Chiral symmetry and Electronic properties of Graphene based on U(1) Strong coupling Lattice gauge theory

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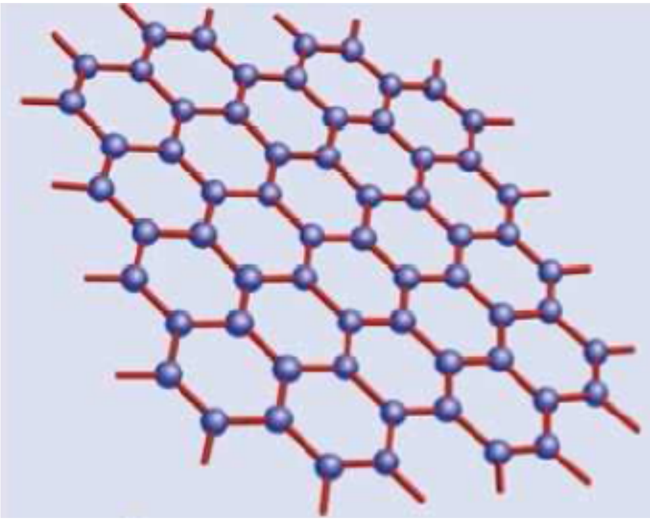
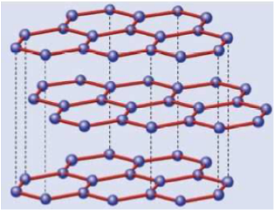
What is Graphene?

Graphene = Single atomic layer of carbon atoms

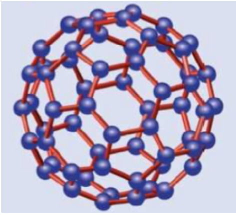
Building block of many kinds of carbon materials:



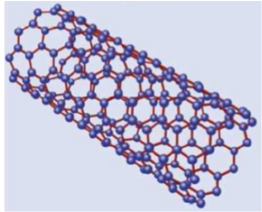
Graphene



graphite



fullerene



nanotube

[Castro Neto *et al.*, 2009]

- Experimentally isolated successfully in 2004 [Novoselov *et al.*,2004]

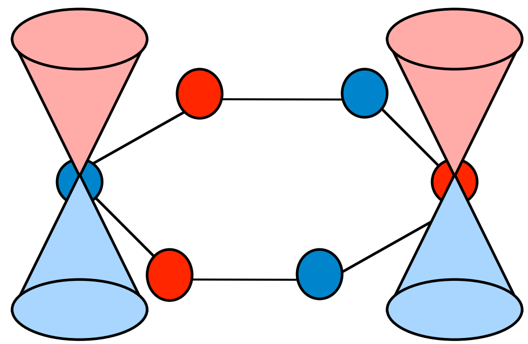
⇒ 2D electron system easy to process and observe

One of the largest research interests

(on both theoretical and experimental side)

Graphene and "Chiral symmetry"

▶ Important feature: [Wallace, 1947]



Dispersion of electrons/holes at low-energy:

Linear around two "Dirac points".

$$E(\mathbf{K}_\pm + \mathbf{p}) = \pm v_F |\mathbf{p}| +O((p/K)^2)$$

⇨ Description as
Massless Dirac fermions

[Semenoff, 1984]

$$\psi_\sigma(\vec{p}) = \begin{pmatrix} a_\sigma(\vec{K}_+ + \vec{p}) \\ b_\sigma(\vec{K}_+ + \vec{p}) \\ b_\sigma(\vec{K}_- + \vec{p}) \\ a_\sigma(\vec{K}_- + \vec{p}) \end{pmatrix}$$

Sublattice symmetry [Z_2]

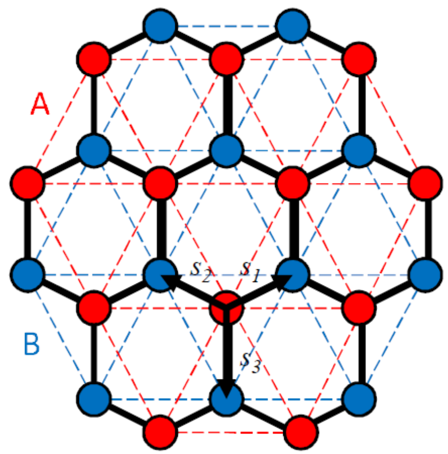
Sublattice on-site energy balance: A⇔B

(On-site energy difference = spectral gap)

⇩ *Low-momentum approx.*

Chiral symmetry [$U(1)_A \in U(4)$]

Generated by γ_5 matrix



(Effectively) Strong gauge coupling

Speed of quasiparticles (fermions)
="Fermi velocity"
v_F (~c/300)

