

Structure of the Thermal Quasi-Particle

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2009.9.3~9.5 熱場の量子論とその応用

有限温度(および有限密度)における相構造や準粒子の構造を硬熱ループ(HTL)近似Dyson-Schwinger方程式に基づく解析をする

- 場の理論に範囲で厳密に導出される非摂動論的な結論を得るための基礎方程式である
- その積分核に対する近似が解析的処方に基づいて逐次的に改善可能である

解を求めるのは大変なので、近似を行う

1. HTL resummed gauge boson propagator

2. Ladder approximation ($Z_1 = 1$) $\delta\Gamma_{\alpha\beta\gamma}^{\mu} = 0$

3. Modified IE (MIE) approximation $G^{\mu\nu}(K)$ で $k_0 = 0$

* transverse modeはIE近似をとらない

これらの近似のもとで、解析を行う

⇒ A, B, Cの6元連立方程式の解を求める

Hard-Thermal-Loop Resummed Dyson-Schwinger Equations

PTP 107 (2002) 759

$$\begin{aligned}
 -i\Sigma_R(P) &= -\frac{e^2}{2} \int \frac{d^4 K}{(2\pi)^4} \\
 &\times [{}^*\Gamma_{RAA}^\mu(-P, K, P-K) S_{RA}(-K, K) {}^*\Gamma_{RAA}^\nu(-K, P, K-P) {}^*G_{RR, \mu\nu}(K-P, P-K) \\
 &+ {}^*\Gamma_{RAA}^\mu(-P, K, P-K) S_{RR}(-K, K) {}^*\Gamma_{AAR}^\nu(-K, P, K-P) {}^*G_{RA, \mu\nu}(K-P, P-K)]
 \end{aligned}$$

$$\begin{aligned}
 {}^*G_R^{\mu\nu}(K) &\equiv {}^*G_{RA}^{\mu\nu}(-K, K) \\
 &= \frac{1}{{}^*\Pi_T^R(K) - K^2 - i\epsilon k_0} A^{\mu\nu} + \frac{1}{{}^*\Pi_L^R(K) - K^2 - i\epsilon k_0} B^{\mu\nu} - \frac{\xi}{K^2 + i\epsilon k_0} D^{\mu\nu} \\
 {}^*G_C^{\mu\nu}(K) &\equiv {}^*G_{RR}^{\mu\nu}(-K, K) \\
 &= (1 + 2n_B(k_0)) [{}^*G_R^{\mu\nu}(K) - {}^*G_A^{\mu\nu}(K)] \\
 n_B(k_0) &= \frac{1}{\exp(k_0/T) - 1}
 \end{aligned}$$

$$A_{\mu\nu} = g^{\mu\nu} - B_{\mu\nu} - D^{\mu\nu}, \quad B_{\mu\nu} = -\frac{\tilde{K}^\mu \tilde{K}^\nu}{K^2}, \quad D_{\mu\nu} = \frac{K^\mu K^\nu}{K^2}$$

$$\tilde{K} = (k, k_0 \vec{k}/k)$$

$$S_R(P) \equiv S_{RA}(-P, P) = \frac{1}{\not{P} + i\epsilon\gamma_0 - \Sigma_R}$$

$$S_C(P) \equiv S_{RR}(-P, P) = (1 - 2n_F(p_0)) [S_R(P) - S_A(P)]$$

$$n_F(p_0) = \frac{1}{\exp(p_0/T) + 1}$$

$$\Sigma_R(P) = (1 - A(P))p_i\gamma^i - B(P)\gamma^0 + C(P)$$

$$*\Gamma_{\alpha\beta\gamma}^\mu = \gamma_{\alpha\beta\gamma}^\mu + \delta\Gamma_{\alpha\beta\gamma}^\mu$$

$$\gamma_{RAA}^\mu = \gamma_{AAR}^\mu = \gamma^\mu, \quad \text{その他はすべて } 0$$

cutoff Λ, Λ' を導入する

$$\int_0^\Lambda dk \int_{-\Lambda'}^{\Lambda'} dk_0 \Rightarrow \int_0^1 dk \int_{-\Lambda_0}^{\Lambda_0} dk_0$$

massの次元を持つ量は Λ を単位ではかる

k, k_0 積分でcutoffを変えて依存性を確認

$$\Lambda_0=1, 2, 3, \dots$$

k, k_0 を等分割し、Iterationで解を求める

$$\begin{aligned} S_R^{-1} &= A(P)p_i\gamma^i + \{p_0 + B(P)\}\gamma^0 - C(P) \\ &= A(p)\cancel{P} + \{p_0 - p_0A(P) + B(P)\}\cancel{\not{p}} - C(P) \end{aligned}$$

- 質量 $M (=C/A)$ の振る舞いと相構造
- 波動繰り込み定数 A^{-1} の値(振る舞い)

