Meson screening mass at finite temperature and density with two-flavor Wilson fermion

ー２フレーバーウィルソンフェルミオンを用いた
格子QCDによる有限温度・密度でのメソン遮蔽質量の研究 ー

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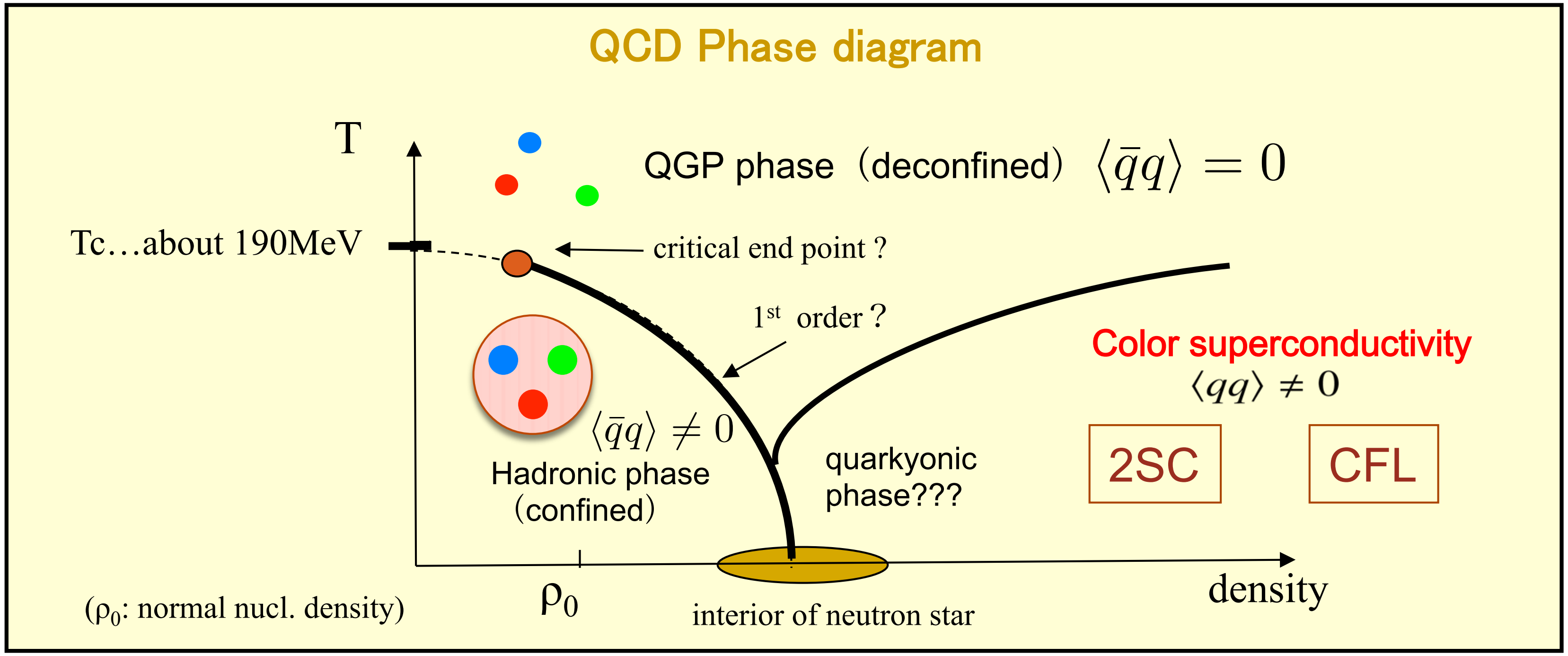
Aim: understanding of QCD phase diagram



Hadronic phase → QGP phase

- Chiral symmetry restoration
- Revelation of color degrees of freedom

… influence not only vacuum property
but also **mesonic excitation mode** ← our study

Hadrons as a probe of environment

- We study screening masses of mesons (PS and V) at finite temperature and density in lattice QCD.

Note) If the single particle picture is satisfied,
screening mass is the mass of a meson for a neutral meson in continuum limit.

1) screening masses at finite temperature (μ =0)

2) screening masses at finite density by Taylor expansion method

- We use the configurations of two-flavor Wilson fermion generated by WHOT-QCD collaboration.

… So far, dynamical calculation of screening mass has been performed
by staggered fermion (QCD-TARO collaboration, RBC-Bielefeld Collaboration,…)

To see (or reduce) the lattice artifact by the choice of lattice fermions,
calculation by dynamical Wilson fermion is important.

※ **Numerical calculation was performed on RIKEN Integrated Cluster of Clusters system. (RICC)**

- Gauge configurations along the lines of constant physics (m_{PS}/m_V constant)

→ Accurate calculation can be performed in the wide range of temperature.

- Action: RG improved gauge action & clover-improved Wilson quark action
- Lattice size & quark masses: $16^3 \times 4$, m_{PS}/m_V=0.65, 0.80
- Temperature: 0.82-4.02 (m_{PS}/m_V=0.65), 0.76-3.01 (m_{PS}/m_V=0.80)
- Number of configurations: 100 confs.

