QCD Thermodynamics on the Lattice: Recent Results

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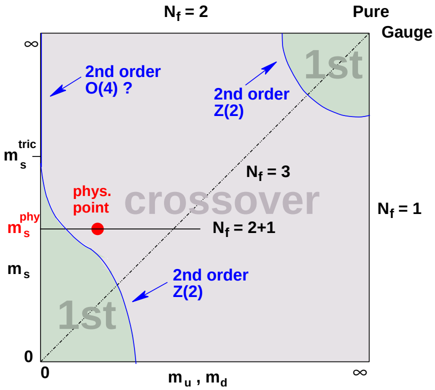
QCD thermal phenomenology: what we know

What we know about QCD:

- We have two interrelated phenomena: deconfinement and chiral symmetry restoration.
- We have confinement at zero temperature and "deconfinement" at high temperature. (Still have spatial confinement.)
- At infinite quark masses (pure SU(3) Yang Mills) the confinement/deconfinement phase transition is first order.
- Chiral symmetry is spontaneously broken at zero temperature and restored at high temperature.
- Chiral models: When quark masses are zero we have a second order phase transition for SU(2) flavor and first order for SU(3) flavor.
- Between these extremes only a nonperturbative calculation can say what happens.
- The present consensus in lattice QCD is that there is only a crossover at physical quark masses.

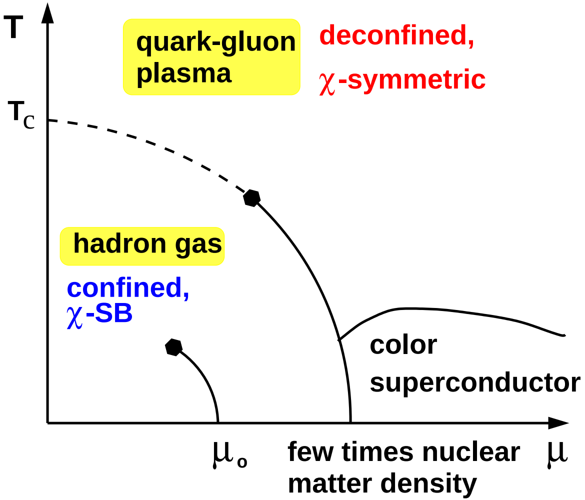
Phase diagram

The diagram below is only a sketch of what we expect with up, down and strange quarks as we vary the masses $m_u = m_d$ and m_s. Lattice calculations aim to check this picture.



Phase diagram

Here is what we might expect if we vary the chemical potential of the up and down quarks. The large μ region has not been reached in lattice calculations.



What we want to know

- Crossover temperature T_c at zero and nonzero baryon number density.
- Equation of state, velocity of sound, etc.
- Transport properties of the plasma.
- Extent of validity of the hadron resonance gas model at low T.
- In-medium hadronic modes (e.g. J/psi), especially above T_c?
- Experimentally accessible critical point at nonzero baryon number density?
- Theoretically interesting critical point at low light quark masses?
- Many more.

A nonperturbative treatment is necessary.
Lattice QCD provides a first-principles nonperturbative treatment.

Lattice QCD at Nonzero Temperature

We start from the quantum grand canonical partition function

$$Z = \mathrm{Tr}\left[\exp\left(-H/T + \sum_i \mu_i N_i/T\right)\right],$$

for temperature T, QCD hamiltonian H, chemical potential μ_i, and conserved charge N_i.

We rewrite it, using the Feynman path integral approach, as

$$Z = \int dA_\mu \, d\psi \, d\bar{\psi} \, \exp[-S(U, \psi, \bar{\psi}, \mu)]$$

The integration is over classical gauge fields A_μ, quark fields ψ, and the classical action S. The classical fields live in Euclidean space time $(\mathbf{x}, \tau)$ with imaginary time

$$\tau \in [0, 1/T]$$

Thermal operator expectation values become

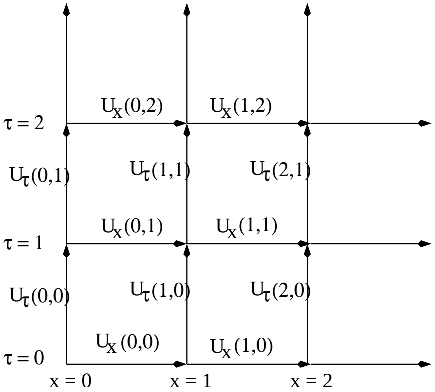
$$\langle \mathcal{O} \rangle = \int dA_\mu \, d\psi \, d\bar{\psi} \, \mathcal{O} \exp[-S(U, \psi, \bar{\psi}, \mu)]/Z$$

