Lattice Coulomb-gauge study of the Gribov-Zwanziger scenario in QGP

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— Outline —

- Introduction

  quark-gluon plasma, aim of this study

- Coulomb gauge QCD

  Instantaneous interaction, transverse gluons

- Gribov-Zwanziger scenario

  Standard gauge fixing, gribov ambiguity, non-perturbative gauge fixing,

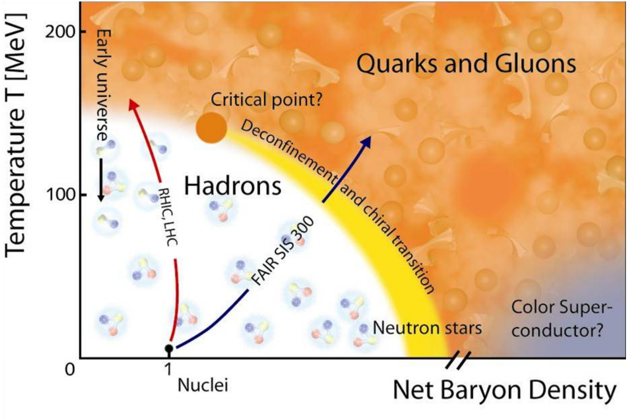
  lattice results

- Remnant of confinement in the quark-gluon plasma phase

  GZ scenario at finite temperatures, lattice results

- Conclusions and outlook

Introduction — quark-gluon plasma

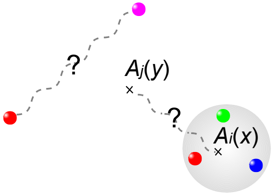


- Elliptic flow, jet quenching, early thermalization
- Success of hydrodynamic model $\Rightarrow$ Not a free gas, but a perfect fluid
- Strongly coupled/interacting quark-gluon plasma (sQGP)

Aim of this study

- Why does the QGP behave as a perfect fluid?
  (What are the properties of the QGP?)
  ⇒ Unravel the infrared dynamics of QCD, especially the infrared (long-range) behavior of the gluons

- Origin of the confining force and the long-range behavior of the gluons in the hadronic phase
  ⇒ Do they show different behavior in the deconfinement phase?



- Fix to Coulomb gauge. Why?
  - Simple confinement scenario is available.
  - The heat-bath provides a preferred Lorentz frame.

Coulomb gauge QCD

- In the Hamiltonian formalism

$$Z = \int \mathscr{D}A_i^{\rm tr}\mathscr{D}E_i^{\rm tr} \exp\left[-\int d^4x\left\{-E_i^{\rm tr}\partial_4 A_i^{\rm tr} + \mathscr{H} + ig_0 J_i A_i^{\rm tr}\right\}\right]$$

- Coulomb gauge Hamiltonian

$$H = \frac{1}{2}\int d^3x\left\{(E_i^{\rm tr})^2 + B_i^2\right\} + \frac{1}{2}\int d^3y d^3z \rho^a(\vec{y},t)\mathcal{V}^{ab}(\vec{y},\vec{z};A^{\rm tr})\rho^b(\vec{z},t)$$

- Physical gauge; elimination of unphysical d.o.f. (unitarity manifest)
- $A_i^{\rm tr}$ describes 'would-be physical' gluons
- Two complementary aspects of QCD (dual role of gluons)
  - There is a long range interaction which confines quarks.

  $\Rightarrow$ Long range correlation?

  - Gluons are absent from the physical spectrum.

  $\Rightarrow$ Short range correlation?

Coulomb gauge QCD

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  - There is a long range interaction which confines quarks.

  $\Rightarrow$ Long range correlation? $\Rightarrow$ instantaneous interaction $\rho\mathcal{V}\rho$

  - Gluons are absent from the physical spectrum.

  $\Rightarrow$ Short range correlation? $\Rightarrow$ A^{tr} has a short range correlation

Non-perturbative gauge fixing (Gribov-Zwanziger)

- The standard GF is valid perturbatively, but not non-perturbatively.
- Gauge fixing à la Gribov; $\int \mathscr{D}A_\mu \to \int_{-\partial_i D_i[A]>0} \mathscr{D}A_\mu \delta(\partial_i A_i)\mathrm{det}(-\partial_i D_i)$
- $M[A] > 0 \Rightarrow$ Add the horizon term γS_h to the YM action with the horizon condition $\Rightarrow$ Local form by introducing auxiliary ghost fields
- Transverse gluon propagator at the tree-level,

$$D^{\mathrm{tr}}(\vec{p}, p_4) = \frac{1}{p_4^2 + E(\vec{p})^2}, \qquad E^2(\vec{p}) = \vec{p}^2 + \frac{m^4}{\vec{p}^2}$$

