Phase Strucure of Thermal QED/QCD: Solution of the DS equation and its Gauge Dependence

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Hard-Thermal-Loop Resummed Dyson-Schwinger Equations(Real Time Formalism)

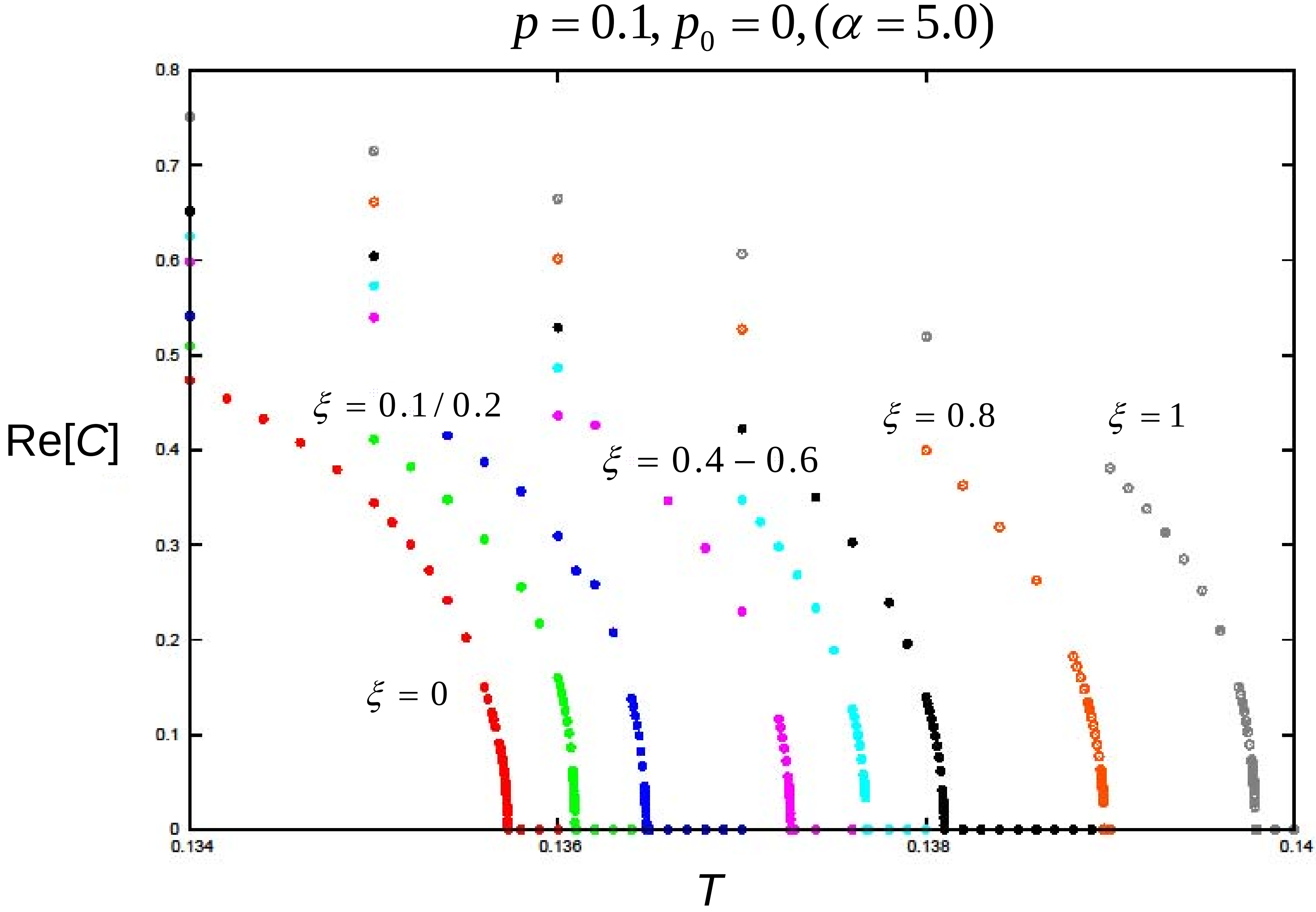
$$-i\Sigma_R(P) = -\frac{e^2}{2}\int \frac{d^4K}{(2\pi)^4} \times \Big[\Gamma^{\mu}_{RAA} S_R(K) \Gamma^{\nu}_{RAA} G_{C,\mu\nu}(P-K)$$
$$+ \Gamma^{\mu}_{RAA} S_C(K)\, \Gamma^{\nu}_{AAR} G_{R,\mu\nu}(P-K) \Big]$$

$$G_R^{\mu\nu}(K) = \frac{1}{\Pi_T^R - K^2 - i\varepsilon k_0} A^{\mu\nu} + \frac{1}{\Pi_L^R - K^2 - i\varepsilon k_0} B^{\mu\nu} - \frac{\xi}{K^2 + i\varepsilon k_0} D^{\mu\nu}$$