Lattice QCD action

- Regular lattice of spacing a and size $N_s^3 \times N_\tau$.
- Gauge field: on links between x and the neighboring site $x + a\hat{\mu}$

$$U_\mu(x) = \exp[igaA_\mu^c(x)\lambda^c/2],$$

  where λ^c are the eight Gell-Mann generators of $SU(3)$.
- The fermion fields are placed on the lattice sites.
- We have the important relation $T = 1/(N_\tau a)$.



QCD on the lattice: gluons

The original Wilson plaquette gauge action is

$$S_G(U) = \sum_{x,\mu,\nu} \frac{\beta}{6} \mathrm{ReTr}[1 - U_{P,\mu\nu}(x)],$$

where $U_{P,\mu\nu}$ is the gauge-invariant product of link matrices surrounding a plaquette (square) of size $a \times a$.
The gauge coupling enters as

$$\beta = 6/g^2.$$

For small a we recover the correct continuum action:

$$\mathrm{ReTr}[1 - U_{P,\mu\nu}(x)] \to \frac{g^2 a^4}{4} \sum_c (F^c_{\mu\nu})^2 + \mathcal{O}(a^6).$$

Improved actions add a 2×1 rectangle to eliminate the $\mathcal{O}(a^6)$ term. Reduces cutoff effects.

QCD on the lattice: fermions

Fermions are represented as Grassmann variable fields $\psi(x)$ occupying the lattice sites. The Dirac action $M(U) = \not{D} + m$ is bilinear in the fields, discretized as

$$S_F(U, \psi) = \sum_{x,y} \bar{\psi}(x) M(U, x, y) \psi(y),$$

(We have suppressed color indices and show only one flavor.) The QCD lattice partition function is

$$Z = \int [dU][d\psi][d\bar{\psi}] \exp[-S_G(U) - S_F(U, \psi)].$$

It is convenient to integrate out the fermion variables:

$$Z = \int [dU] \exp[-S_G(U)] \det[M(U)].$$

Strengths and weaknesses of lattice QCD

- Huge strength: We can do *ab initio* nonperturbative QCD calculations. We know of no alternative for QCD.
- Weaknesses:
  - Static thermodynamic equilibrium only.
  - Euclidean time: Real time properties are difficult to extract. Transport properties are not easy.
  - Requires a real, positive weight e^{-S}. Nonzero quark number density is difficult.
- We need phenomenological models to extrapolate from lattice results!

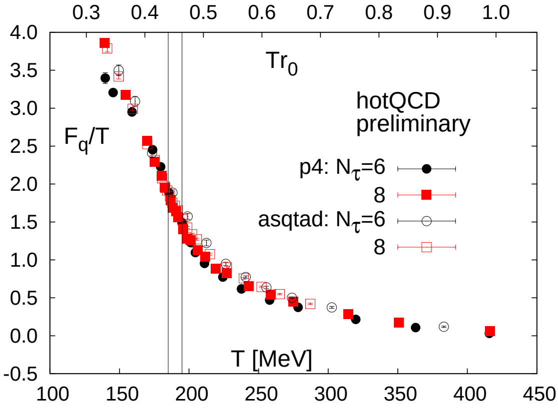
Basic lattice thermal phenomenology

- Traditional deconfinement probe: "Polyakov loop" L or static quark free energy F_q:

$$
\begin{aligned}
L &= \left\langle \mathrm{Tr} P \exp\left(ig \int_0^{1/T} d\tau\, A_0(\tau)\right) \right\rangle \\
&\sim \exp[-F_q(T)/T]
\end{aligned}
$$

Static quark free energy

In the case illustrated below, the quarks are not infinitely massive but $F_q(T)$ still decreases with increasing T, suggesting deconfinement.



Clearly, this quantity is not a strong indicator of the crossover.

