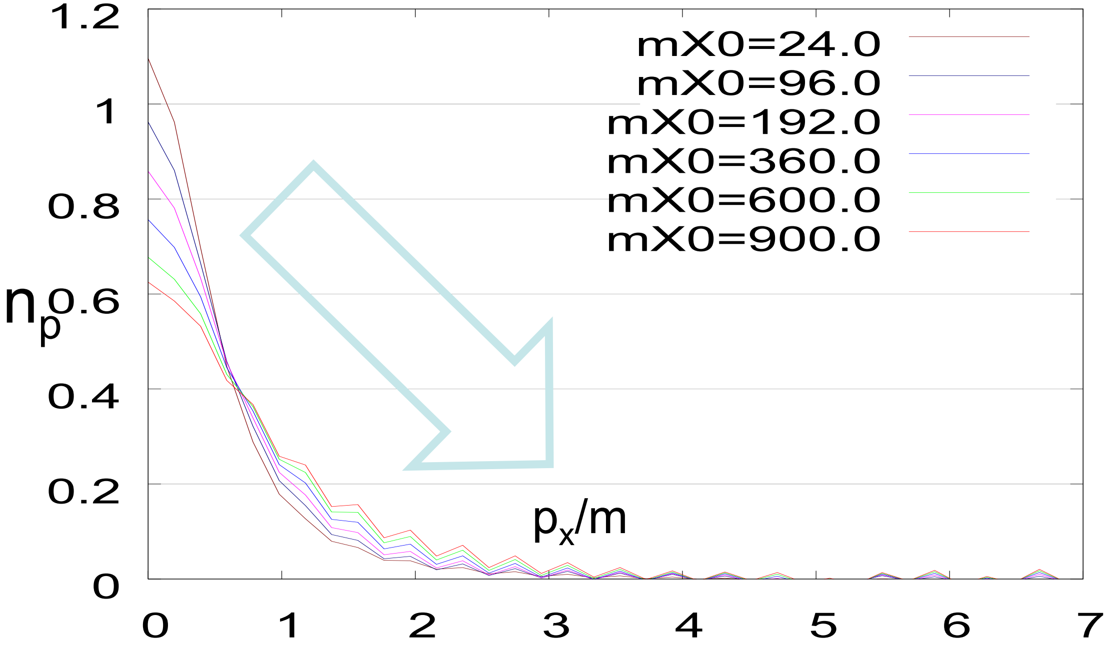
Change of distribution function



In the case of T=m,

0⇔3 and 1⇔2 processes produce particles about around 0 momentum mode during energy transfer from field to particles.

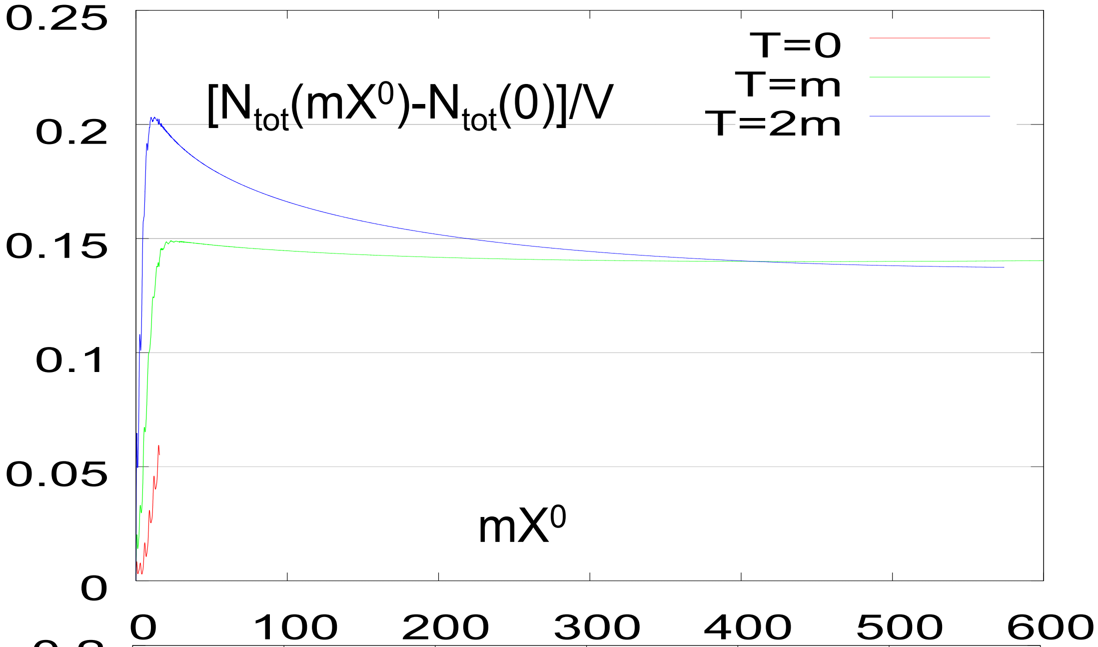
Slow reheating

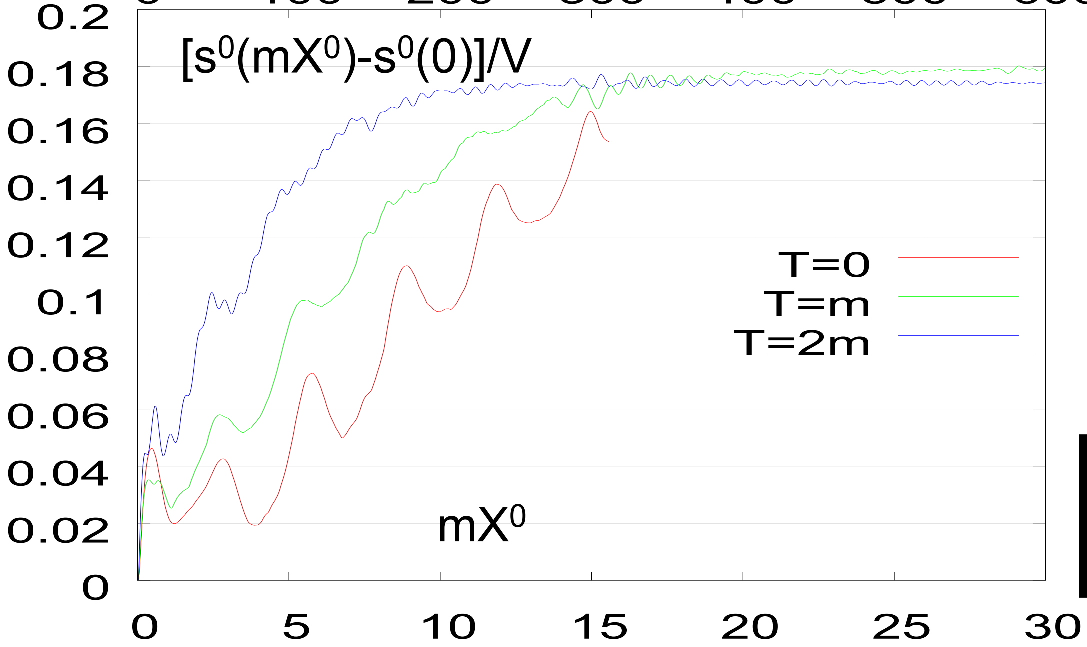


After field damps, distribution around 0 mode is transferred to higher momentum mode due to KB dynamics.