$\Rightarrow$ Infrared vanishing gluon propagator (confinement of gluons)

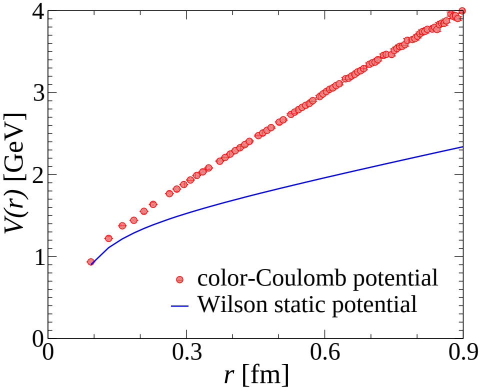
- Color-Coulomb potential (instantaneous interaction)

$$V_c(r) \sim \frac{\Lambda_{\mathrm{QCD}}^2 r}{\ln(c' \Lambda_{\mathrm{QCD}} r)}$$

asymptotically at large r (confinement of quarks).

V.N.Gribov 1978, D.Zwanziger 1993, 2006

Lattice study of the color-Coulomb potential



- Linearly rising potential at large quark separations
- 2-3 times larger string tension than that of the static potential
- Zwanziger's inequality (Zwanziger 2003)

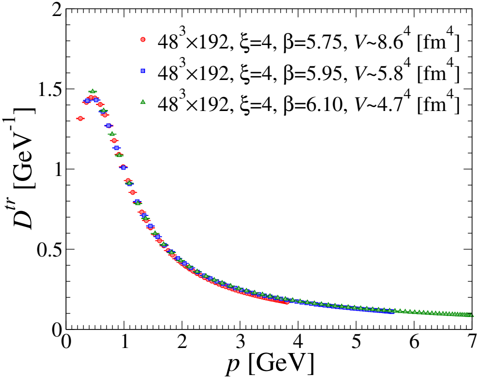
$$V_c(R) \geq V_W(R)$$

- Necessary condition for color confinement

  (cf. U(1) group, SU(2) with fund. Higgs in the Higgs (broken) phase)

J.Greensite and S.Olejnik 2003, A.Nakamura and T.Saito 2006

Equal-time transverse gluon propagator in p-space



- Equal-time propagator

$$\left\langle A_i^a(\vec{x},t)A_j^b(\vec{y},t)e^{-i\vec{k}\cdot(\vec{x}-\vec{y})}\right\rangle = \delta^{ab}\left(\delta^{ij}-\frac{p_ip_j}{|\vec{p}|^2}\right)D^{\mathrm{tr}}(|\vec{p}|)$$

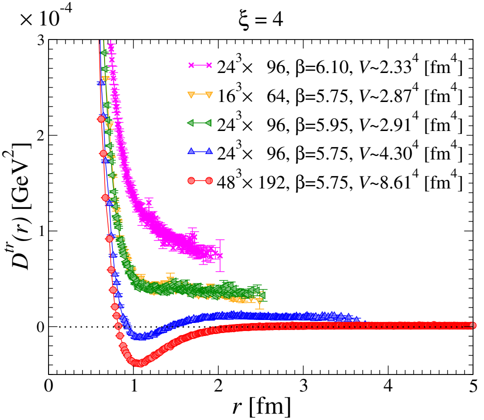
- IR suppressed and shows a turnover at about 500 [MeV]

- Corresponds to the inverse of the dispersion relation;

$$D(\vec{p}) \equiv \int \frac{dp_4}{2\pi} D(\vec{p},p_4) = \int \frac{dp_4}{2\pi}\frac{1}{p_4^2+E(|\vec{p}|)} = \frac{1}{2E(|\vec{p}|)}$$

Y.Nakagawa, A.Nakamura, T.Saito, H.Toki, in preparation

Equal-time transverse gluon propagator in x-space



- Correlation function

$$\left\langle A_i^a(\vec{x},t) A_j^b(\vec{y},t) \right\rangle = \delta^{ab} \left(\delta^{ij} - \frac{\partial_i \partial_j}{\partial_i^2} \right) D^{\mathrm{tr}}(|\vec{x} - \vec{y}|)$$

- Rapidly decreases with distance, develops a negative dip, and vanishes at large distances.

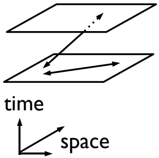
- No correlation between gluon fields beyond the hadronic scale, indicating the confinement of gluons.

