Netsuba workshop, Aug 30, 2010

Lattice Study of Quark Self-Energy in the Deconfined Phase

非閉じ込め相におけるクォーク自己エネルギーの格子QCDによる解析

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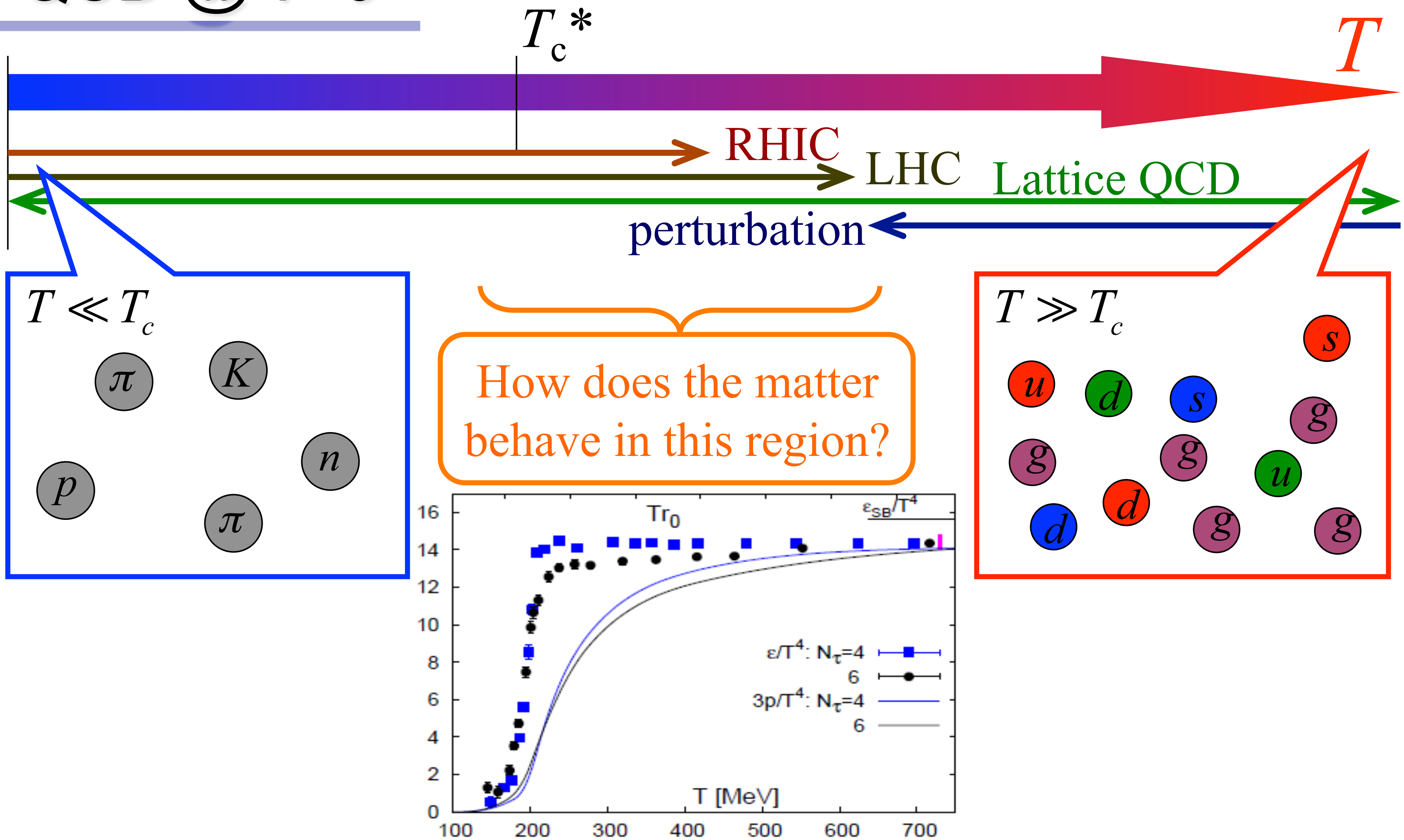
(Osaka U.)

O.Kaczmarek, F.Karsch, MK, W.Soeldner, in preparation;
MK, et al., in preparation.

QCD @ T>0



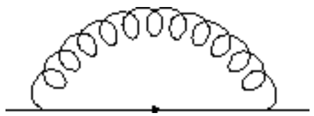
How do **hadrons** cease to exist?

How do **quarks and gluons** disappear?

Quarks at Extremely High T

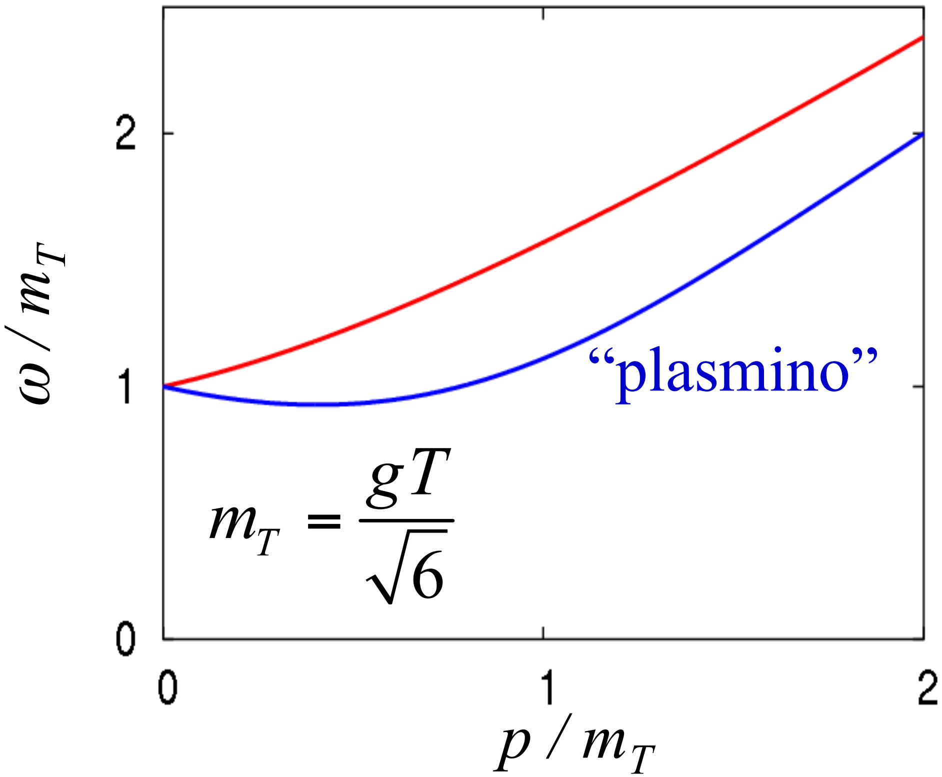
Klimov '82, Weldon '83
Braaten, Pisarski '89

- Hard Thermal Loop approx. ($\boldsymbol{p}$, ω, $m_q \ll T$)
- 1-loop ($g \ll 1$)

$\Sigma(\omega,\mathbf{p}) =$ 

$$S(\omega,\mathbf{p}) = \frac{1}{\omega\gamma_0 - \mathbf{p}\boldsymbol{\gamma} - \Sigma(\omega,\mathbf{p})}$$

- Gauge independent spectrum
- 2 collective excitations having a "thermal mass" $\sim gT$
- width $\sim g^2 T$
- The plasmino mode has a minimum at finite $\boldsymbol{p}$.



