強く相互作用するフェルミ粒子系の熱力学

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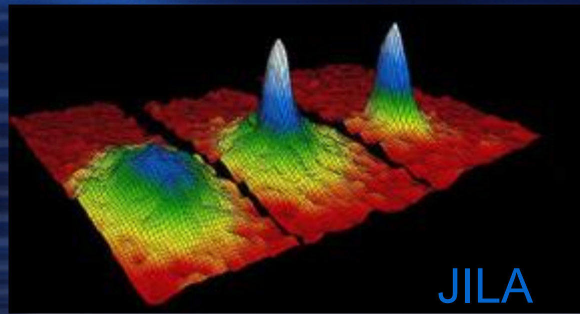
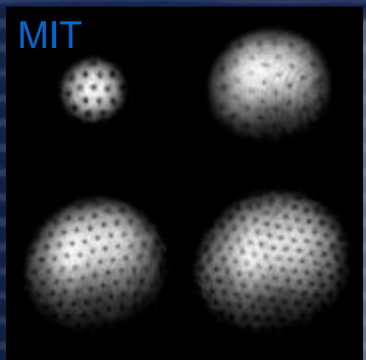
JST-ERATOマクロ量子制御プロジェクト

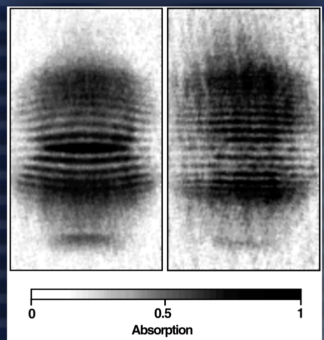
BEC in a cold atom system

1995 Realization of atomic gas Bose-Einstein condensation



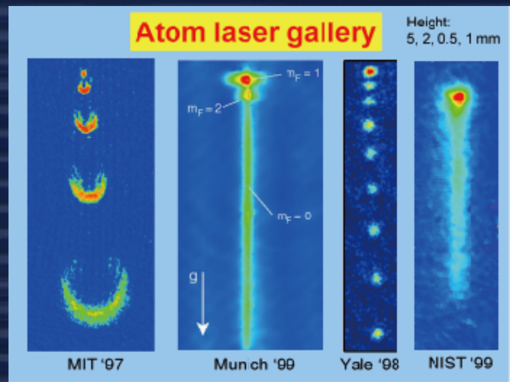


vortices

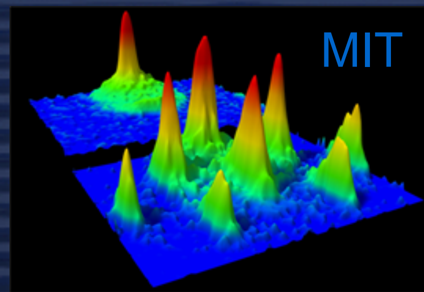


Science 275, 637 (1997).

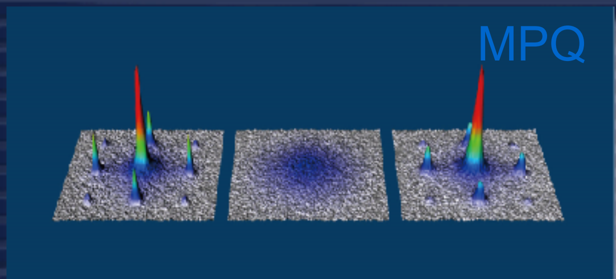
interference



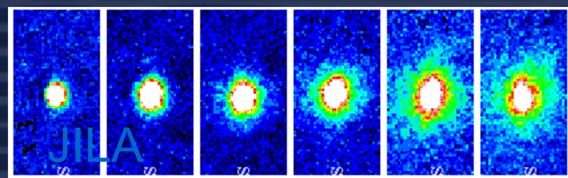
Atom laser



Super-radiance



Mott-insulator phase



Bose nova

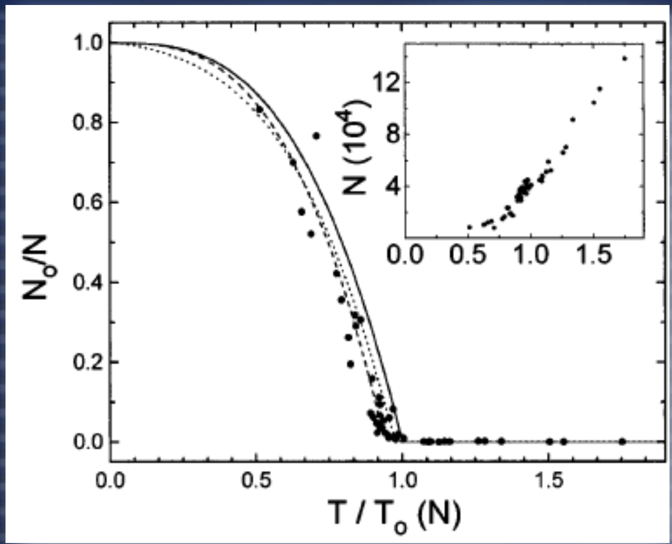
BEC in a cold atom system

Cold atoms are

- very dilute (10^{11}~10^{14} cm^{-3}),
- with no impurities, no defects.

→ Amenable to simple theoretical description

Condensate fraction



J. R. Ensher, et al.,
Phys. Rev. Lett. **77**, 4984
(1996).

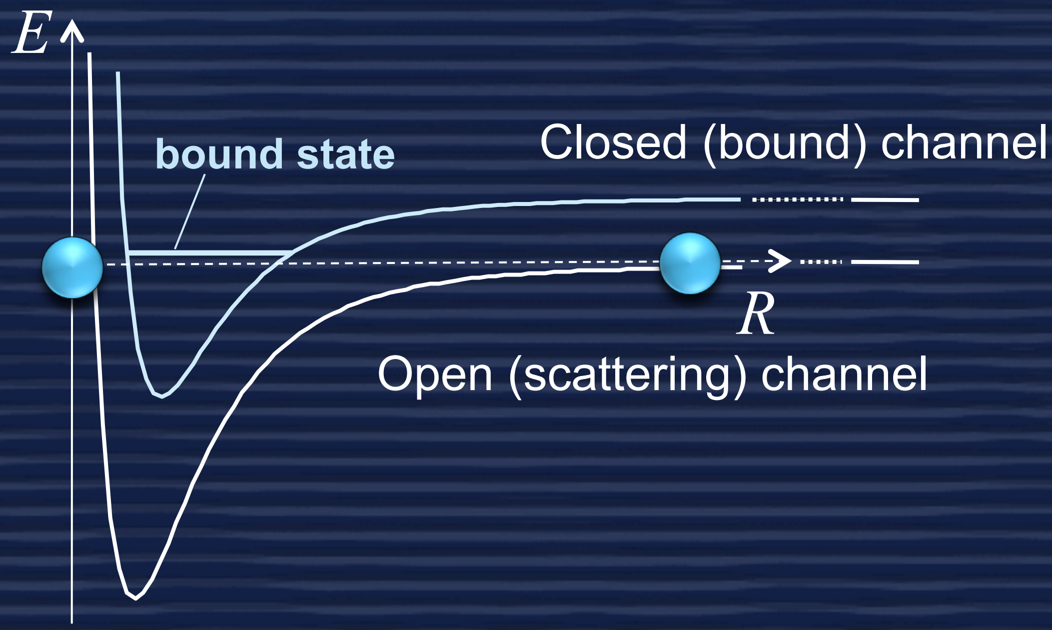
5% deviation of critical temperature from theoretical predictions → ▪3% shift due to finite number correction
▪2% shift due to interaction

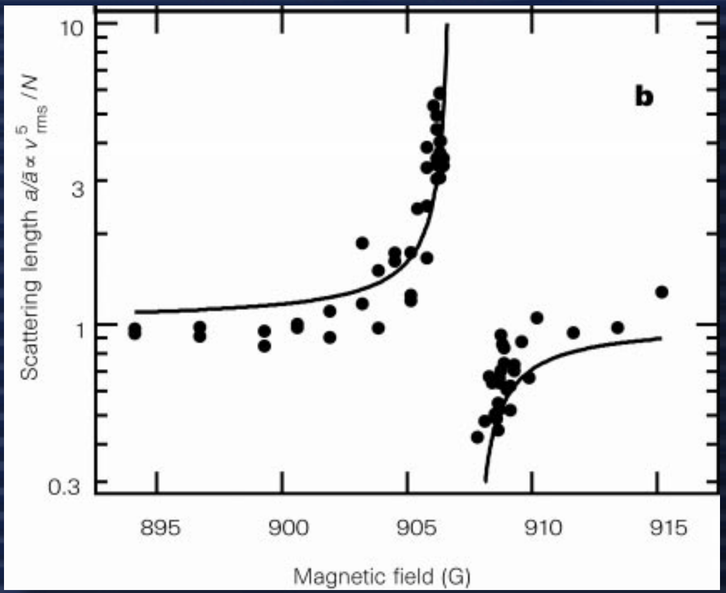
Inter-atomic interaction is tunable !!

Feshbach resonance

There are two channels corresponding to different spin states.

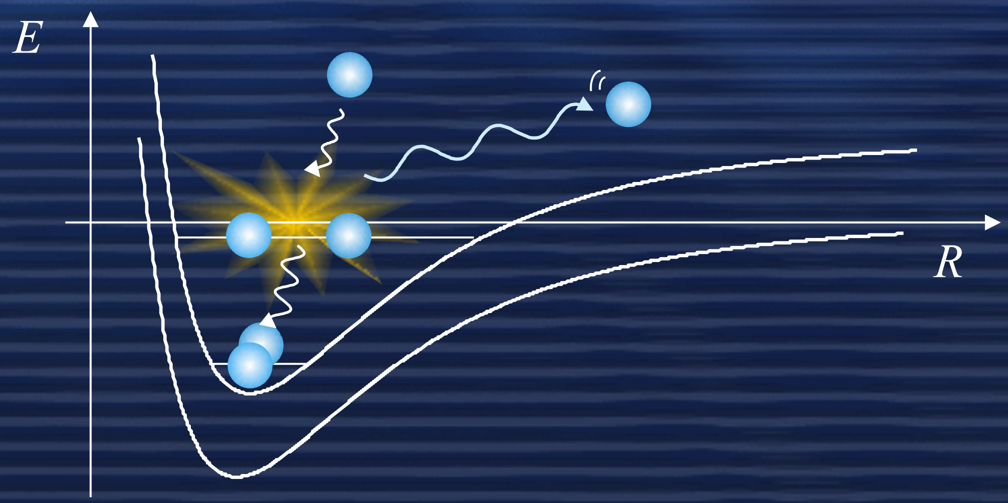
Resonance occurs when open and closed channel are energetically degenerate.

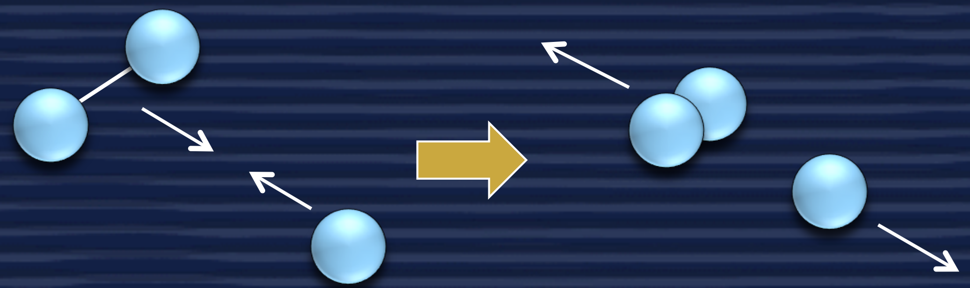


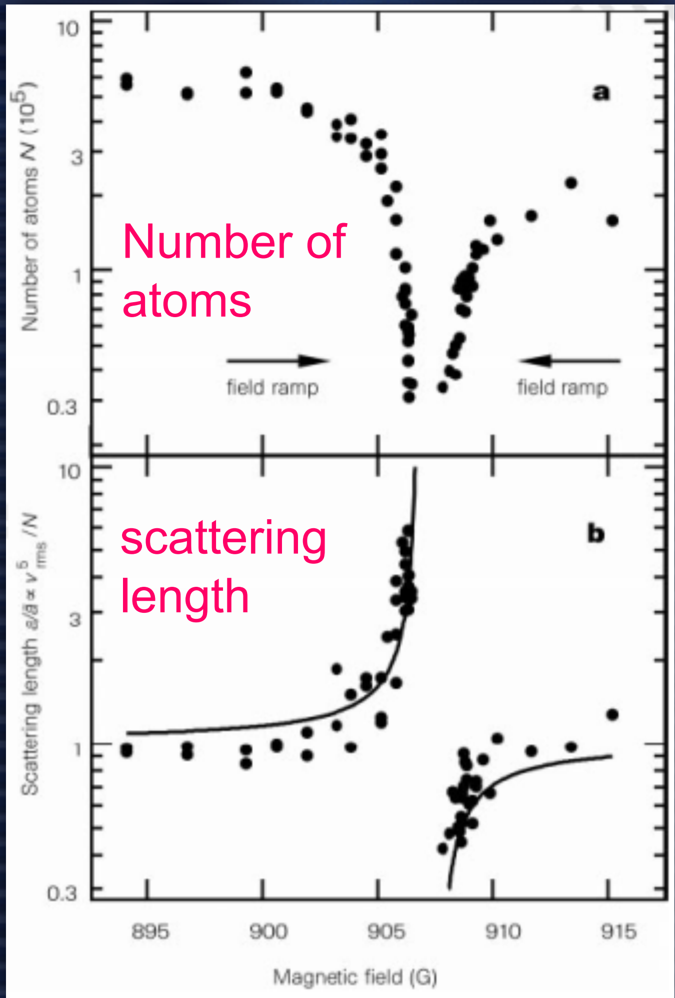


S. Inouye, et al.,
Nature **392**, 151 (1998).

Loss near Feshbach resonance



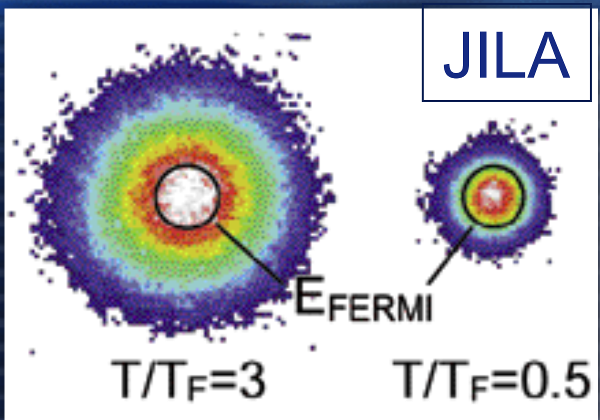




S. Inouye et al.,
Nature, **392**, 151 (1998).

ultracold fermionic atoms

1999 Fermi degenerate gas



Collision channel

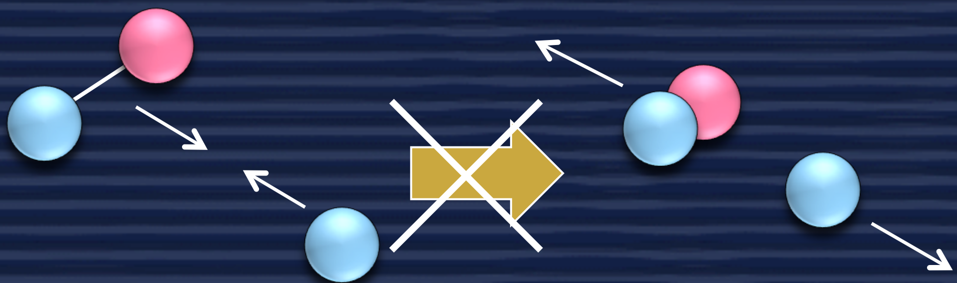
Identical bosons : l=0(s-wave), l=2 (d-wave), …
Identical fermions: l=1 (p-wave), l=3 (f-wave), …

ultracold : s-wave is the dominant collision channel.