Thermalization

Particle and Entropy production





More total particle number density is produced <u>at early time $mX^0<30$</u> for larger T, but final total number density seems to be the same.

> This must be confirmed for other types of initial conditions !

Similar evolution for entropy density <u>at early time $mX^0<30$.</u>

But monotonically increasing?

Within 1st order of gradient expansion ?

> We should confirm whether Proof of H-theorem is possible or not !

However

- Singularity of longitudinal part of Green's function.
- Gauge invariance or controlled gauge dependence of kinetic entropy far from equilibrium.

However

- Singularity in longitudnal mode

$$\Pi_L(X,\omega,\mathbf{k}) = \frac{\omega^2}{\mathbf{k}^2}\Sigma(X,\omega,\mathbf{k})$$

If the Ward identity holds.

$$D_R = -\frac{\mathbf{k}^2}{\omega^2}\frac{1}{(\mathbf{k}^2 - \mathrm{Re}\Sigma_R) - \frac{1}{2}\Sigma_\rho}$$

singularity at $\omega \to 0$. Breakdown of gradient expansion.

- Gauge dependence of s^μ

That is controlled at thermal equilibrium. (Next page) In far from equilibrium ?

$$s^0 - \mathcal{S}_L^{(0)} = \mathcal{S} = -\frac{\delta\Omega}{\delta T}$$

Blaiziot, Iancu and Rebhan (1999)

Independent of T

$$\mathcal{S} = \int \frac{d^{d+1}k}{(2\pi)^{d+1}}(d-1)\left[-\mathrm{Im}\log D_{T,R}^{-1} + \mathrm{Im}\Pi_{T,R}\mathrm{Re}D_{T,R}\right]\frac{\partial f^{eq}}{\partial T}$$
$$+\int \frac{d^{d+1}k}{(2\pi)^{d+1}}\left[-\mathrm{Im}\log\left(\frac{\mathbf{k}^2}{(k^0+i\epsilon)^2}D_{L,R}^{-1}\right) + \mathrm{Im}\Pi_{L,R}\mathrm{Re}D_{L,R}\right]\frac{\partial f^{eq}}{\partial T} + \mathcal{S}'$$

$$\mathcal{S}' = 0$$

For LO skeleton expansion.

Controlled gauge dependence

Exact $\Gamma_{\rm 2PI}$ **(Thermodynamic potential)** Nielsen (1975), Fukuda and Kugo (1976)

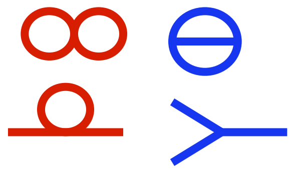
Gauge invariant **at** $\dfrac{\delta\Gamma}{\delta D} = 0 \Leftrightarrow$ **Schwinger-Dyson equation**

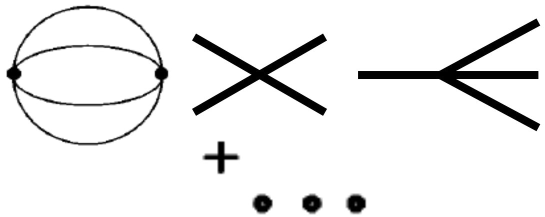
$\Gamma_{\rm 2PI}$ **⇒ Gauge invariant Energy, Pressure and Entropy derived from δΓ/δT**

Truncated effective action (Smit and Arrizabaraga (2002), Carrington et al (2005))

$$\Gamma_{\rm 2PI} = \Gamma_L + \Gamma_{\rm ex} \qquad \Gamma_L \sim O(g^{2L-2}) \, / \, \Gamma_{\rm ex} = O(g^{2L})$$

Expansion of coupling of self energy





Stationary point

$\dfrac{\delta\Gamma_L}{\delta D} = 0$ **⇔ SD equation with truncation**

Under gauge transformation $\delta\Gamma_L \sim g^2\Gamma_L$ **Higher order gauge dependence**

Γ_L **⇒ Energy, Pressure and Entropy derived from δΓ/δT has controlled gauge dependence. Gauge invariance is reliable in the truncated order.**

Kita's Entropy

$$s \equiv \hbar k_{\mathrm{B}} \int \frac{d^3p\, d\varepsilon}{(2\pi\hbar)^4} \sigma \left[A \frac{\partial (G_0^{-1} - \mathrm{Re}\Sigma^{\mathrm{R}})}{\partial \varepsilon} + A_{\Sigma} \frac{\partial \mathrm{Re} G^{\mathrm{R}}}{\partial \varepsilon} \right],$$

$$\boldsymbol{j}_s \equiv \hbar k_{\mathrm{B}} \int \frac{d^3p\, d\varepsilon}{(2\pi\hbar)^4} \sigma \left[-A \frac{\partial (G_0^{-1} - \mathrm{Re}\Sigma^{\mathrm{R}})}{\partial \boldsymbol{p}} - A_{\Sigma} \frac{\partial \mathrm{Re} G^{\mathrm{R}}}{\partial \boldsymbol{p}} \right],$$

$$\frac{\partial s_{\mathrm{coll}}}{\partial t} \equiv \hbar k_{\mathrm{B}} \int \frac{d^3p\, d\varepsilon}{(2\pi\hbar)^4} \mathcal{C} \ln \frac{1 \pm \phi}{\phi}.$$

$$\sigma[\phi] \equiv -\phi \ln \phi \pm (1 \pm \phi) \ln(1 \pm \phi).$$

Equilibrium at

$$\ln \frac{1 \pm \phi_1}{\phi_1} = \alpha + \beta(\varepsilon_1 - \boldsymbol{v} \cdot \boldsymbol{p}_1),$$

Gauge invariance (QED)

R. L. Stratonovich (1956), Fujita (1966).

Let us define the following Green's function, (Fermions)

$$\check{G}(p\varepsilon, r_{12}t_{12}) \equiv \int d^3\bar{r}_{12}\, d\bar{t}_{12}\, \check{G}(1,2)\, e^{-iI(1,2)}\, e^{-i(p\cdot\bar{r}_{12}-\varepsilon\bar{t}_{12})/\hbar}$$

$$I(1,2) \equiv \frac{e}{\hbar c}\int_{\vec{r}_2}^{\vec{r}_1} \vec{A}(\vec{s})\cdot d\vec{s}$$