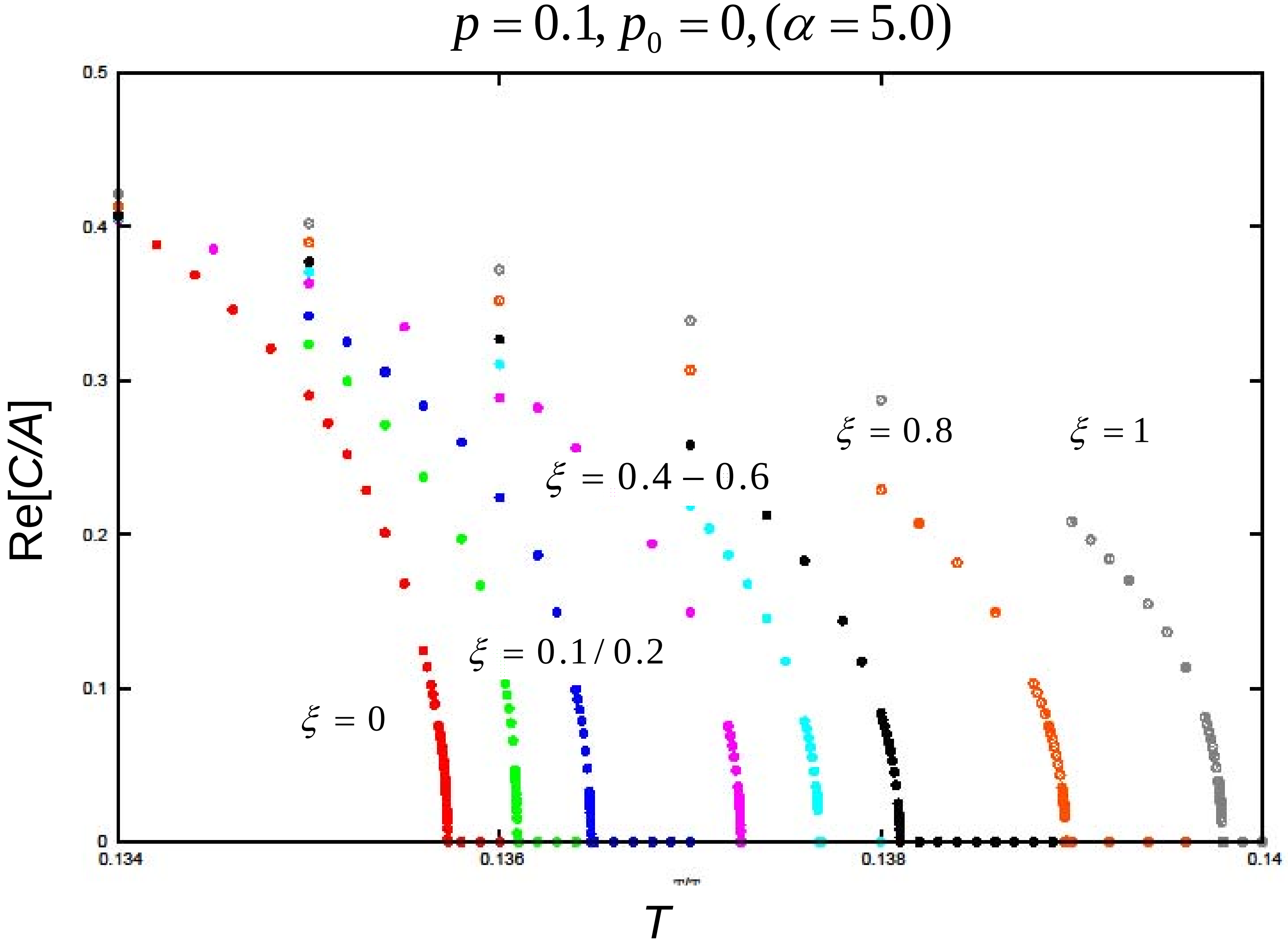
$$A^{\mu\nu} = g^{\mu\nu} - B^{\mu\nu} - D^{\mu\nu},\; B^{\mu\nu} = -\frac{\tilde{K}^{\mu}\tilde{K}^{\nu}}{K^2},\; D^{\mu\nu} = \frac{K^{\mu}K^{\nu}}{K^2},\; \tilde{K} = (k, -k_0\vec{k})$$