→ Identical fermions do not collide.

Think about two-component fermions 

At the Feshbach resonance for  and  ,
no loss occurs due to Pauli exclusion principle.

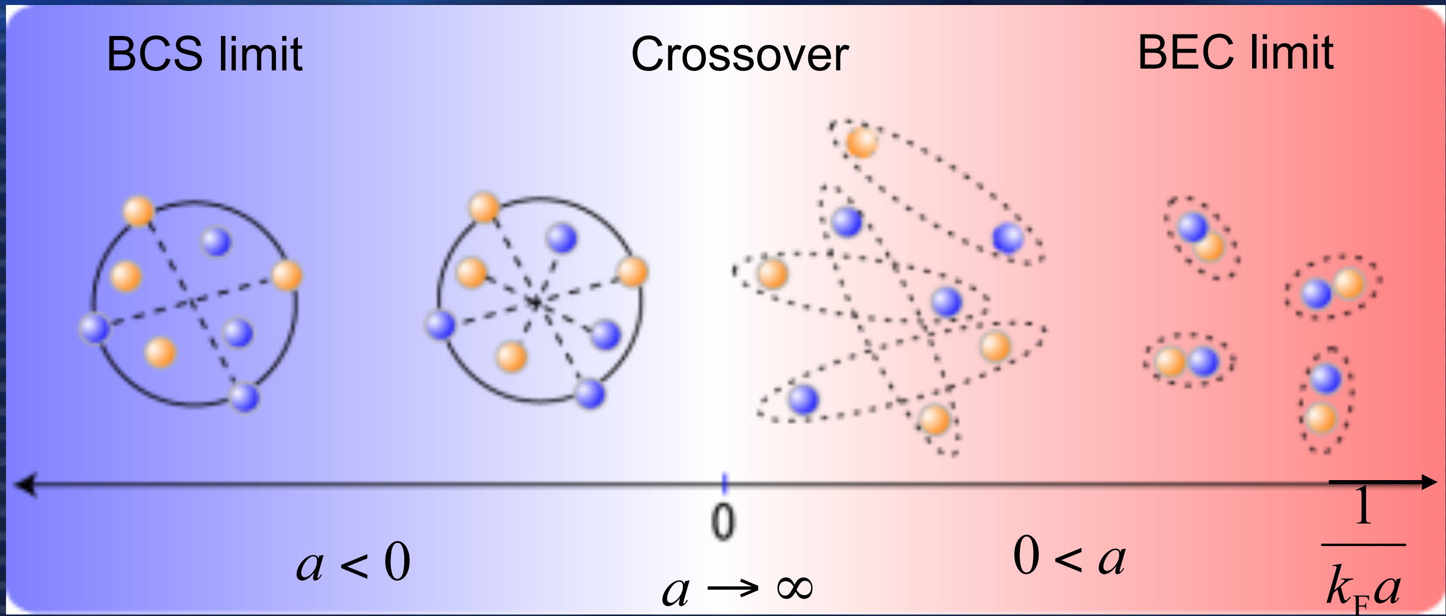


Therefore two-component fermions are stable even at
a Feshbach resonance.

We are able to prepare an interacting (reasonably stable) two-component Fermi gas of atoms with an arbitrary interaction strength !!

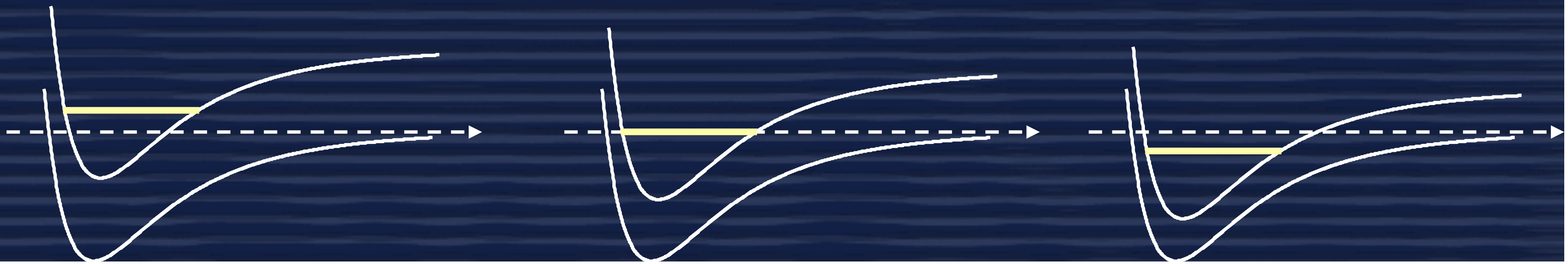
BCS–BEC crossover

Idea : BEC and BCS type superfluid are the opposite extreme of the same phase



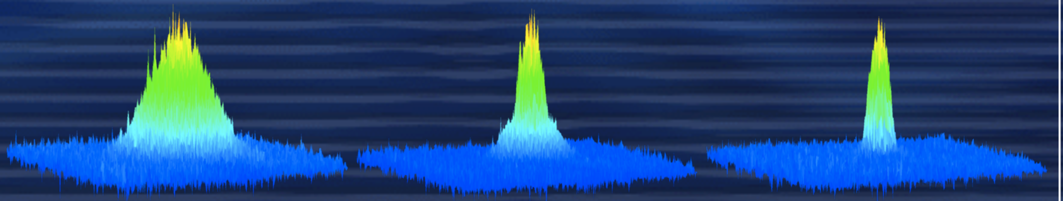
Pairs of fermions - Cooper pairs
Momentum space pairing

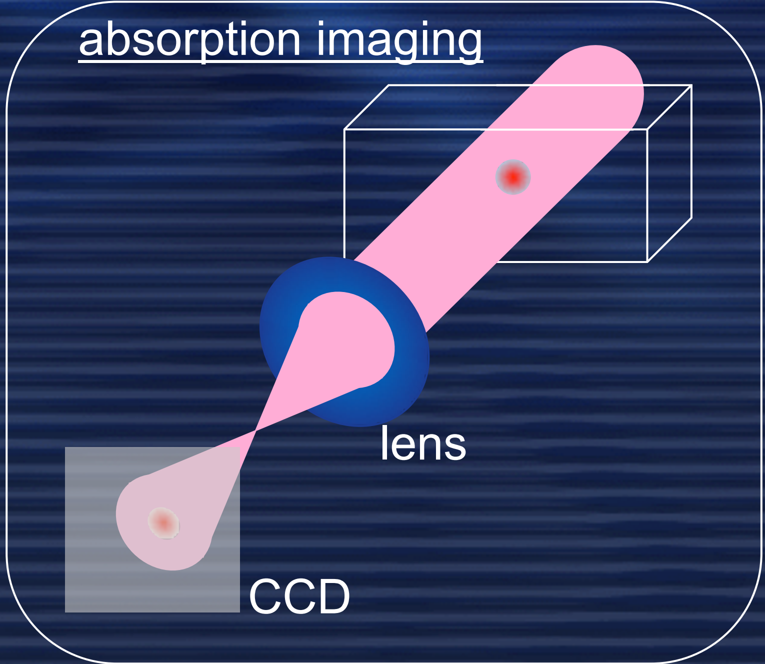
Tightly bound bosons - molecules
Real space pairing



momentum distribution measurement

Bosonic case

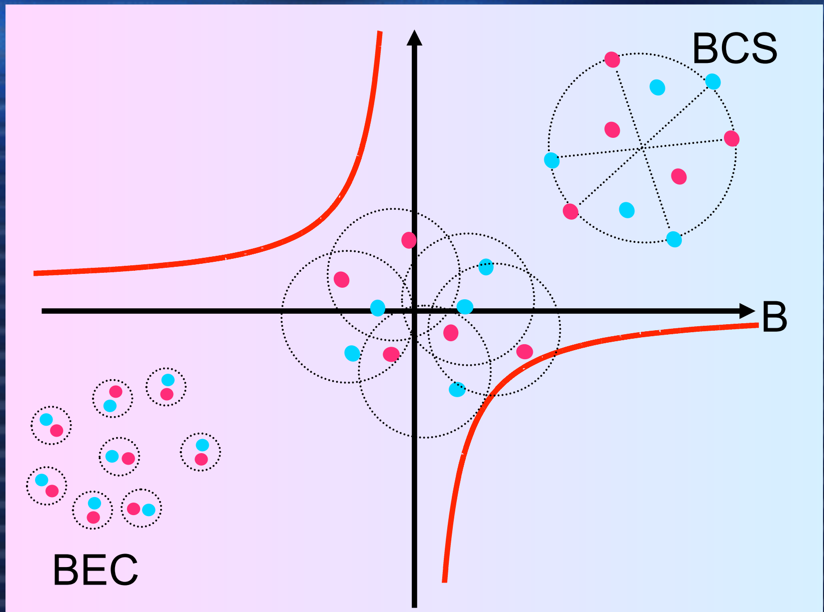


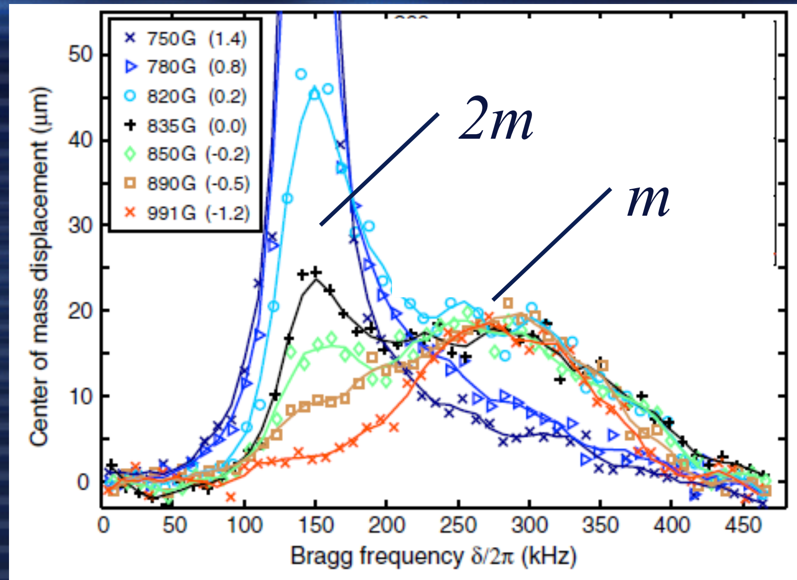


Condensed or not?
See the bimodal profile !!

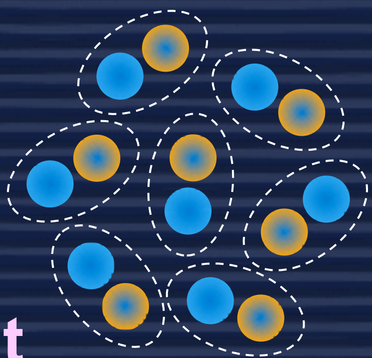
… unfortunately this scheme does not work.