Ward恒等式とゲージ依存性

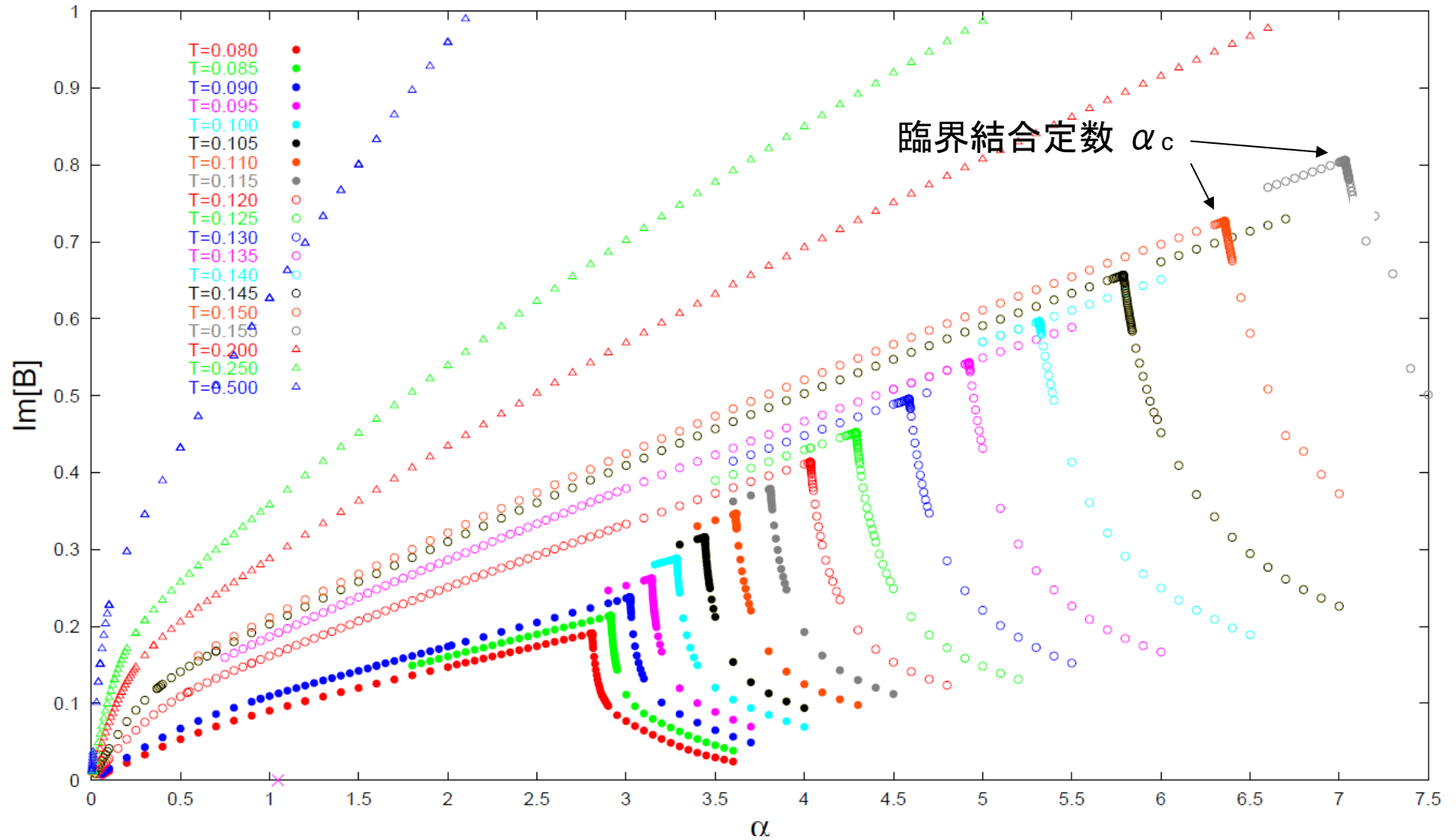
⇒ 吉田講演

- $\text{Im}[B]$ の振る舞いとスペクトラル関数の構造

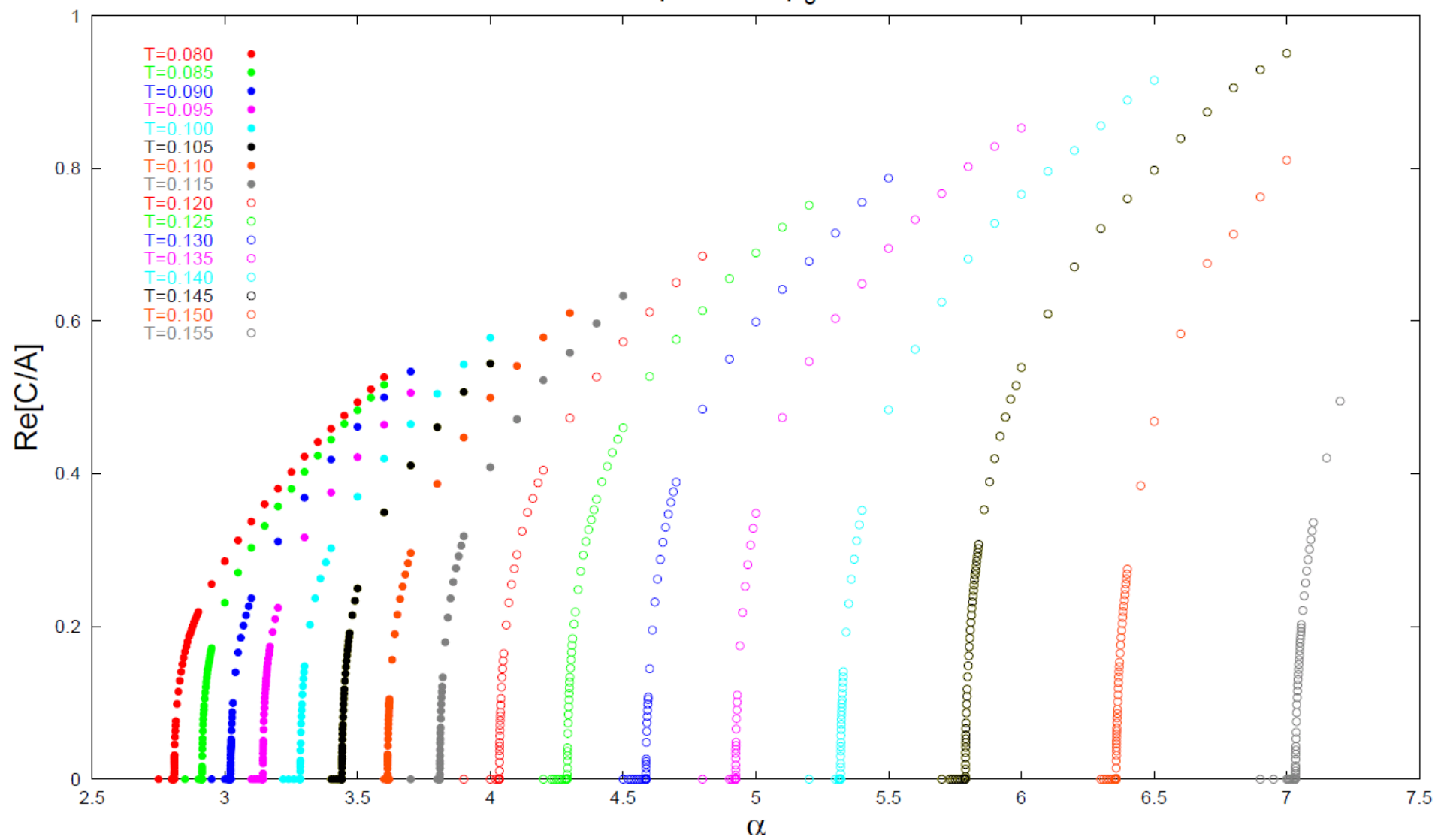
⇒ 本講演

$\text{Im}[B(p_0=0, p=0.1 \wedge)]$ の振る舞い

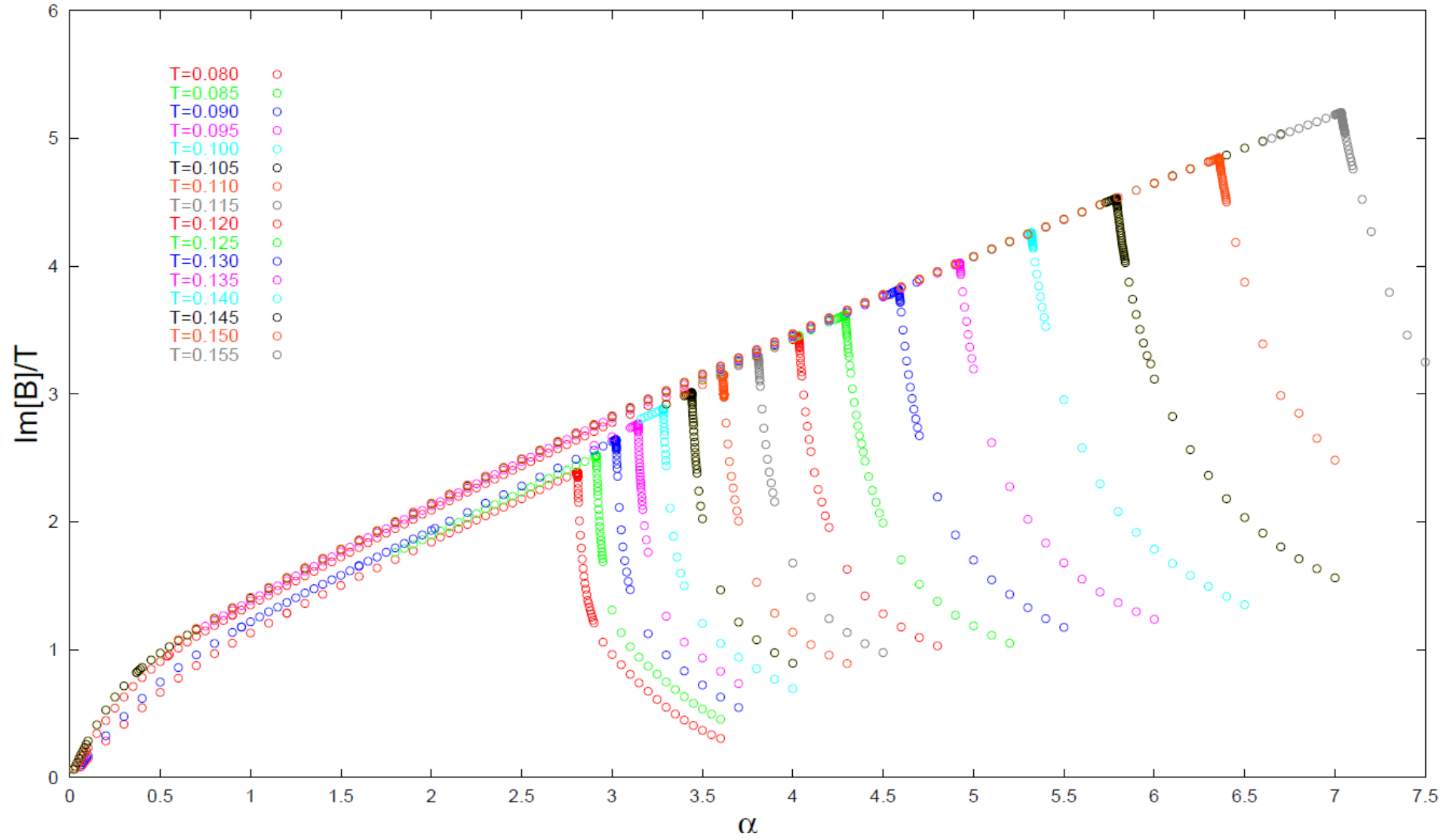
$p=0.1 \wedge, p_0=0$