$$\Gamma^{\mu} = \gamma^{\mu}$$

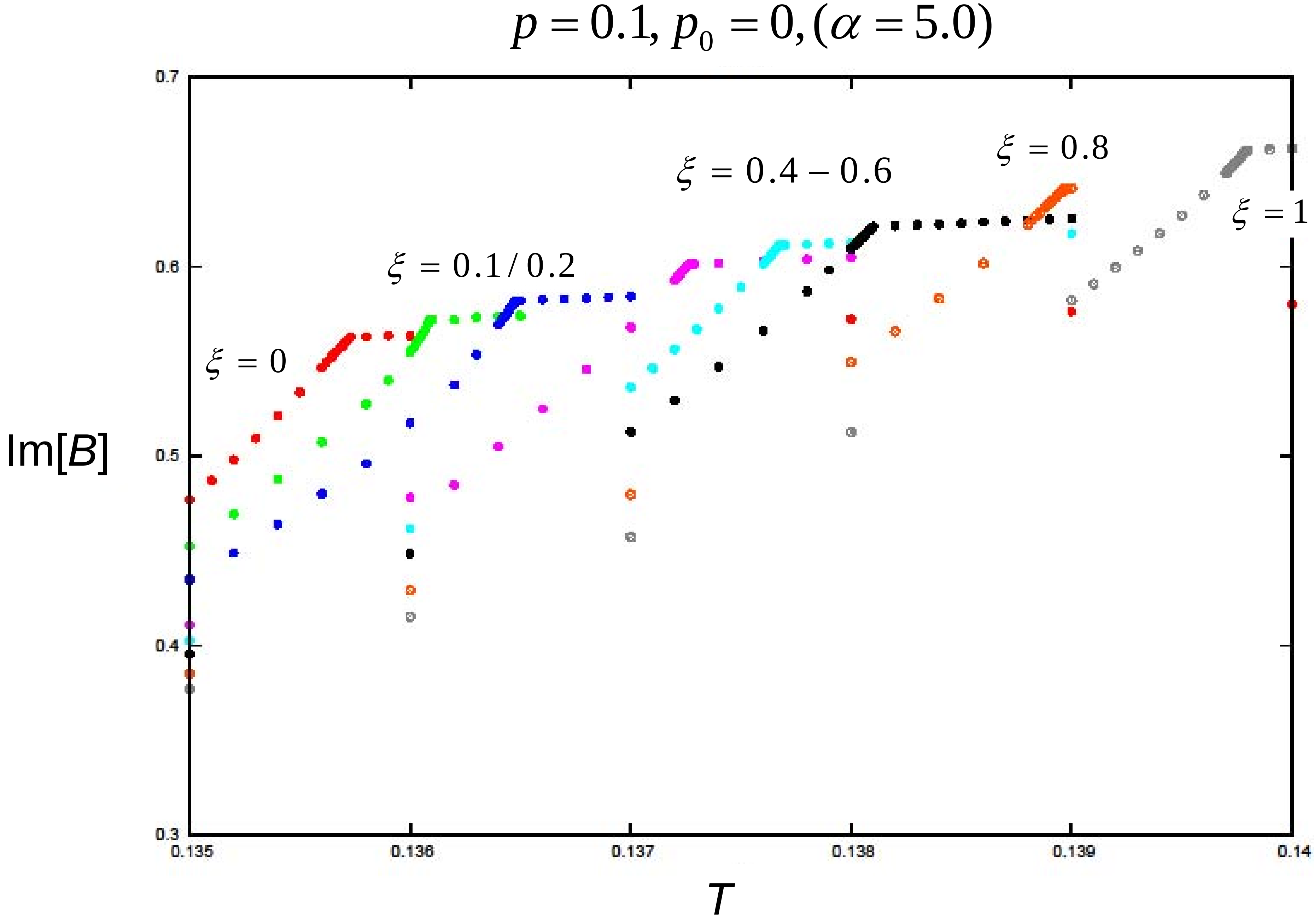
$$\Sigma_R(P) = (1 - A(P))\, p_i \gamma^i - B(P)\gamma_0 + C(P)$$



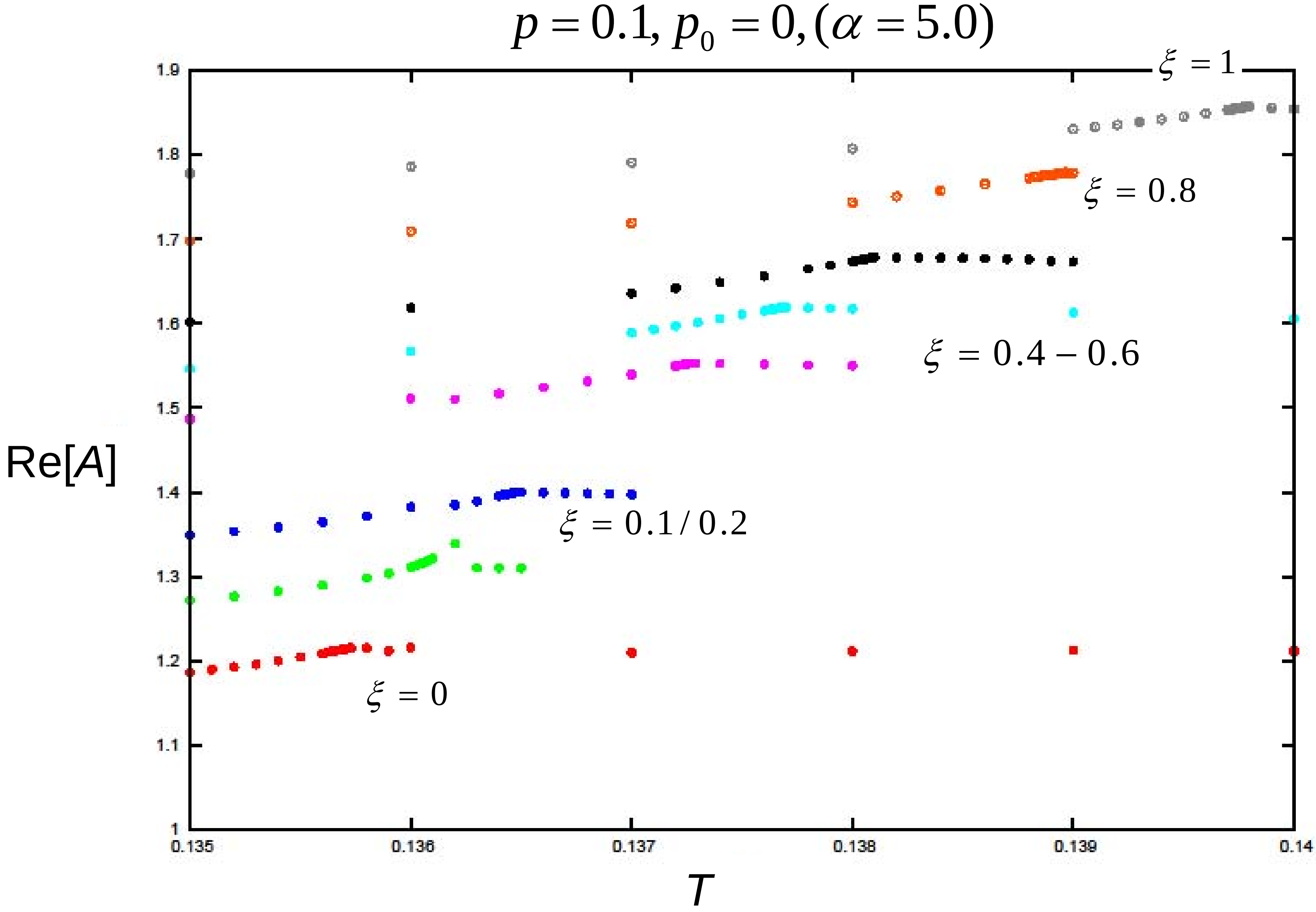
$p = 0.1, p_0 = 0, (\alpha = 5.0)$
Re[C]
T
$\xi = 0.1/0.2$
$\xi = 0.4 - 0.6$
$\xi = 0.8$
$\xi = 1$
$\xi = 0$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.134
0.136
0.138
0.14