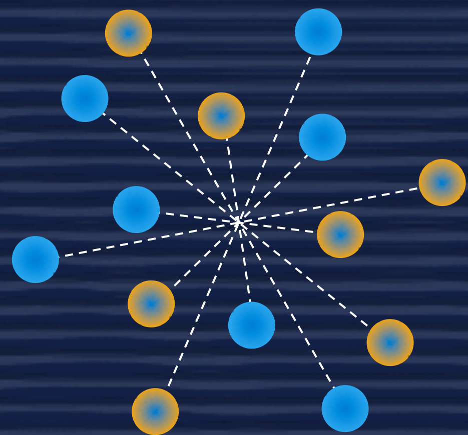
Fermion pair condensate





G. Veeravalli et al.
Phys. Rev. Lett.
101, 250403 (2008)





BEC limit

spatially
correlated pair

momentum
correlated pair

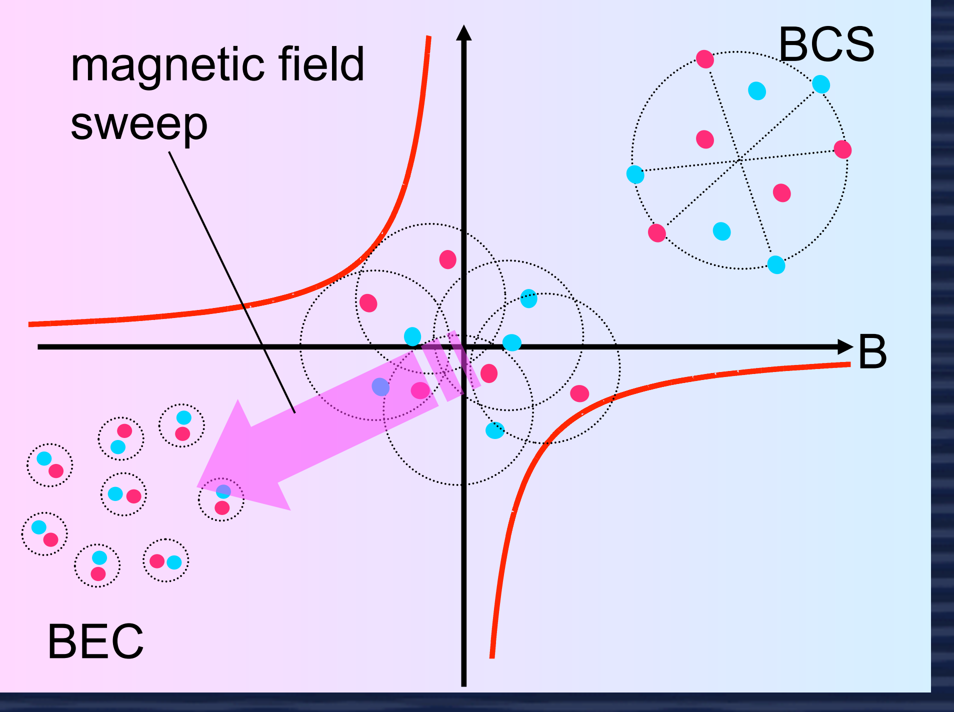
BCS limit

"projection"

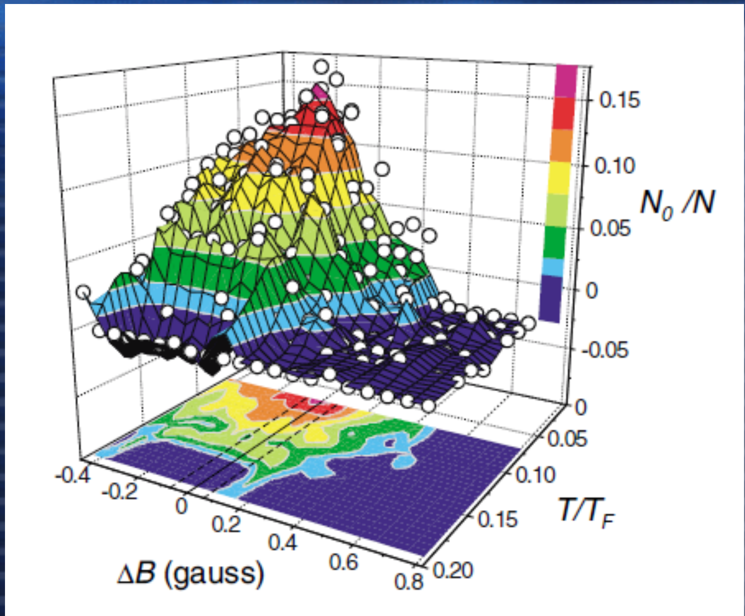
C. A. Regal et al.
Phys. Rev. Lett., **92**, 040403 (2004)

If we sweep the magnetic field

- slow enough to convert atom pairs into molecules
- fast enough such that the momentum distribution of the projected molecules reflects that of pairs prior to the sweep



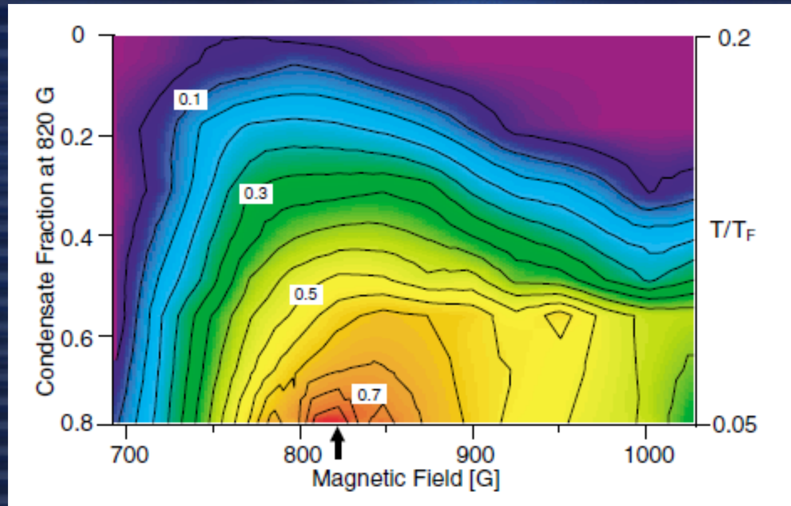
BCS-BEC crossover (experiment)



C. A. Regal *et al.*, PRL 92, 040403 (2004)

T/T_F is measured under ideal gas condition (magnetic field where a=0).

Adiabatic does keep entropy constant, but not T/T_F.

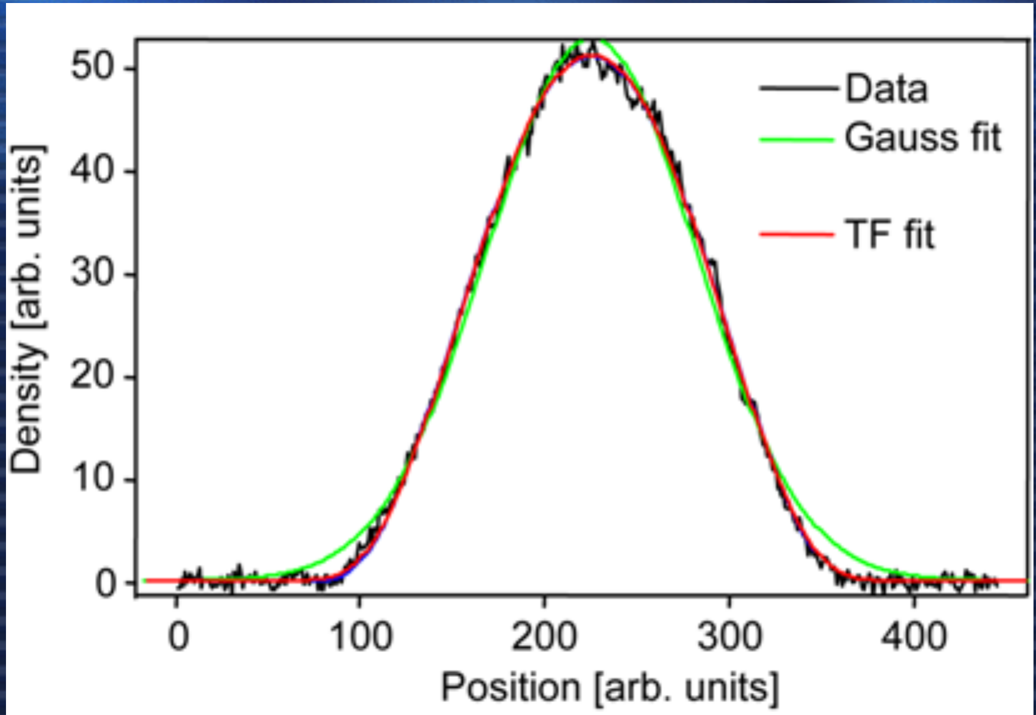


M. W. Zwierlein *et al.*, PRL 92, 120403 (2004)

T/T_F was measured at 1025 G (magnetic field where a=-4000a_0).

Again, T/T_F is not the one measured at unitarity, but somewhere different.

Conventional thermometry



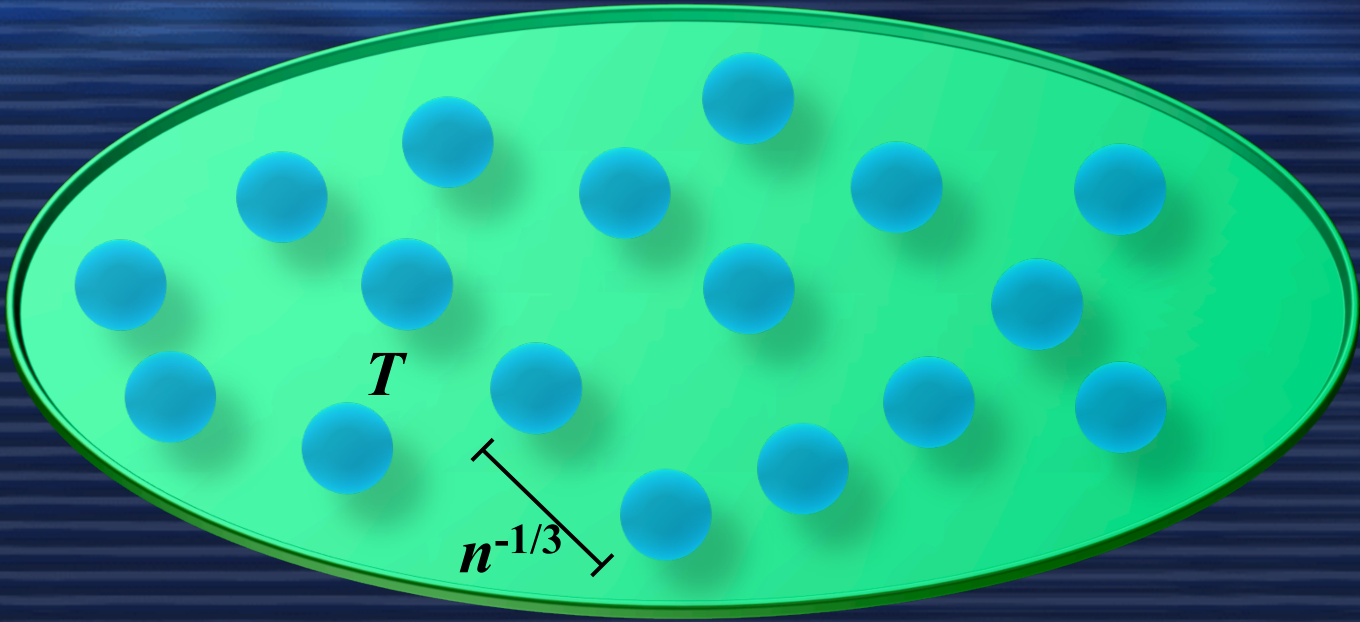
Let the cloud expand. → Fit the profile with a reasonable function.

$$n(z) = -\frac{3N}{\sqrt{\pi}\sigma_{TF,z}}\left(\frac{T}{T_F}\right)^{5/2} \cdot Li_{5/2}\left[-\xi \cdot \exp\left(-\frac{1}{2}\frac{z^2}{\sigma_z^2}\right)\right]$$

This scheme is good only when interaction energy << kinetic energy.

Thermodynamic
of an ideal Fermi gas

Thermodynamics of an ideal Fermi gas



Thermodynamic behavior of an ideal Fermi gas is described by its temperature T and density n.

Thermodynamic of an ideal Fermi gas

Fermi-Dirac distribution

$$n(\varepsilon) = \frac{1}{z^{-1}e^{\beta\varepsilon}+1} \quad \left(z \equiv e^{\beta\mu},\ \beta = (k_{\mathrm{B}}T)^{-1} \right)$$

$$N = \int_0^\infty \frac{D(\varepsilon)}{z^{-1}e^{\beta\varepsilon}+1} d\varepsilon \Rightarrow \left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)^{-3/2} = -\frac{3\sqrt{\pi}}{4} Li_{3/2}\left[-\exp\left(\frac{\mu/E_{\mathrm{F}}}{k_{\mathrm{B}}T/E_{\mathrm{F}}} \right) \right]$$

$$\left(-Li_s(-z) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1}}{z^{-1}e^t+1} dt \right)$$

$$\boxed{\frac{\mu}{E_{\mathrm{F}}} = f_\mu\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)}$$

Thermodynamic of an ideal Fermi gas

$$E = \int_0^\infty \frac{\varepsilon D(\varepsilon)}{z^{-1} e^{\beta\varepsilon} + 1} d\varepsilon$$

$$\frac{E}{N E_\mathrm{F}} = -\frac{3\sqrt{\pi}}{4} \left(\frac{k_\mathrm{B} T}{E_\mathrm{F}} \right)^{5/2} Li_{5/2}(-z)$$

$$\Rightarrow \quad \boxed{\frac{E}{N E_\mathrm{F}} = f_E \left(\frac{k_\mathrm{B} T}{E_\mathrm{F}} \right)}$$

Other thermodynamic functions also have this similarity.

$$\frac{S}{k_\mathrm{B}} = f_S \left(\frac{k_\mathrm{B} T}{E_\mathrm{F}} \right)$$

$$\frac{F}{N E_\mathrm{F}} = f_F \left(\frac{k_\mathrm{B} T}{E_\mathrm{F}} \right)$$

Thermodynamic of an ideal Fermi gas

Internal energy : $$\frac{E}{NE_\mathrm{F}} = f_{E,ideal}\left(\frac{k_\mathrm{B}T}{E_\mathrm{F}}\right)$$

Helmholtz free energy : $$\frac{F}{NE_\mathrm{F}} = f_{F,ideal}\left(\frac{k_\mathrm{B}T}{E_\mathrm{F}}\right)$$

Chemical potential : $$\frac{\mu}{E_\mathrm{F}} = f_{\mu,ideal}\left(\frac{k_\mathrm{B}T}{E_\mathrm{F}}\right)$$

Entropy : $$\frac{s}{Nk_\mathrm{B}} = f_{S,ideal}\left(\frac{k_\mathrm{B}T}{E_\mathrm{F}}\right)$$

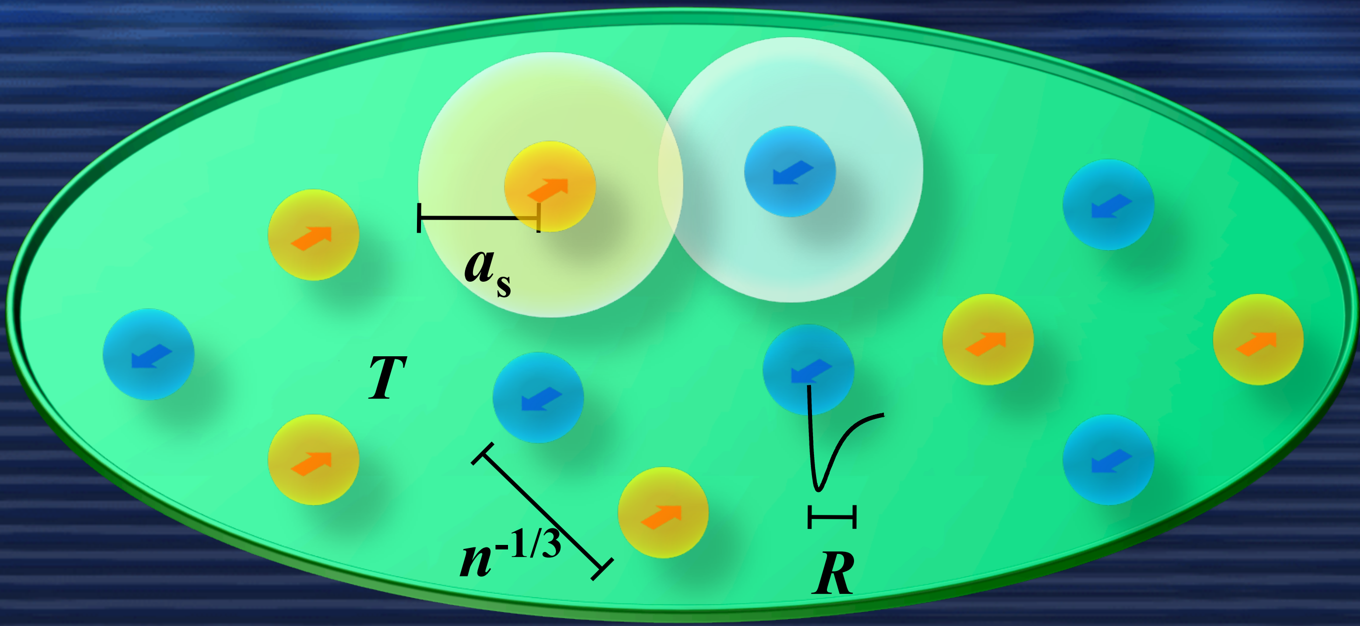
Dimensionless functions

Material specific parameter, such as m, is taken up by E_F (T_F).
(Shape of the functions do not depend on the particle's nature.)