Decomposition of Quark Propagator

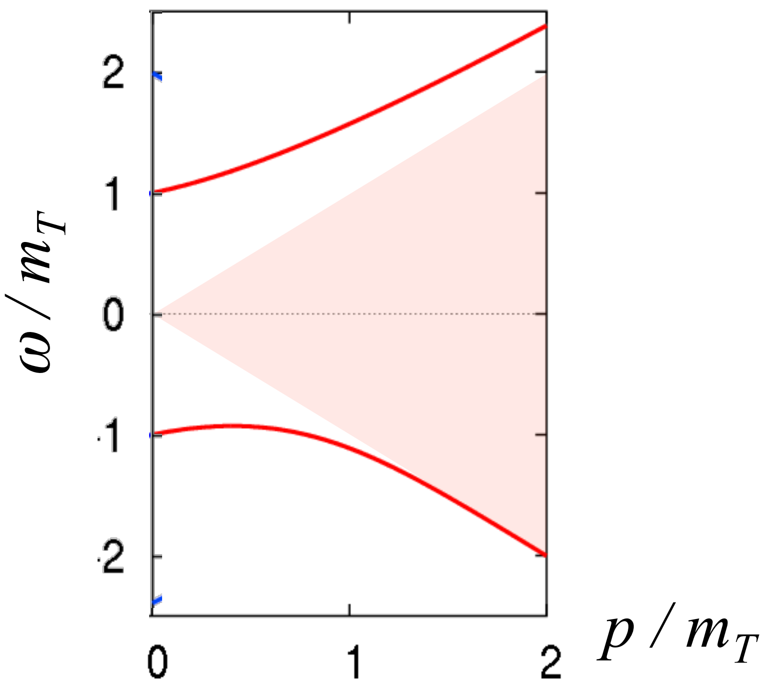
$$S(\omega,\mathbf{p}) = S_+(\omega,\mathbf{p})\Lambda_+(\vec{\mathbf{p}})\gamma^0 + S_-(\omega,\mathbf{p})\Lambda_-(\vec{\mathbf{p}})\gamma^0$$

$$\Lambda_\pm(\mathbf{p}) = \frac{E_\mathbf{p} \pm \gamma_0(\mathbf{p}\cdot\vec{\gamma} + m)}{2E_\mathbf{p}}$$

$$E_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$$

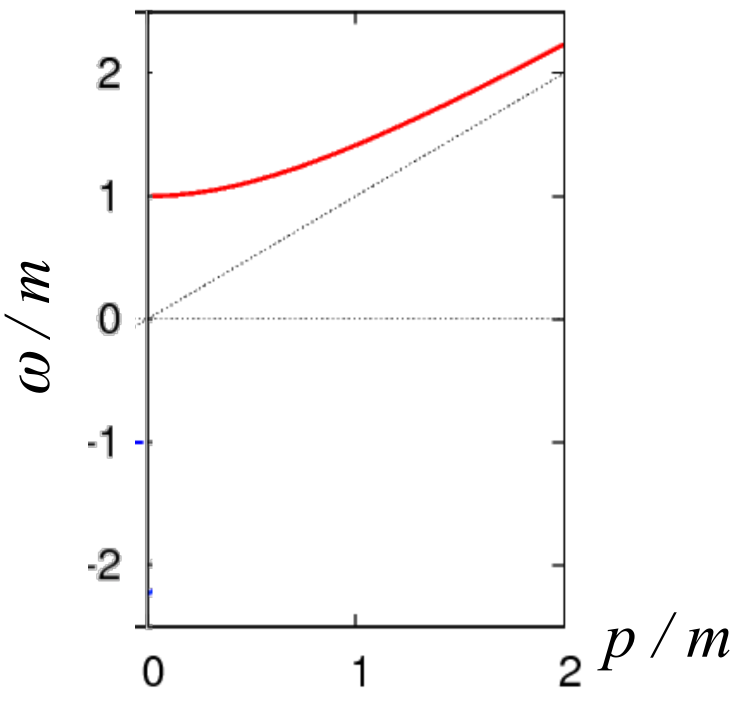
HTL (high T limit)

$$S_{\mathrm{HTL}}(\omega,\mathbf{p}) = \frac{L_+(\mathbf{p})\gamma^0}{\omega - p - \Sigma_+} + \frac{L_-(\mathbf{p})\gamma^0}{\omega + p - \Sigma_-}$$



Free quark with mass m

$$S_{\mathrm{free}}(\omega,\mathbf{p}) = \frac{\Lambda_+(\mathbf{p})\gamma^0}{\omega - E_\mathbf{p}} + \frac{\Lambda_-(\mathbf{p})\gamma^0}{\omega + E_\mathbf{p}}$$



Lattice Study of Quarks above T_c

Karsch, MK, '07, '09

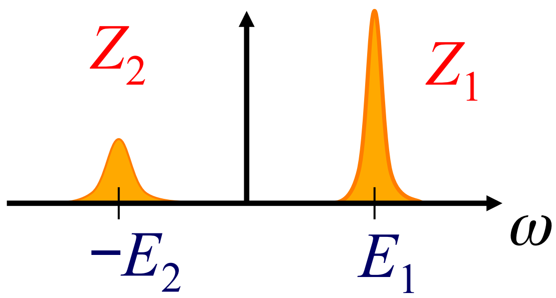
- quenched approximation
- clover improved Wilson
- Landau gauge fixing