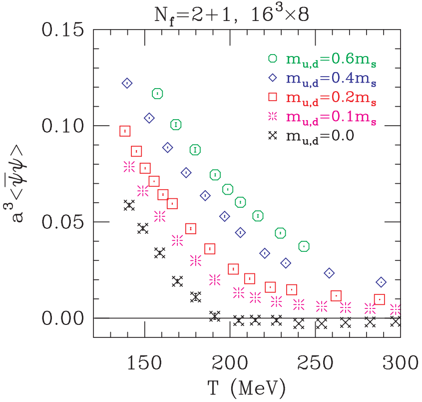
Basic lattice thermal phenomenology

- Traditional chiral symmetry probe: the light quark chiral condensate

$$\langle \bar{\psi}\psi \rangle = (T/V)\partial \log Z/\partial m.$$

Chiral condensate

- $\langle \bar{\psi}\psi \rangle$ is nonzero when chiral symmetry is spontaneously broken and zero when it is restored.
- We expect restoration at high T.
- When all sea quark masses are nonzero, chiral symmetry is not exact, so we don't get zero, exactly.
- The example below shows results for a range of sea quark masses.



Varying T and taking the continuum limit

$$T = 1/(aN_\tau)$$

Two methods:

1. Fixed N_τ method:
   Note that through the renormalization group, a depends on $\beta \propto 1/g^2$, so as β increases, g decreases, a decreases, and T increases.
   Low T implies larger cutoff effects!
2. Fixed scale method (WHOT):
   As N_τ decreases, T increases. Cutoff effects are uniform in T.

In both cases we want to take the continuum limit. For method 1 we do this by increasing N_τ. With method 2 we reduce a explicitly. Most results presented here use the fixed N_τ method.

We work along "lines of constant physics"; *i.e.* we tune the bare quark masses so as to keep (zero-temperature) meson masses fixed in physical units as T is varied.

We extrapolate $m_{u,d}$ to its physical value.

Setting the scale

To get T in MeV we set the lattice scale a at zero temperature (same hamiltonian parameters). Requires matching one dimensionful lattice result with one experimental result. Two common methods:

1. Kaon decay constant (f_K) scale. Straightforward: Measure af_K at zero temperature. From experimental f_K, we then know a.
2. Heavy quark potential. Indirect: r_0 or r_1 scale.
   Find R such that
   $$-R^2 dV(R)/dR = 1.$$
   This defines $R = r_1$. The Sommer r_0 scale is similar. To get its value, on the same ensemble we measure both r_1/a and the splitting of the Υ spectrum. From the experimental splittings we get a and, therefore, $r_1 \approx 0.31$ fm $r_0 \approx 0.47$ fm. Measuring r_1/a on any other ensemble then tells us a.

All scale definitions must agree at zero lattice spacing and physical quark masses. We can get $\sim 2\%$ accuracy in T. At nonzero a and unphysical quark masses, agreement is not required.

Lattice fermion doubling problem

With a naive discretization in three space and one time dimension we get 2^4 species of the same mass. This is called "doubling" (should be "sixteen-ing"). Remedies

- Wilson fermions: Add a (mass)-dimension-5 term to the action to lift the degeneracy. All unwanted fermions get masses of order $1/a$. Breaks chiral symmetry explicitly.
- Domain wall and overlap fermions: From Wilson fermions, construct a chirally symmetric action. Rigorous, elegant, but computationally expensive.
- Staggered fermions: Partially diagonalize the fermion matrix to reduce the degeneray from 16 to 4. Call them "tastes". (Then each flavor comes in four tastes.) Take the fourth root of the fermion determinant to get an approximately correct counting of sea quarks. The approximation is exact in the continuum limit (?).

Fermion actions in results presented here

Staggered fermions are the most thoroughly studied, so most results I will present use them. Wilson (clover) fermions are next. Domain wall and overlap fermions are only beginning.

Also, most results presented here are for $2 + 1$ flavors, *i.e.* for degenerate u and d quarks ($m_u = m_d = m_\ell$) plus a strange quark (m_s).

