Transport coefficients of causal dissipative relativistic hydrodynamics in lattice gauge simulations

格子ゲージシミュレーションによる因果的散逸流体力学における輸送係数の計算

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**Introduction**

Properties of **Quark-Gluon Plasma**
- study early universe after Big Bang
- Heavy-ion collision experiments in RHIC and LHC

To extract significant information after the collision,

**Relativistic hydrodynamics**: indispensable

Key issue for reliable calculations
- **Transport coefficients** (viscosity, relaxation time ...)
  - determined from a microscopic theory

Propose: calculation of the transp. coeff. from lattice QCD simulations

**Topic of this talk**
- Introduction of causal dissipative relativistic hydrodynamics
  - how to determine trans. coeff. from microscopic theory
- Ratio of trans. coeff. $\frac{\eta}{\tau_\pi (\varepsilon + p)} \Rightarrow$ lattice gauge simulations
What's causal dissipative relativistic hydrodynamics?

**IF**: relativistic fluid is described by the **Navier-Stokes (NS) equation**.

Relativistic extension of the NS equation

\[ \pi^{\mu\nu} = \eta_{GKN} P^{\mu\nu\alpha\beta} \partial_\alpha u_\beta \]

\( \pi^{\mu\nu} \): share viscous pressure
\( \partial_\alpha u_\beta \): velocity gradient
\( \eta_{GKN} \): shear viscosity

In general, \( \pi^{\mu\nu} \propto \partial_\alpha u_\beta \): Newtonian

Transport coefficients: calculated by **Green-Kubo-Nakano (GKN) formula**.

\[ \eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\epsilon(t_1-t)} \int_{-\infty}^{t_1} dt' \langle T_{12}(x,t)T_{12}(x',t') \rangle_{\text{ret}} \]

Problems:
1) the NS eq. describes a signal propagation with infinite velocity.
2) the NS eq. is **unstable** and cannot describe equilibration processes.

→ Relativistic fluid is non-Newtonian
What's causal dissipative relativistic hydrodynamics?

Introduction of relaxation time $\tau_\pi$ : avoid the problems

Causal dissipative relativistic hydrodynamics

$$\tau_\pi P^{\mu\nu\alpha\beta} \frac{d}{d\tau} \pi_{\alpha\beta} + \pi^{\mu\nu} = \eta P^{\mu\nu\alpha\beta} \partial_\alpha u_\beta$$

note: $\eta$ does not correspond to $\eta_{GKN}$.

Microscopic formulae for transp. coeffs.


$$\frac{\eta}{\beta(\epsilon + p)} = \frac{\eta_{GKN}}{\beta^2 \int d^3x \langle T^{0x}(\vec{x}), T^{0x}(\vec{0}) \rangle}$$

$\eta_{GKN}$ : viscosity in GKN formula

$$\frac{\tau_\pi}{\beta} = \frac{\eta_{GKN}}{\beta^2 \int d^3x \langle T^{yx}(\vec{x}), T^{yx}(\vec{0}) \rangle}$$

$T^{\mu\nu}$ : energy-momentum tensor

$$\langle A,B \rangle = \int_0^\beta \frac{d\lambda}{\beta} \langle A(-i\lambda)B \rangle_{eq}$$

Determination of $\eta$ and $\tau_\pi$ from lattice QCD simulations
**GKN formula and our approach**

Studies of viscosity from GKN formula in lattice simulations


Temporal correlator of energy-momentum tensor

\[ G_{12}(t) = \left\langle \int d^3x T_{12}(x,t)T_{12}(0,0) \right\rangle \]

extract spectral function from large \( t \) behavior

very noisy and difficult!

Our “first” approach: focus on a ratio of transp. coeffs.

\[
\frac{\eta}{\sigma_\pi (\varepsilon + p)} = \frac{G^{yx}(T)}{G^{0x}(T)}
\]

\[ G^{\mu\nu}(T) \equiv T \int d^3x \int_0^{1/T} d\lambda \left\langle T_{\mu\nu}(-i\lambda, x)T_{\mu\nu}^\dagger (0,0) \right\rangle_{\text{eq}} \]

note: independent on the GKN formula

\[ \text{defined as an integral to spatial and temporal direction,} \]

\[ \text{we may extract clear signal without large statistics.} \]
Energy-momentum tensor on lattice

We consider pure gauge theory without dynamical quarks.

**EM tensor of gluon field**

\[
T_{\mu\nu}(x) = 2 \sum_\alpha \text{tr}[F_{\mu\alpha}F_{\nu\alpha}] - \frac{1}{2} \delta_{\mu\nu} \left( 1 + \frac{\beta(g)}{2g} \right) \sum_{\rho,\sigma} \text{tr}[F_{\rho\sigma}F_{\rho\sigma}]
\]

\( \beta(g) \): beta function on lattice

Boyd et al., NPB469 (1996) 419.