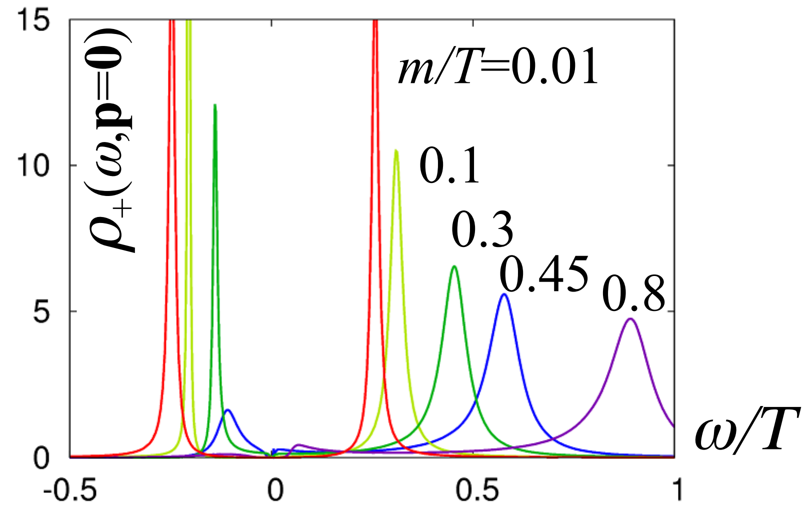
T/T_c	β	$N_x^3 \mathrm{x} N_t$
3	7.45	128^3x16
		64^3x16
		48^3x16
	7.19	48^3x12
1.5	6.87	128^3x16
		64^3x16
		48^3x16
	6.64	48^3x12
1.25	6.72	64^3x16
		48^3x16

- 2-pole ansatz for $\rho_+(\omega)$.

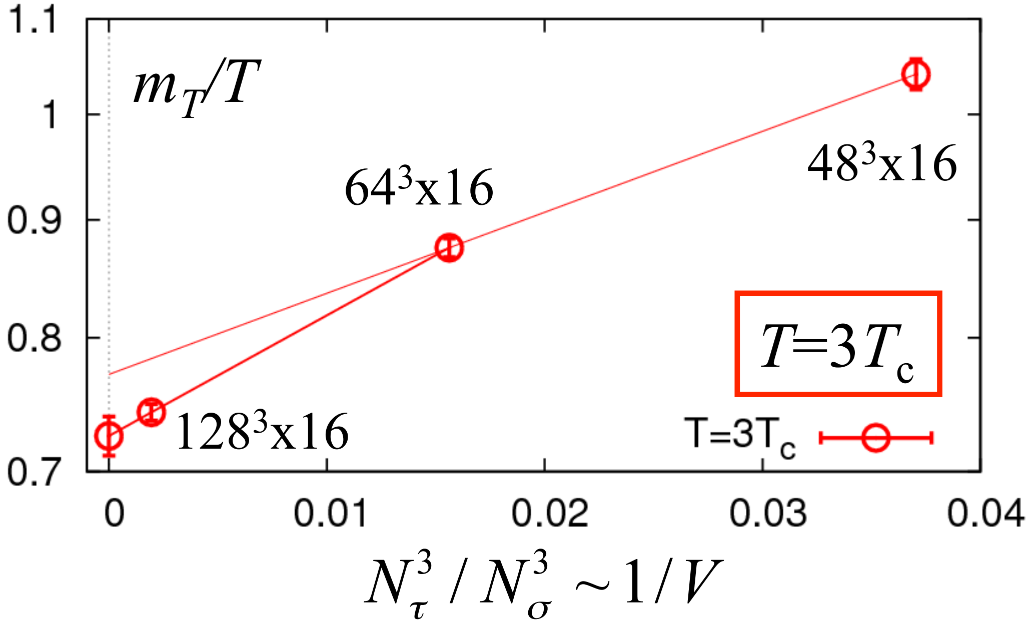
$$\rho_+(\omega) = Z_1\delta(\omega - E_1) + Z_2\delta(\omega + E_2)$$

4-parameter fit E_1, E_2, Z_1, Z_2





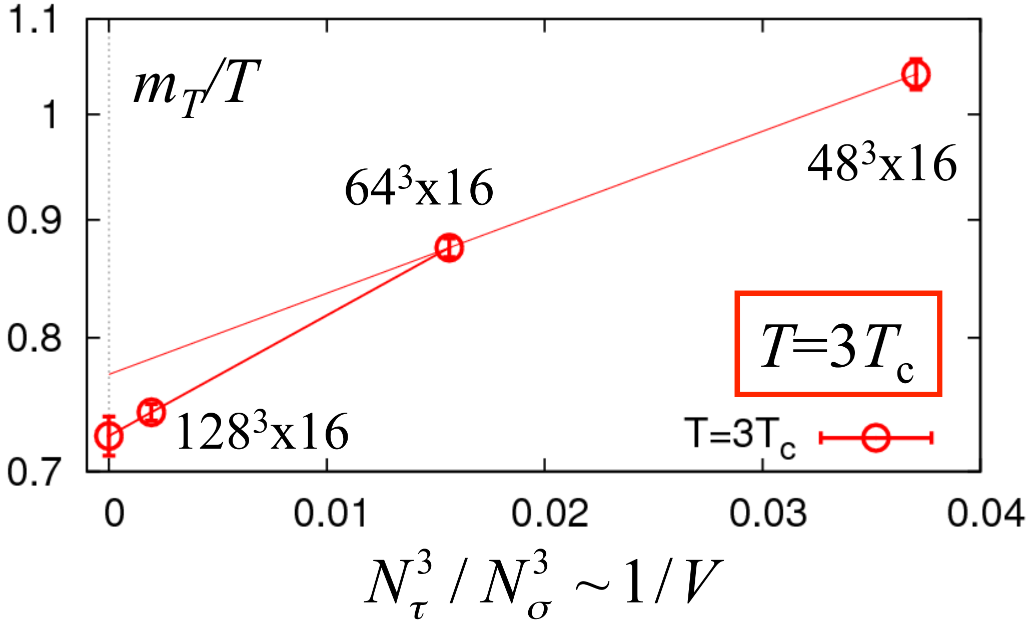
Spatial Volume Dependence of m_T



- Strong spatial volume dependence of m_T.

$m_T/T=0.725(14)$

Spatial Volume Dependence of m_T



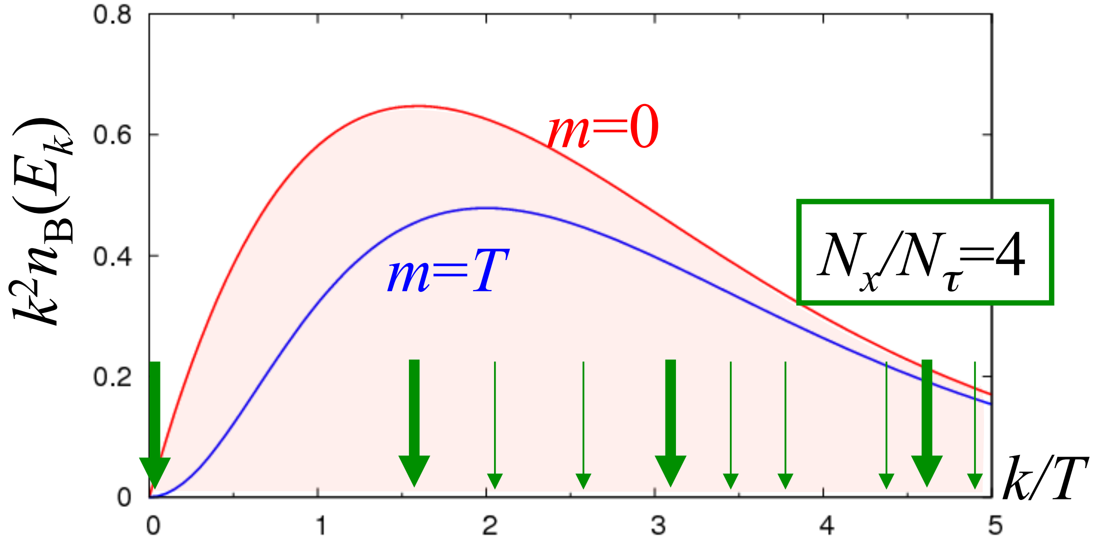
- Strong spatial volume dependence of m_T.