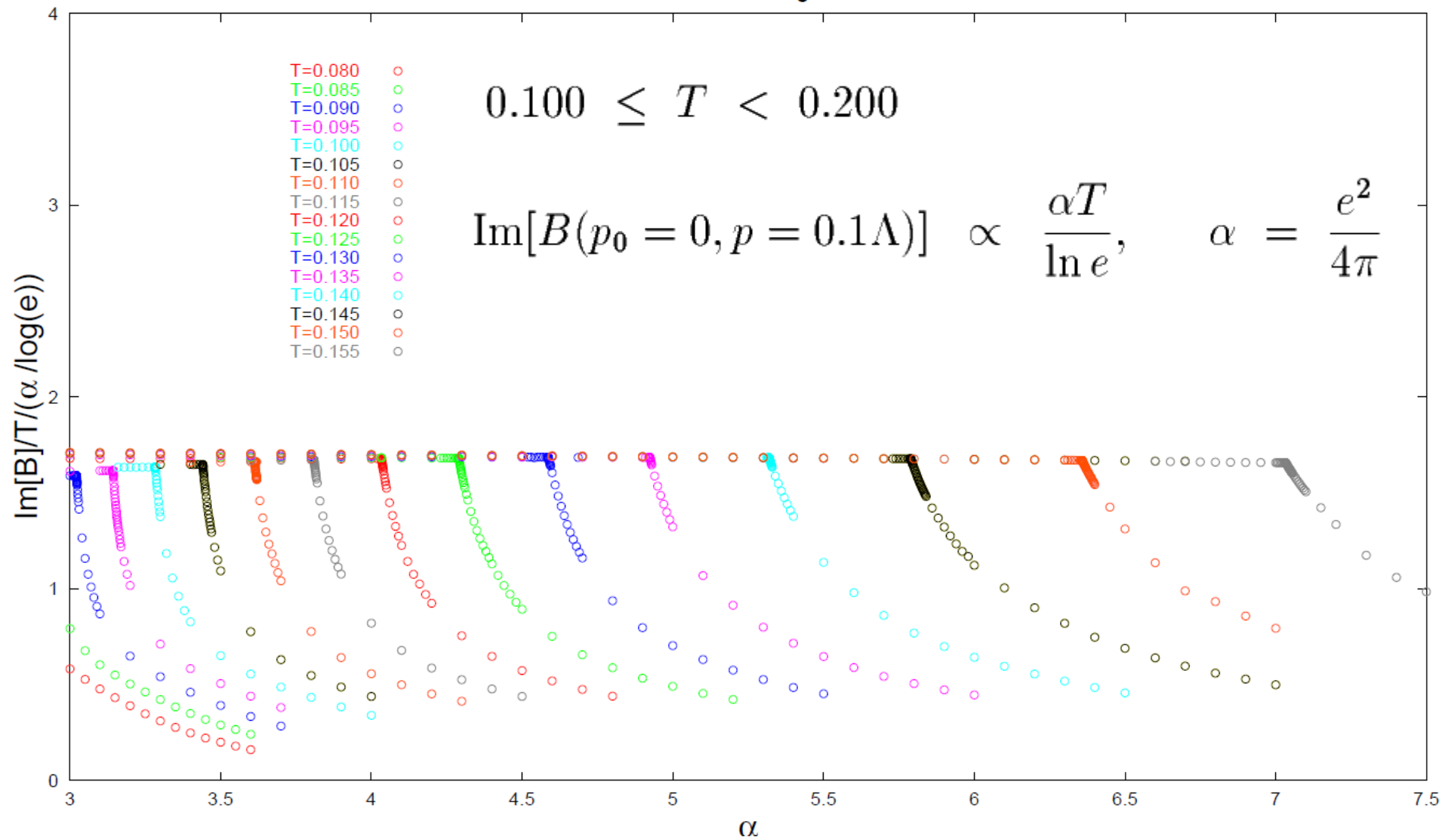
$\rho=0.1 \Lambda, p_0=0$



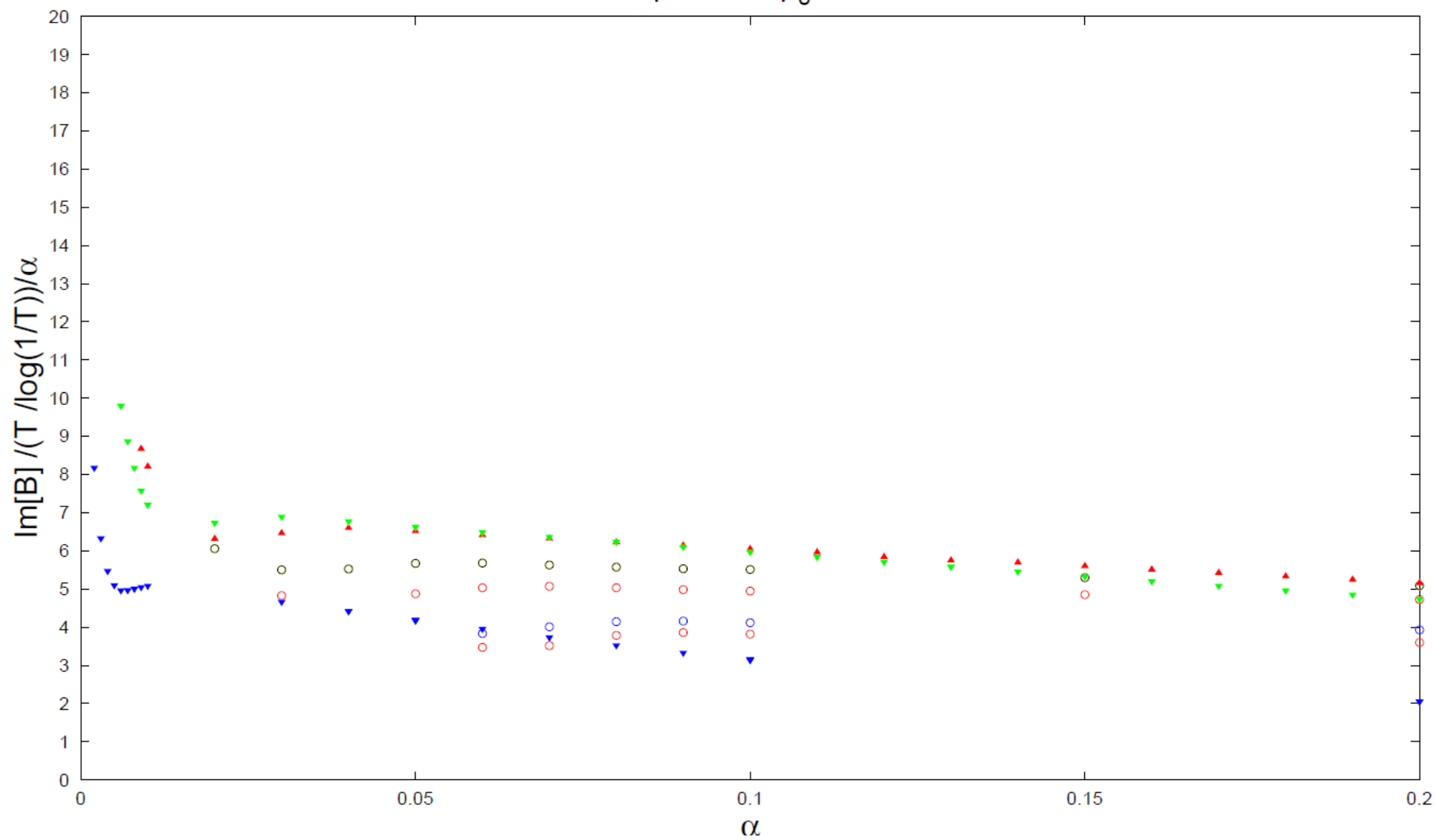
$p=0.1 \quad \Lambda, p_0=0$



$p=0.1 \Lambda, p_0=0$



$p=0.1 \quad \Lambda, p_0=0$



スペクトラル関数 (symmetric phase)

$$\rho_{\pm}(p_0, p) = -\frac{1}{\pi} \text{Im} \frac{1}{D_{\pm}(p_0, p)} = -\frac{1}{\pi} \text{Im} \frac{1}{p_0 + B(p_0, p) \mp pA(p_0, p)}$$

$$S_R = \frac{1}{2} \left[\frac{1}{D_+} \left(\gamma^0 + \frac{p_i \gamma^i}{p} \right) + \frac{1}{D_-} \left(\gamma^0 - \frac{p_i \gamma^i}{p} \right) \right]$$