m_{PS}/m_V=0.65

β	K	T/Tpc	Traj.
1.50	0.150290	0.82(3)	5000
1.60	0.150030	0.86(3)	5000
1.70	0.148086	0.94(3)	5000
1.75	0.146763	1.00(4)	5000
1.80	0.145127	1.07(4)	5000
1.85	0.143502	1.18(4)	5000
1.90	0.141849	1.32(5)	5000
1.95	0.140472	1.48(5)	5000
2.00	0.139411	1.67(6)	5000
2.10	0.137833	2.09(7)	5000
2.20	0.136596	2.59(9)	5000
2.30	0.135492	3.22(12)	5000
2.40	0.134453	4.02(15)	5000

m_{PS}/m_V=0.80

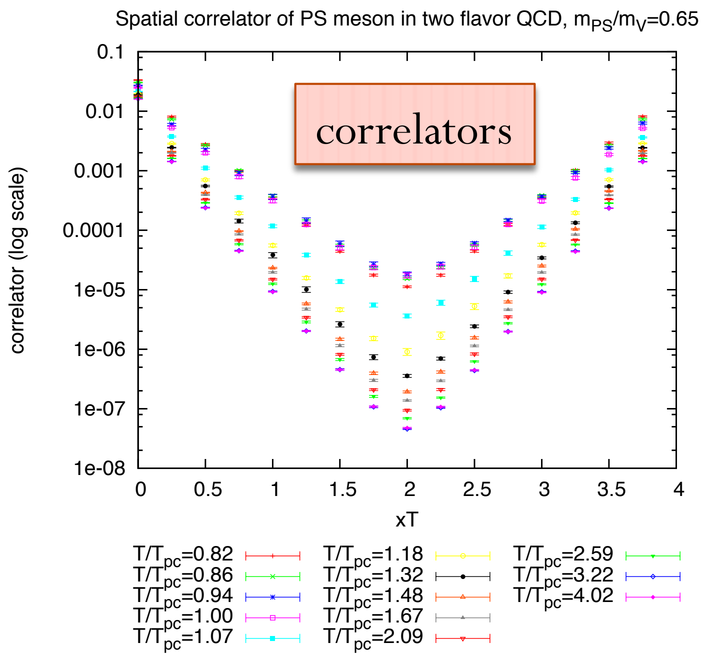
β	K	T/Tpc	Traj.
1.50	0.143480	0.76(4)	5500
1.60	0.143749	0.80(4)	6000
1.70	0.142871	0.84(4)	6000
1.80	0.141139	0.93(5)	6000
1.85	0.140070	0.99(5)	6000
1.90	0.138817	1.08(5)	6000
1.95	0.137716	1.20(6)	6000
2.00	0.136931	1.35(7)	5000
2.10	0.135860	1.69(8)	5000
2.20	0.135010	2.07(10)	5000
2.30	0.134194	2.51(13)	5000
2.40	0.133395	3.01(15)	5000

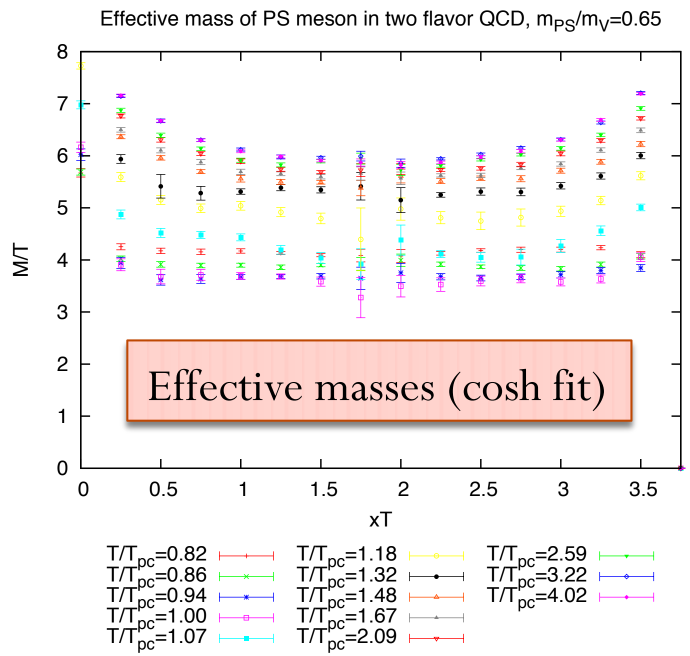
1) Finite temperature (μ=0)

We measure spatial correlators of mesons (M: meson operator):

$$G(x) \equiv \sum_{y,z,t} \langle M(x,y,z,t) M(0,0,0,0)^\dagger \rangle \quad (M(x,y,z,t) \equiv \bar{q}(x,y,z,t)\Gamma q(x,y,z,t))$$

Fitting to the functional form, $G(x) = A(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x - \hat{x})})$, we obtain the meson screening mass $\hat{M}$.