$\ll$

Speed of photons (U(1) gauge field)
= speed of light
c (in vacuum)

Coupling of fermions to U(1) gauge field (=Coulomb coupling) is effectively enhanced:

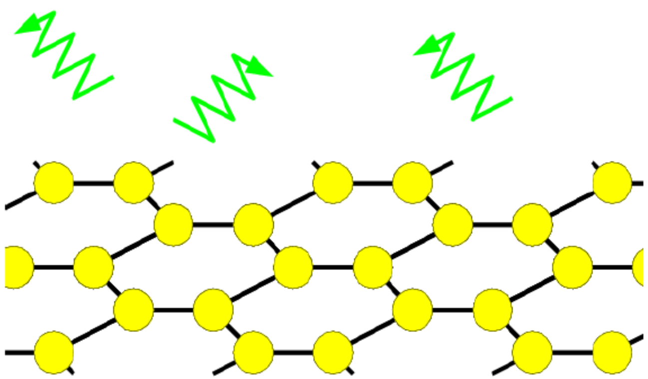
$$\alpha_* = \frac{\alpha_{\mathrm{QED}}}{v_F} \quad (\sim 300\, \alpha_{\mathrm{QED}})$$

- Continuum effective theory: "braneworld", "Reduced QED" [Gorbar *et al.*,2002]

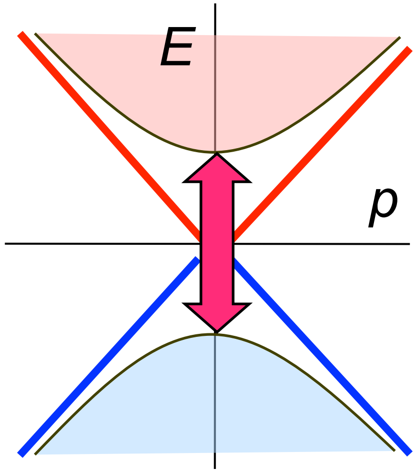
Photons (U(1) gauge field): (3+1)-dim.

Strong coupling U(1) Gauge theory

Fermions: (2+1)-dim.



What will occur at Strong coupling?



Strong coupling ⇒ spontaneous chiSB …?

<u>Graphene:</u> Dynamical gap generation
(Semimetal-insulator transition)

Similar mechanism ⇕

<u>QCD:</u> Dynamical quark mass

cf. A. H. Castro Neto, Physics **2**, 30 (2009).

<u>This work:</u>

Strong coupling expansion of U(1) lattice gauge theory *("reduced QED")*

⇒ analytic calculations of

- ▶ Fermion dynamical gap at/around

  $\beta=0$ (strong coupling), $m=0$ (chiral limit), $V=\infty$ (infinite volume)

- ▶ Collective excitations

  e.g.) (pseudo-)NG mode (~ pion)

Regularization on a square lattice

Regularize the continuum effective model on a square lattice.

► Fermions:

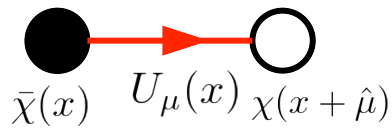
[Hands & Strouthos, 2008]
[Drut & Lahde, 2008-2010]

- Described by a single staggered fermion χ

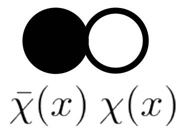
$$S_F = \sum_{x^{(3)}} \left[\frac{1}{2} \sum_{\mu=1,2,4} \left(V_\mu^+(x) - V_\mu^-(x) \right) + m_* M(x) \right]$$

(2+1) dim.

hopping (kinetic) term



Bare (external) mass



► Gauge field:

U(1) Link variables: $U_4(x) = e^{i\theta(x)}$ $(-\pi \leq \theta < \pi)$ $\quad \vec{U}(x) = 1$

(instantaneous approx.)

- Two types of formulation:

Compact: $$S_G^{\mathrm{C}} = \beta \sum_{x^{(4)}} \sum_{j=1,2,3} \left[1 - \mathrm{Re}\left(U_4(x) U_4^\dagger(x+\hat{j}) \right) \right]$$

plaquette

Non-compact: $$S_G^{\mathrm{NC}} = \frac{\beta}{2} \sum_{x^{(4)}} \sum_{j=1,2,3} \left[\theta(x) - \theta(x+\hat{4}) \right]^2$$

⇨ difference …?