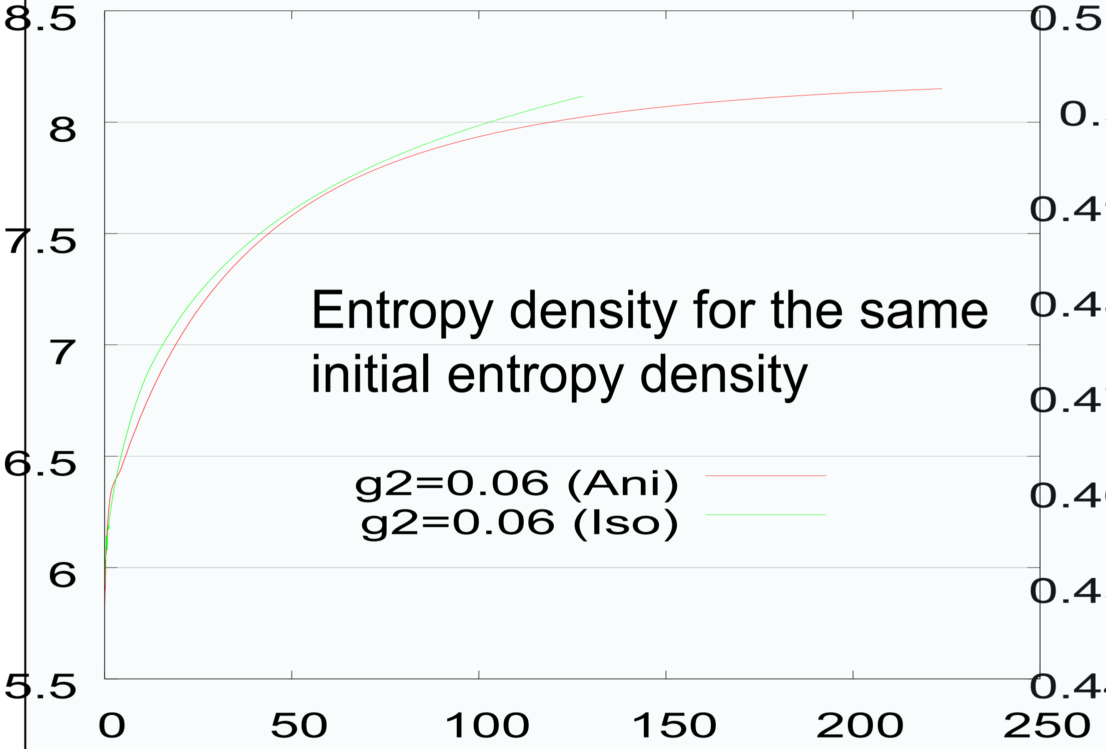
Under the gauge transformation

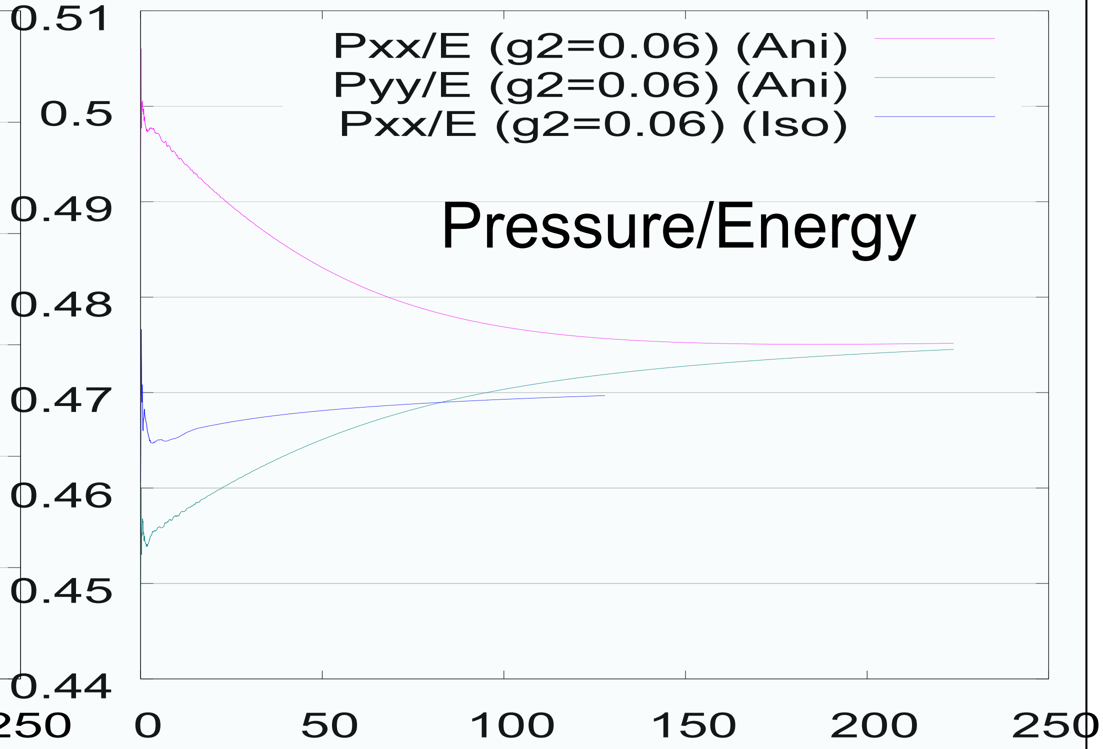
$$\boldsymbol{A}(1) \to \boldsymbol{A}(1) + \boldsymbol{\nabla}_1\chi(1), \qquad A_4(1) \to A_4(1) - \frac{1}{c}\frac{\partial\chi(1)}{\partial t_1},$$

The above Fourier transformed Green's function is gauge invariant. Similar analysis might be possible in QCD.

Anisotropic vs. Isotropic IC

Isotropic initial distribution function is prepared to have the same entropy density.