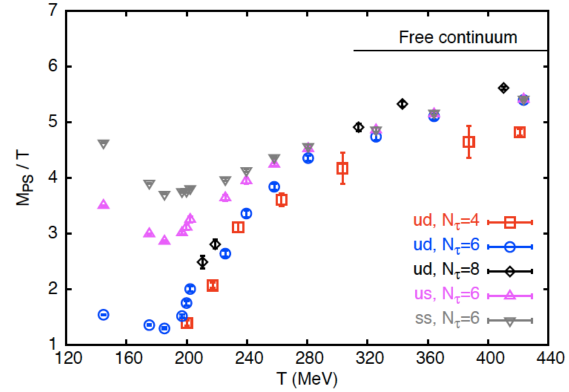




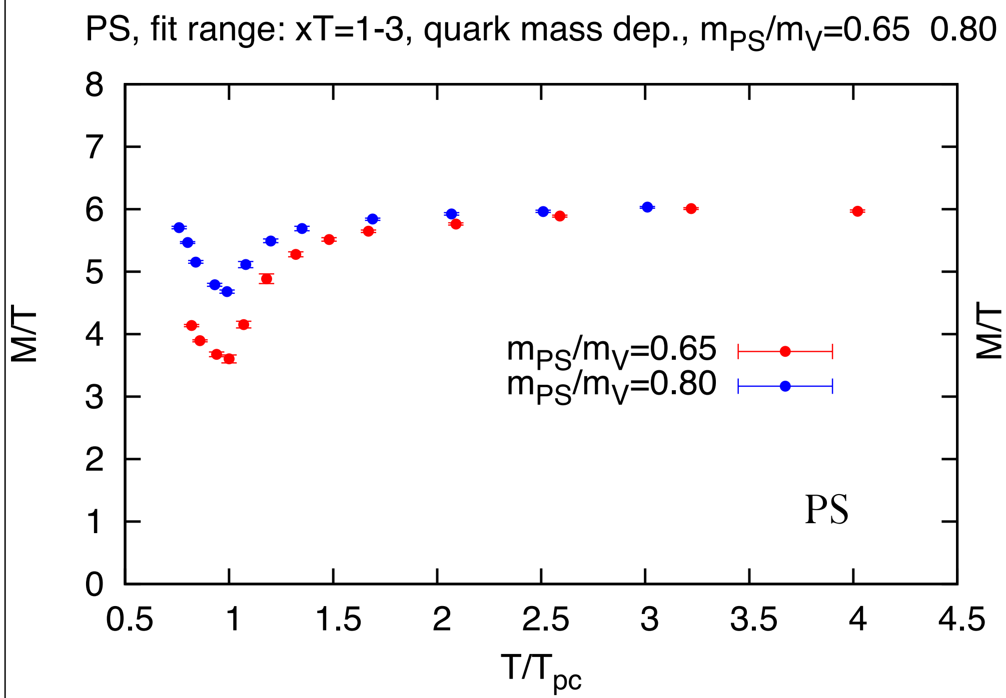
… Signals are clean at all temperatures

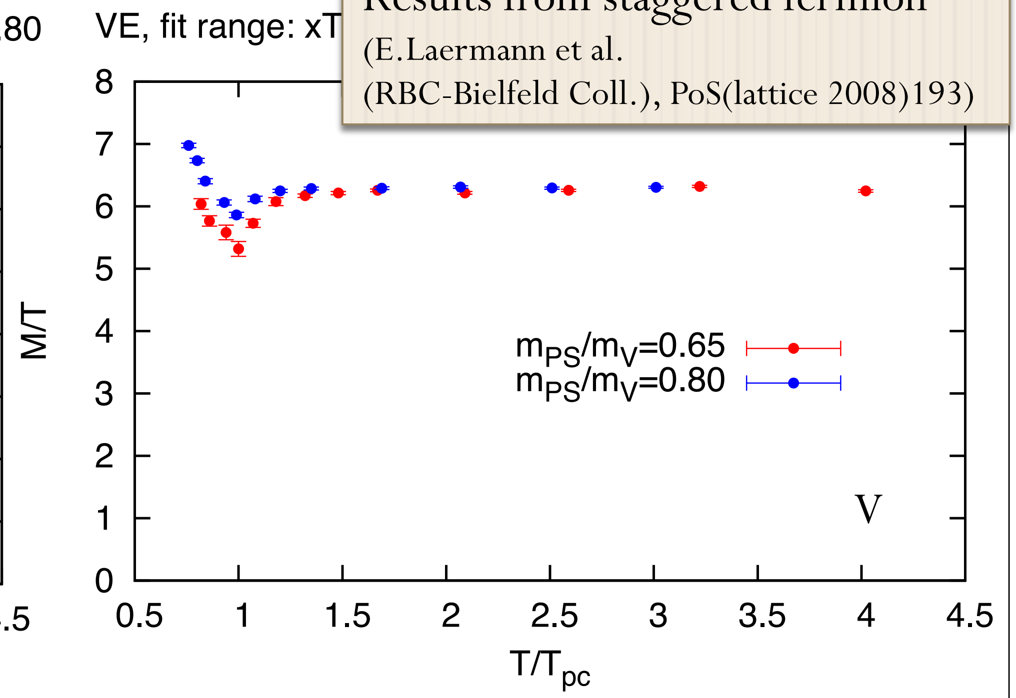
Results

- Temperature dependence of meson mass



Results from staggered fermion
(E.Laermann et al.
(RBC-Bielfeld Coll.), PoS(lattice 2008)193)

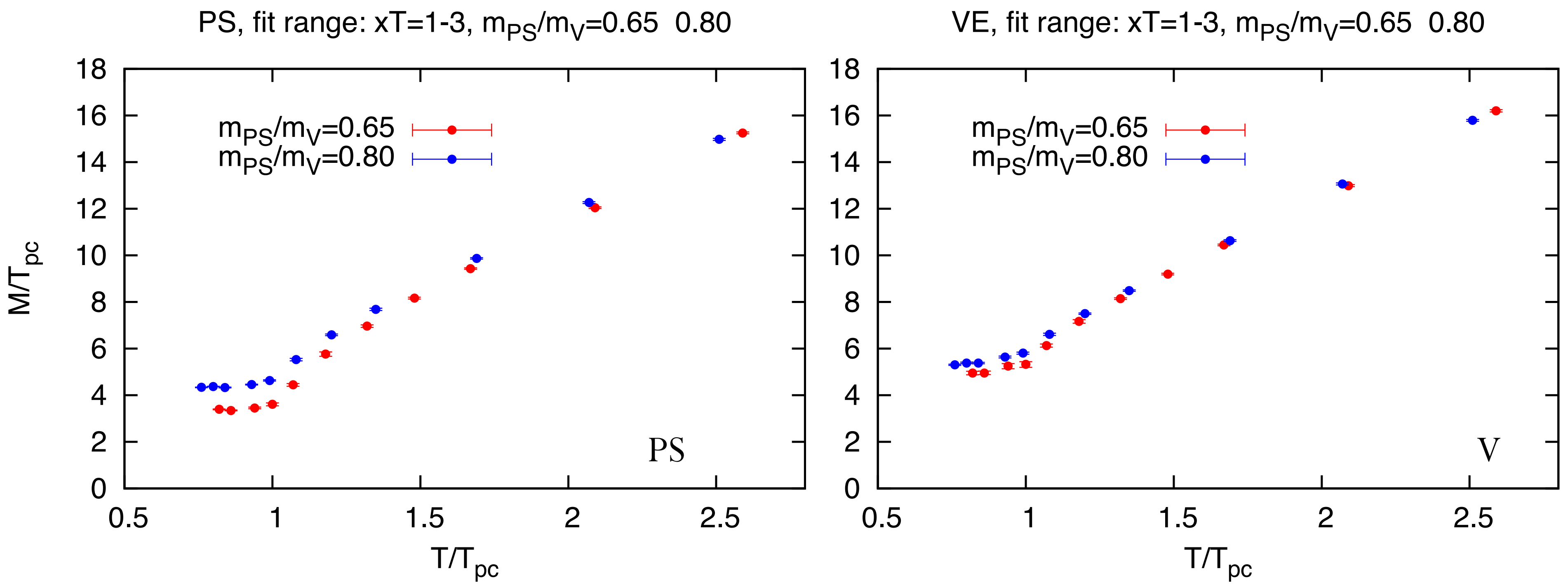




- There is a specific structure around Tc (in the plot of M/T)
- Meson masses become $2\pi T$ at high temperature
- Quark mass dependence of meson masses is larger in PS channel than V channel

