Gauge-Higgs Unification in Randall-Sundrum Spacetime at Finite Temperature

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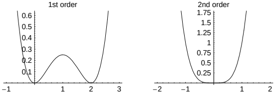
Electroweak phase transition and thermal effects

- Our universe : baryon asymmetric (no anti-proton in our daily life)
- Baryogenesis - Sakhalov's three conditions
  1. B violation process
  2. C and CP symmetry is broken
  3. out of thermal equiriblium
- Electroweak baryogenesis - the 3rd condition requires the first-order phase transition and the expanding bubbles (inside : broken phase)

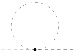


- Order of thermal EWPT
  1. Standard Model - 2nd
  2. SUSY - 1st for some models [Funakubo et.al.]

For other extension of the SM (little higgs, gauge-Higgs unification, etc.), we need to clarify the order of thermal EWPT.

Gauge-Higgs unification (GHU)

- Hierarchy Problem $\Leftarrow$ quadratic divergent δm_h^2

$$m_h^2 = m_{\text{bare}}^2 + ($$  $$\sim g\Lambda^2), \quad \Lambda: \text{cutoff} \tag{1}$$

$$m_h = \mathcal{O}(100\text{GeV}), \quad m_{\text{bare}}, \Lambda \sim M_{GUT} \gg m_h \tag{2}$$

$\rightarrow$ fine-tuning between m_{bare} and Λ.

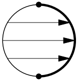
- Gauge-Higgs unification [N.S.Manton(1983), ...]:
  - extra-dimensional component of the gauge field = the Higgs field

$$A_M = (A_\mu, A_y = h) \tag{3}$$

  - gauge symmetry is spontaneously broken by nonzero $\langle A_y \rangle$
  - Effective potential and the Higgs-mass is finite, thanks to the gauge symmetry in the higher-dimensional spacetime
    $\rightarrow$ solve the fine-tuning problem [Inami-Lim-HH (1998)]

GHU Example - SU(3) model

[Kubo-Lim-Yamashita (2001)]



- $SU(3)$ gauge theory in 5D (ExD is compactified on S^1/Z_2):
  - boundary conditions at $y_0 = 0$, $y_1 = \pi R$

$$
\begin{aligned}
A_\mu(x^\mu, y_i - y) &= +P_i A_\mu(x^\mu, y_i + y) P_i^{-1}, && (4)\\
A_y(x^\mu, y_i - y) &= -P_i A_y(x^\mu, y_i + y) P_i^{-1}, && (5)\\
\psi_{\rm fd}(x^\mu, y_i - y) &= \eta\gamma_5 P_i \psi_{\rm fd}(x^\mu, y_i + y), \quad \eta = \pm 1. && (6)\\
P_i &= {\rm diag}(+1, +1, -1) && (7)
\end{aligned}
$$

  - zero modes:

$$
\begin{pmatrix} A_\mu^{3,8} & A_\mu^{1,2} & \\ A_\mu^{1,2} & A_\mu^{3,8} & \\ & & A_\mu^{8} \end{pmatrix}, \quad \begin{pmatrix} & & A_y^{4,5} = H_1 \\ & & A_y^{6,7} = H_2 \\ A_y^{4,5} = H_1^\dagger & A_y^{6,7} = H_2^\dagger & \end{pmatrix}. \qquad (8)
$$

$SU(2) \times U(1)$ gauge theory with doublet Higgs

  - Higgs VEV $\to$ Wilson-line phase

$$
\langle W \rangle = \exp\left(\oint dy\, g \langle A_y \rangle\right) \qquad (9)
$$

- Fermions
  - chiral zero modes

$$\begin{pmatrix} u_L \\ d_L \\ D_R \end{pmatrix} \tag{10}$$

  - mass term from gauge coupling : $g\bar{\Psi}\langle A_y\rangle\Psi$
- massive vector boson (e.g. W_μ, Z_μ) - KK mass from $\partial_y A_\mu - ig\left[\langle A_y\rangle, A_\mu\right]$
- Higgs potential
  - In 5D GHU, there are no Higgs potential at tree level ($\because F_{\mu y} = 0$).
  - Higgs potential is generated as quantum corrections [Hosotani (1983)]