m_T/T=0.725(14)

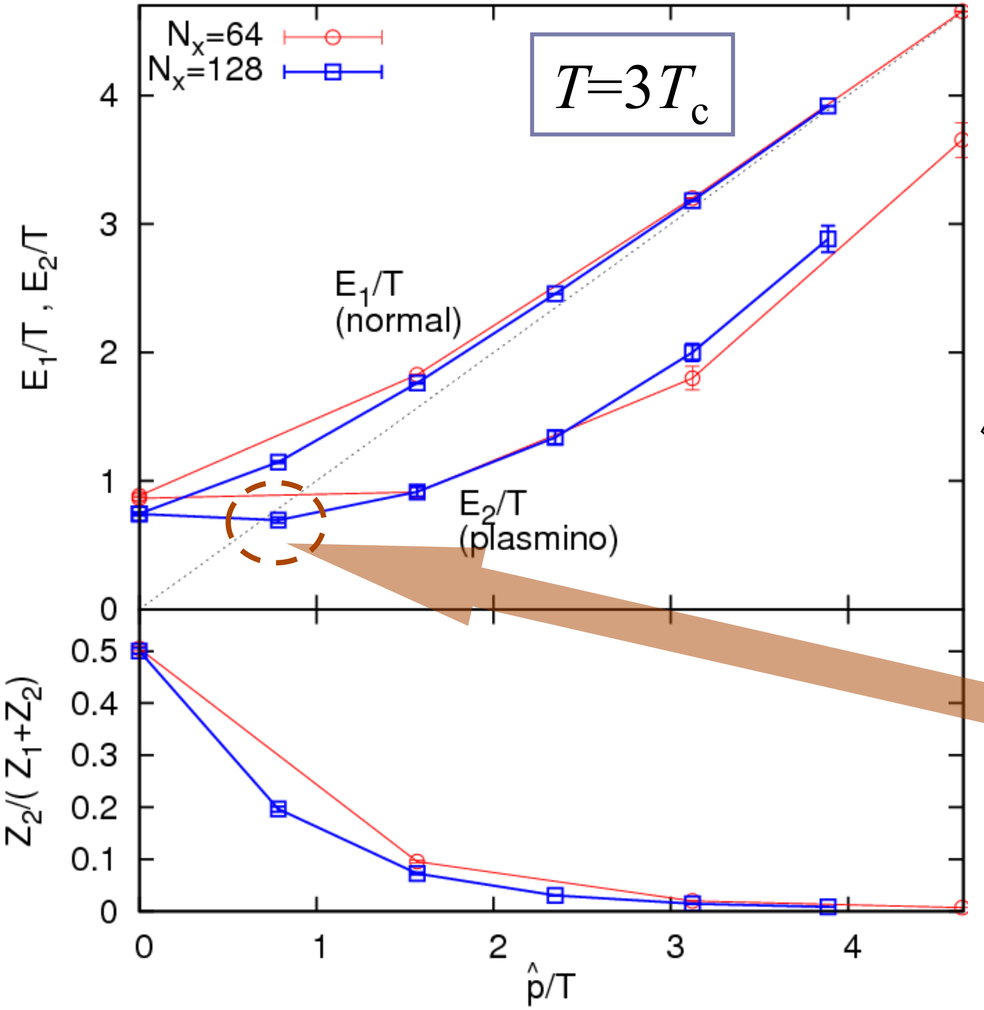
- Discretization of p

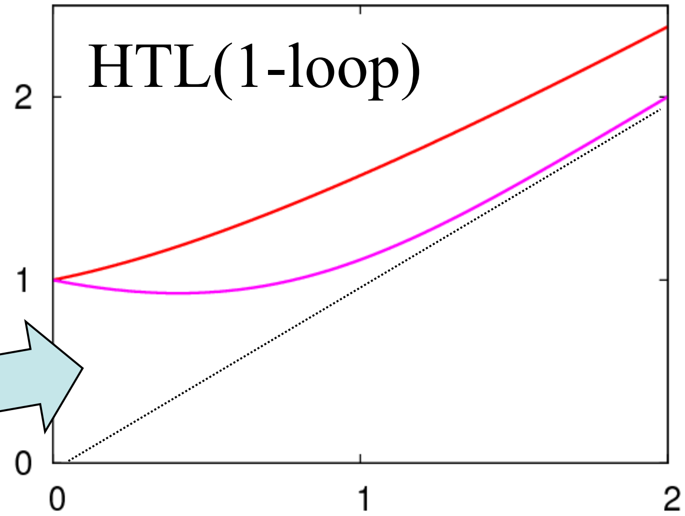
$$p_{\min} = \frac{2\pi}{L_x} = 2\pi T \frac{N_\tau}{N_x}$$

- $N_x/N_\tau=4 \rightarrow p_{\min}\sim 1.57T$
- $N_x/N_\tau=8 \rightarrow p_{\min}\sim 0.79T$



Quark Dispersion on 128^3x16 Lattice





E_2 has a minimum at p>0

- Existence of the plasmino minimum is strongly indicated.
- E_2, however, is not the position of plasmino pole.