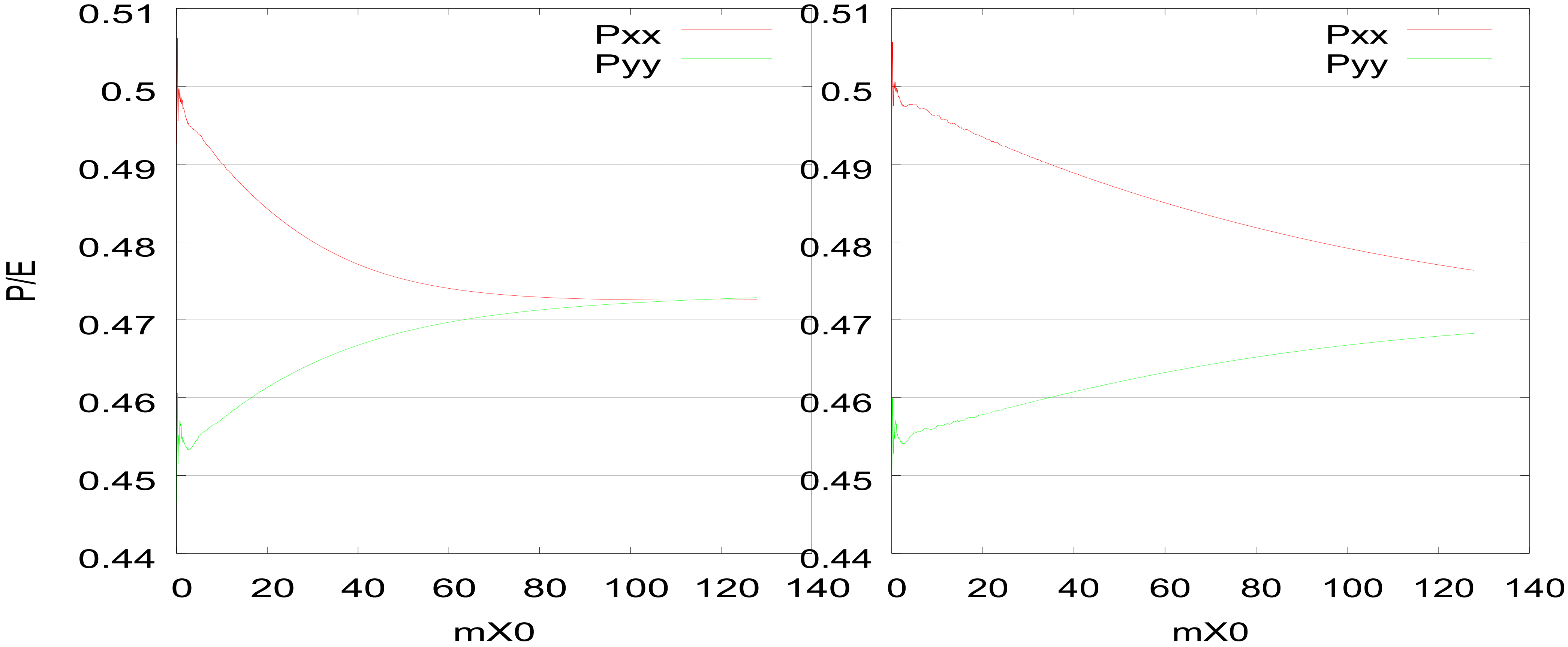




Isotropization might not be so effective for entropy saturation in g⇔gg dynamics.

→ We should investigate initial condition dependence.

Isotropization



$g^2/m=0.08$ $g^2/m=0.06$

Time scale of isotropization is nearly the scale of entropy saturation.

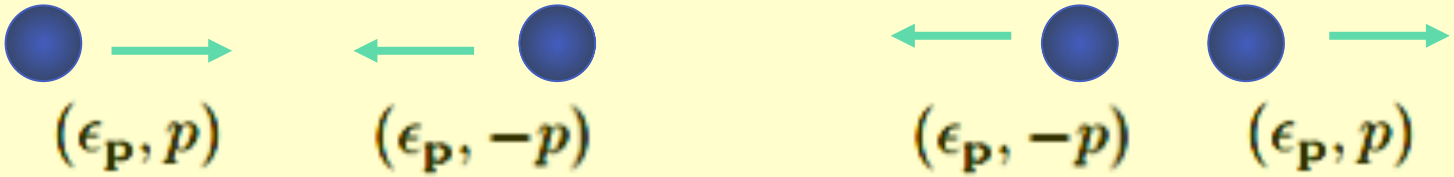
Thermalization Time ≒ Isotropization Time

Boltzmann vs. KB (O(N) in 1+1 dimension)

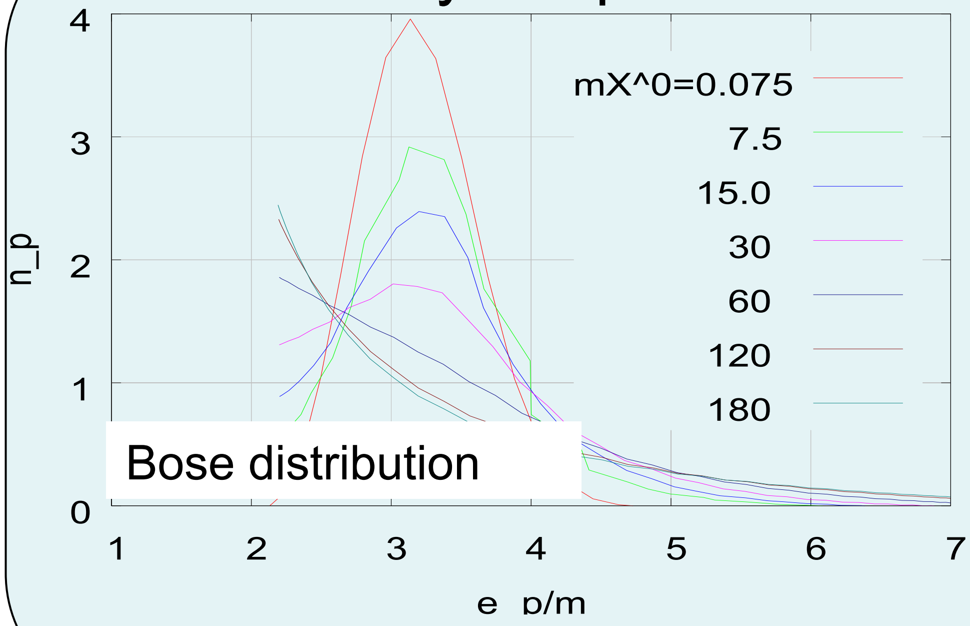
Boltzmann eq in 1+1 dimension. (On-shell)