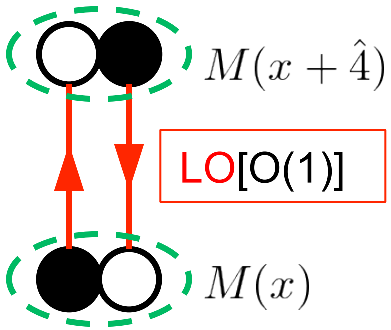
Strong coupling expansion

Expansion parameter: $\beta \equiv \dfrac{1}{4\pi\alpha_*}$ (inverse effective coupling)

Link integration is performed order by order: $[S_G \sim O(\beta)]$

$$Z = \int [d\chi d\bar{\chi}][d\theta] \left[\sum_{n=0}^{\infty} \frac{(-S_G)^n}{n!} e^{-S_F} \right] = \int [d\chi d\bar{\chi}] e^{-S_\chi}$$

► 4-fermi couplings are induced by the link integration.



Linearize by Stratonovich-Hubbard transf.

Introduce <u>complex</u> auxiliary field:

$$\phi(x) \equiv \phi_\sigma + i\epsilon(x)\phi_\pi$$

Scalar
$M(x) = \bar{\chi}(x)\chi(x)$

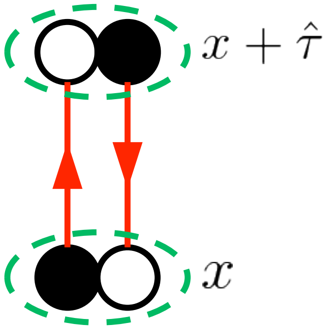
Pseudoscalar
$P(x) = \bar{\chi}(x) i\epsilon(x)\chi(x)$

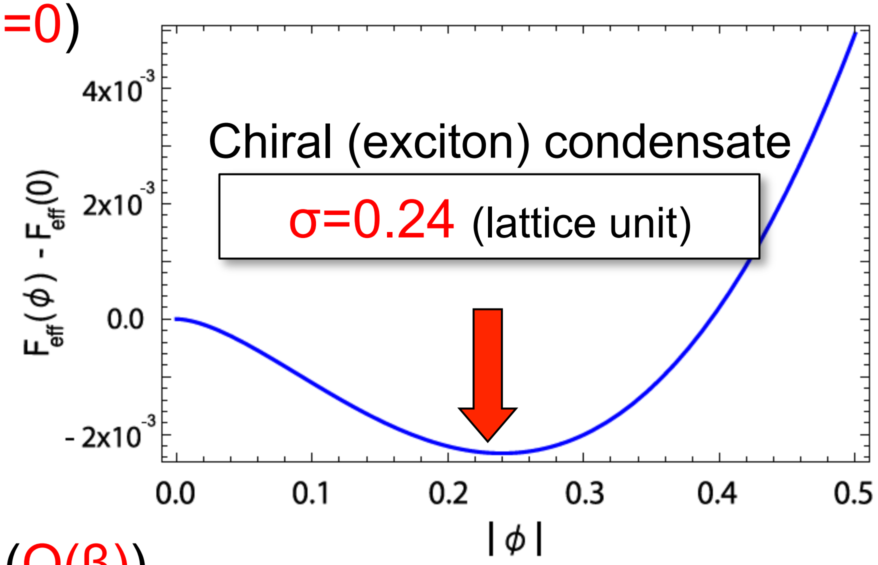
<u>Order parameter of chiSB.</u>

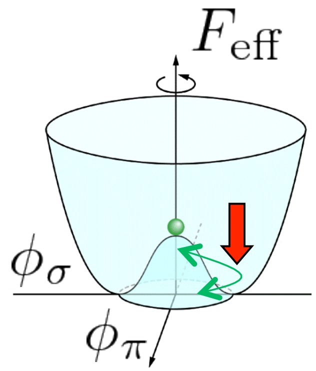
Effective potential and Chiral condensate

► <u>Strong coupling limit</u> (β=0)

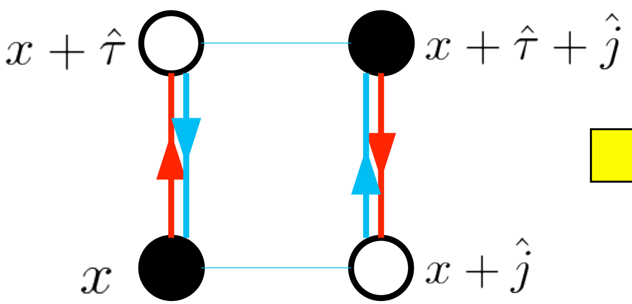
induced 4-fermi:

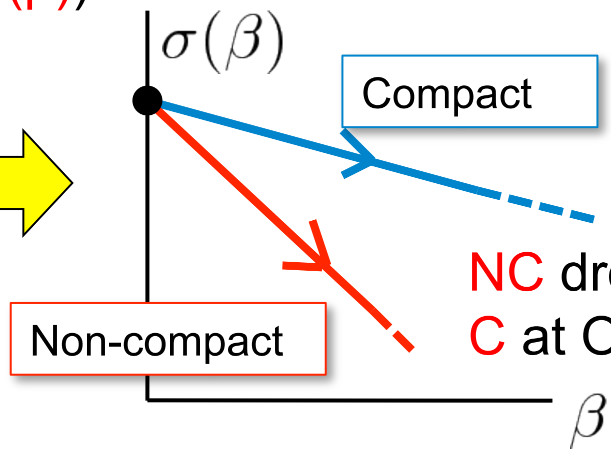






► <u>*N*ext-to-*L*eading Order</u> (O(β))





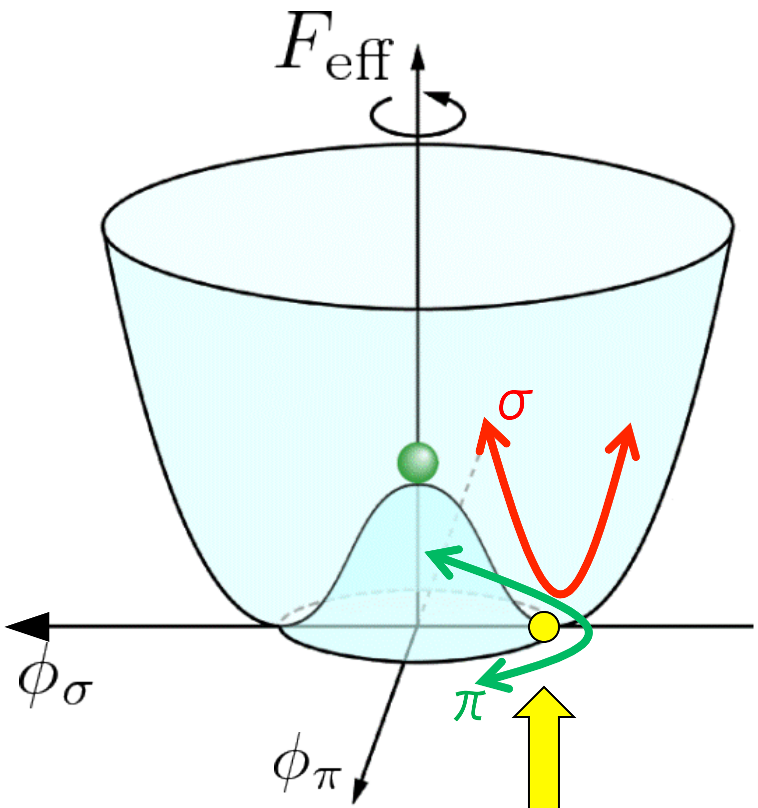
$\sigma^{C}(\beta) = 0.24 - 0.30\beta$

NC drops twice faster than C at O(β).

The value of $\sigma(\beta=0)$ and $\sigma^{NC}(\beta) < \sigma^{C}(\beta)$ agree with MC simulation results. [Drut, Lahde & Suoranta, 2010]

Collective excitations (excitons)

►Two excitation modes: fluctuations of the order parameter ϕ



$$\langle \phi_\sigma \rangle = -\sigma$$

► π-mode (phase fluctuation mode)

$$M_\pi \simeq \frac{2v_F}{a}\sqrt{\frac{m_{\rm bare}}{M_F(m=0)}} \xrightarrow{m=0} 0$$

"NG boson" from chiSB

Light mode (pion-like)

Similar to pions in QCD (GMOR relation):

$$\boxed{M_\pi = \sqrt{m_{\rm bare}\sigma}/f_\pi}$$

► σ-mode (amplitude fluctuation mode)

$$M_\sigma \simeq (5.47 - 1.97\beta)\mathrm{eV}$$

beyond low-energy approx.

Analysis without low-energy EFT…?

Conclusion

► Monolayer graphene ⇨ "reduced QED" model (gapless)

⇨ Lattice strong coupling expansion (analytic)

► Strong coupling: spontaneous chiSB

⇨ Dynamical spectral gap: $M_F = \frac{\sigma}{2} \quad \left[\rightarrow \frac{v_F}{a} \frac{\sigma a^2}{2} \right]$

► Compact vs. Non-compact formulation:

⇨ Difference starts from NLO: $\sigma^{NC}(\beta) < \sigma^{C}(\beta)$

► Collective excitation modes:

π (phase fluctuation) ⇨ Pion-like behavior (GMOR rel.)

Future prospects:

- Taste breaking? (compare with overlap fermions, …)
- Anisotropy effects? (graphene under uniaxial strain)
- Transport properties? (effect on electric conductivity, …)

Thank you.