Quark Self-Energy

$$S(\omega,\mathbf{p}) = \frac{1}{p\cdot\gamma - m - \Sigma(\omega,\mathbf{p})}$$

Im. part

Spectral function

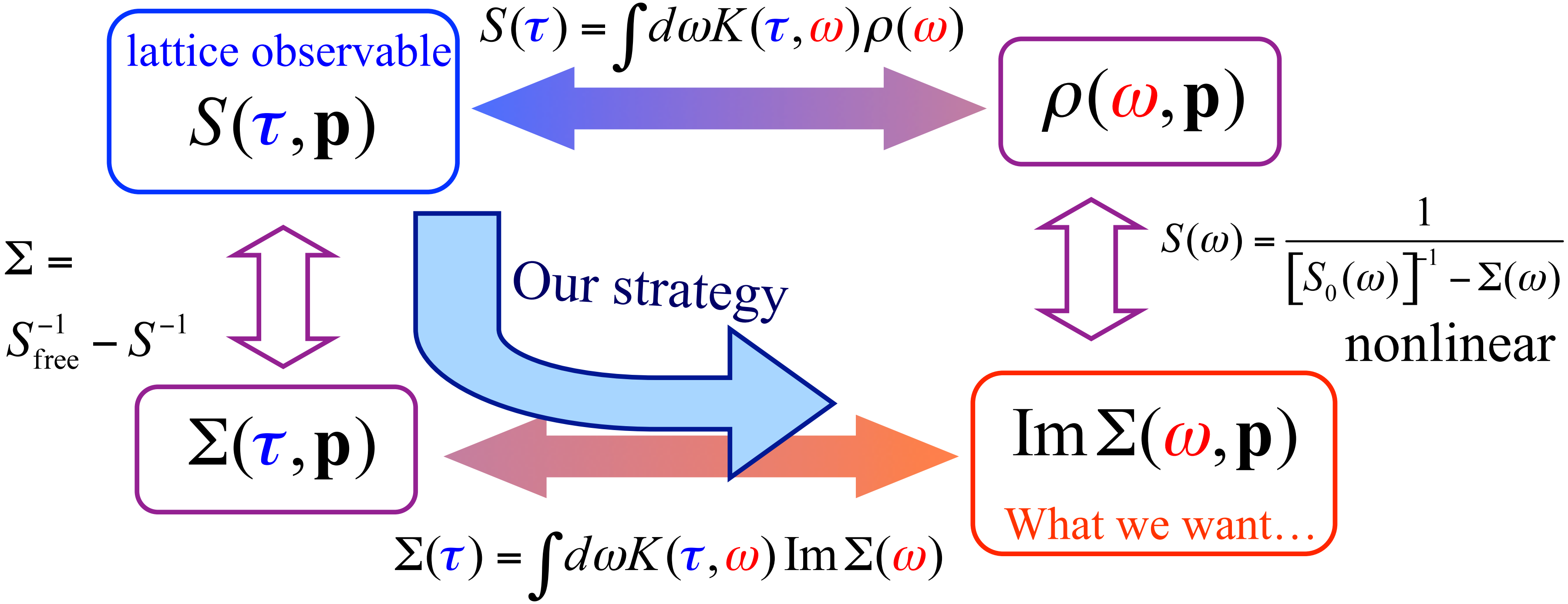
$$\rho(\omega,\mathbf{p}) = -\frac{1}{\pi}\,\mathrm{Im}\,S(\omega,\mathbf{p})$$

- experimental observable

Self-energy (ImΣ)

- decay rate of quasi-quark @pole
- Theoretically, ImΣ is useful to understand spectral properties.

Correlator & Self-Energy



$S(\tau,\mathbf{p}) \Rightarrow \Sigma(\tau,\mathbf{p})$

Schwinger-Dyson eq.: $S = S_{\rm free} + S_{\rm free}\Sigma S$

$\Sigma = S_{\rm free}^{-1} - S^{-1}$

$$\Sigma(\tau,\mathbf{p}) = -\left[S(\tau,\mathbf{p})\right]^{-1} + \left[S_{\rm free}(\tau,\mathbf{p})\right]^{-1}$$

Inverse matrix of lattice correlator (in τ space)

~~$S_{\rm free}(\tau) = Z\exp[-E\tau]$~~

~~$[S_{\rm free}(\tau)]^{-1} \sim (E + d/d\tau)\delta(\tau)$~~

~~vanish, unless $\tau = 0$~~

An algorithm:

$S(\tau) \overset{\rm F.T.}{\Leftrightarrow} S(i\omega_n)$

$\Downarrow$ inversion

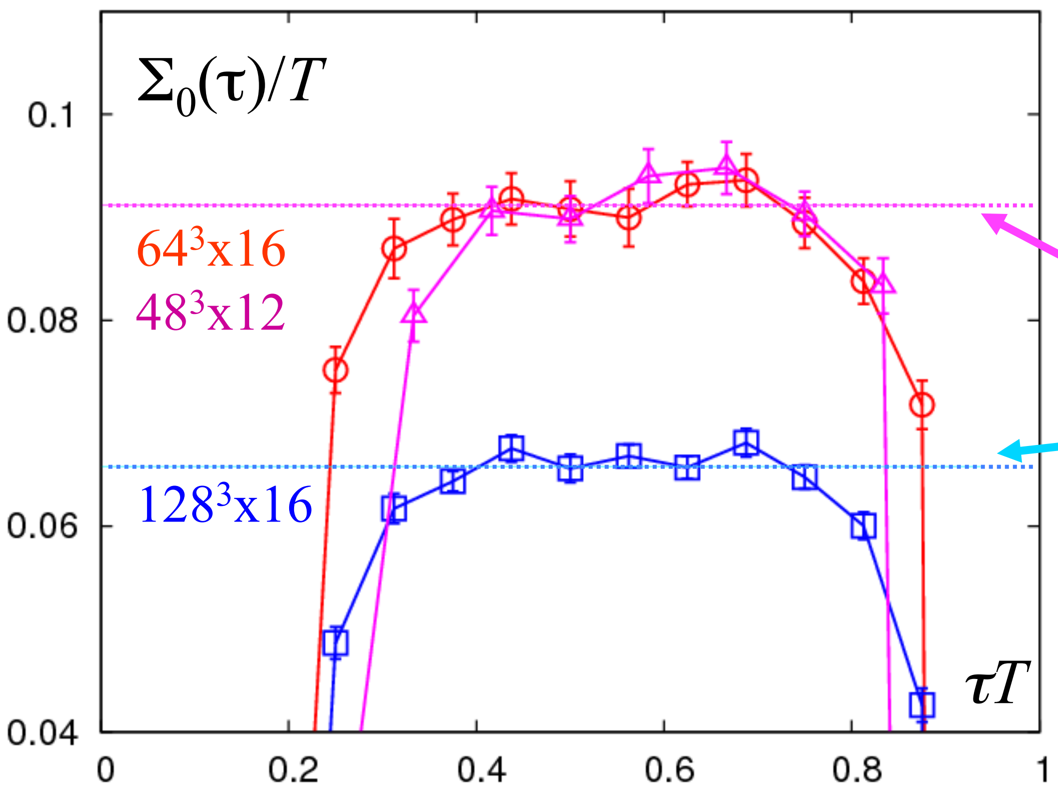
$\Sigma(\tau) \overset{\rm F.T.}{\Leftrightarrow} \Sigma(i\omega_n)$

Note: Σ(τ) is not the fermion matrix. Inverse, after statistical average.