Y.Nakagawa, A.Nakamura, T.Saito, H.Toki, in preparation

Gribov-Zwanziger scenario at finite temperature

- Introducing temperature in field theory
  $=$ Compactify the Euclidean time direction with the period $\beta = 1/T$.

- The Coulomb gauge condition $\partial_i A_i(\vec{x}, \tau) = 0$
  holds at each time.



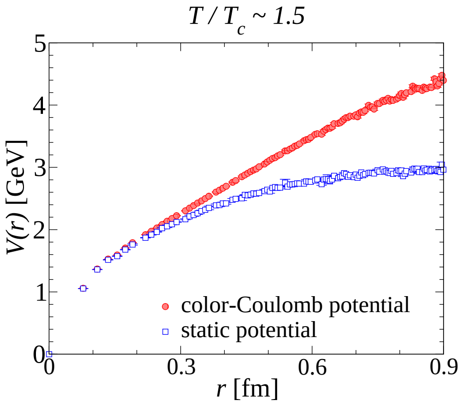
- Reduction of configuration space independently of β:

$$\int \mathscr{D}A_\mu = \int \prod_{\vec{x},\tau} A_\mu(\vec{x}, \tau) \to \int_{-\partial_i D_i[A]>0} \prod_{\vec{x},\tau} A_\mu(\vec{x}, \tau)\delta(\partial_i A_i)\mathrm{det}(-\partial_i D_i)$$

- Gap equation in one-loop approx. leads $m(T) \sim g^2(T)T$ as $T \to \infty$,

$$D^{\mathrm{tr}}(\vec{p}, \omega_n) = \frac{1}{\omega_n^2 + \vec{p}^2 + \frac{m^4(T)}{\vec{p}^2}}, \quad V_c(\vec{p}) \sim \frac{m^2(T)}{|\vec{p}|^4 \ln\left(c\frac{|\vec{p}|}{m(T)}\right)}$$

Color-Coulomb potential in deconfinement phase

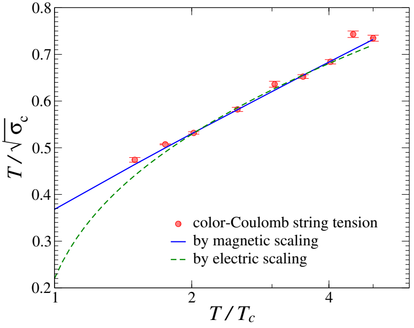


- confining potential even in the deconfinement phase!
- The color-Coulomb string tension does not serve as an order parameter for the deconfinement phase transition.

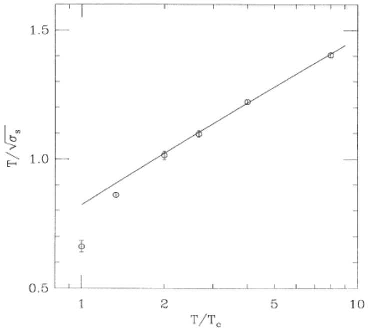
- The deconfinement phase transition is not necessarily accompanied by the disappearance of the origin of confinement.

J.Greensite, S.Olejnik, and D.Zwanziger 2004
Nakamura and Saito 2006
Y.Nakagawa, A.Nakamura, T.Saito, H.Toki, and D.Zwanziger 2006

Magnetic scaling



from: Y.Nakagawa, A.Nakamura, T.Saito, H.Toki, and D.Zwanziger, PRD73, 094504 (2006)

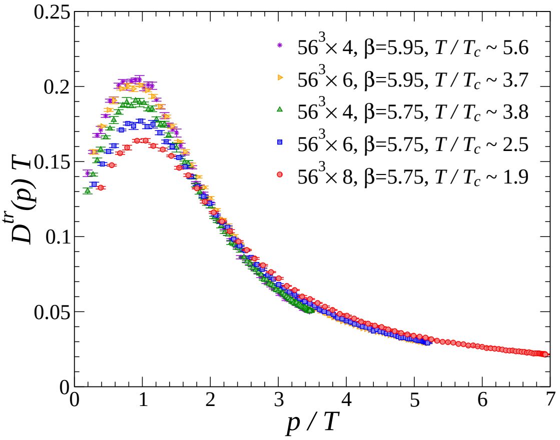


from: G.S.Bali, J.Fingberg, Urs M.Heller, F.Karsch, F.Schilling, PRL71 3059 (1993)

- The temperature dependence of the color-Coulomb string tension is comparable with the magnetic scaling $\sqrt{\sigma_c} \sim g^2(T)T$, where

$$\frac{1}{g^2(T)} = 2b_0 \ln\left(\frac{T}{\Lambda}\right) + \frac{b_1}{b_0} \ln\left[2\ln\left(\frac{T}{\Lambda}\Lambda\right)\right]$$