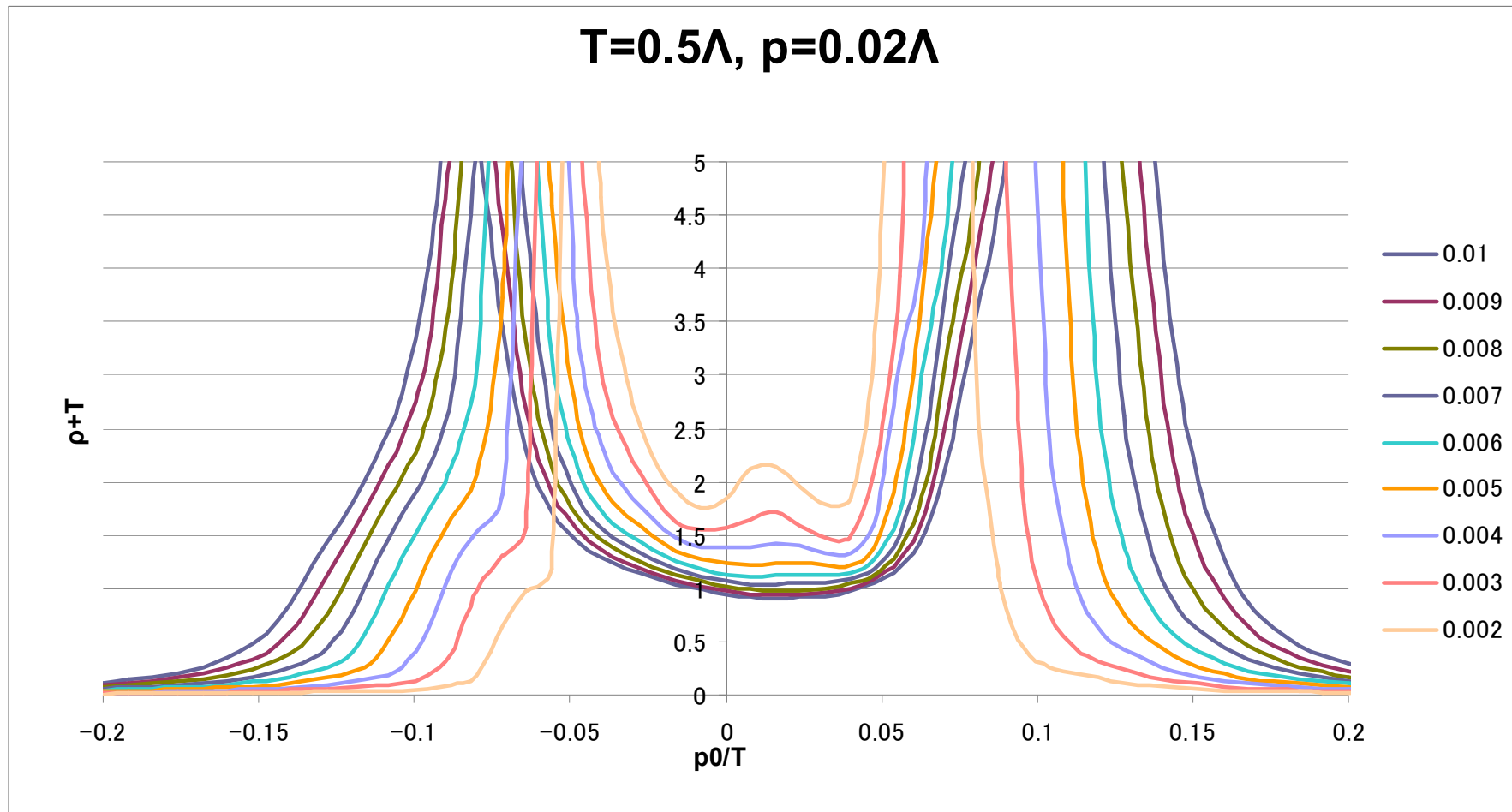
Charge Conjugationを満足すべき

$$\rho_-(p_0, p) = \rho_+(-p_0, p)$$

$$\text{Re}[A(p_0, p)] = \text{Re}[A(-p_0, p)], \quad \text{Im}[A(p_0, p)] = -\text{Im}[A(-p_0, p)]$$