Numerical Result: $\Sigma(\tau)$

$T=3T_c$, $p=0$, chiral limit

$$S = S_0\gamma^0 - S_V \vec{p}\cdot\vec{\gamma} + S_S$$



$N_\sigma/N_\tau=4$

$N_\sigma/N_\tau=8$

2 poles (HTL)

$$\Sigma_0(\omega) = m_T^2\delta(\omega) \Leftrightarrow \Sigma_0(\tau) = m_T^2/T$$

- $\Sigma(\tau)$ is consistent with the values expected from the pole ansatz (HTL self-energy) for $\tau T \sim 1/2$.
- Strong deviation near the source.

Summary

We analyzed the quark self-energy in imaginary time above T_c in quenched lattice QCD.

⇨ The behavior of $\Sigma(\tau)$ is consistent with the one predicted by the two-pole ansatz.

Future Plans

- MEM analysis of Im$\Sigma(\omega)$ with much larger N_τ.
- Calculation of decay rates of quasi-particles.
- Exploit the combined information of $\rho(\omega)$ and $\Sigma(\omega)$ in the analytic continuation of the propagator.
- Application to other channels, such as gluons, mesons, and etc.

Choice of Source

- Wall source, instead of point source

- point: $S(\mathbf{p}=\mathbf{0},\tau)=\sum_{\mathbf{x}}\langle\psi(\mathbf{x},\tau)\overline{\psi}(\mathbf{0},0)\rangle$
- wall : $S(\mathbf{p}=\mathbf{0},\tau)=\frac{1}{V}\sum_{\mathbf{x},\mathbf{y}}\langle\psi(\mathbf{x},\tau)\overline{\psi}(\mathbf{y},0)\rangle$

•same (or, less) numerical cost
•quite effective to reduce error!!

Quality of data on **128^{3}x16** lattice is about **3** times better than on **64^{3}x16**.

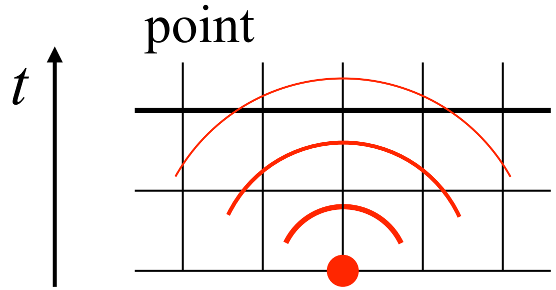
$\sim\sqrt{8}$

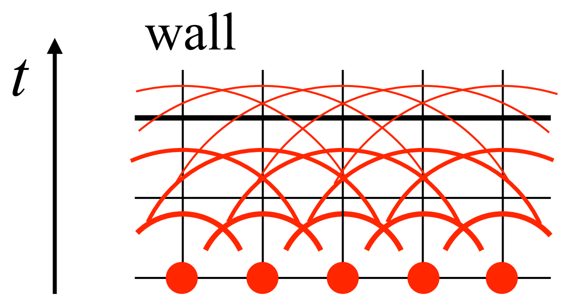
What's the source?

$$K = \not{D} - m_0$$

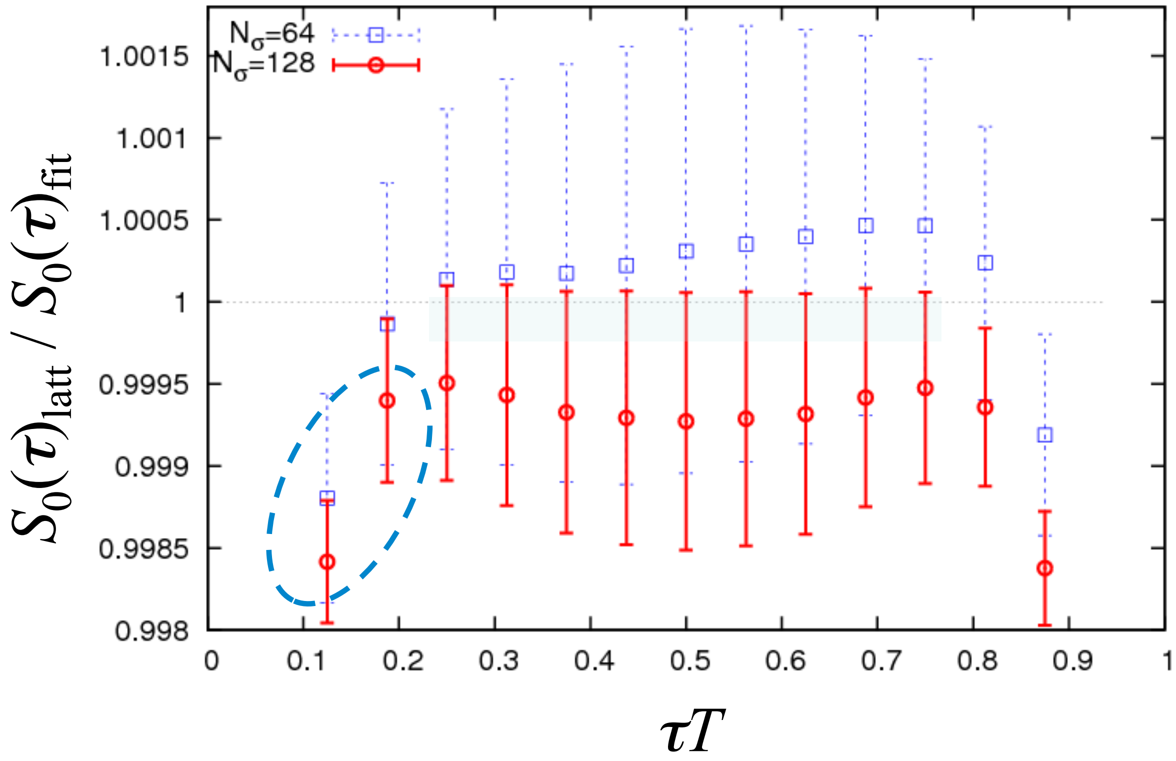
$$K\phi_{\text{result}} = \phi_{\text{source}}$$

$$\phi_{\text{result}} = K^{-1}\phi_{\text{source}}$$





Check 3 : Correlator & Fitting Result



128^3x16, 64^3x16,
T=3T_c, chiral limit,
N_{data}=9 for $S_0(\tau)_{fit}$

- It seems that there exists a contribution of a pole with an energy of order a^{-1} and with a negative residue.
- Due to this lattice artifact, datapoints near the source cannot be used.
- Much finer lattice is needed to improve N_{data}.