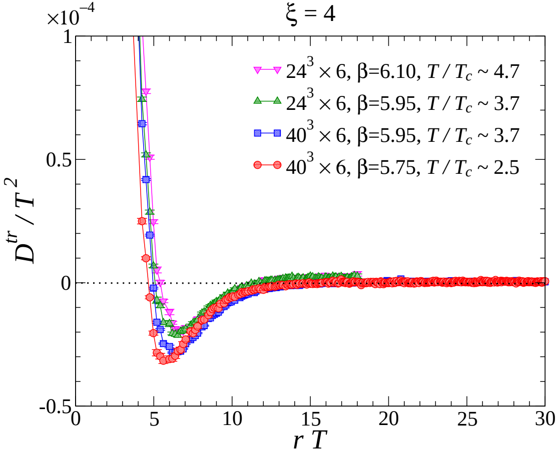
D^{tr} in p-space in the deconfinement phase



Gribov-Zwanziger scenario works in the deconfinement phase;
the infrared suppression and the turnover even above T_c.

Y.Nakagawa, A.Nakamura, T.Saito, H.Toki, in preparation

D^{tr} in x-space in the deconfinement phase



Not a simple power-law decay;

rapid decrease, negative dip, vanishing at large rT

as in the confinement phase.

Y.Nakagawa, A.Nakamura, T.Saito, H.Toki, in preparation

— Conclusions —

Gribov-Zwanziger scenario in Coulomb gauge QCD

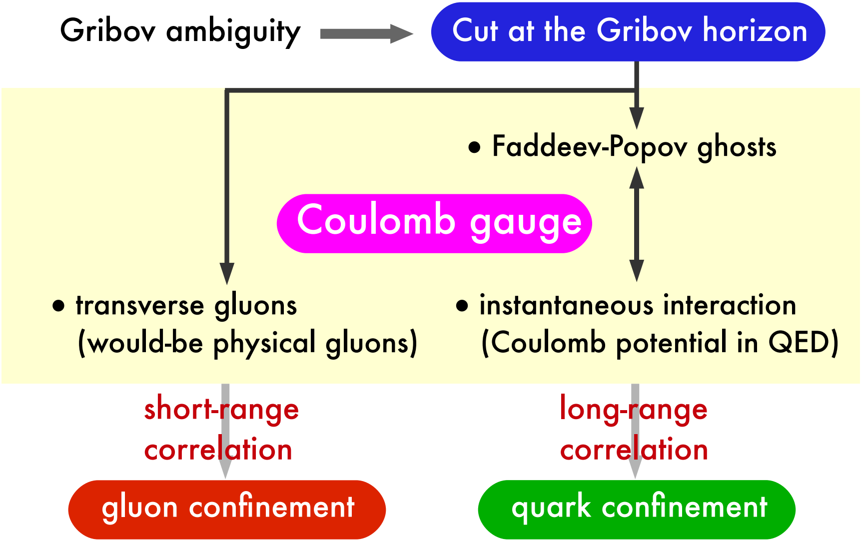
- Cut at the Gribov horizon seriously alter the IR dynamics of QCD
- Coexistence of the complementary aspects of QCD
  - Confining color instantaneous interaction
  - IR suppression of the transverse gluon propagator

    (No correlation between gauge fields beyond the hadronic scale)
- Gribov-Zwanziger scenario works even in the deconfinement phase:
  - Linear rising behavior of the color-Coulomb potential,

    and its string tension shows the *magnetic scaling*.
  - Confinement of the magnetic gluons persists above T_c;

    the magnetic gluons are *permanently* confined.

Confinement scenario in Coulomb gauge

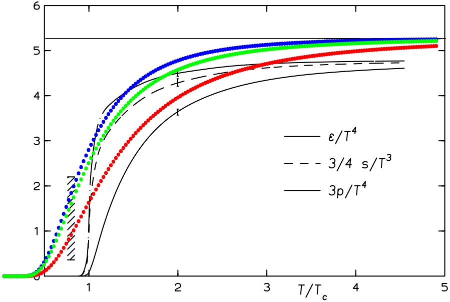


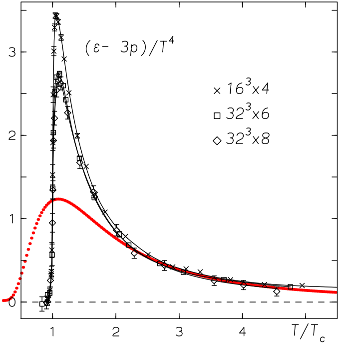
FMR (fundamental modular region) gas

- The gas of noninteracting quasiparticles with modified dispersion relation

$$E(p) = \sqrt{p^2 + \frac{m^4}{p^2}}$$

(m=705 [MeV] by fitting the trace anomaly at high T)





from: D. Zwanziger, PRL94:182301 (2005)