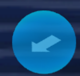
→ Universal thermodynamics

Thermodynamic of an interacting Fermi gas

Ultracold, dilute, interacting Fermi gases



- ultracold : s-wave is the dominant channel.

  ⇒  collide only with 

- dilute : details of the potential is much smaller than $n^{-1/3}$

  ⇒ The collision process can be described by a single parameter, so-called scattering length a_s.

Thermodynamic of an interacting Fermions

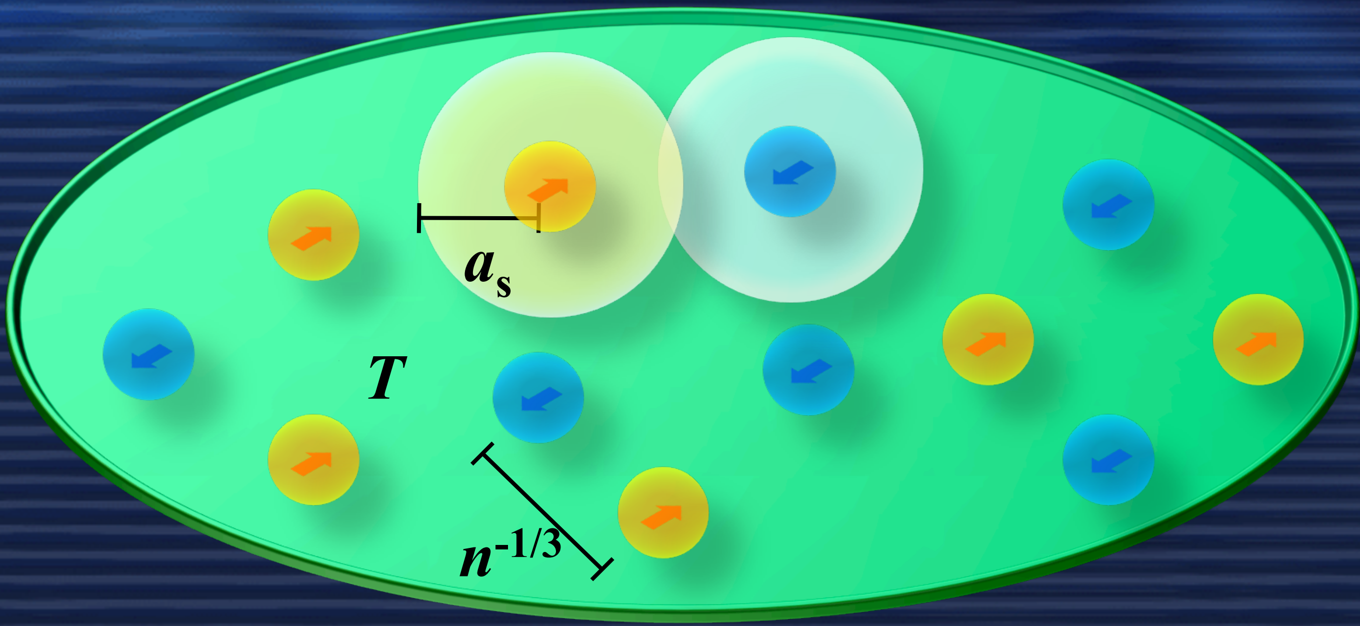
Ideal Fermi gas

$$\frac{E}{NE_\mathrm{F}} = f_{E,ideal}\left(\frac{k_\mathrm{B}T}{E_\mathrm{F}}\right)$$

Fermi gas with interaction

$$\frac{E}{NE_\mathrm{F}} = f_E\left(k_\mathrm{B}T, E_\mathrm{F}, E_\mathrm{int}\left(a_\mathrm{s}\right)\right)$$

Ultracold dilute Fermi gas



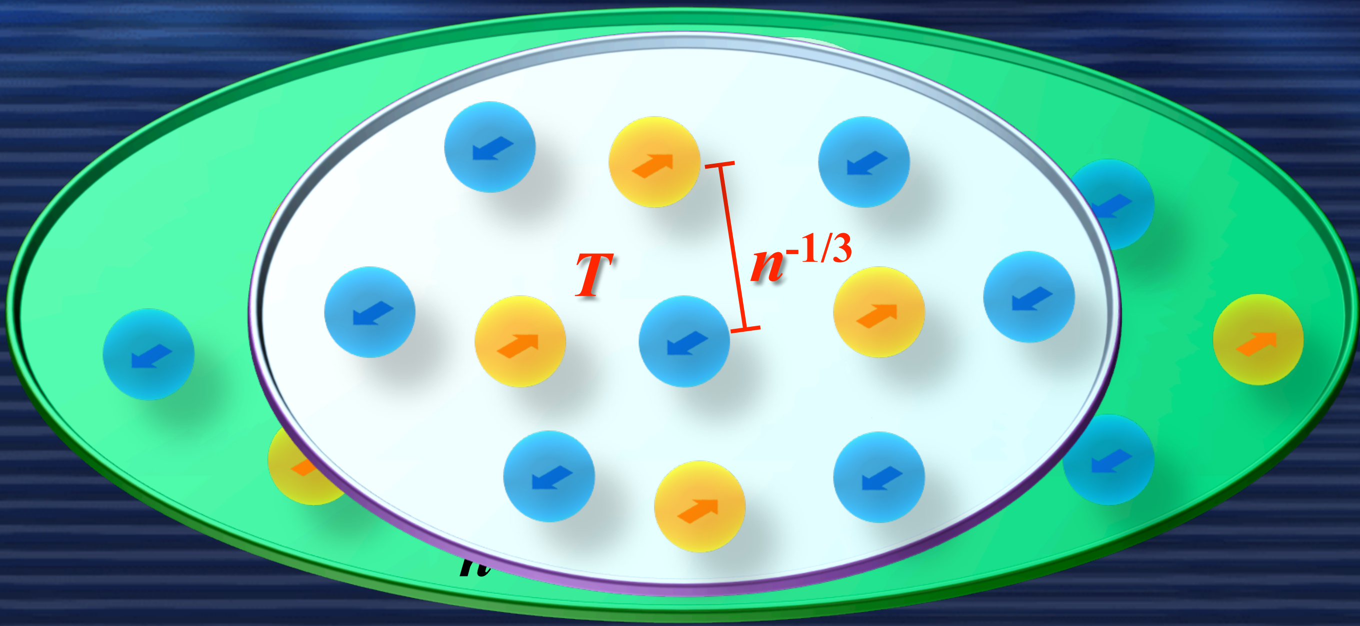
Remember the fact that a_s ***is tunable!!***

Then, what happens when…

$$|a_s| \longrightarrow \infty$$

This situation is called unitarity limit.

Unitarity limit and Universality



a_s drops out of the description of the thermodynamics.

Thermodynamics depends only on the density n and temperature T.

Universal thermodynamics holds again…?

Thermodynamic of an interacting Fermions

Ideal Fermi gas

$$\frac{E}{NE_{\mathrm{F}}} = f_{E,ideal}\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Fermi gas with interaction

$$\frac{E}{NE_{\mathrm{F}}} = f_E\left(k_{\mathrm{B}}T, E_{\mathrm{F}}, U(a)\right)$$

When the scattering length diverges…

$$\frac{E}{NE_{\mathrm{F}}} = f_E\left(k_{\mathrm{B}}T, E_{\mathrm{F}}, \cancel{U(a)}\right) \Rightarrow f_{E,|a|=\infty}\left(k_{\mathrm{B}}T, E_{\mathrm{F}}\right) = f_{E,|a|=\infty}\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

There is a hypothesis that the thermodynamic functions again have the universal form.

Universal hypothesis

Universal thermodynamics

According to universal hypothesis, all thermodynamics should obey the universal functions:

Internal energy : $$\frac{E}{NE_{\mathrm{F}}} = f_E\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Helmholtz free energy : $$\frac{F}{NE_{\mathrm{F}}} = f_F\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Chemical potential : $$\frac{\mu}{E_{\mathrm{F}}} = f_\mu\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Entropy : $$\frac{S}{Nk_{\mathrm{B}}} = f_S\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$$

Dimensionless universal functions, (shape of the function is different from those for an ideal gas)

System looks like a non-interacting Fermi gas.
(no other dimensional parameters involved in the problem)

Universal thermodynamics

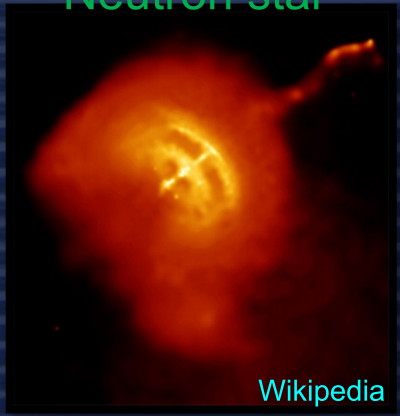
Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction.

$$E_{gs} = f\left(N, V, m\right) = N \cdot E_{\mathrm{F}} \times \xi$$

pure number

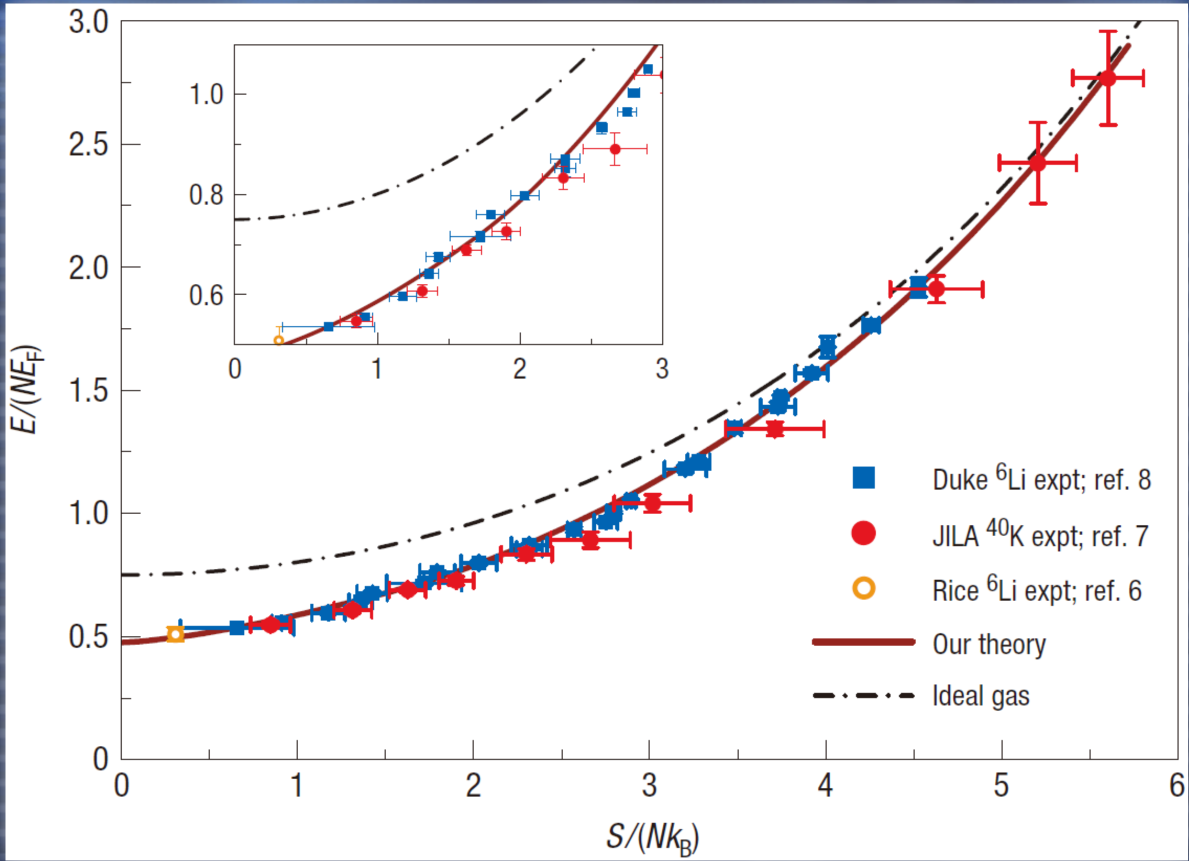
Neutron star



Besides pure theoretical curiosity, this problem is relevant to neutron stars!

Universal thermodynamics

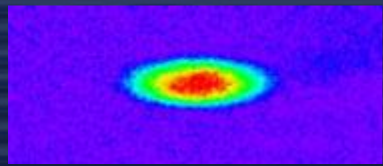
H. Hu, P. D. Drummond & X.-J. Liu,
Nature Physics **3**, 469 - 472 (2007)

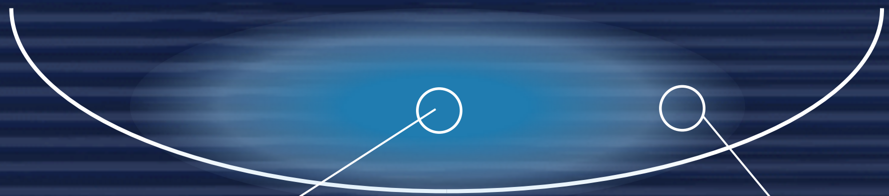


$f_E\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right)$ is still not known···

T is constant over the cloud (thermal equilibrium).
E_{F} depends on the position (local density).