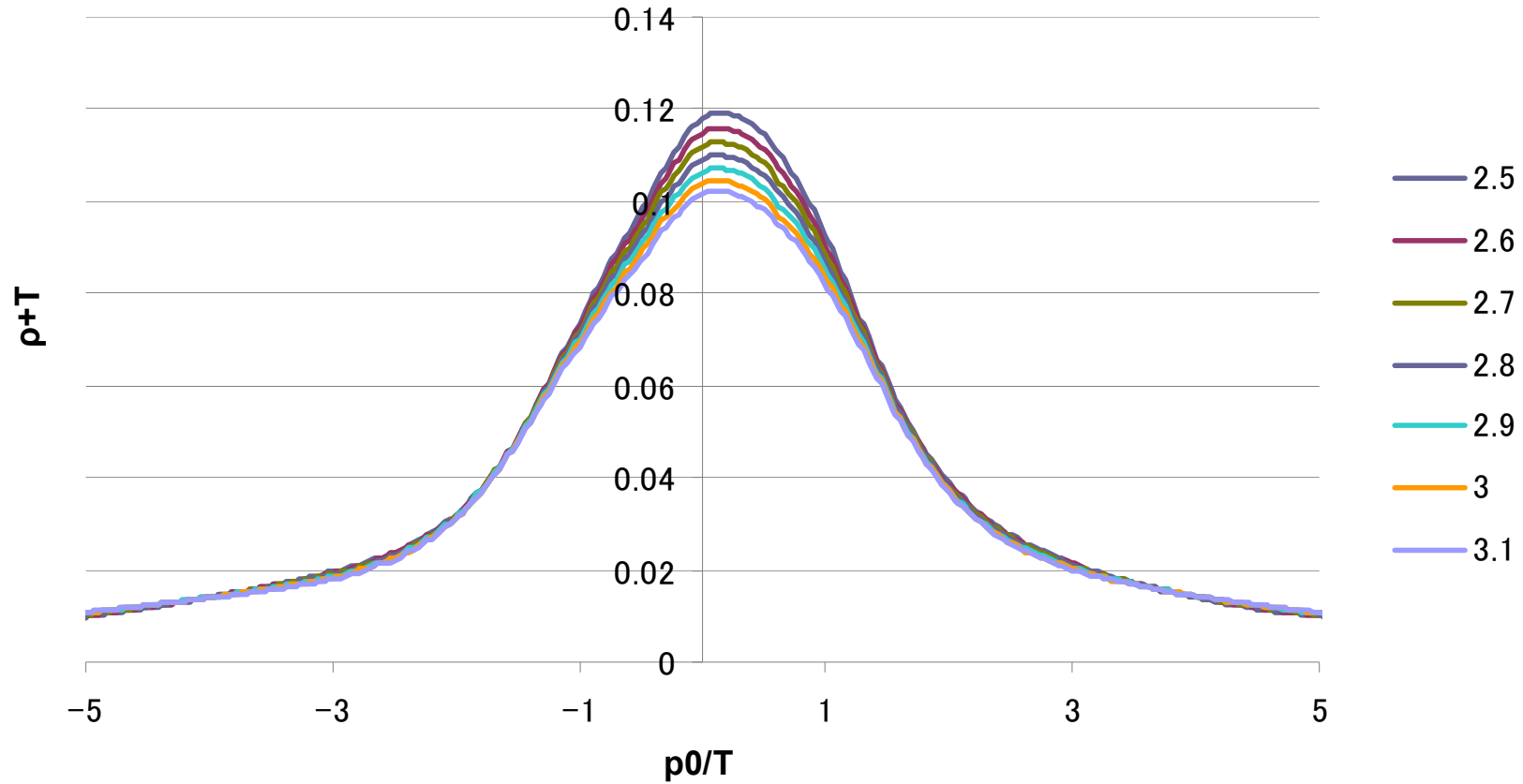
$$\text{Re}[B(p_0, p)] = -\text{Re}[B(-p_0, p)], \quad \text{Im}[B(p_0, p)] = \text{Im}[B(-p_0, p)]$$

得られた解は満たしている (満たさないときは、収束しない)