Lattice setup

- Update: Wilson plaquette action

$$S = \frac{\beta}{\xi_B} \sum_{n,i<j\leq 3} \mathfrak{Re}\,\mathrm{Tr}(1 - U_{ij}(n)) + \beta\xi_B \sum_{n,i\leq 3} \mathfrak{Re}\,\mathrm{Tr}(1 - U_{i4}(n)).$$

- The renormalized anisotropy $\xi = a_s/a_\tau$ differs from ξ_B
  $\Rightarrow$ use the relation obtained by Klassen
  (T.R. Klassen, NPB533:557, 1998)
- Adopt the values of lattice spacing given in
  Y. Namekawa et al., PRD64:074507 (2001) for $\xi = 2$
  H. Matsufuru, T. Onogi and T. Umeda, PRD64:114503 (2001) for $\xi = 4$
- Gauge fixing: Wilson-Mandula method with Fourier acceleration
  $\Rightarrow$ stop the iteration when $(\partial_i A_i)^2 < 10^{-14}$

Standard gauge fixing (Faddeev-Popov)

- Gauge degrees of freedom = too many degrees of freedom
  $\Rightarrow$ Integrate over an infinite gauge volume

- Gauge fixing needed
  $\Rightarrow$ Pick up the representatives from each gauge orbit
  (gauge orbit = a set of physically equivalent configurations)

- Gauge fixing à la Faddeev and Popov;

$$\int \mathscr{D}A_\mu \to \int \mathscr{D}A_\mu \delta(\partial_i A_i)\mathrm{det}(-\partial_i D_i)$$

- Faddeev-Popov ghosts

$$S = S_{\mathsf{YM}} + S_{\mathsf{FP}}, \quad S_{\mathrm{FP}} = \int d^4x(ib\partial_i A_i - \bar{c}Mc)$$

Gribov ambiguity

- The gauge transformation

$$ {}^{g}A_i(\vec{x},t) = g(\vec{x}) \left(A_i(\vec{x},t) + \frac{1}{ig_0}\partial_i \right) g^{-1}(\vec{x}) $$

- There are Gribov copies satisfying the Coulomb gauge condition if

$$ D_i \left(g^{-1}(\vec{x})\partial_i g(\vec{x}) \right) = 0 $$

- Under the infinitesimal gauge transformation $g(\vec{x}) = 1 + i\theta(\vec{x})$,

$$ M\theta(\vec{x}) = 0 $$

$\Rightarrow$ Gribov copies exist if the FP operator has the zero modes.

- $M > 0$ for small gA, but develops a zero eigenvalue for large gA.

$\Rightarrow$ FP prescription is valid perturbatively, but not non-perturbatively.

V.N.Gribov 1978, I.M.Singer, 1978

Non-perturbative gauge fixing (Gribov-Zwanziger)

- Gauge fixing à la Gribov;

$$\int \mathscr{D}A_\mu \to \int_{M[A]>0} \mathscr{D}A_\mu \delta(\partial_i A_i)\mathrm{det}(M)$$

- Gribov-Zwanziger action

$$Z = \int \mathscr{D}A_\mu \delta(\partial_i A_i)\mathrm{det}(M)\exp(-S_{\mathrm{YM}}-\gamma S_h)$$

$$S_h = \int d^3x\, g^2 f^{abc} A_i^b (M^{-1})^{ad} f^{dec} A_i^e - 3(N_c^2 - 1)$$

with the horizon condition (gap equation)

$$\frac{\partial \ln Z}{\partial \gamma} = -\langle S_h \rangle = 0 \quad \text{(minimize the free energy)}$$

D.Zwanziger 1993

Non-perturbative gauge fixing (Gribov-Zwanziger)

- Gauge fixing à la Gribov;

$$\int \mathscr{D}A_\mu \to \int_{M[A]>0} \mathscr{D}A_\mu \delta(\partial_i A_i)\mathrm{det}(M)$$