$\Rightarrow$ $\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}$ is position-dependent.





$$\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}(n_0)}$$

$$\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}\left[n(\mathbf{r})\right]} \geq \frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}(n_0)}$$

Global measurement only gives the integration of all the different phases.

Goal of this experiment

Measurement of local thermodynamic quantities

and

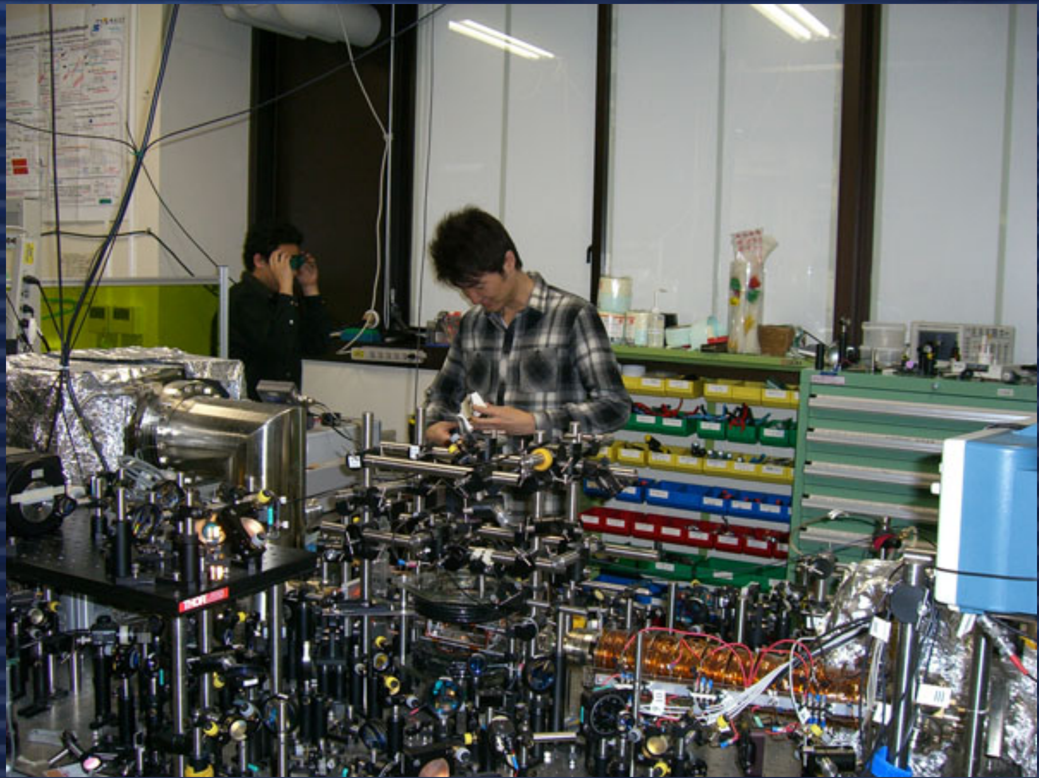
the determination of the universal thermodynamic function.

$$\frac{E}{NE_{\mathrm{F}}} = f_E\left(\frac{k_{\mathrm{B}}T}{E_{\mathrm{F}}}\right) \Rightarrow \frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})E_{\mathrm{F}}\left[n(\mathbf{r})\right]} = f_E\left(\frac{T}{T_{\mathrm{F}}\left[n(\mathbf{r})\right]}\right)$$

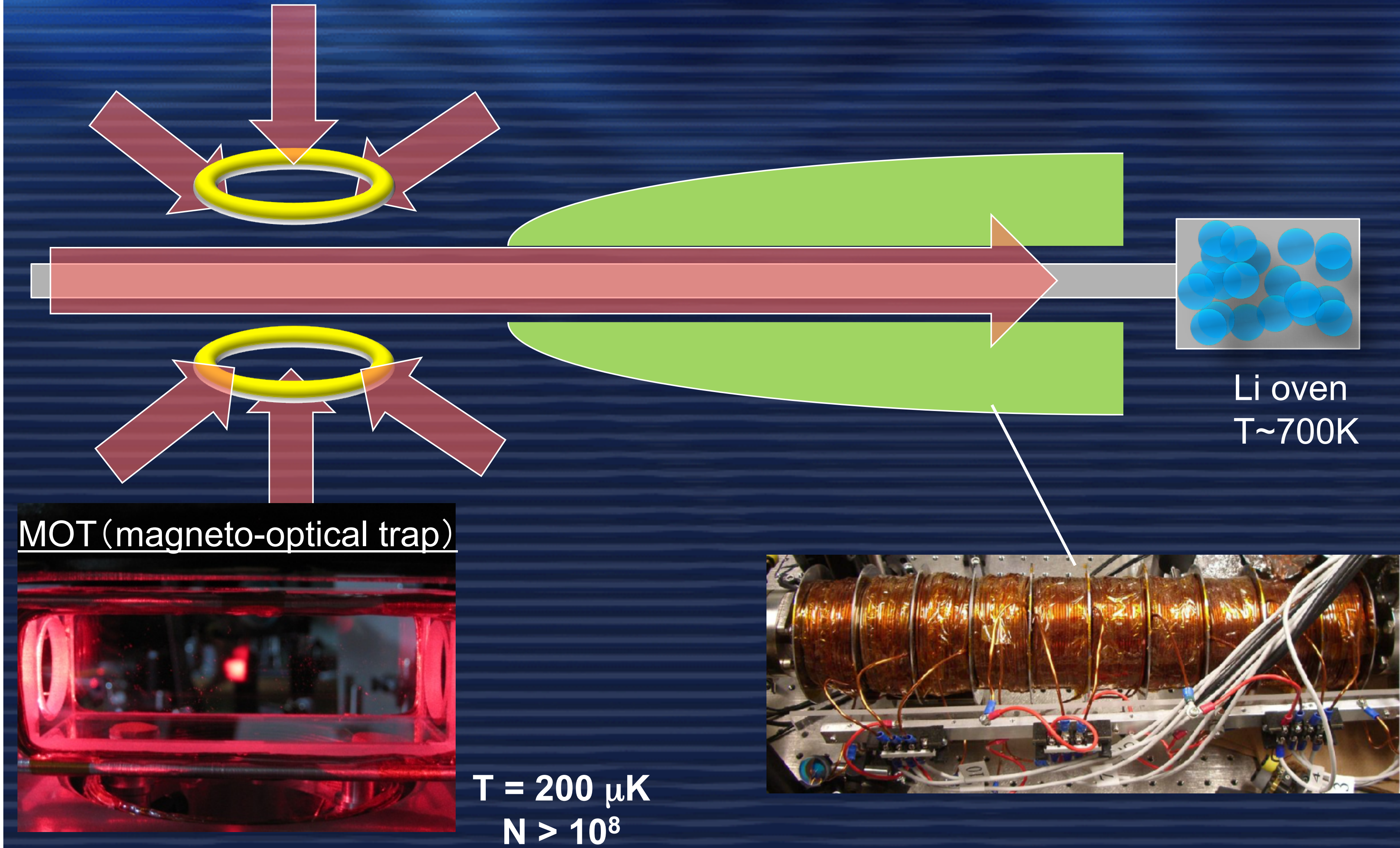
ε : local energy density

$$E_{\mathrm{F}} = k_{\mathrm{B}}T_{\mathrm{F}}$$

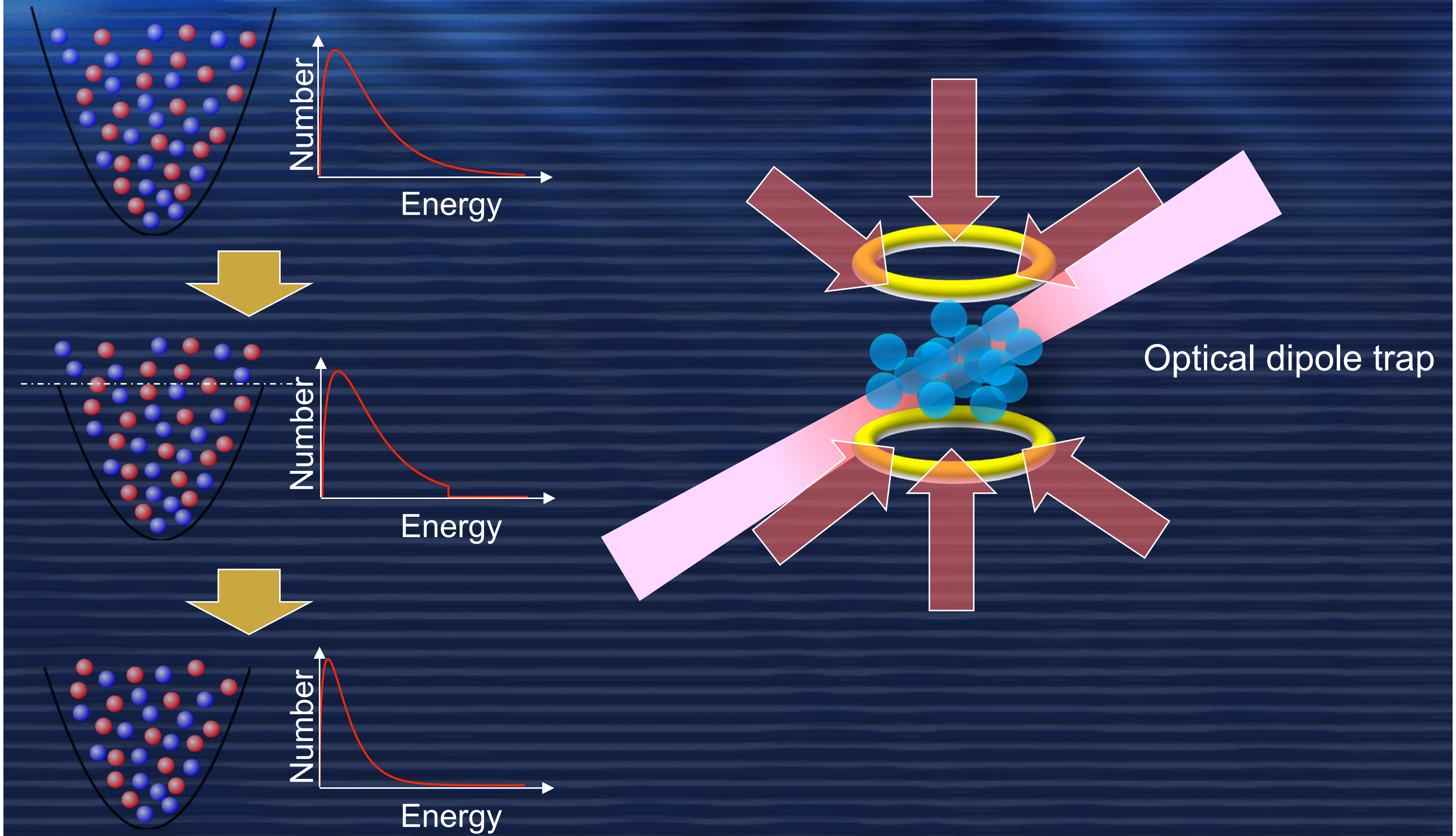
Experiment setup

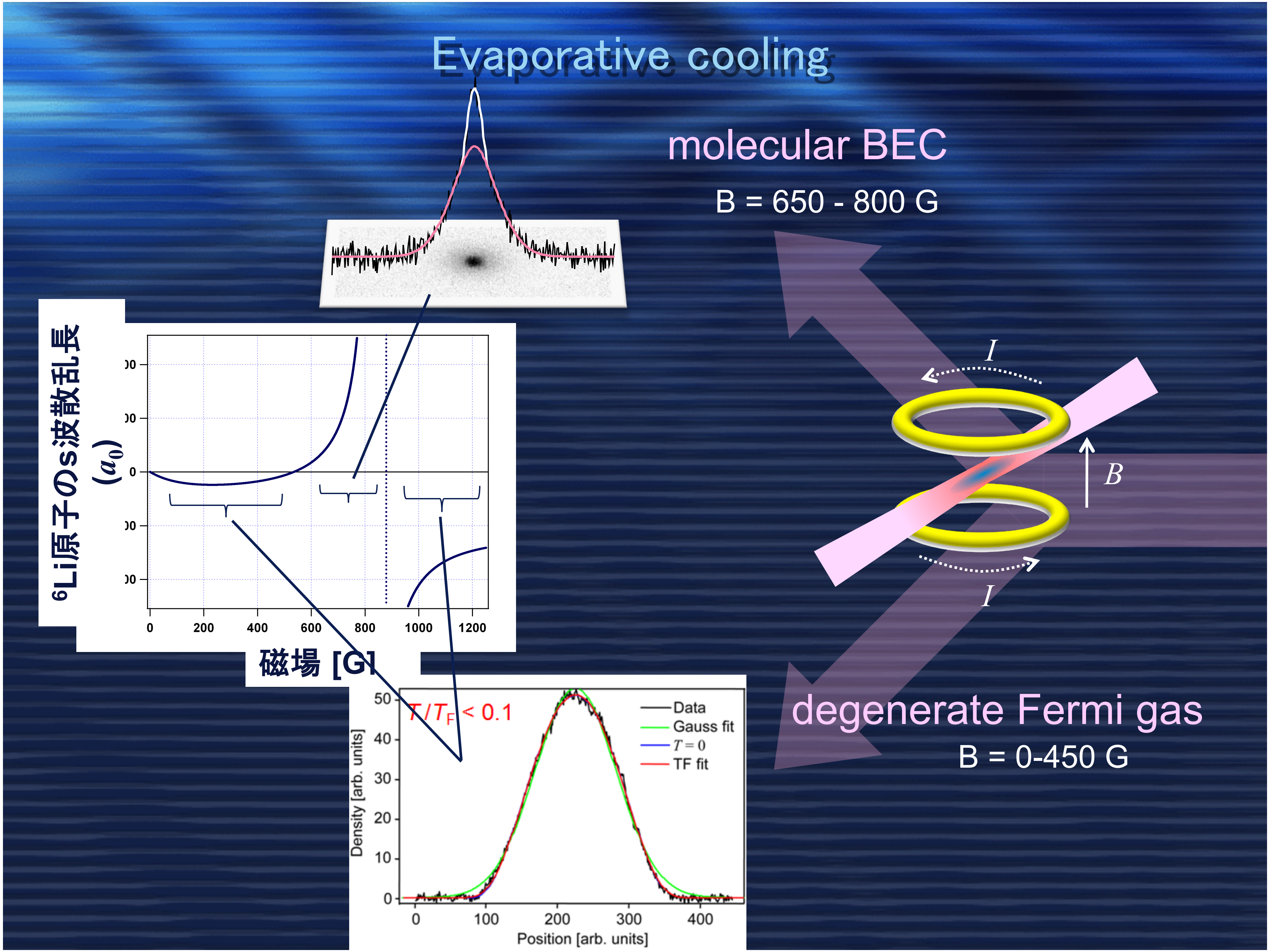


Deceleration and trapping



Optical dipole trap

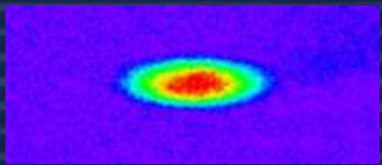




Evaporative cooling
molecular BEC
B = 650 - 800 G
I
B
I
degenerate Fermi gas
B = 0-450 G
^{6}Li原子のs波散乱長
(a_0)
0
0 200 400 600 800 1000 1200
磁場 [G]
$T/T_F < 0.1$
Data
Gauss fit
$T = 0$
TF fit
Density [arb. units]
0 10 20 30 40 50
0 100 200 300 400
Position [arb. units]