Results

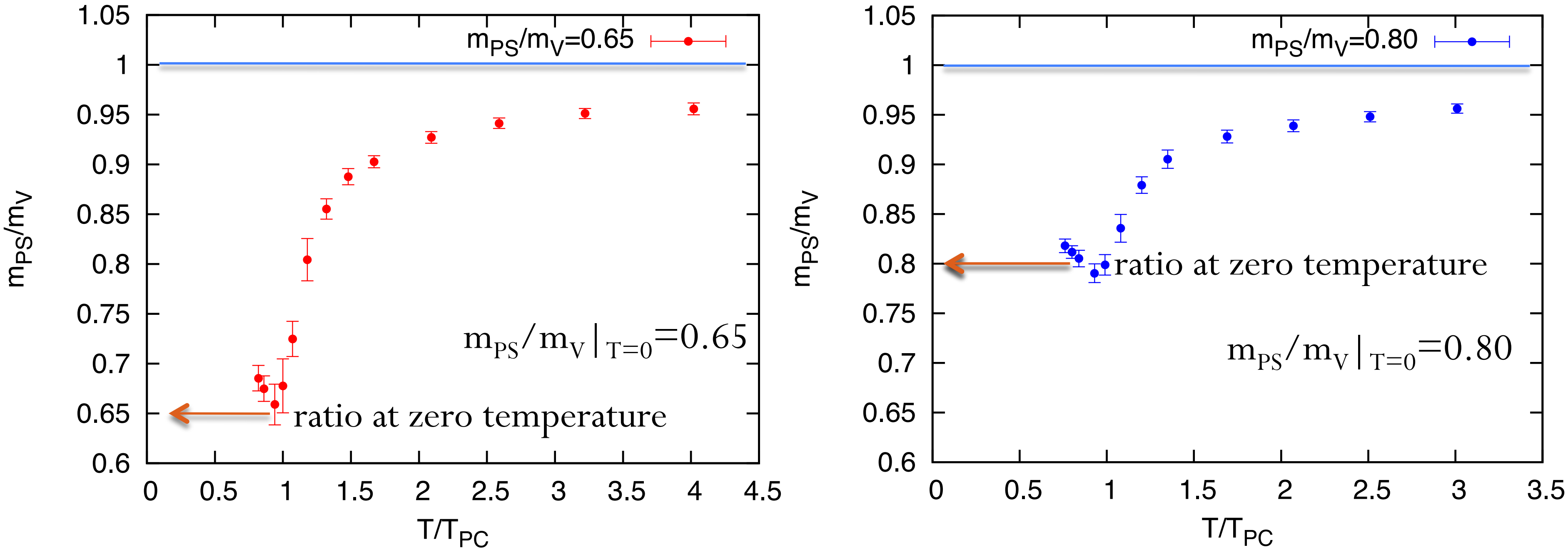
- Temperature dependence (M/Tpc).



- Meson screening mass increases very slowly below Tc, and rapidly above Tc.

Results

- Temperature dependence.



- At low temperature, the ratio is about 0.65 and 0.80 (ratio at T=0).
- Above Tc, the ratio quickly increases and approaches one.

→ Mesons are composed of a free quark and a free anti-quark, each of which has the thermal mass πT at high temperature.

Lattice QCD at finite density

- Difficulty at finite density…sign problem

$$Z = \int \mathcal{D}U \, \mathcal{O}(U,\mu) \det D(U,\mu) e^{-S_g(U)}$$

$(\det D(\mu))^{\dagger} = \det D(-\mu)$ … imaginary in general

  - (naïve) Important sampling is impossible due to imaginary part of determinant factor.
  - Due to the large fluctuation of fermionic determinant, huge amount of Monte Carlo samples are needed (sign problem).

We use Taylor expansion by quark chemical potential

…expansion by μ /T

2) Finite density (preliminary)

- We calculate **second response of meson masses to the isoscalar chemical potential** in **two-flavor Wilson fermion by Taylor expansion method**

Taylor expansion method ref.) S.Choe et al., PRD65, 054501 (2002)…staggered

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}U e^{-S} (\det D(\mu))^2 \mathcal{O}}{\int \mathcal{D} e^{-S} (\det D(\mu))^2} \qquad (\ \mu \equiv \mu_u = \mu_d \ \ldots \text{ isoscalar chemical potential })$$

$$= \frac{\langle (\mathcal{O} + \dot{\mathcal{O}}\mu + \frac{1}{2}\ddot{\mathcal{O}}\mu^2 + O(\mu^3))(1 + \frac{\dot{\Delta}}{\Delta}\mu + \frac{\ddot{\Delta}}{\Delta}\mu^2 + O(\mu^3)) \rangle}{1 + \langle \frac{\dot{\Delta}}{\Delta} \rangle \mu + \frac{1}{2} \langle \frac{\ddot{\Delta}}{\Delta} \rangle \mu^2 + O(\mu^3)} \qquad (\ \Delta \equiv (\det \mathrm{D}(\mu))^2|_{\mu=0}\ )$$