- GZ action in a local form by introducing the auxiliary ghosts

$$\begin{aligned} S &= S_{\mathrm{YM}} \\ &+ \int d^4x (ib^a \partial_i A_i^a - \bar{c}^a (Mc)^a) \\ &+ \int d^4x \left(\bar{\phi}_\mu^{ab} (M\phi_\mu)^{ab} - \bar{\omega}_\mu^{ab} (M\omega_\mu)^{ab} - (\partial_i \bar{\omega}_\mu)^{ab} (gD_i c \times \phi_\mu)^{ab} \right) \\ &+ \gamma^2 \int d^4x \left(D_i^{ab} \phi_i^{ba} - D_i^{ab} \bar{\phi}_i^{ba} + (gD_i c \times \bar{\omega}_i)^{aa} - 3\gamma^2 (N_c^2 - 1) \right) \end{aligned}$$

D.Zwanziger 1993

Presence of Gribov copies

- Decompose the FP operator as

$$-\partial_i D_i^{ab} = -\partial^2 \delta^{ab} - g f^{abc} A_i^{\mathrm{tr}\ c} \partial_i = M_0^{ab} + M_1^{ab}(gA)$$

- M_1 is traceless since f^{abc} is antisymmetric,

$$\mathrm{Tr}\, M_1(gA) = \sum_{a=1}^{N_c^2-1} M_1^{aa}(gA) = 0 = \sum (\text{eigenvalue})$$

$\Rightarrow M_1$ has both positive eigenvalues and negative eigenvalues.

- Let one of negative eigenvalues of M_1 be λ and the corresponding eigenfunction ϕ,

$$(\phi, M_1(gA)\phi) = \lambda(gA) < 0$$

- Let μ be a scaled factor, then it follows that

$$(\phi, M(\mu gA)\phi) = (\phi, -\partial_i^2 \phi) + \mu(\phi, M_1(gA)\phi) = (\partial_i \phi, \partial_i \phi) + \mu\lambda(gA)$$

- The first term is strictly positive, but the second term is negative.

Color-Coulomb potential

- The instantaneous interaction energy between color charges

$$V_c(\vec{x}-\vec{y}) = g^2 \vec{T}^a \cdot \vec{T}^b \langle \mathcal{V}^{ab}(\vec{x},\vec{y};A^{\text{tr}}) \rangle$$
$$= g^2 \vec{T}^a \cdot \vec{T}^b \left\langle \int d^3z (M^{-1}[A])^{ac}_{\vec{x},\vec{z}} (-\nabla^2_{\vec{z}}) (M^{-1}[A])^{cb}_{\vec{z},\vec{y}} \right\rangle .$$

- In QED (abelian gauge theory)

$$V(\vec{x}-\vec{y}) = \frac{e^2}{4\pi|\vec{x}-\vec{y}|} \quad \text{(Coulomb potential)}$$

- Time-time gluon propagator

$$D_{44}(x-y) = V_c(\vec{x}-\vec{y})\delta(x_4-y_4) + P(x-y)$$

$\Rightarrow$ Time-like gluons mediate the instantaneous interaction.

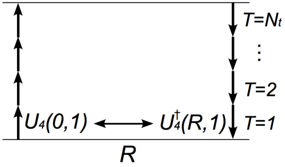
A. Cucchieri and D.Zwanziger, Phys.Rev.D65:014002 (2001)

Lattice study of the Color-Coulomb potential

- Partial-length Polyakov line (PPL)

$$L(\vec{x}, T) = \prod_{t=1}^{T} U_4(\vec{x}, t)$$

$\Rightarrow L(\vec{x}, N_t)$ is the untraced Polyakov loop.



- Define the correlator of PPL

$$G(R, T) = \frac{1}{3}\langle \mathrm{Tr}\left[L(R, T)L^{\dagger}(0, T)\right]\rangle, \quad R = |\vec{x} - \vec{y}|$$

$$V(R, T) = \log \frac{G(R, T)}{G(R, T+1)}$$

- $V(R, 0) = -\log G(R, 1)$ corresponds to the color-Coulomb potential.

No confinement without color-Coulomb confinement

- Consider a trial state obtained by adding $Q\bar{Q}$-singlet pair at separation R to the vacuum,

$$|\Phi_{Q\bar{Q}}\rangle = |Q\bar{Q}\text{-singlet}\rangle \otimes |\text{vac}\rangle$$

(no vacuum polarization by the sources)

- Note that $|\Phi_{Q\bar{Q}}\rangle$ is not an eigenstate of the Hamiltonian!
- Energy expectation value of the state $|\Phi_{Q\bar{Q}}\rangle$

$$E_{Q\bar{Q}} = \langle \Phi_{Q\bar{Q}}|H|\Phi_{Q\bar{Q}}\rangle - \langle \text{vac}|H|\text{vac}\rangle = V_c(R) + E_{\text{self}}$$

- R-dependent part of $E_{Q\bar{Q}}$ is precisely the color-Coulomb potential.