Improved fermion actions

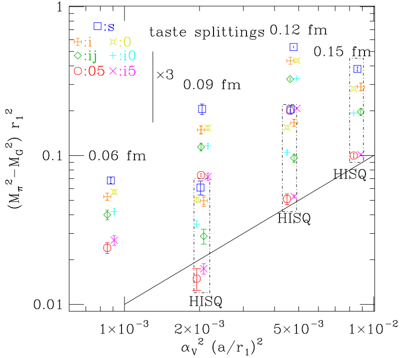
Goal: Reduce cutoff effects at a given a. Do this by adding irrelevant higher-dimensional terms to the action. (Symanzik.)
The traditional staggered fermion action is "unimproved": good to $\mathcal{O}(a^2)$.

- Reduce taste-splitting effects (2010: very important). Listed in order of increasing improvement:
  - p4 (RBC)
  - asqtad (MILC)
  - stout (Bielefeld-Wuppertal)
  - HISQ (HPQCD)
- Reduce remaining cutoff effects. Listed in order of increasing improvement:
  - stout $\mathcal{O}(a^2)$
  - p4, asqtad, HISQ $\mathcal{O}(a^2\alpha_s)$

I will show some very new results from HotQCD based on the HISQ action $N_\tau = 6, 8$ and asqtad $N_\tau = 12$.

Staggered taste splitting

Four tastes of quarks and four of antiquarks yield sixteen of each pion. Taste symmetry is broken at nonzero a, resulting in a multiplet structure. The splitting decreases approximately as $a^2\alpha_V^2$.
Compare HISQ with asqtad splitting [MILC, 2010].



HISQ has much smaller splitting.

HotQCD collaboration

I will review some results from others, but most of the results presented come from work of the HotQCD collaboration:

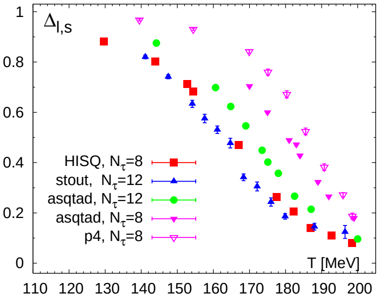
A. Bazavov, T. Bhattacharya, M. Cheng, N.H. Christ, C. D., S. Gottlieb, R. Gupta, U.M. Heller, C. Jung, F. Karsch, E. Laermann, L. Levkova, C. Miao, R.D. Mawhinney, S. Mukherjee, P. Petreczky, D. Renfrew, C. Schmidt, R.A. Soltz, W. Soeldner, R. Sugar, D. Toussaint, W. Unger and P. Vranas

Subtracted chiral condensate

Need a renormalization-independent definition:

$$\Delta_{\ell,s} = [\left\langle\bar{\psi}\psi\right\rangle_\ell(T) - m_\ell/m_s\left\langle\bar{\psi}\psi\right\rangle_s(T)]/[\left\langle\bar{\psi}\psi\right\rangle_\ell(T=0) - m_s/m_\ell\left\langle\bar{\psi}\psi\right\rangle_s(T=0)]$$

Removes an additive UV divergence and multiplicative renormalization factors. Here $m_\ell/m_s = 0.2$.



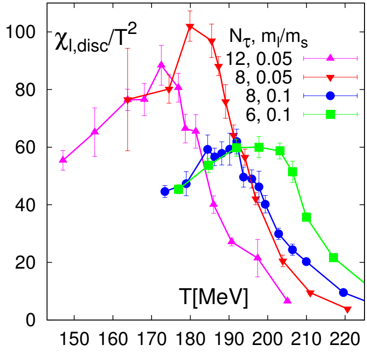
[Bazavov and Petreczky, HotQCD 2010]

- Conclude: Reducing taste-symmetry breaking shifts curve to lower T.

Chiral susceptibility

$$\chi_\ell = \frac{T}{V}\frac{\partial^2}{\partial m^2}\log Z = \chi_{\ell,disc} + 2\chi_{\ell,conn}$$

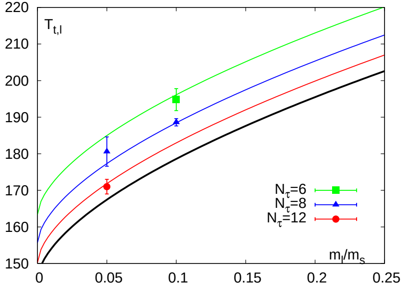
Related to fluctuations in the order parameter $\langle\bar{\psi}\psi\rangle$. Peaks at the crossover temperature. Becomes infinite at the critical point. (All asqstad values here.)