<u>No thermalization occurs.</u>

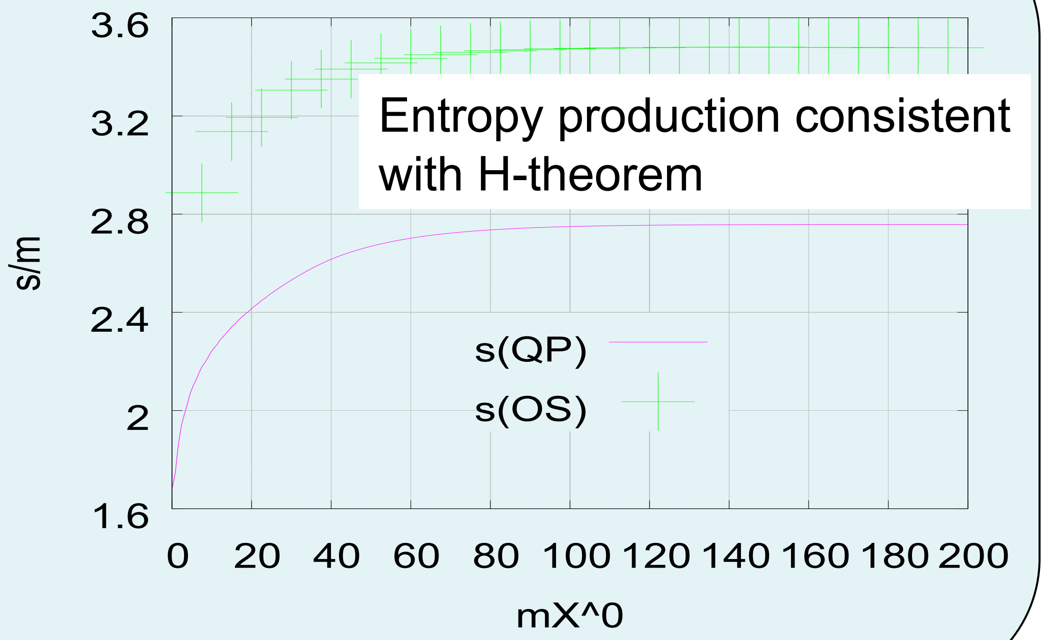
Energy momentum conservation ⇒No change of momentum.



Kadanoff-Baym eq. in 1+1 dimension. (Off-shell) (Numerical results)

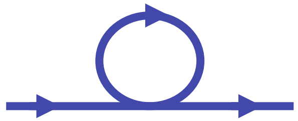


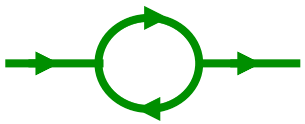
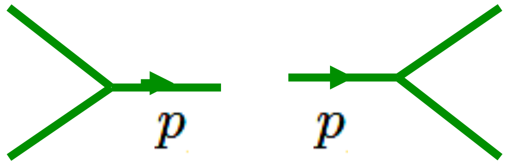
J. Berges (2002)



Non-Abelian Gauge Theory

- **No classical field $\langle A \rangle = 0$**
- **Leading Order Self Energy of coupling**

LO (local)  $O(g^2)$

LO (nonlocal)  ⇔ 

$O(g^2)$

Scalar Theory as a toy model

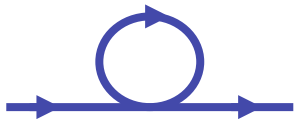
Application for BEC, Cosmology (or reheating) and DCC dynamics?

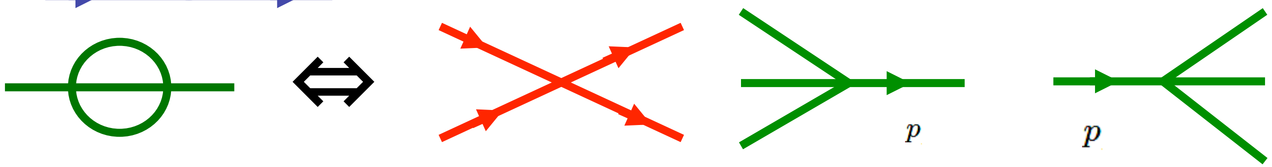
- **ϕ^4 theory with no condensate $<\phi>=0$**

$$\mathcal{L}_{\text{int}} = -\frac{1}{4!}\lambda\hat{\phi}^4$$

- **Next Leading Order Self-Energy of coupling**

Full Green's function $G=1/[G_0^{-1}-\Sigma(G)]$

LO $\Sigma=$  $O(\lambda)$

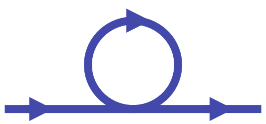
NLO 

$O(\lambda^2)$

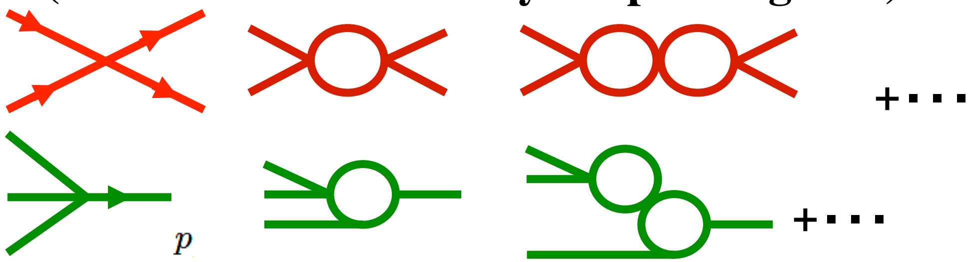
- **O(N) theory with no condensate**

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!N}(\hat{\phi}_a\hat{\phi}_a)^2$$

- **Next Leading Order Self-Energy in 1/N expansion** (not restricted in weakly coupled regimes)