−We take $\mathcal{O}$ as the meson correlator G for isoscalar chemical potential:

$$G \equiv \mathrm{tr}(D^{-1}_{x0}(\mu)\Gamma D^{-1}_{0x}(\mu)\Gamma^\dagger) = \mathrm{tr}(D^{-1}_{x0}(\mu)\Gamma\gamma_5(D^{-1}(-\mu))^\dagger_{x0}\gamma_5\Gamma^\dagger)$$

$$\rightarrow \ \text{2nd order}: \langle \dot{G}\frac{\dot{\Delta}}{\Delta} \rangle + \frac{1}{2}\langle \ddot{G} \rangle + \frac{1}{2}\langle G\frac{\ddot{\Delta}}{\Delta} \rangle - \frac{1}{2}\langle G \rangle \langle \frac{\ddot{\Delta}}{\Delta} \rangle \qquad (\text{Note: } \langle \frac{\dot{\Delta}}{\Delta} \rangle = 0,\ \langle G\frac{\dot{\Delta}}{\Delta} \rangle = 0\ )$$

Finite density

- **Leading:**

$$\langle G\rangle|_{\mu=0} = \langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger]\rangle \ldots \text{ already shown}$$

- **Second derivative:**

$$\begin{aligned}\frac{d^2}{d\mu^2}\mathrm{Re}\langle G\rangle|_{\mu=0} =& 4\langle \mathrm{Retr}[(D^{-1}\dot D D^{-1}\dot D D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger]\rangle \\ &-2\langle \mathrm{Retr}[(D^{-1}\ddot D D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger]\rangle \\ &-2\langle \mathrm{Retr}[(D^{-1}\dot D D^{-1})_{x0}\Gamma\gamma_5(D^{-1}\dot D D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger]\rangle \\ &+8\langle \mathrm{Imtr}[(D^{-1}\dot D D^{-1})_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger]\cdot \mathrm{ImTr}(D^{-1}\dot D)\rangle \\ &+2\mathrm{Re}\{\langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger](2(\mathrm{Tr}(D^{-1}\dot D))^2 - \mathrm{Tr}(D^{-1}\dot D D^{-1}\dot D) + \mathrm{Tr}(D^{-1}\ddot D))\rangle \\ &\quad -\langle \mathrm{tr}[D^{-1}_{x0}\Gamma\gamma_5(D^{-1})^\dagger_{x0}\gamma_5\Gamma^\dagger]\rangle\langle 2(\mathrm{Tr}(D^{-1}\dot D))^2 - \mathrm{Tr}(D^{-1}\dot D D^{-1}\dot D) + \mathrm{Tr}(D^{-1}\ddot D)\rangle\}\end{aligned}$$

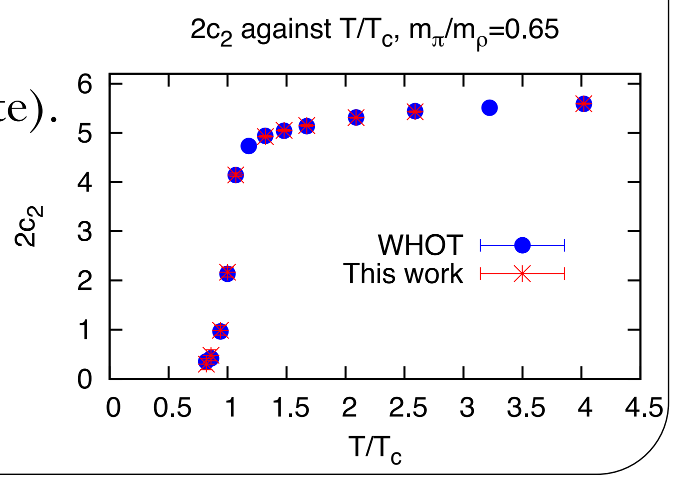
(first three terms) $\frac{1}{2}\langle\ddot G\rangle$; (fourth term) $\langle \dot G \frac{\dot\Delta}{\Delta}\rangle$; (last terms) $\frac{1}{2}\left(\langle G\frac{\ddot\Delta}{\Delta}\rangle - \langle G\rangle\langle\frac{\ddot\Delta}{\Delta}\rangle\right)$

※ Tr denotes **trace including space-time coordinate**.

Noise method is adopted (to take trace for spatial coordinate).

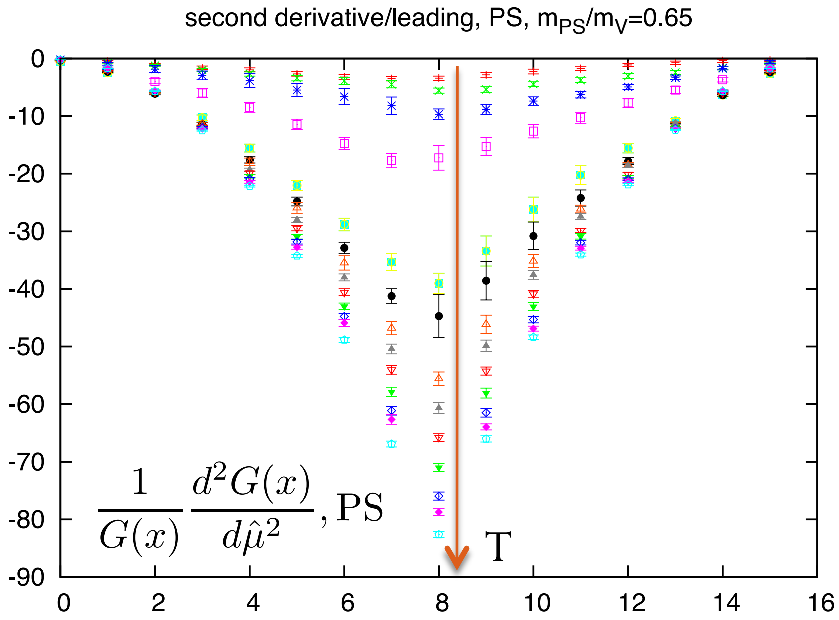
$$\mathrm{Tr}(A) \simeq \frac{1}{N_{\rm noise}}\sum_{i=1}^{N_{\rm noise}}\sum_{it,a,\alpha}^{N_t,3,4}\eta^\dagger_{i,it,a,\alpha}A\,\eta_{i,it,a,\alpha}$$