[HotQCD Preliminary]

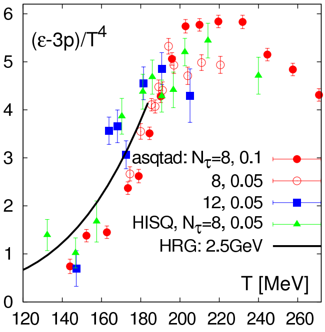
T_c extrapolation

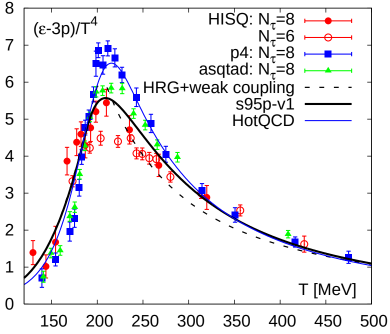
- Plot the position of the peak in T in susceptibility *vs.* light quark mass ratio m_ℓ/m_s at fixed N_τ.
- Continuum limit means $N_\tau \to \infty$.
- Fit to $T_c = T_c(0) + a(m_\ell/m_s)^{1/(\beta\delta)} + b/N_\tau^2$ where $1/(\beta\delta) = 1.08$ is an $O(4)$ critical exponent.
- Take continuum limit and $m_\ell/m_s = 1/27$ physical.
- Asqtad $T_c(\mathrm{phys}) \approx 164(6)$ MeV (HotQCD preliminary, 2010). Budapest-Wuppertal result for a closely related observable: 147(2)(3).



Equation of state (trace anomaly)

Energy density ϵ and pressure p *vs.* T. First look at the "trace anomaly" $I = \epsilon - 3p$: (HotQCD Preliminary)



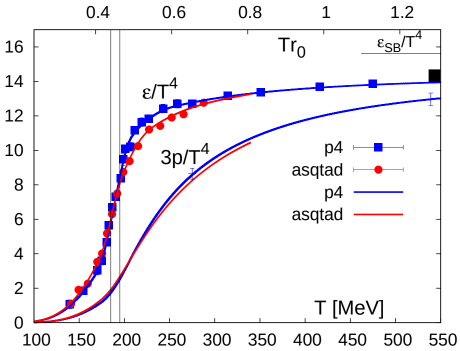


- Low T results lie below the HRG prediction. Taste-splitting effect?
- At high temperature actions agree.
- Lines on right represent different parameterizations.

Energy density and pressure

We get pressure p and energy density ϵ from I.

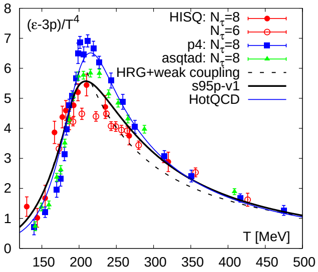
$$
\begin{aligned}
I &= \epsilon - 3p \\
p &= \frac{T}{V}\int_{T_0}^{T} dT' \, \frac{1}{T'^5} I(T')
\end{aligned}
$$



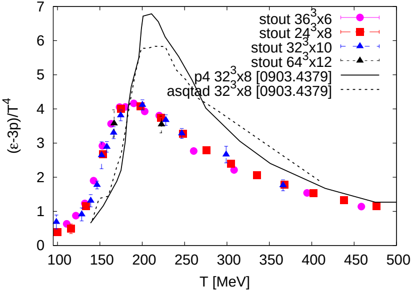
[HotQCD, 2009]

Equation of state (trace anomaly)

Stout *vs.* HISQ/asqtad trace anomaly.
The discrepancy is large and not understood (2010).
Note the stout result has been multiplied by a "tree-level improvement" factor (as much as a 50% correction).