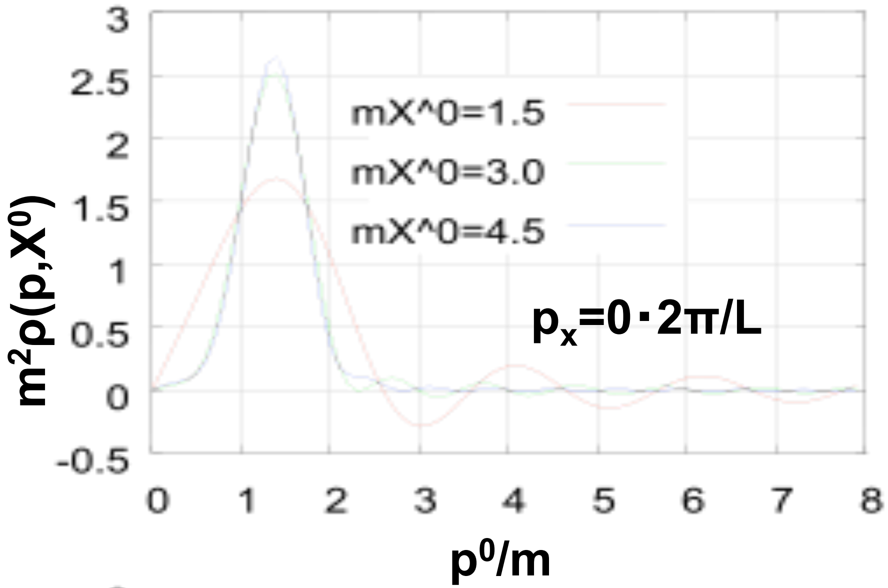
$\Sigma=$ 

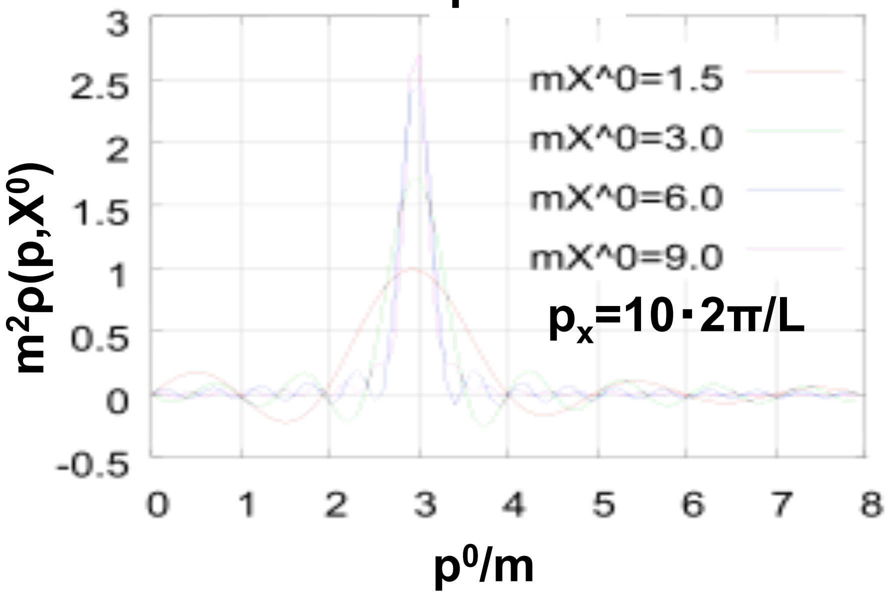
O(1)

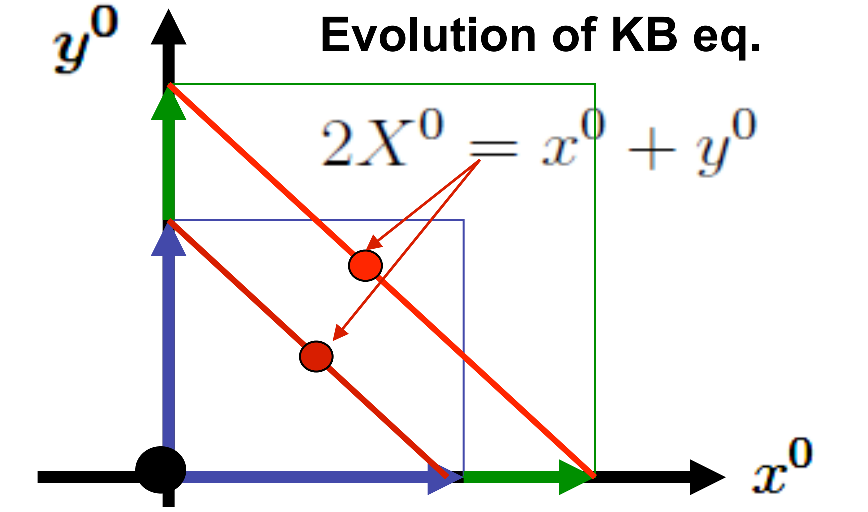


$O(1/N)$

To plot our kinetic entropy, Fourier transformed Green's function with x^0-y^0 is necessary.