No confinement without color-Coulomb confinement

- The ground state of the $Q\bar{Q}$-singlet system, $|\Phi^0_{Q\bar{Q}}\rangle$, is different from $|\Phi_{Q\bar{Q}}\rangle = |Q\bar{Q}\text{-singlet}\rangle \otimes |\text{vac}\rangle$ since the flux tube is formed.

$$|\Phi^0_{Q\bar{Q}} >= \lim_{t\to\infty} \frac{\exp(-tH)|\Phi_{Q\bar{Q}} >}{< \Phi_{Q\bar{Q}}|\exp(-2tH)|\Phi_{Q\bar{Q}} >}$$

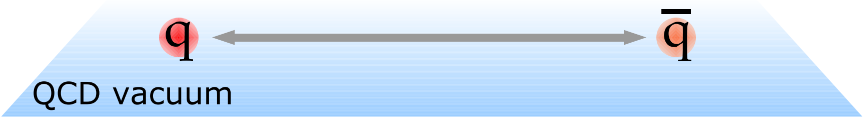
$$E^0_{Q\bar{Q}} = \langle\Phi^0_{Q\bar{Q}}|H|\Phi^0_{Q\bar{Q}}\rangle - \langle\text{vac}|H|\text{vac}\rangle = V(R) + E^0_{\text{self}}$$

- $V(R)$ is the static potential obtained from the Wilson loop

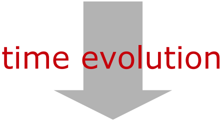
$$V(R) = -\lim_{T\to\infty} \frac{d}{dT} \log\langle W(R,T)\rangle$$

- Since $E^0_{Q\bar{Q}} \leq E_{Q\bar{Q}}$, it follows that $V_c(R) \geq V(R)$.
- If $V_c(R)$ is non-confining, $V(R)$ is also non-confining. (Zwanziger 2003)

No confinement without color-Coulomb confinement



$$|\Phi_{Q\bar{Q}}\rangle = |Q\bar{Q}\text{-singlet}\rangle \otimes |\text{vac}\rangle$$



$$|\Phi^0_{Q\bar{Q}} > \sim \lim_{t\to\infty} \exp(-tH)|\Phi_{Q\bar{Q}} >$$

Spectral sum for the color-Coulomb potential

- Color-Coulomb potential

$$V_c(\vec{x}-\vec{y})\delta^{ab} = g^2\,\vec{T}^a\cdot\vec{T}^b\left\langle [M^{-1}(-\nabla^2)M^{-1}]^{ab}_{\vec{x},\vec{y}}\right\rangle$$

- The Green's function M^{-1} of the F-P operator $M = -\partial_i D_i$ is given by

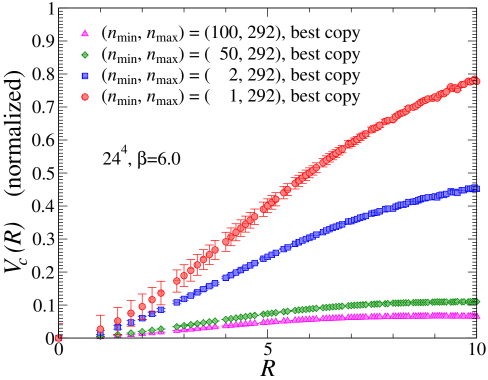
$$(M^{-1}[A])^{ab}(\vec{x},\vec{y}) = \sum_n \frac{\phi_n^{*a}(\vec{x})\phi_n^b(\vec{y})}{\lambda_n}$$

- Spectral sum for the color-Coulomb potential

$$V_c(\vec{x}-\vec{y}) = g^2\frac{-C_f}{N_c^2-1}\left\langle \sum_{n,m}^{n_{\max}} \phi_n^{*a}(\vec{x})\phi_m^a(\vec{y})\frac{\int d^3z\,\phi_n^c(\vec{z})(-\vec{\nabla}^2)\phi_m^{*c}(\vec{z})}{\lambda_n\lambda_m}\right\rangle$$

- Evaluate low-lying 300 λ_n and ϕ_n by the Lanzos method and reconstruct the color-Coulomb potentital

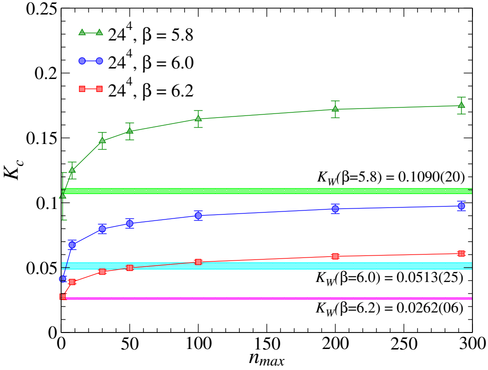
Spectral sum for the color-Coulomb potential



- # of F-P eigenvalues $24^3 \times 8 \sim 110000$
- Dropping the lowest eigenmode significantly reduces the slope of the color-Coulomb potential.
- The color-Coulomb potential becomes flat at large R by removing the low-lying 50 eigenmodes.