**Field strength tensor on lattice**

**Gluon field on lattice**

\[
U_\mu(n) = \exp[iaA_\mu(n)]
\]

\[
U_{\mu\nu}(n) = \exp[ia^2 F_{\mu\nu}(n_c) + O(a^3)]
\]

**Clover combination of plaquettes**

\[
Q_{\mu\nu}(n) = U_{\mu\nu}(n) + U_{\nu-\mu}(n) + U_{-\mu-\nu}(n) + U_{-\nu\mu}(n)
\]

\[
\frac{1}{8} \left[ Q_{\mu\nu}(n) - Q^{\dagger}_{\mu\nu}(n) \right] = ia^2 F_{\mu\nu}(n) + O(a^3)
\]
Lattice simulations at finite temperature

Parameter details

- Quenched simulation with standard plaquette action
- Lattice size: $N_s^3 \times N_t = 24^3 \times 4, 5, 6, 7, 8, 16, 24$
- Lattice coupling: $\beta = 6.0$ ($a = 0.093$ fm)
- $T / T_c = 0.5—2.0$ ($N_t = 24$ is regarded as $T = 0$)
- Statistics: 1000—4000 configurations

Behavior of EM tensor

Diagonal part: trace anomaly

$$\epsilon - 3p = \frac{T}{V} \int d^3x \int_0^{1/T} d\tau \sum_{\mu=1}^{4} \langle T_{\mu\mu}(x) \rangle_T$$

$\Rightarrow$ Zero $T$ subtraction

$$\epsilon - 3p = \left( \frac{T}{V} \int d^3x \tau \sum_{\mu=1}^{4} T_{\mu\mu}(x) \right)_{T} - \left( \frac{T}{V} \int d^3x \tau \sum_{\mu=1}^{4} T_{\mu\mu}(x) \right)_{T=0}$$

Typical behavior for trace anomaly, $\epsilon$ and $p$ is obtained.

Off-diagonal part: $\mu \neq \nu$

$$\frac{T}{V} \int d^3x \int_0^{1/T} d\tau \langle T_{\mu\nu}(x) \rangle_T \approx 0$$ due to discretized rotational symmetry on lattice.
Ratio of transport coefficients at finite temperature

\[ \frac{\eta}{\tau_\pi (\epsilon + p)} = \frac{G^{ij}_\pi(T)}{G_{i4}^{\pi}(T)} \]

\[ G^{\mu\nu}(T) \equiv T \int d^3x \int_0^{1/T} d\lambda \langle T^{\mu\nu}(-i\lambda, x) T^{\mu\nu}(0,0) \rangle_{\text{eq}} \]

\[ G_{\mu\nu}(T) \equiv V \left( \langle T^{\pi\mu}_\pi T^{\pi\nu}_\pi \rangle_T - \langle T^{\pi\mu}_\pi T^{\pi\nu}_\pi \rangle_{T=0} \right) \]

where \( T^{\pi\mu}_\pi \equiv \frac{T}{V} \int dx \int_0^{1/T} d\tau T^{\mu\nu}_\pi (\tau, x) \)

Correlator of EM tensor

\[ G_{ij}(T) = \frac{1}{3} \left[ G_{12} + G_{13} + G_{23} \right] \]

\[ G_{i4}(T) = \frac{1}{3} \left[ G_{41} + G_{24} + G_{34} \right] \]

\[ G^{ij}_\pi(T) \approx G_{i4}^{\pi}(T) \]

- \( \frac{\eta}{\tau_\pi (\epsilon + p)} \sim 1 \)
- enhancement at \( T = T_c \)?
Comparison with free gas theory

Free gas theory for boson


\[ m_B = 140 \text{ MeV} \]

In free boson gas case,

\[ T \int d^3x \int_0^{1/T} d\lambda \left\langle T^{yx}(-i\lambda, x) T^{yx}(0, 0) \right\rangle_{\text{eq}} = \frac{p}{\beta} \]

\[ T \int d^3x \int_0^{1/T} d\lambda \left\langle T^{0x}(-i\lambda, x) T^{0x}(0, 0) \right\rangle_{\text{eq}} = \frac{\epsilon + p}{\beta} \]

\[ \frac{\eta}{\tau_\pi(\epsilon + p)} \rightarrow \frac{p}{\epsilon + p} \quad \frac{\eta}{\tau_\pi(\epsilon + p)} \rightarrow \frac{1}{4} \]

at high \( T \) limit

Behavior is similar, but magnitude is large.

1. Ratio goes to zero at \( T \rightarrow 0 \)?
2. Ratio goes to 1/4 at high \( T \)?

Lattice gauge simulations

This work

\[ \frac{G_{ij}}{G_{ij}} \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \quad \circ \]

Simulations at wider \( T \) range.
Summary and outlook

Relativistic fluid should be a non-Newtonian fluid because of causality and stability.

→ the GKN formula is not applicable to calculate the transp. coeffs.


\[
\frac{\eta}{\beta(\varepsilon + p)} = \frac{\eta_{\text{GKN}}}{\beta^2 \int d^3x \left( T^{0x}(\vec{x}), T^{0x}(0) \right)} \quad \frac{\tau_\pi}{\beta} = \frac{\eta_{\text{GKN}}}{\beta^2 \int d^3x \left( T^{yx}(\vec{x}), T^{yx}(0) \right)}
\]

→ \( \frac{\eta}{\tau_\pi(\varepsilon + p)} \) can be calculated independently of the GKN formula.

Calculations of the ratio from correlators of EM tensor in lattice gauge simulations.

→ temperature dependence of the ratio
  • enhancement around \( T_c \)?
  • behavior is similar to free boson gas, magnitude is large.

Outlook

• Simulations at wider \( T \) range for a comparison with the free boson gas theory.

• Calculations of \( \eta_{\text{GKN}} \) from the GKN formula to determine \( \eta \) and \( \tau_\pi \), separately.