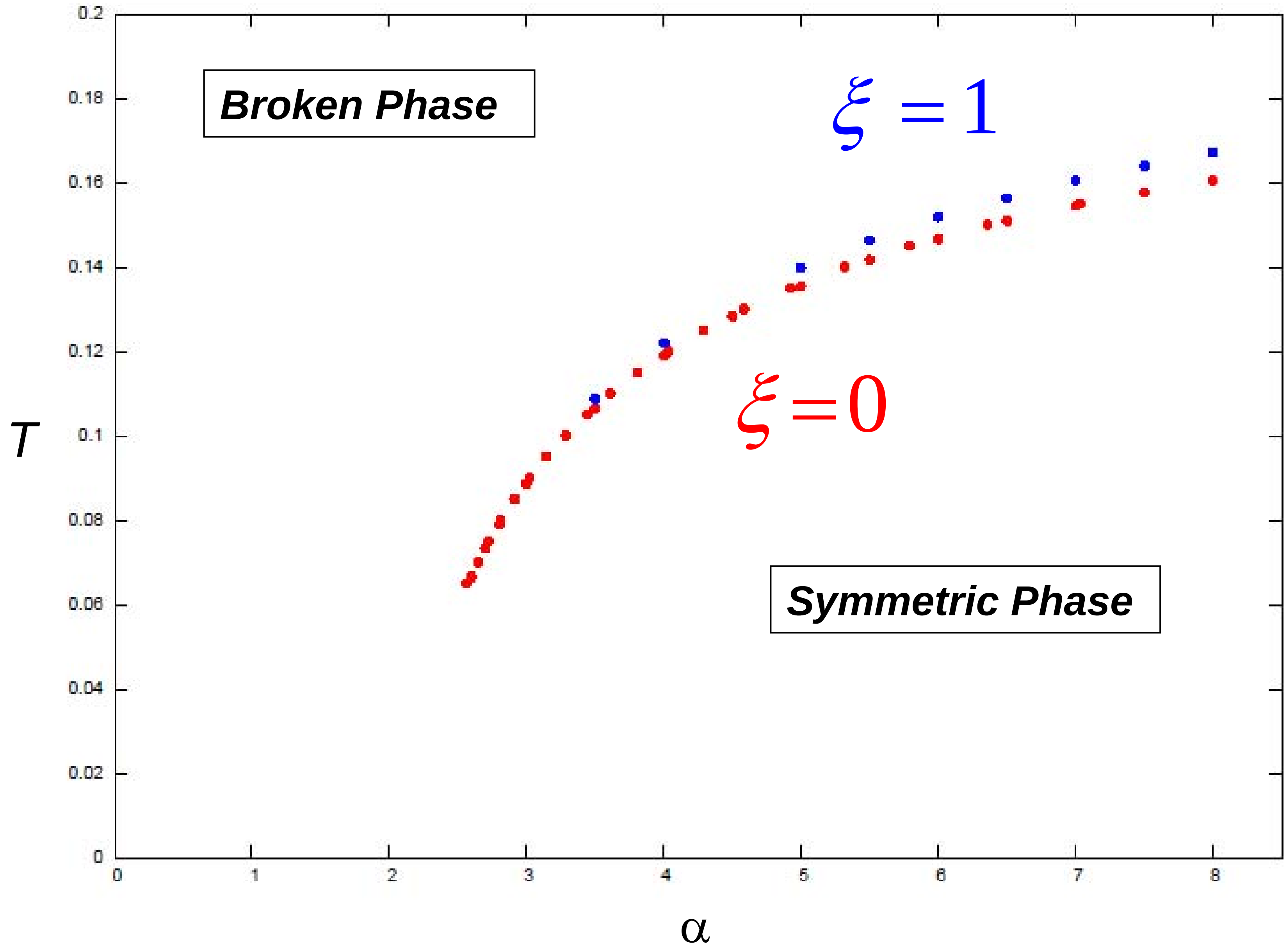
$p = 0.1, p_0 = 0, (\alpha = 5.0)$
Re[C/A]
T
0.5
0.4
0.3
0.2
0.1
0
0.134
0.136
0.138
0.14
$\xi = 0$
$\xi = 0.1/0.2$
$\xi = 0.4 - 0.6$
$\xi = 0.8$
$\xi = 1$