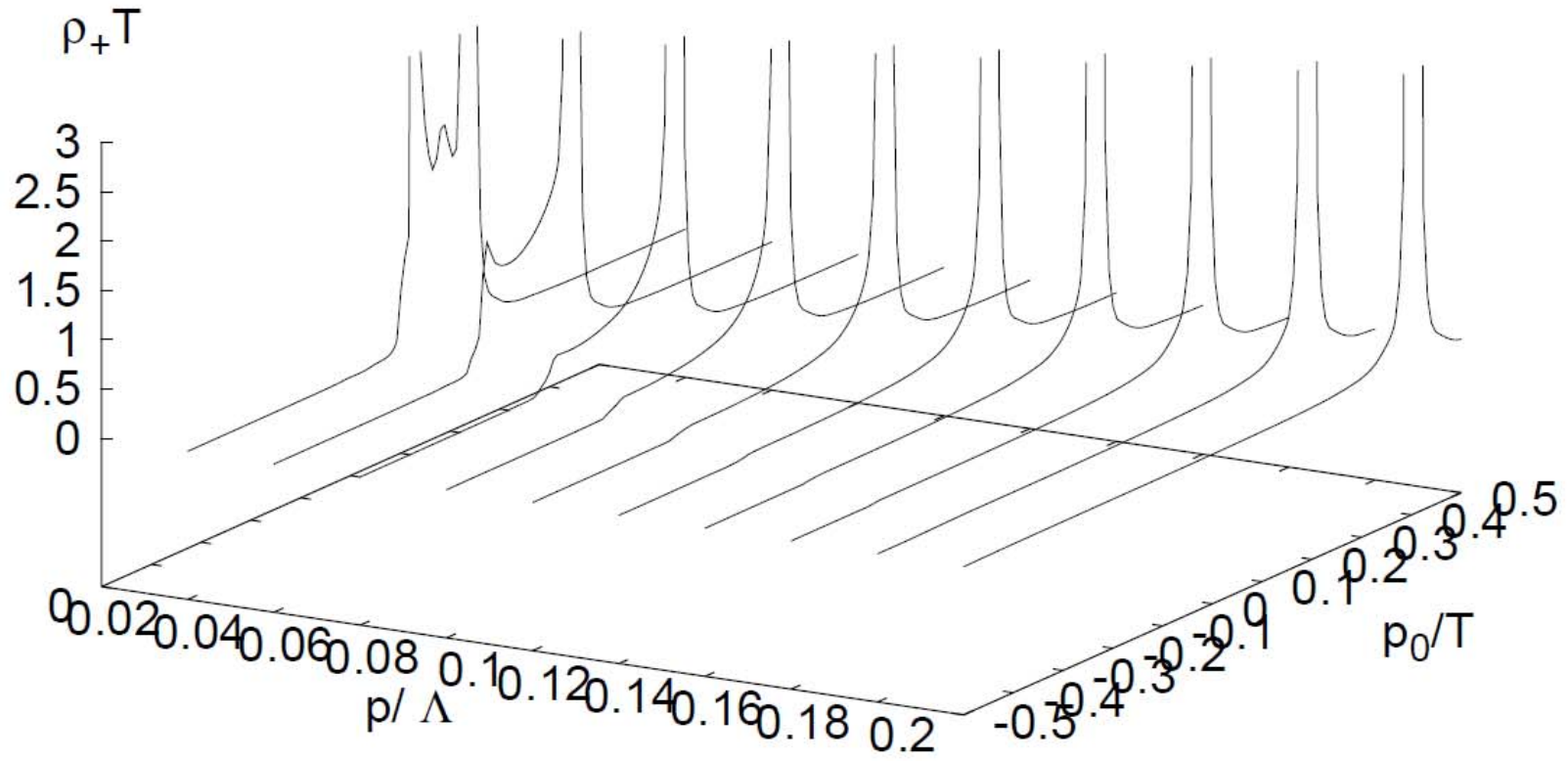
small couplong で three-peak構造

$T=0.08\Lambda$, $p=0.02\Lambda$

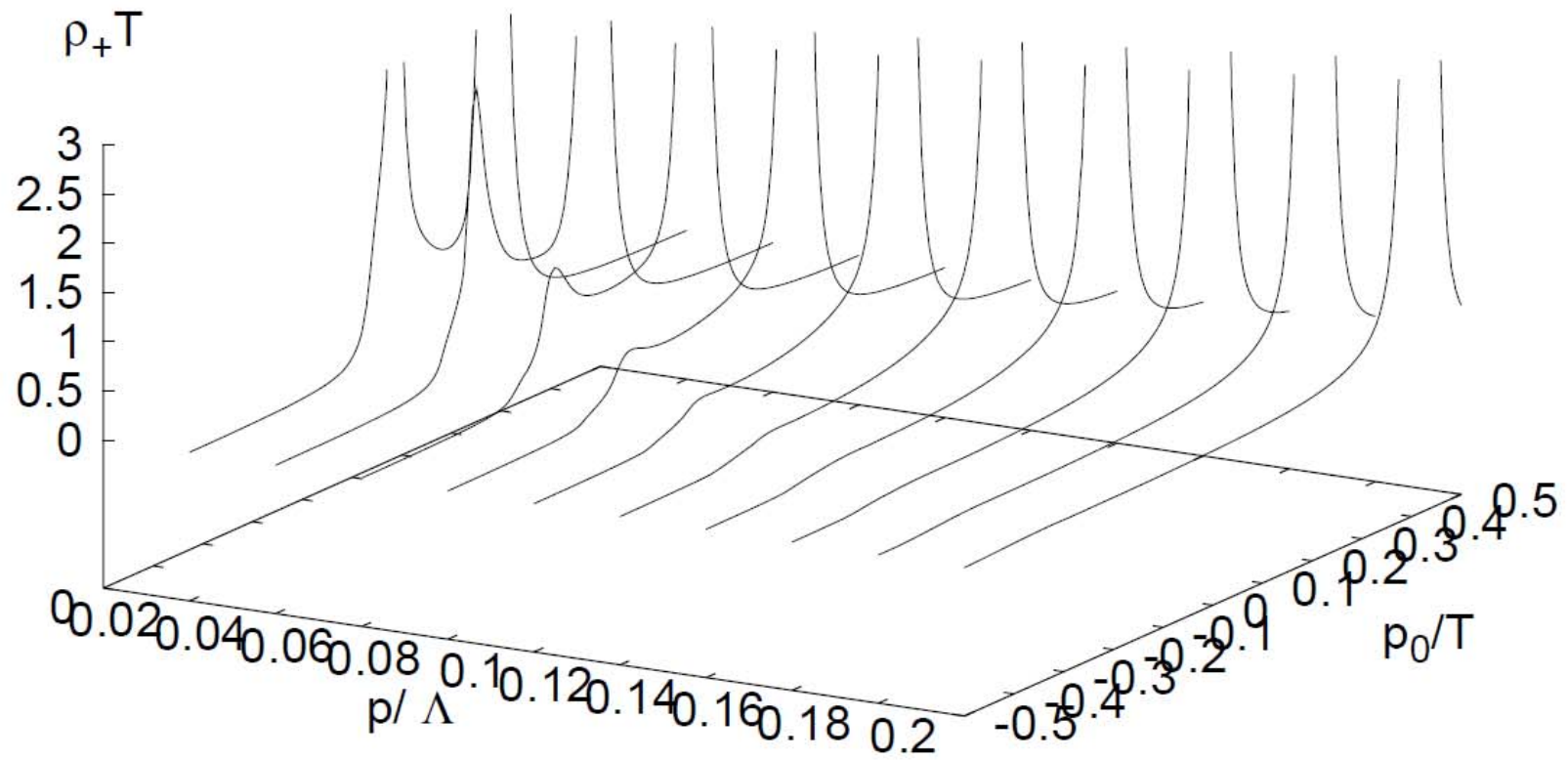


one-peak構造

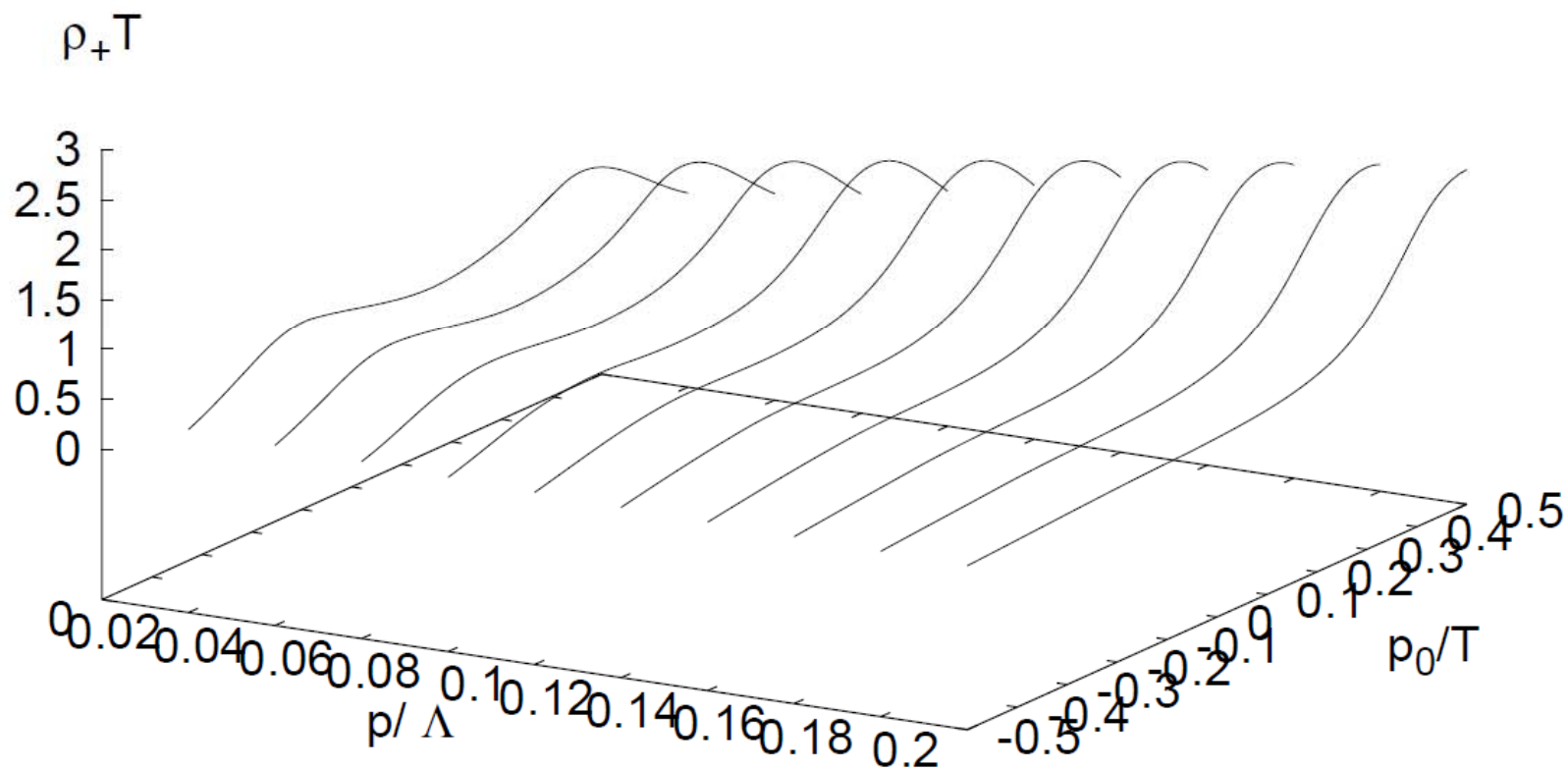
$T=0.5 \Lambda, \alpha=0.002$



$T=0.5 \Lambda, \alpha=0.01$



$T=0.5 \Lambda, \alpha=0.1$



まとめ

**Im[B] は、symmetric phase と broken phase で振る舞い
が変わる**

Symmetric phase では

$0.100 \leq T < 0.200$ のとき, ほぼ αT の振る舞いをする
($3.5 < \alpha < 7.5$)

$$\text{Im}[B(p_0 = 0, p = 0.1\Lambda)] \propto \frac{\alpha T}{\ln e}, \quad \alpha = \frac{e^2}{4\pi}$$

weak coupling での振る舞いは、解明できていない

スペクトラル関数

weak coupling ($\alpha < 0.1?$)では、

一般に two-peak構造で、

$p \sim 0 (< 0.04)$ で three-peak構造 になる

strong coupling では、 one-peak構造である

Ladder approximation

$$\begin{aligned}
 & p^2[1 - A(P)] \\
 &= e^2 \int \frac{d^4 K}{(2\pi)^4} \left[\{1 + 2n_B(p_0 - k_0)\} \text{Im} [{}^* G_R^{\rho\sigma}(P - K)] \times \right. \\
 & \quad \left[\{K_\sigma P_\rho + K_\rho P_\sigma - p_0(K_\sigma g_{\rho 0} + K_\rho g_{\sigma 0}) - k_0(P_\sigma g_{\rho 0} + P_\rho g_{\sigma 0}) + pkz g_{\sigma\rho} \right. \\
 & \quad \left. + 2p_0 k_0 g_{\sigma 0} g_{\rho 0} \} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right. \\
 & \quad \left. + \{P_\sigma g_{\rho 0} + P_\rho g_{\sigma 0} - 2p_0 g_{\sigma 0} g_{\rho 0}\} \frac{k_0 + B(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right] \\
 & + \{1 - 2n_F(k_0)\} {}^* G_R^{\rho\sigma}(P - K) \times \\
 & \quad \text{Im} \left[\{K_\sigma P_\rho + K_\rho P_\sigma - p_0(K_\sigma g_{\rho 0} + K_\rho g_{\sigma 0}) - k_0(P_\sigma g_{\rho 0} + P_\rho g_{\sigma 0}) + pkz g_{\sigma\rho} \right. \\