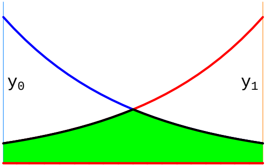
$$V_{\text{eff}}^{\text{T}=0} = \frac{(-1)^{2\eta}}{2}\sum_\ell \int \frac{d^4p}{i(2\pi)^4} \ln\left[p^2 + m_\ell^2\right], \tag{11}$$

- Problems
  - Difficulty in obtaining large top-quark mass
  - One-loop Higgs potential (and mass) is too small ($m_h \sim 10\text{GeV!}$)

GHU on Randall Sundrum space-time

- Fermions in S^1/Z_2 extra space:
  - zero-mode wave-function : domain-wall profile due to bulk mass term
  - Yukawa-coupling : overlap of wave-functions of fermions and gauge zero modes:

$$H(x)\bar{\psi}_R(x)\psi_L(x) = \int dy \bar{\psi}_R(x,y) A_y(x,y) \psi_L(x,y) \tag{12}$$



→ lightest-mode mass depends exponentially on the bulk mass parameter!

- Higgs effective potential (and Higgs mass) are enhanced [Hosotani et.al,2007, HH 2007].

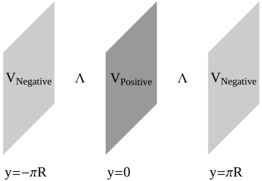
$$m_h \sim \mathcal{O}(100\text{GeV}) \tag{13}$$

Randall-Sundrum space-time

- non-factorizable metric:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \quad k : AdS_5 \text{ curvature} \qquad (14)$$

- circle with identification : $y \to -y$ fundamental region : $[0, \pi R]$ fixed points : $y_0 = 0, \quad y_1 = \pi R$



- Hierarchy
  1. Planck (hidden brane) scales : $\Lambda, M_5, k, R \sim M_{pl}$
  2. Kaluza-Klein (visible brane) scales : $m_{KK} = \pi k e^{-kR\pi} \frac{1}{1-e^{-\pi k R}}$
  3. $kR \simeq 12 \to e^{kR\pi} \simeq M_{pl}/\text{TeV}$

Finite-Temperature Effects

- 1-loop correction for effective potential (per d.o.f of the field):

$$V_{\rm eff}^{1-{\rm loop}} = \frac{(-1)^{2\eta}}{2}\frac{1}{\beta}\int\frac{d^3p}{(2\pi)^3}\sum_{n=-\infty}^{\infty}\sum_{\ell}\ln\left[\left(\frac{2\pi(n+\eta)}{\beta}\right)^2+\vec{p}^2+m_\ell^2\right],$$
$$\eta = 0({\rm boson}), \tfrac{1}{2}({\rm fermion}), \quad \beta \equiv 1/T. \qquad (15)$$

- When the extra dimension is compactified on S^1 (radius R),

$$m_\ell^2 = \left(\frac{2\pi\ell+\theta}{2\pi R}\right)^2 + M^2, \quad M : \text{bulk mass} \qquad (16)$$

→ one may use many tricks (Poisson sum formula, etc...)

- after Poisson re-summation, we have

$$V_{\rm eff} = V_{\rm eff}^{T=0} + 2(-1)^{2\eta-1}\sum_\ell\sum_{\tilde{n}=1}^{\infty}(-)^{2\eta\tilde{n}}\frac{(n|m_\ell|\beta)^2K_2(\tilde{n}|m_\ell|\beta)}{(\sqrt{2\pi}\tilde{n}\beta)^4} \qquad (17)$$

- KK conditions (Hosotani-Noda etal, 2005):

$$0 = \lambda_n^2 z_1 F_{\alpha-1,\alpha-1}(\lambda_n, z_1) F_{\alpha,\alpha}(\lambda_n, z_1) - \frac{4}{\pi^2}\sin^2\frac{\theta}{2},$$

$$m_n = k\lambda_n, \quad \alpha = \frac{1}{2} \pm (M/k), \quad \theta : \text{Wilson-line phase}$$

$$F_{\alpha,\beta}(\lambda, z) = Y_\beta(\lambda) J_\alpha(\lambda z) - J_\beta(\lambda) Y_\alpha(\lambda z) \qquad (18)$$