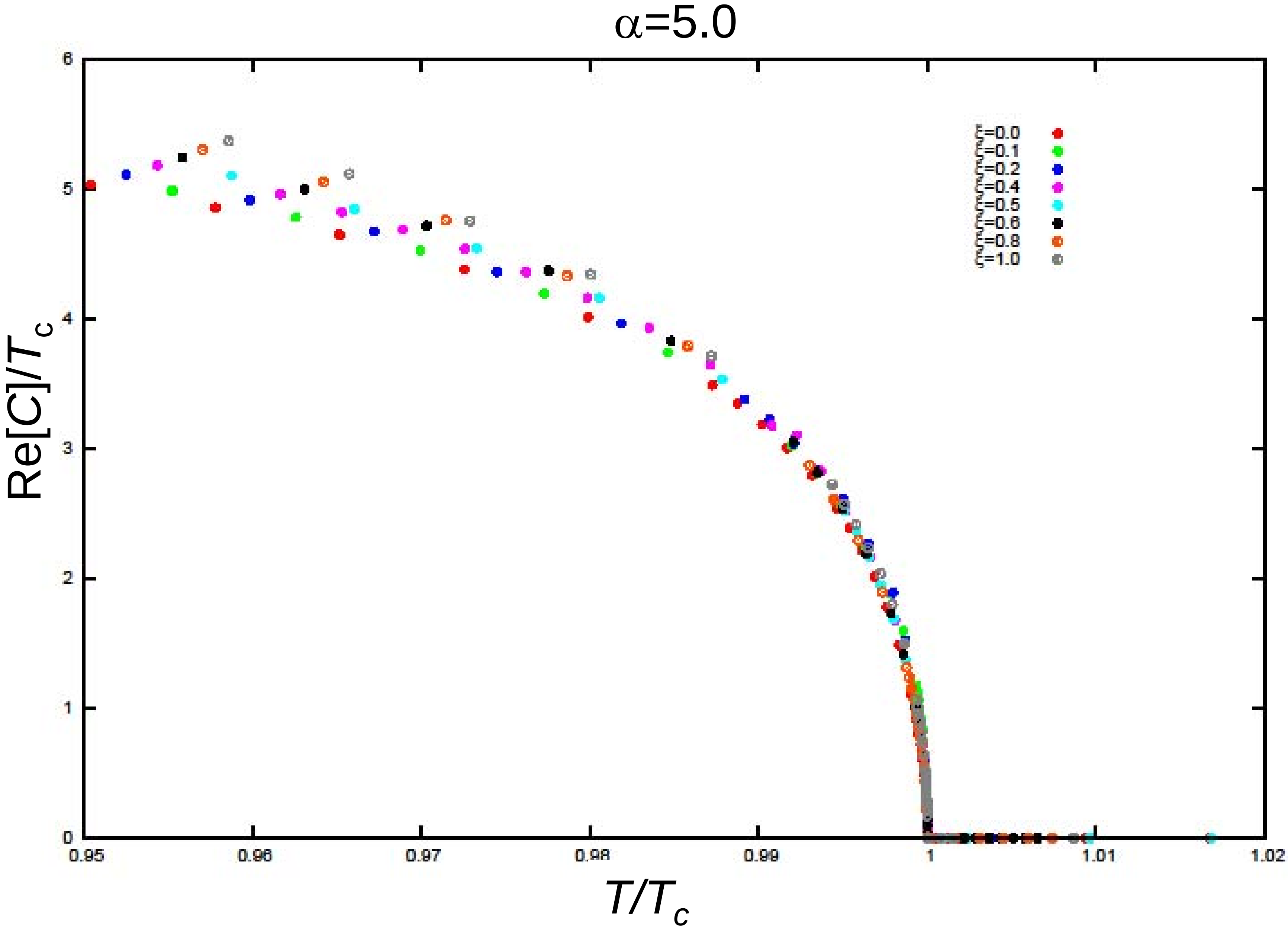
$p = 0.1, p_0 = 0, (\alpha = 5.0)$
Im[B]
T
$\xi = 0$
$\xi = 0.1/0.2$
$\xi = 0.4 - 0.6$
$\xi = 0.8$
$\xi = 1$
0.7
0.6
0.5
0.4
0.3
0.135
0.136
0.137
0.138
0.139
0.14



p = 0.1, p_0 = 0, (α = 5.0)
ξ = 1
ξ = 0.8
ξ = 0.4 – 0.6
ξ = 0.1 / 0.2
ξ = 0
Re[A]
T
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.135
0.136
0.137
0.138
0.139
0.14



Broken Phase
$\xi = 1$
$\xi = 0$
Symmetric Phase
T
α
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0
1
2
3
4
5
6
7
8


α=5.0
Re[C]/T_c
T/T_c
ξ=0.0
ξ=0.1
ξ=0.2
ξ=0.4
ξ=0.5
ξ=0.6
ξ=0.8
ξ=1.0
0
1
2
3
4
5
6
0.95
0.96
0.97
0.98
0.99
1
1.01
1.02

Gauge invariance (Ward Identity)