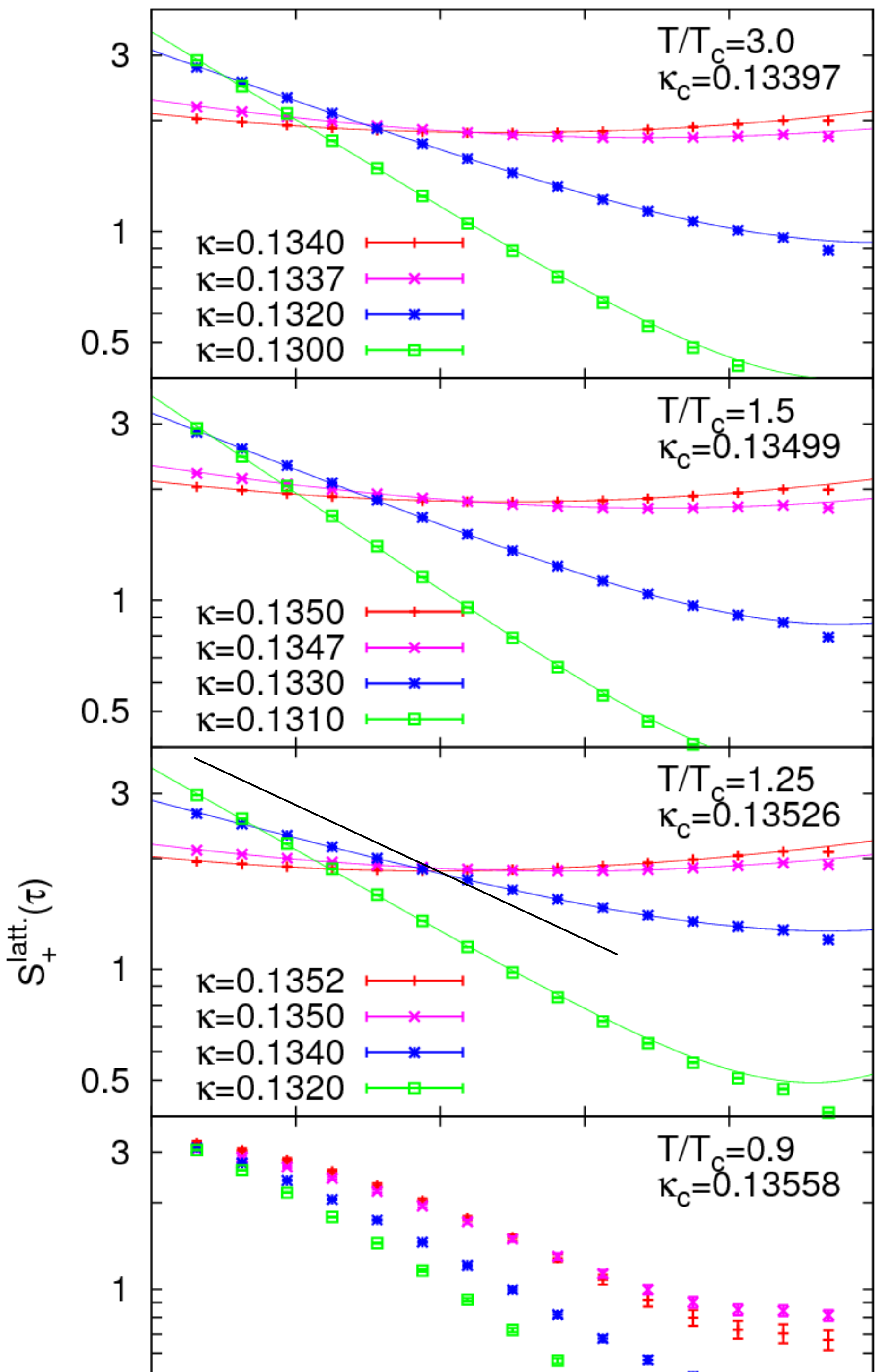
Below T_c

$C_+(\tau)$ for 48^3x16 lattices

T/T_c= 3, 1.5, 1.25,
0.93, 0.55.

Quark correlator below T_c

- is concave upward.
- indicates a negative norm.
- does not approach the chiral symm. one in the chiral limit.



Dirac Structure of Spectral Function

$$\rho(\mathbf{p},\omega) = \rho_0(\mathbf{p},\omega)\gamma^0 - \rho_V(\mathbf{p},\omega)\vec{p}\cdot\vec{\gamma} + \rho_S(\mathbf{p},\omega)$$

even / odd / odd

zero momentum

$$\rho(\mathbf{p},\omega) = \rho_0(\mathbf{p},\omega)\gamma^0 + \rho_S(\mathbf{p},\omega)$$
$$= \rho_+^M L_+\gamma^0 + \rho_-^M L_-\gamma^0$$

$$\rho_\pm^M = \rho_0 \pm \rho_S \qquad L_\pm = \frac{1\pm\gamma^0}{2}$$

chirally symmetric

$$\rho(\mathbf{p},\omega) = \rho_0(\mathbf{p},\omega)\gamma^0 - \rho_V(\mathbf{p},\omega)\vec{p}\cdot\vec{\gamma}$$
$$= \rho_+^P P_+\gamma^0 + \rho_-^P P_-\gamma^0$$

$$\rho_\pm^P = \rho_0 \pm \rho_V \qquad P_\pm = \frac{1\pm\gamma^0\vec{p}\cdot\vec{\gamma}}{2}$$