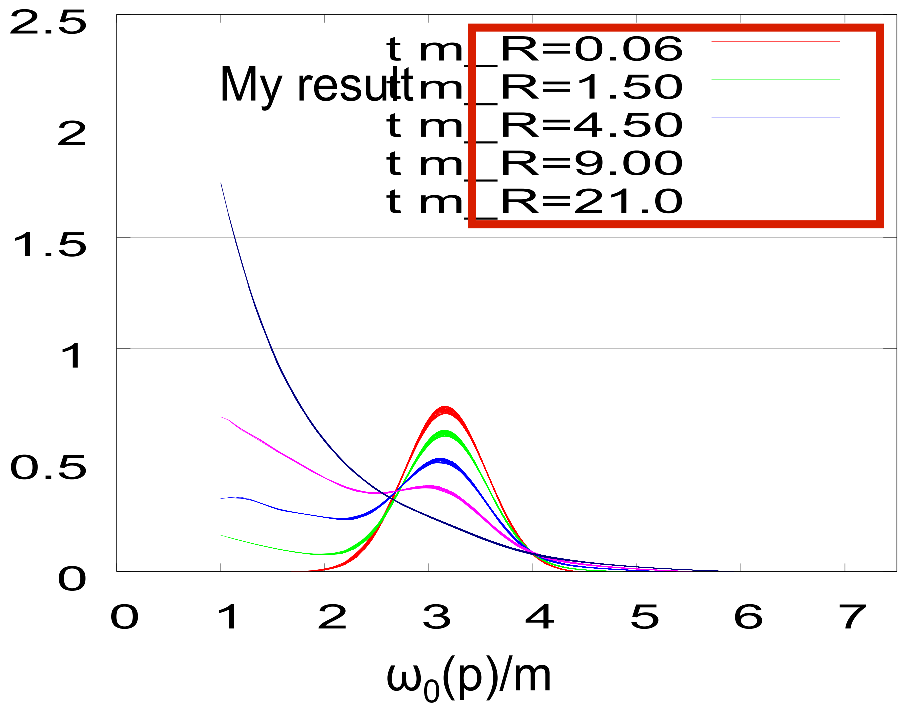


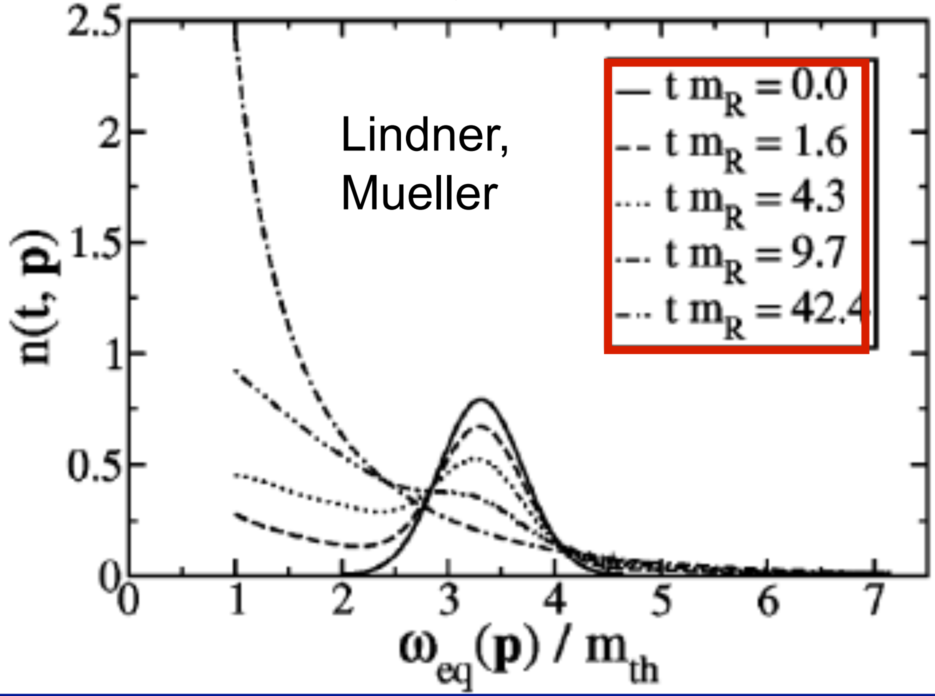
$$-2X^0 \le x^0 - y^0 = z^0 \le 2X^0$$

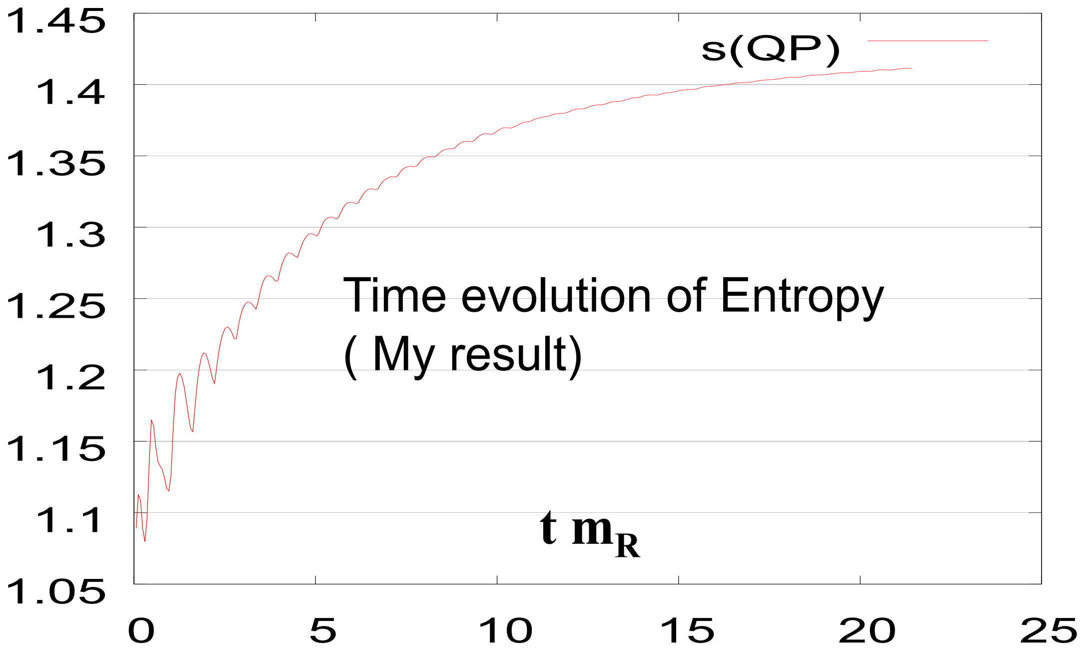
The interval x^0-y^0=z^0 is finite for Fourier transformation. Then we can not resolve narrower peak of spectral function than ~1/X^0. This is the origin of the oscillation around the peak.

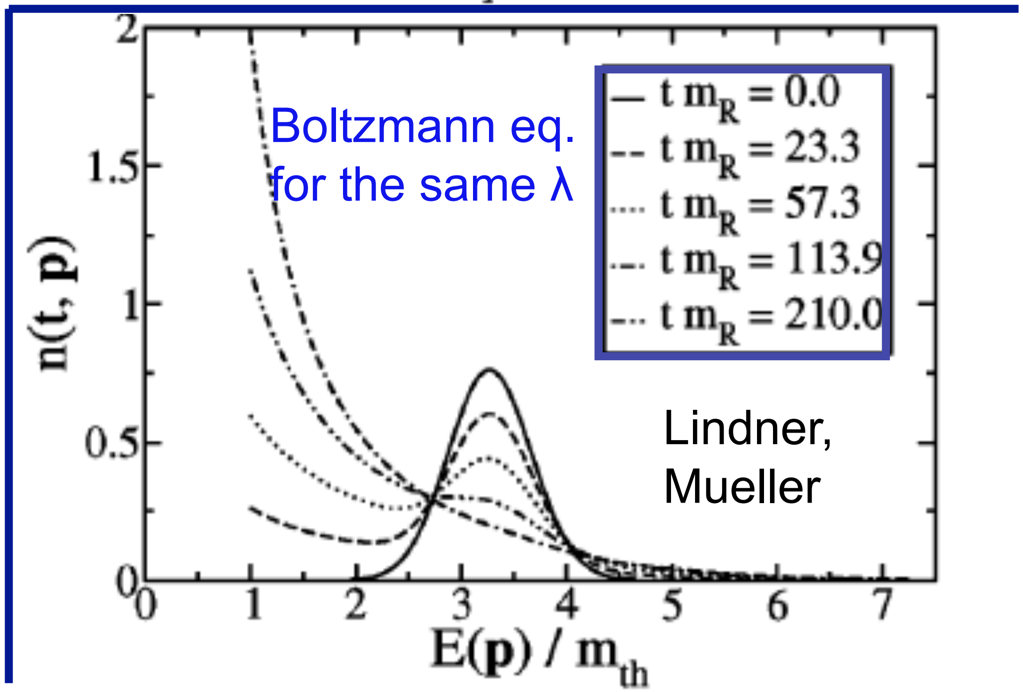
Numerical artifact at very early time X^0~1/m.

Time evolution of distribution function with Kadanoff-Baym eq.