T=0 Landau gauge($\xi = 0$) $Z_1 = Z_2$

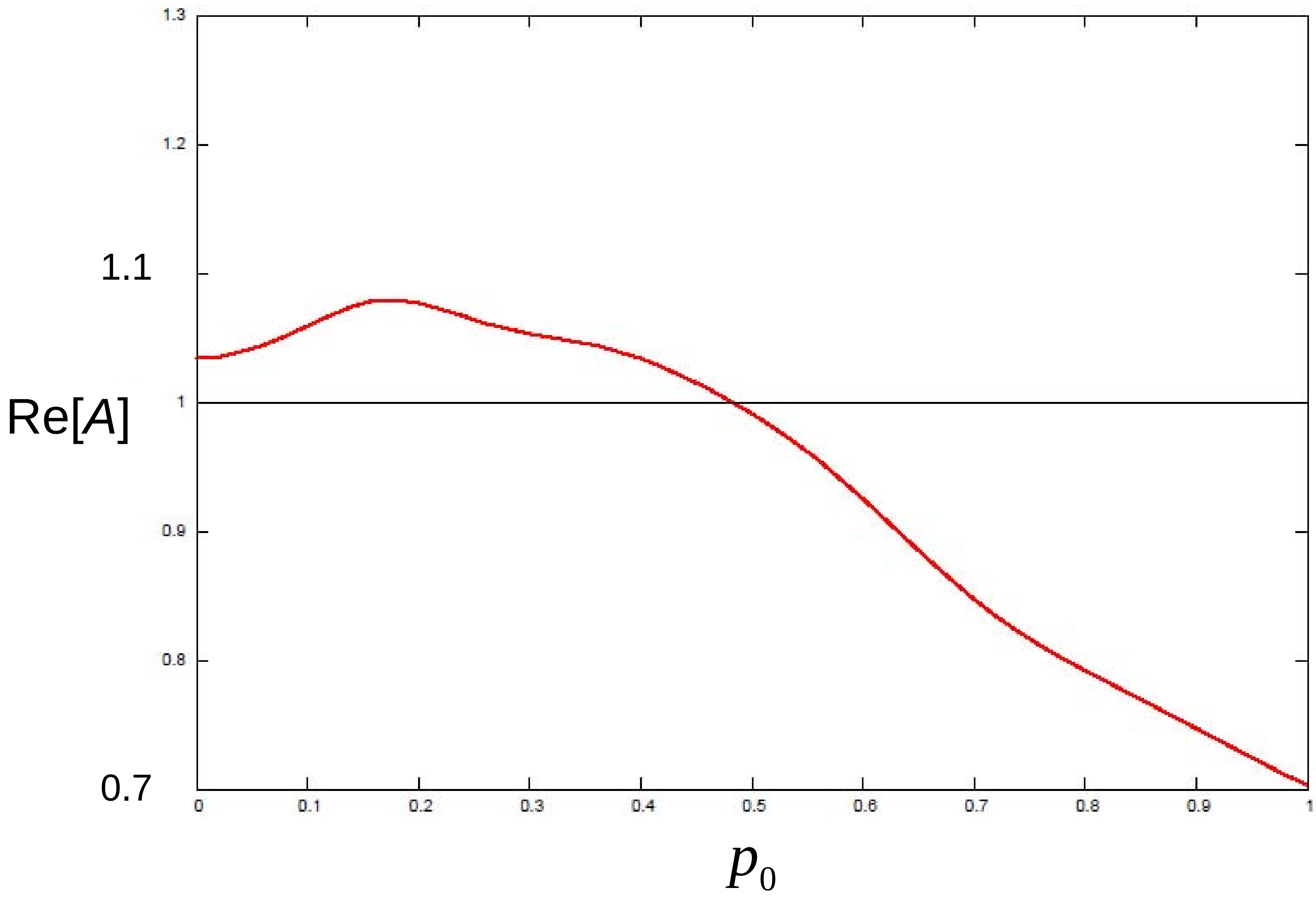
A(P)=1 for the point vertex

T.Maskawa and H.Nakajima,PTP 52,1326(1974)

PTP 54,860(1975)

$T \neq 0 \quad A(P) \neq 1$

$T \neq 0$ Find the gauge ξ such that *A(P)=1*



1.3
1.2
1.1
Re[A]
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p_0

$\xi \to \xi(q_0, q)$ depends on momentum

$A(P) = 1$: integral equation of $\xi(q_0, q)$

Expand $\xi(q_0, q)$ by a series of functions $F_m(q_0), G_n(q)$

$$\xi(q_0, q) = \sum_{mn} C_{mn} F_m(q_0)\, G_n(q)$$

C_{mn} : Expansion coefficients

Minimize $\delta A = \int d^4 P \left| A - 1 \right|^2, \ \Rightarrow \ \dfrac{\partial \delta A}{\partial C_{mn}} = 0$