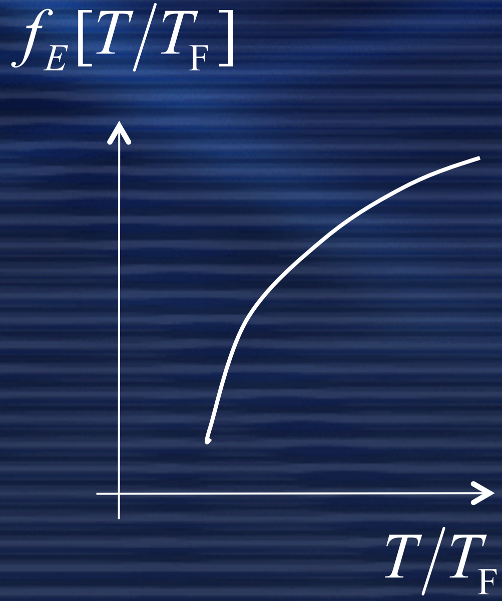
Determination of local energy $\varepsilon(\mathbf{r})$

$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_\mathrm{F}\left[n(\mathbf{r})\right]} = f_E[T/T_\mathrm{F}]$$



density profile

$$n(\mathbf{r})$$



Useful equations :

- **Equation of state of unitary gas :** $p(\mathbf{r}) = \frac{2}{3}\varepsilon(\mathbf{r})$
- **mechanical equilibrium (eq. of force balance) :**

$$\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_\mathrm{Trap}(\mathbf{r}) = 0$$

$$n(\mathbf{r}) \Rightarrow p(\mathbf{r}) \Rightarrow \varepsilon(\mathbf{r})$$

Determination of temperature T

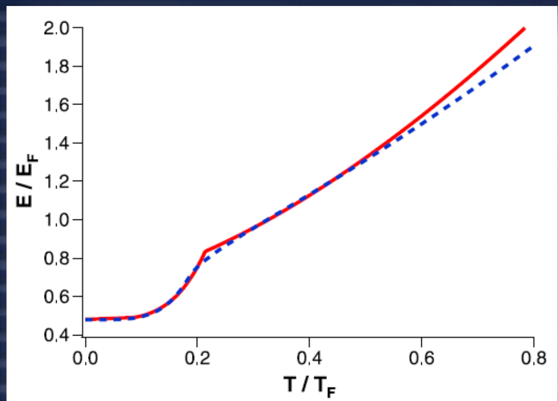
$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_{\mathrm{F}}\left[n(\mathbf{r})\right]} = f_E[T/T_{\mathrm{F}}]$$

$p(\mathbf{r}) = \frac{2}{3}\varepsilon(\mathbf{r})$ and $\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{\mathrm{Trap}}(\mathbf{r}) = 0$ $\Rightarrow$ $E_{\mathrm{total}} = 2\times E_{\mathrm{potential}}$

Adiabatic B-field sweep to turn off the interaction $\Rightarrow$ entropy S

$\boxed{E_{\mathrm{total}} \text{ vs } S} \xrightarrow{} E_{\mathrm{total}}$ vs T

$1/T = \partial S/\partial E$



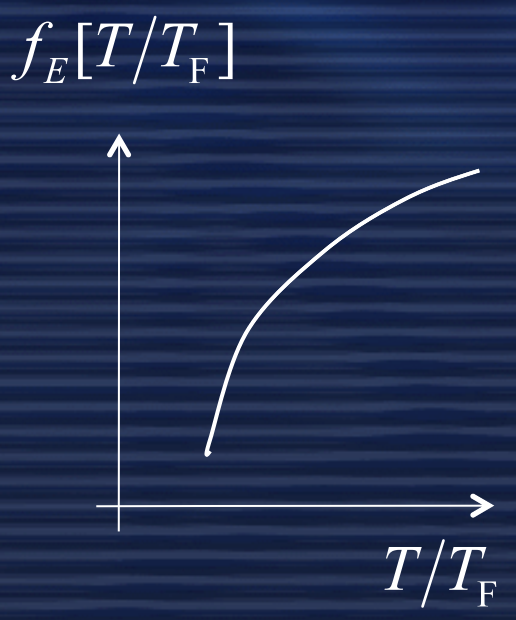
Le Luo and J.E. Thomas,
J Low Temp Phys **154,** 1 (2009).

Our scheme

$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_{\mathrm{F}}\left[n(\mathbf{r})\right]} = f_E[T/T_{\mathrm{F}}]$$

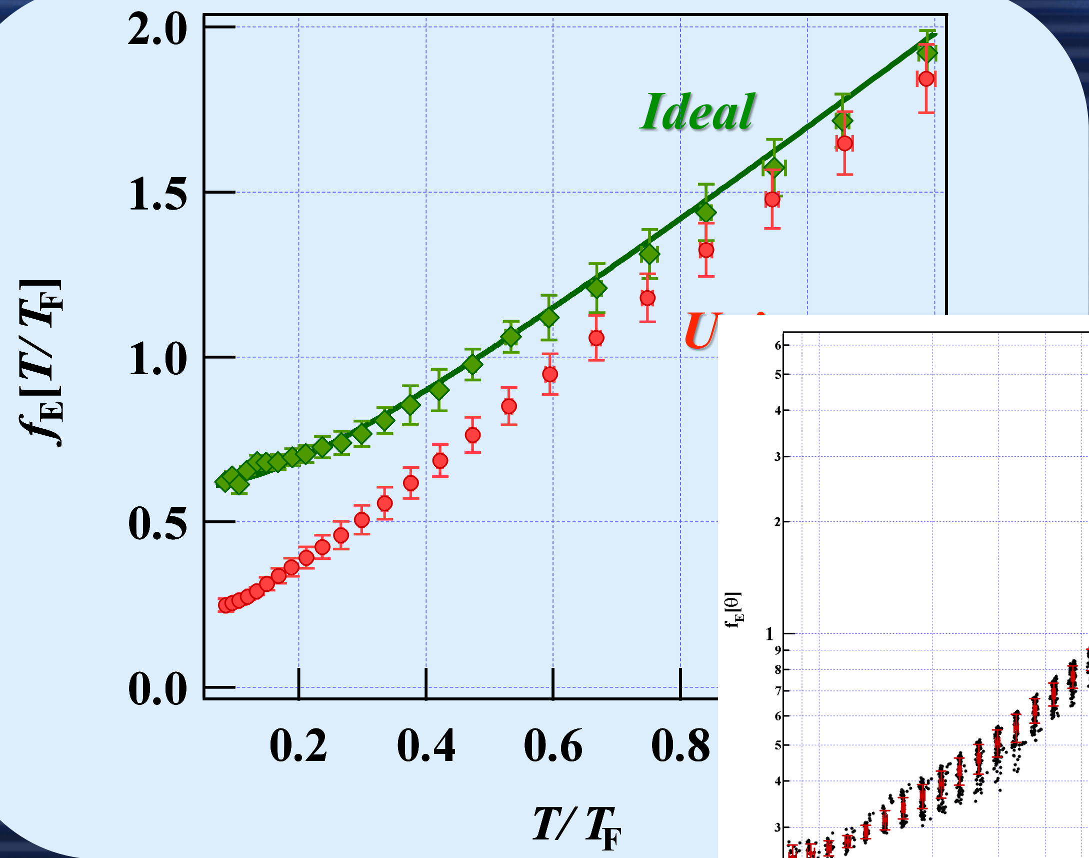
$$p = \frac{2}{3}\varepsilon$$

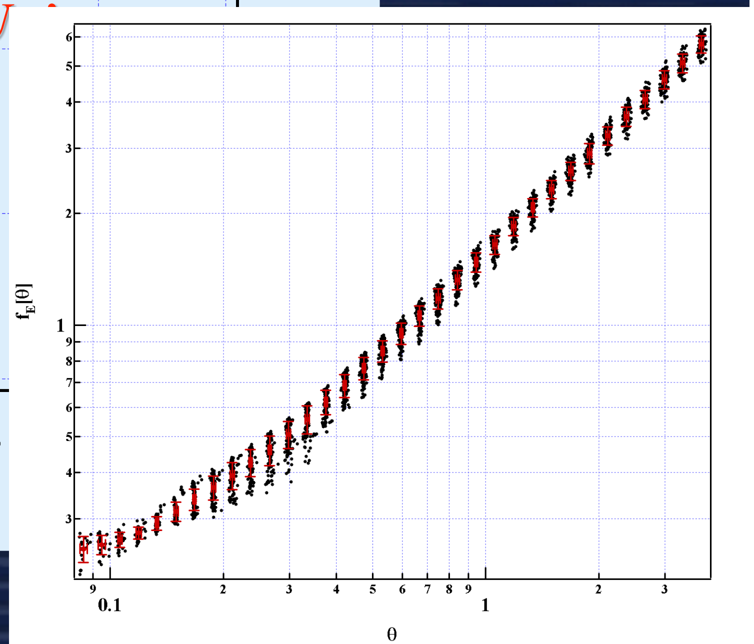
$$\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{\mathrm{Trap}}(\mathbf{r}) = 0$$



Experimental determination of $f_E[T/T_F]$

$$\frac{\varepsilon(\mathbf{r})}{n(\mathbf{r})\,E_F\left[n(\mathbf{r})\right]} = f_E[T/T_F]$$





M. Horikoshi, S. Nakajima,
M. Ueda and T. Mukaiyama,
Science, **327**, 442 (2010).

About 800 images are analyzed.

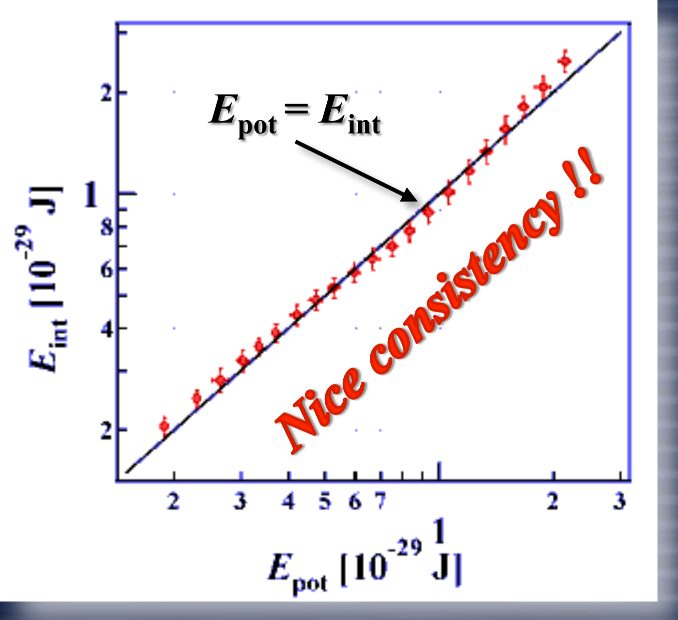
Verification of the determined $f_E[T/T_F]$

1. Energy comparison

$$E_{\text{total}} = 2\times E_{\text{potential}} \Rightarrow E_{\text{pot}} = E_{\text{internal}}$$

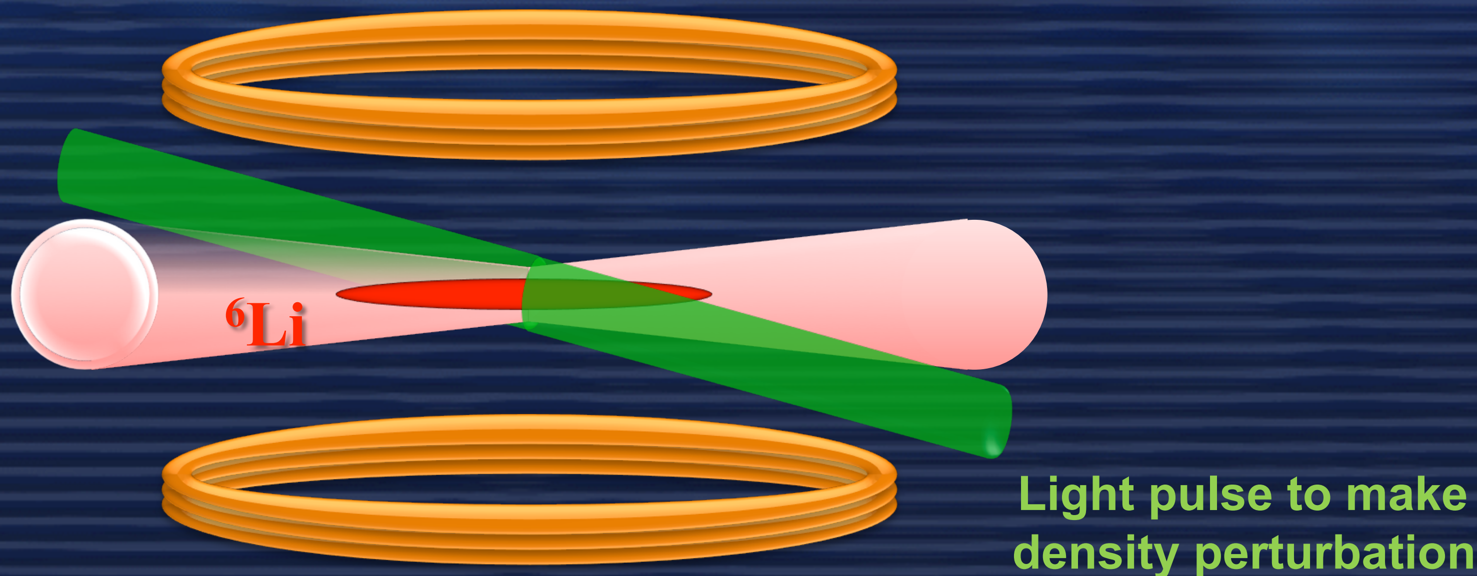
Comparison

- ✓ **Potential energy par particle :** $E_{\text{pot}} = \frac{3}{2} m\omega_z^2 < z^2 >$
- ✓ **Internal energy par particle :** $E_{\text{internal}} = \int n\varepsilon_F(n)\,\underline{\underline{f_E[\theta]}}\,\mathrm{d}V \Big/ N$