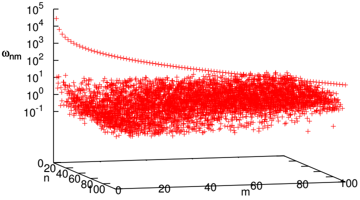
YN, A. Nakamura, T. Saito, H. Toki, PRD81:054509 (2010)

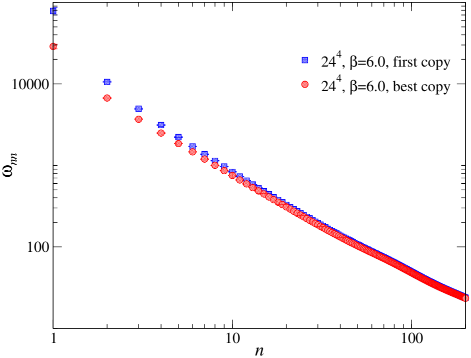
Saturation of the color-Coulomb string tension



- The contribution from the lowest mode is comparable to the Wilson string tension.
- The lowest mode accounts for the large portion of K_c.
- The further inclusion of the higher eigenmodes does not alter V_c.

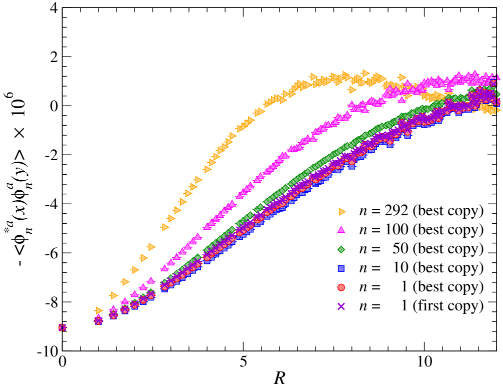
Weight factor ω_{nm}





$$V_c(\vec{x}-\vec{y}) = g^2 \frac{-C_f}{N_c^2-1} \left\langle \sum_{n,m=n_{\min}}^{n_{\max}} \phi_n^{*a}(\vec{x})\phi_m^a(\vec{y}) \underbrace{\frac{\int d^3z \phi_n^c(\vec{z})(-\vec{\nabla}^2)\phi_m^{*c}(\vec{z})}{\lambda_n \lambda_m}}_{\omega_{nm}} \right\rangle .$$

Correlator of the F-P eigenfunctions



- Linearly rising behavior for the lowest eigenmode
- The distance dependence of the correlation function does not so much differ for $n \leq 50$.
- For $n = 292$, the correlation function becomes non-confining at large R.

$$V_c(\vec{x}-\vec{y}) = g^2 \frac{-C_f}{N_c^2-1} \left\langle \sum_{n,m=n_{\min}}^{n_{\max}} \phi_n^{*a}(\vec{x})\phi_m^a(\vec{y}) \frac{\int d^3z\, \phi_n^c(\vec{z})(-\vec{\nabla}^2)\phi_m^{*c}(\vec{z})}{\lambda_n \lambda_m} \right\rangle .$$

Lattice study of the equal-time transverse gluon propagator

- Consider the equal-time transverse gluon propagator

$$D_{ij}^{ab}(\vec{p}) = \left\langle A_i^a(\vec{x},t) A_j^b(\vec{y},t) e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \right\rangle = \delta^{ab} \left(\delta^{ij} - \frac{p_i p_j}{|\vec{p}|^2} \right) D^{\text{tr}}(|\vec{p}|)$$

- Linear definition of the lattice gauge field

$$A_\mu(x+\hat{\mu}/2) = \frac{1}{2i} \left[(U_\mu(x) - U_\mu^\dagger(x)) - \frac{\mathbf{1}}{N_c} \text{Tr}(U_\mu(x) - U_\mu^\dagger(x)) \right].$$

- Renormalization

$$aD^{tr}(a^2\vec{p}^2) = Z_{tr}(\mu^2, a^a) D_R^{tr}(\vec{p}^2; \mu^2), \quad D_R^{tr}(\vec{p}^2 = \mu^2) = \frac{1}{\mu}$$