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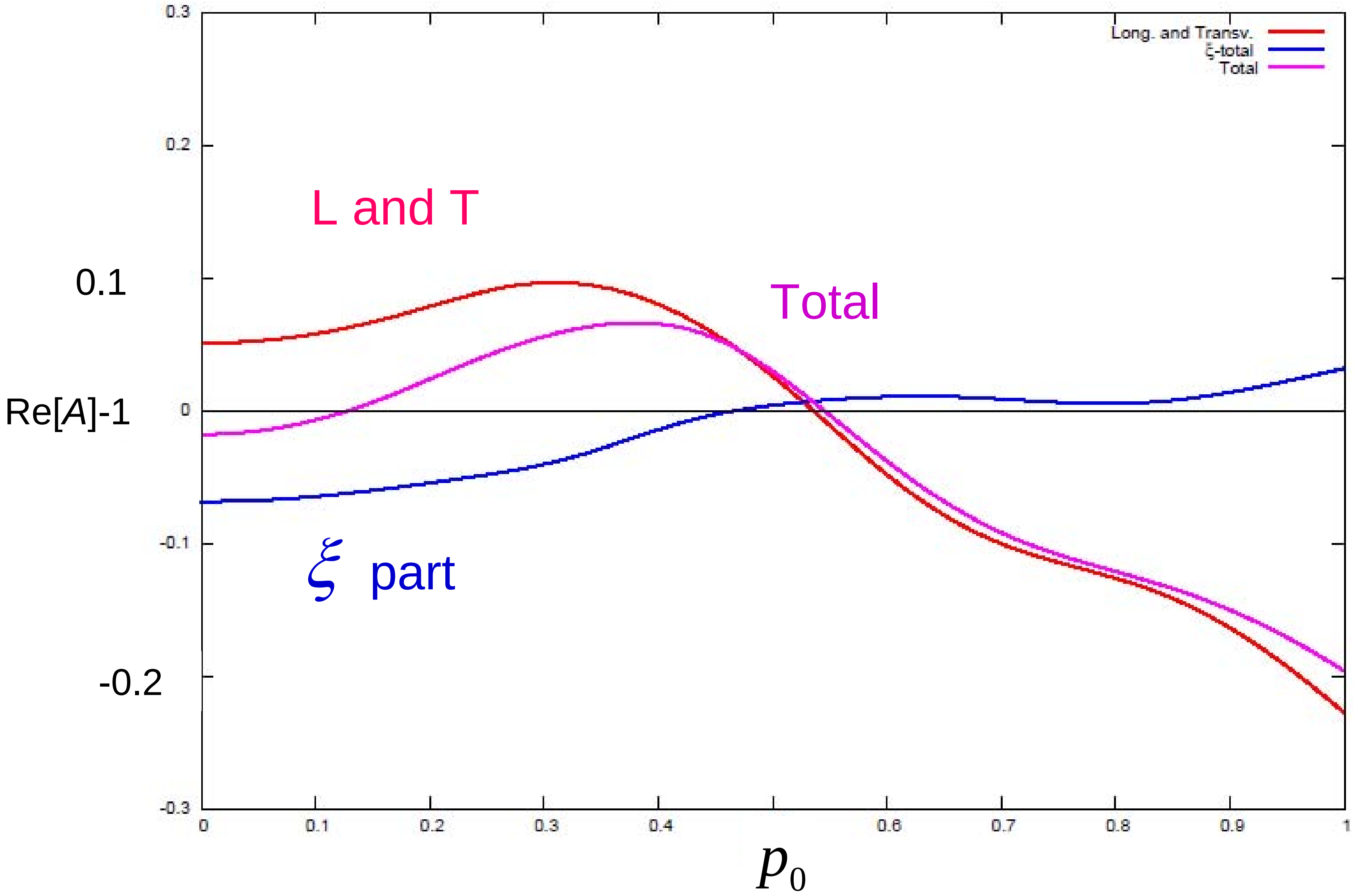
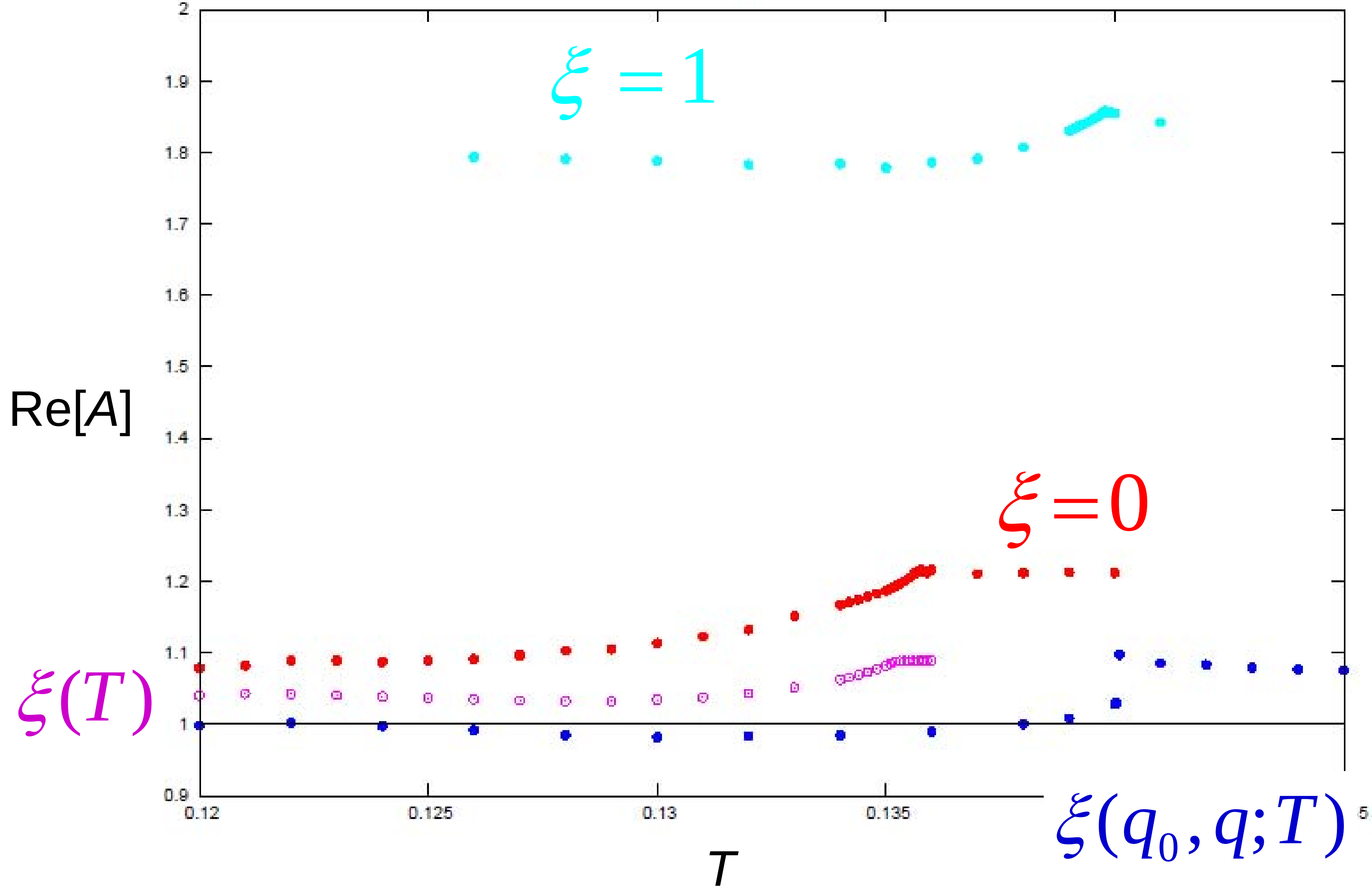
$$G_R^{\mu\nu}(K) = \frac{1}{\Pi_T^R - K^2 - i\varepsilon k_0} A^{\mu\nu} + \frac{1}{\Pi_L^R - K^2 - i\varepsilon k_0} B^{\mu\nu} - \frac{\xi(k_0, k)}{K^2 + i\varepsilon k_0} D^{\mu\nu}$$