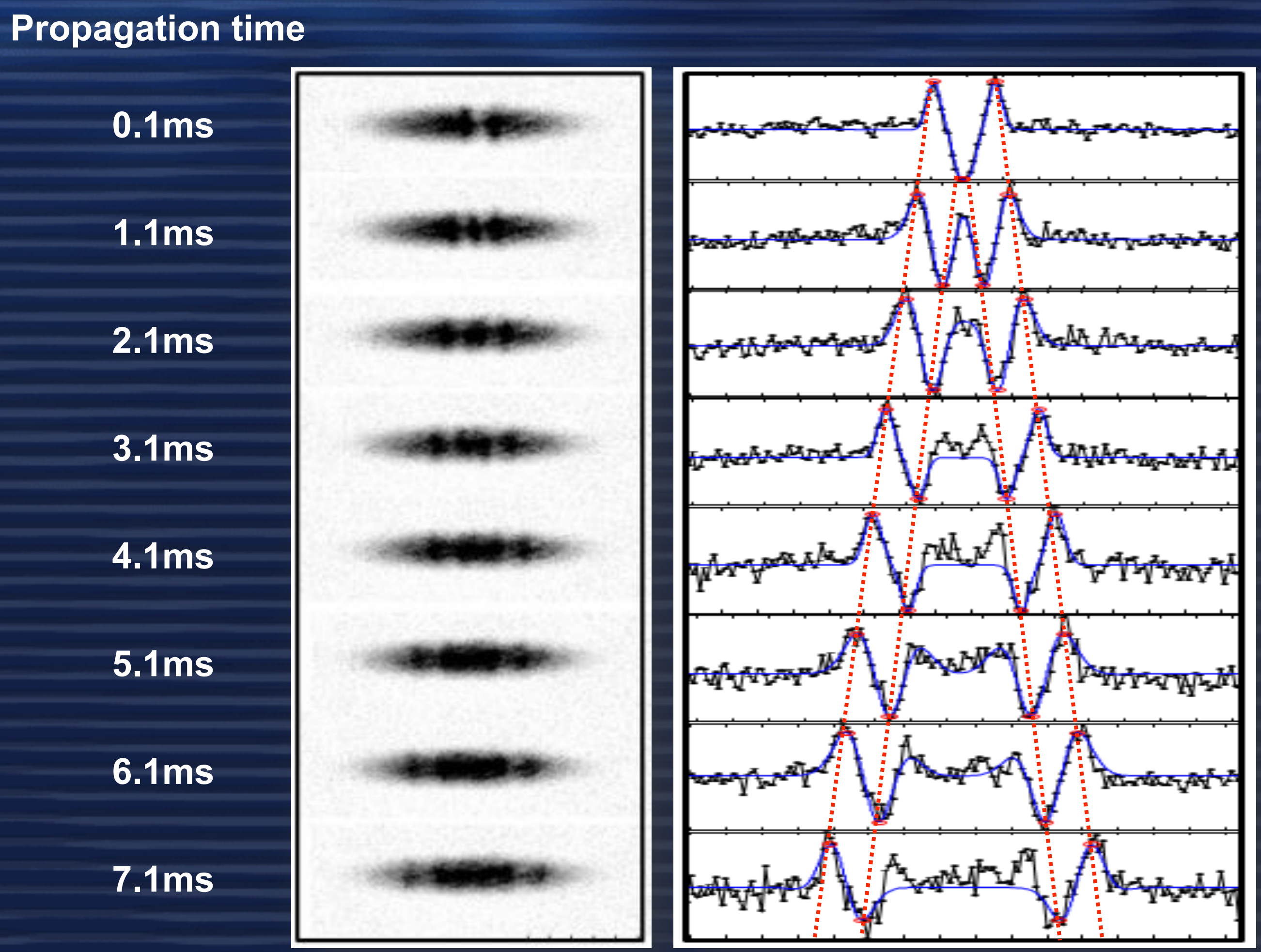
Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound



Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound



Verification of the determined $f_E[T/T_\mathrm{F}]$

2. Effective speed of the first sound

Unitary gas shows hydrodynamic behavior due to the large collision rate

Effective speed of the first sound : $\bar{u}_1^2[n,\theta] = \left. \dfrac{\iint n \, \mathrm{d}x\mathrm{d}y}{m \iint n \left(\dfrac{\partial p}{\partial n} \right)^{-1} \mathrm{d}x\mathrm{d}y} \right|_{z=0}$

$$p = \frac{2}{3}\varepsilon \propto f_E[T/T_\mathrm{F}]$$

[P. Capuzzi, PRA **73**, 021603(R) (2006)]

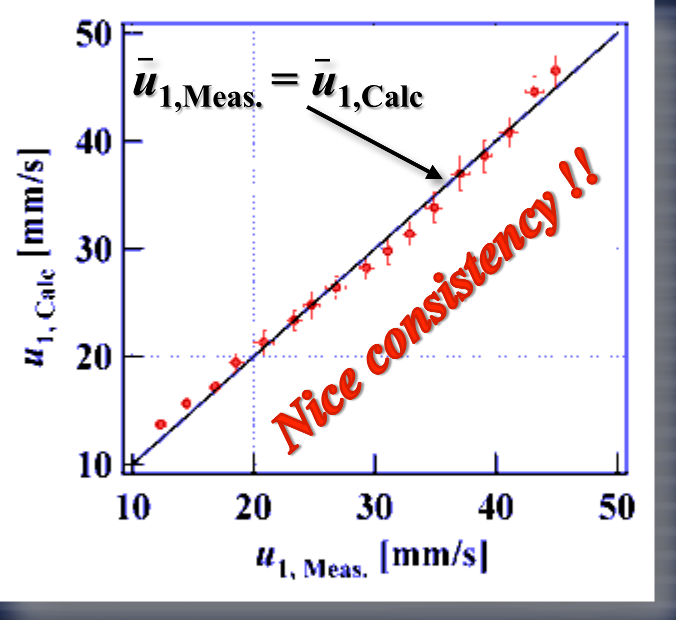
Comparison

Experiment

Verification of the determined $f_E[T/T_F]$

2. Effective speed of the first sound

Experimental values vs. calculated values from $f_E[\theta]$

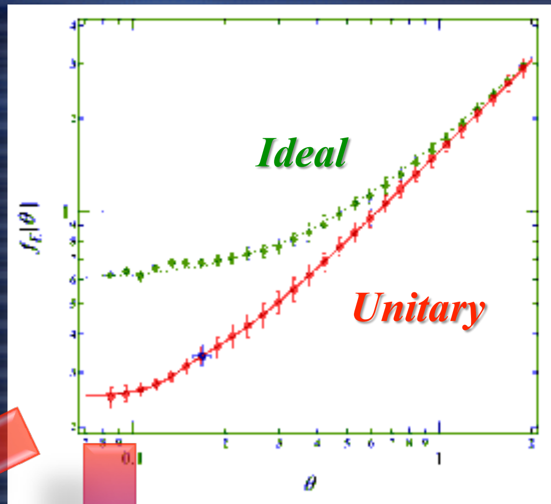


The universal function of the internal energy $f_E[T/T_F]$

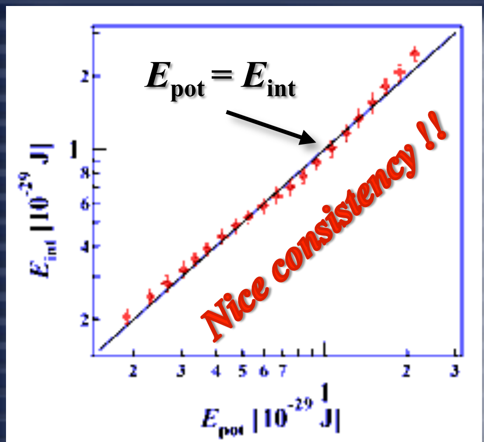
Universal hypothesis : $\dfrac{\varepsilon}{nE_F} = f_E[T/T_F]$

Equation of state : $p = \dfrac{2}{3}\varepsilon$

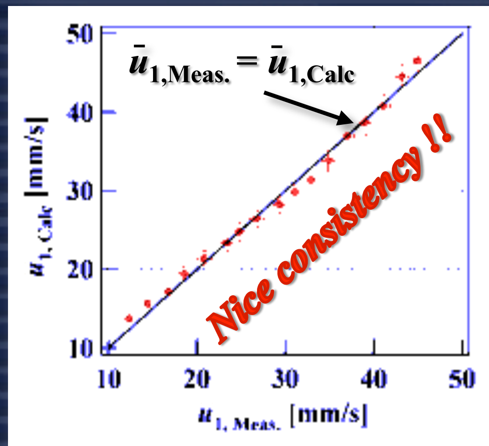
Mechanical equilibrium : $\nabla p(\mathbf{r}) + n(\mathbf{r})\nabla V_{\mathrm{Trap}}(\mathbf{r}) = 0$