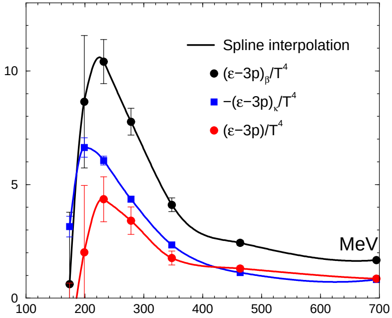
[HotQCD, preliminary]

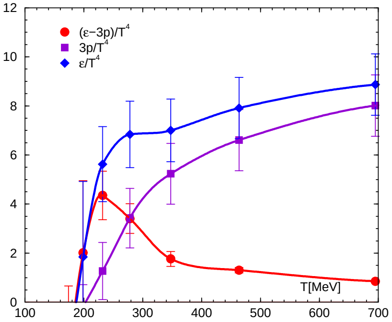


[Budapest-Wuppertal, preliminary]

Equation of state (trace anomaly)

Fixed scale approach.

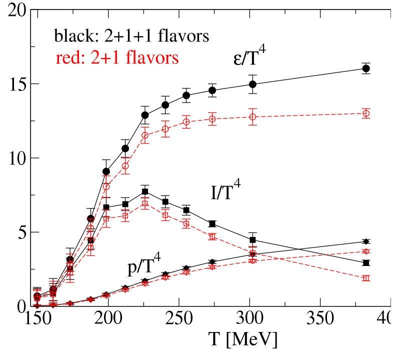




[WHOT, preliminary, 2010]

- 2-flavor case was successful. 2+1 above are very new.
- Comparisons with staggered and fixed N_τ will be very useful.
- Do results agree better with HRG at low T?

Charm contribution to EOS



[MILC, 2010]

- Valence charm only. No charm sea quarks.
- Larger effect than expected.
- Stout and p4 action results seems consistent with this.

Nonzero density

Use Taylor expansion at zero temperature

$$\frac{p}{T^4} = \sum_{n,m=0}^{\infty} c_{nm}(T) \left(\frac{\mu_{ud}}{T}\right)^n \left(\frac{\mu_s}{T}\right)^m ,$$

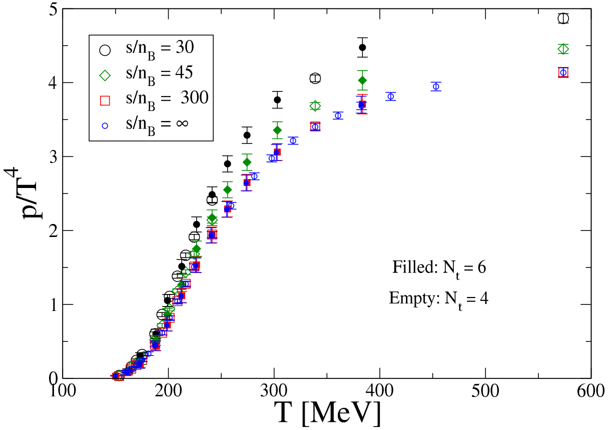
The coefficients c_{nm} are evaluated at zero chemical potential

$$c_{nm}(T) = \frac{1}{n!} \frac{1}{m!} \frac{1}{T^3 V} \frac{\partial^{n+m} \ln Z}{\partial(\mu_{ud}/T)^n \partial(\mu_s/T)^m} \bigg|_{\mu_{ud,s}=0} .$$

For interesting alternative methods, see Ejiri *et al.* (2009).

Isentropic pressure

Constant entropy per baryon s/n_B.



[MILC, 2009]

In-medium hadronic modes

Deducing thermal real-time response from lattice data?

- Thermal (Matsubara) correlator

$$C(x_0, \mathbf{x}, T) = \langle \mathcal{O}(x_0, \mathbf{x}) \mathcal{O}(0, 0) \rangle \,.$$

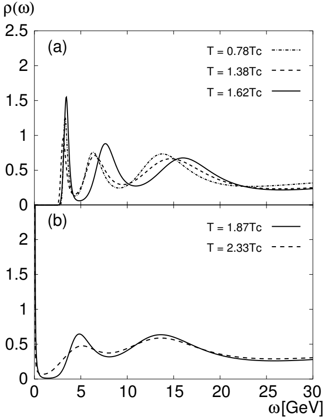
- Real-frequency spectral density $\rho(\omega, \mathbf{q}, T)$ is then obtained by inverting the Kubo formula for the partial Fourier transform:

$$C(x_0, \mathbf{q}, T) = \frac{1}{2\pi} \int_0^\infty d\omega \, \rho(\omega, \mathbf{q}, T) K(\omega, x_0, T),$$