$$\Sigma_R(P) = (1 - A(P))\, p_i \gamma^i - B(P)\gamma_0 + C(P)$$

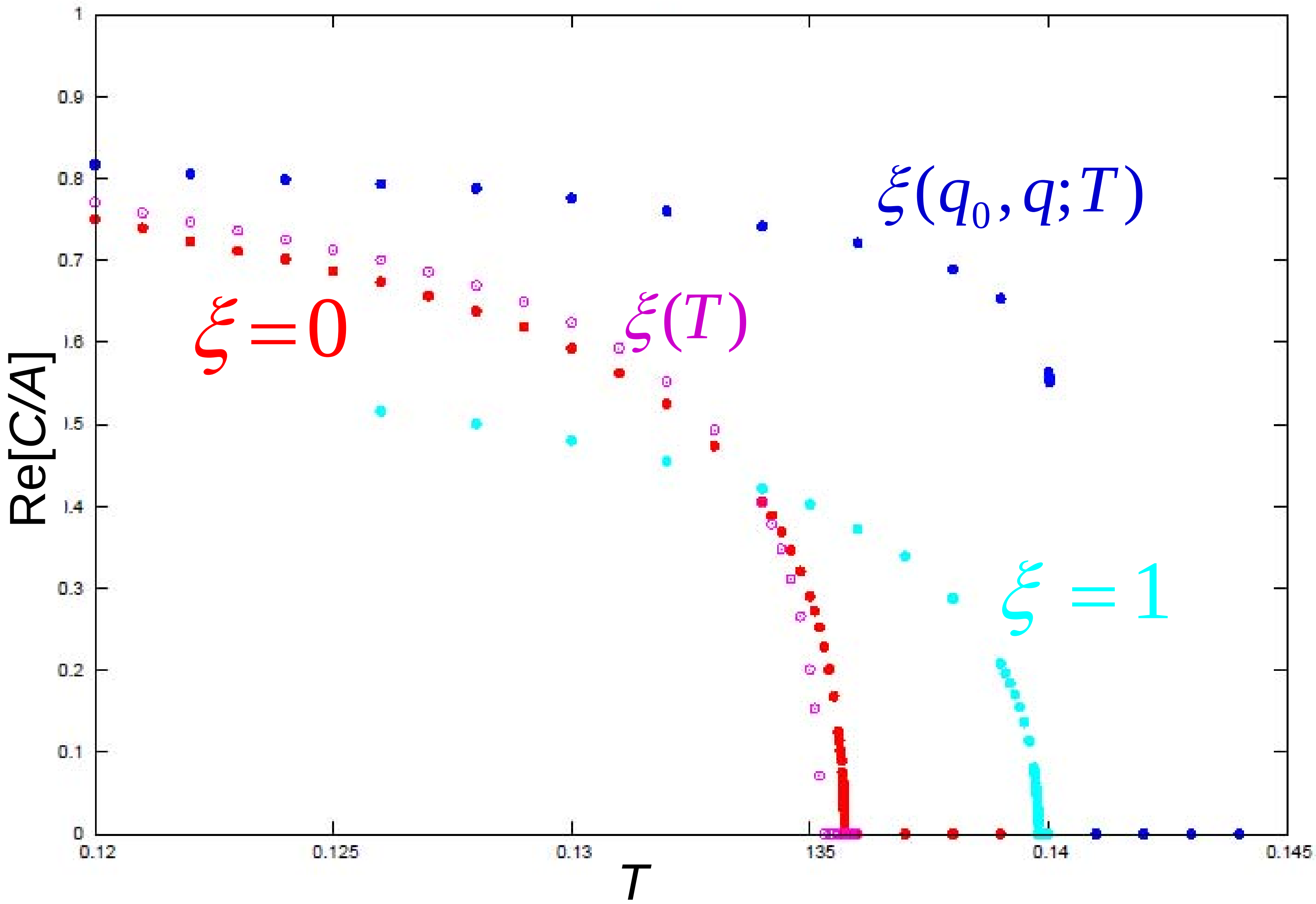


Long. and Transv.
ξ-total
Total
L and T
Total
ξ part
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Re[A]-1
0
0.1
0.2
0.3
0.4
0.6
0.7
0.8
0.9
1
p0

$p = 0.1,\ p_0 = 0, (\alpha = 5.0)$



$$p = 0.1, p_0 = 0, (\alpha = 5.0)$$



Conclusion

- T_c の gauge dependence は大きくはない。

- WT identity： $A(P) \neq 1$ （Linear gauge)

- Nonlinear gauge: $\xi(q_0, q)$ $\Rightarrow$ $A(P) \approx 1$

  （最適な関数と数によってより広い範囲で）