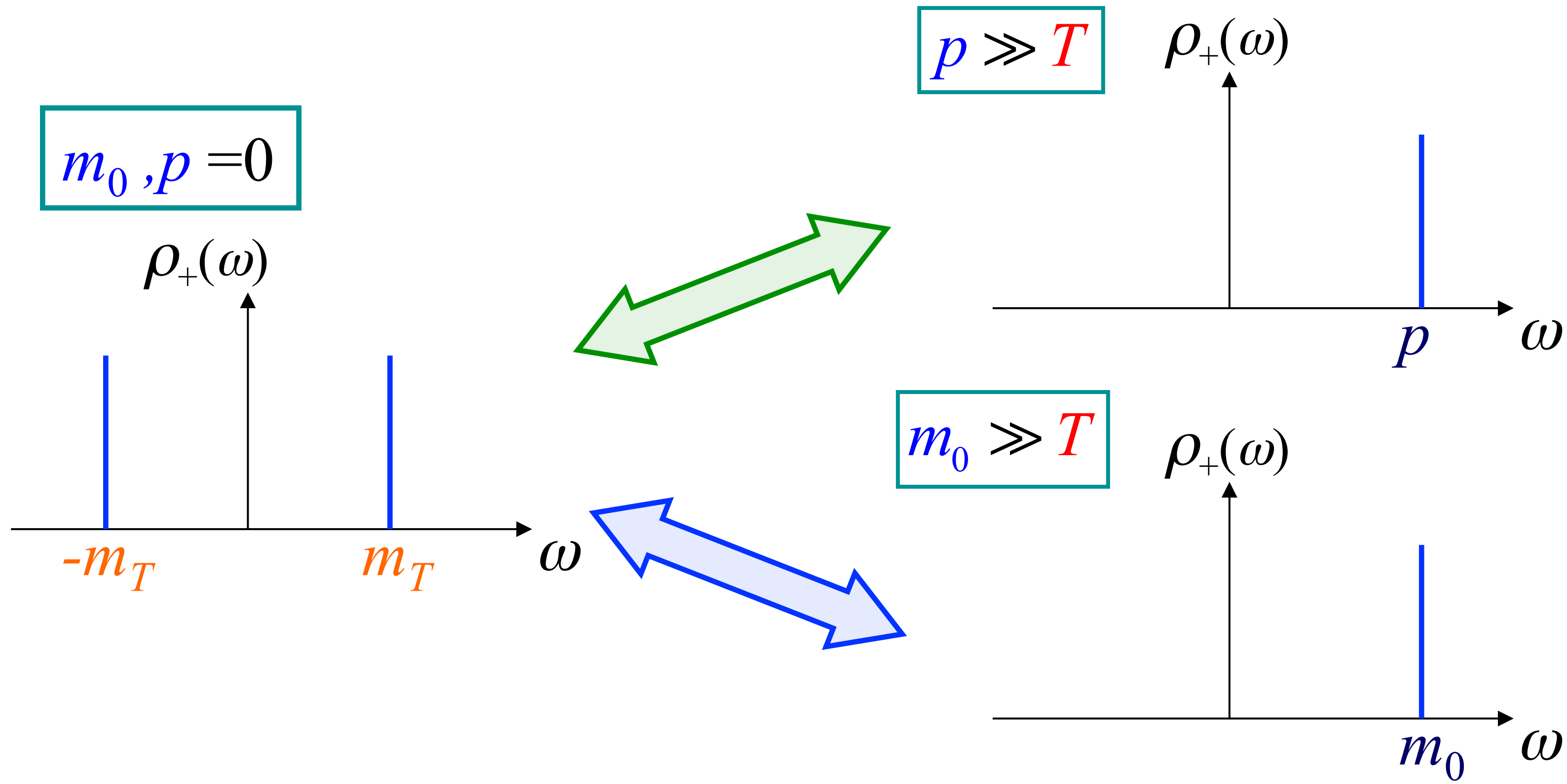
- positive definite
- not even nor odd
- even only with chiral symm.

Lattice Study of Quarks above T_c

Karsch, MK, '07, '09

$$\rho(\omega) = \rho_+(\omega)\Lambda_+\gamma^0 + \rho_-(\omega)\Lambda_-\gamma^0$$

- ✓ success of 2-pole ansatz
- ✓ emergence of thermal mass m_T~$0.8T$
- ✓ dispersion relation (p dependence)
- ✓ bare quark mass (m_0) dependence



Moment Expansion

$$S(\tau) = \int_{-\infty}^{+\infty} d\omega \frac{\cosh \omega(\tau - 1/2T)}{\cosh \omega / 2T} \rho(\omega)$$

Taylor expand $\cosh\omega(\tau\text{-}1/2T)$

$$S(\tau) = \sum_n (\tau - 1/2T)^n \frac{1}{n!} \int_{-\infty}^{+\infty} d\omega \left(\frac{\omega}{T}\right)^n \frac{\rho(\omega)}{\cosh \omega / 2T}$$

- If $\rho(\omega)$ vanishes for $|\omega|>\Lambda$, higher order terms behave $< \frac{1}{n!}\left(\frac{\Lambda}{2T}\right)^n$

Moment Expansion

$$S_0(\tau) = \int_{-\infty}^{+\infty} d\omega \frac{\cosh \omega(\tau - 1/2T)}{\cosh \omega/2T} \rho_0(\omega)$$

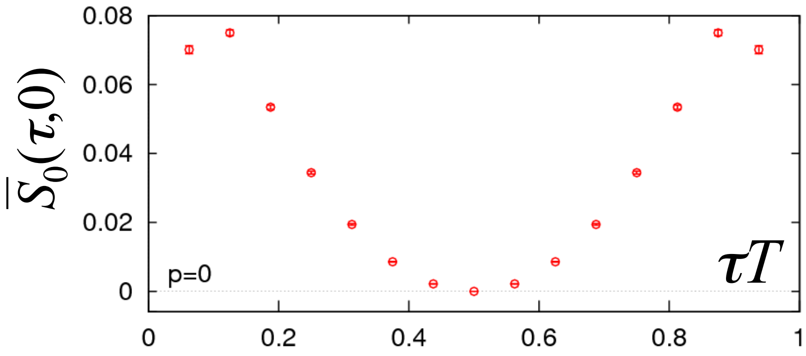
Taylor expand $\cosh\omega(\tau\text{-}1/2T)$

$$S_0(\tau) = \sum_n (\tau - 1/2T)^n \frac{1}{n!} \int_{-\infty}^{+\infty} d\omega \left(\frac{\omega}{T}\right)^n \frac{\rho_0(\omega)}{\cosh \omega/2T}$$

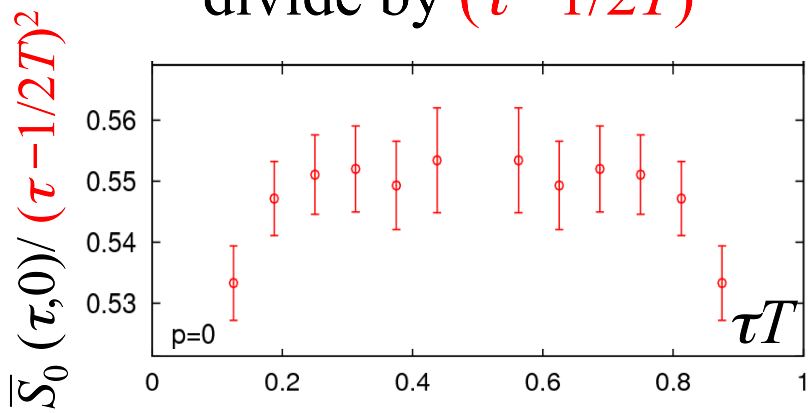
- If $\rho(\omega)$ vanishes for $|\omega|>\Lambda$, higher order terms behave $< \frac{1}{n!}\left(\frac{\Lambda}{2T}\right)^n$

Quark correlator for p=0, m_0=0

midpoint subtracted



divide by $(\tau - 1/2T)^2$



Quark Quasi-Particle Peaks

- 2-pole ansatz well reproduces $S(\tau)$ over wide ranges of m_0 & p.
  ⟺ Gaussian ansatz does not.
- MEM analysis also supports the existence of quasi-particle peaks.

➤Quark spectrum near but above T_c most probably has two sharp peaks (normal & plasmino).
➤Analysis with the 2-pole ansatz would grasp qualitative behavior of the spectrum.