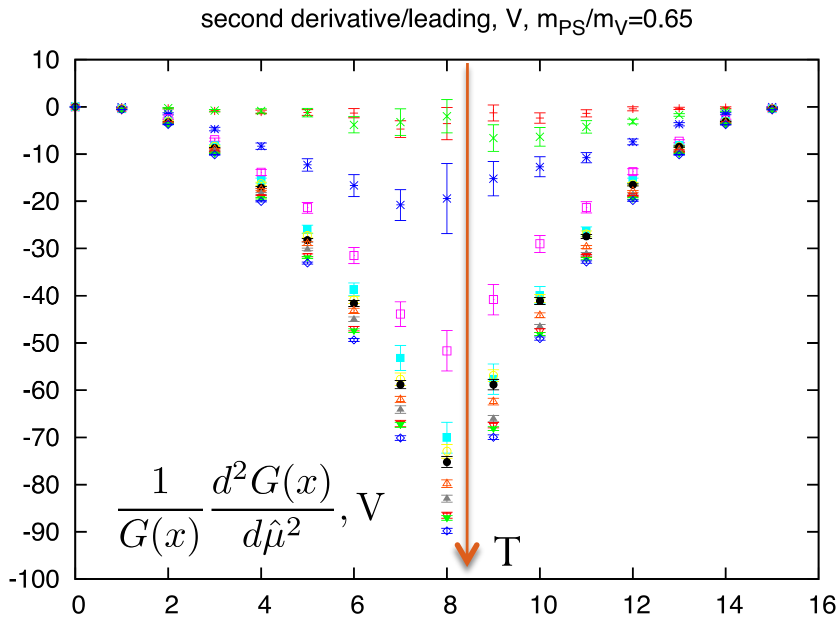
$$\frac{1}{N_{\rm noise}}\sum_{i=1}^{N_{\rm noise}}\eta(i,x)\eta^*(i,y)\simeq\delta_{x,y}$$ …100 noises, U(1)



Second derivatives of correlators

Preliminary





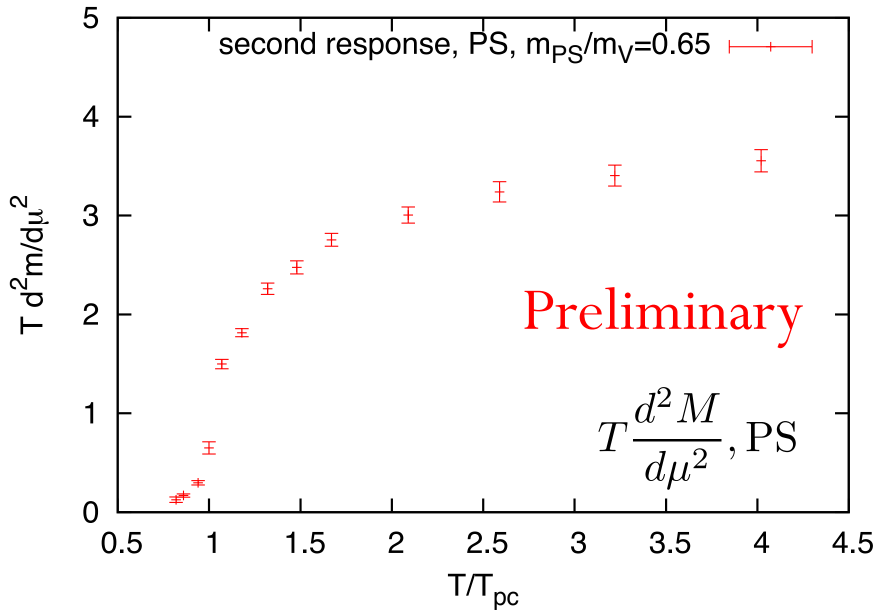
We fit the correlators by the following functional form:

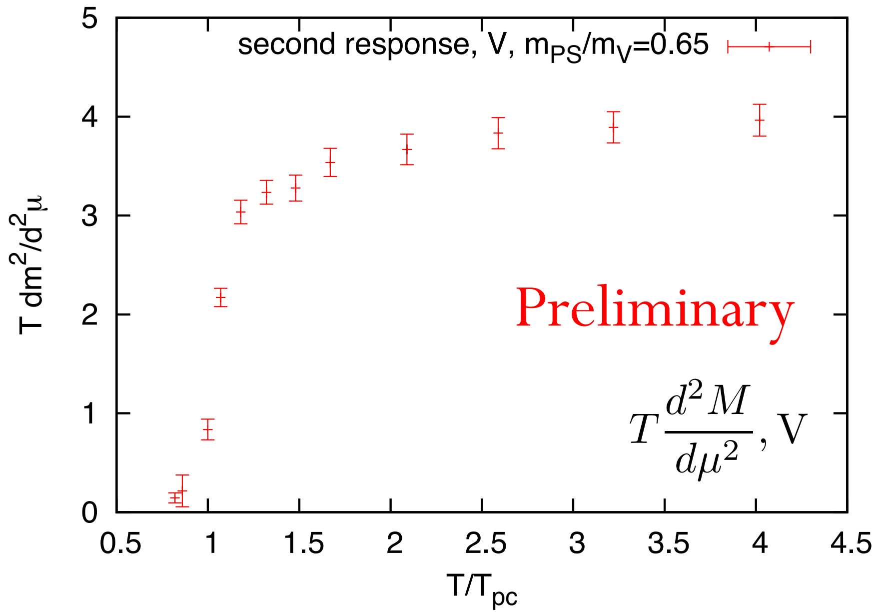
Leading order: $G(x) = A(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x-\hat{x})})$

2^{nd} order: $\frac{1}{G(x)}\frac{d^2G(x)}{d\hat{\mu}^2} = \frac{1}{A}\frac{d^2A}{d\hat{\mu}^2} + \frac{d^2\hat{M}}{d\hat{\mu}^2}\left\{\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left[\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right]-\frac{L_x}{2}\right\}$

$\rightarrow$ Second derivative of meson masses, $\frac{d^2\hat{M}}{d\hat{\mu}^2}(= N_t T\frac{d^2M}{d\mu^2})$,is obtained.