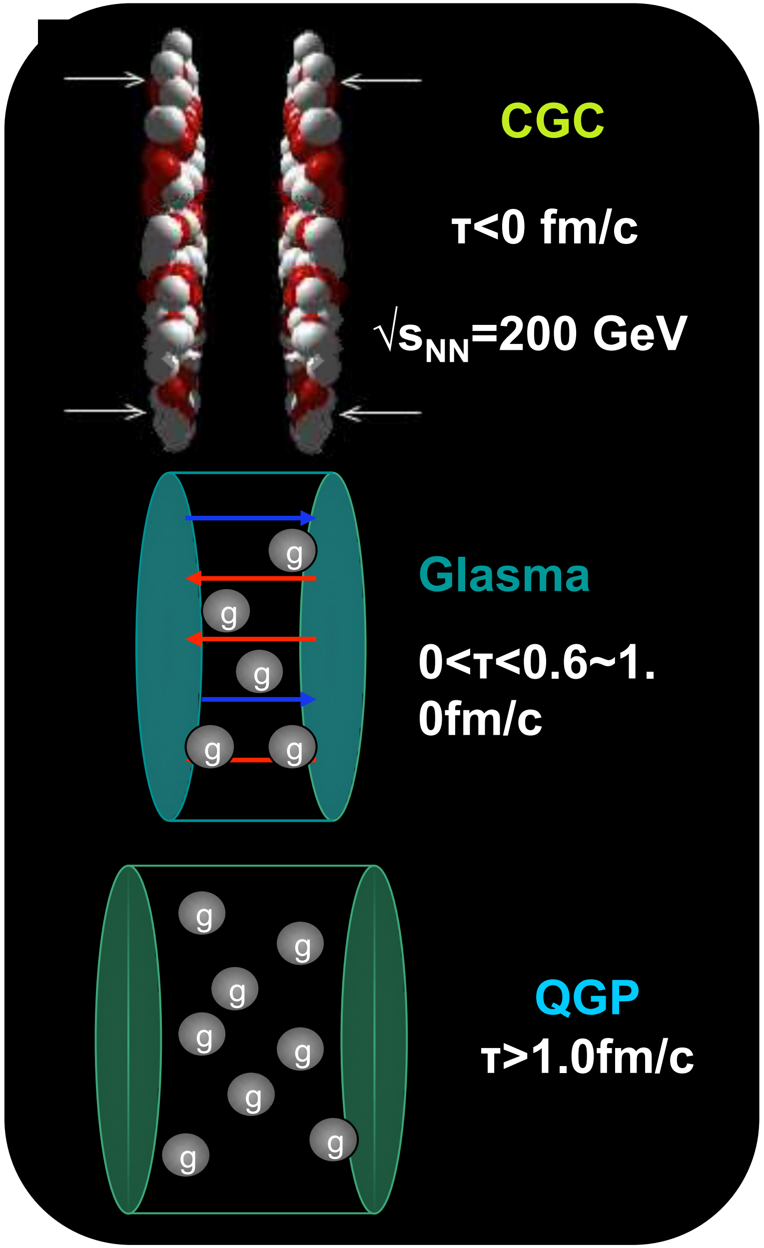






Relativistic Heavy Ion Collision

Brookhaven National Laboratory



Background 1

Creation of Quark-Gluon Plasma (QGP)

Success of ideal hydrodynamics after thermalization.

Assumption: Early Thermalization of gluons (1fm/c)!

It is necessary to understand thermalization processes for dense gluonic system.

Classical Boltzmann eq. should not be applied.

Background 2

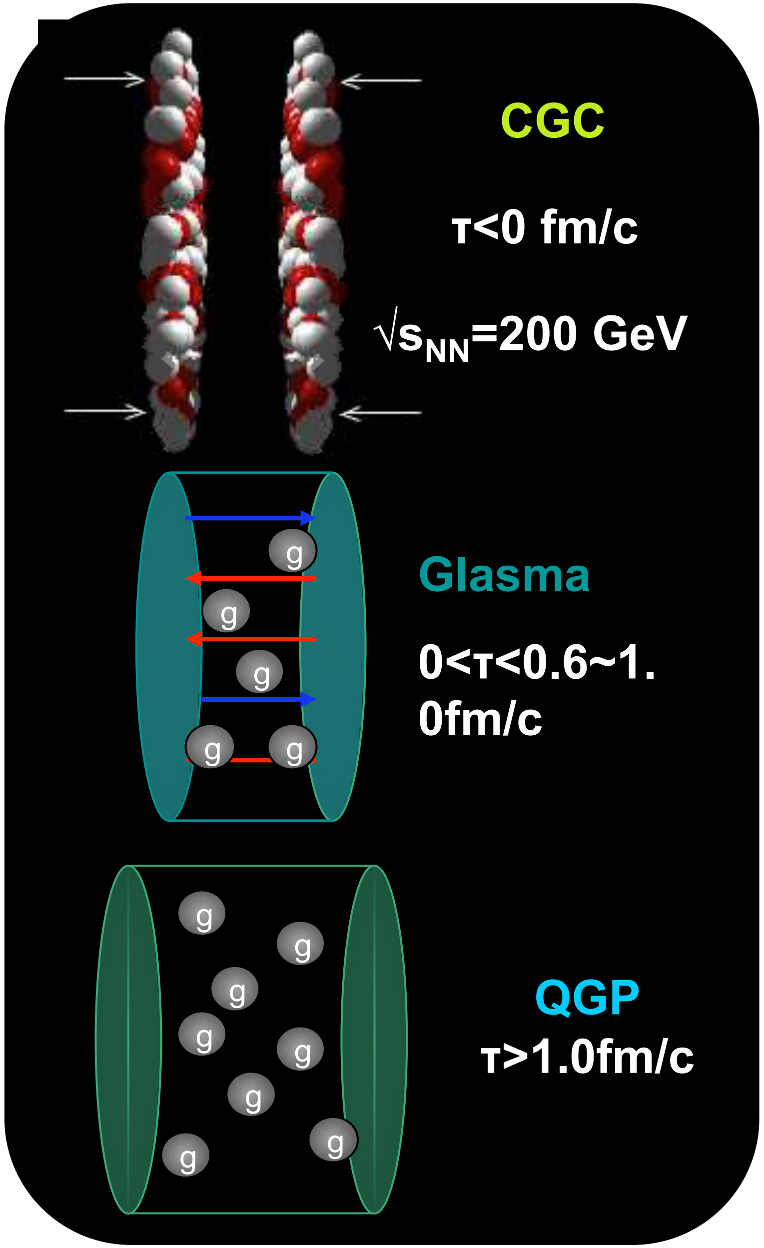
Nonequilibrium phenomena for dense system at the early Universe (Cosmology)

Bose-Einstein Condensation near critical point (Condensed matter physics)

Quantum nonequilibrium processes based on <u>field theory</u>

Relativistic Heavy Ion Collision

Brookhaven National Laboratory



Figures from P. SORENSEN

Background

Formation of Quark-Gluon Plasma (QGP)

Success of ideal hydrodynamics after thermalization.

Assumption: Early Thermalization of gluons (1fm/c)!

It is necessary to understand thermalization processes for dense gluonic system.

Semi-Classical Boltzmann eq. should not be applied.

Quantum nonequilibrium processes

based on field theory

Application of Kadanoff-Baym eq. to early thermalization of gluons.