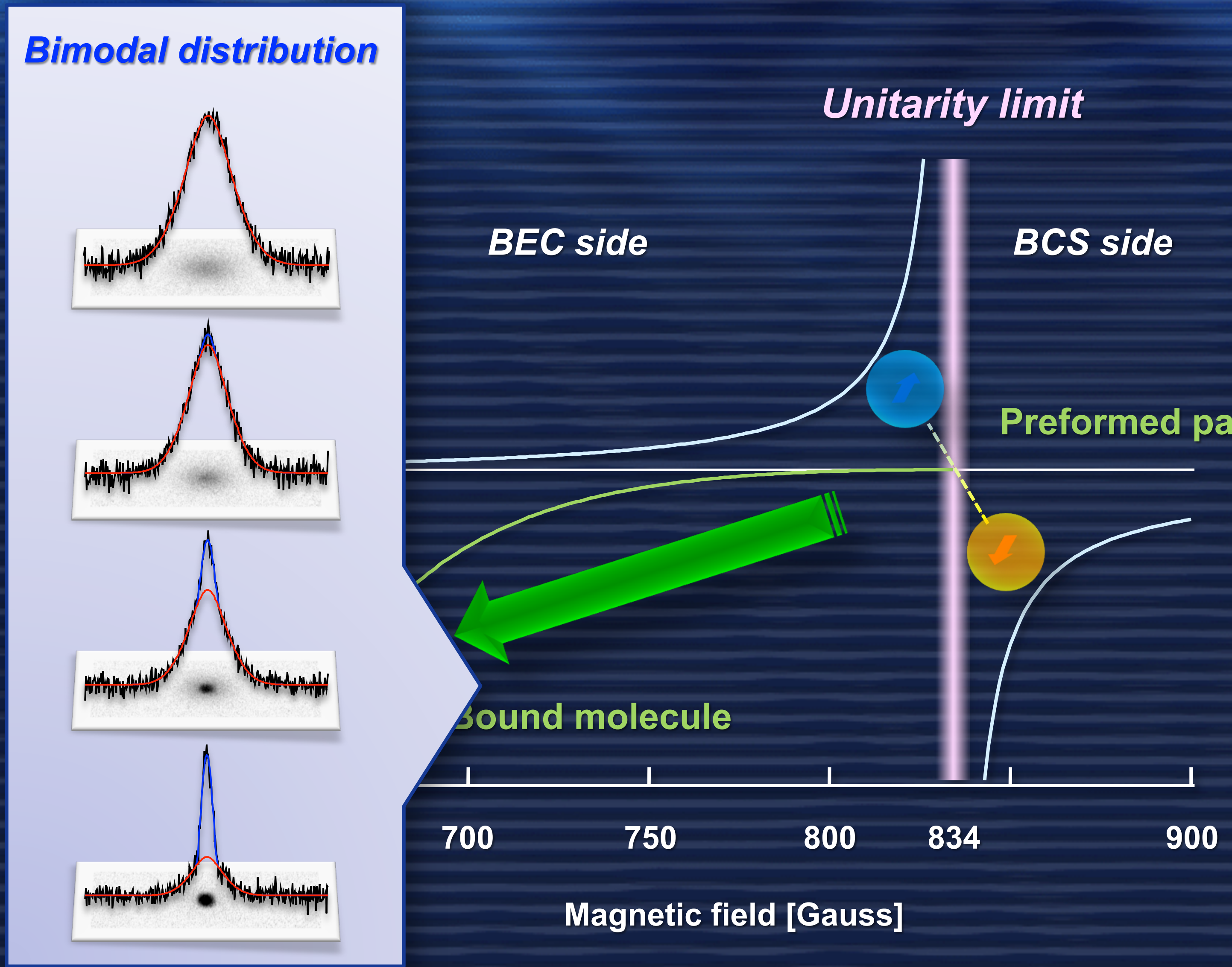
Energy comparison



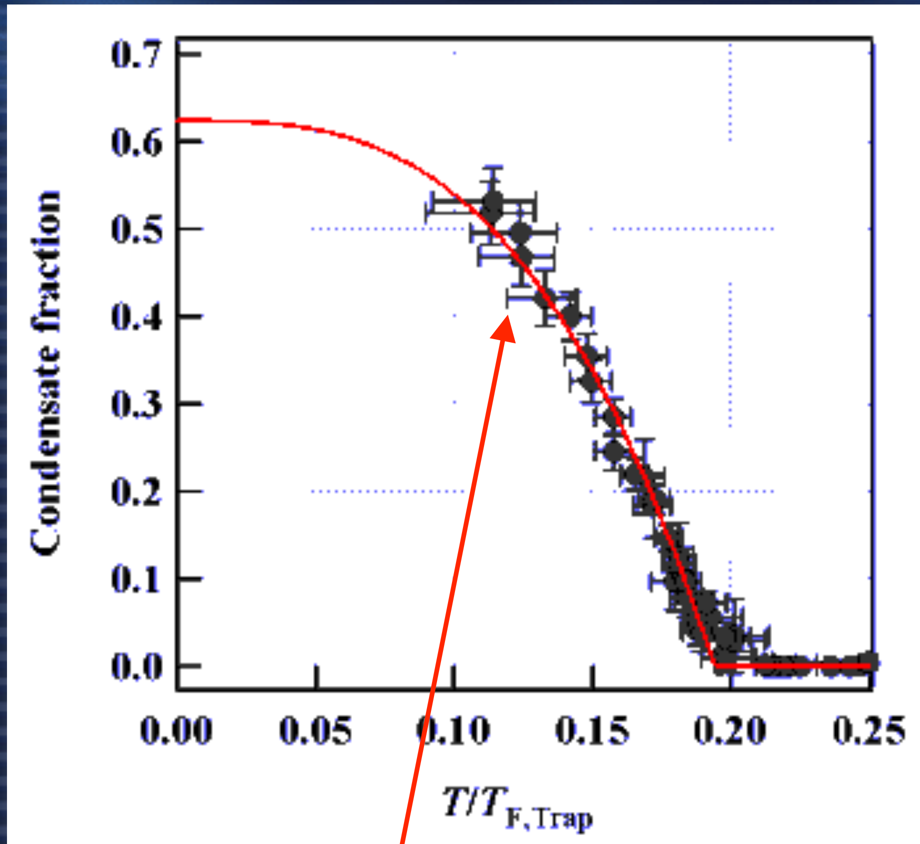
Speed of the first sound



Bimodal distribution of a fermion pair condensate



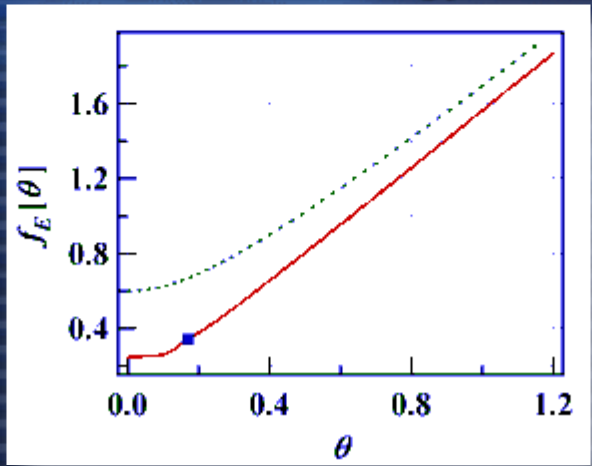
Condensate fraction vs Temperature



$$f(x) = CF_{\mathrm{Max}} \cdot \left\{ 1 - \left(\frac{T}{T_C} \right)^{3.0(1)} \right\}$$

Universal thermodynamic functions

Internal energy

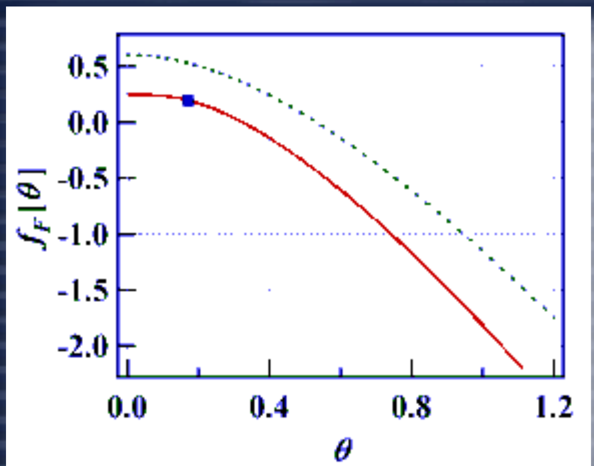


$$f_E = f_F - \theta f_F'$$

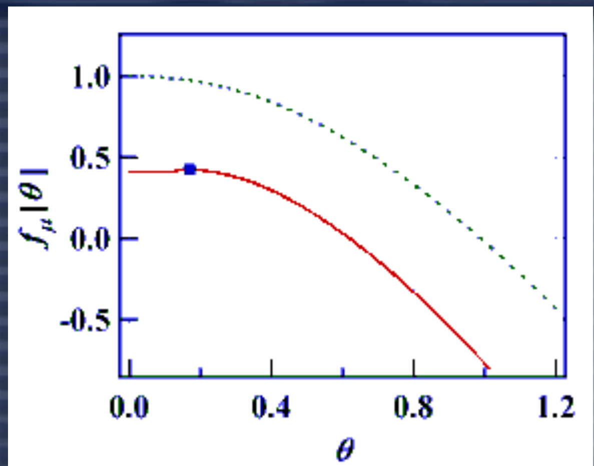
$$f_\mu = \left(5 f_E - 2\theta f_F'\right)/3$$

$$f_S = -f_F'$$

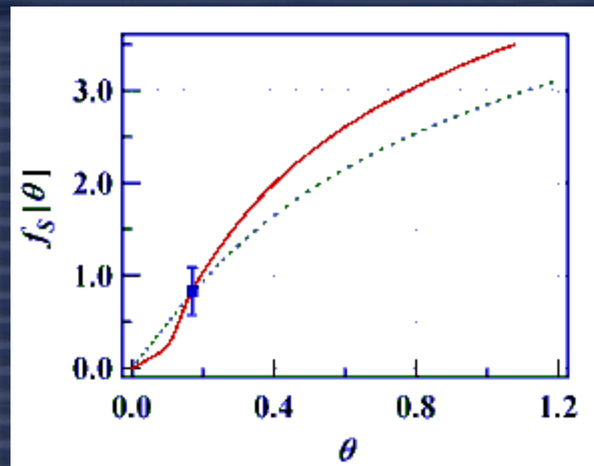
Helmholtz free energy



Chemical potential



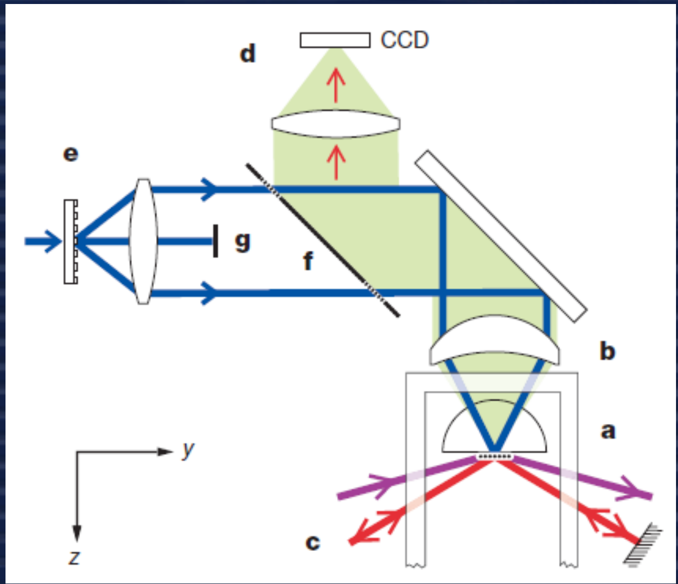
Entropy

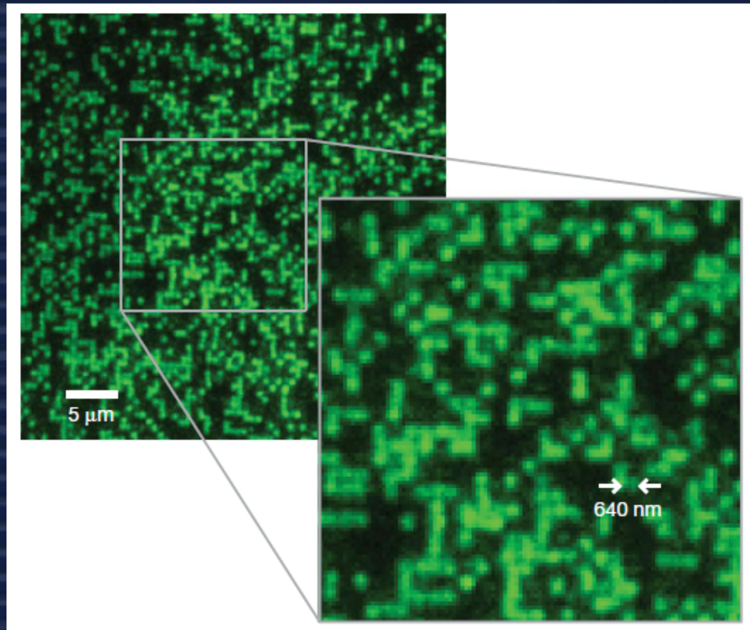


In the case of unitary gas, equation of state $p(\mathbf{r}) = 2\varepsilon(\mathbf{r})/3$ is available (exceptional case !!) which enable us to measure local thermodynamic quantities.

Then, how can we determine local thermodynamic quantities without help of equation of state ?

- Box potential
- High resolution local probe

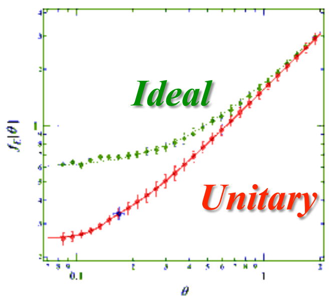




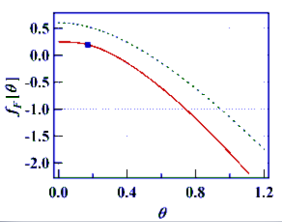
W. S. Bakr et al.Nature 462, 74 (2009).

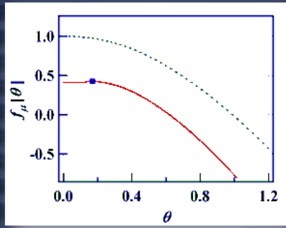
Summary

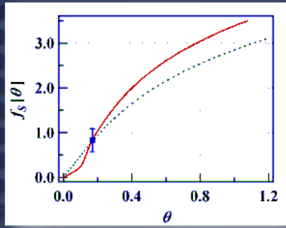
- The universal function of the internal energy was determined at the unitarity limit



- The other thermodynamic functions were derived from the thermodynamic relationship
- The critical parameters were determined at the superfluid transition temperature

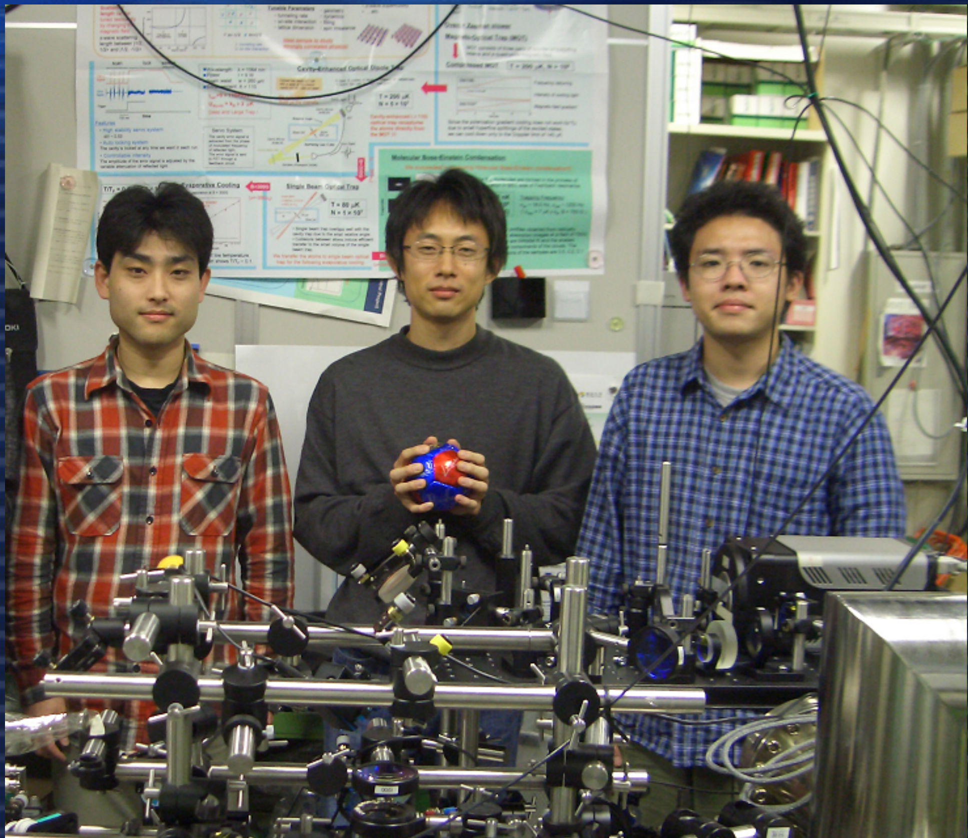






M. Horikoshi, S. Nakajima, M. Ueda and T. Mukaiyama, Science, **327**, 442 (2010).

The team (ERATO project)



T. Mukaiyama
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M. Horikoshi
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S. Nakajima
(Ph.D student)

Unitary gas

Efimov physics

Masahito Ueda
(project leader)

Equation of state for a unitary Fermi gas

$$\Delta E = \Delta N E_{\mathrm{F}}(n) f_E(T/T_{\mathrm{F}})$$
$$\Delta S = \Delta N k_{\mathrm{B}} f_S(T/T_{\mathrm{F}})$$

$$E_{\mathrm{F}}(n) = \frac{\hbar^2}{2m}(6\pi^2 n)^{2/3}$$

$$p = -\left(\frac{\partial(\Delta E)}{\partial(\Delta V)}\right)_{\Delta N, \Delta S}$$

ΔS=一定 ⟹ T/T_{F}=一定

$$p = -\left(\frac{\partial(\Delta E)}{\partial(\Delta V)}\right)_{\Delta N, T/T_{\mathrm{F}}} = -\Delta N f_E(T/T_{\mathrm{F}})\left(\frac{\partial(E_{\mathrm{F}}(n))}{\partial(\Delta V)}\right)_{\Delta N, T/T_{\mathrm{F}}}$$

$$= \frac{2}{3} n E_{\mathrm{F}}(n) f_E(T/T_{\mathrm{F}})$$

$$= \frac{2}{3}\varepsilon(n)$$