- zero-temperature [HH(2007), Yamashita-Haba-Okada-Matsumoto (2008)]
  $a \equiv e^{-k\pi R}$

$$V_{\rm eff}^{T=0} = \frac{k^4 a^4}{16\pi^4}\int_0^\infty dt\, t^3 \ln[\tilde{F}_{\rm warped}] \qquad (19)$$

$$\tilde{F}_{\rm warped}(\theta, \nu) = 2I_\nu(t) I_{\nu-1}(t) K_\nu(at) K_{\nu-1}(at) + \cdots$$
$$+2I_\nu(at) I_{\nu-1}(at) K_\nu(t) K_{\nu-1}(t) - \frac{\cos\theta}{at^2} \qquad (20)$$

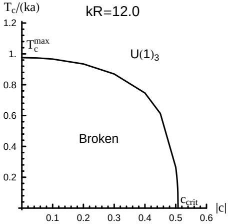
where

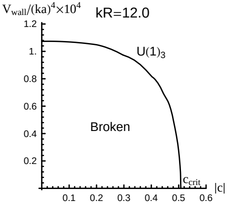
$$\nu = 0, 1 \quad : \quad \text{gauge-higgs fields} \qquad (21)$$

$$\nu = \tfrac{1}{2} \pm \tfrac{M}{k} \quad : \quad \text{fermions with bulk mass } M \qquad (22)$$

Numerical study for $SU(2)$ model (preliminary)

- Result
  - ($SU(2)$ is broken to $U(1)_3$ by orbifold b.c.)
  - pure gauge field : $U(1)_3$ unbroken
  - gauge field + fundamental fermion : $U(1)_3$ unbroken
  - gauge field + adjoint fermion :





- (C.f.) Flat space-time
  [Shiraishi (1989), Ho-Hosotani (1990), Takenaga et.al]:
  - gauge field + adjoint fermion : $SU(2) \to U(1)_3$
  - critical temperature : $T_c = 0.812/L$,
  - potential barrier height: $0.176/L^4$

$SO(5) \times U(1)$ GHU model

As application to the particle physics, we study the finite-temperature effect on the model proposed by Hosotani et.al (2008-),

- $m_{KK} (\simeq \pi k a) \sim 1.5\text{TeV}$ for $kR = 12$
- $m_h \sim 70\text{GeV}$
- The model have "H-parity"
  - $P_H = -1$ is assigned for h and $+1$ for other SM fields
  - All P_H-odd interactions $(hWW, hZZ, h\bar{f}f, hhh)$ vanish.
    $\rightarrow$ the model can avoid the LEP constraint $(m_h \leq 114\text{GeV})$
  - h is stable and can be the candidate of dark matter (higgs dark-matter).

Numerical Study

- Effective potential - we adopt the following approximation:

$$V_{\rm eff} \simeq \underbrace{V_W + V_Z}_{W,Z\text{boson}} + \underbrace{4V_{\rm top}}_{\text{top quark}}, \qquad (23)$$

$$V_W + V_Z \sim 3V_W = 3 \cdot 3V(1, 2\theta), \qquad (24)$$

$$V_{\rm top} = -4V(0.063, 2\theta) \qquad (25)$$

  other fermions' contribution (b, c, s, d, u, e, μ, τ, and non-SM heavy particles) are negligible.

- Result for $kR = 12$ (preliminary)
  - critical temperature : $\beta_c = 1.52/ka \to T_c \sim 330\text{GeV}$
  - height of potential barrier : $7.2 \times 10^{-5}(ka)^4 \sim (50\text{GeV})^4$

Summary

- Numerically studied Hosotani mechanism at finite-temperature
  - correction from the zero-temperature is obtained by summing up hundreds of Kaluza-Klein masses
  - obtain critical temperature and the height of the potential wall
- Apply to the gauge-Higgs unification ($SO(5) \times U(1)$ model) [preliminary]
  - Critical temperature $\sim$ a few hundred GeV
  - Height of the potential wall $\sim \mathcal{O}(10^6)\mathrm{GeV}^4$

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