Second derivative of meson mass

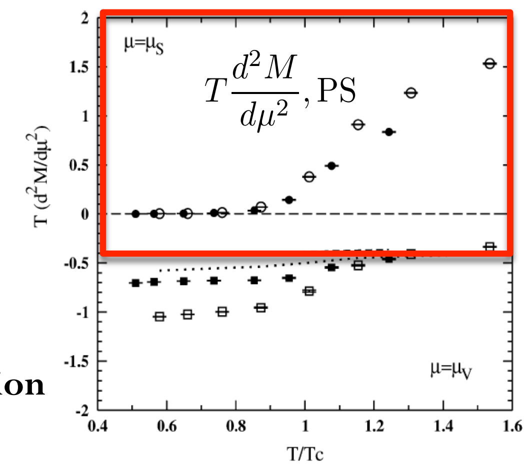




- **Second response is positive**
- **Becomes large after phase transition**
- **Jump around Tc**

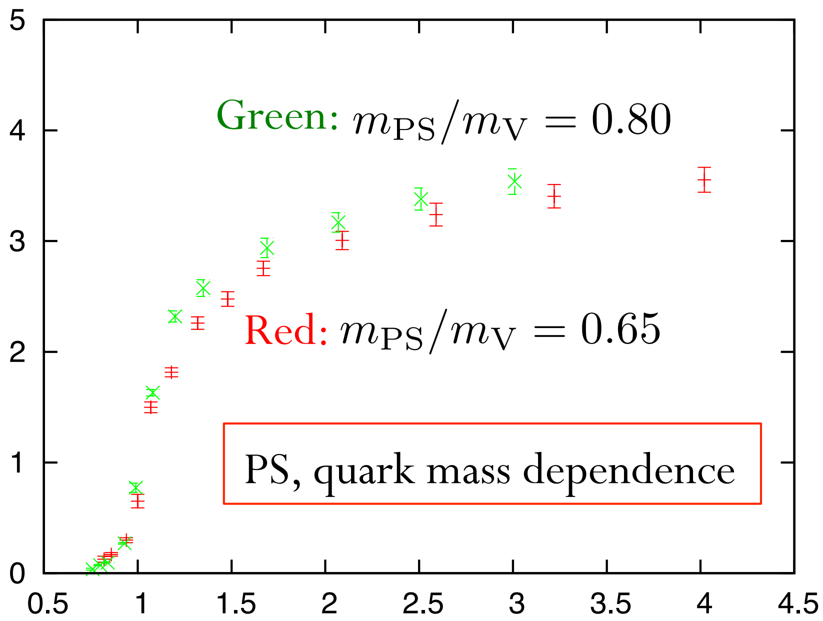
… mostly due to operator part

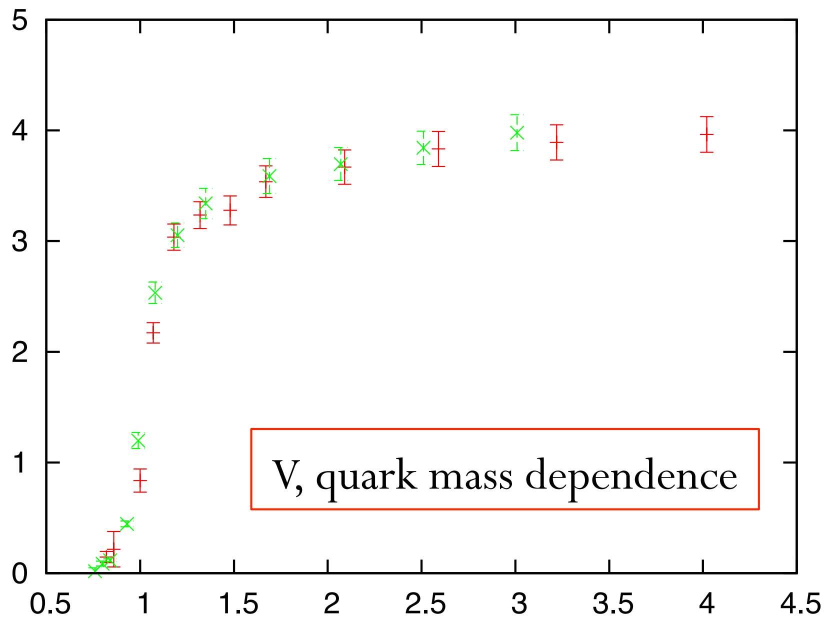
cf) staggered fermion



Results by staggered fermion
I.Pushkina et al.(QCD-TARO collab.)PLB609 (2005)