$$K(\omega, x_0, T) = \frac{\cosh \omega (x_0 - 1/2T)}{\sinh(\omega/2T)}.$$

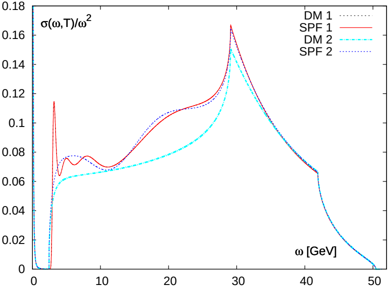
- Difficult inverse problem. Requires additional assumptions – *e.g.* maximum "entropy" plus default model.

J/psi spectral density vs T



Pioneering work by Asakawa-Hatsuda (2004)
The ground state peak is visible up to $1.62\,T_c$. These results are obtained in a quenched simulation.

J/psi spectral density vs T

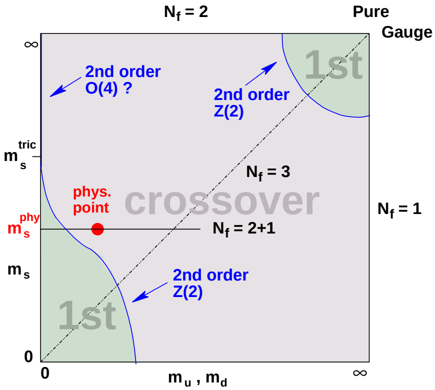


[Ding et al, Lattice 2009]

- Concerns raised by T. Umeda (2004).
- Quenched $N_\tau = 32$ here.
- Two very different spectral functions fit the same $T = 1.5\,T_c$ data.
- Depends on the "default model" for MEM analysis.
- DM is the default model. SPF is the output from MEM.
- One (DM 1 SPF 1) has a J/psi peak and one (DM 2 SPF 2) does not.
- Need $N_\tau = 100$??

Phase diagram

We repeat this sketch to orient the discussion of the "magnetic equation of state".



Scaling of chiral order parameter (Magnetic equation of state)

At zero quark mass we expect universal $O(4)$ critical behavior at the chiral-symmetry-restoring phase transition. Define

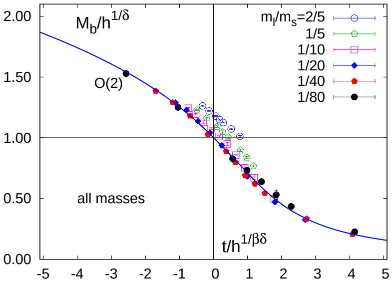
$$t = (T - T_c)/T_c \quad \text{and} \quad h = (m_\pi/m_K)^2 \approx m_\ell/m_s$$

For small h and t we have

$$M(t,h) = m_s/T^4 \left\langle \bar{\psi}\psi(t,h) \right\rangle \to t^\beta f(z) + \mathrm{regular}$$

where $z = z_0 t/h^{1/(\beta\delta)}$ and $f(z)$ is the universal scaling function for $O(2)$ or $O(4)$.

Scaling of chiral order parameter (Magnetic equation of state)



[S. Ejiri et al. (2009)].

- Tests predictions of effective chiral models.
- Surprisingly good scaling already demonstrated long ago for Wilson fermions. [Iwasaki et al. (1997)].
- This time we are testing staggered (p4) fermions.
- Seems to be approaching the O(2) or O(4) scaling function at small quark mass.

Conclusions

- Lattice calculations are teaching us a great deal about high T QCD.
- Cutoff effects in staggered fermion simulations seem now to be under control.
- Improved staggered fermion actions (HISQ) are very promising.
- Wilson fermion simulations are catching up.
- Communication between phenomenologists and lattice physicists is important.

References for QCD thermodynamics

- Recent monograph: K. Yagi, T. Hatsuda, and Y. Miake, “From big bang to little bang.”
- R. Hwa and X.N. Wang, Quark Gluon Plasma 4
- C.D. and U. Heller, QCD Thermodynamics from the Lattice, Eur. Phys. J. A41:405-437,2009; arXiv:0905.2949 [hep-lat]

Tsukubasan Shrine, photo by Laurel Casjens