 & \quad \left. + 2p_0 k_0 g_{\sigma 0} g_{\rho 0} \} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right. \\
 & \quad \left. + \{P_\sigma g_{\rho 0} + P_\rho g_{\sigma 0} - 2p_0 g_{\sigma 0} g_{\rho 0}\} \frac{k_0 + B(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right] \Big]
 \end{aligned}$$

$B(P)$

$$\begin{aligned}
&= e^2 \int \frac{d^4 K}{(2\pi)^4} \left[\{1 + 2n_B(p_0 - k_0)\} \text{Im} [{}^*G_R^{\rho\sigma}(P - K)] \times \right. \\
&\quad \left[\{K_\sigma g_{\rho 0} + K_\rho g_{\sigma 0} - 2k_0 g_{\sigma 0} g_{\rho 0}\} \frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right. \\
&\quad \left. + \{2g_{\rho 0} 2g_{\sigma 0} - g_{\sigma\rho}\} \frac{k_0 + B(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right] \\
&\quad + \{1 - 2n_F(k_0)\} {}^*G_R^{\rho\sigma}(P - K) \times \\
&\quad \text{Im} \left[\frac{A(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \{K_\sigma g_{\rho 0} + K_\rho g_{\sigma 0} - 2k_0 g_{\sigma 0} g_{\rho 0}\} \right. \\
&\quad \left. + \frac{k_0 + B(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \{2g_{\rho 0} 2g_{\sigma 0} - g_{\sigma\rho}\} \right] \left. \right]
\end{aligned}$$

$C(P)$

$$\begin{aligned}
&= -e^2 \int \frac{d^4 K}{(2\pi)^4} g_{\sigma\rho} \left[\{1 + 2n_B(p_0 - k_0)\} \text{Im} [{}^*G_R^{\rho\sigma}(P - K)] \times \right. \\
&\quad \frac{C(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} + \{1 - 2n_F(k_0)\} \times \\
&\quad \left. {}^*G_R^{\rho\sigma}(P - K) \text{Im} \left[\frac{C(K)}{[k_0 + B(K) + i\epsilon]^2 - A(K)^2 k^2 - C(K)^2} \right] \right]
\end{aligned}$$

$\rho=0.1 \Lambda, \rho_0=0$