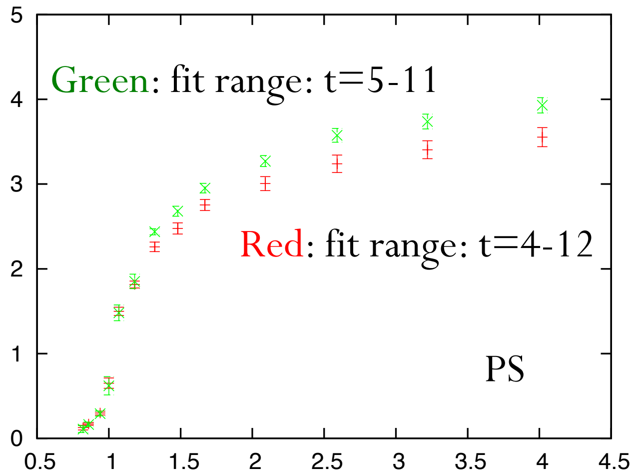
Second derivative of meson mass

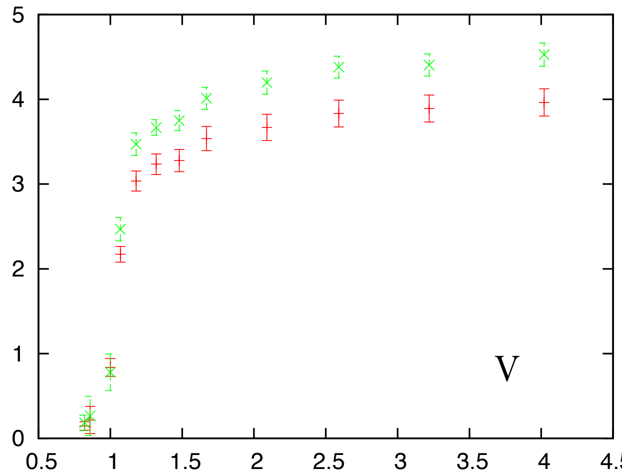




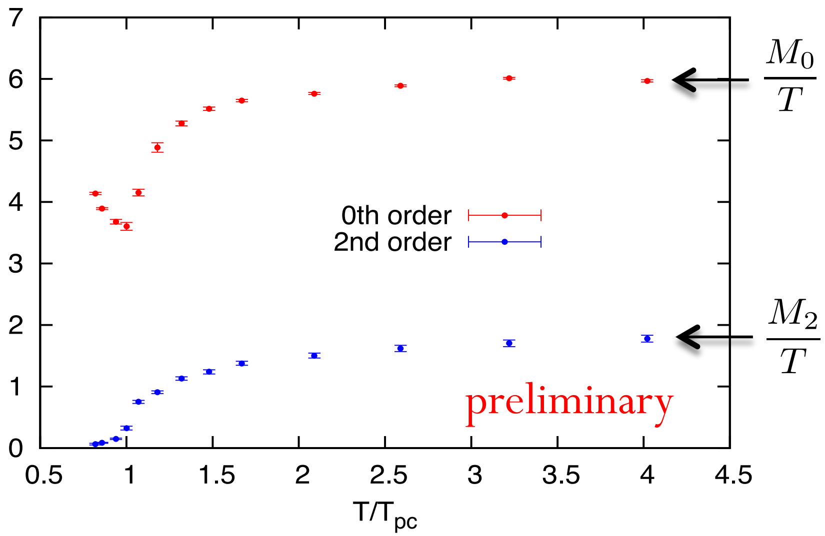
. . . The response becomes slightly large as quark mass becomes large

[Note]
fit range dependence

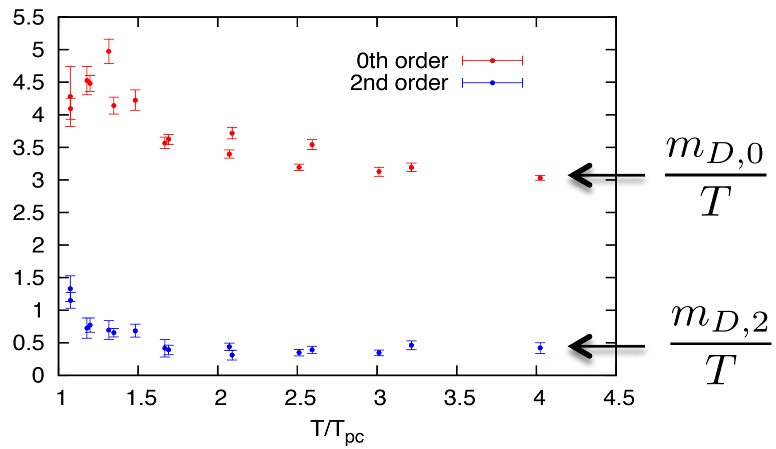




Comparison with gluon screening mass



cf) Second derivative of gluon screening mass (Y.Maezawa et al., PRD**75**, 074501 (2007))



$$\frac{M(\mu)}{T} = \frac{M_0}{T} + \frac{M_2}{T}\left(\frac{\mu}{T}\right)^2 + O(\mu^4)$$

$$\frac{m_D(\mu)}{T} = \frac{m_{D,0}}{T} + \frac{m_{D,2}}{T}\left(\frac{\mu}{T}\right)^2 + O(\mu^4)$$

- **Behavior of screening masses are opposite between mesons and gluons:**

$\frac{M_0}{T}$ and $\frac{M_2}{T}$ $\rightarrow$ large, for T $\rightarrow$ large $\Leftrightarrow$ $\frac{m_{D,0}}{T}$ and $\frac{m_{D,2}}{T}$ $\rightarrow$ small, for T $\rightarrow$ large

- The ratio of 2nd order to 0th order is larger for mesons than gluons above Tc:

$$\frac{M_2}{M_0} > \frac{m_{D,2}}{m_{D,0}} \quad \text{above Tc}$$

(mesons…20-30% above Tc; gluons…about 10% above Tc)

(Note: quarks couple to μ directly $\Leftrightarrow$ gluons couple to μ only through quark loops)

Summary

- We have studied meson screening masses (PS and V) at finite temperature and density in lattice QCD with two-flavor Wilson fermion generated by WHOT-QCD collaboration.

- Finite temperature, $\mu=0$:
  - Below and around Tc : meson masses increase very slowly.
  - Above Tc: increase rapidly and approach to $2\pi T$, where the mesons may become two free quarks.

- Finite μ :
  - $T\frac{d^2M}{d\mu^2}$ is very small below Tc
  - $T\frac{d^2M}{d\mu^2}$ is positive and increases above Tc

  - Meson screening masses have qualitatively different behavior compared to gluon screening mass, which feels μ effect only through quark loops.